The Quark-Diquark Structure of Baryons

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UMONS
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FREEDOM OF RESEARCH

Comparison of Two Frameworks

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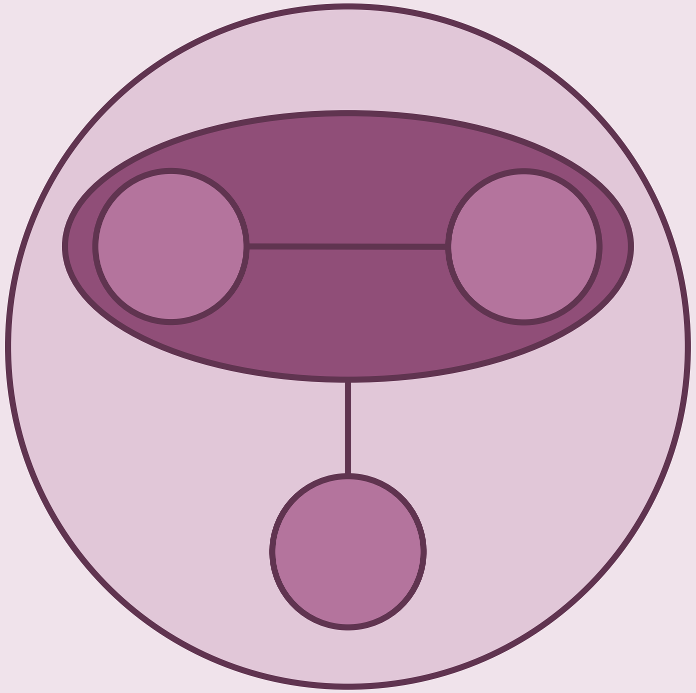
Quark-diquark approximation:



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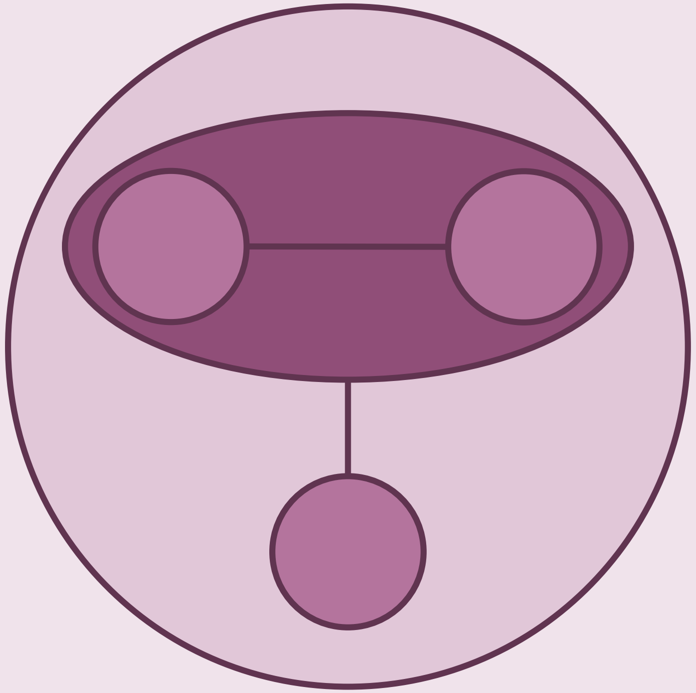
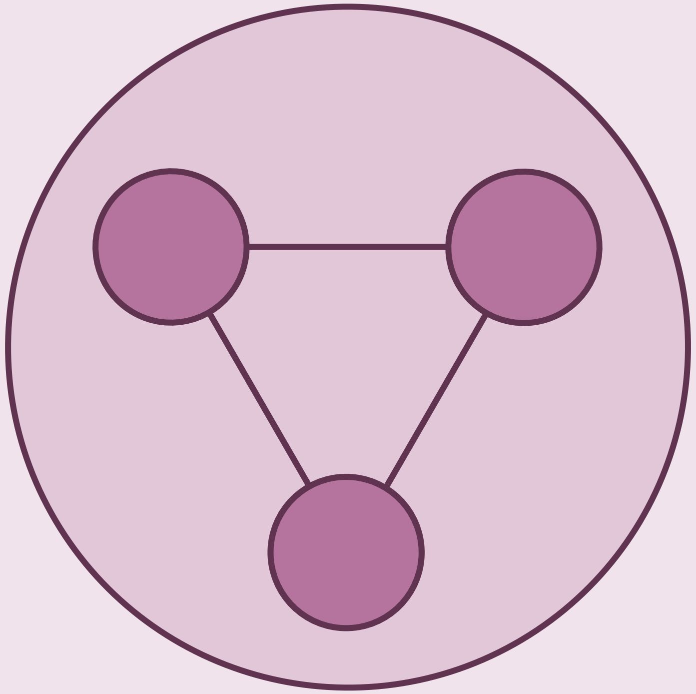


Three-body model:



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Constituent picture:

- Baryon = 3 interacting valence quarks,
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Studied systems:

Baryons:	States:
• bbb,	• Ground states,
• bbn,	• L=8,
• nnb,	• All spin states,
• nnn.	• All isospin states.

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Constituent picture:

- Baryon = 3 interacting valence quarks,
- Solve Schrödinger-like equations,
- Interactions = potential.

Computed quantities:

- Masses,
- Wave functions,
- Characteristic distances.

Studied systems:

Baryons:

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States:

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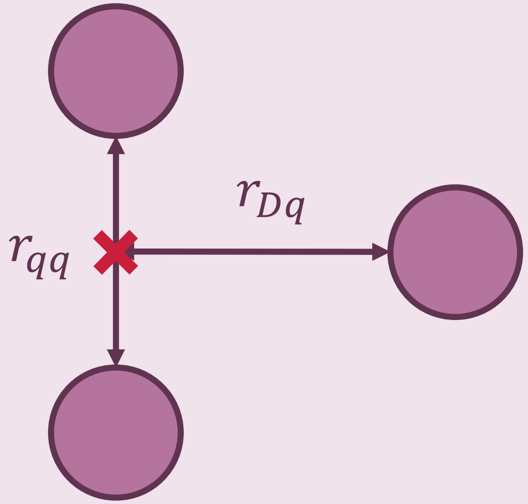


The Three-Body Hamiltonian

The Three-Body Hamiltonian

A general Hamiltonian for two-body forces:

$$H = \sum_i T_i(p_i) + \sum_{i<j} V_{ij}\left(r_{ij}\right),$$

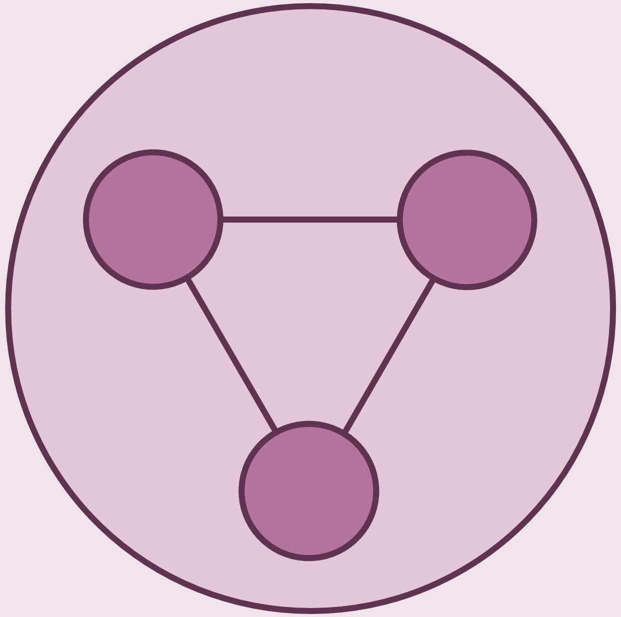
Semi-relativistic kinetic part:

$$T_i(p_i) = \sqrt{{p_i}^2 + m_i^2},$$

Potential:

$$V_{ij}\left(r_{ij}\right) = C r_{ij} - \frac{2b}{3r_{ij}} + V_0 + \frac{\alpha_s}{9 m_i m_j} \Lambda^2 \, \frac{\mathrm{e}^{-\Lambda r_{ij}}}{r_{ij}} 4\, \boldsymbol{s}_i \cdot \boldsymbol{s}_j,$$

→ Interactions between two quarks.



p_i: quark i momentum,
r_{ij}: relative distance between quarks i and j,
m_i: quark i mass,
$\boldsymbol{s}_i$: quark i spin operator,
C, b, α_s, Λ, V_0: constants.

