Another look at the $\Lambda(1405)$: the $\pi\Sigma - \bar{K}N$ chiral approach meets the lattice QCD

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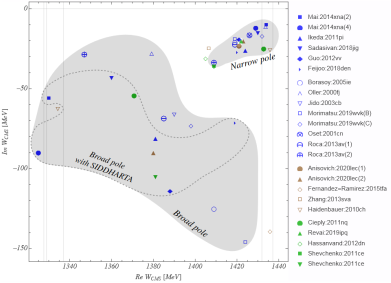
Outline

1. Introduction
2. Chirally motivated $\pi\Sigma - \bar{K}N$ coupled channels model
3. Finite volume formulation
4. LQCD connection
5. Summary

Introduction - ChPT predictions

- chirally motivated $\pi\Sigma - \bar{K}N$ coupled channels models provide two dynamically generated poles on the "second Riemann sheet" of the (isoscalar) scattering amplitude between the $\pi\Sigma$ and $\bar{K}N$ thresholds [two pole structure of $\Lambda(1405)$]
- two $1/2^-$ resonances in PDG tables: $\Lambda(1380)$ $(\star\star)$ and $\Lambda(1405)$ $(\star\star\star\star)$
- the position of the $\Lambda(1380)$ pole is not well restricted by the experimental data, with $\Gamma_{\Lambda(1380)} \approx 100 - 200$ MeV



M. Mai - Eur. Phys. J. Spec. Top. 230 (2021) 6, 1593

Introduction - recent LQCD result

tremendous lattice QCD progress: BaSc collaboration - first simulation of the $\pi\Sigma - \bar{K}N$ coupled channels interactions

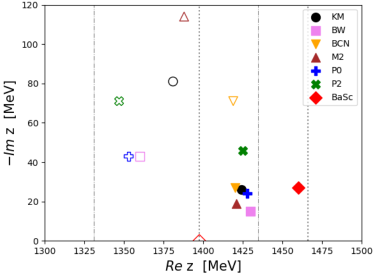
J. Bulava et al. [BaSc Collaboration] - Phys. Rev. Lett. 132 (2024) 051901
J. Bulava et al. [BaSc Collaboration] - Phys. Rev. D 109 (2024) 014511

- $N_\mathrm{f} = 2 + 1$ dynamical quark flavors, $M_\pi \approx 200\ \mathrm{MeV}$ and $M_K \approx 485$ MeV
- $\pi\Sigma$-$\bar{K}N$ scattering K-matrix utilized to obtain the scattering amplitudes from the finite-volume spectrum
- an extrapolation to the complex energy plane reveals a virtual bound state just below the $\pi\Sigma$ threshold and a resonance pole below the $\bar{K}N$ threshold
- Lattice QCD supports the two-pole nature of $\Lambda(1405)$!
- in near future, LQCD may serve as another data source, e.g. simulating the $\pi\Sigma$ interactions at $\bar{K}N$ subthreshold energies

The work is a huge achievement by itself but can we relate its results to the chirally motivated $\pi\Sigma$-$\bar{K}N$ models? Yes we can! (At least it looks so.)

Introduction - $\Lambda(1405)$ poles

- Predictions of chirally motivated NLO coupled channel models by several groups. The models reproduce the low-energy K^-p reactions data.



- P0 model - *P. C. Bruns, A. C., Nucl. Phys. A 1019 (2022) 122378*
  P2 model - *A. C., P. C. Bruns - Nucl. Phys. A 1043 (2024) 122819*
  simultaneous fit to K^-p data and to the $\pi\Sigma$ photoproduction spectra by CLAS collaboration, the $\gamma p \longrightarrow K^+\pi\Sigma$ reaction
- BaSc $\Lambda(1405)$ pole shifted due to $\bar{K}N$ threshold shift. No obvious link of the BaSc virtual state to the $\Lambda(1380)$ poles.

