

TOWARDS A DESCRIPTION OF DIBARYONS FROM FUNCTIONAL METHODS

JOSHUA HOFFER

9. SEPTEMBER 2026
NSTAR 2026, SEVILLE, SPAIN

HADRONS

HADRONS

Conventional

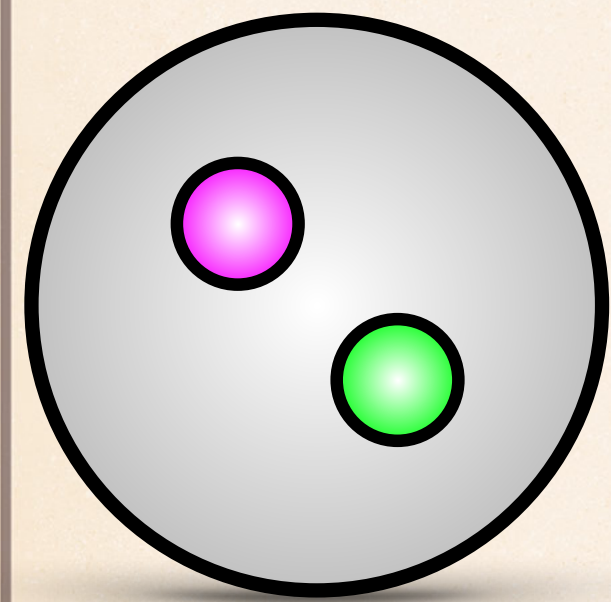
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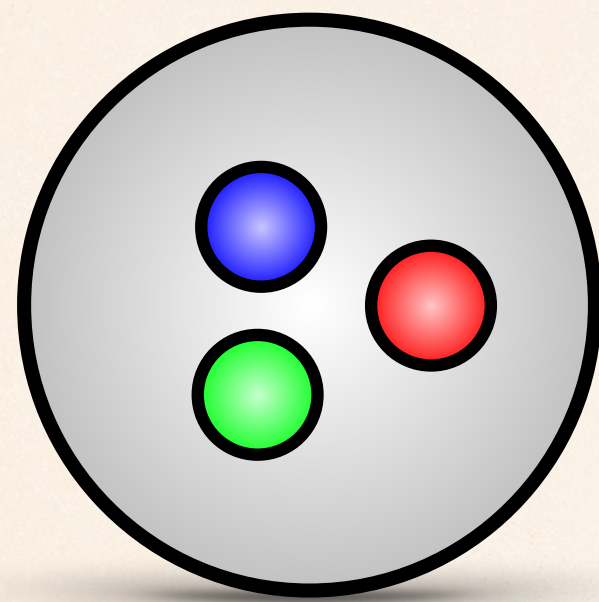
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HADRONS

Conventional



Meson

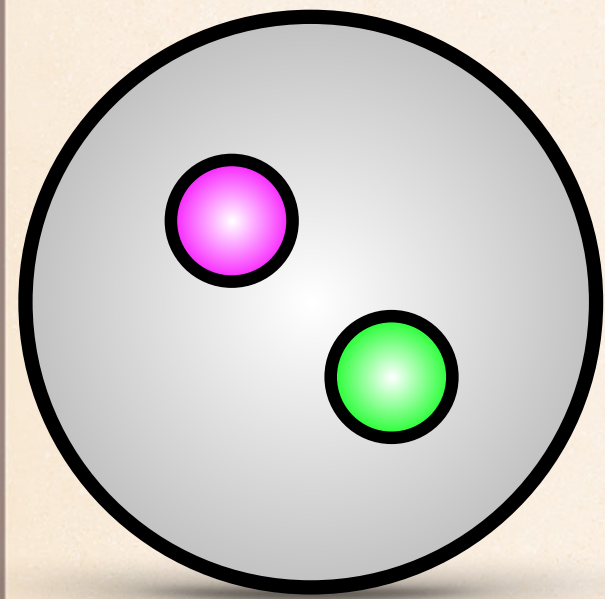


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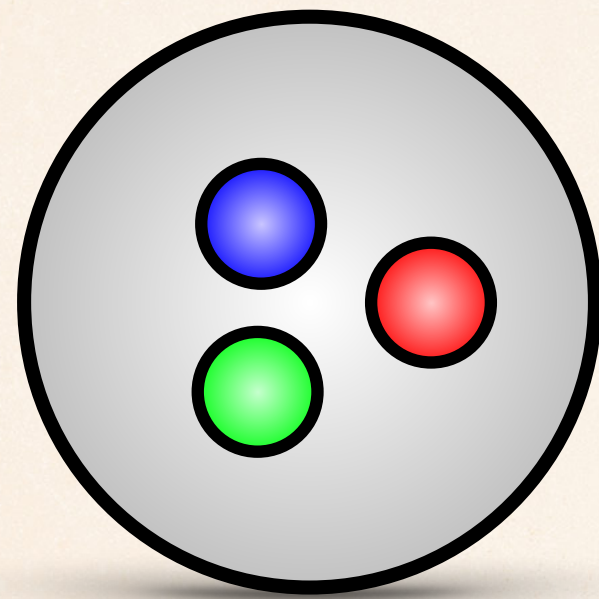
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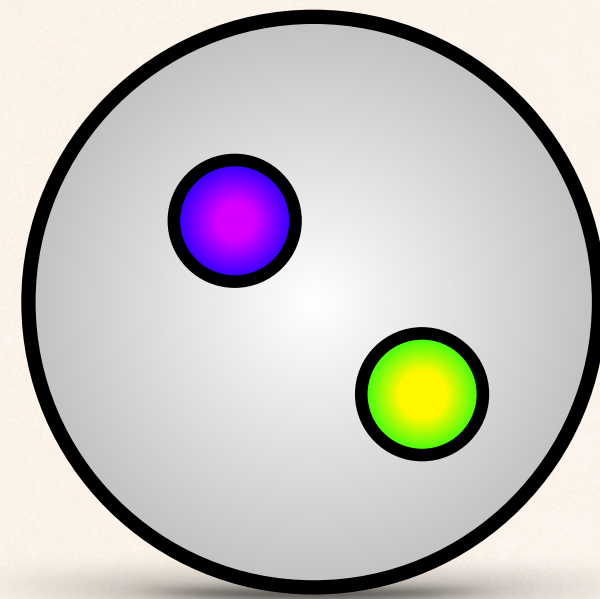
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Meson



Baryon

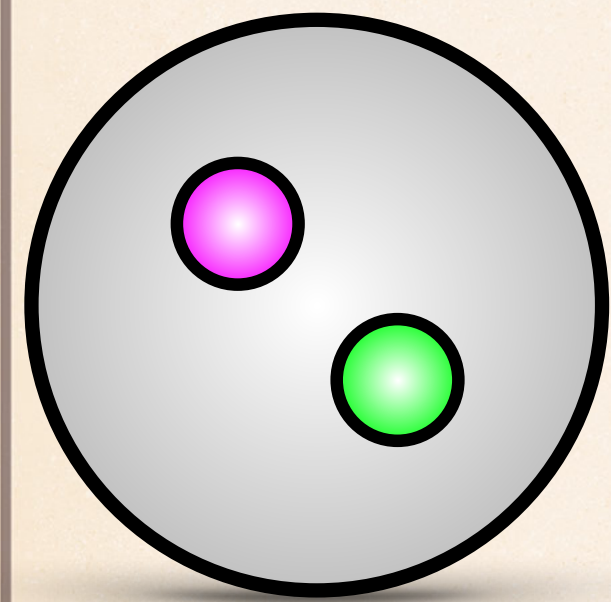


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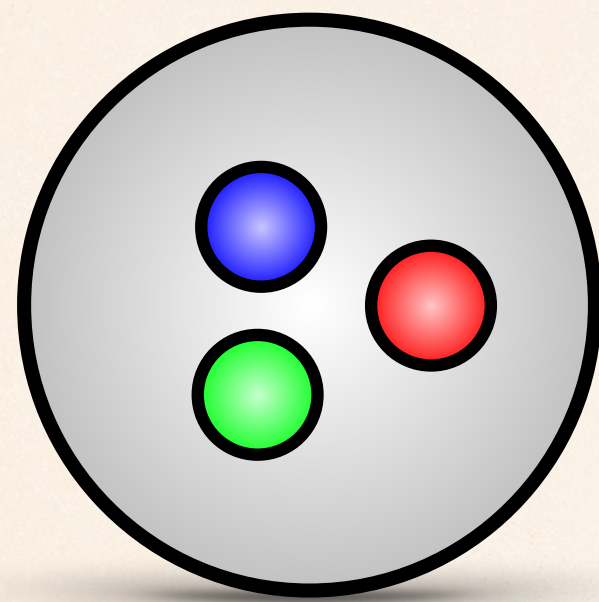
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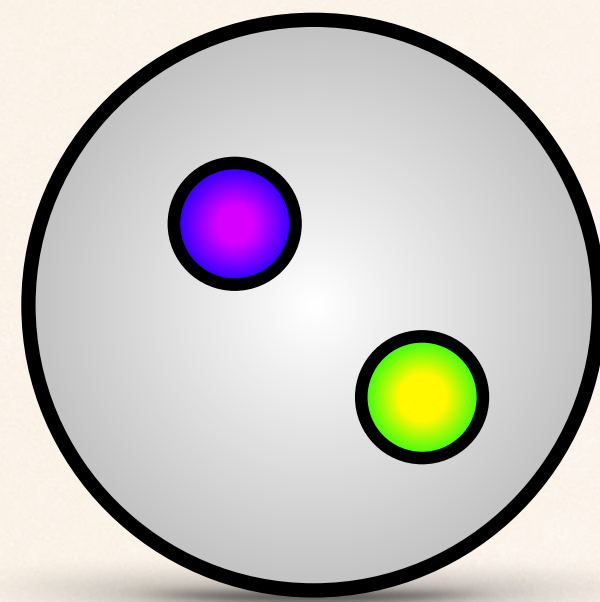


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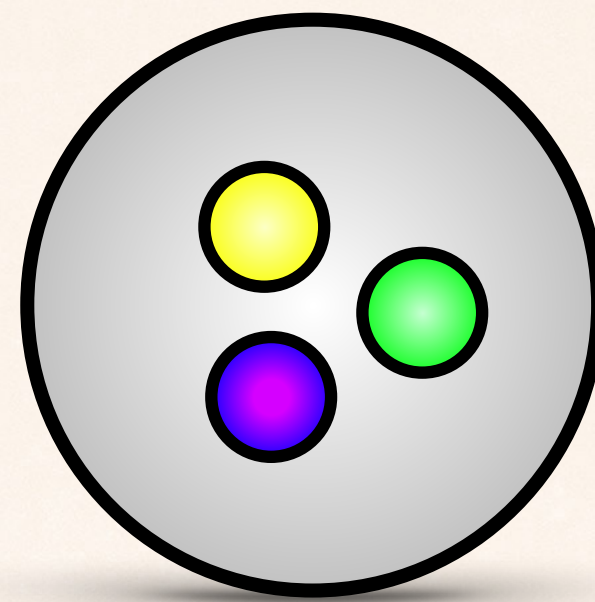


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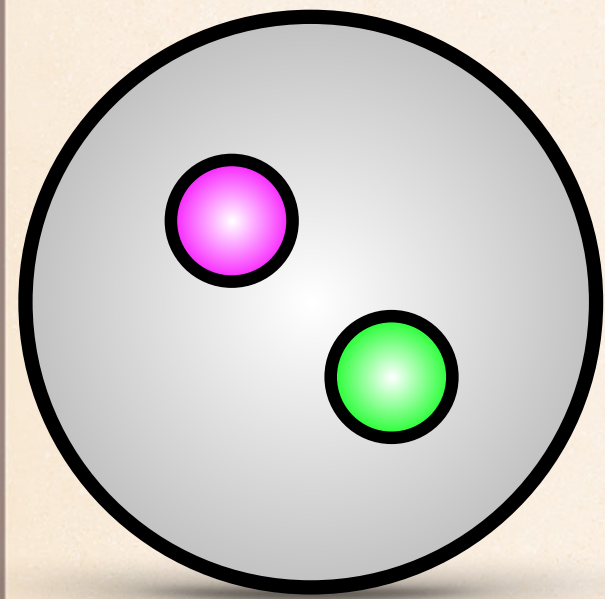
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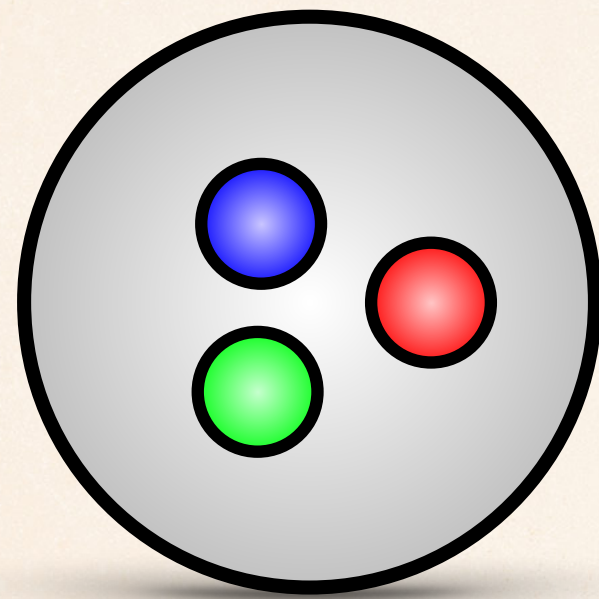
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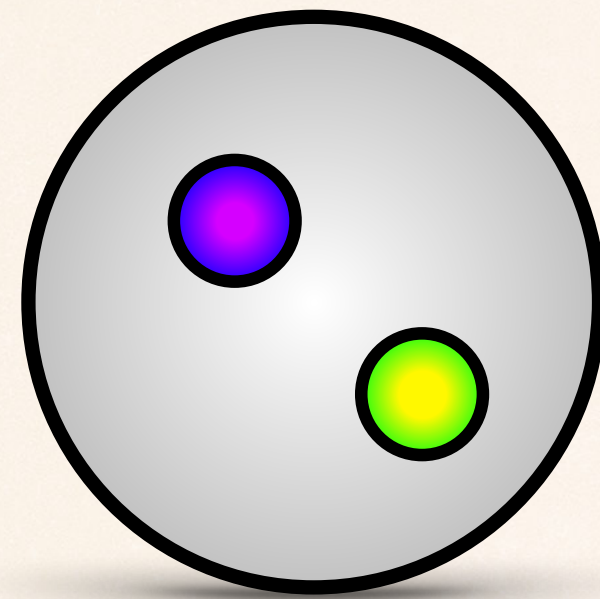
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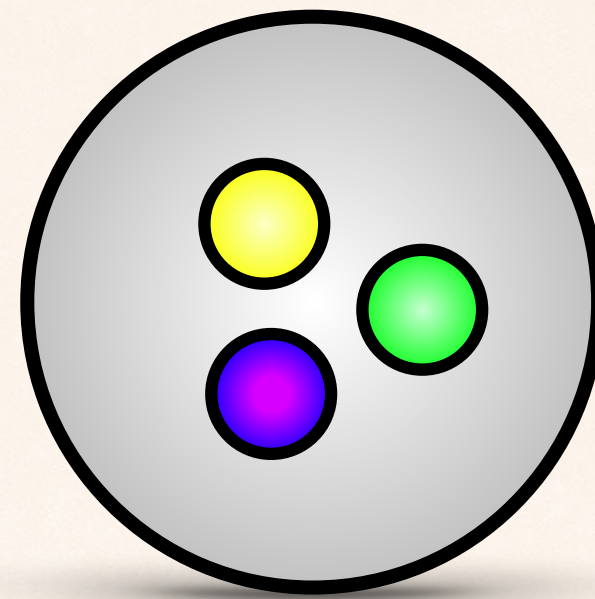
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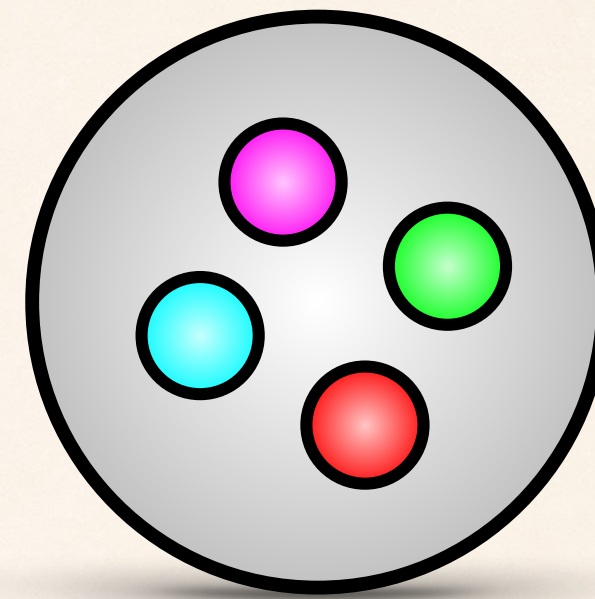
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Glueball



