Analysis of Heavy Baryon-Baryon Interactions Using the Quark Cluster Model

Gento Yoshikawa

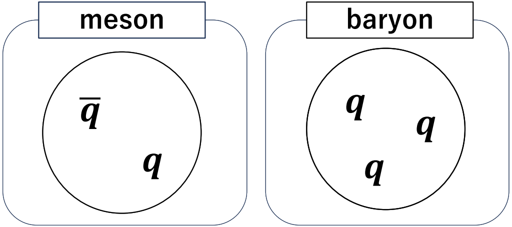
Nagoya University
In collaboration with
Sachiko Takeuchi (RCNP, Univ of Osaka),
Takayasu Sekihara (Kyoto Prefectural Univ),
Yasuhiro Yamaguchi (Tokyo Metropolitan Univ)

September 9, 2026

Contents

Introduction

Conventional hadrons : meson($\pi, \rho, \omega, \ldots$)、baryon($N, \Delta, \Omega, \ldots$)



q:quarks $\bar{q}$:anti-quarks

Hadrons that cannot be explained within the conventional quark model are referred to as exotic hadrons.

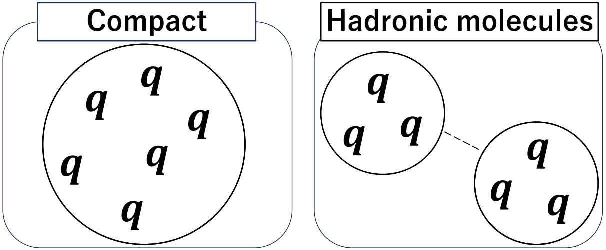
In recent years, many exotic hadrons containing charm quarks have been reported.

- $T_{cc}^{+}(cc\bar{u}\bar{d}), P_c(c\bar{c}qqq), X(3872)(c\bar{c}q\bar{q})$

Roel Aaij et al. Nature Phys. 18.7 (2022) 751-754., Roel Aaij et al. PRL 122.22 (2019) 222001., S. K. Choi et al. PRL 91 (2003) 262001.

Internal Structure of Exotic Hadrons

- Possible internal structures of exotic hadrons
  - Compact multiquark states
  - Hadronic molecules: bound states of hadrons

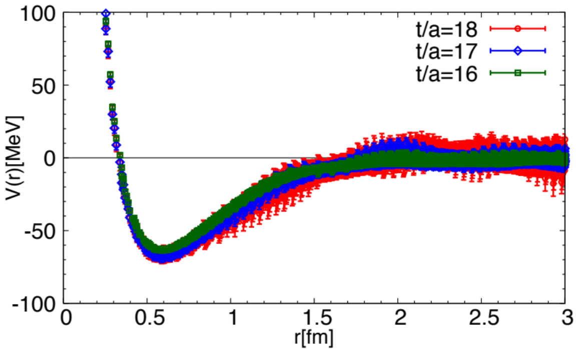


q:quarks

- Many recently reported exotic hadrons are considered candidates for hadronic molecules.
  $\because$ found near the thresholds.
- Understanding hadron–hadron interactions is essential for describing hadronic molecules.

Studies of Hadron–Hadron Interactions

- In recent years, lattice QCD simulations of hadron–hadron interactions have made remarkable progress.
- Numerical results for hadron–hadron interactions are becoming increasingly available.

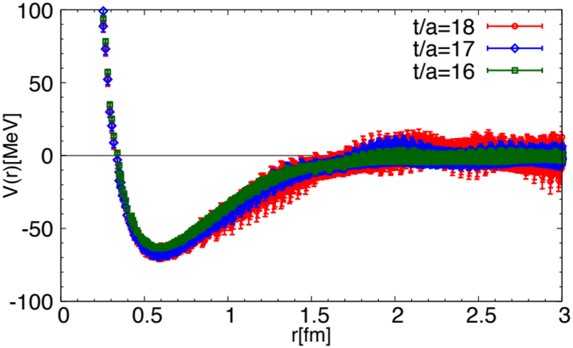


Interaction potential of the Ω–Ω system calculated by the HAL QCD Collaboration. [1]

[1] Shinya Gongyo et al. In: Phys. Rev. Lett. 120.21 (2018), p. 21. 2001.

Studies of Hadron–Hadron Interactions

- In recent years, lattice QCD simulations of hadron–hadron interactions have made remarkable progress.
- Numerical results for hadron–hadron interactions are becoming increasingly available.
- However, lattice QCD simulations alone do not reveal the underlying interaction mechanism.



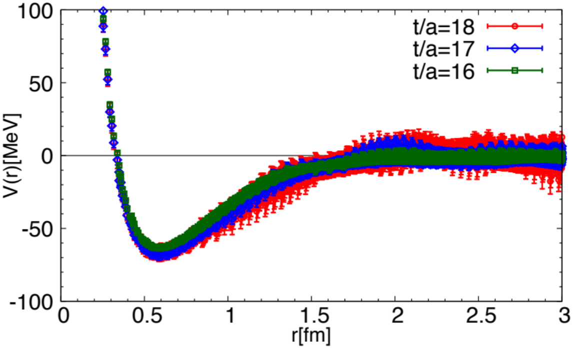
Interaction potential of the Ω–Ω system calculated by the HAL QCD Collaboration. [1]

[1] Shinya Gongyo et al. In: Phys. Rev. Lett. 120.21 (2018), p. 21. 2001.

Studies of Hadron–Hadron Interactions

- In recent years, lattice QCD simulations of hadron–hadron interactions have made remarkable progress.
- Numerical results for hadron–hadron interactions are becoming increasingly available.
- However, lattice QCD simulations alone do not reveal the underlying interaction mechanism.

$\rightarrow$ Theoretical analysis are essential for understanding the mechanism.

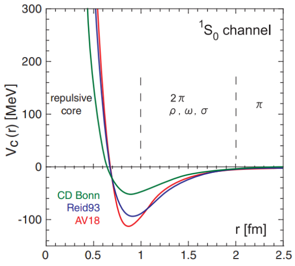


Interaction potential of the Ω–Ω system calculated by the HAL QCD Collaboration. [1]

[1] Shinya Gongyo et al. In: Phys. Rev. Lett. 120.21 (2018), p. 21. 2001.

Hadron–Hadron Interactions

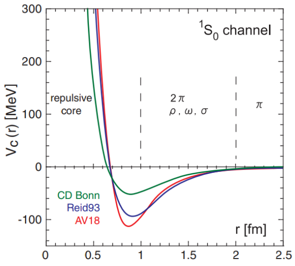
- The interaction between nucleons is well understood.
  - $r > 2\,\mathrm{fm}$:
    Described by a Yukawa-type potential due to π exchange.
  - $1\,\mathrm{fm} < r < 2\,\mathrm{fm}$:
    Complex structure arising from the exchange of 2π, ρ, ω, and σ mesons.
  - $r < 1\,\mathrm{fm}$:
    Dominated by quark degrees of freedom inside hadrons.



