TOWARDS A DESCRIPTION OF DIBARYONS FROM FUNCTIONAL METHODS

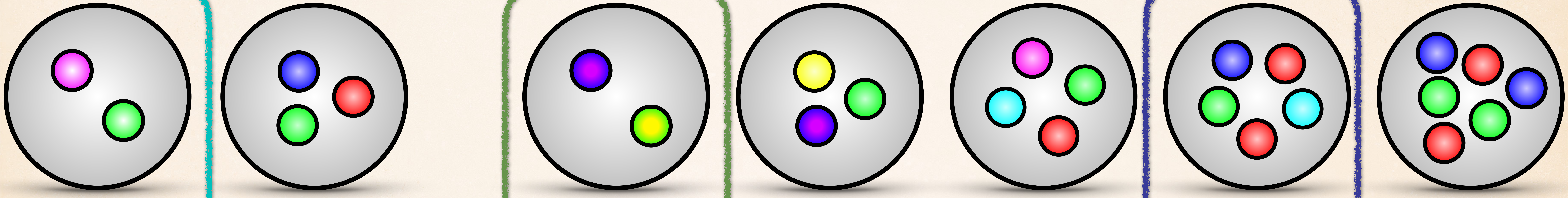
Joshua Hoffer

9. September 2026

NSTAR 2026, Seville, Spain

HADRONS

Conventional | Exotic



Meson | Baryon | Glueball | Hybrid | Tetra-quarks | Penta-quarks | Hexaquark

see talk Angel Miramontes

see talk Markus Huber

see talk Raúl Torres

Overview see talk Gernot Eichmann

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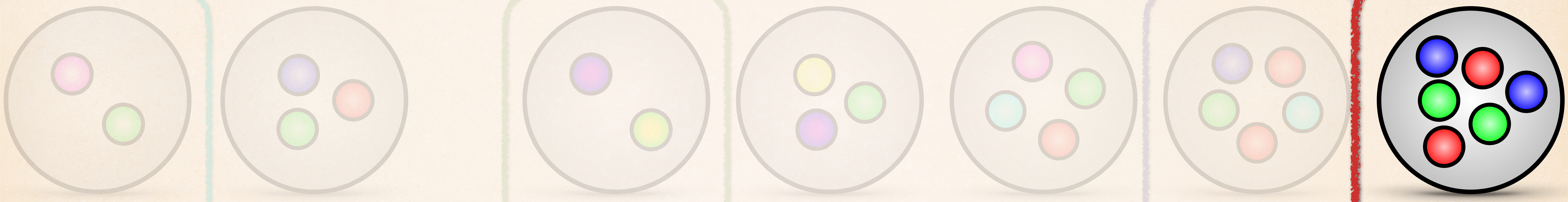

Meson

Baryon

Glueball

Hybrid

Tetra-quarks

Penta-quarks

Hexaquark

see talk Markus Huber

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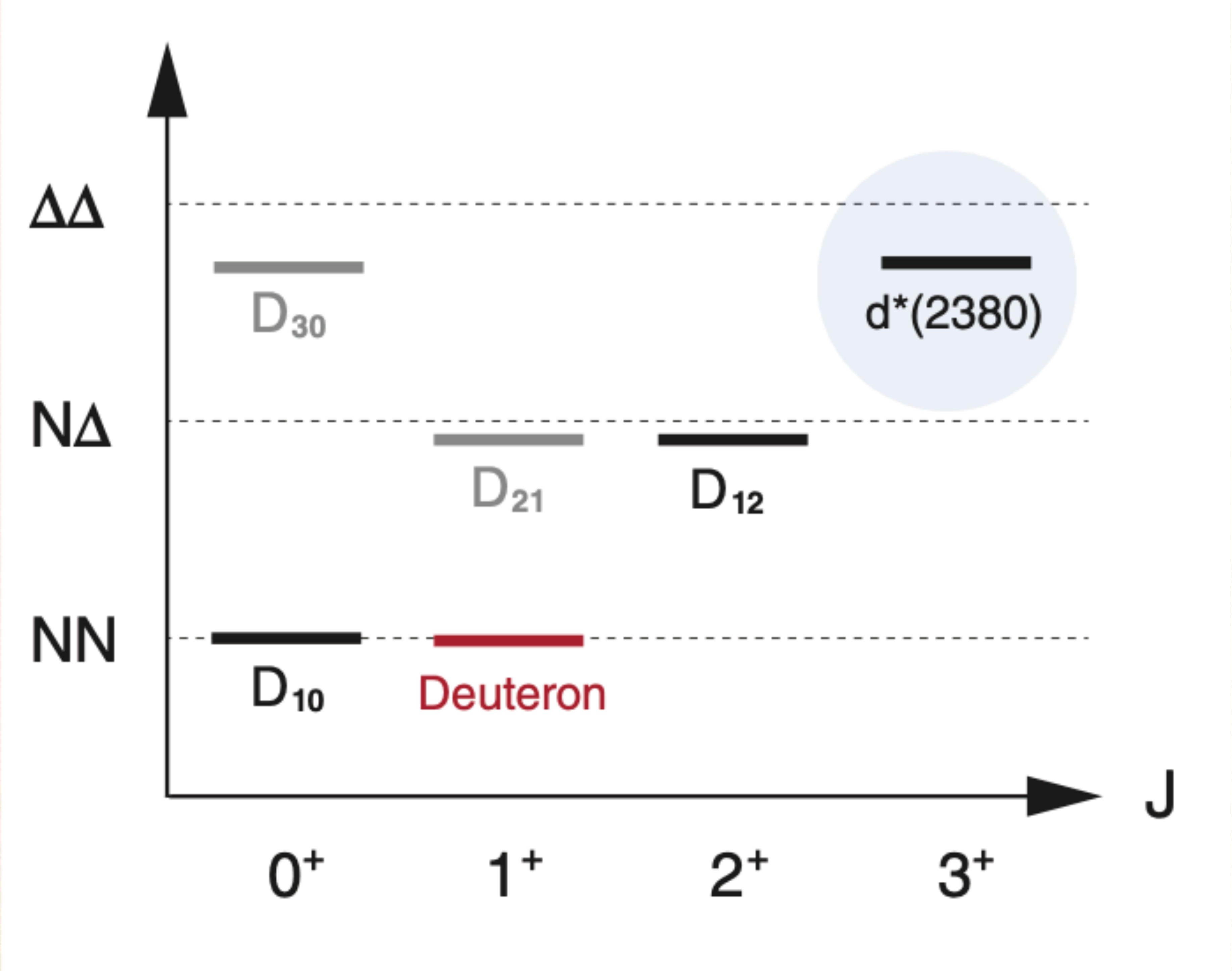
Overview see talk Gernot Eichmann

HEXAQUARKS

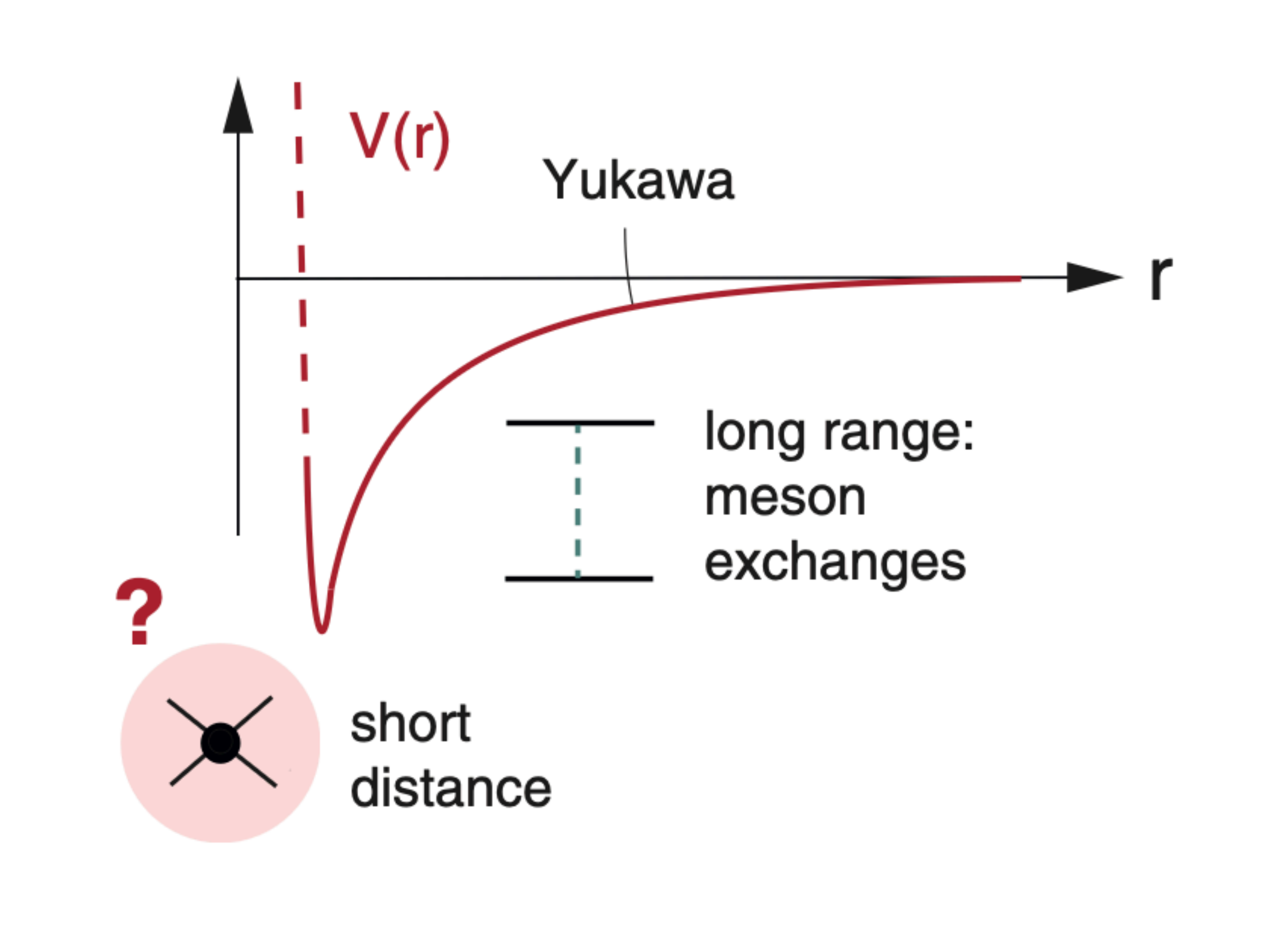
- Many interesting particles:
  - Deuteron and its cousins, Diproton (Helium 2), Neutronium
  - H-dibaryon, all-charm dibaryons
- Study spectrum from quark and gluon level



Dyson, Xuong, PRL 13 (1964)
Adlarson et al., PRL 106 (2011), PRL 112 (2014),
Bashkanov, Brodsky, Clement, PLB 727 (2013),
Gal, Garcilazo, PRL 111(2013), . . .