The Quark-Quark Potential

Standard interactions:

$$V_{ij}(r_{ij}) = \boxed{Cr_{ij} - \frac{2b}{3r_{ij}} + V_0} + \boxed{\frac{\alpha_s}{9m_i m_j}\Lambda^2 \frac{\mathrm{e}^{-\Lambda r_{ij}}}{r_{ij}} 4\, \boldsymbol{s}_i \cdot \boldsymbol{s}_j.}$$

Central
-
Cornell

Spin
-
Yukawa

The Quark-Diquark Hamiltonians

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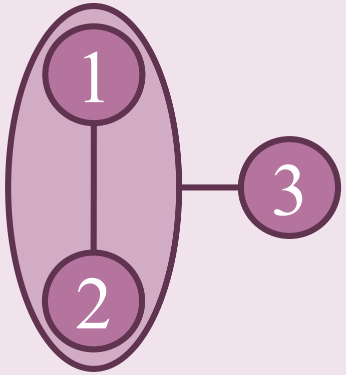
Same form as for the three-body model

The Quark-Diquark Hamiltonians



Same form as for the three-body model

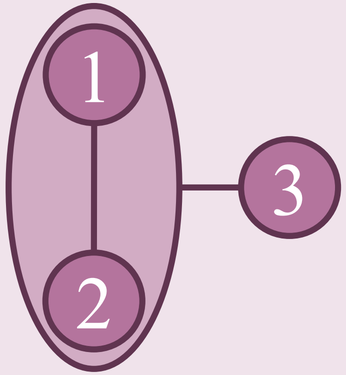
Diquark:

$$H_D = \sqrt{{p_1}^2 + m_1^2} + \sqrt{{p_2}^2 + m_2^2} + V_{qq}(r),$$

$$V_{qq}(r) = Cr - \frac{2b}{3r} + V_0 + \frac{\alpha_s}{9m_i m_j}\Lambda^2 \frac{\mathrm{e}^{-\Lambda r}}{r} 4\, \boldsymbol{s}_1 \cdot \boldsymbol{s}_2\,.$$

$$H_D \rightarrow M_D \;\&\; \Psi_D$$

The Quark-Diquark Hamiltonians



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$$H_D \to M_D \; \& \; \Psi_D$$

Quark-diquark:

$$H_B = \sqrt{p_D{}^2 + M_D^2} + \sqrt{p_3{}^2 + m_3^2} + V_{Dq}(r),$$

What is the potential $V_{Dq}(r)$?

The Quark-Diquark Potential

1. Colour considerations

A diquark = two quarks,

$$3 \otimes 3 = \mathbf{\bar{3}} \oplus 6.$$

The baryon = colour singlet,

$$3 \otimes \bar{3} = \mathbf{1} \oplus 8.$$

The diquark transforms as an antiquark under the colour group SU(3).

First assumption:

$$V_{Dq} = V_{\bar{q}q} = 2\, V_{qq}.$$

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$\boxed{\text{Diquark not point-like}}$

The Quark-Diquark Potential

2. Spatial extension considerations

$\boxed{\text{Convolution with colour density.}}$

The Quark-Diquark Potential

2. Spatial extension considerations

Convolution with colour density.

Test of two convolution formulas:

$$V^1_{Dq}(\boldsymbol{R}) = \int \mathrm{d}^3 r\, |\Psi_D(\boldsymbol{r})|^2\, 2\, V_{qq}(|\boldsymbol{r} + \boldsymbol{R}|),\ [1]$$

$$V^2_{Dq}(\boldsymbol{R}) = 8 \int \mathrm{d}^3 r\, |\Psi_D(2\, \boldsymbol{r})|^2\, 2\, V_{qq}(|\boldsymbol{r} + \boldsymbol{R}|).$$

New procedure including the colour density from [2].

[1] Giannuzzi, Phys. Rev. D **79**, 094002 (2009), [2] Nazarov, Phys. Rev. B **87**, 165125 (2013).

Methods

For the quark-diquark approximation

Lagrange mesh method (LMM)

For the three-body model

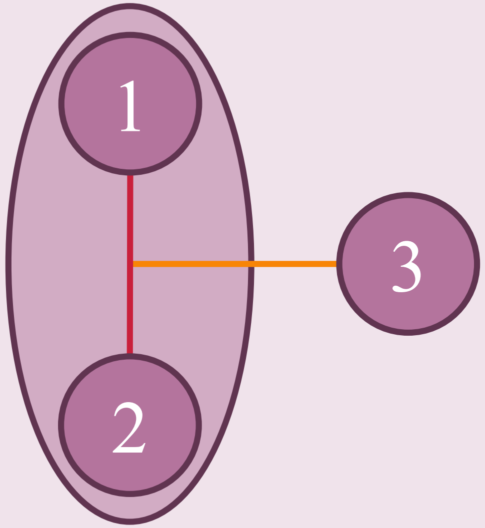
Oscillator basis expansion (OBE)

Methods

For the quark-diquark approximation

Lagrange mesh method (LMM)

$$\left(n_D, l_D, n_{Dq}, l_{Dq}, S_D, I_D\right)$$



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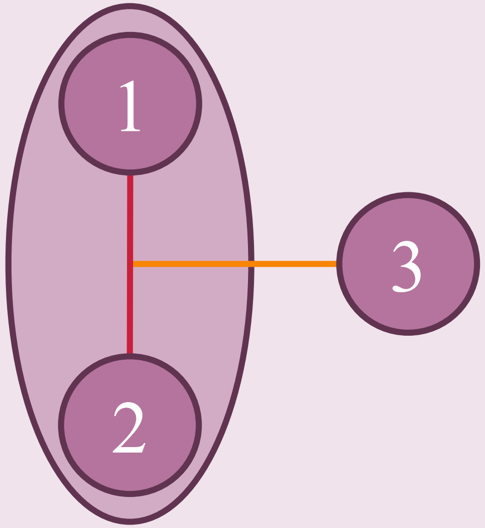
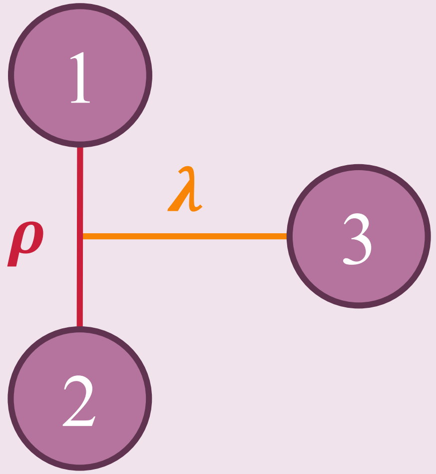


For the three-body model

Oscillator basis expansion (OBE)

$$\left(n_\rho, l_\rho, n_\lambda, l_\lambda, S_{12}, I_{12}\right)$$



Methods: Practical Application

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3-body

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Baryon properties

↓

3-body

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↓

M_{qqq}

Methods: Practical Application

Baryon properties

↓

3-body → **Eigenstates**

↓

M_{qqq}

Methods: Practical Application

Baryon properties

↓

3-body → **Eigenstates**

↓ ↓

M_{qqq}

$$\alpha|\Phi_\alpha\rangle + \beta|\Phi_\beta\rangle + \cdots$$

$$|\alpha|^2 \geq |\beta|^2 \geq \cdots$$

Methods: Practical Application

Baryon properties

↓

3-body → **Eigenstates** → **Dominant components**

↓ ↓ ↓

M_{qqq}

$\alpha|\Phi_\alpha\rangle + \beta|\Phi_\beta\rangle + \cdots$

$|\Phi_\alpha\rangle, |\Phi_\beta\rangle$

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Methods: Practical Application

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3-body → **Eigenstates** → **Dominant components** → **Quark-diquark**

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Methods: Practical Application

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$|\alpha|^2 \geq |\beta|^2 \geq \cdots$

$|\Phi_\alpha\rangle, |\Phi_\beta\rangle$

$M_{Dq}^{\alpha}, M_{Dq}^{\beta}$

Results: Characteristic Distances

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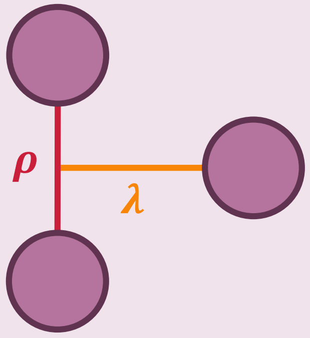
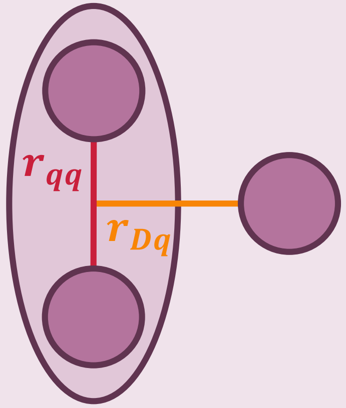
Study of baryon structures:

Mean values of $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$ for 3-body $\Leftrightarrow$ $\boldsymbol{r_{qq}}$ and $\boldsymbol{r_{Dq}}$ for quark-diquark.