Nucleon–nucleon interaction potential
N. Ishii, S. Aoki, and T. Hatsuda. In: Phys. Rev. Lett. 99 (2007), p. 022001.

Hadron–Hadron Interactions

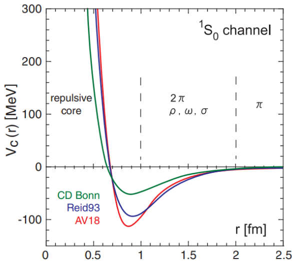
- The interaction between nucleons is well understood.
  - $r > 2\,\mathrm{fm}$:
    Described by a Yukawa-type potential due to π exchange.
  - $1\,\mathrm{fm} < r < 2\,\mathrm{fm}$:
    Complex structure arising from the exchange of 2π, ρ, ω, and σ mesons.
  - $r < 1\,\mathrm{fm}$:
    Dominated by quark degrees of freedom inside hadrons.



Nucleon–nucleon interaction potential
N. Ishii, S. Aoki, and T. Hatsuda. In: Phys. Rev. Lett. 99 (2007), p. 022001.

Hadron–Hadron Interactions

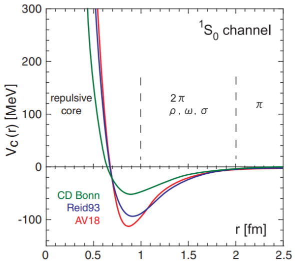
- The interaction between nucleons is well understood.
  - $r > 2\,\mathrm{fm}$:
    Described by a Yukawa-type potential due to π exchange.
  - $1\,\mathrm{fm} < r < 2\,\mathrm{fm}$:
    Complex structure arising from the exchange of 2π, ρ, ω, and σ mesons.
  - $r < 1\,\mathrm{fm}$:
    Dominated by quark degrees of freedom inside hadrons.



Nucleon–nucleon interaction potential
N. Ishii, S. Aoki, and T. Hatsuda. In: Phys. Rev. Lett. 99 (2007), p. 022001.

Hadron–Hadron Interactions

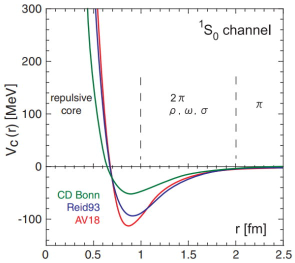
- The interaction between nucleons is well understood.
  - $r > 2\,\mathrm{fm}$:
    Described by a Yukawa-type potential due to π exchange.
  - $1\,\mathrm{fm} < r < 2\,\mathrm{fm}$:
    Complex structure arising from the exchange of 2π, ρ, ω, and σ mesons.
  - $r < 1\,\mathrm{fm}$:
    Dominated by quark degrees of freedom inside hadrons.



Nucleon–nucleon interaction potential
N. Ishii, S. Aoki, and T. Hatsuda. In: Phys. Rev. Lett. 99 (2007), p. 022001.

Hadron–Hadron Interactions

- The interaction between nucleons is well understood.
  - $r > 2\,\mathrm{fm}$:
    Described by a Yukawa-type potential due to π exchange.
  - $1\,\mathrm{fm} < r < 2\,\mathrm{fm}$:
    Complex structure arising from the exchange of 2π, ρ, ω, and σ mesons.
  - $r < 1\,\mathrm{fm}$:
    Dominated by quark degrees of freedom inside hadrons.

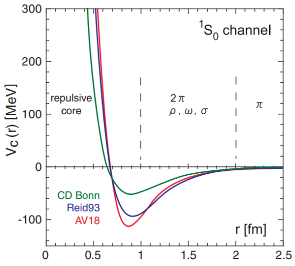


Nucleon–nucleon interaction potential
N. Ishii, S. Aoki, and T. Hatsuda. In: Phys. Rev. Lett. 99 (2007), p. 022001.

Short-range interactions between general hadrons, especially those containing c or b quarks, remain poorly understood.

Hadron–Hadron Interactions

- The interaction between nucleons is well understood.
  - $r > 2\,\mathrm{fm}$:
    Described by a Yukawa-type potential due to π exchange.
  - $1\,\mathrm{fm} < r < 2\,\mathrm{fm}$:
    Complex structure arising from the exchange of 2π, ρ, ω, and σ mesons.
  - $r < 1\,\mathrm{fm}$:
    Dominated by quark degrees of freedom inside hadrons.

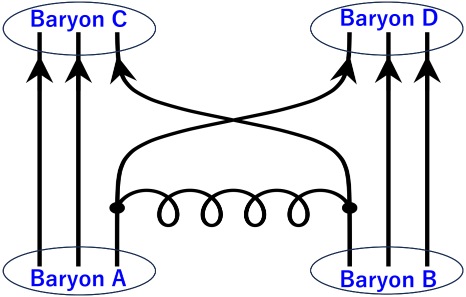


Nucleon–nucleon interaction potential
N. Ishii, S. Aoki, and T. Hatsuda. In: Phys. Rev. Lett. 99 (2007), p. 022001.

Short-range interactions between general hadrons, especially those containing c or b quarks, remain poorly understood.
$\rightarrow$ Focus on the short-range region.

Quark Cluster Model

- Description of the short-range region of hadron–hadron interactions



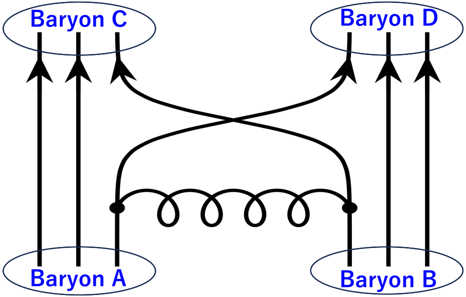
Schematic picture of quark exchange

[1] M. Oka, K. Shimizu and K. Yazaki, Prog. Theor. Phys. Suppl. **137**, 1-20 (2000)

Quark Cluster Model

- Description of the short-range region of hadron–hadron interactions

⇒ Quark Cluster Model



Schematic picture of quark exchange

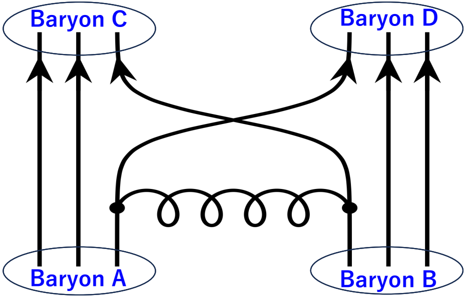
[1] M. Oka, K. Shimizu and K. Yazaki, Prog. Theor. Phys. Suppl. **137**, 1-20 (2000)