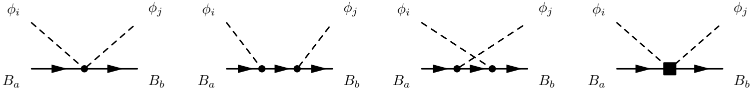
Chirally motivated $\pi\Sigma - \bar{K}N$ model

involved channels	$\pi\Lambda$	$\pi\Sigma$	$\bar{K}N$	$\eta\Lambda$	$\eta\Sigma$	$K\Xi$
thresholds (MeV)	1250	1330	1435	1660	1740	1810

- strongly interacting multichannel system with an s-wave resonance, the $\Lambda(1405)$, just below the $K^- p$ threshold
- modern theoretical treatment based on effective chiral Lagrangians and effective potentials constructed to match the tree-level meson-baryon amplitudes derived from ChPT up to NLO order

$$v_{0+}(s) := f_{0+}^{\mathrm{tree}}(s) = \frac{\sqrt{E+m}}{F_\Phi} \left(\frac{C(s)}{8\pi\sqrt{s}} \right) \frac{\sqrt{E+m}}{F_\Phi}$$

- the matrix $C(s)$ comprises contributions from the Weinberg-Tomozawa, direct and crossed Born terms, and from NLO corrections to the WT term



$\mathcal{O}(q^1)$ contact | direct s-term | crossed u-term | $\mathcal{O}(q^2)$ contact

Chirally motivated $\pi\Sigma - \bar{K}N$ model

- we follow the original idea by Kaiser, Siegel and Weise (1995) introducing separable form of the effective potentials

$$v_{c'c}(W=\sqrt{s}) = g_{c'}(k_{c'})\,[f_{0+}^{\mathrm{tree}}(W)]_{c'c}\,g_c(k_c)$$

  with Yamaguchi form factors $g_c(k) = 1/[1+(k/\alpha_c)^2]$ that feature regulator scales (*soft cutoffs*) α_c

- Lippman-Schwinger equation solved algebraically

$$f = v + v\,G\,f$$

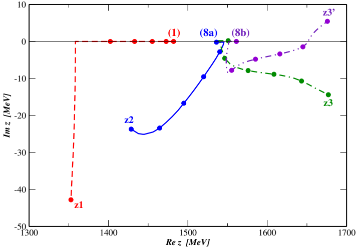
$$f_{c'c}(W) = g_{c'}(k_{c'})\left[(1 - v(W)\cdot G(W))^{-1}\cdot v\right]_{c'c}\,g_c(k_c)$$

$$G_c(W) = -4\pi \int \frac{d^3p}{(2\pi)^3}\,\frac{g_c^2(p^2)}{k_c^2 - p^2 + \mathrm{i}0} = \frac{(\alpha_c + \mathrm{i}q_c)^2}{2\alpha_c}\,g_c^2(q_c)$$

- parameters: scales α_c, LECs (NLO couplings) from the chiral Lagrangian
- K^-p data fits: low energy x-sections, threshold BRs, kaonic hydrogen 1s level; plus $\pi\Sigma$ photoproduction mass spectra in case of our P2 model

SU(3) flavor restoration

in the isoscalar sector, the P0 model generates 3 poles: $\Lambda(1380)$, $\Lambda(1405)$, $\Lambda(1670)$
a singlet and two octet states when the SU(3) flavor symmetry is gradually restored



$$M_\pi^2(x) = \frac{1}{3}\left(2M_K^2 + M_\pi^2\right) - \frac{2x}{3}\left(M_K^2 - M_\pi^2\right)$$
$$M_K^2(x) = \frac{1}{3}\left(2M_K^2 + M_\pi^2\right) + \frac{x}{3}\left(M_K^2 - M_\pi^2\right)$$
$$m_N(x) = \frac{1}{3}\left(m_N + m_\Sigma + m_\Xi\right) - \frac{x}{3}\left(m_\Sigma + m_\Xi - 2m_N\right)$$
$$m_\Sigma(x) = \frac{1}{3}\left(m_N + m_\Sigma + m_\Xi\right) + \frac{x}{3}\left(2m_\Sigma - m_N - m_\Xi\right)$$
$$F_\pi(x) = \frac{1}{3}\left(F_\pi + 2F_K\right) - \frac{2x}{3}\left(F_K - F_\pi\right)$$
$$\alpha_c(x) = \alpha_0 + x\left(\alpha_c - \alpha_0\right)$$