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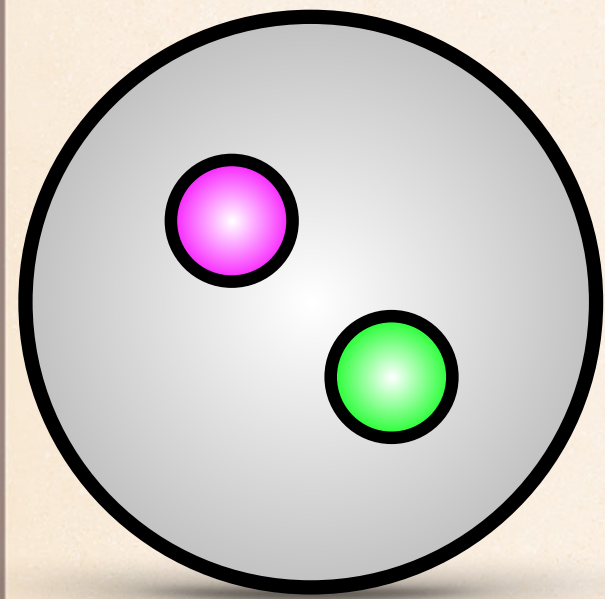


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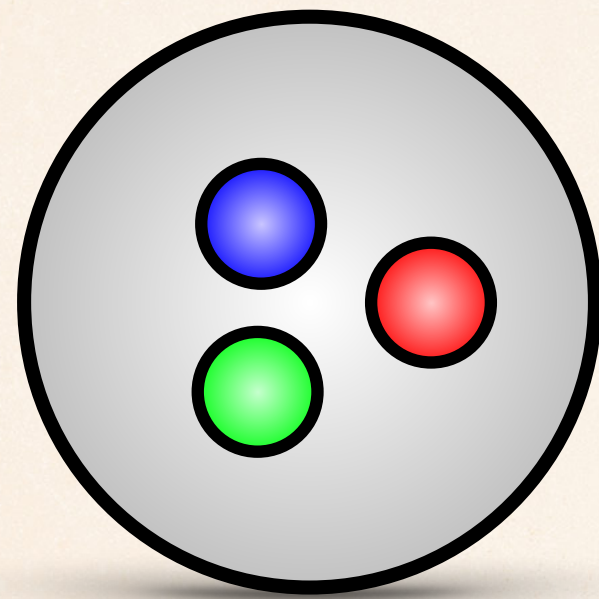
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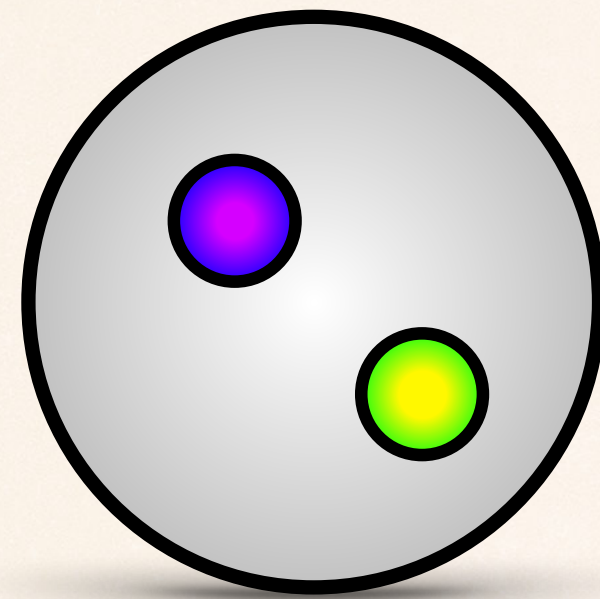
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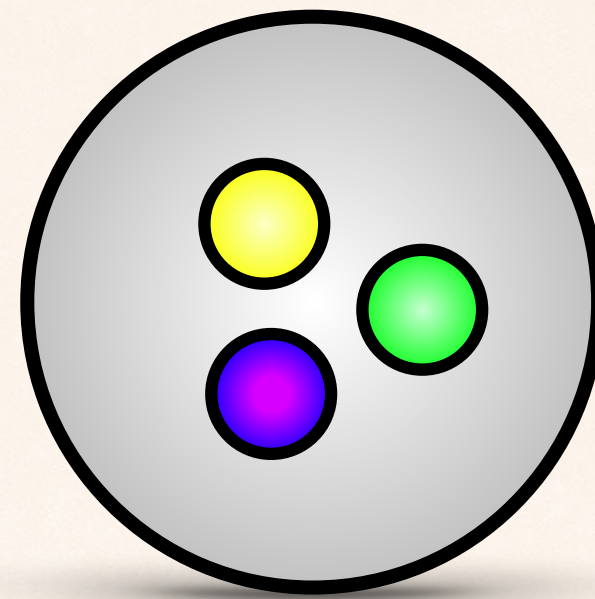
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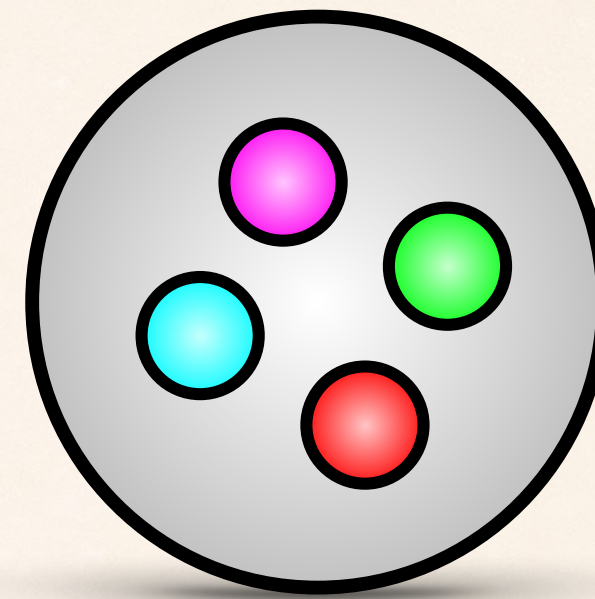


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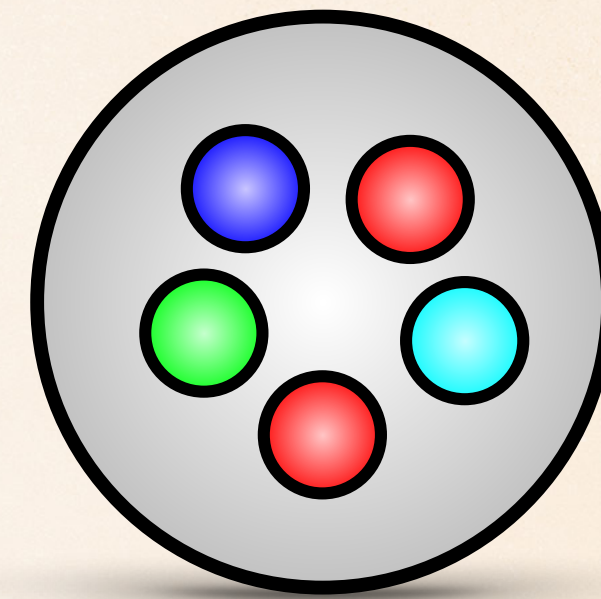


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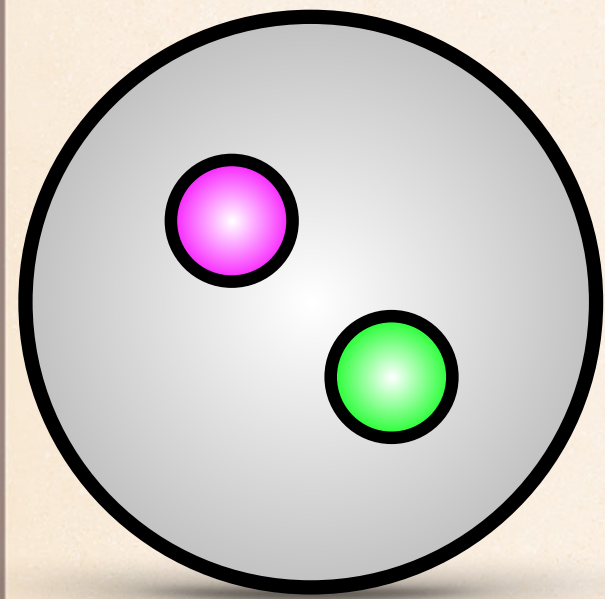
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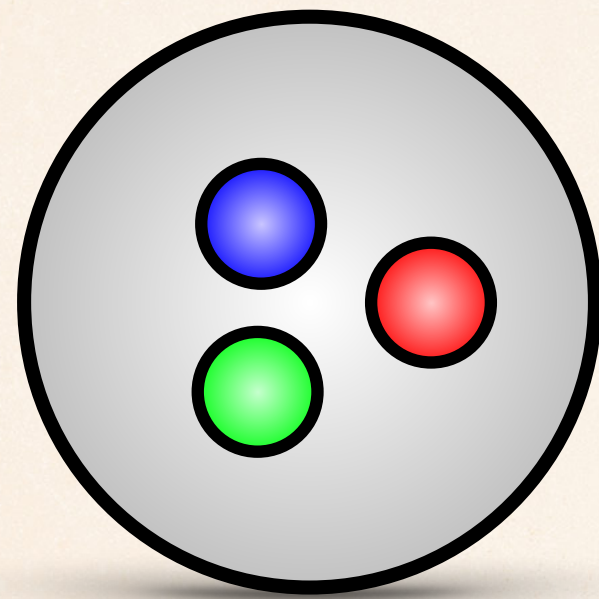
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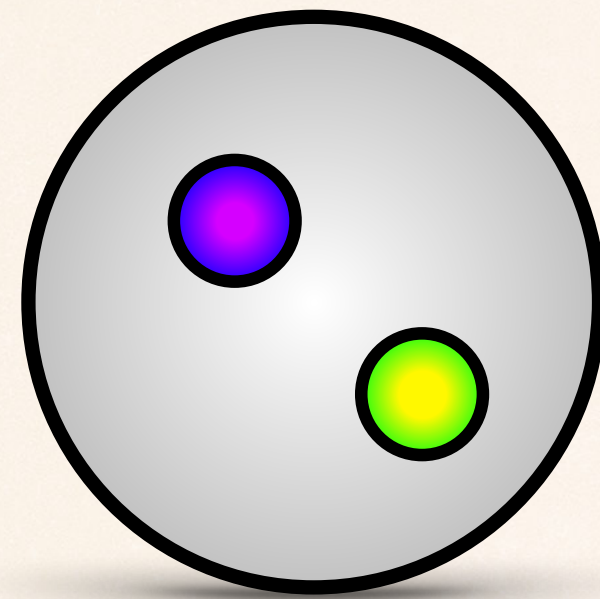
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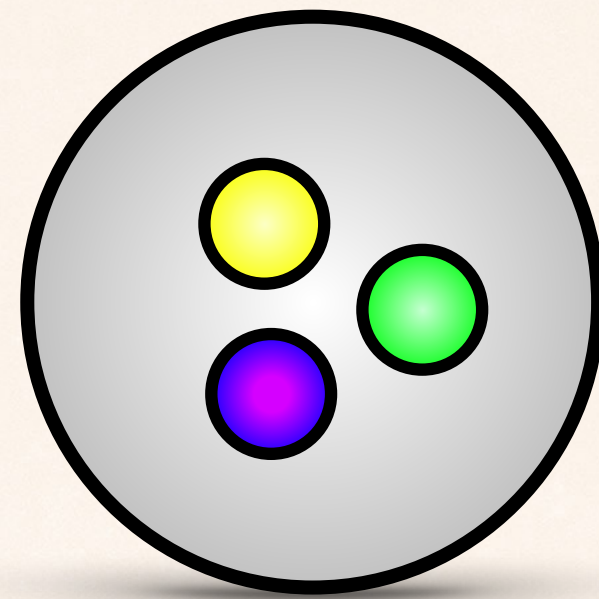
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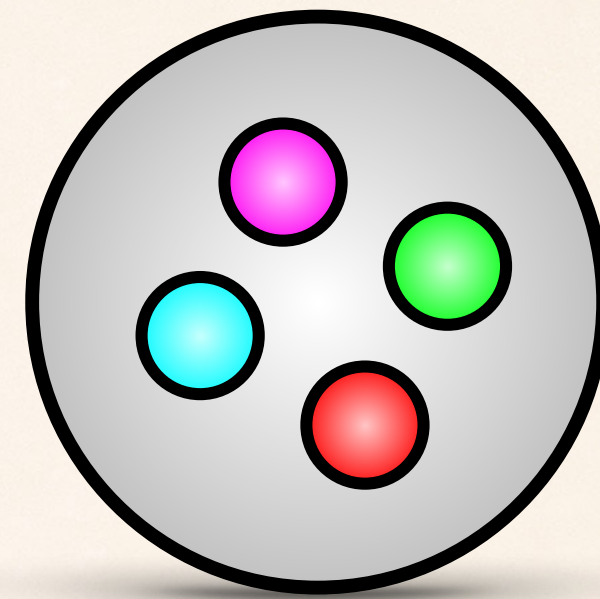


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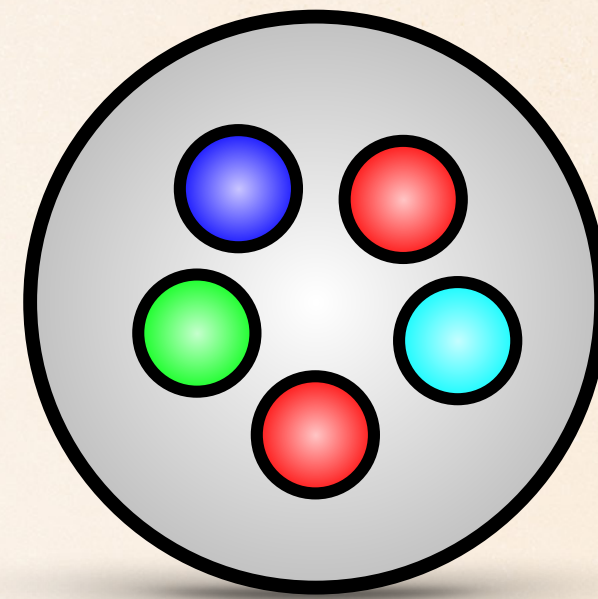