Quark Cluster Model

- Description of the short-range region of hadron–hadron interactions

⇒ Quark Cluster Model

- ▷ A model that explicitly incorporates quark degrees of freedom
- ▷ Hadrons are described as clusters of quarks
- ▷ Effects of quark exchange are taken into account
- ▷ Applicable to arbitrary hadron–hadron systems



Schematic picture of quark exchange

[1] M. Oka, K. Shimizu and K. Yazaki, Prog. Theor. Phys. Suppl. **137**, 1-20 (2000)

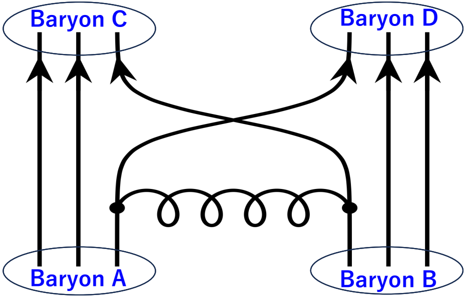
Quark Cluster Model

- Description of the short-range region of hadron–hadron interactions

⇒ Quark Cluster Model

- ▷ A model that explicitly incorporates quark degrees of freedom
- ▷ Hadrons are described as clusters of quarks
- ▷ Effects of quark exchange are taken into account
- ▷ Applicable to arbitrary hadron–hadron systems

Previous studies[1] successfully described the repulsive core in the nucleon–nucleon interaction.



Schematic picture of quark exchange

[1] M. Oka, K. Shimizu and K. Yazaki, Prog. Theor. Phys. Suppl. **137**, 1-20 (2000)

Quark Cluster Model

- Description of the short-range region of hadron–hadron interactions

⇒ Quark Cluster Model

▷ A model that explicitly incorporates quark degrees of freedom

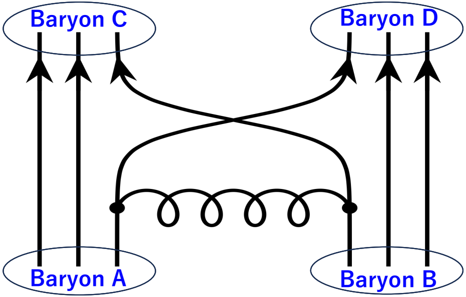
▷ Hadrons are described as clusters of quarks

▷ Effects of quark exchange are taken into account

▷ Applicable to arbitrary hadron–hadron systems

Previous studies[1] successfully described the repulsive core in the nucleon–nucleon interaction.

⇒ Extension to systems containing c, b quarks

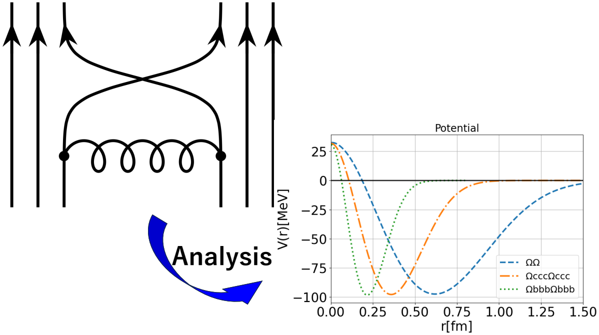


Schematic picture of quark exchange

[1] M. Oka, K. Shimizu and K. Yazaki, Prog. Theor. Phys. Suppl. **137**, 1-20 (2000)

Research Objective

- Research objective:
  Development of a framework to analyze hadron–hadron systems containing c and b quarks, which frequently appear in exotic hadrons.

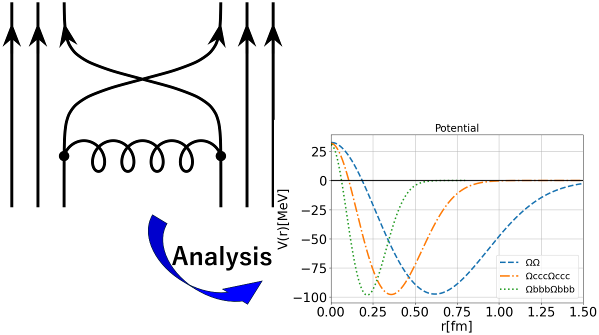


[1] T. Sekihara, T. Hashiguchi. Phys. Rev. C 108 (2023) 065202

Research Objective

- Research objective:
  Development of a framework to analyze hadron–hadron systems containing c and b quarks, which frequently appear in exotic hadrons.

- Previous study[1]
  Analysis of the $\Omega\Omega$ interaction



[1] T. Sekihara, T. Hashiguchi. Phys. Rev. C 108 (2023) 065202

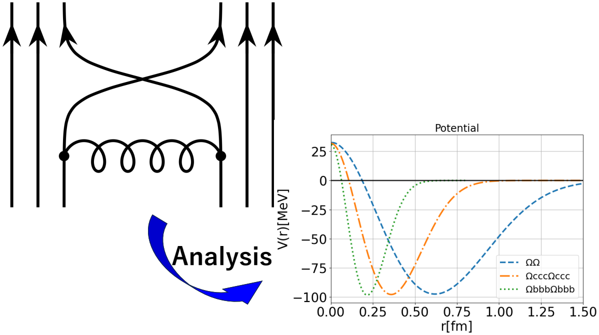
Research Objective

- Research objective:
  Development of a framework to analyze hadron–hadron systems containing c and b quarks, which frequently appear in exotic hadrons.

- Previous study[1]
  Analysis of the $\Omega\Omega$ interaction

$\Downarrow$ Extension

- This work
  Analysis of the interactions among $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ systems



[1] T. Sekihara, T. Hashiguchi. Phys. Rev. C 108 (2023) 065202

Model construction

- Baryon wave function

→ 3-body Schrödinger equation in the quark model:

$$\left[\sum_i^3 m_i - \frac{1}{2\mu_\rho}\frac{\partial^2}{\partial \boldsymbol{\rho}^2} - \frac{1}{2\mu_\lambda}\frac{\partial^2}{\partial \boldsymbol{\lambda}^2} + V_{23}(\rho_1) + V_{31}(\rho_2) + V_{12}(\rho_3)\right]\phi^{(B)}(\boldsymbol{\rho},\boldsymbol{\lambda}) = M_B\phi^{(B)}(\boldsymbol{\rho},\boldsymbol{\lambda})$$

▷ quark-quark potential T. Sekihara, T. Hashiguchi. Phys. Rev. C 108 (2023) 065202

$$V_{ij}(r) = \frac{\vec{\lambda}_i}{2}\cdot\frac{\vec{\lambda}_j}{2}\left[\frac{\alpha_{ij}}{r} - \frac{3}{4}kr + D - \frac{2\pi\alpha_{ss}}{3}\frac{\vec{\sigma}_i\cdot\vec{\sigma}_j}{m_i m_j}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3\exp\left(-\sigma^2 r^2\right)\right]$$

Coulomb term confinement term Spin-spin term

The parameters are determined to reproduce baryon masses.