$x = 0$ - restored SU(3) flavor symmetry point, $x = 1$ - physical point

specific path chosen to guarantee constant average quark mass by keeping constant

$$X_M^2 := \frac{1}{3}\left(2M_K^2(x) + M_\pi^2(x)\right) \quad \text{and} \quad X_B := \frac{1}{3}\left(m_N(x) + m_\Sigma(x) + m_\Xi(x)\right)$$

more details in *P. C. Bruns, A. C. - Nucl. Phys. A 1019 (2022) 122378*

Note on dynamically generated resonances/poles

coupling matrices for the WT term:

$$
C_{ij}^{I=0} = \begin{array}{c}
\begin{array}{cccc} \pi\Sigma & \bar{K}N & \eta\Lambda & K\Xi \end{array} \\
\begin{pmatrix}
4 & -\sqrt{3/2} & 0 & \sqrt{3/2} \\
 & 3 & 3\sqrt{1/2} & 0 \\
 & & 0 & -3\sqrt{1/2} \\
 & & & 3
\end{pmatrix}
\end{array}
\qquad
C_{ij}^{I=1} = \begin{array}{c}
\begin{array}{ccccc} \pi\Lambda & \pi\Sigma & \bar{K}N & \eta\Sigma & K\Xi \end{array} \\
\begin{pmatrix}
0 & 0 & -\sqrt{3/2} & 0 & -\sqrt{3/2} \\
 & 2 & -1 & 0 & 1 \\
 & & 1 & -\sqrt{3/2} & 0 \\
 & & & 0 & -\sqrt{3/2} \\
 & & & & 1
\end{pmatrix}
\end{array}
$$

For both isospins, the poles can originate from the $\pi\Sigma$, $\bar{K}N$ and $K\Xi$ channels.

three dynamically generated Λ^* and three Σ^* poles

However, the $\pi\Sigma$ related $I = 1$ pole is located too far from the real axis.

A.C., M. Mai, U.-G. Meißner, J. Smejkal - On the pole content of coupled channels chiral approaches used for the $\bar{K}N$ system, Nucl. Phys. A 954 (2016) 17

$\pi\Sigma - \bar{K}N$ interactions in a finite volume

our *MB* loop function:

$$G_c(k_c) = -4\pi \int \frac{d^3p}{(2\pi)^3} \frac{g_c^2(p^2)}{k_c^2 - p^2 + \mathrm{i}0} = [g(k_c)]^2 \left(\frac{\alpha_c^2 - k_c^2}{2\alpha_c} + \mathrm{i}\, k_c \right), \quad g(k) = \frac{\alpha^2}{\alpha^2 + k^2}$$

simple generalization to the cubic finite-volume $V = L^3$ presented in
P. C. Bruns, A. C. - Eur. Phys. J. A 61 (2025) 44

possible three momenta: $\vec{p}_n = \frac{2\pi}{L}\vec{n}$, with $\vec{n} \in \mathbb{Z}^3$

$$G^{\mathrm{FV}}(q, L) = \frac{4\pi}{L^3} \sum_{\vec{n}\in\mathbb{Z}^3} \frac{[g(p_n)]^2}{p_n^2 - q^2}$$

$$G^{\mathrm{FV}}(q; L) = [g(q)]^2 \left[\tilde{G}(q) + \Delta\tilde{G}_{\mathcal{R}}(q; L) + \Delta\tilde{G}_{\mathcal{I}}(q; L) \right],$$

$$\Delta\tilde{G}_{\mathcal{R}}(q; L) = \sum_{\vec{0}\neq\vec{n}\in\mathbb{Z}^3} e^{-n\alpha L},$$

$$\Delta\tilde{G}_{\mathcal{I}}(q; L) = \sum_{\vec{0}\neq\vec{n}\in\mathbb{Z}^3} \frac{1}{nL} \left[e^{\pm\mathrm{i}qnL} - (1 + n\alpha L)e^{-n\alpha L} \right]$$

one can sum over n^2 with weight factors $w(n^2)$ counting the multiplicity of lattice points $\vec{n}$ with a given n^2: $w(n^2) = 1, 6, 12, 8, 6 \ldots$ for $n^2 = 0, 1, 2, 3, 4 \ldots$