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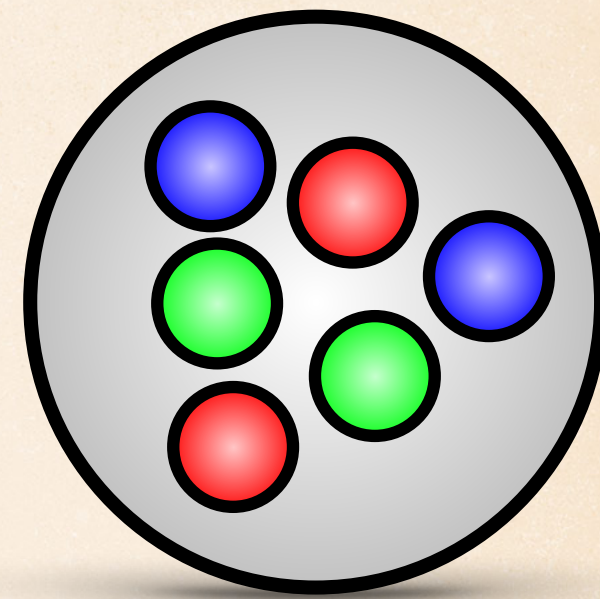
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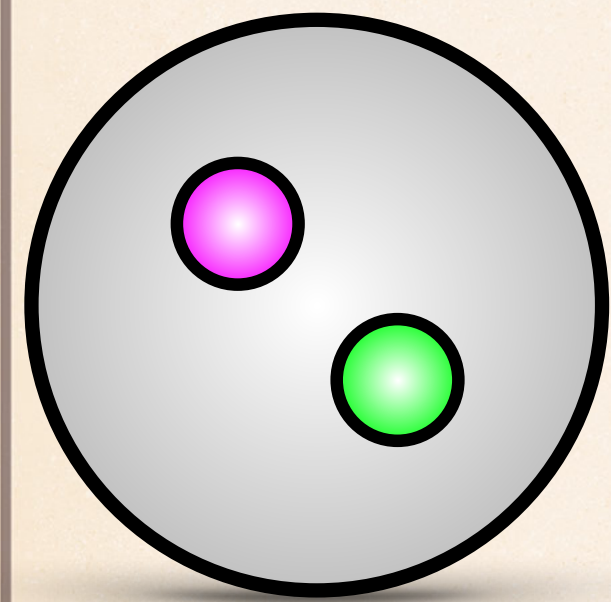
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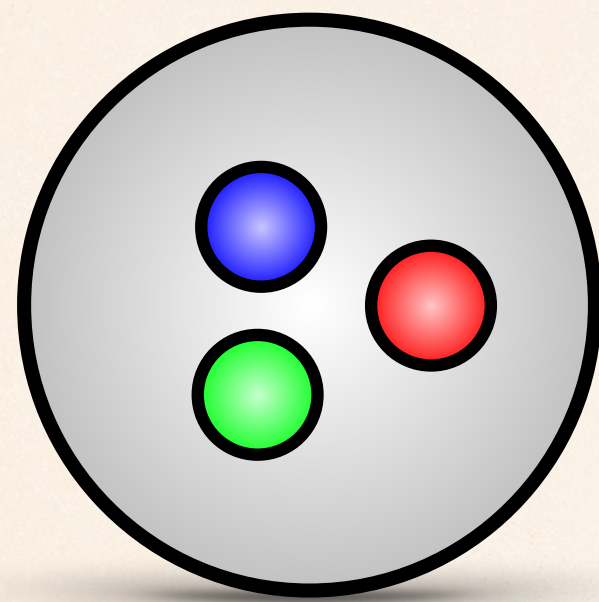
Hexaquark

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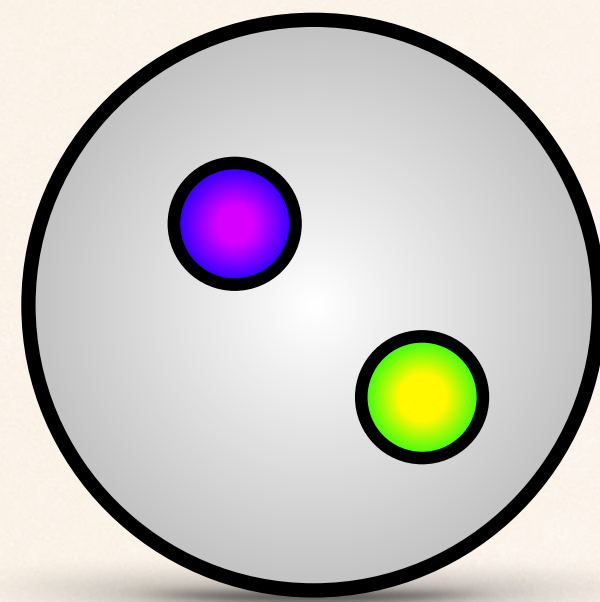


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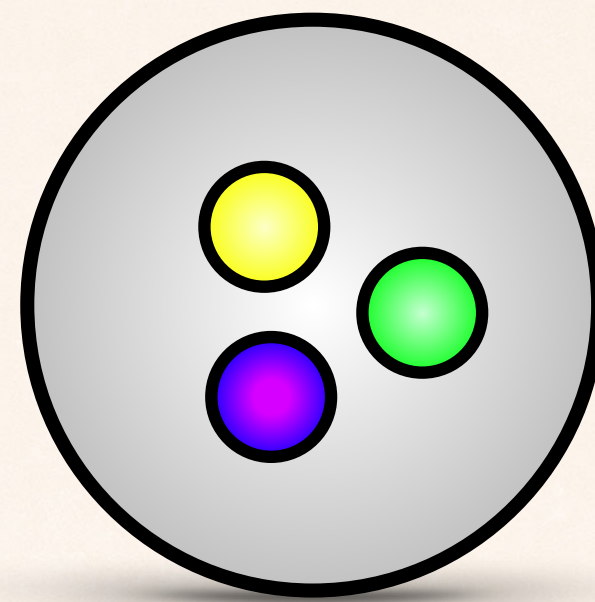


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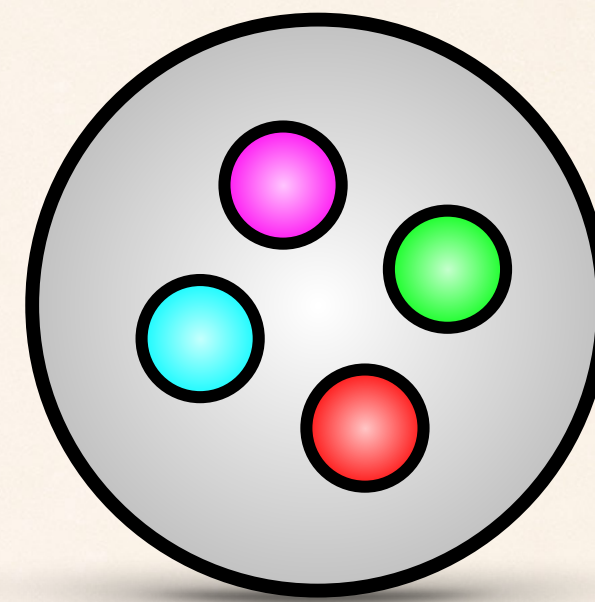
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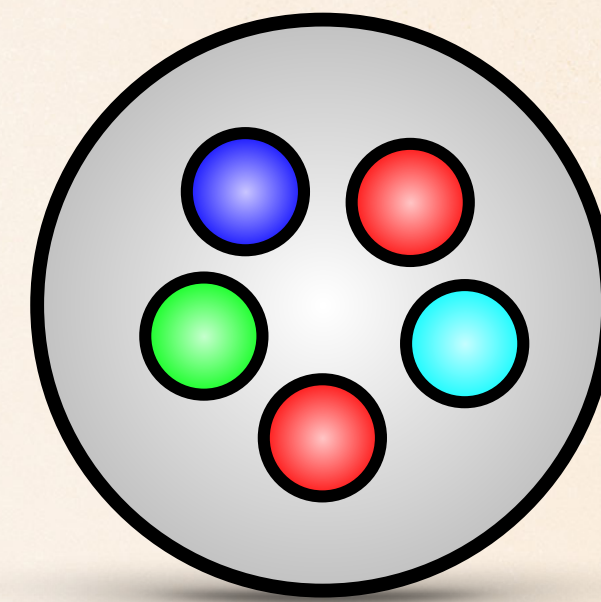
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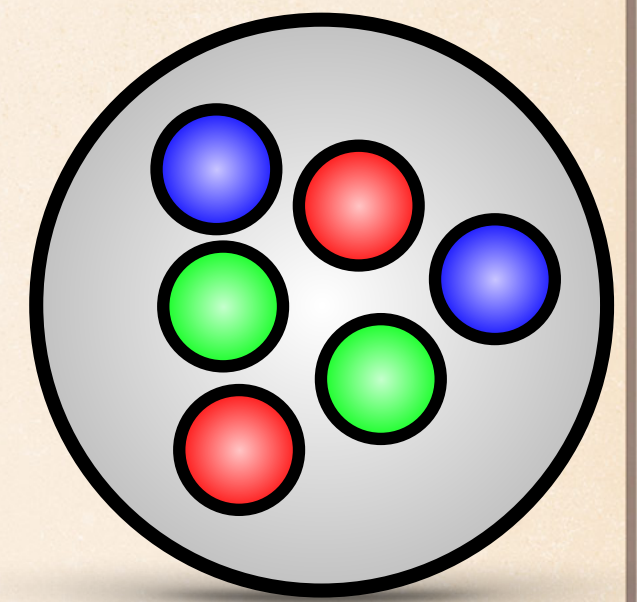
Hybrid



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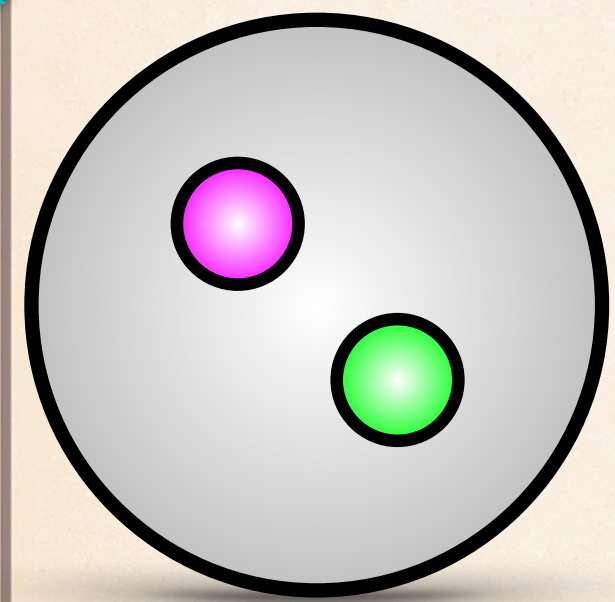


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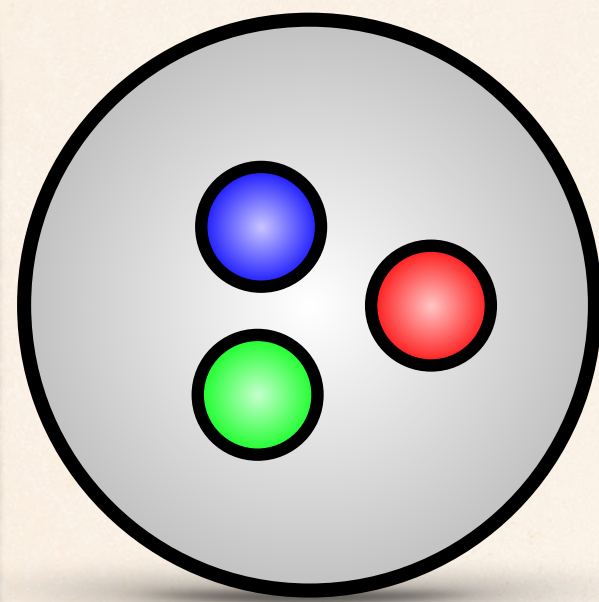
Overview see talk Gernot Eichmann

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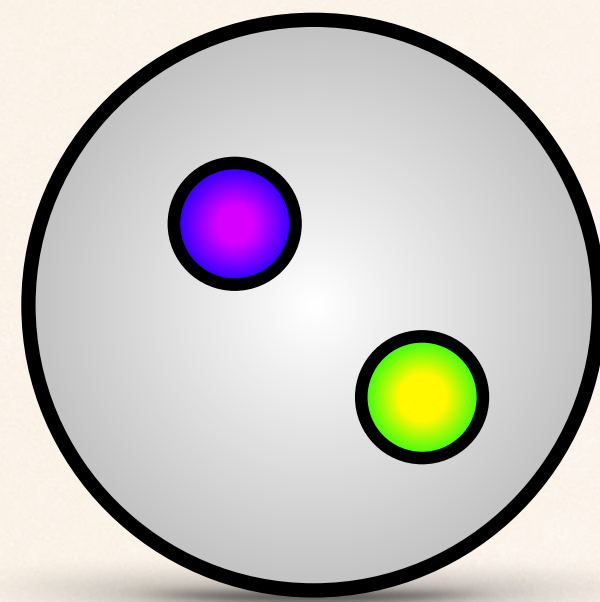


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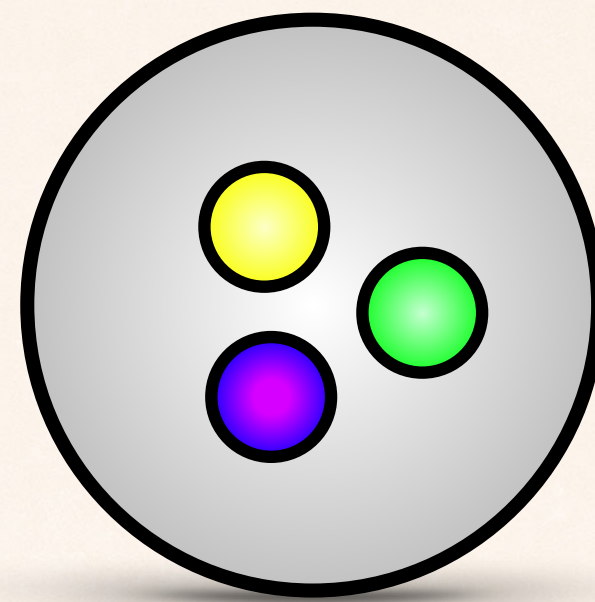