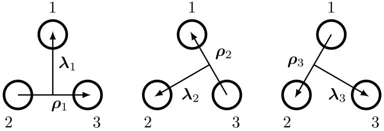
$\alpha_{ij} = \frac{K}{\mu_{ij}}$	$\mu_{ij} = \frac{m_i m_j}{m_i + m_j}$	K	184 MeV
α_{ss}	0.785	k	0.755 GeV/fm
σ	3.50 fm^{-1}	D	381 MeV
$m_u = m_d$	336 MeV	m_s	509 MeV

Baryon wave function

- Baryon wave function

$\rightarrow$ 3body Schrödinger equation in the quark model:

$$\left[\sum_i^3 m_i - \frac{1}{2\mu_\rho}\frac{\partial^2}{\partial \boldsymbol{\rho}^2} - \frac{1}{2\mu_\lambda}\frac{\partial^2}{\partial \boldsymbol{\lambda}^2} + V_{23}(\rho_1) + V_{31}(\rho_2) + V_{12}(\rho_3)\right]\phi^{(B)}(\boldsymbol{\rho},\boldsymbol{\lambda}) = M_B\phi^{(B)}(\boldsymbol{\rho},\boldsymbol{\lambda})$$



The baryon mass M_B and internal wave function $\phi^{(B)}(\boldsymbol{\rho},\boldsymbol{\lambda})$ are determined using the variational method.

trial function : $\phi^{(B)}(\boldsymbol{\rho},\boldsymbol{\lambda}) = N\exp\left(-\frac{\boldsymbol{\rho}^2}{2b_\rho^2} - \frac{\boldsymbol{\lambda}^2}{2b_\lambda^2}\right)$

$$b_\rho = \sqrt{\frac{m}{\mu_\rho}}b, \quad b_\lambda = \sqrt{\frac{m}{\mu_\lambda}}b$$

Baryon	b [fm]
Ω	0.419
Ω_{ccc}	0.244
Ω_{bbb}	0.147

2 baryon state

2-baryon system is described as a 6-quark system.

$$|B_aB_b\rangle = \frac{1}{3!}\sum_{\mu_1,\mu_2,\mu_3}\sum_{\mu_4,\mu_5,\mu_6}\omega^{(B_a)}_{\mu_1,\mu_2,\mu_3}\omega^{(B_b)}_{\mu_4,\mu_5,\mu_6}\int d\xi\Psi(\boldsymbol{R}_{\text{tot}})\psi(\boldsymbol{r})\phi^{(B_a)}(\boldsymbol{\lambda}_a,\boldsymbol{\rho}_a)\phi^{(B_b)}(\boldsymbol{\lambda}_b,\boldsymbol{\rho}_b)$$
$$\times\, \hat{b}^\dagger_{\mu_1}(\boldsymbol{r}_1)\hat{b}^\dagger_{\mu_2}(\boldsymbol{r}_2)\hat{b}^\dagger_{\mu_3}(\boldsymbol{r}_3)\hat{b}^\dagger_{\mu_4}(\boldsymbol{r}_4)\hat{b}^\dagger_{\mu_5}(\boldsymbol{r}_5)\hat{b}^\dagger_{\mu_6}(\boldsymbol{r}_6)\,|0\rangle$$

$\hat{b}^\dagger_{\mu_i}(\boldsymbol{r}_i)$: Creation operator for the i-th quark
$\mu_i = (\text{flavor}, \text{spin}, \text{color}), \boldsymbol{r}_i$: Coordinate of the i-th quark

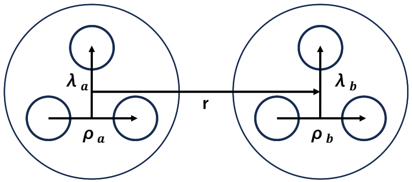
$$\left\{\hat{b}_\mu(\boldsymbol{r}),\hat{b}^\dagger_\nu(\boldsymbol{r}')\right\} = \delta_{\mu\nu}\delta(\boldsymbol{r}-\boldsymbol{r}'),\quad \left\{\hat{b}_\mu(\boldsymbol{r}),\hat{b}_\nu(\boldsymbol{r}')\right\} = 0,\quad \left\{\hat{b}^\dagger_\mu(\boldsymbol{r}),\hat{b}^\dagger_\nu(\boldsymbol{r}')\right\} = 0$$

2 baryon state

2-baryon system is described as a 6-quark system.

$$|B_aB_b\rangle = \frac{1}{3!}\sum_{\mu_1,\mu_2,\mu_3}\sum_{\mu_4,\mu_5,\mu_6}\omega^{(B_a)}_{\mu_1,\mu_2,\mu_3}\omega^{(B_b)}_{\mu_4,\mu_5,\mu_6}\int d\xi\,\Psi(\boldsymbol{R}_{\text{tot}})\psi(\boldsymbol{r})\phi^{(B_a)}(\boldsymbol{\lambda}_a,\boldsymbol{\rho}_a)\phi^{(B_b)}(\boldsymbol{\lambda}_b,\boldsymbol{\rho}_b)$$
$$\times\,\hat{b}^\dagger_{\mu_1}(\boldsymbol{r}_1)\hat{b}^\dagger_{\mu_2}(\boldsymbol{r}_2)\hat{b}^\dagger_{\mu_3}(\boldsymbol{r}_3)\hat{b}^\dagger_{\mu_4}(\boldsymbol{r}_4)\hat{b}^\dagger_{\mu_5}(\boldsymbol{r}_5)\hat{b}^\dagger_{\mu_6}(\boldsymbol{r}_6)\,|0\rangle$$

- $\Psi(\boldsymbol{R}_{\text{tot}})$: Center-of-mass wave function of 2 baryons
- $\psi(\boldsymbol{r})$: Relative wave function between 2 baryons, what we want
- $\phi^{(B_a)}(\boldsymbol{\lambda}_a,\boldsymbol{\rho}_a)$ and $\phi^{(B_b)}(\boldsymbol{\lambda}_b,\boldsymbol{\rho}_b)$: internal wave functions of baryons B_a and B_b, obtained by the three-body Schrödinger equation (known)