HEXAQUARKS

- Many interesting particles:
  - Deuteron and its cousins, Diproton (Helium 2), Neutronium
  - H-dibaryon, all-charm dibaryons
- Study spectrum from quark and gluon level
- Investigate Microscopic origins of short-range nuclear force

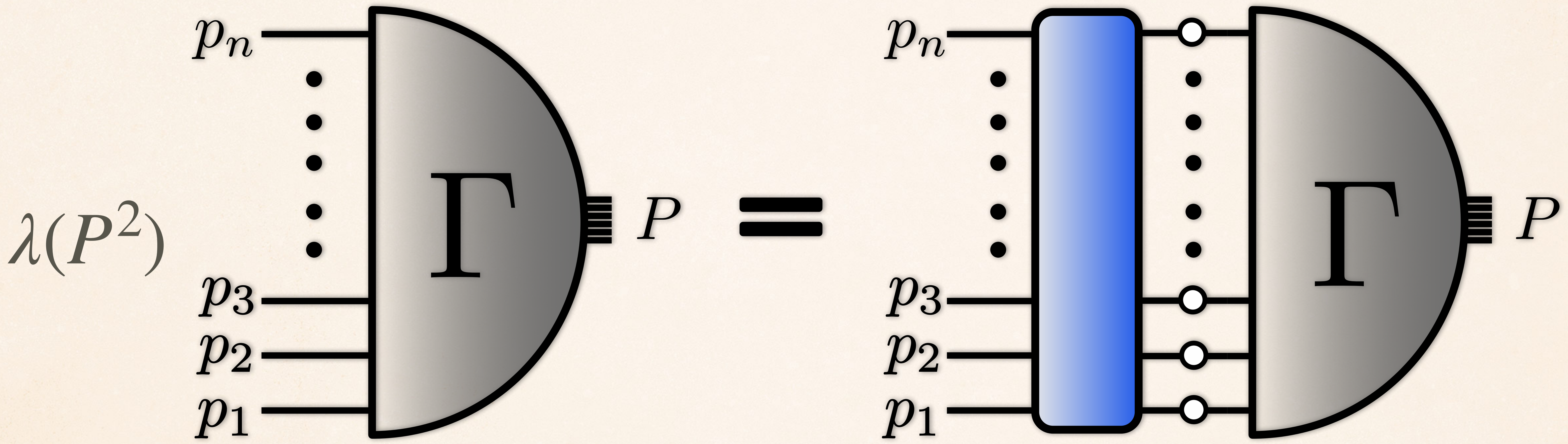


FUNCTIONAL FRAMEWORK

- Fully relativistic
- Non-perturbative
- Exact QCD n-point functions (need to be truncated later)
- Flexible framework
  - Easily adjustable regarding different quark flavours and tensors
  - Evaluate importance of different diagrams for the equation

FUNCTIONAL FRAMEWORK

- Properties of Hadrons from Hadronic bound state equations



$$\boxed{\lambda(P^2)\,\Gamma = KG\,\Gamma}$$

with $\lambda(P^2 = -M^2) = 1$

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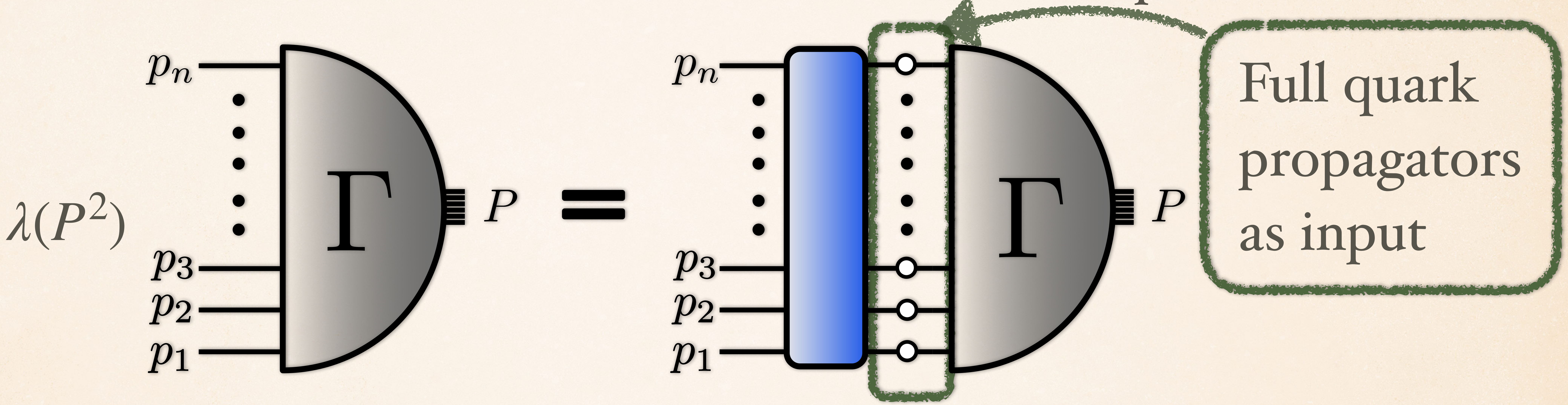

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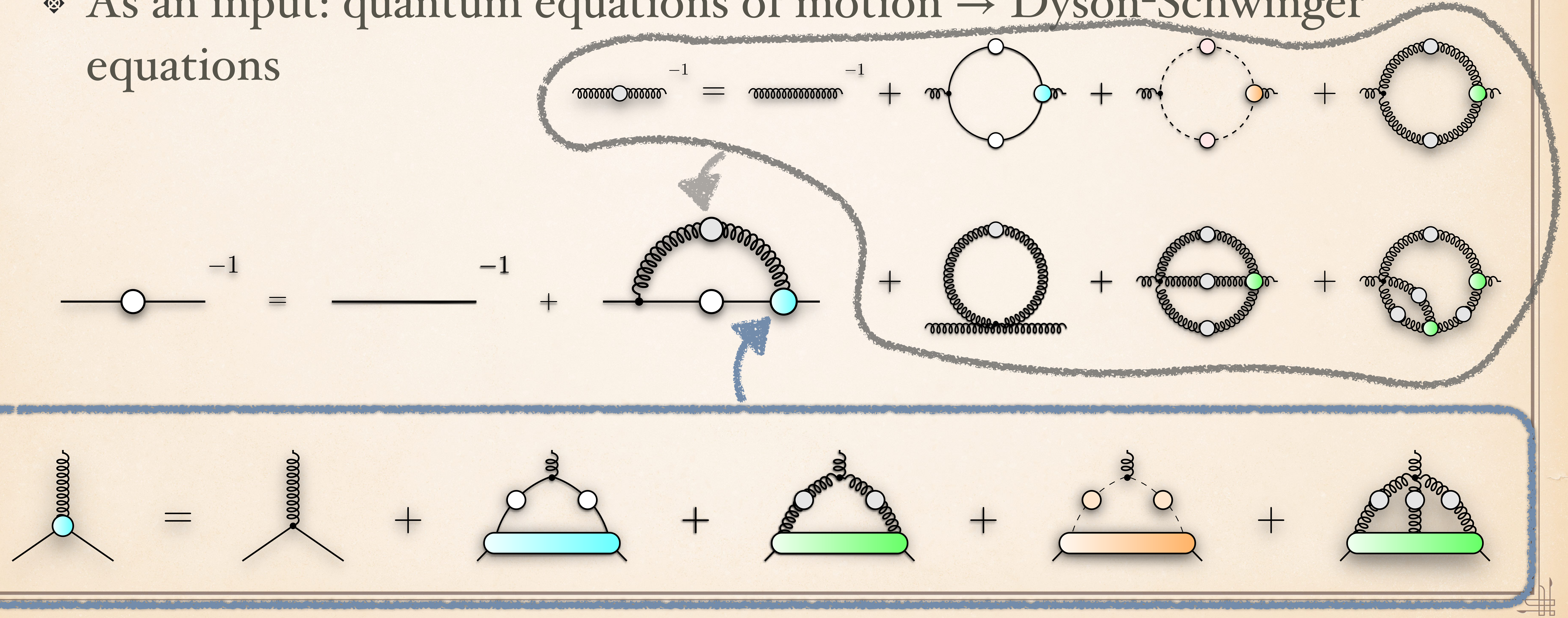
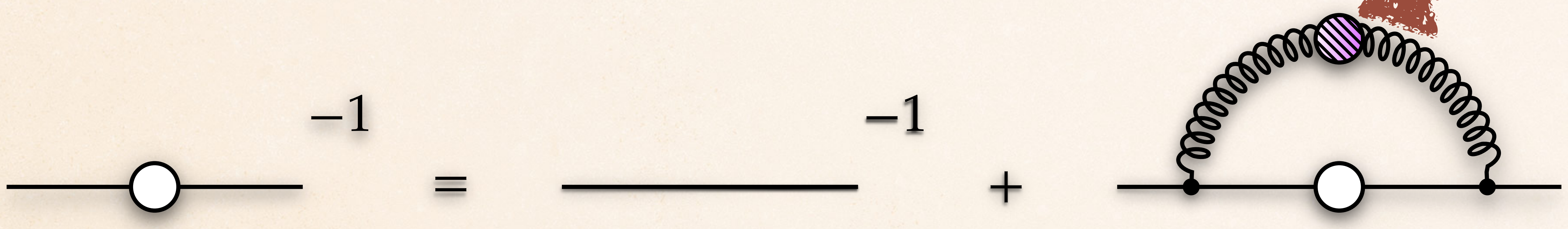
FUNCTIONAL FRAMEWORK