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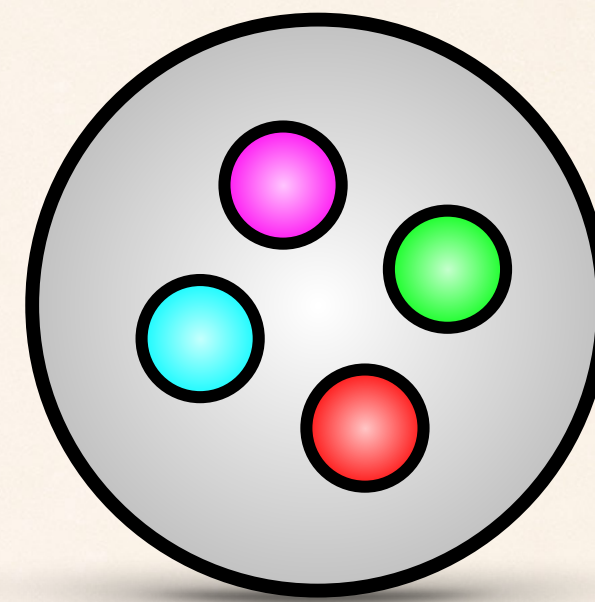
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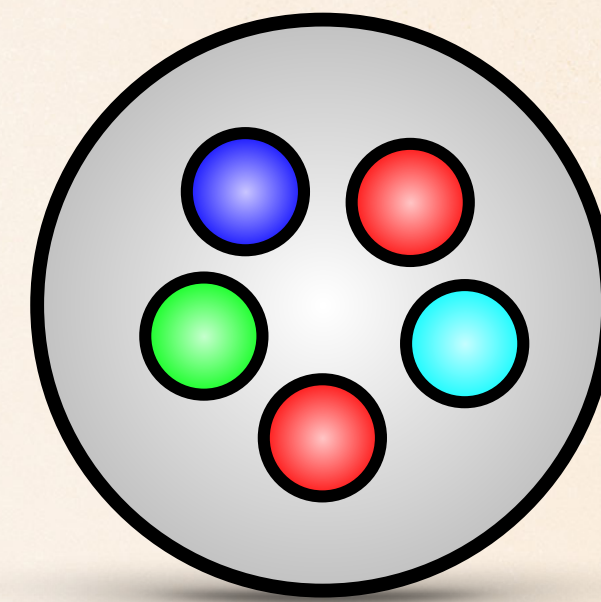
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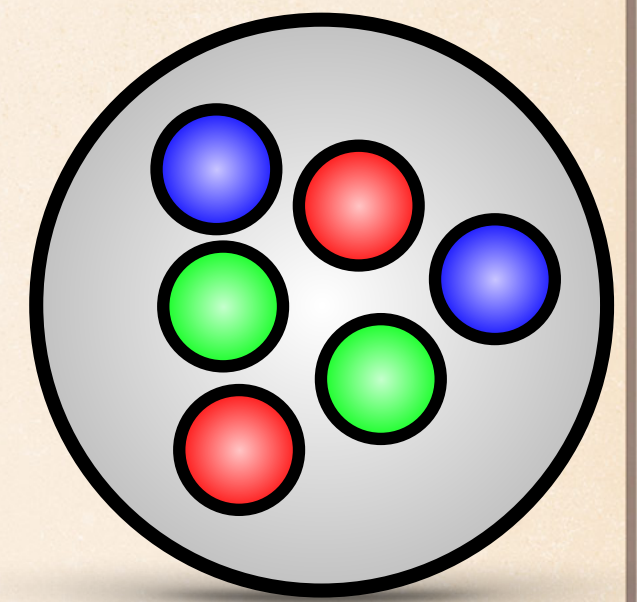
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Penta-
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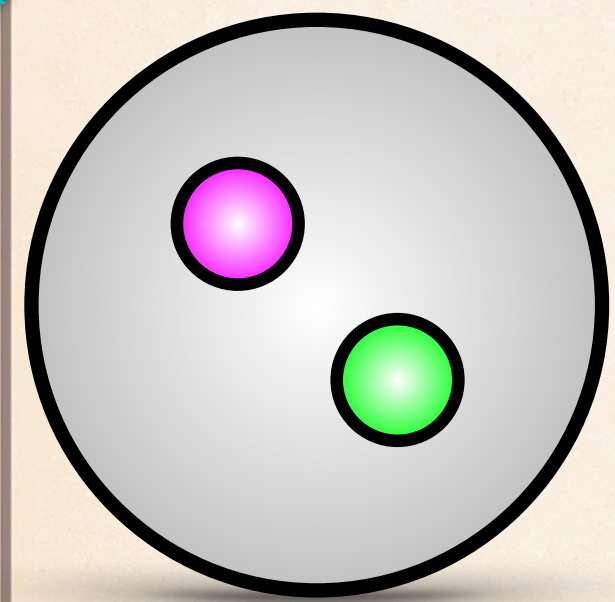
Hexaquark

see talk Angel Miramontes

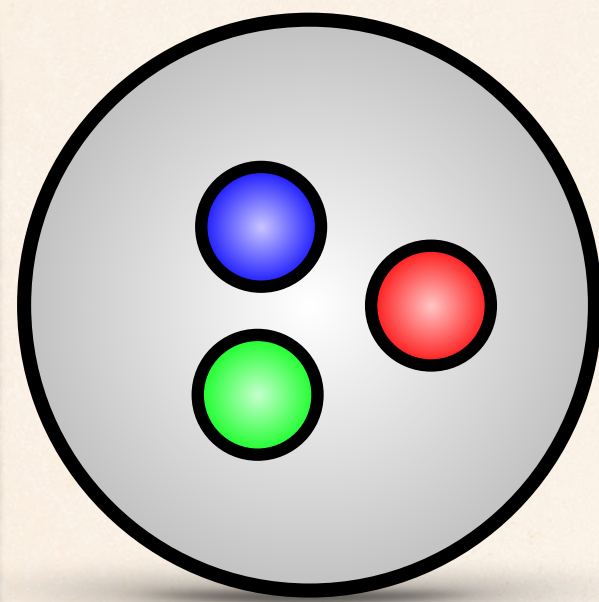
Overview see talk Gernot Eichmann

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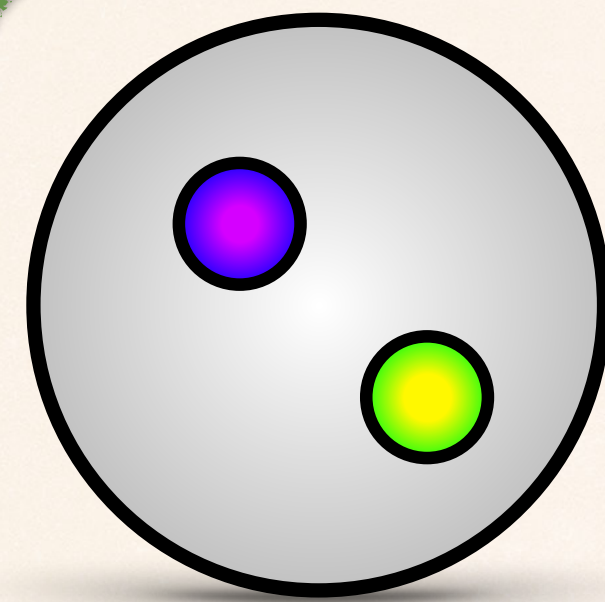
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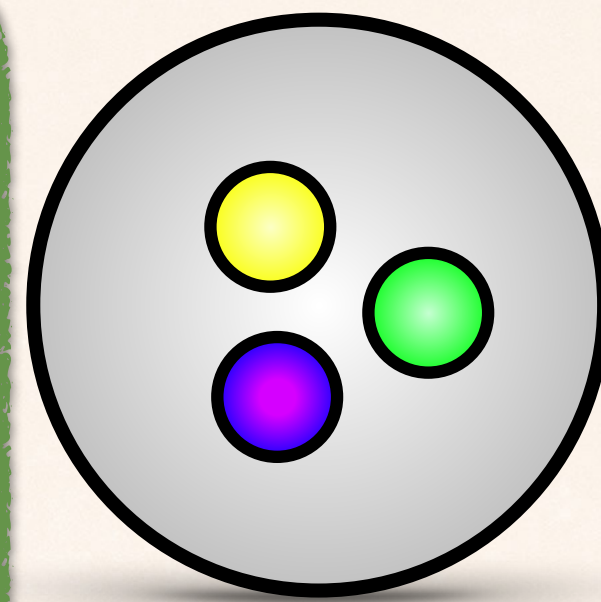
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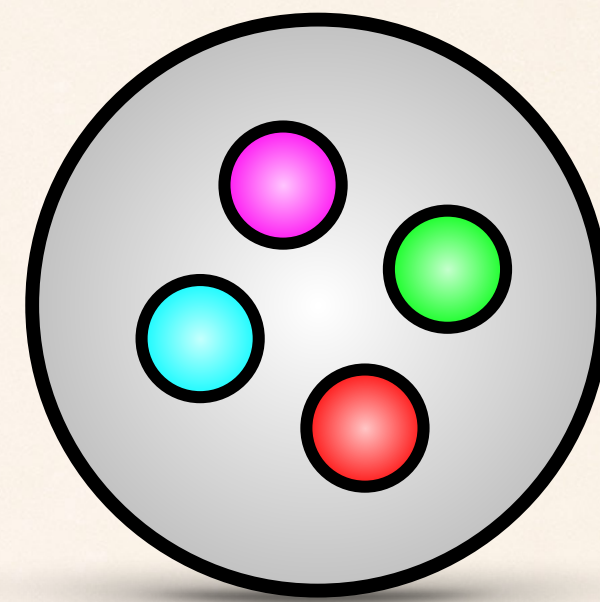


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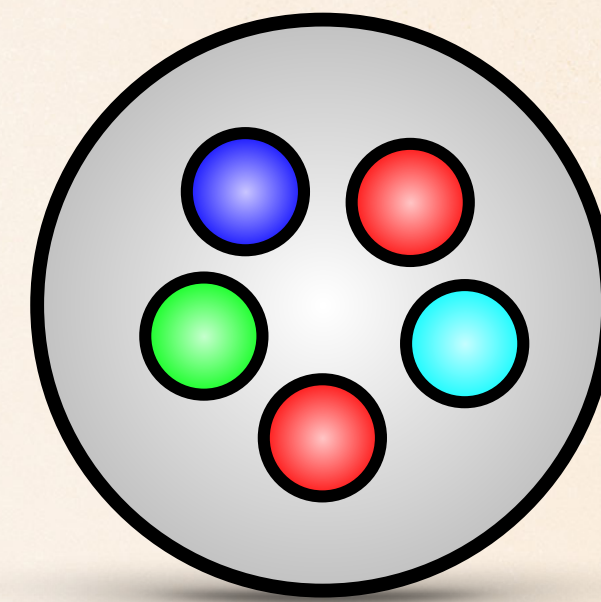


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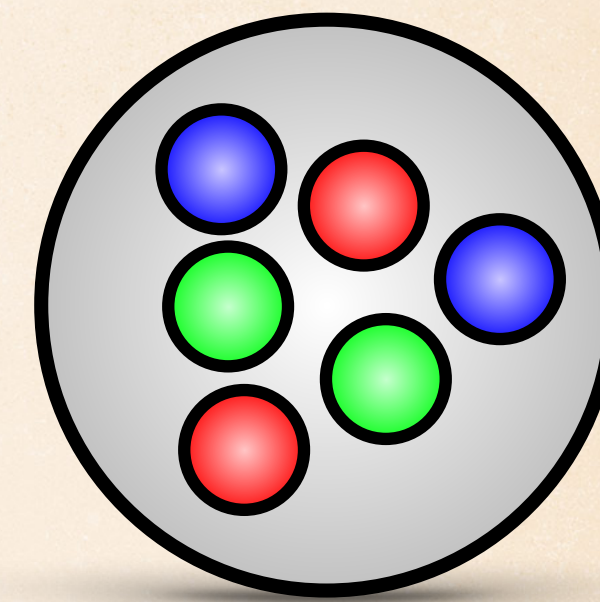
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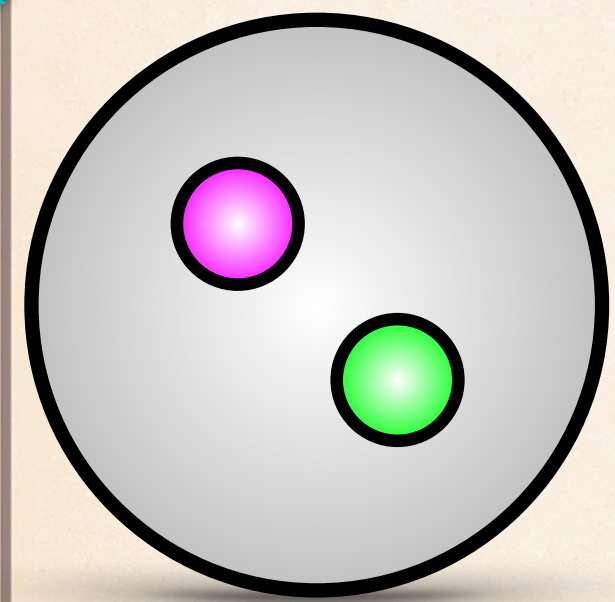
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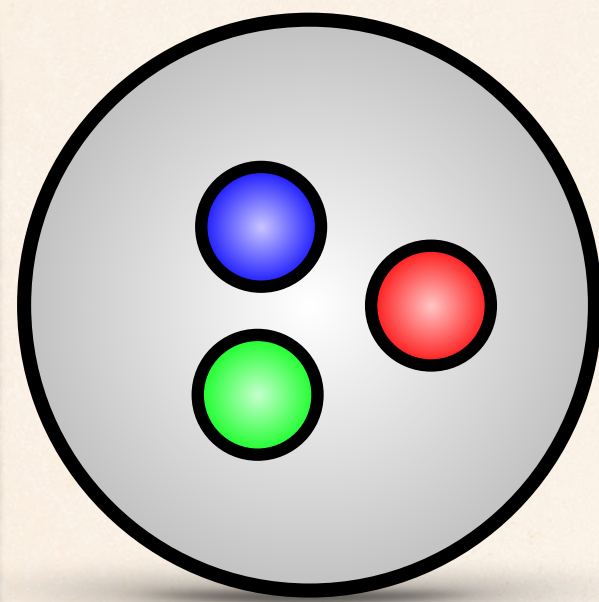
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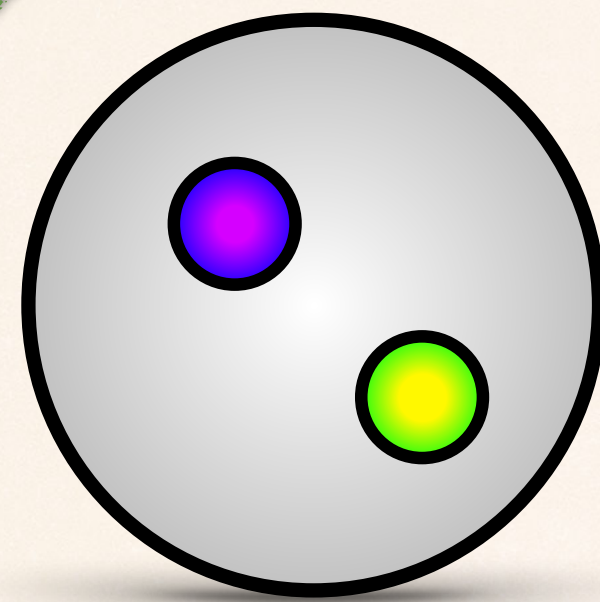


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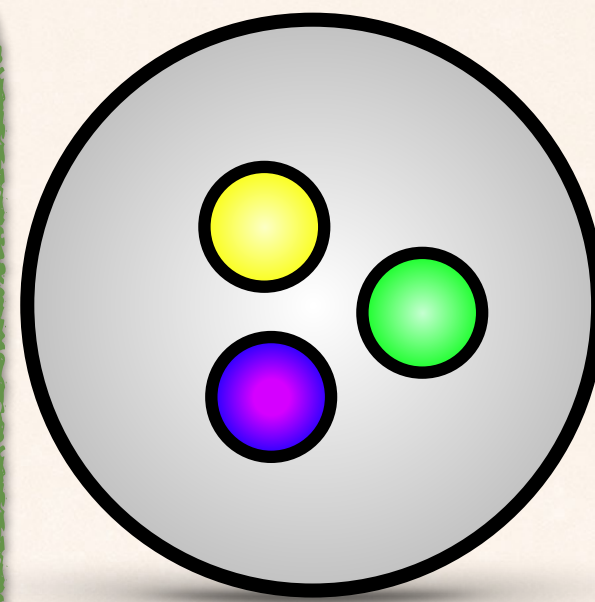