$\pi\Sigma - \bar{K}N$ interactions in a finite volume

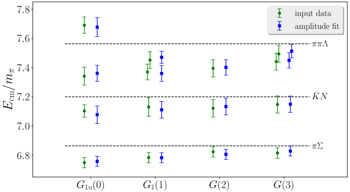
The convergence of the generated pole positions to their infinite volume ones demonstrated for increasing values of the lattice spatial extent L:

L [1/GeV]	$z_{\Lambda(1380)}$ [MeV]	$z_{\Lambda(1405)}$ [MeV]
20	(1365.8, -72.2)	(1569.8, -30.9)
40	(1350.7, -46.4)	(1425.0, -24.5)
60	(1352.0, -42.4)	(1426.2, -21.1)
80	(1352.8, -42.6)	(1429.1, -26.1)
100	(1352.7, -42.8)	(1429.5, -22.7)
∞	(1352.7, -42.8)	(1428.5, -23.7)

oscillations caused by the $\exp(\pm \mathrm{i} q L n)$ term in $\Delta\tilde{G}_{\mathcal{I}}(q; L)$ are better suppressed at complex energies further from the real axis

LQCD connection - stationary energies spectrum

BaSc finite volume spectrum - stationary-state energies obtained from correlation matrices involving the meson-baryon operators



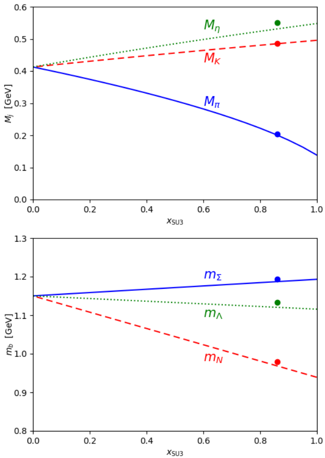
- 4 irreps (symmetry channels) $G(\vec{d})$ involving s-wave for the total momentum $\vec{P}^2 = (2\pi/L)^2\vec{d}^2$
- $\vec{d}$ - three-vector of integers
  L - extent of the L^3 spatial lattice
- lattice spacing $a = 0.0633$ fm, spatial extend $L = 64a \approx 4$ fm

matching the BaSc $G_{1u}(0)$ irrep spectrum for $L = 20\ \mathrm{GeV}^{-1}$:

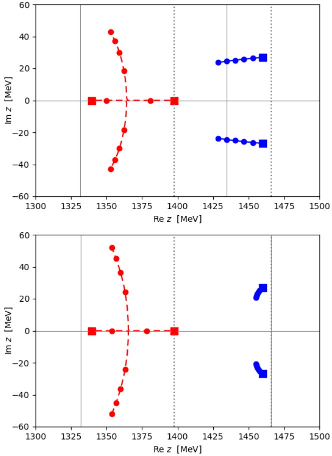
BaSc energies [MeV]	P0 energies [MeV]	P2 energies [MeV]
1374 ± 8	1375 ± 2	1371 ± 2
1447 ± 9	1432 ± 1	1429 ± 1
1495 ± 13	1494 ± 6	1509 ± 8
1566 ± 12	1556 ± 12	1594 ± 16

Nice agreement for the P0 model without readjusting any parameters!

LQCD connection - BaSc and our $\bar{K}N$ model



hadron masses upon restoring SU(3) flavor symmetry; BaSc values marked at $x_{\mathrm{SU3}} = 0.86$



pole trajectories upon varying the hadron masses and meson decay constants from their BaSc values to those in the physical limit; only M_π varied in the bottom

Summary

- Despite being broken, the SU(3) flavor and chiral symmetries govern the meson-baryon interactions at low energies. Our chirally motivated coupled channel approach works well in the $(S=-1)$ $\bar{K}N$ sector.
- Three dynamically generated $\Lambda^*(1/2)^-$ states are predicted, two in the $\Lambda(1405)$ energy region, the third one assigned to $\Lambda(1670)$. The pertinent poles go to a singlet and two SU(3) octet states as the SU(3) flavor symmetry is restored.
- The finite-volume generalization of our $\pi\Sigma - \bar{K}N$ model can be quite well related (without re-adjusting any parameters!) to the recent LQCD results by the BaSc collaboration.
- The two-pole structure of $\Lambda(1405)$ is confirmed by recent lattice QCD simulations. The relatively large pion mass $M_\pi \approx 200$ MeV was found responsible for the transformation of the $\Lambda(1380)$ pole into the virtual state reported by the BaSc collaboration.