- As an input: quantum equations of motion → Dyson-Schwinger equations

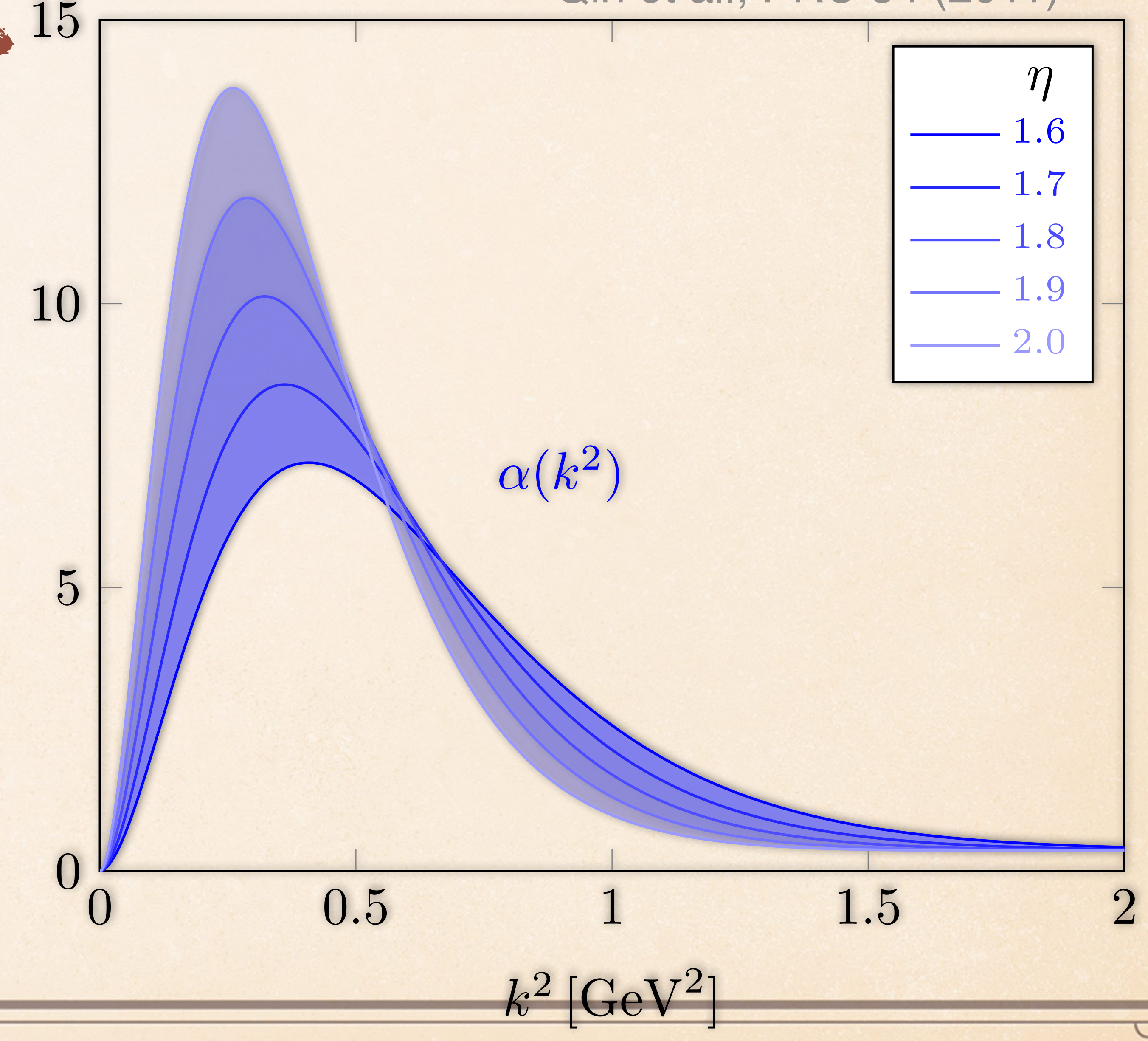


FUNCTIONAL FRAMEWORK

- As an input: quantum equations of motion → Dyson-Schwinger equations
  - Quark-Gluon vertex + Gluon DSE → effective gluon exchange

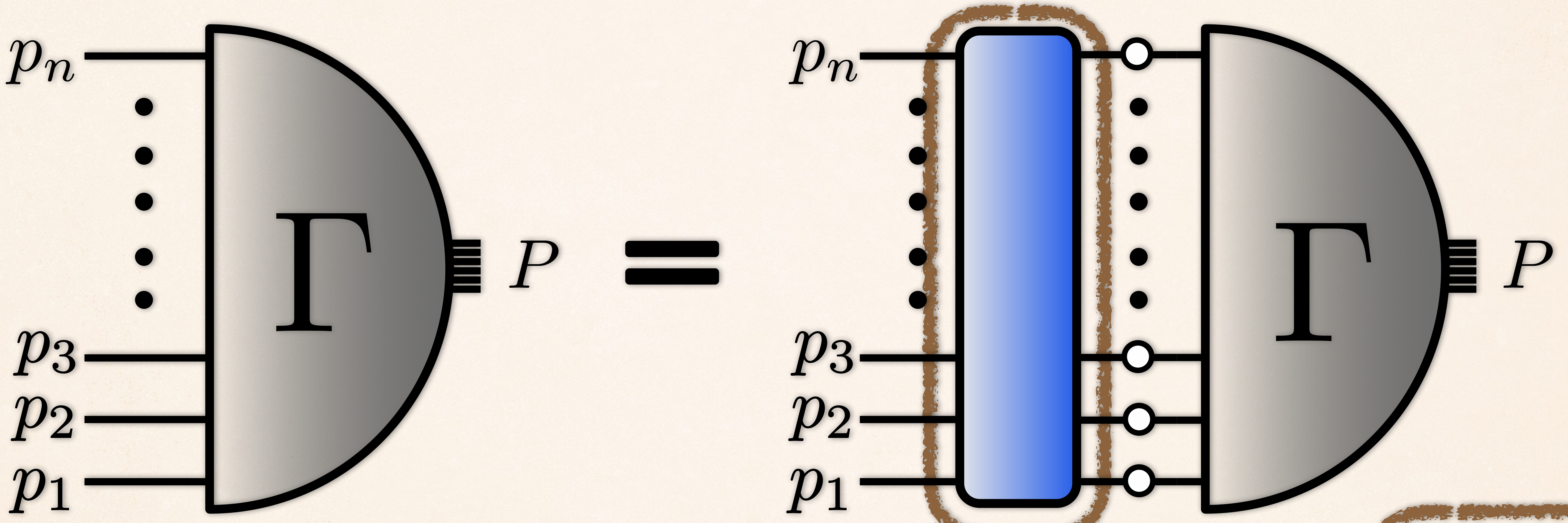


Maris, Tandy, PRC 60 (1999)
Qin et al., PRC 84 (2011)