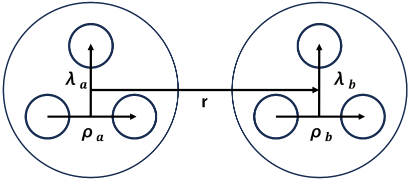
Relative coordinates of the 6-quark system:$\boldsymbol{r},\boldsymbol{\rho}_a,\boldsymbol{\lambda}_a,\boldsymbol{\rho}_b,\boldsymbol{\lambda}_b$

2 baryon state

2-baryon system is described as a 6-quark system.

$$|B_aB_b\rangle = \frac{1}{3!}\sum_{\mu_1,\mu_2,\mu_3}\sum_{\mu_4,\mu_5,\mu_6}\omega^{(B_a)}_{\mu_1,\mu_2,\mu_3}\omega^{(B_b)}_{\mu_4,\mu_5,\mu_6}\int d\xi\Psi(\boldsymbol{R}_{\text{tot}})\psi(\boldsymbol{r})\phi^{(B_a)}(\boldsymbol{\lambda}_a,\boldsymbol{\rho}_a)\phi^{(B_b)}(\boldsymbol{\lambda}_b,\boldsymbol{\rho}_b)$$
$$\times\,\hat{b}^\dagger_{\mu_1}(\boldsymbol{r}_1)\hat{b}^\dagger_{\mu_2}(\boldsymbol{r}_2)\hat{b}^\dagger_{\mu_3}(\boldsymbol{r}_3)\hat{b}^\dagger_{\mu_4}(\boldsymbol{r}_4)\hat{b}^\dagger_{\mu_5}(\boldsymbol{r}_5)\hat{b}^\dagger_{\mu_6}(\boldsymbol{r}_6)\,|0\rangle$$

- $\Psi(\boldsymbol{R}_{\text{tot}})$: Center-of-mass wave function of 2 baryons
- $\psi(\boldsymbol{r})$: Relative wave function between 2 baryons, what we want
- $\phi^{(B_a)}(\boldsymbol{\lambda}_a,\boldsymbol{\rho}_a)$ and $\phi^{(B_b)}(\boldsymbol{\lambda}_b,\boldsymbol{\rho}_b)$: internal wave functions of baryons B_a and B_b, obtained by the three-body Schrödinger equation (known)



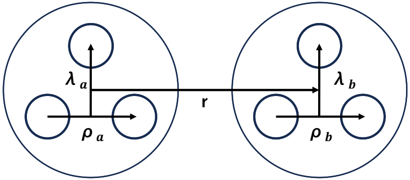
Relative coordinates of the 6-quark system: $\boldsymbol{r},\boldsymbol{\rho}_a,\boldsymbol{\lambda}_a,\boldsymbol{\rho}_b,\boldsymbol{\lambda}_b$

2 baryon state

2-baryon system is described as a 6-quark system.

$$|B_a B_b\rangle = \frac{1}{3!}\sum_{\mu_1,\mu_2,\mu_3}\sum_{\mu_4,\mu_5,\mu_6}\omega^{(B_a)}_{\mu_1,\mu_2,\mu_3}\omega^{(B_b)}_{\mu_4,\mu_5,\mu_6}\int d\xi\Psi(\boldsymbol{R}_{\text{tot}})\psi(\boldsymbol{r})\phi^{(B_a)}(\boldsymbol{\lambda}_a,\boldsymbol{\rho}_a)\phi^{(B_b)}(\boldsymbol{\lambda}_b,\boldsymbol{\rho}_b)$$
$$\times\,\hat{b}^\dagger_{\mu_1}(\boldsymbol{r}_1)\hat{b}^\dagger_{\mu_2}(\boldsymbol{r}_2)\hat{b}^\dagger_{\mu_3}(\boldsymbol{r}_3)\hat{b}^\dagger_{\mu_4}(\boldsymbol{r}_4)\hat{b}^\dagger_{\mu_5}(\boldsymbol{r}_5)\hat{b}^\dagger_{\mu_6}(\boldsymbol{r}_6)\,|0\rangle$$

- $\Psi(\boldsymbol{R}_{\text{tot}})$: Center-of-mass wave function of 2 baryons
- $\psi(\boldsymbol{r})$: Relative wave function between 2 baryons, what we want
- $\phi^{(B_a)}(\boldsymbol{\lambda}_a,\boldsymbol{\rho}_a)$ and $\phi^{(B_b)}(\boldsymbol{\lambda}_b,\boldsymbol{\rho}_b)$: internal wave functions of baryons B_a and B_b, obtained by the three-body Schrödinger equation (known)



Relative coordinates of the 6-quark system: $\boldsymbol{r}, \boldsymbol{\rho}_a, \boldsymbol{\lambda}_a, \boldsymbol{\rho}_b, \boldsymbol{\lambda}_b$

2 baryon state

2-baryon system is described as a 6-quark system.

$$|B_aB_b\rangle = \frac{1}{3!}\sum_{\mu_1,\mu_2,\mu_3}\sum_{\mu_4,\mu_5,\mu_6} \omega^{(B_a)}_{\mu_1,\mu_2,\mu_3}\omega^{(B_b)}_{\mu_4,\mu_5,\mu_6}\int d\xi\Psi(\boldsymbol{R}_{\text{tot}})\psi(\boldsymbol{r})\phi^{(B_a)}(\boldsymbol{\lambda}_a,\boldsymbol{\rho}_a)\phi^{(B_b)}(\boldsymbol{\lambda}_b,\boldsymbol{\rho}_b)$$
$$\times\,\hat{b}^\dagger_{\mu_1}(\boldsymbol{r}_1)\hat{b}^\dagger_{\mu_2}(\boldsymbol{r}_2)\hat{b}^\dagger_{\mu_3}(\boldsymbol{r}_3)\hat{b}^\dagger_{\mu_4}(\boldsymbol{r}_4)\hat{b}^\dagger_{\mu_5}(\boldsymbol{r}_5)\hat{b}^\dagger_{\mu_6}(\boldsymbol{r}_6)\,|0\rangle$$

$\omega^{(B_a)}_{\mu_1,\mu_2,\mu_3}, \omega^{(B_b)}_{\mu_4,\mu_5,\mu_6}$: (Color, flavor, spin) weights for each baryon

Example: $B = \Omega_{(S_z=+3/2)}$

$\omega^{(B)}_{\mu_1,\mu_2,\mu_3} =$ (color) $\times$ (flavor-spin)