EXTRAS

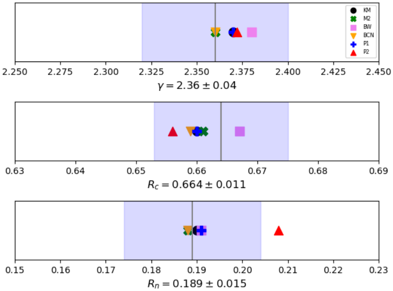
$K^- p$ threshold data

threshold branching ratios:

$$\gamma = \frac{\Gamma(K^- p \to \pi^+ \Sigma^-)}{\Gamma(K^- p \to \pi^- \Sigma^+)} = 2.36 \pm 0.04$$

$$R_c = \frac{\Gamma(K^- p \to \text{charged})}{\Gamma(K^- p \to \text{all})} = 0.664 \pm 0.011$$

$$R_n = \frac{\Gamma(K^- p \to \pi^0 \Lambda)}{\Gamma(K^- p \to \text{neutral})} = 0.189 \pm 0.015$$



kaonic atoms:

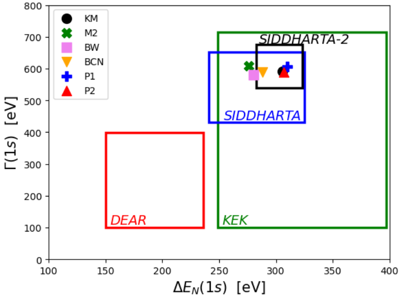
an electron replaced by the K^- meson

$$E(nl) = E_{\text{el.mag.}}(nl) + \Delta E_N(nl) - \frac{i}{2}\,\Gamma_N(nl)$$

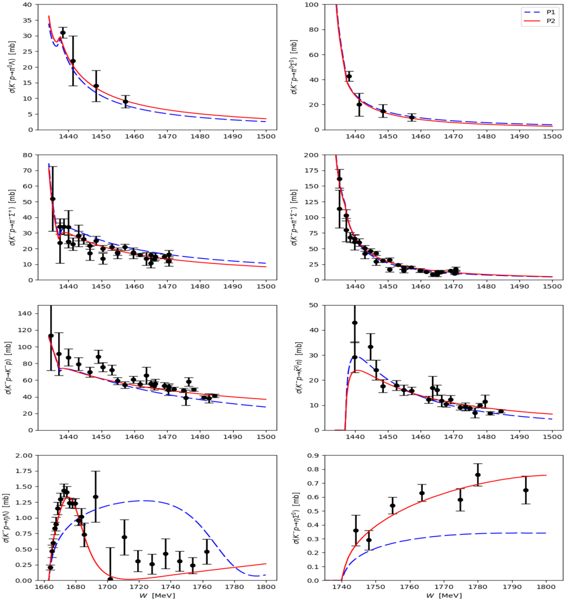
kaonic hydrogen (2026):

$\Delta E_N(1s) = 303 \pm 17(stat.) \pm 2.5(syst.)$ eV

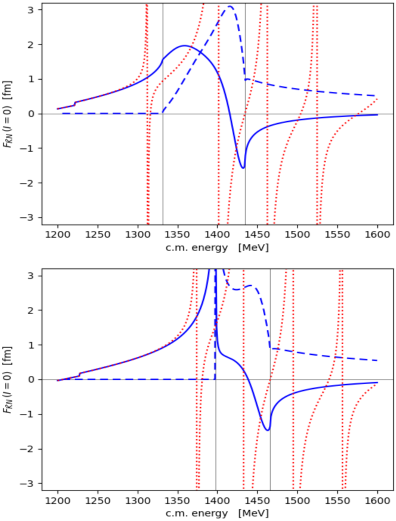
$\Gamma(1s) = 607 \pm 62(stat.) \pm 6(syst.)$ eV



$K^- p \to MB$ total cross sections



I=0 $\bar{K}N$ amplitude in FV



$L = 20\ \mathrm{GeV}^{-1}$, top - physical masses, bottom - BaSc masses