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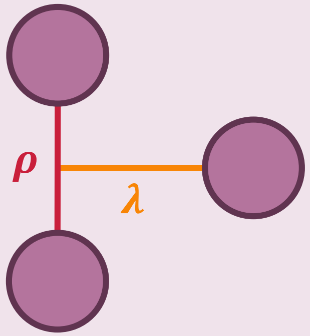

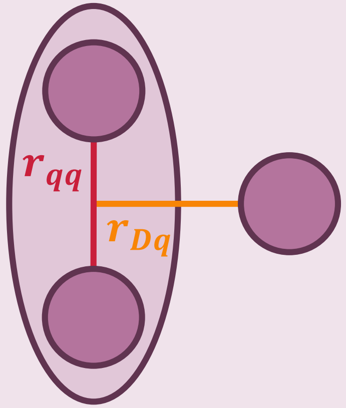

Relative difference with 3-body: from 10% to 70%,

Limitation of the quark-diquark model.

→ Study of the structure based on 3-body values.

Results: Convolution for V_{Dq}

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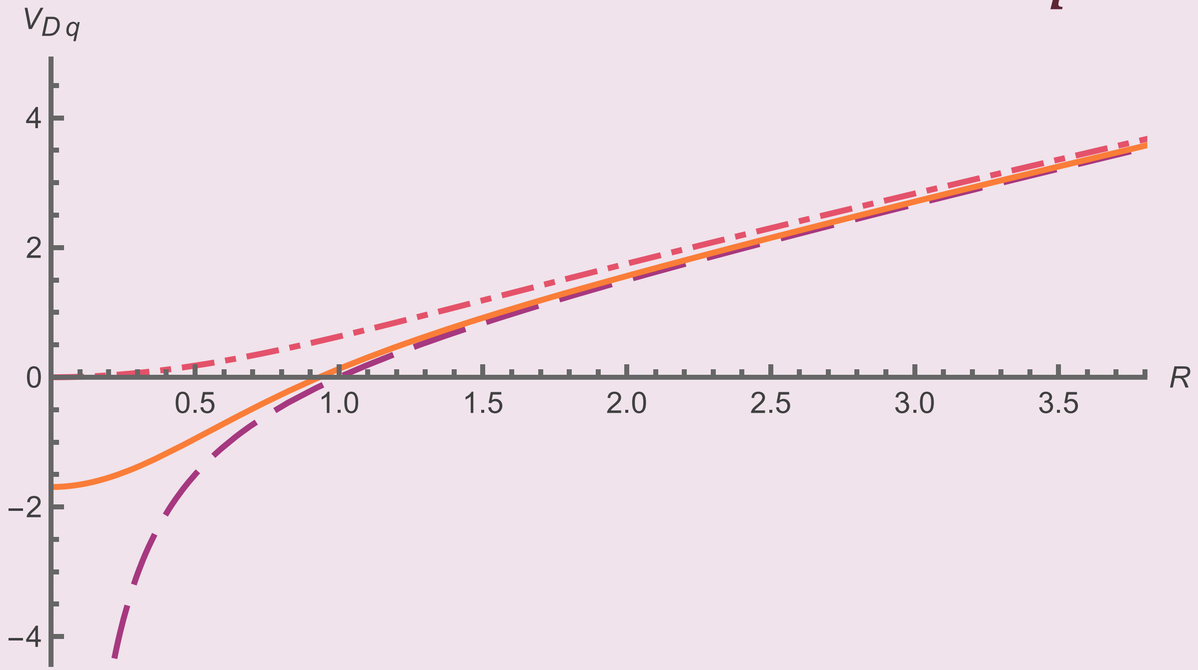

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$$V_{Dq}^{1}(\boldsymbol{R}) = \int \mathrm{d}^3 r\, |\Psi_D(\boldsymbol{r})|^2\, 2\,V_{qq}(|\boldsymbol{r}+\boldsymbol{R}|)\,,$$

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Only for identical particles in the diquark!

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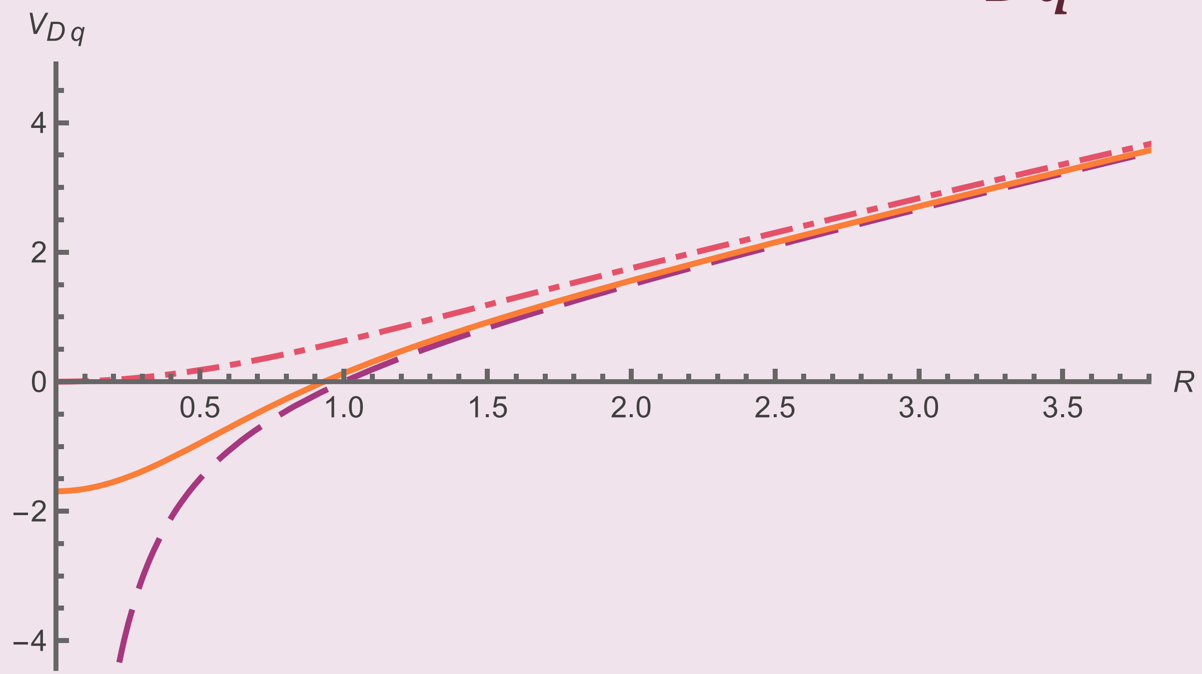

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Only for identical particles in the diquark!

Relative differences in masses:

Ground states, $S = 3/2$.

- bbb:

  $\approx 4\%$ $\approx 3\%$ $\approx 1\%$

- bbn:

  $\approx 2\%$ $\approx 2\%$ $\approx 0.2\%$

- nnb:

  $\approx 6\%$ $\approx 11\%$ $\approx 3\%$

- nnn:

  $\approx 14\%$ $\approx 36\%$ $\approx 6\%$

Results: Three Identical Particles

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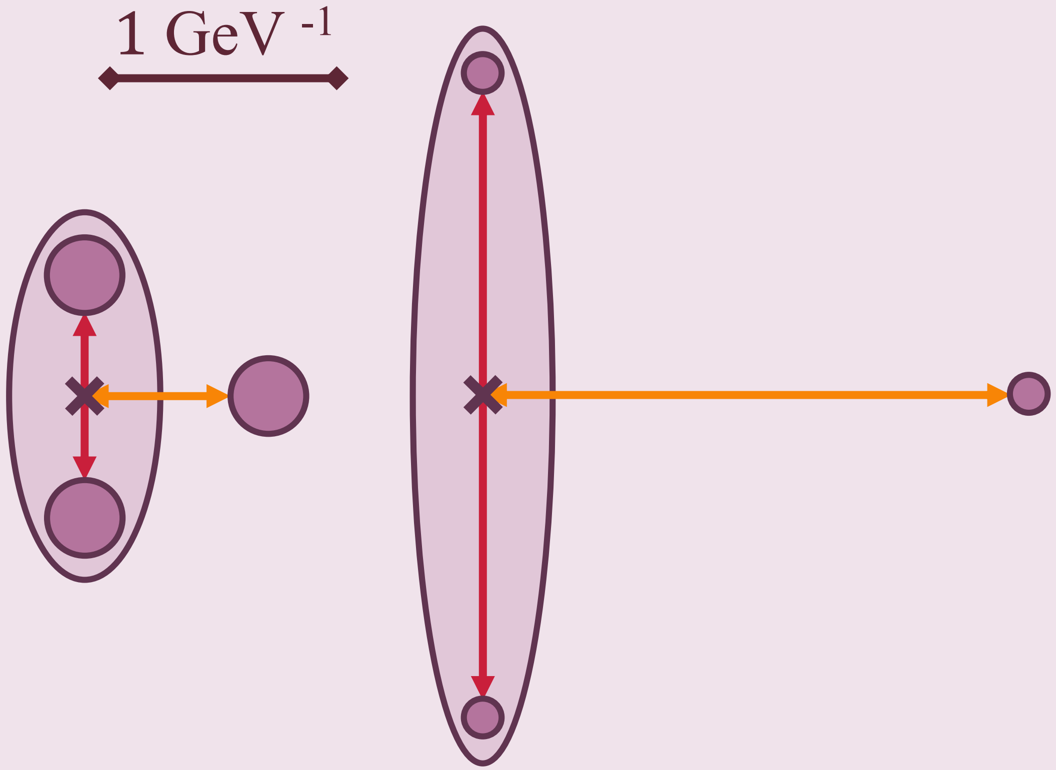
- Ground states structures ($S = 3/2$):

Symmetries → Equilateral triangles.