(color) : $(R,G,B) = (G,B,R) = (B,R,G) = -(R,B,G) = -(G,R,B) = -(B,G,R) = \dfrac{1}{\sqrt{6}}$

(flavor-spin) : $(s_\uparrow, s_\uparrow, s_\uparrow) = 1$

RGM equation

The two-baryon state is an eigenstate of the 6-quarks Hamiltonian with eigenvalue E:

$$\hat{H}_{6q} \left| B_a B_b \right\rangle = E \left| B_a B_b \right\rangle$$

RGM equation

The two-baryon state is an eigenstate of the 6-quarks Hamiltonian with eigenvalue E:

$$\hat{H}_{6q} \left| B_a B_b \right\rangle = E \left| B_a B_b \right\rangle$$

Multiply from the left by $\langle B_c B_d(\boldsymbol{r})|$, compute matrix element.

$$\langle B_c B_d(\boldsymbol{r})| \, \hat{H}_{6q} \left| B_a B_b \right\rangle = E \, \langle B_c B_d(\boldsymbol{r}) | B_a B_b \rangle$$

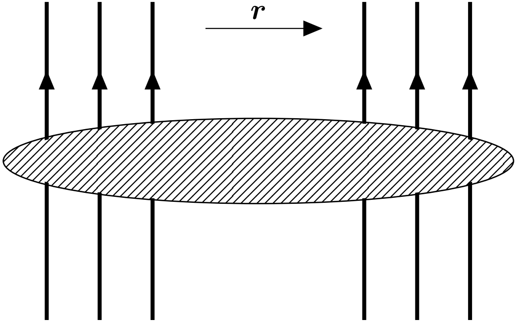
RGM equation

The two-baryon state is an eigenstate of the 6-quarks Hamiltonian with eigenvalue E:

$$\hat{H}_{6q} \left| B_a B_b \right\rangle = E \left| B_a B_b \right\rangle$$

Multiply from the left by $\langle B_c B_d(\boldsymbol{r})|$, compute matrix element.

$$\langle B_c B_d(\boldsymbol{r})| \hat{H}_{6q} \left| B_a B_b \right\rangle = E \left\langle B_c B_d(\boldsymbol{r}) | B_a B_b \right\rangle$$



RGM equation

The two-baryon state is an eigenstate of the 6-quarks Hamiltonian with eigenvalue E:

$$\hat{H}_{6q} \left| B_a B_b \right\rangle = E \left| B_a B_b \right\rangle$$

Multiply from the left by $\langle B_c B_d(\boldsymbol{r})|$, compute matrix element.

$$\langle B_c B_d(\boldsymbol{r})| \hat{H}_{6q} \left| B_a B_b \right\rangle = E \left\langle B_c B_d(\boldsymbol{r}) | B_a B_b \right\rangle$$

$$\to \int d^3 r' \left[K(\boldsymbol{r}, \boldsymbol{r}') + V_{\text{int}}(\boldsymbol{r}, \boldsymbol{r}') + \left(M_{B_a} + M_{B_b} - E \right) N(\boldsymbol{r}, \boldsymbol{r}') \right] \psi(\boldsymbol{r}') = 0$$

$K(\boldsymbol{r}, \boldsymbol{r}')$: kinetic term, $\quad V_{\text{int}}(\boldsymbol{r}, \boldsymbol{r}')$: intaraction term

RGM equation

The two-baryon state is an eigenstate of the 6-quarks Hamiltonian with eigenvalue E:

$$\hat{H}_{6q} \left| B_a B_b \right\rangle = E \left| B_a B_b \right\rangle$$

Multiply from the left by $\langle B_c B_d(\boldsymbol{r})|$, compute matrix element.

$$\langle B_c B_d(\boldsymbol{r})| \hat{H}_{6q} \left| B_a B_b \right\rangle = E \langle B_c B_d(\boldsymbol{r}) | B_a B_b \rangle$$

$$\to \int d^3 r' \left[K(\boldsymbol{r}, \boldsymbol{r}') + V_{\text{int}}(\boldsymbol{r}, \boldsymbol{r}') + (M_{B_a} + M_{B_b} - E) N(\boldsymbol{r}, \boldsymbol{r}') \right] \psi(\boldsymbol{r}') = 0$$

$K(\boldsymbol{r}, \boldsymbol{r}')$: kinetic term, $\quad V_{\text{int}}(\boldsymbol{r}, \boldsymbol{r}')$: intaraction term

This equation is called the Resonating Group Method (RGM) equation.

RGM kernel

RGM equation：

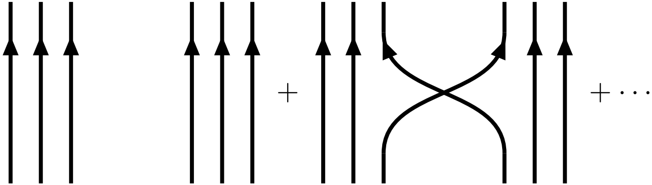
$$\int d^3r' \left[K(\boldsymbol{r},\boldsymbol{r}') + V_{\mathsf{int}}(\boldsymbol{r},\boldsymbol{r}') + (M_{B_a} + M_{B_b} - E)\, N(\boldsymbol{r},\boldsymbol{r}')\right] \psi(\boldsymbol{r}') = 0$$

RGM kernel

RGM equation：

$$\int d^3r' \left[K(\boldsymbol{r},\boldsymbol{r}') + V_{\text{int}}(\boldsymbol{r},\boldsymbol{r}') + (M_{B_a} + M_{B_b} - E)\, N(\boldsymbol{r},\boldsymbol{r}')\right] \psi(\boldsymbol{r}') = 0$$

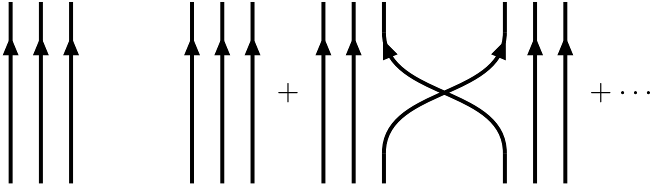
$$\left.\begin{array}{l} N(\boldsymbol{r},\boldsymbol{r}') \\ K(\boldsymbol{r},\boldsymbol{r}') \end{array}\right\}$$