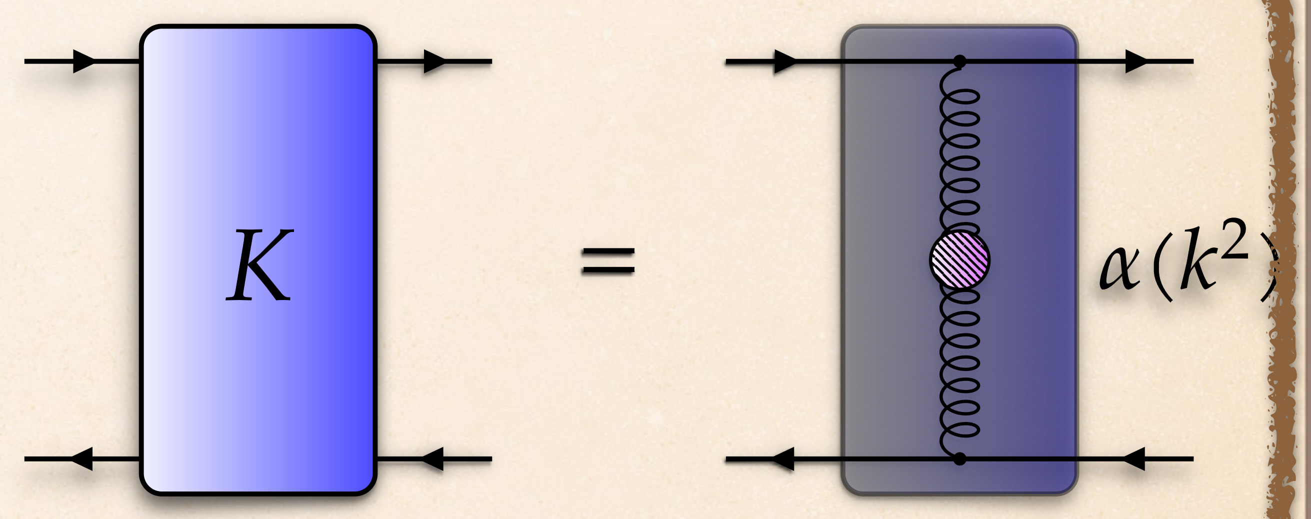


FUNCTIONAL FRAMEWORK

- Properties of Hadrons from Hadronic bound state equations

$\lambda(P^2)$ 

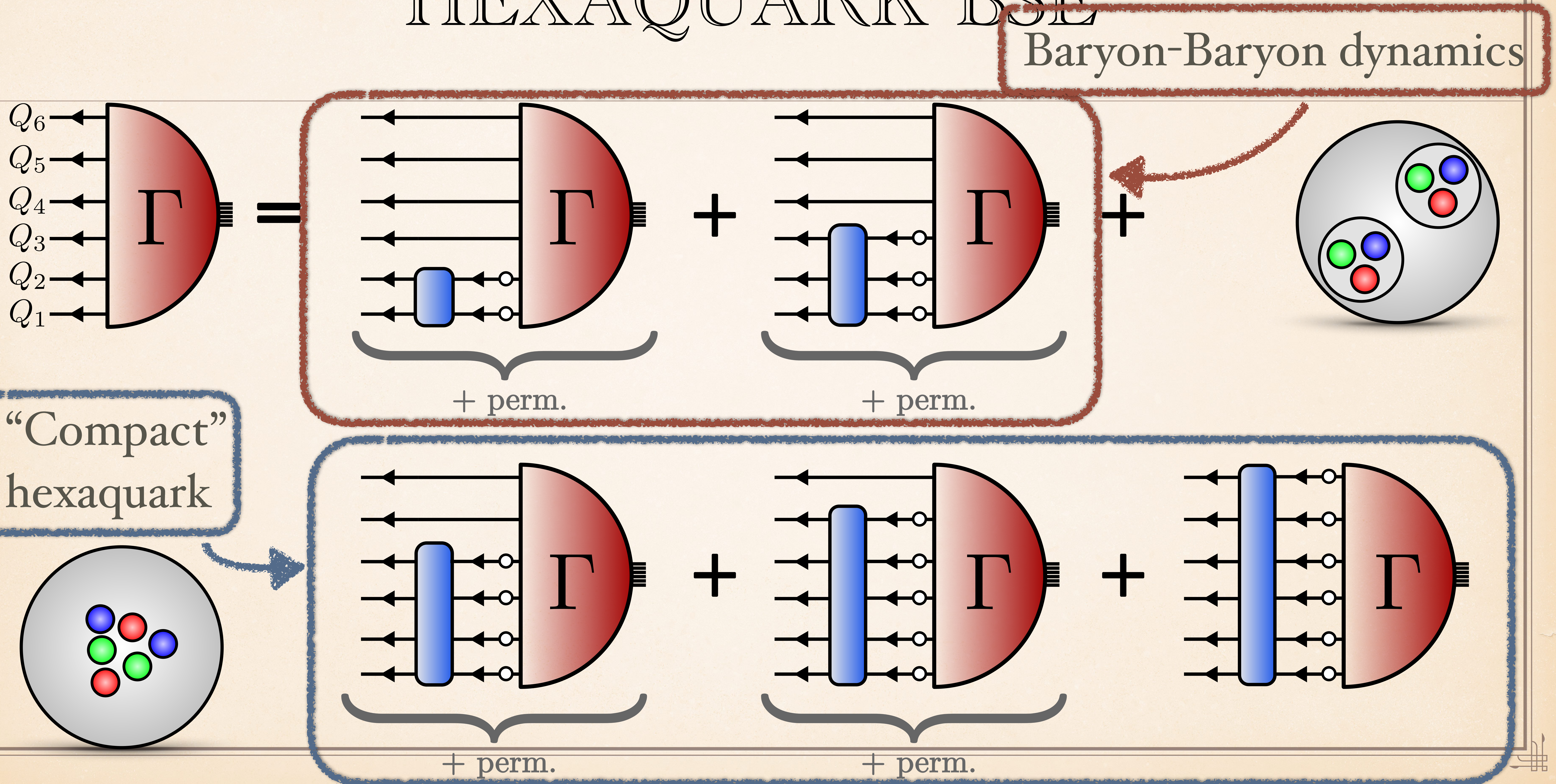
Two-body interactions



$$\boxed{\lambda(P^2)\,\Gamma = KG\Gamma}$$

with $\lambda(P^2 = -M^2) = 1$

HEXAQUARK BSE



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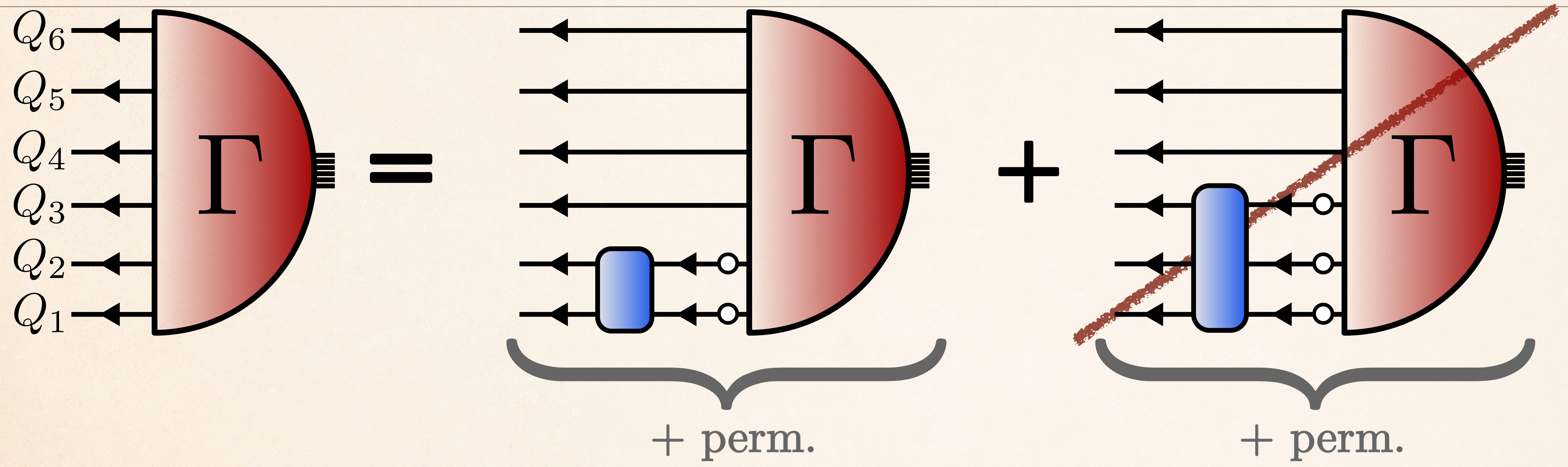

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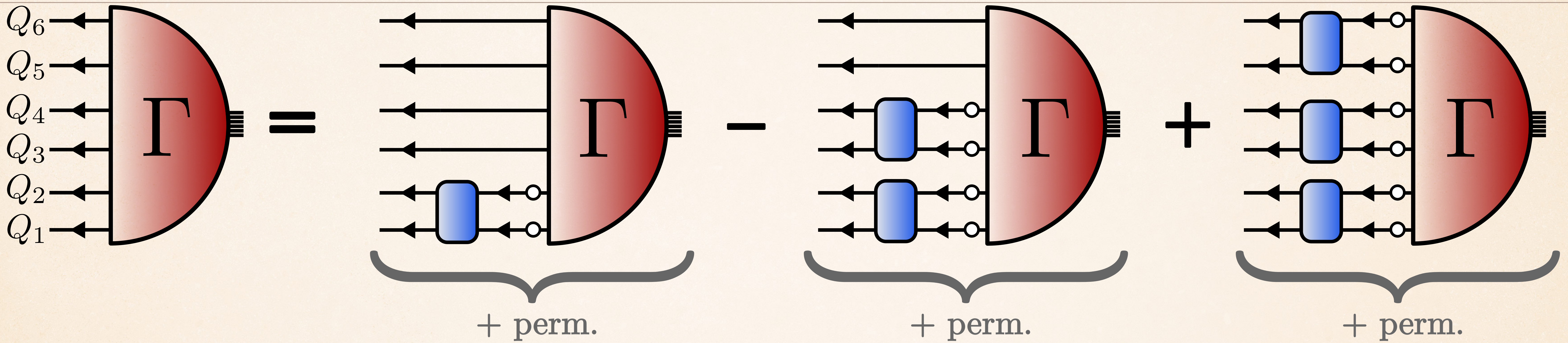

Huang, Weldon, PRD 11 (1975)
Kvinikhidze, Khvedelidze, Theor.Math.Phys. 90 (1992)

10 Baryon-Baryon combinations:

(123)(456)	(125)(346)	(134)(256)	(136)(245)	(146)(235)
(124)(356)	(126)(345)	(135)(246)	(145)(236)	(156)(234)

SIX-QUARK BSA

$$\Gamma^{(\mu)}(k,q,p,r,l,P) = \sum_{i=1}^{N} f_i(\Omega)\,\tau_i^{(\mu)}(k,q,p,r,l,P) \otimes \Gamma_\mathrm{C} \otimes \Gamma_\mathrm{F}$$

- Overall BSA needs to be antisymmetric under quark exchange $\rightarrow$ Express in terms of irreducible representations of S_6 $\rightarrow$ antisinglet $\bar{\mathbf{1}}$

PERMUTATION GROUP S_6

- 720 elements: f_{123456} , ...
- Transposition T: $1 \leftrightarrow 2$
- Cyclic permutation C:
  $1 \to 2 \to \ldots \to 6 \to 1$
- Generate group $S_6 = \langle T, C \rangle$