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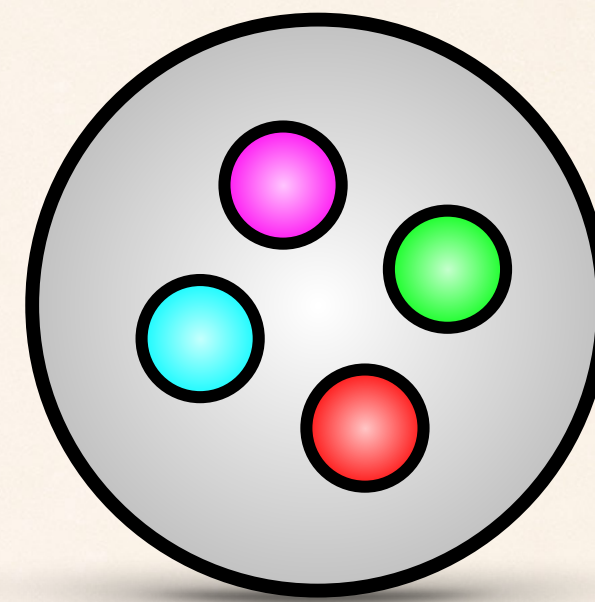
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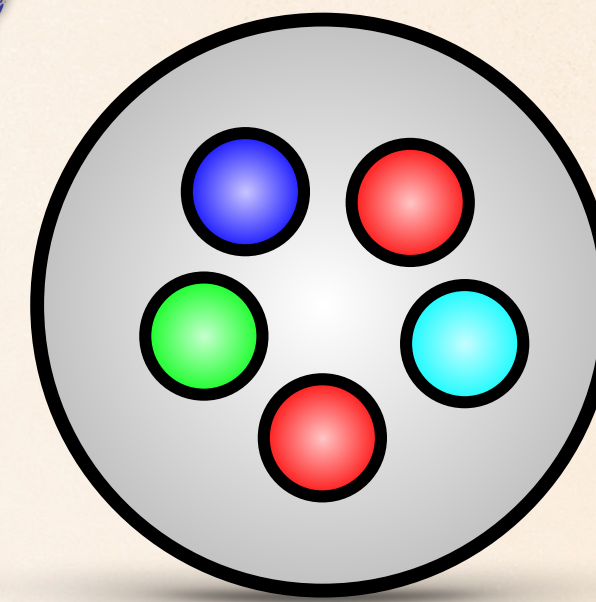
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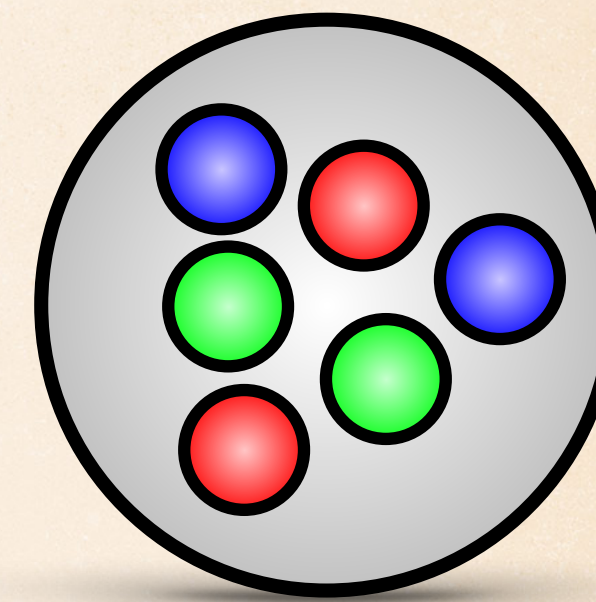
Hybrid



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quarks



Penta-
quarks



Hexaquark

see talk Markus Huber

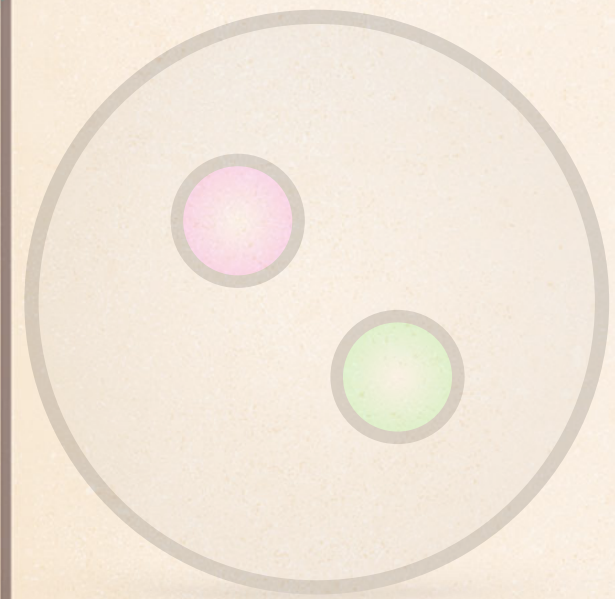
see talk Raúl Torres

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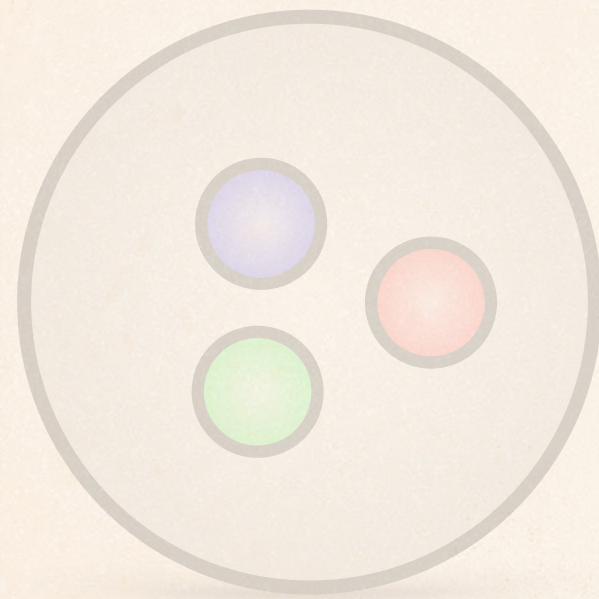
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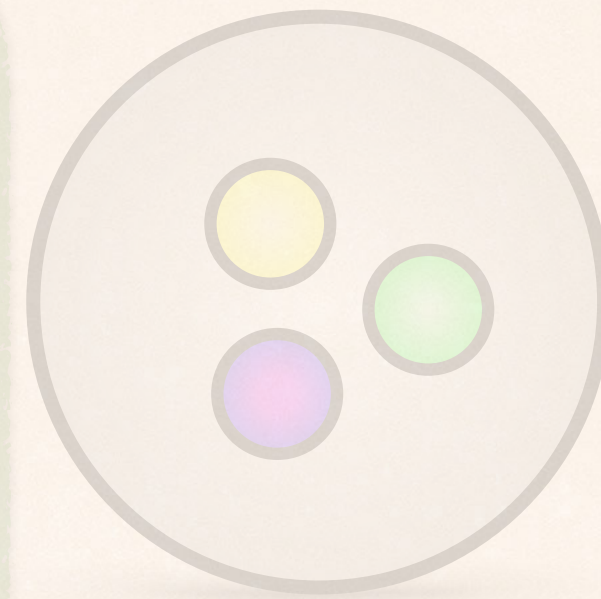


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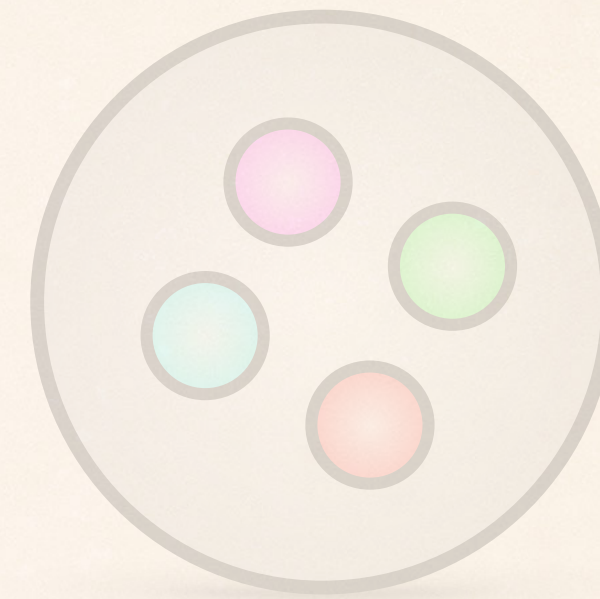
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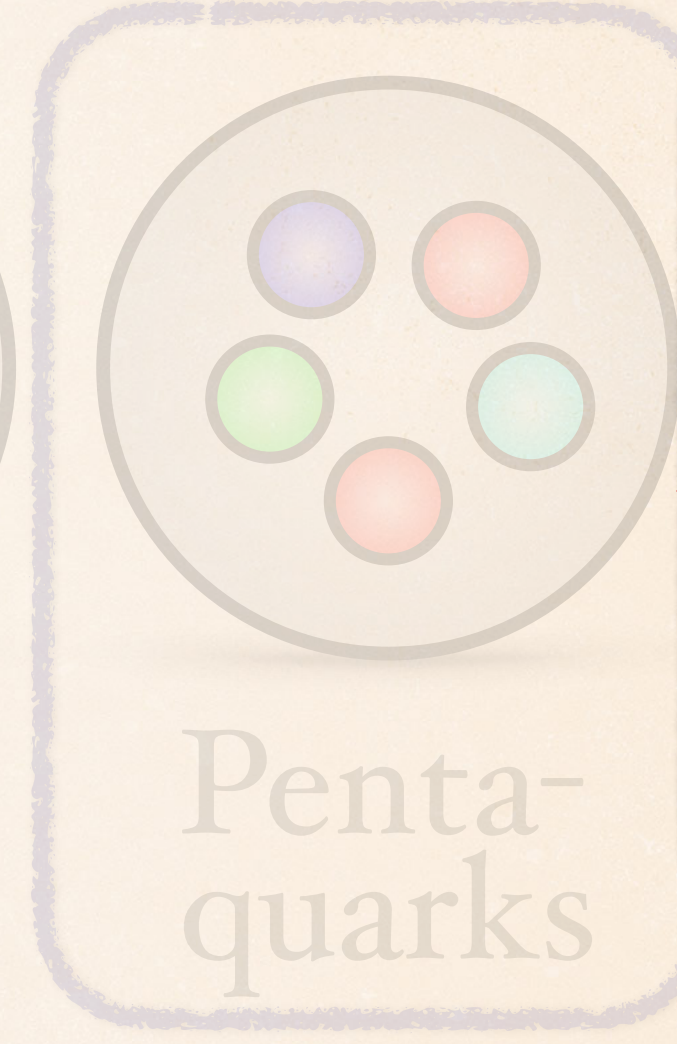
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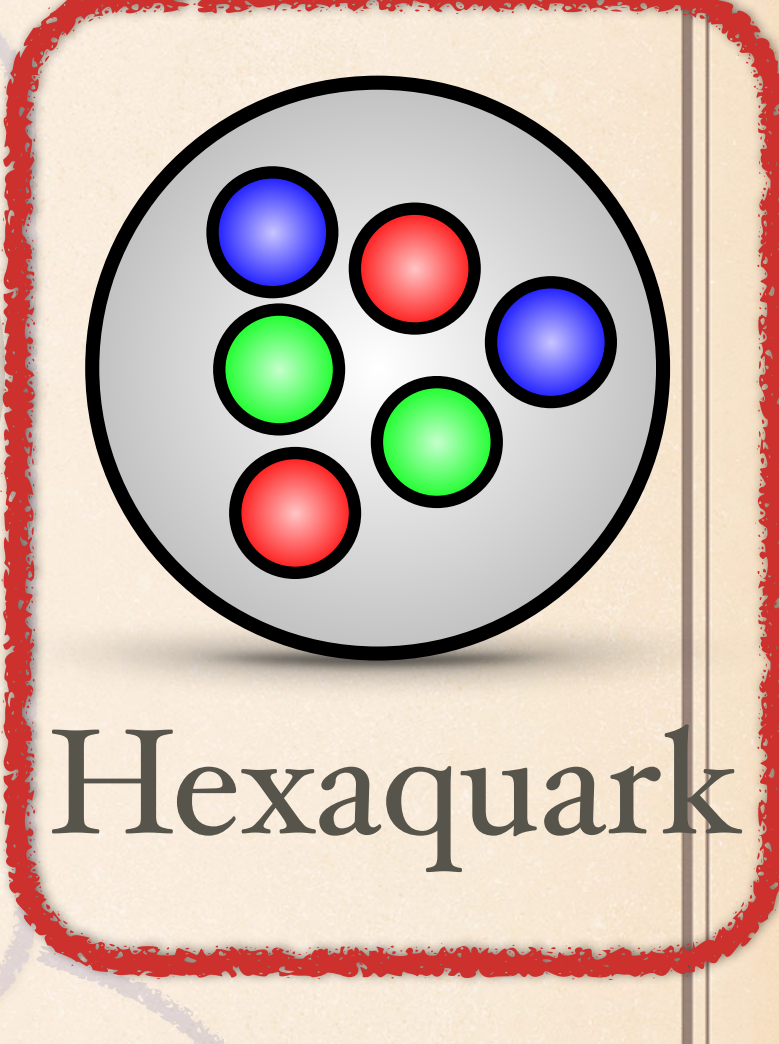
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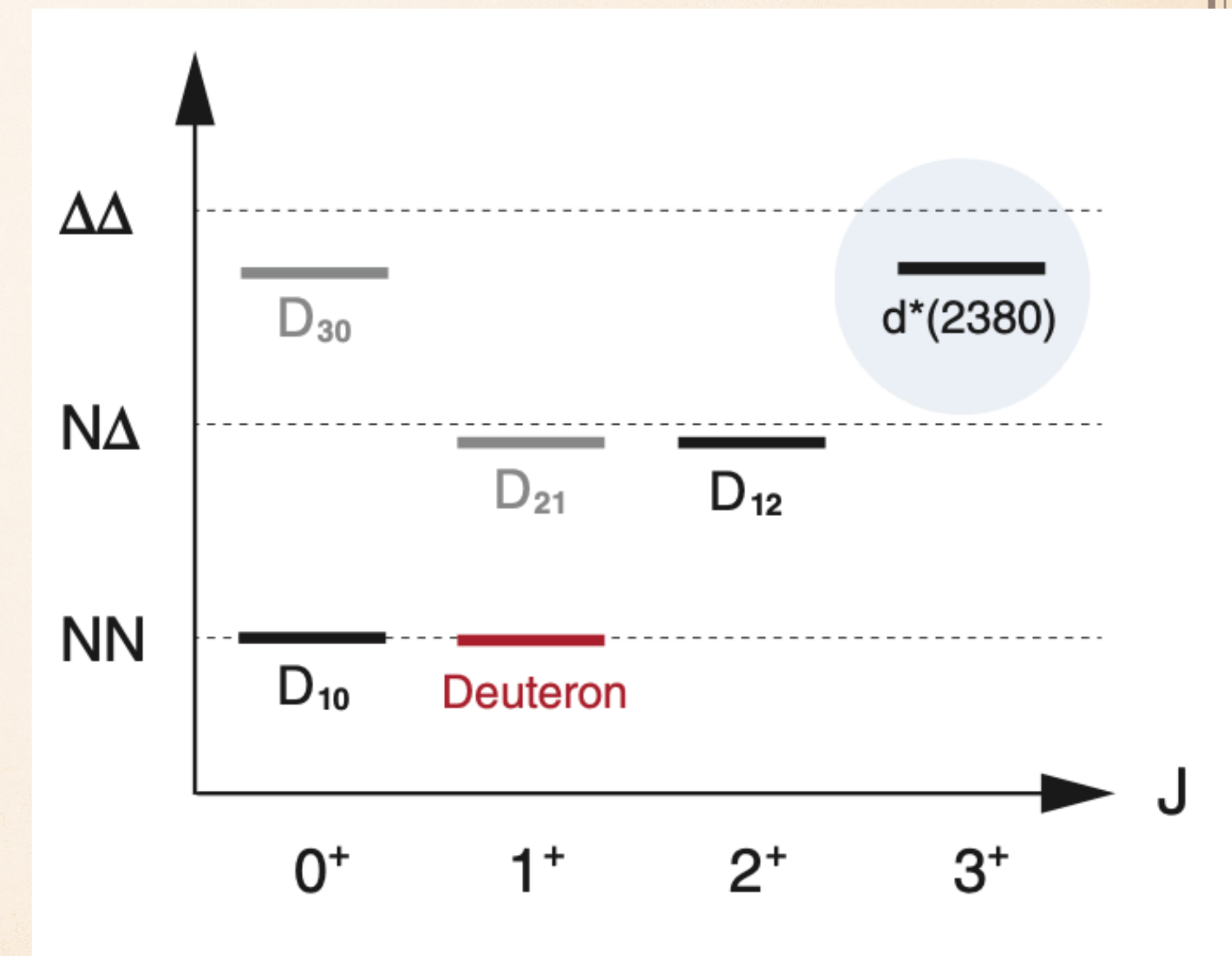
see talk Angel Miramontes