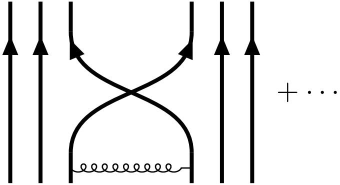
RGM equation：

$$\int d^3r' \left[K(\boldsymbol{r},\boldsymbol{r}') + V_{\text{int}}(\boldsymbol{r},\boldsymbol{r}') + (M_{B_a} + M_{B_b} - E)\, N(\boldsymbol{r},\boldsymbol{r}')\right] \psi(\boldsymbol{r}') = 0$$

$$\left.\begin{array}{l} N(\boldsymbol{r},\boldsymbol{r}') \\ K(\boldsymbol{r},\boldsymbol{r}') \end{array}\right\}$$



$V_{\text{int}}(\boldsymbol{r},\boldsymbol{r}')$:



Derivation of the Baryon-Baryon Potential

RGM equation：

$$\int d^3r' \left[K(\boldsymbol{r},\boldsymbol{r}') + V_{\mathsf{int}}(\boldsymbol{r},\boldsymbol{r}') - \mathcal{E}N(\boldsymbol{r},\boldsymbol{r}')\right] \psi(\boldsymbol{r}') = 0$$

$$\mathcal{E} \equiv E - M_{B_a} - M_{B_b}$$

Solving the RGM equation gives $(\mathcal{E}, \psi(r))$.

Derivation of the Baryon-Baryon Potential

RGM equation：

$$\int d^3r' \left[K(\boldsymbol{r},\boldsymbol{r}') + V_{\text{int}}(\boldsymbol{r},\boldsymbol{r}') - \mathcal{E}N(\boldsymbol{r},\boldsymbol{r}')\right]\psi(\boldsymbol{r}') = 0$$

$$\mathcal{E} \equiv E - M_{B_a} - M_{B_b}$$

Solving the RGM equation gives $(\mathcal{E}, \psi(r))$.

$\rightarrow$ The local baryon–baryon potential is derived from $(\mathcal{E}, \psi(r))$.

Derivation of the Baryon-Baryon Potential

RGM equation：

$$\int d^3r' \left[K(\boldsymbol{r},\boldsymbol{r}') + V_{\mathsf{int}}(\boldsymbol{r},\boldsymbol{r}') - \mathcal{E}N(\boldsymbol{r},\boldsymbol{r}')\right] \psi(\boldsymbol{r}') = 0$$

$$\mathcal{E} \equiv E - M_{B_a} - M_{B_b}$$

Solving the RGM equation gives $(\mathcal{E}, \psi(r))$.
$\rightarrow$ The local baryon–baryon potential is derived from $(\mathcal{E}, \psi(r))$.

$$V(r) = \mathcal{E} + \frac{1}{2\mu\chi}\frac{\mathrm{d}^2\chi}{\mathrm{d}r^2}, \quad \chi(r) = r\psi(r)$$

Derivation of the Baryon-Baryon Potential

RGM equation：

$$\int d^3r' \left[K(\boldsymbol{r},\boldsymbol{r}') + V_{\text{int}}(\boldsymbol{r},\boldsymbol{r}') - \mathcal{E}N(\boldsymbol{r},\boldsymbol{r}')\right]\psi(\boldsymbol{r}') = 0$$

$$\mathcal{E} \equiv E - M_{B_a} - M_{B_b}$$

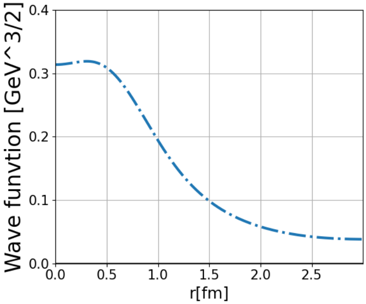
Solving the RGM equation gives $(\mathcal{E}, \psi(r))$.
$\rightarrow$ The local baryon–baryon potential is derived from $(\mathcal{E}, \psi(r))$.

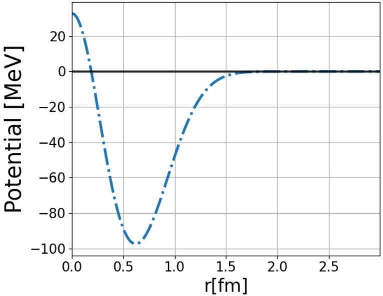
$$V(r) = \mathcal{E} + \frac{1}{2\mu\chi}\frac{\mathrm{d}^2\chi}{\mathrm{d}r^2}, \quad \chi(r) = r\psi(r)$$

In this talk, we show results without quark-quark interactions. $(V_{\text{int}}(\boldsymbol{r},\boldsymbol{r}') = 0)$
Even without the interaction potential, the baryon–baryon potential is generated by quark exchange effects originating from the kinetic term.

Results

- Result for the $\Omega\Omega(S=0)$ system with $V_{int}(\boldsymbol{r},\boldsymbol{r}')=0$





$$\mathcal{E} = -14.54 \text{ MeV}$$

- A baryon–baryon potential is generated at $r \lesssim 1.5\,\mathrm{fm}$ even without quark–quark interactions $\rightarrow$ originated from quark exchange effects
- $\mathcal{E} < 0 \rightarrow$ bound state
  - $\because$ Absence of Pauli-forbidden states $\rightarrow$ quark exchange induces an attractive interaction

Comparison with previous studies

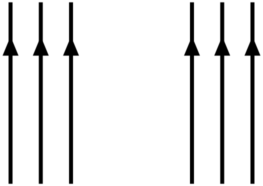
Comparison with T. Sekihara and T. Hashiguchi, PRC108(2023)065202
kinetic term：

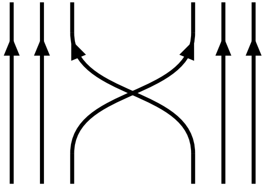
$$K(\boldsymbol{r}, \boldsymbol{r}') = K^{d}(\boldsymbol{r}, \boldsymbol{r}') + K^{ex}(\boldsymbol{r}, \boldsymbol{r}')$$

Comparison with previous studies

Comparison with T. Sekihara and T. Hashiguchi, PRC108(2023)065202
kinetic term :

$$K(\boldsymbol{r},\boldsymbol{r}') = K^d(\boldsymbol{r},\boldsymbol{r}') + K^{ex}(\boldsymbol{r},\boldsymbol{r}')$$

$K^d(\boldsymbol{r},\boldsymbol{r}') =$ 

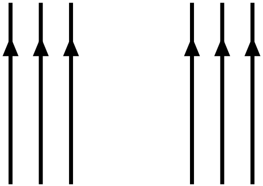
$K^{ex}(\boldsymbol{r},\boldsymbol{r}') =$  $+ \cdots$