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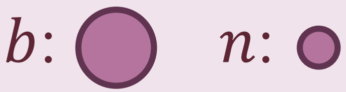

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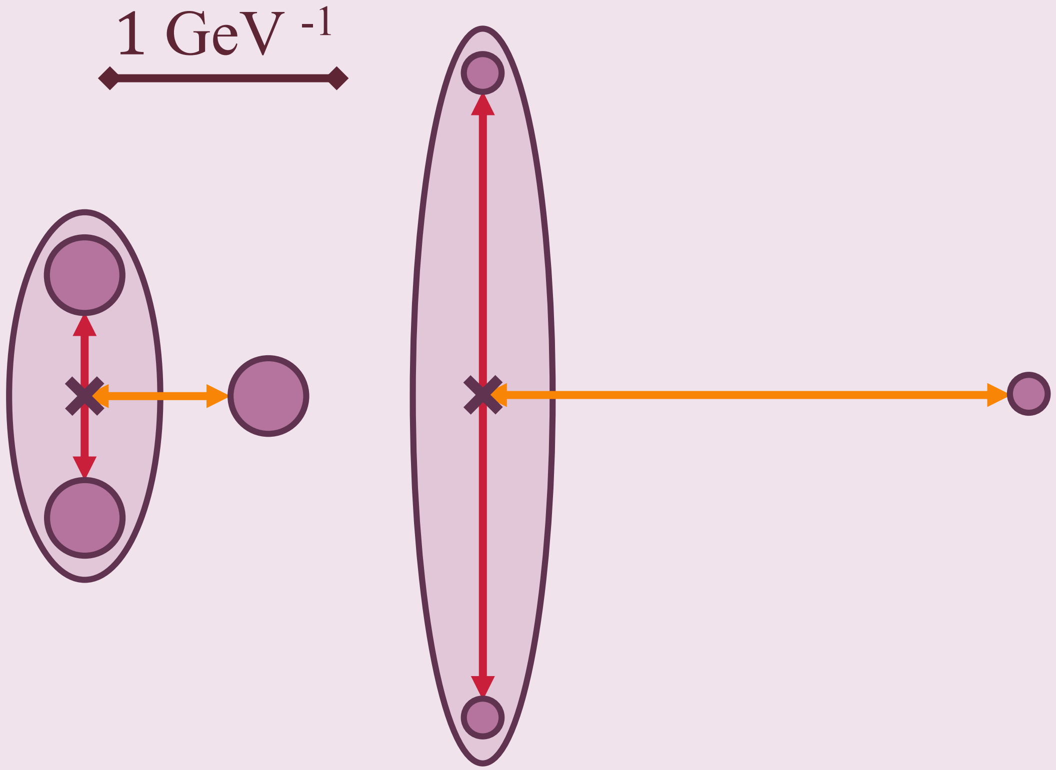
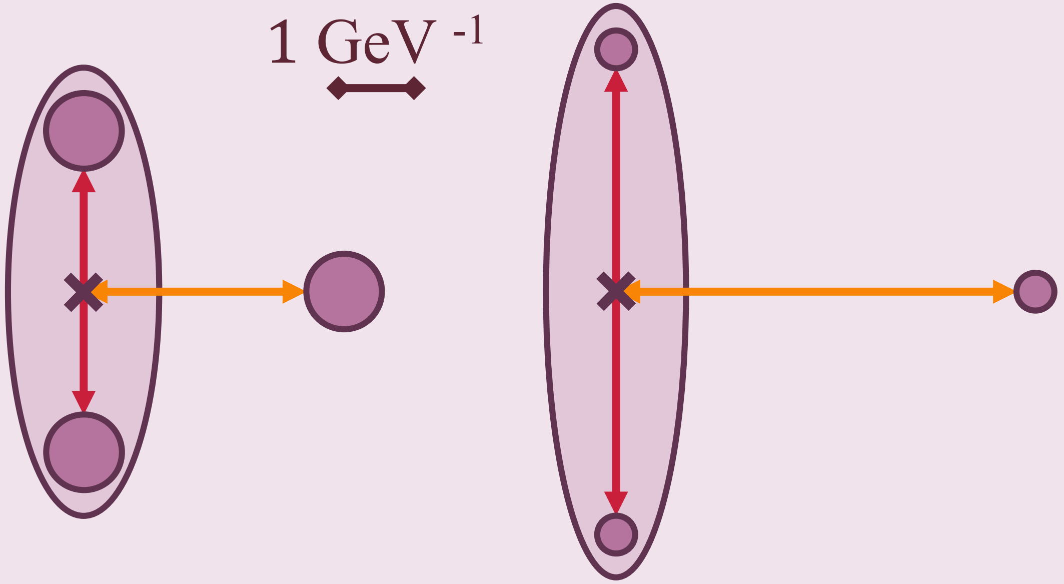


- $L = 8$ states structures ($S = 1/2$):

3-body states: many HO basis components → complex systems.

Still equilateral triangle structure → Excitation balance.



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$$\boxed{V^{\text{spin}} \propto (m_i m_j)^{-1}}$$

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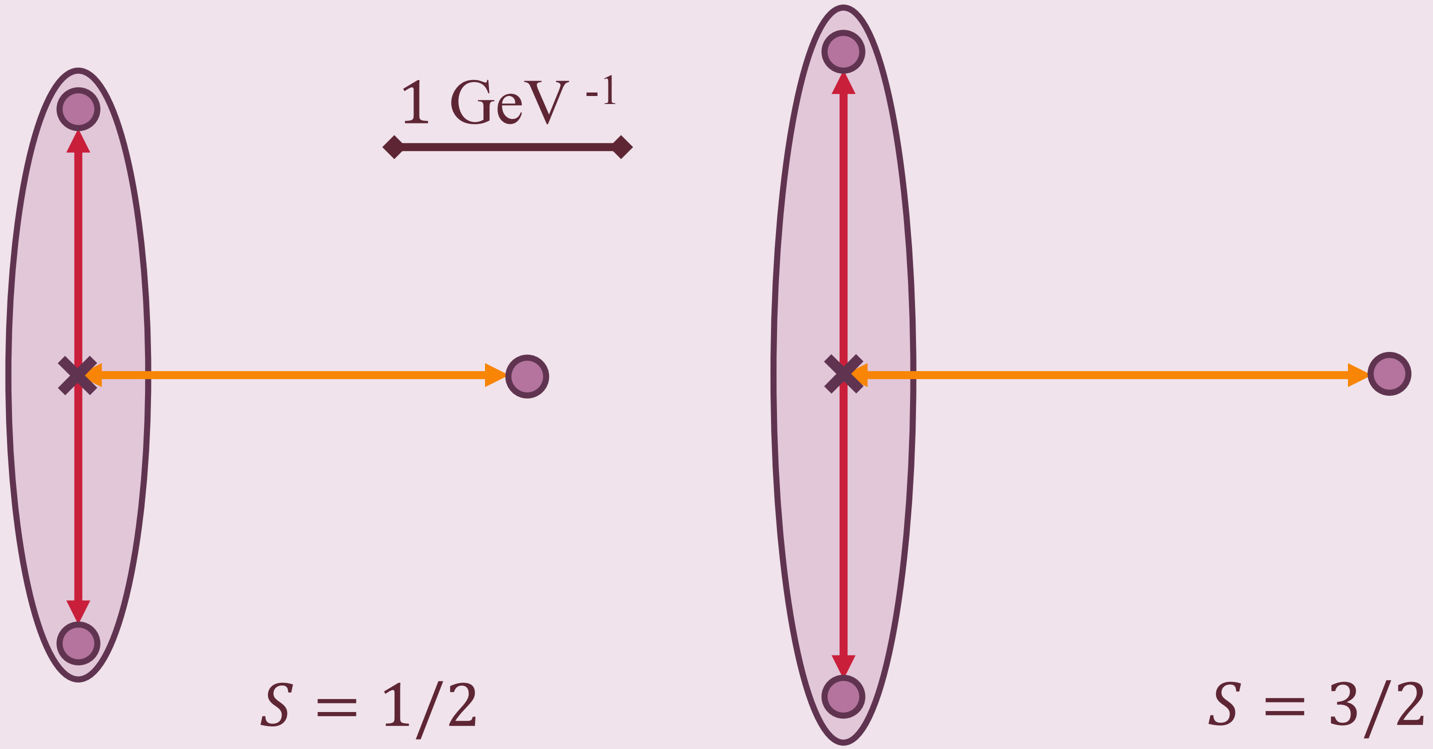