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HEXAQUARKS

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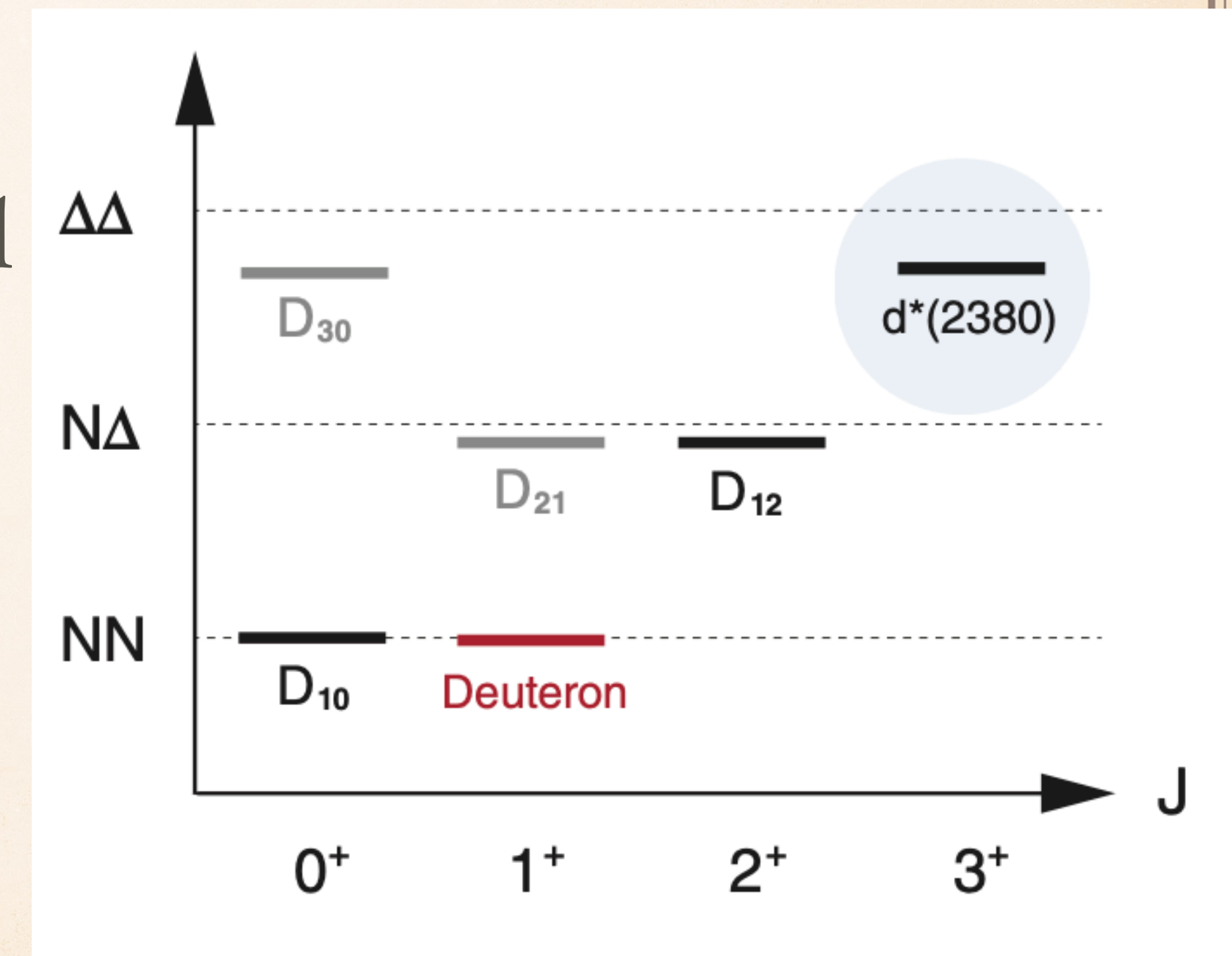
- ❖ Many interesting particles:
 - ❖ Deuteron and its cousins, Diproton (Helium 2), Neutronium
 - ❖ H-dibaryon, all-charm dibaryons



Dyson, Xuong, PRL 13 (1964)
Adlarson et al., PRL 106 (2011), PRL 112 (2014),
Bashkanov, Brodsky, Clement, PLB 727 (2013),
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HEXAQUARKS

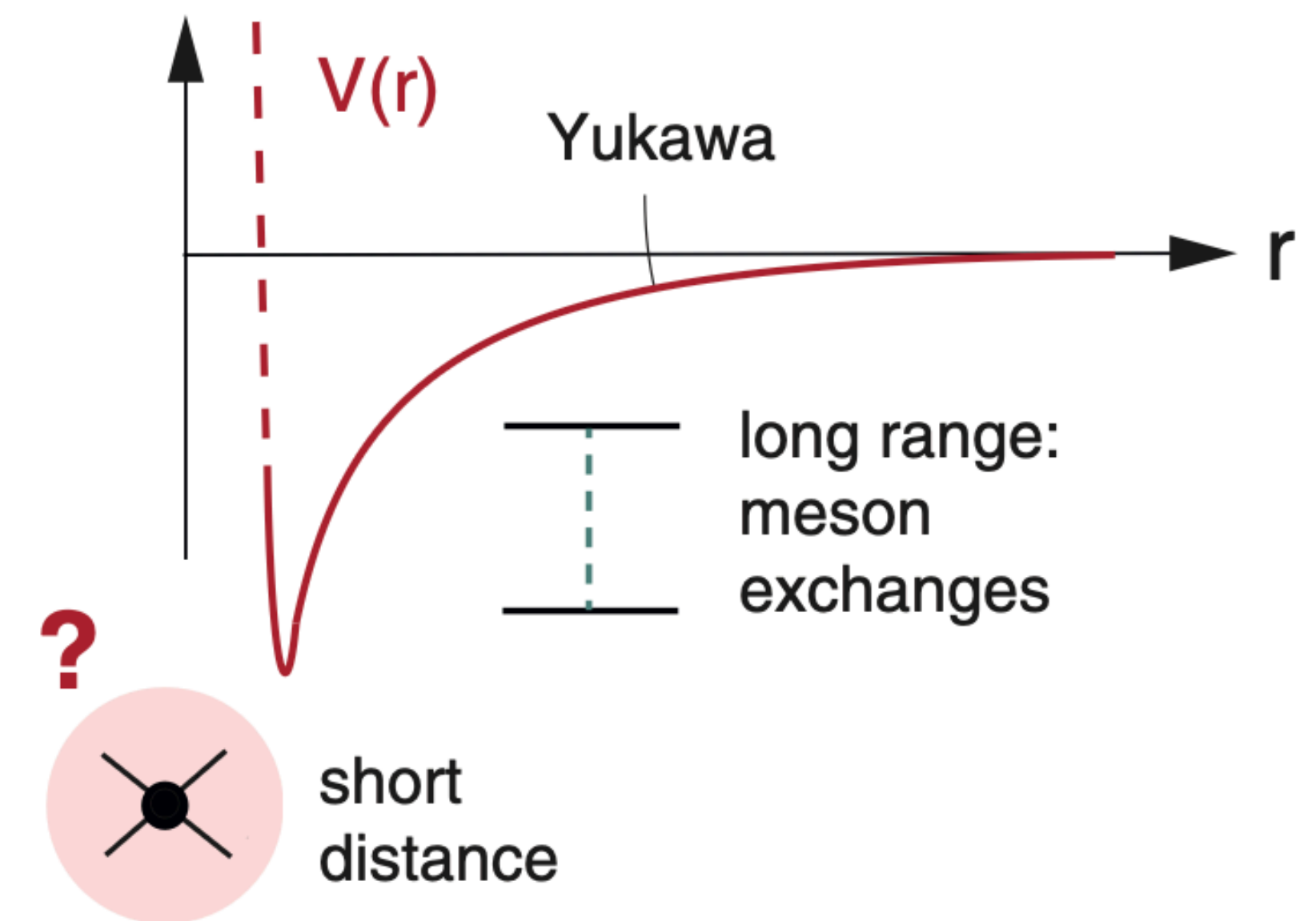
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- ❖ Study spectrum from quark and gluon level
- ❖ Investigate Microscopic origins of short-range nuclear force

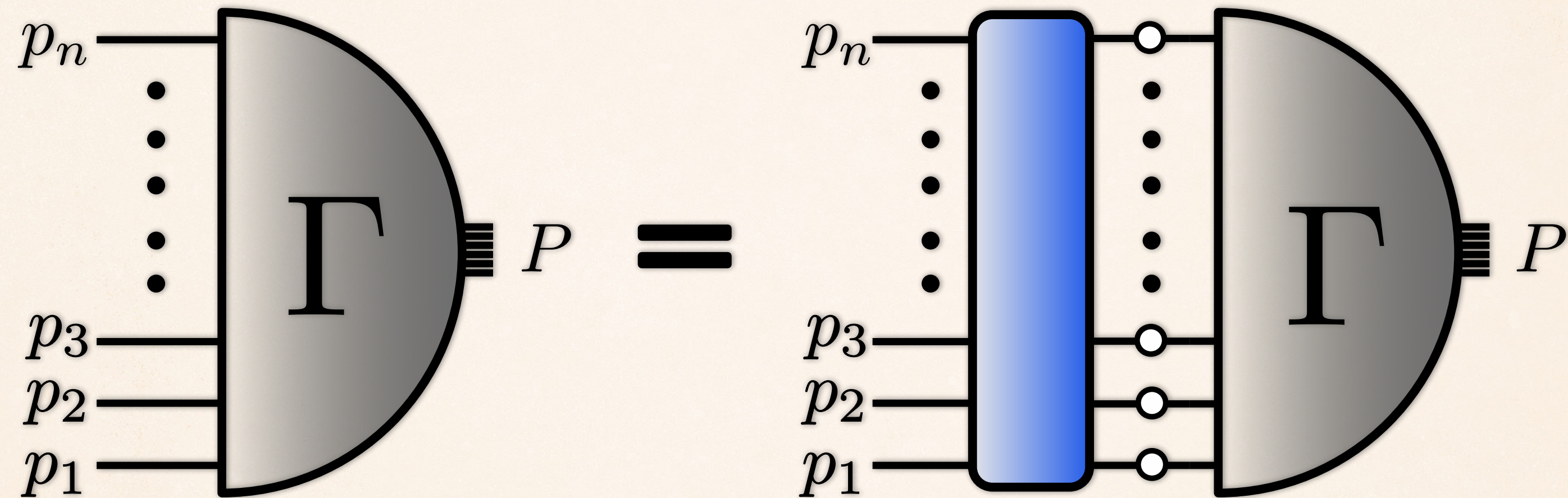


FUNCTIONAL FRAMEWORK

- ❖ Fully relativistic
- ❖ Non-perturbative
- ❖ Exact QCD n-point functions (need to be truncated later)
- ❖ Flexible framework
 - ❖ Easily adjustable regarding different quark flavours and tensors
 - ❖ Evaluate importance of different diagrams for the equation