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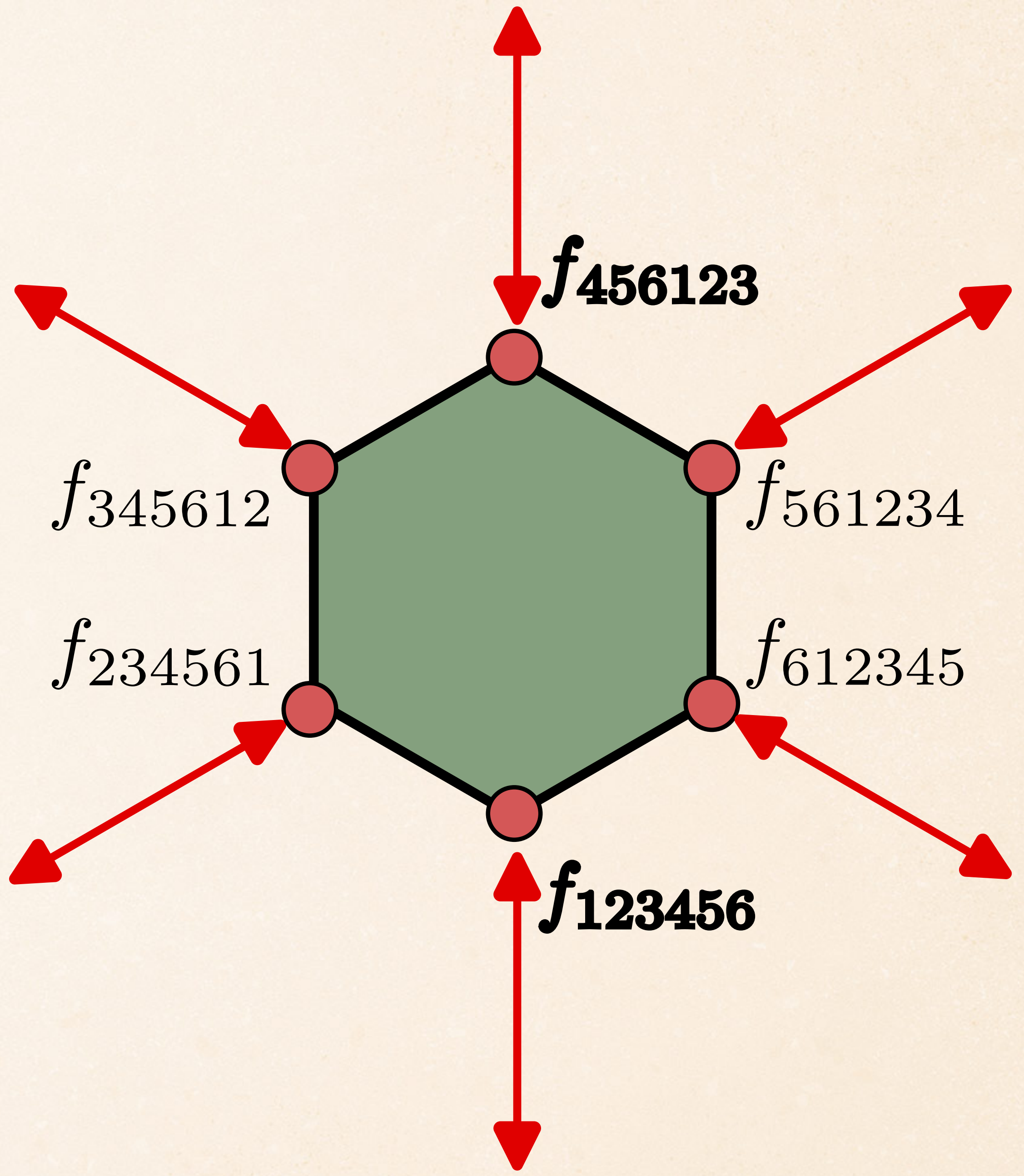
- Generate group $S_6 = \langle T, C \rangle$
- Use C^6 →**120 Hexagons** connected by T



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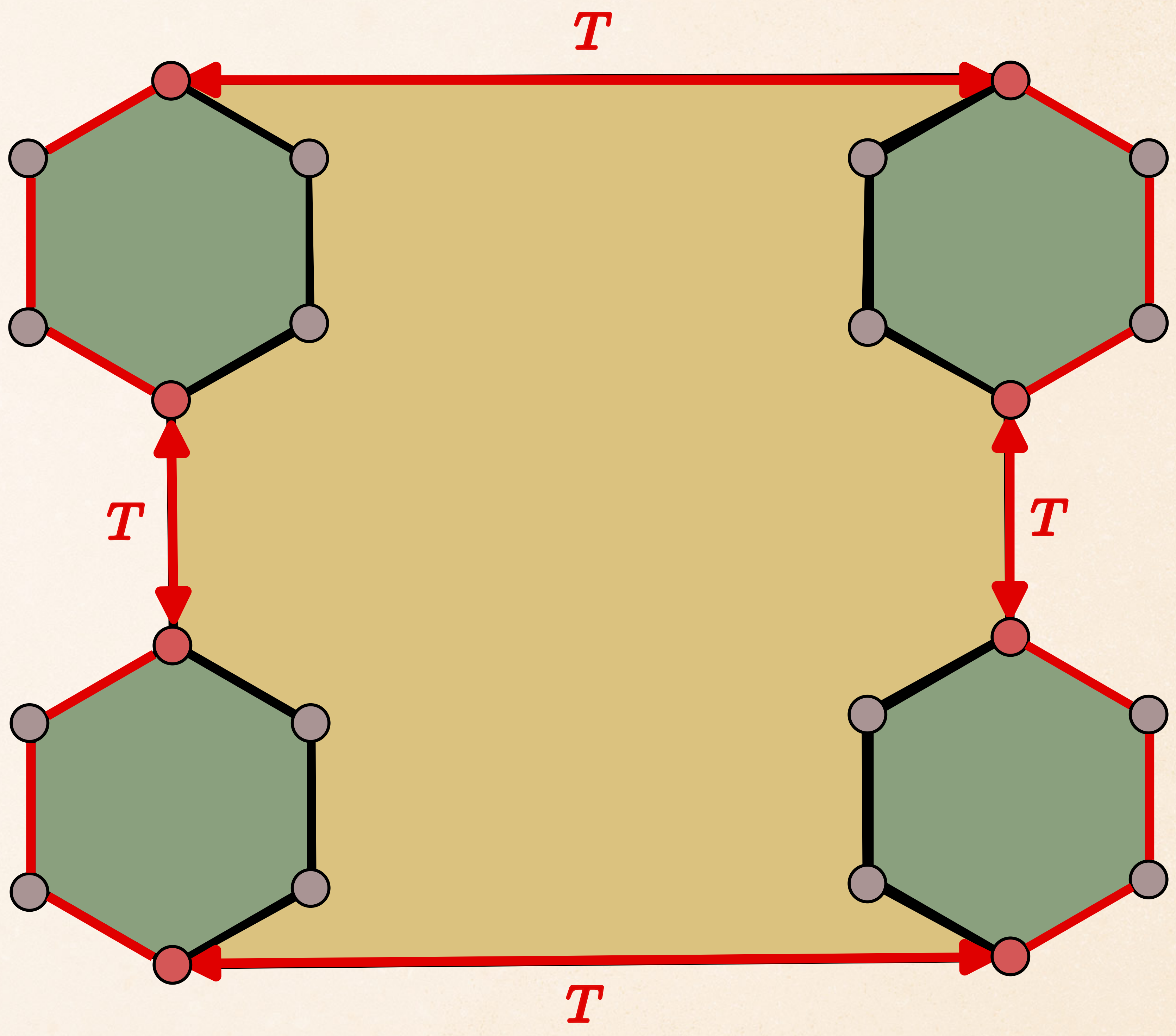
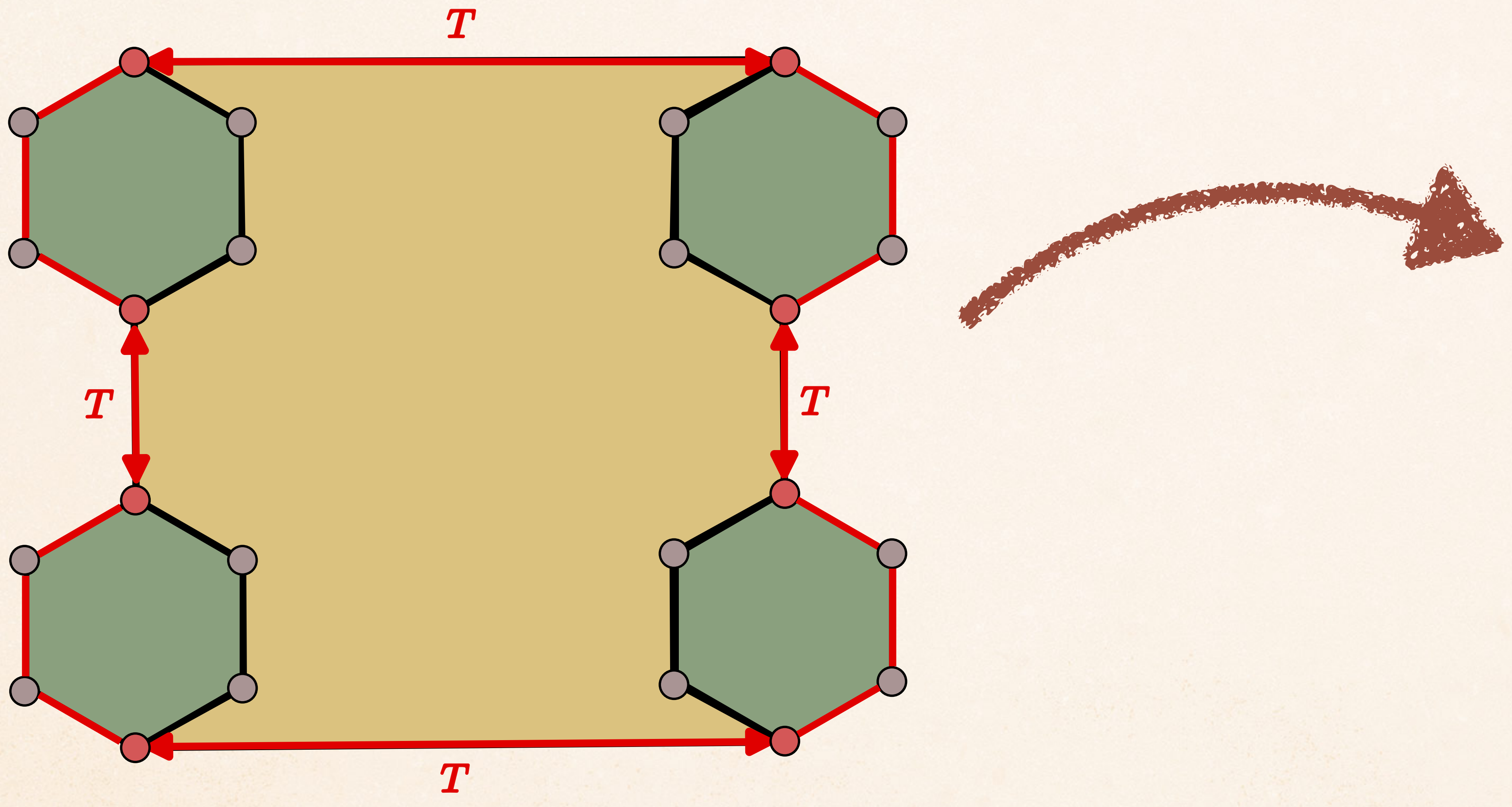
- Transposition T: $1 \leftrightarrow 2$
- Cyclic permutation C:
  $1 \to 2 \to \ldots \to 6 \to 1$
- Generate group $S_6 = \langle T, C \rangle$
- 120 Hexagons connected by T
- Repeat $C^3\, T$
  → **90 "plaquettes" with 8 elements**