Results: *bbn*

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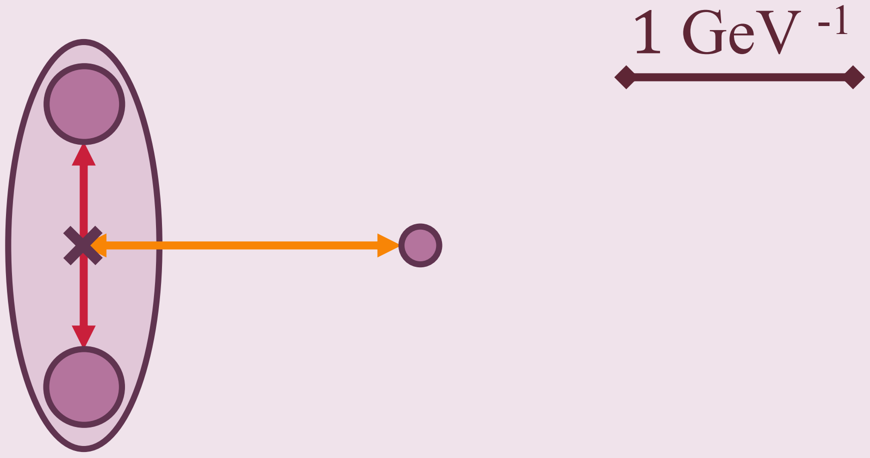
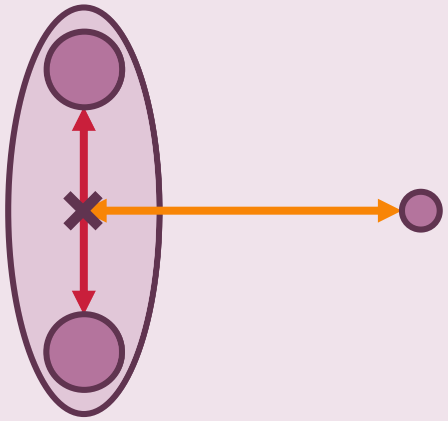


Ground state, $S = 3/2$

- Ground state:
  Relatively compact diquark.
  Relative difference 0.2%.

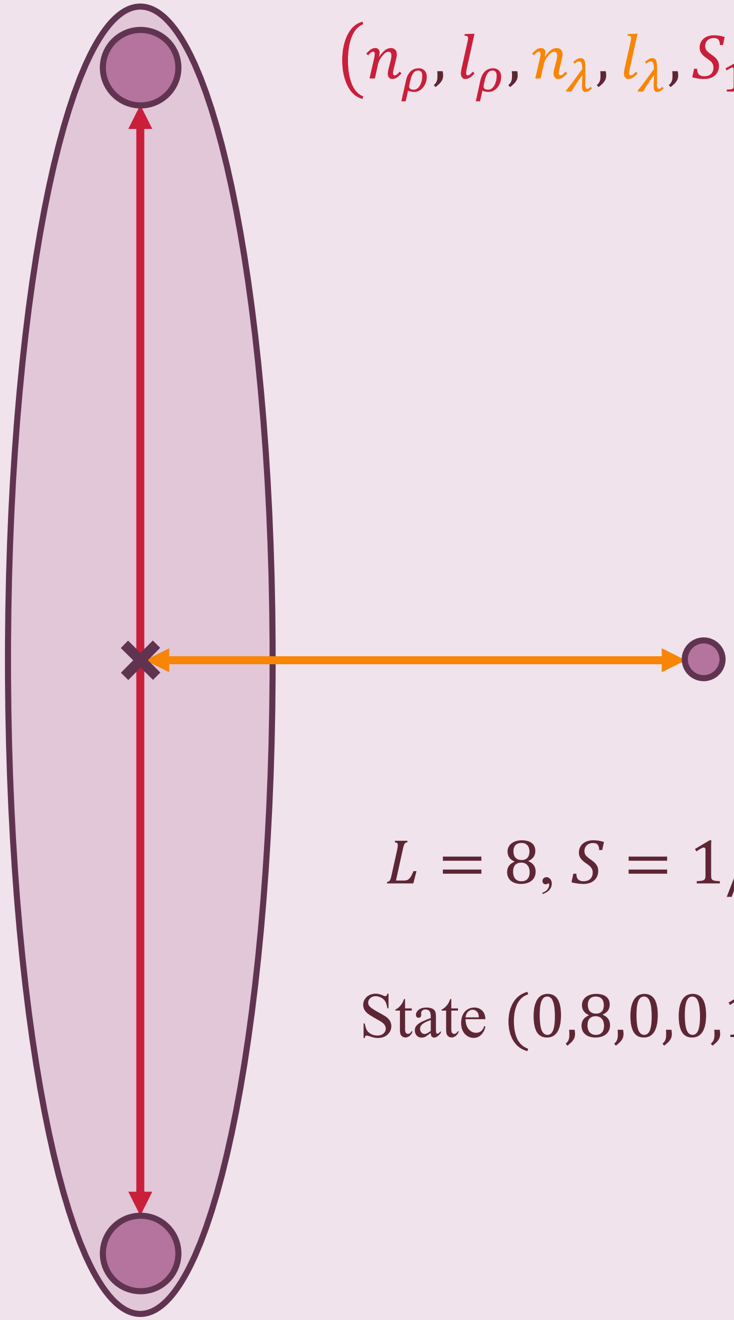
Results: *bbn*

$(n_\rho, l_\rho, n_\lambda, l_\lambda, S_{12}, I_{12})$



Ground state, $S = 3/2$

1 GeV $^{-1}$



- Ground state:
  Relatively compact diquark.
  Relative difference 0.2%.
- $L = 8$:
  Extended diquark
  Relative differences 1%.

$L = 8$, $S = 1/2$,

State (0,8,0,0,1,0)

Results: *nnb*

Results: nnb

Highlight of intra-diquark spin interactions:

Ground states with $S = 1/2$.

Results: nnb

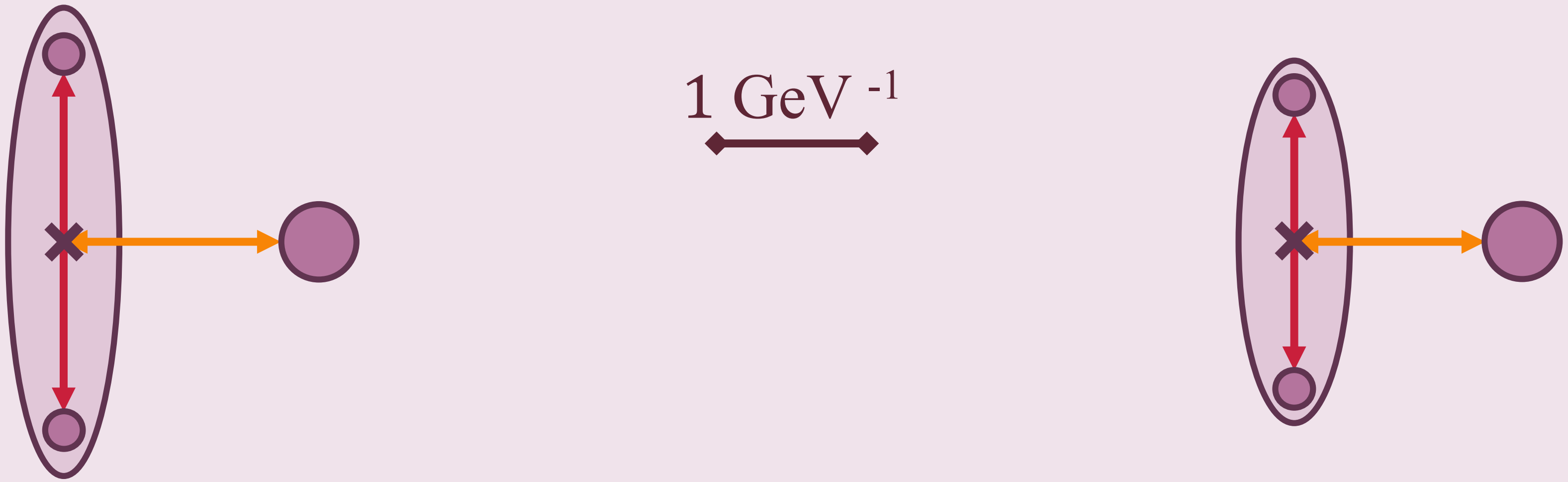
$(n_\rho, l_\rho, n_\lambda, l_\lambda, S_{12}, I_{12})$

Highlight of intra-diquark spin interactions:

Ground states with $S = 1/2$.