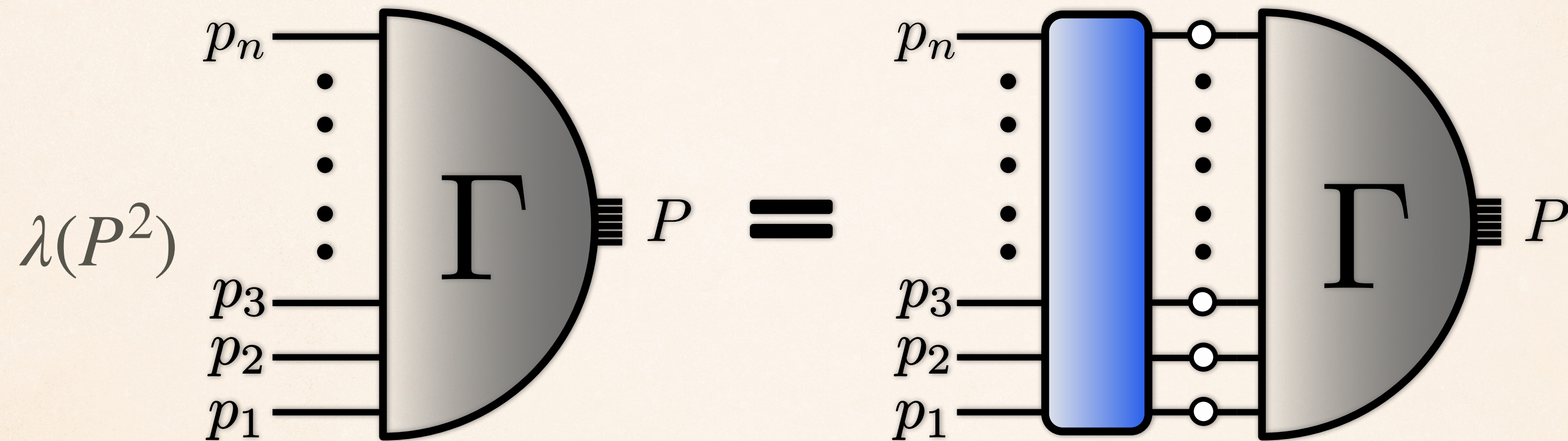
FUNCTIONAL FRAMEWORK

- ❖ Properties of Hadrons from Hadronic bound state equations



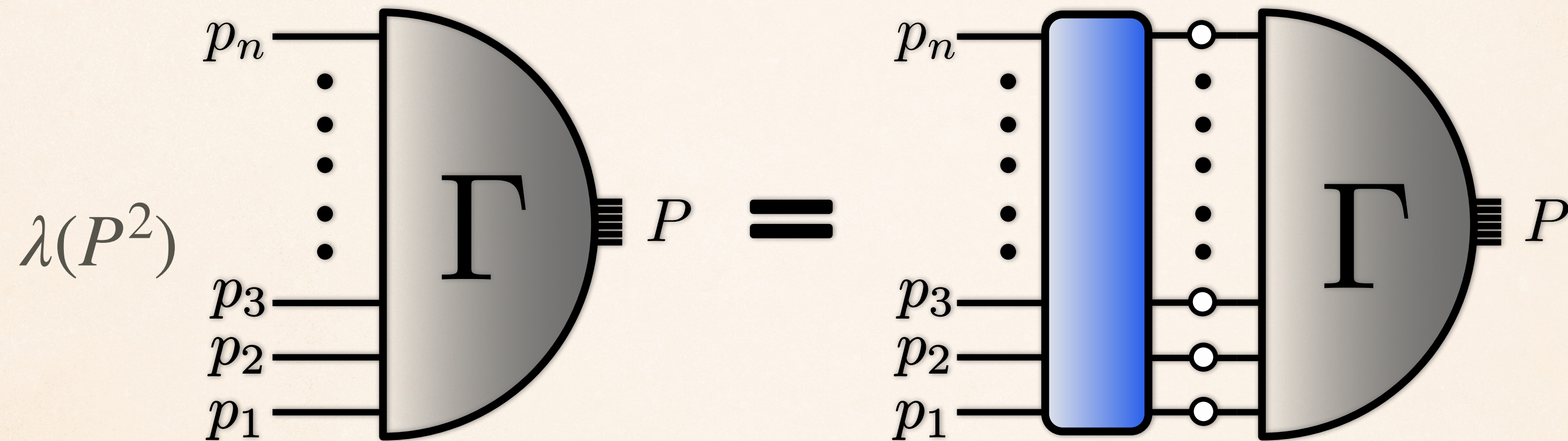
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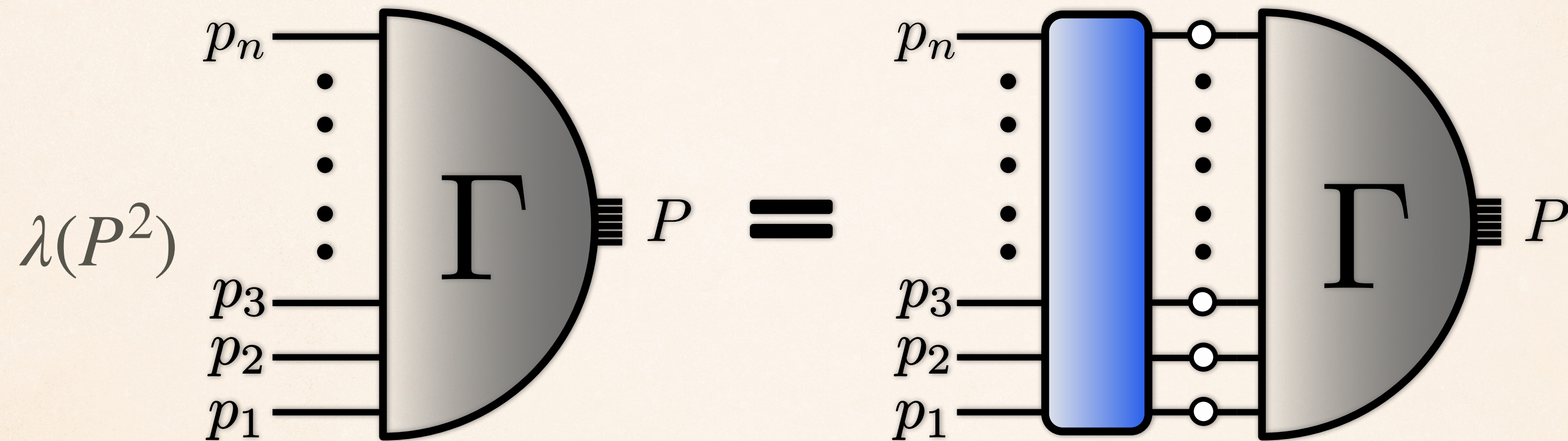
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$$\lambda(P^2) \Gamma = K G \Gamma$$

FUNCTIONAL FRAMEWORK

- ◆ Properties of Hadrons from Hadronic bound state equations

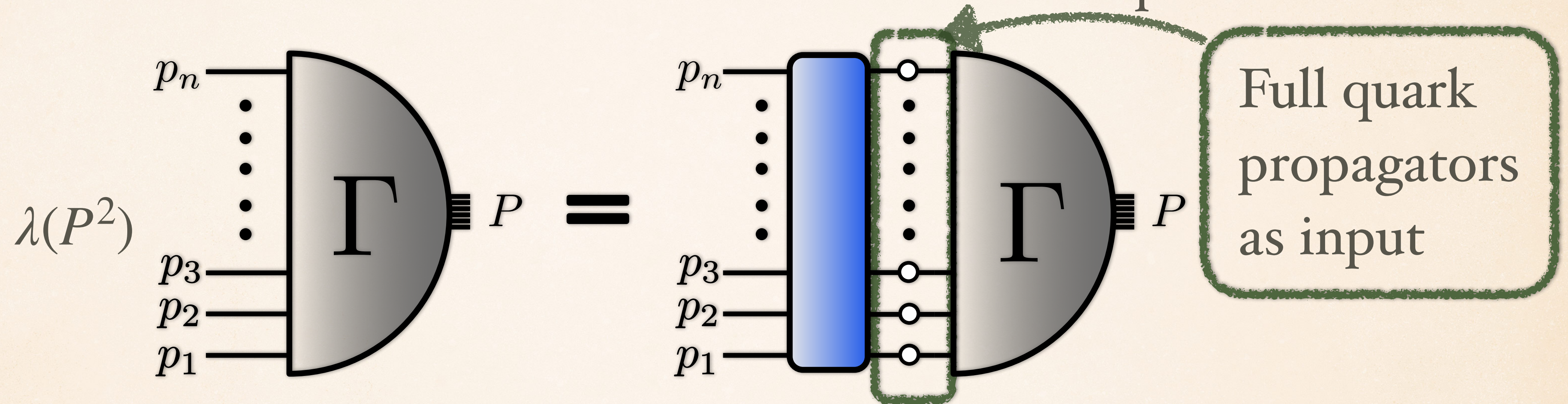


$$\lambda(P^2) \Gamma = K G \Gamma$$

with $\lambda(P^2 = -M^2) = 1$

FUNCTIONAL FRAMEWORK

- ❖ Properties of Hadrons from Hadronic bound state equations



$$\lambda(P^2) \Gamma = K G \Gamma$$

$$\text{with } \lambda(P^2 = -M^2) = 1$$

FUNCTIONAL FRAMEWORK

- ❖ As an input: quantum equations of motion $\rightarrow$ Dyson-Schwinger equations

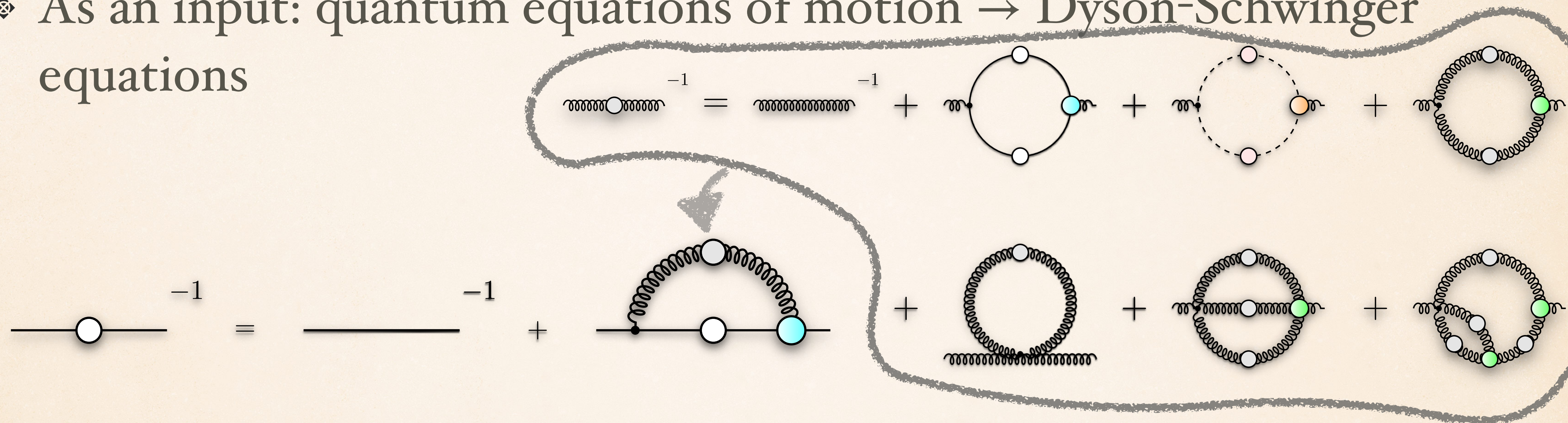
FUNCTIONAL FRAMEWORK

- ❖ As an input: quantum equations of motion $\rightarrow$ Dyson-Schwinger equations

The diagram illustrates the Dyson-Schwinger equation for a fermion propagator. On the left, a horizontal line with a white circular vertex is labeled with -1 above it. This is followed by an equals sign. To the right of the equals sign is a horizontal line labeled with -1 above it, followed by a plus sign. To the right of the plus sign is a more complex diagram: a horizontal line with three vertices. The leftmost vertex is a black dot, the middle is a white circle, and the rightmost is a cyan circle. A wavy line (representing a boson) connects the black dot and the white circle, with a white circle at its peak. Another wavy line connects the white circle and the cyan circle, also with a white circle at its peak.

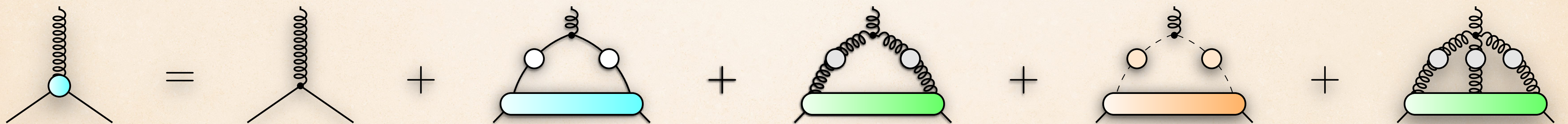
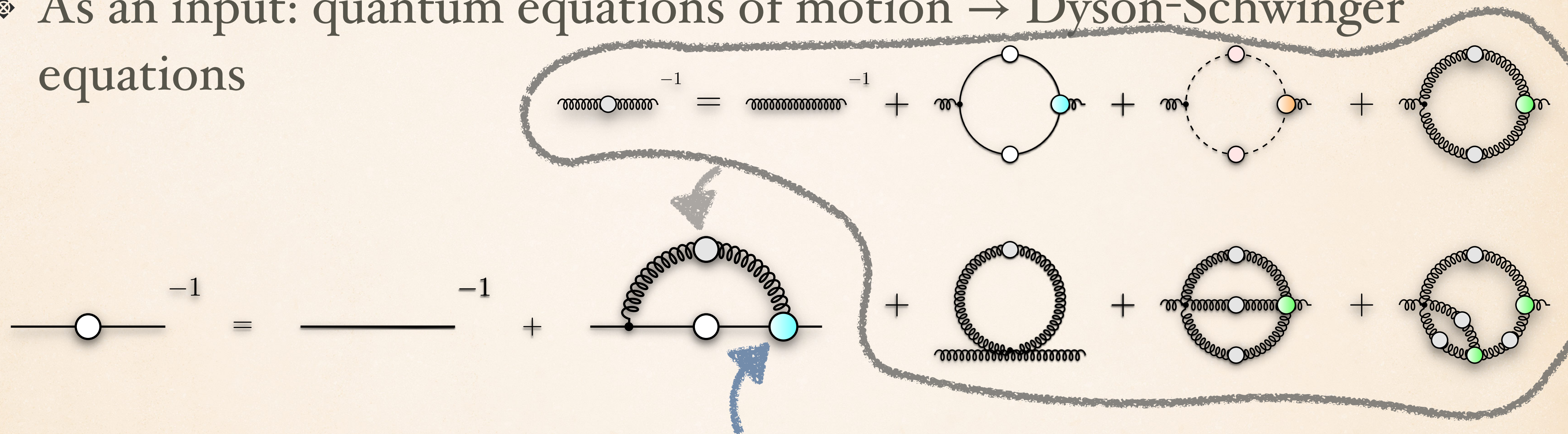
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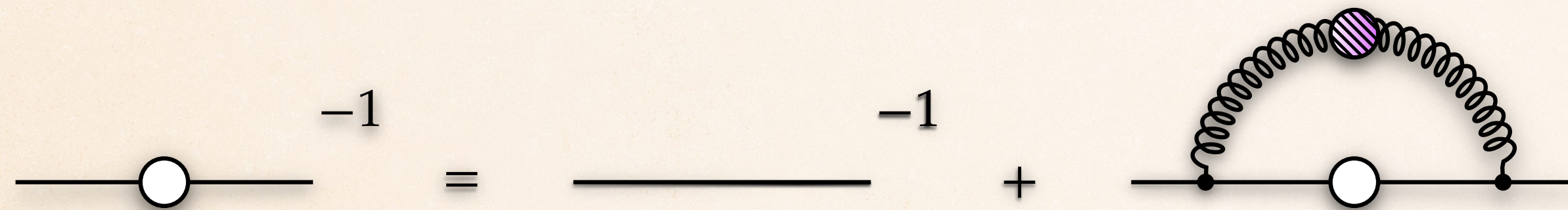
FUNCTIONAL FRAMEWORK

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FUNCTIONAL FRAMEWORK

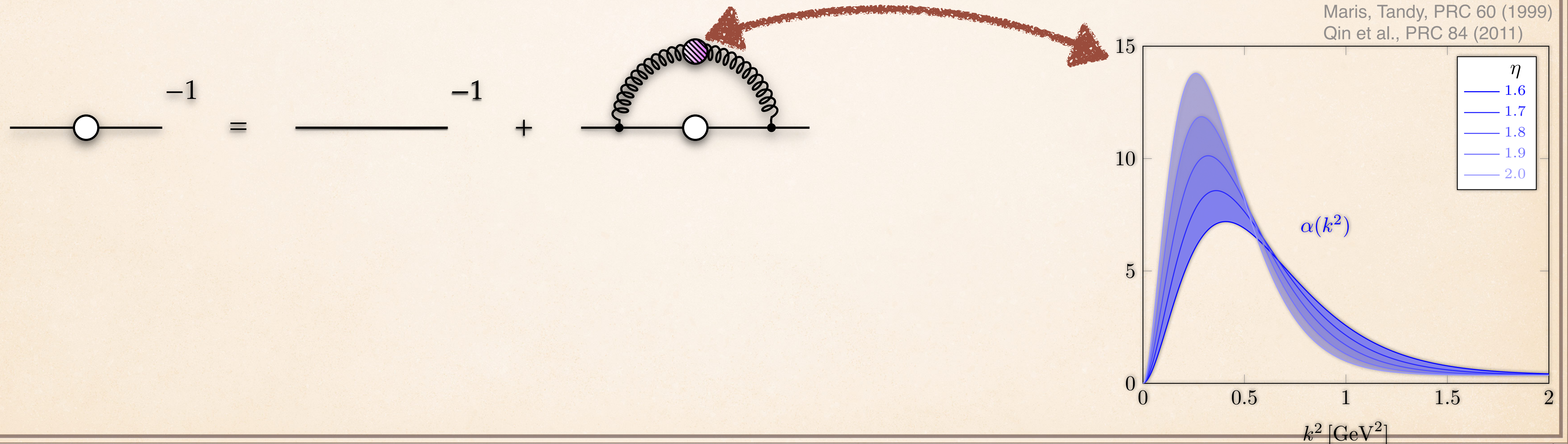
- ❖ As an input: quantum equations of motion $\rightarrow$ Dyson-Schwinger equations
- ❖ Quark-Gluon vertex + Gluon DSE $\rightarrow$ effective gluon exchange



The diagram illustrates the Dyson-Schwinger equation for the quark propagator. On the left, a horizontal line with a white circle in the middle is labeled with a superscript -1 . This is followed by an equals sign. To the right of the equals sign is a horizontal line with a superscript -1 , followed by a plus sign. To the right of the plus sign is a diagram representing a self-energy correction: a horizontal line with a white circle in the middle, and a loop of two wavy lines (gluons) connected to the line at two points. The top vertex of the loop is a shaded circle.