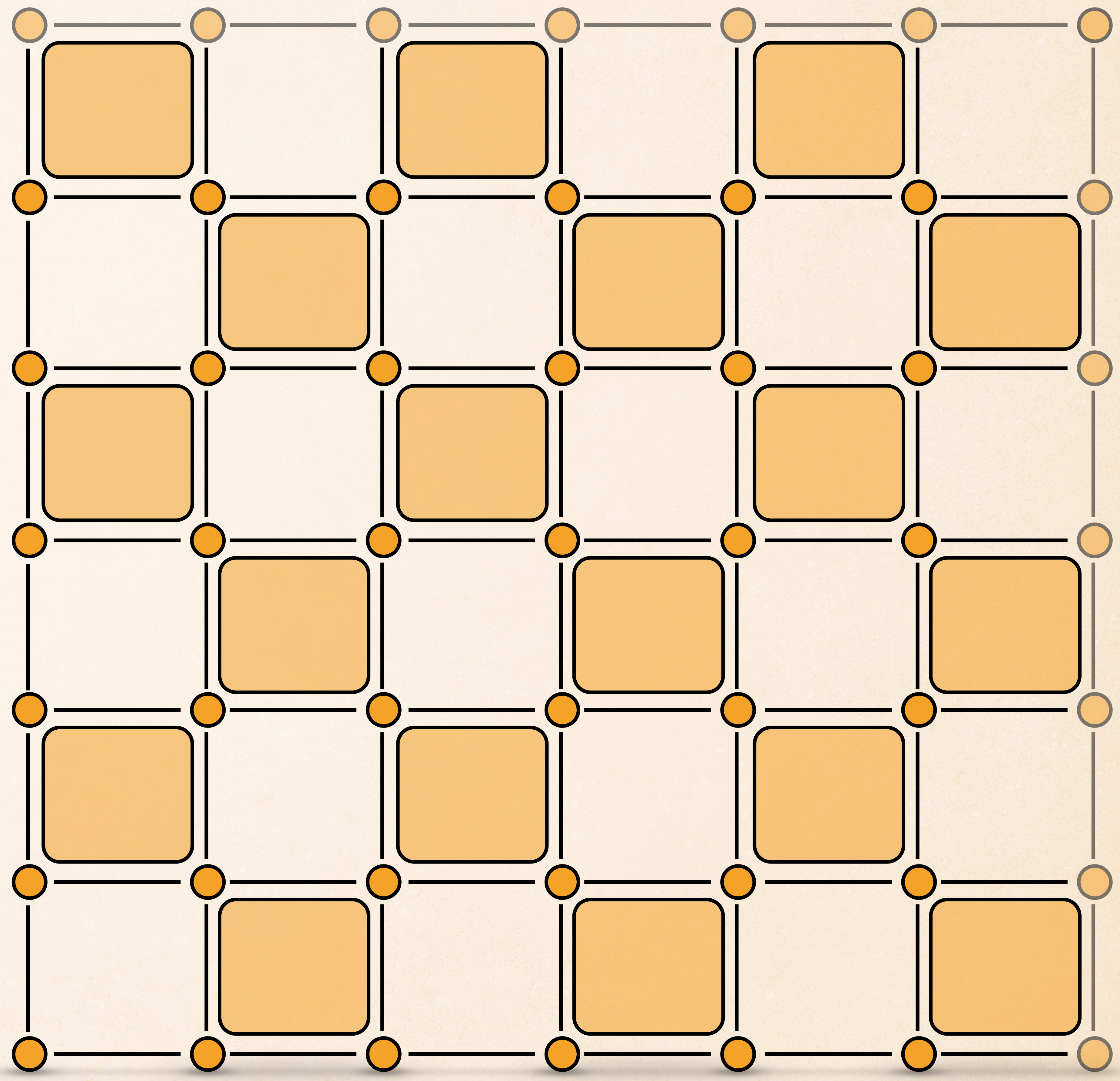


Eichmann, JH, in preparation

CHESSBOARD PLANES FOR S_6

- Chessboard Planes
  - 18 plaquettes $\rightarrow$ 144 elements





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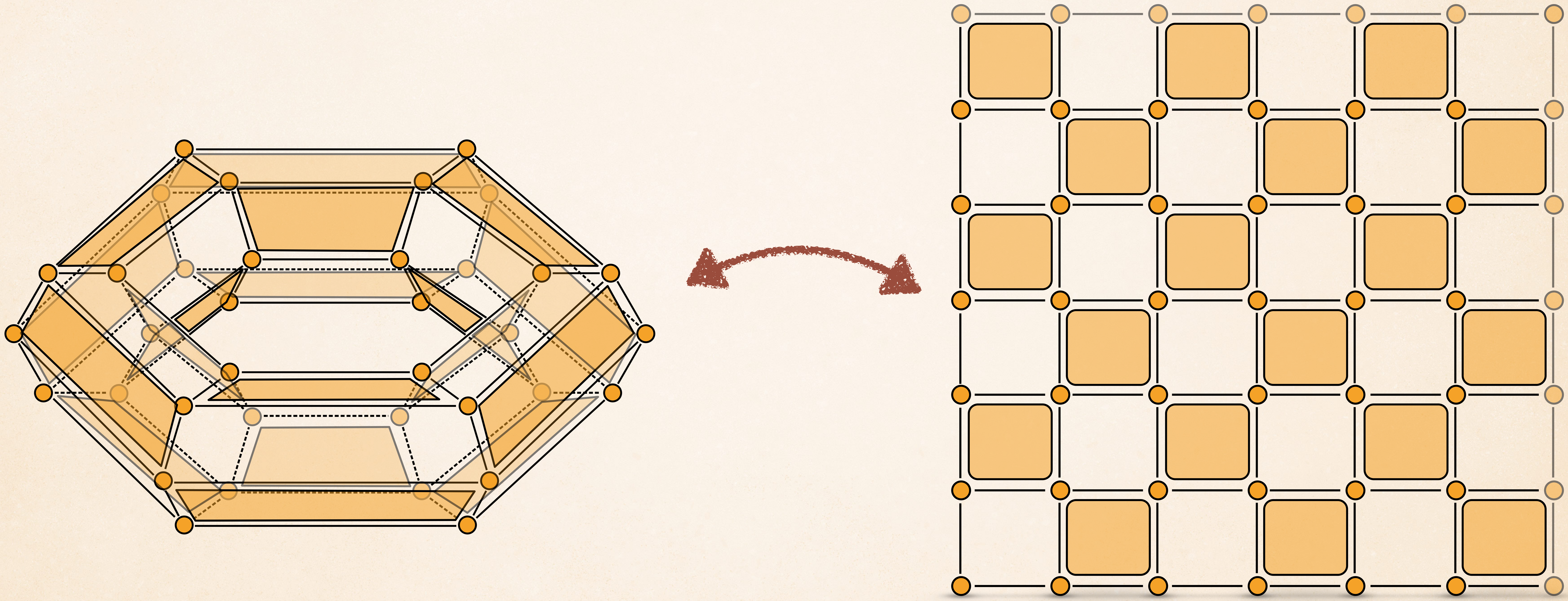

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CHESSBOARD PLANES FOR S_6

- 10 unique Chessboard Planes with 18 “plaquettes”
- Pick 9 “plaquettes” per Chessboard Plane → get all 90 once

1

	4		58		77
24		72		12	
	6		60		74
21		69		9	
	1		54		82
19		67		7	

2

	79		51		12
15		3		63	
	80		50		11
13		1		61	
	81		49		10
14		2		62	

3

	39		63		40
90		60		59	
	33		70		34
81		42		41	
	18		85		19
84		48		47	

4

	73		7		59
11		23		71	
	78		5		57
6		18		66	
	83		2		53
8		20		68	

5

83		46		44	
	16		86		21
80		40		38	
	31		71		36
89		58		56	
	37		62		42

6

	82		45		43
20		17		87	
	79		39		37
35		32		72	
	88		57		55
41		38		61	

7

8		55		76	
	70		10		22
9		56		75	
	65		5		17
3		52		84	
	64		4		16

8

	27		65		48
87		54		51	
	25		68		46
76		34		28	
	13		88		24
78		36		30	

9

47		28		64	
	86		53		50
45		26		69	
	75		33		27
23		14		89	
	77		35		29

10

44		30		66	
	85		52		49
43		29		67	
	73		31		25
22		15		90	
	74		32		26

Eichmann, JH, in preparation

CHESSBOARD PLANES FOR S_6

- 10 unique Chessboard Planes with 18 “plaquettes”
- Pick 9 “plaquettes” per Chessboard Plane → get all 90 once
- Sum elements to get explicit expressions of irreducible representations of S_6