$S_{12} = 1 \rightarrow$ Repulsive spin interactions.

$S_{12} = 0 \rightarrow$ Attractive spin interactions.



State (0,0,0,0,1,1)

State (0,0,0,0,0,0)

Results: nnb

$(n_\rho, l_\rho, n_\lambda, l_\lambda, S_{12}, I_{12})$

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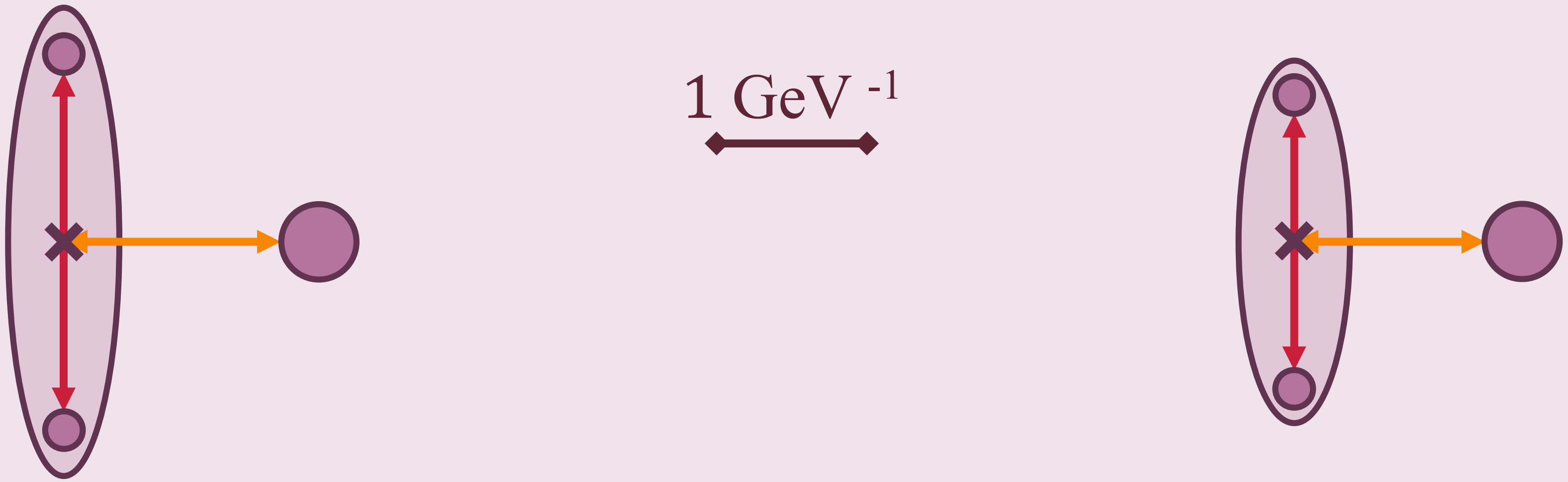


State (0,0,0,0,1,1)
3.7%

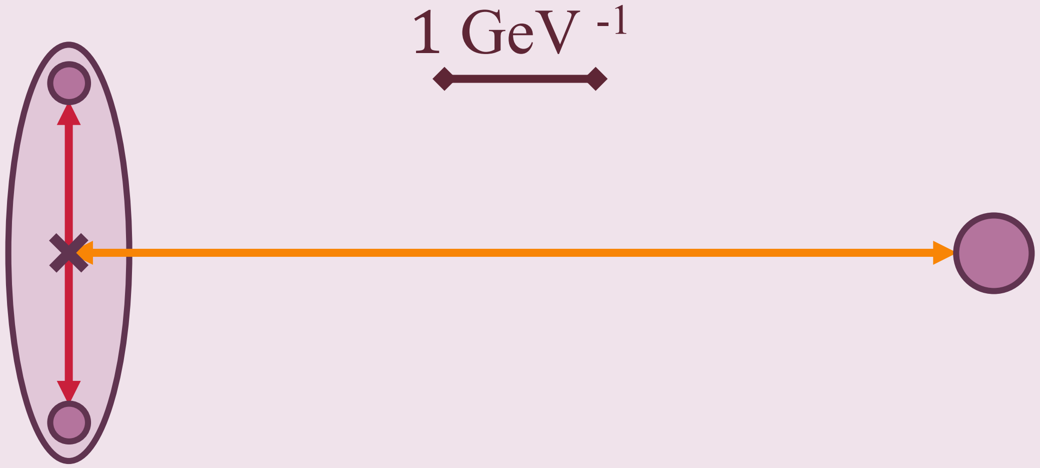
State (0,0,0,0,0,0)
4.8%

Results: nnb

$(n_\rho, l_\rho, n_\lambda, l_\lambda, S_{12}, I_{12})$

$L = 8$ state:

- Very compact diquark,
- Relative difference 3.3%.



State (0,0,0,8,0,0)

Take Home Message

Key Results:

- Direct computation not working for distances,
- New convolution formula,
- Spin impact on quark-diquark structure,
- Globally reasonable results for masses,
- Compact diquark not necessary.

Take Home Message

Outlooks:

- New procedure to compute distances,
- Improvement of spin interactions,
- Diquark in a superposition of states,
- Application to more complex systems.

Main Reference:

C. Tourbez, C. Chevalier, and C. Semay, Eur. Phys. J. A **62**, 101 (2026).

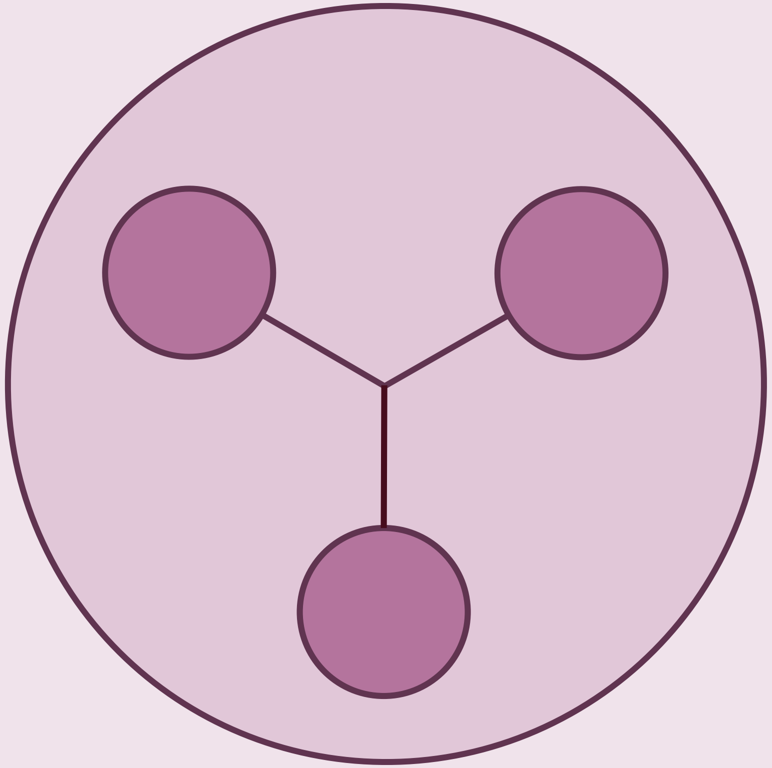
Introduction

Constituent approach:

- Baryon = 3 interacting valence quarks,
- Solve Schrödinger-like equations,
- Interactions = potential.

Types of interactions between quarks:

- Central-colour short range,
- Central-colour long range (confinement),
- Spin-colour.



Methods: Lagrange Mesh

For the quark-diquark approximation

H_D & H_B

$$H|\psi\rangle = E|\psi\rangle.$$

Expansion of $|\psi\rangle$:

$$|\psi\rangle = \sum_j c_j \, |f_j\rangle \, .$$

To simplify the computations.

Methods: Oscillator Basis Expansion

For the three-body model

$$H|\psi\rangle = E|\psi\rangle,$$

$$|\psi\rangle = \sum_j c_j\,|\Phi_j\rangle,$$

$|\Phi_j\rangle$ eigenstates of HO.

$$\rightarrow \sum_j \left[H_{ij} - E\,\delta_{ij}\right] c_j = 0,$$

where $H_{ij} = \langle \Phi_i | H | \Phi_j \rangle$.