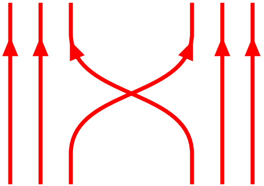
Comparison with previous studies

Comparison with T. Sekihara and T. Hashiguchi, PRC108(2023)065202

kinetic term：

$$K(\boldsymbol{r},\boldsymbol{r}') = K^{d}(\boldsymbol{r},\boldsymbol{r}') + K^{ex}(\boldsymbol{r},\boldsymbol{r}')$$

$K^{d}(\boldsymbol{r},\boldsymbol{r}') =$ 

$K^{ex}(\boldsymbol{r},\boldsymbol{r}') =$  $+ \cdots$

$K^{ex}_{\boldsymbol{i}}(\boldsymbol{r},\boldsymbol{r}')$: Kinetic term for coordinate $\boldsymbol{i}$

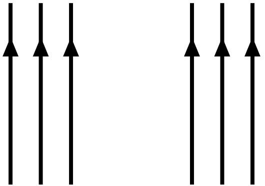
previous study：$K^{ex}(\boldsymbol{r},\boldsymbol{r}') = K^{ex}_{\boldsymbol{r}}(\boldsymbol{r},\boldsymbol{r}')$ (Only the relative kinetic term between clusters)

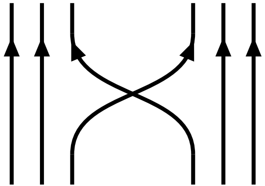
Comparison with previous studies

Comparison with T. Sekihara and T. Hashiguchi, PRC108(2023)065202

kinetic term：

$$K(\boldsymbol{r},\boldsymbol{r}') = K^{d}(\boldsymbol{r},\boldsymbol{r}') + K^{ex}(\boldsymbol{r},\boldsymbol{r}')$$

$K^{d}(\boldsymbol{r},\boldsymbol{r}') =$ 

$K^{ex}(\boldsymbol{r},\boldsymbol{r}') =$  $+ \cdots$

$K^{ex}_{\boldsymbol{i}}(\boldsymbol{r},\boldsymbol{r}')$: Kinetic term for coordinate $\boldsymbol{i}$

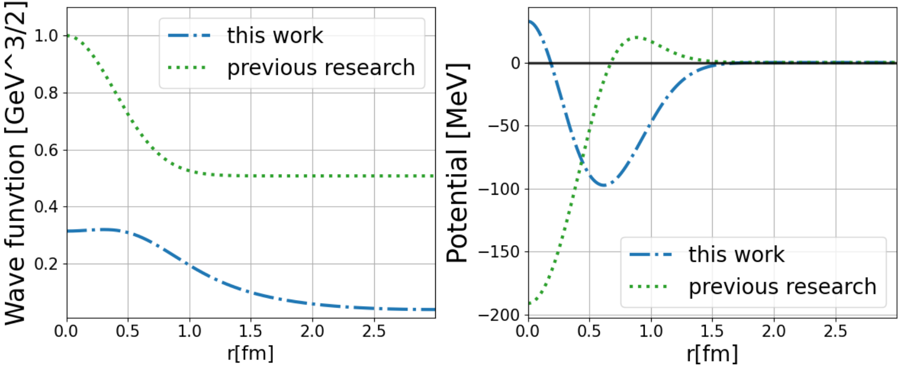
Previous study: $K^{ex}(\boldsymbol{r},\boldsymbol{r}') = K^{ex}_{\boldsymbol{r}}(\boldsymbol{r},\boldsymbol{r}')$ missing contributions

This work: $K^{ex}(\boldsymbol{r},\boldsymbol{r}') = K^{ex}_{\boldsymbol{r}}(\boldsymbol{r},\boldsymbol{r}') + K^{ex}_{\boldsymbol{\lambda}}(\boldsymbol{r},\boldsymbol{r}') + K^{ex}_{\boldsymbol{\rho}}(\boldsymbol{r},\boldsymbol{r}')$

We improved the kinetic kernel.

Comparison with previous studies

- Comparison with previous work: $\Omega\Omega(S=0)$ system with only the kinetic term



Previous study: $\mathcal{E} = 3.663 \times 10^{-11}$ MeV, : No bound state

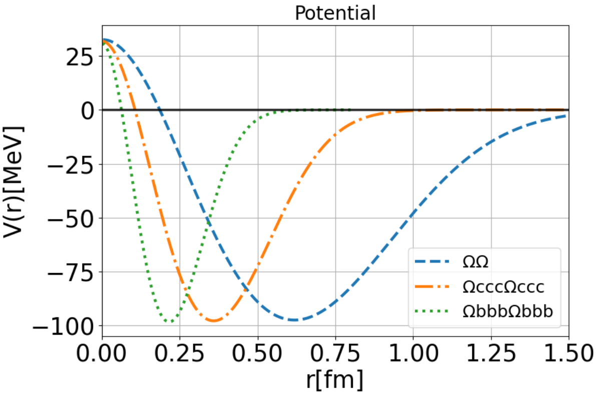
This work: $\mathcal{E} = -14.54$ MeV, : Bound state

- A significant difference is observed.

Extension to c, b quarks: $\Omega\Omega \to \Omega_{ccc}\Omega_{ccc}, \Omega_{bbb}\Omega_{bbb}$ (S wave, $S = 0$)

without quark–quark interactions：
$V_{\text{int}}(\boldsymbol{r}, \boldsymbol{r}') = 0$

- No Pauli-forbidden states exist in the six-quark system for $S = 0$
  $\to$ Attractive interaction at the baryon level
- Quark mass increases
  $\to$ Interaction range decreases
  $\to$ Binding energy increases
- The values at $r = 0$ and the minimum values of the potentials are identical for all three systems
  $\to$ Under investigation (future work)



Baryon-Baryon	energy eigenvalues $\mathcal{E}$ [MeV]
$\Omega\Omega$	-14.54
$\Omega_{ccc}\Omega_{ccc}$	-16.52
$\Omega_{bbb}\Omega_{bbb}$	-19.04

Summary

- Summary
  - Many exotic hadrons considered as hadronic molecules have been reported recently, and understanding of baryon–baryon interactions is essential.
  - Lattice QCD calculations have provided interaction data for various hadron systems.
  - We have constructed a framework to analyze baryon–baryon potentials using the quark cluster model.
  - We investigated the effect of quark exchange without quark-quark interactions, considering only the kinetic term.
  - The baryon–baryon potentials of $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ were analyzed, and their quark-mass dependence was investigated.
- Future prospects
  - Solve the RGM equation including quark–quark interactions and analyze the resulting local potentials.
  - Apply the developed framework to various baryon–baryon systems.
  - Compare with Lattice QCD results to clarify the mechanism of baryon–baryon interactions.