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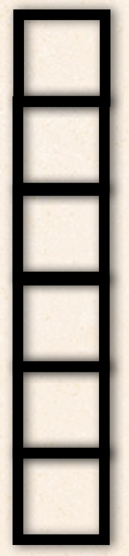
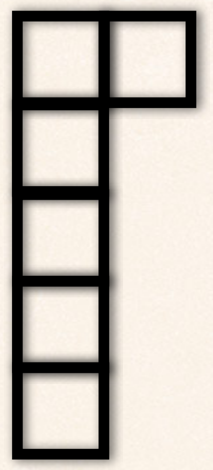
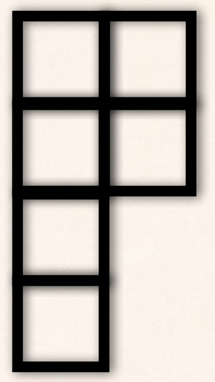
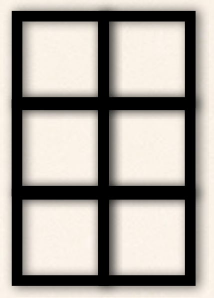
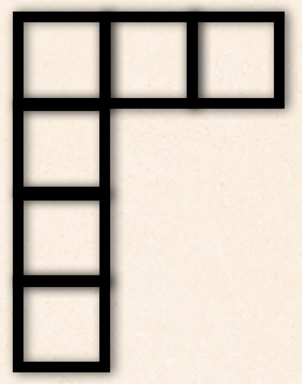
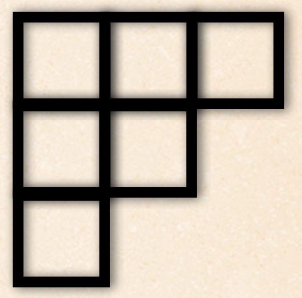
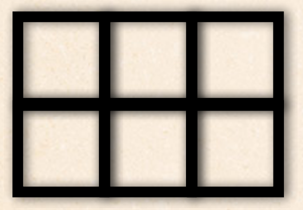
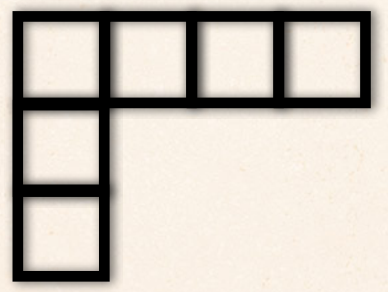
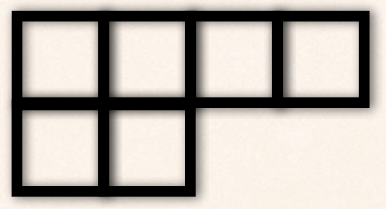
IRREDUCIBLE REPRESENTATIONS OF S_6

					
$\bar{\mathbf{1}}$	$\bar{\mathbf{5}}$	$\bar{\mathbf{9}}$	$\bar{\mathbf{5}}'$	$\overline{\mathbf{10}}$	$\mathbf{16}$
$\mathscr{A}$	$\mathscr{V}^-$	$\mathscr{N}^-$	$\mathscr{Q}^-$	$\mathscr{D}^-$	$\mathscr{X}$

				
$\mathbf{5}'$	$\mathbf{10}$	$\mathbf{9}$	$\mathbf{5}$	$\mathbf{1}$
$\mathscr{Q}^+$	$\mathscr{D}^+$	$\mathscr{N}^+$	$\mathscr{V}^+$	$\mathscr{S}$

SIX-QUARK BSA

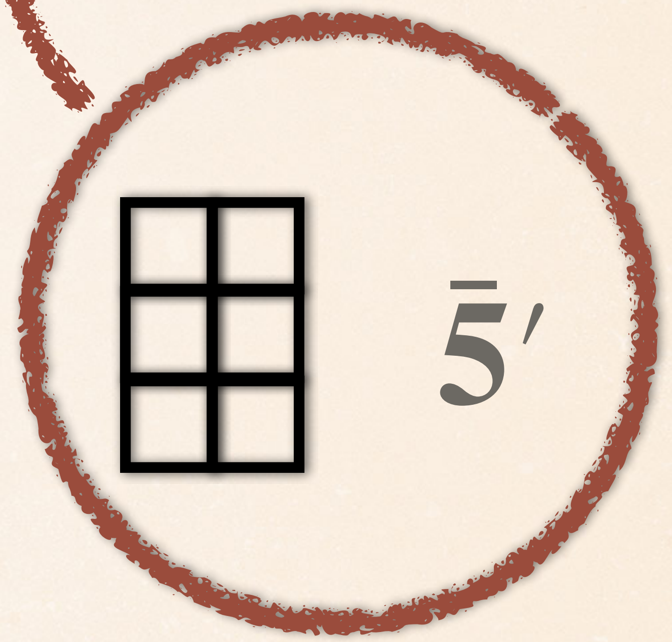
$$\Gamma^{(\mu)}(k,q,p,r,l,P) = \sum_{i=1}^{N} f_i(\Omega)\, \tau_i^{(\mu)}(k,q,p,r,l,P) \otimes \Gamma_\mathrm{C} \otimes \Gamma_\mathrm{F}$$

- Overall BSA needs to be antisymmetric under quark exchange $\rightarrow$ Express in terms of irreducible representations of S_6 $\rightarrow$ antisinglet $\bar{\mathbf{1}}$
- $\mathbf{M}_\mathrm{D} \otimes \mathbf{M}_\mathrm{C} \otimes \mathbf{M}_\mathrm{F} = \bar{\mathbf{1}}$

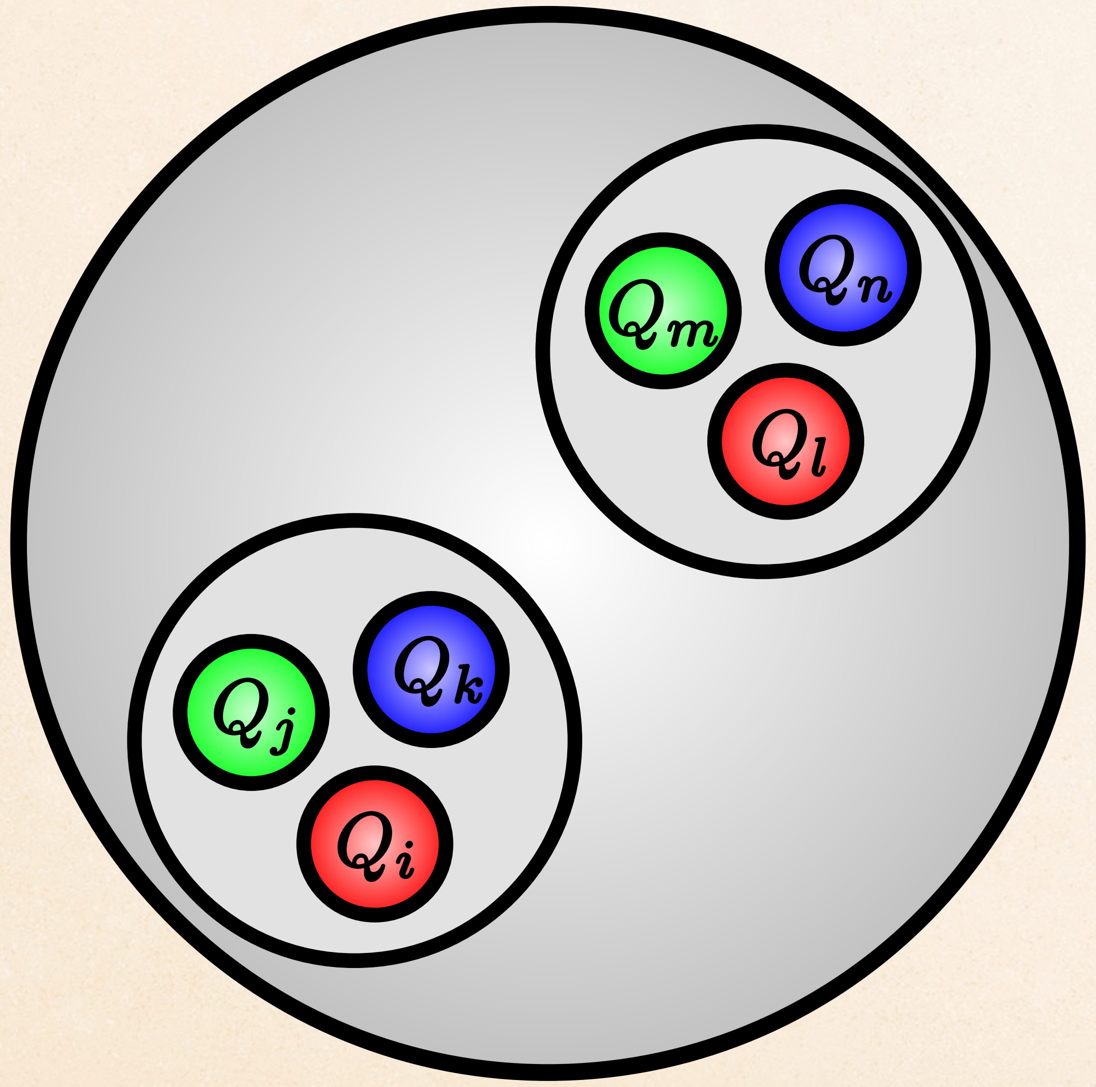
Eichmann, JH, in preparation

SIX-QUARK BSA

$$\Gamma^{(\mu)}(k,q,p,r,l,P) = \sum_{i=1}^{N} f_i(\Omega)\,\tau_i^{(\mu)}(k,q,p,r,l,P) \otimes \boxed{\Gamma_{\mathrm{C}}} \otimes \Gamma_{\mathrm{F}}$$