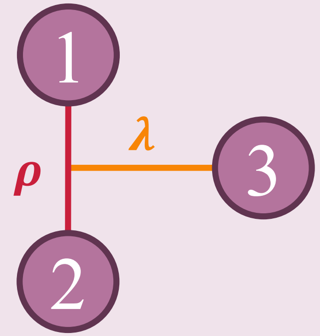
1. Basis Construction.

$$\Phi = \phi_{\text{colour}}\,\phi^I_{\text{isospin}}\,\phi^S_{\text{spin}}\,\phi^L_{\text{space}},$$

colour singlet, I, S, L and parity fixed.

One state i of the basis is described by

$$\Big(\underbrace{n_{\rho_i}, l_{\rho_i}}_{\boldsymbol{\rho}}, \underbrace{n_{\lambda_i}, l_{\lambda_i}}_{\boldsymbol{\lambda}}, S_{12_i}, I_{12_i}\Big).$$



2. Matrix element computation.

3. Hamiltonian matrix diagonalisation.

Methods: Oscillator Basis Expansion

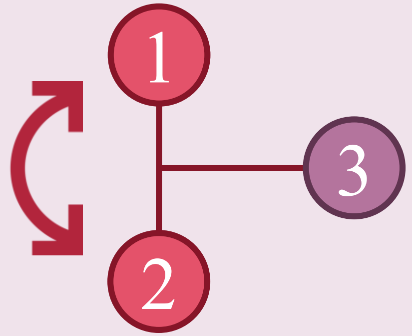
$bbn,\quad bbb,$
$nnb,\quad nnn.$

What about the symmetries ? $\rightarrow$ Pauli principle

Two identical quarks:

$$[H_{\mathrm{ho}}\,, P_{12}] = 0, \ ✓$$

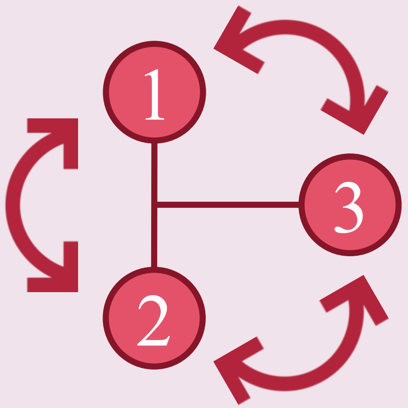
Just one condition on the eigenvalues.



Three identical quarks:

✓ $[H_{\mathrm{ho}}\,, P_{12}] = 0$ & $[H_{\mathrm{ho}}\,, P_{23}] = 0,$ ✗

P_{23} matrix diagonalisation.



Results: Quantitative Considerations

Approximated mass values for 3-body model:

Ground state / $L = 8$

bbb:	14.5 GeV	16.5 GeV	bbn:	10 GeV	12 GeV
nnb:	6 GeV	8 GeV	nnn:	1 GeV	3.5 GeV

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Outlooks:

- New procedure to compute distances,
- Convolution for different particles,
- Diquark in a superposition of states,
- Application to more complex systems.

Main References:

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Potential:

- R. K. Bhaduri, L. E. Cohler, and Y. Nogami, Il Nuovo Cimento A **65**, 376 (1981),
- L. Theußl, R. F. Wagenbrunn, B. Desplanques, and W. Plessas, Eur. Phys. J. A **12**, 91 (2001).

Convolution:

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Colour density:

- V. U. Nazarov, Phys. Rev. B **87**, 165125 (2013).

Lagrange Mesh:

- D. Baye, Phys. Rep. **565**, 1 (2015).

OBE:

- C. Chevalier and S. Youcef Khodja, Few-Body Syst. **65**, 86 (2024).

The Quark Interactions

Types of interactions studied:

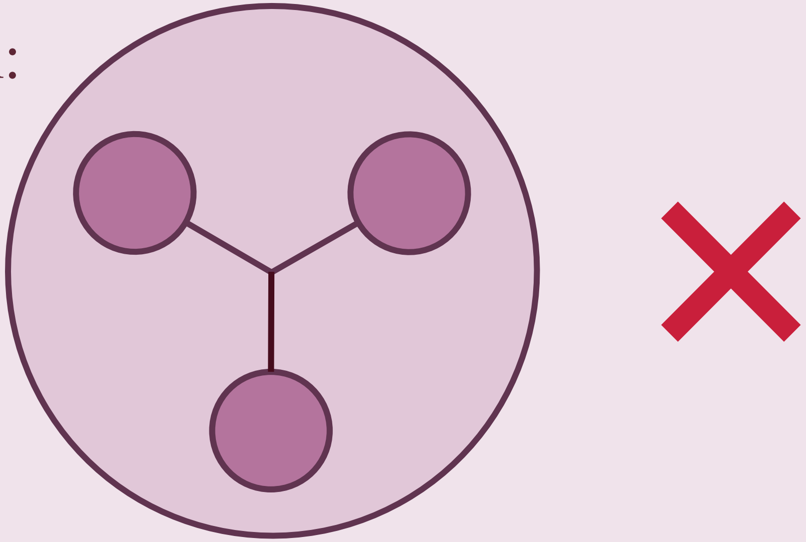
- Central-colour short range,
- Central-colour long range (confinement),
- Spin-colour.

Two-body interactions.

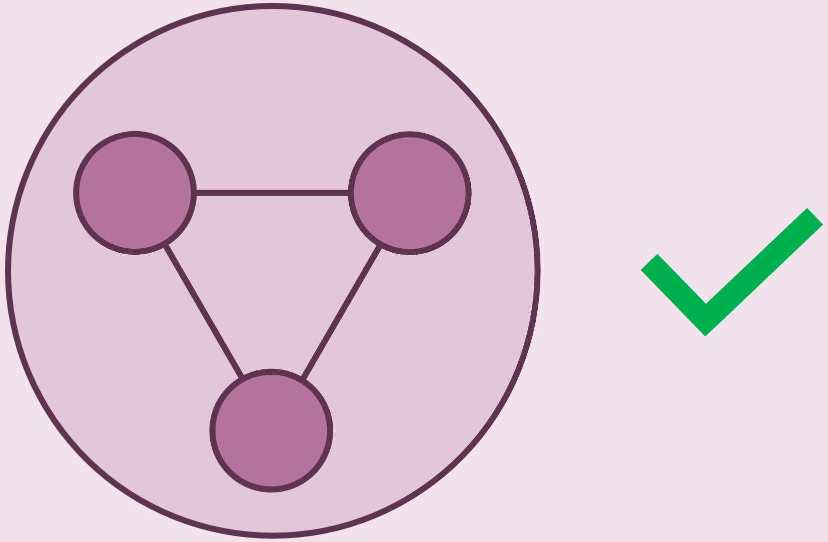
Example for confinement:

- Y-junction model,
- Delta model.

Y-junction:



Delta:

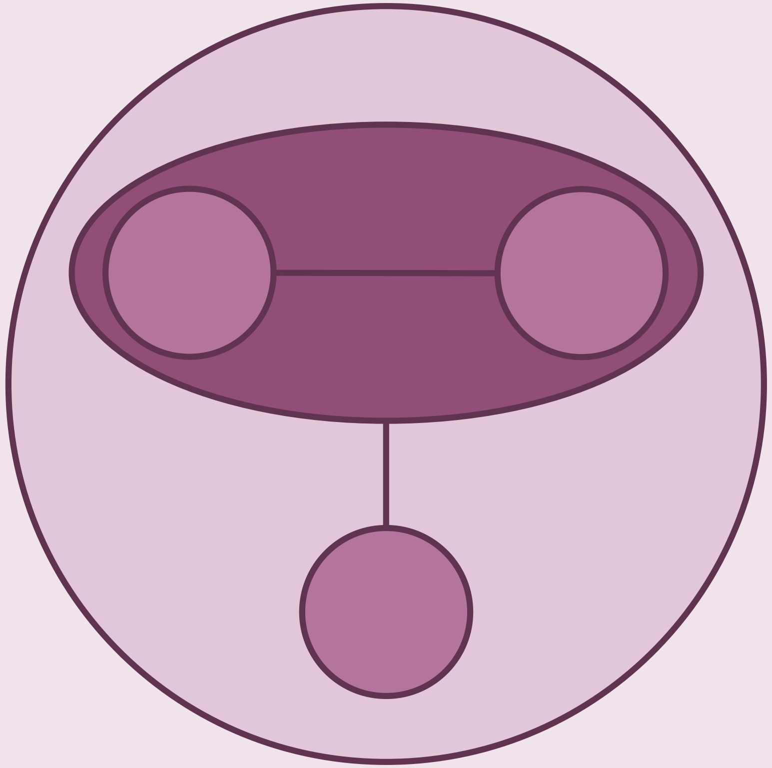


The Quark-Diquark Model

Concept:
Split the three-body system
in two two-body systems
→ Approximative model

1. A two-quark system
   = The diquark

2. A quark-diquark system



The Quark-Quark Potential

$$V_{ij}(r_{ij}) = Cr_{ij} - \frac{2b}{3r_{ij}} + V_0 + \frac{\alpha_s}{9m_i m_j}\Lambda^2 \frac{\mathrm{e}^{-\Lambda r_{ij}}}{r_{ij}} 4\,\boldsymbol{s}_i \cdot \boldsymbol{s}_j,$$

Cornell's potential:

$$V^{\text{Cornell}}(r) = Cr - \frac{2b}{3r} + V_0.$$

Linear part: long range central-colour,

Coulombian part: short range central-colour,

Constant: To reproduce experimental data.

Spin potential:

$$V^{\text{Spin}}(r) = \frac{\alpha_s}{9m_i m_j}\Lambda^2 \frac{\mathrm{e}^{-\Lambda r}}{r} 4\,\boldsymbol{s}_i \cdot \boldsymbol{s}_j.$$

Yukawa: short range,

Spin operator:

$$\boldsymbol{s}_i \cdot \boldsymbol{s}_j = \frac{1}{2}\left(\boldsymbol{S}_{ij}^2 - \boldsymbol{s}_i^2 - \boldsymbol{s}_j^2\right),$$

with $\boldsymbol{S}_{ij}^2 = \left(\boldsymbol{s}_i + \boldsymbol{s}_j\right)^2$.

Methods: Lagrange Mesh

For the quark-diquark model

$$H|\psi\rangle = E|\psi\rangle,$$

$H = H_D$ or H_B.

$$|\psi\rangle = \sum_j c_j \, |f_j\rangle, \longleftarrow \langle \boldsymbol{r}|f_j\rangle = \frac{f_j(r/h)}{\sqrt{h}\, r} Y_{lm}(\hat{r})$$

Mesh method:

- Mesh points x_k, $k \in \{1, \ldots, N\}$,

$$L_N(x_k) = 0,$$

L_N: Laguerre polynomial of degree N.

- Set of orthonormal functions $f_j(x)$,

$$f_j(x_k) \propto \delta_{jk},$$

f_j: regularized Lagrange functions.

$$f_i(x) = (-1)^i x_i^{-1/2}\, x(x - x_i)^{-1}\, L_N(x) \exp\left(-\frac{x}{2}\right)$$

Finally, we have to solve:

Gauss approx.
$\langle f_i|f_j\rangle \approx \delta_{ij}$

$$\sum_{j=1}^{N} \left[T_{ij} + V(hx_i)\delta_{ij} - E\delta_{ij}\right] c_j = 0,$$

with $T_{ij} \approx \langle f_i|T|f_j\rangle$ computable.

Methods: Oscillator Basis Expansion

2. Matrix element computation.

$$\langle \Phi_i | H | \Phi_j \rangle = ?$$

$$H_{ij} = \langle \Phi_i | V_{12} | \Phi_j \rangle + \langle \Phi_i | V_{23} | \Phi_j \rangle + \langle \Phi_i | V_{13} | \Phi_j \rangle + \langle \Phi_i | T_1 | \Phi_j \rangle + \langle \Phi_i | T_2 | \Phi_j \rangle + \langle \Phi_i | T_3 | \Phi_j \rangle.$$

$$H = \sum_k T_k + \sum_{k<l} V_{kl} \,,$$

$$T_k(p) = \sqrt{p^2 + m_k^2},$$

$$V_{kl}(r) = Cr - \frac{2b}{3r} + V_0 + \frac{\alpha_s}{9 m_k m_l} \Lambda^2 \frac{e^{-\Lambda r}}{r} 4\, \mathbf{s_k} \cdot \mathbf{s_l}.$$

3. Hamiltonian matrix diagonalisation.

Eigenstate example:

$$\frac{1}{\sqrt{2}} (0,0,0,0,1,1) + \frac{1}{\sqrt{2}} (0,0,0,0,0,0)$$

$$(n_i, l_i, \nu_i, \lambda_i, S_{12_i}, I_{12_i})$$

Methods: Oscillator Basis Expansion

What about the diquark symmetries ?

Type	Isospin (I_{12})	Spin (S_{12})	Space (L_{12})
bb	S (0)	S (1)	S (pair)
	S (0)	A (0)	A (impair)
nn	S (1)	S (1)	S (pair)
	A (0)	S (1)	A (impair)
	S (1)	A (0)	A (impair)
	A (0)	A (0)	S (pair)

Spin factors

$$\boldsymbol{s}_i \cdot \boldsymbol{s}_j = \frac{1}{2}\left(\boldsymbol{S}_{ij}^2 - \boldsymbol{s}_i^2 - \boldsymbol{s}_j^2\right), \qquad \boldsymbol{S}_{ij}^2 = \left(\boldsymbol{s}_i + \boldsymbol{s}_j\right)^2.$$

Diquark:

$$\boldsymbol{s}_1 \cdot \boldsymbol{s}_2 = \frac{1}{2}\left(\boldsymbol{S}_D^2 - \boldsymbol{s}_1^2 - \boldsymbol{s}_2^2\right).$$

S_D	$s_1 \cdot s_2$
1	$\frac{1}{4}$
0	$-\frac{3}{4}$

Quark-diquark:

$$\boldsymbol{S}_D \cdot \boldsymbol{s}_3 = \frac{1}{2}\left(\boldsymbol{S}_B^2 - \boldsymbol{S}_D^2 - \boldsymbol{s}_3^2\right).$$

S_B	S_D	$S_D \cdot s_3$
$\frac{3}{2}$	1	$\frac{1}{2}$
$\frac{1}{2}$	1	-1
$\frac{1}{2}$	0	0

Results: *nnb*

$(n_i, l_i, \nu_i, \lambda_i, S_{12_i}, I_{12_i})$

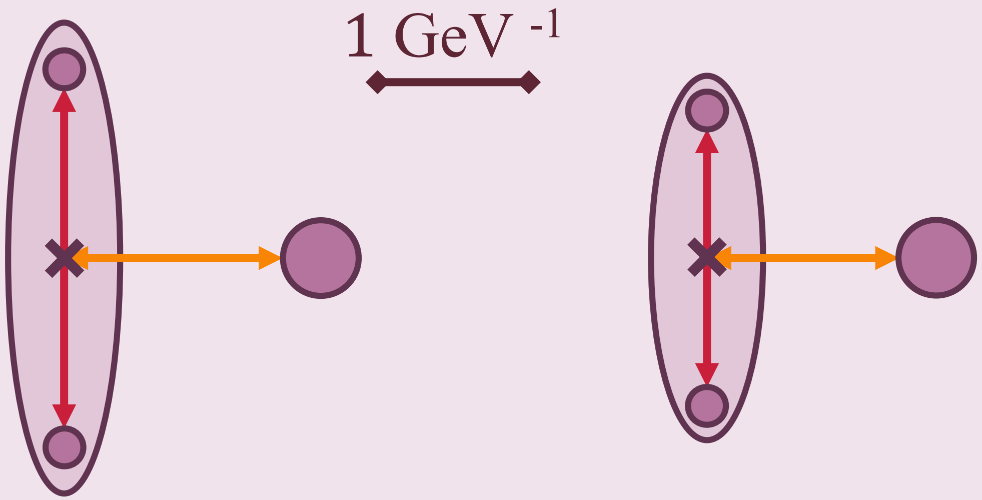
Highlight of intra-diquark spin interactions:

- Fundamental states with $S = {}^1/_2$,
- Total isospin $I = I_{12} = 1$ or 0.
- Symmetry constraints:

  Space (L): S (0) & Colour: A.

Isospin (I_{12})	**Spin (S_{12})**
S (1)	S (1)
A (0)	A (0)

$$V^{spin} \propto \boldsymbol{s_1} \cdot \boldsymbol{s_2} = \begin{cases} \frac{1}{4} \text{ if } S_{12} = 1, \\ -\frac{3}{4} \text{ if } S_{12} = 0. \end{cases}$$



State (0,0,0,0,1,1)

≈ 0.7%

State (0,0,0,0,0,0)

≈ 2.4%

Results: Summary

Three Identical particles:
- Equilateral triangle structure,
- $L = 8$ complex states.

bbb:
- Globally more compact,
- Good correspondence,
- Minimal spin impact.

nnn:
- Extended,
- Not very favourable,
- More spin impact.

bbn:
- Compact diquark for fundamental,
- Very extended diquark for $L = 8$,
- Good correspondence for both.

nnb:
- Important intra-diquark spin interactions,
- Non-compact diquark for fundamental,
- Very compact diquark for $L = 8$,
- Better results for less compact diquark.