$$\Gamma_{\mathrm{C}} = \varepsilon^{A_i A_j A_k}\,\varepsilon^{A_l A_m A_n}$$



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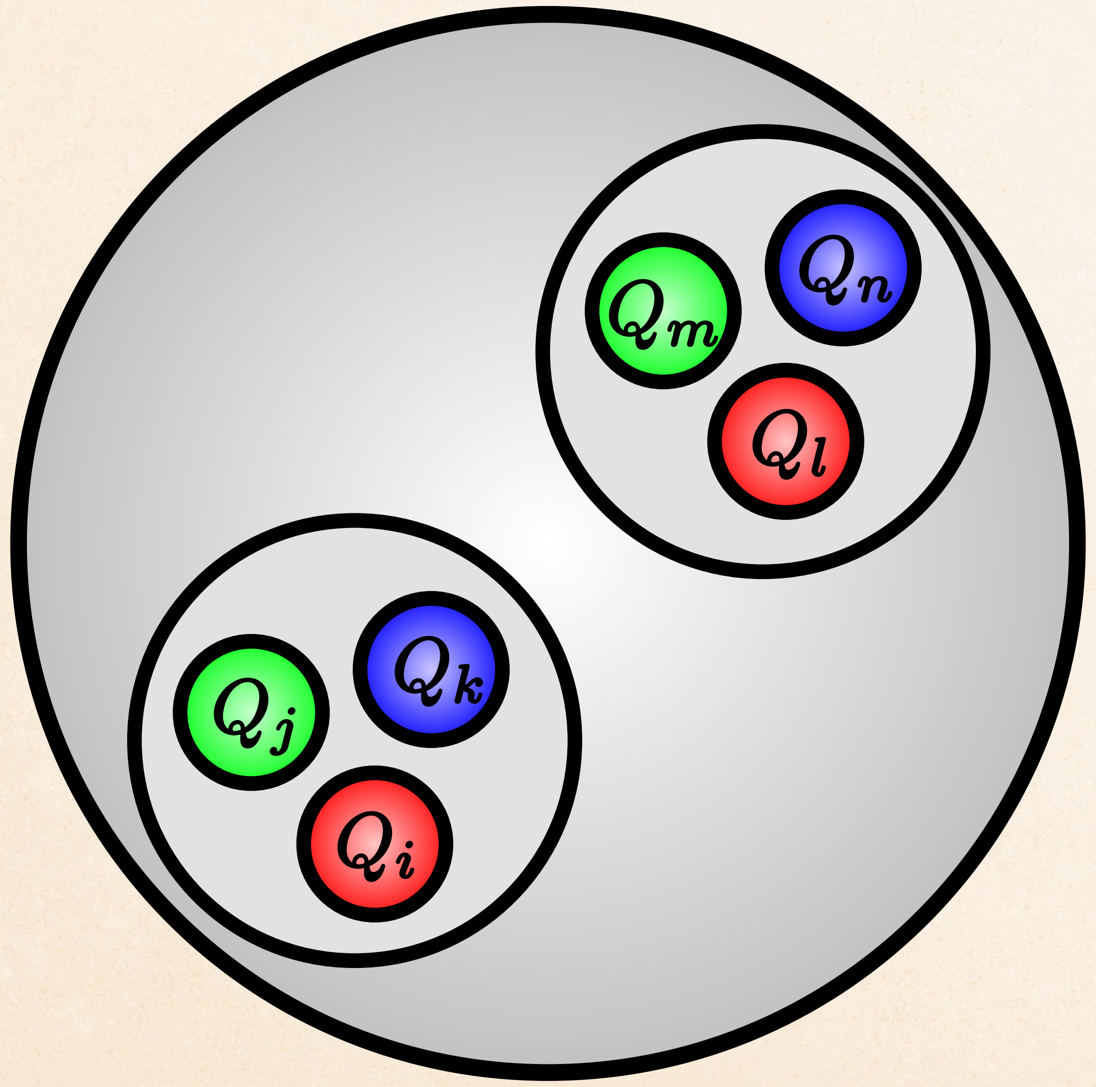
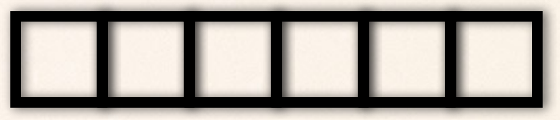
$$Q_i\, Q_j\, Q_k\, Q_l\, Q_m\, Q_m = u\,u\,d\,u\,d\,d$$



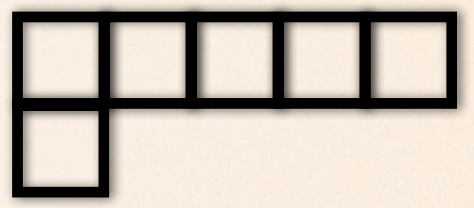
Deuteron

1



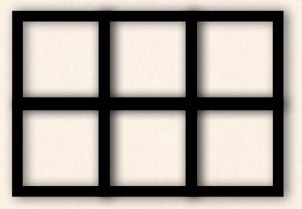
$I = 3$

5



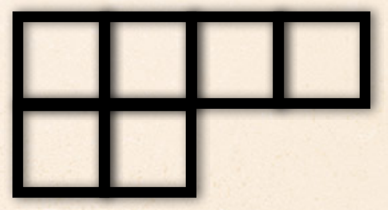
$I = 2$

5′



$I = 0$

9



$I = 1$

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- Example Deuteron:
  - $\mathbf{M}_\mathrm{D} \otimes \mathbf{M}_\mathrm{C} \otimes \mathbf{M}_\mathrm{F} = \mathbf{M}_\mathrm{D} \otimes \bar{\mathbf{5}}' \otimes \mathbf{5}' \stackrel{!}{=} \bar{\mathbf{1}}$
  - Restrictions on representations $\mathbf{M}_\mathrm{D}$ for Dirac part of BSA

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SIX-QUARK BSA

$$\Gamma^{(\mu)}(k,q,p,r,l,P) = \sum_{i=1}^{N} f_i(\Omega)\,\tau_i^{(\mu)}(k,q,p,r,l,P) \otimes \Gamma_C \otimes \Gamma_F$$

- Number of needed Dirac tensors N
  - 4096 for $J = 0 \to 320$ S-wave
  - 12288 for $J = 1 \to 576$ S-wave
- Lorentz invariants Ω:
  - Singlet: $\mathcal{S}_0$ Quintets: $\mathcal{V}_1^+, \mathcal{V}_2^+$ Nonet: $\mathcal{N}^+$

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SIX-QUARK BSA

$$\Gamma^{(\mu)}(k,q,p,r,l,P) = \boxed{\sum_{i=1}^{N} f_i(\Omega)\, \tau_i^{(\mu)}(k,q,p,r,l,P)} \otimes \Gamma_C \otimes \Gamma_F$$

- Strategy for solution:
  - Neglect "subleading" Lorentz invariants
    - $f_i(\mathcal{S}_0, \mathcal{V}_1^+, \mathcal{V}_2^+, \mathcal{N}^+) \;\to\; f_i(\mathcal{S}_0, \mathcal{N}^+)$
  - Dynamic three-body pole generation in $\mathcal{N}^+$
  - "Physical basis" Ansatz:
    - $f_i(\mathcal{S}_0, \mathcal{N}^+) \;\to\; f_i(\mathcal{S}_0) \cdot \mathrm{P}_1 \cdot \mathrm{P}_2$

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SIX-QUARK BSA

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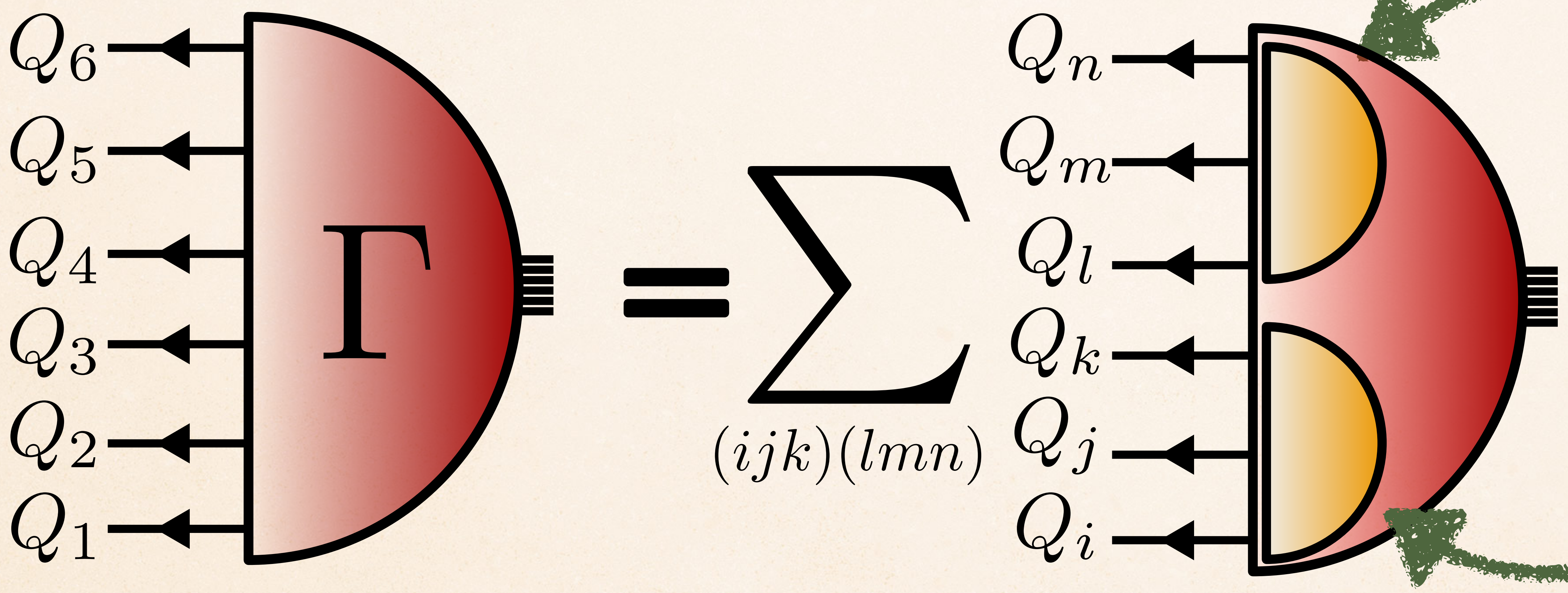
- "Physical" BSA:



Combine dominant S-wave tensors of baryons, e.g., nucleon and Δ

with $i < j < k$ & $l < m < n$

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CONCLUSION & OUTLOOK

- Construct hexaquark BSA using the "physical" Ansatz and in terms of the S_6 multiplets
- Calculate the dibaryon spectrum numerically
  - E.g., Deuteron and cousins, H-dibaryon, all-charm dibaryons
- Investigate the internal structure