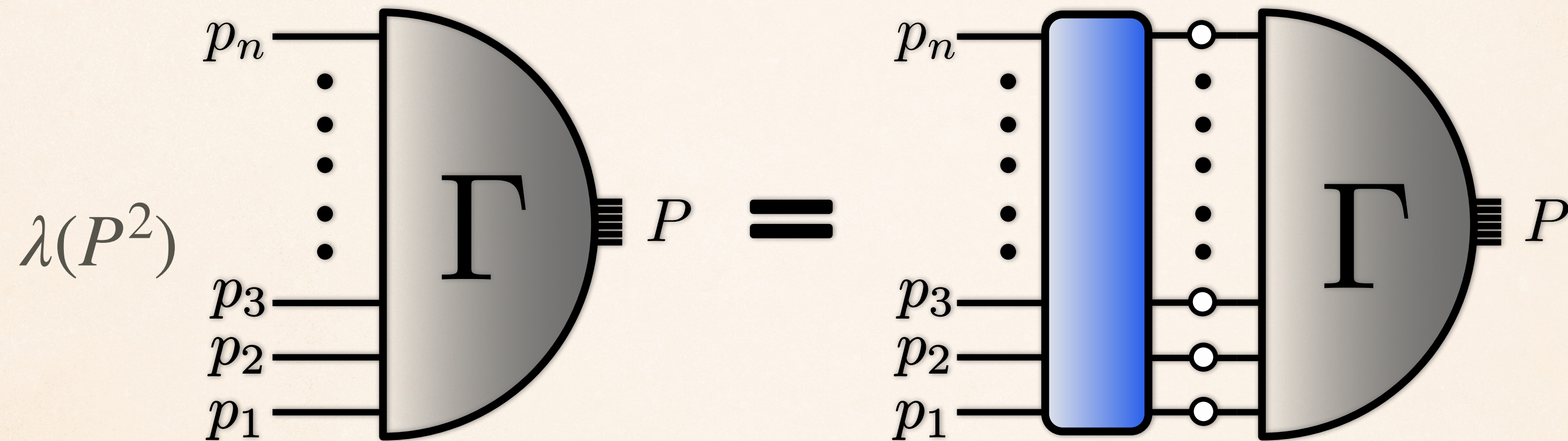
FUNCTIONAL FRAMEWORK

- ❖ As an input: quantum equations of motion $\rightarrow$ Dyson-Schwinger equations
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FUNCTIONAL FRAMEWORK

- ◆ Properties of Hadrons from Hadronic bound state equations

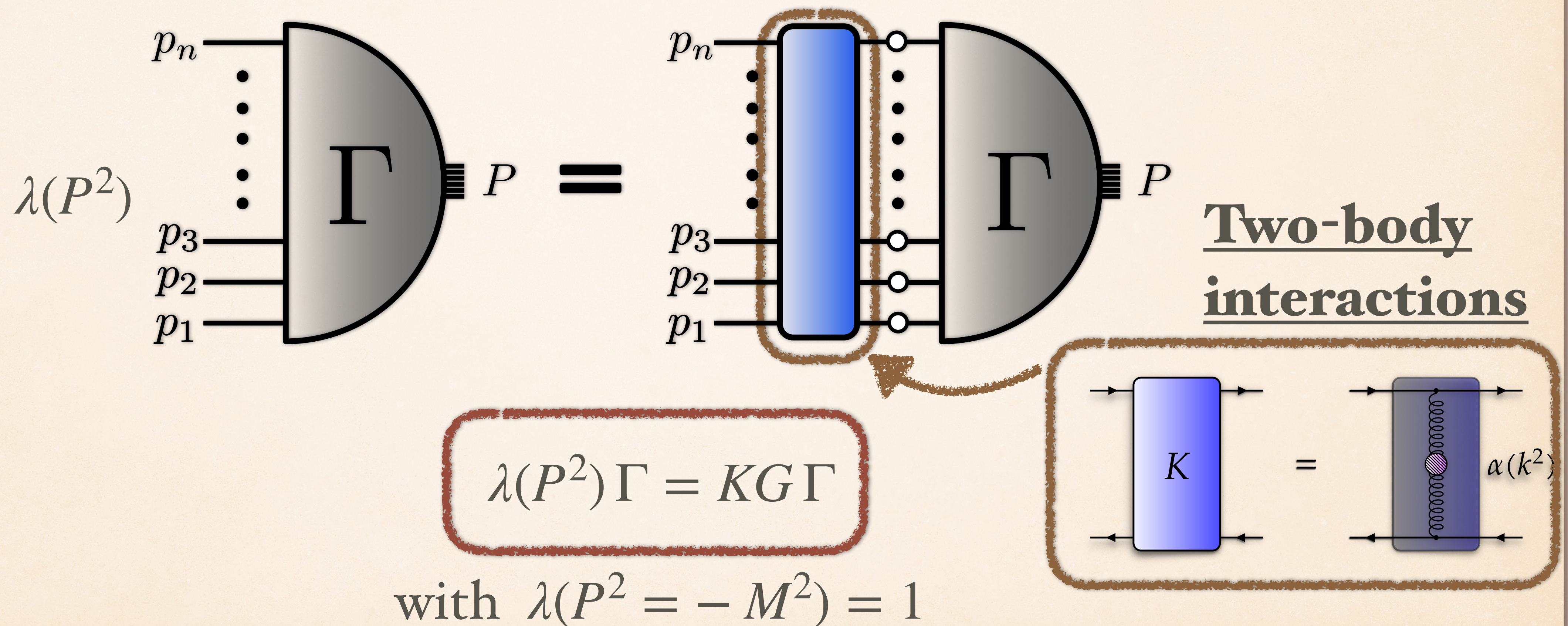


$$\lambda(P^2) \Gamma = K G \Gamma$$

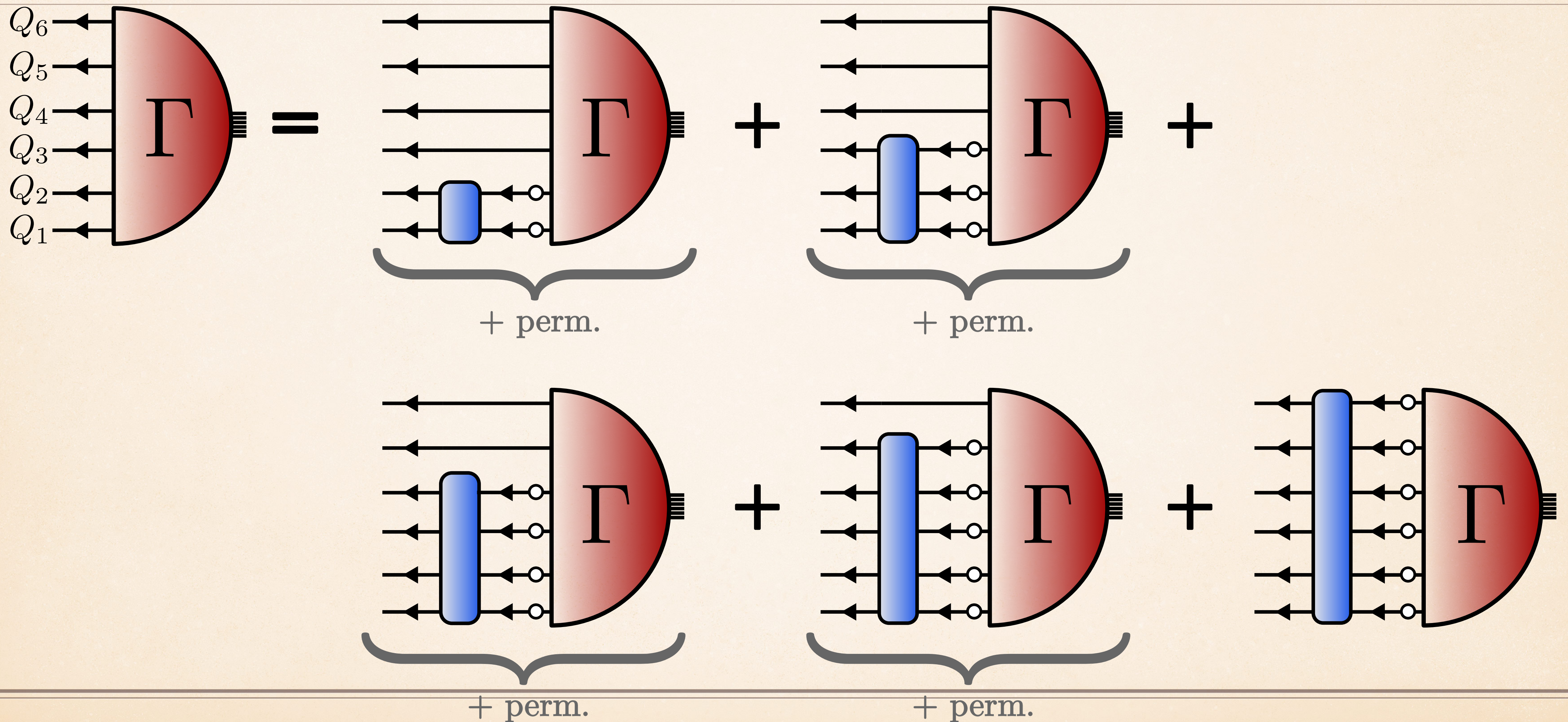
$$\text{with } \lambda(P^2 = -M^2) = 1$$

FUNCTIONAL FRAMEWORK

- ❖ Properties of Hadrons from Hadronic bound state equations

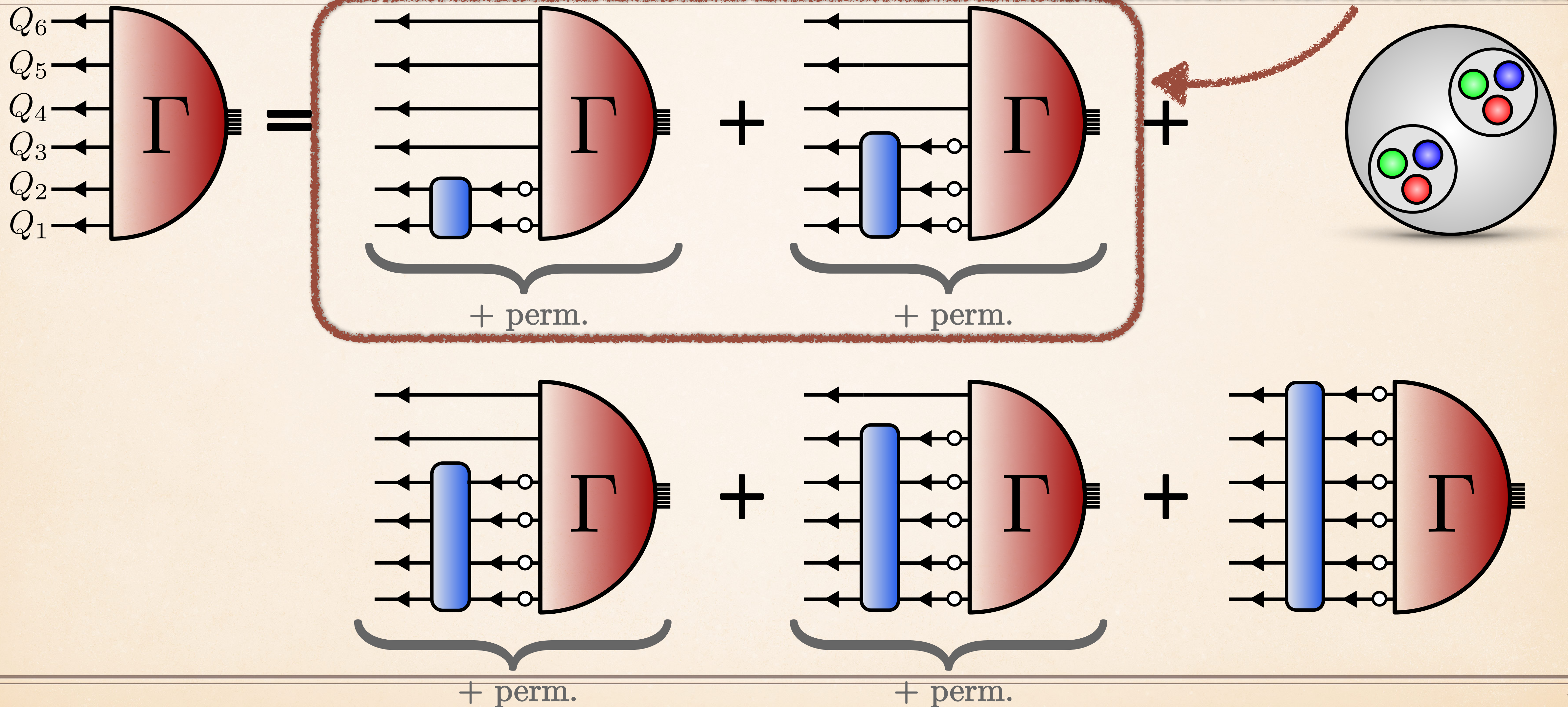


HEXAQUARK BSE



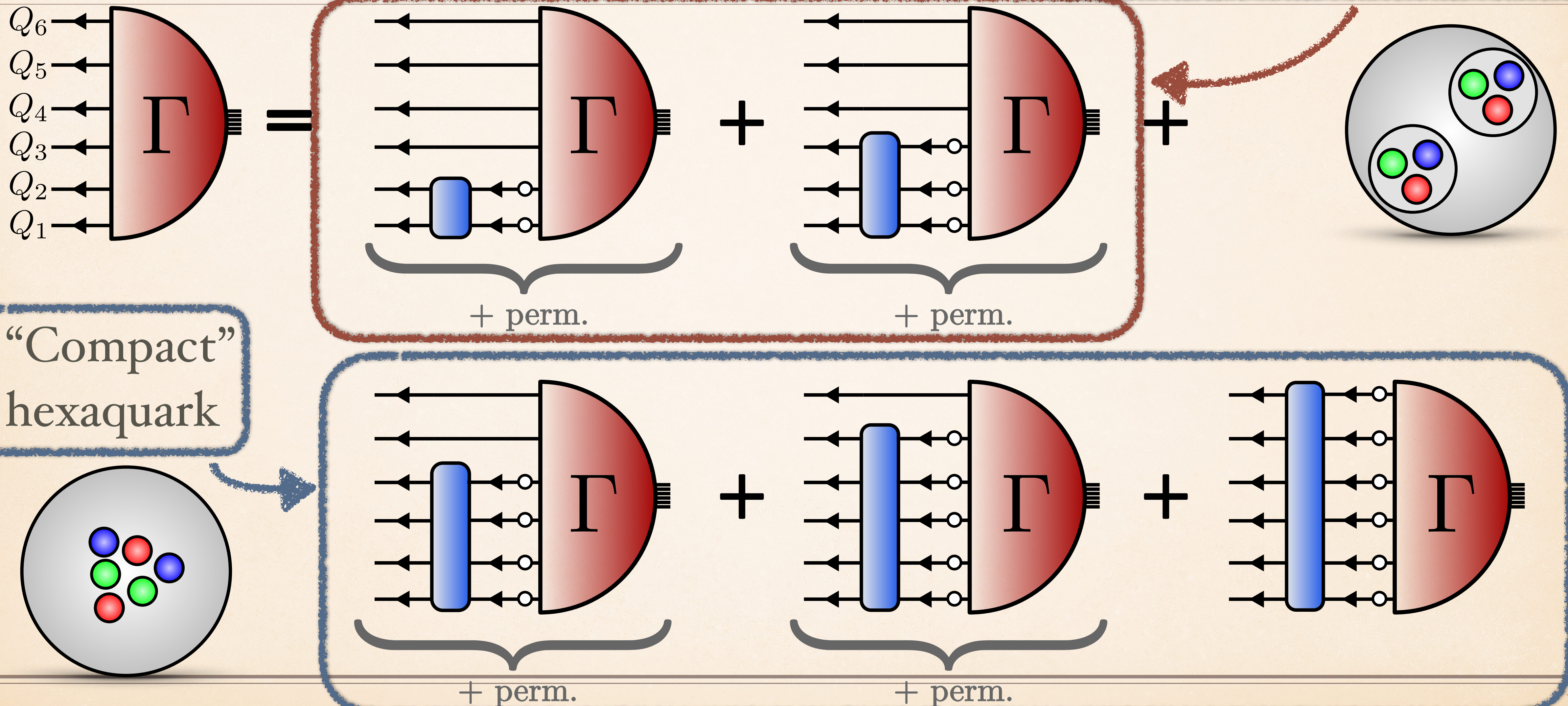
HEXAQUARK BSE

Baryon-Baryon dynamics

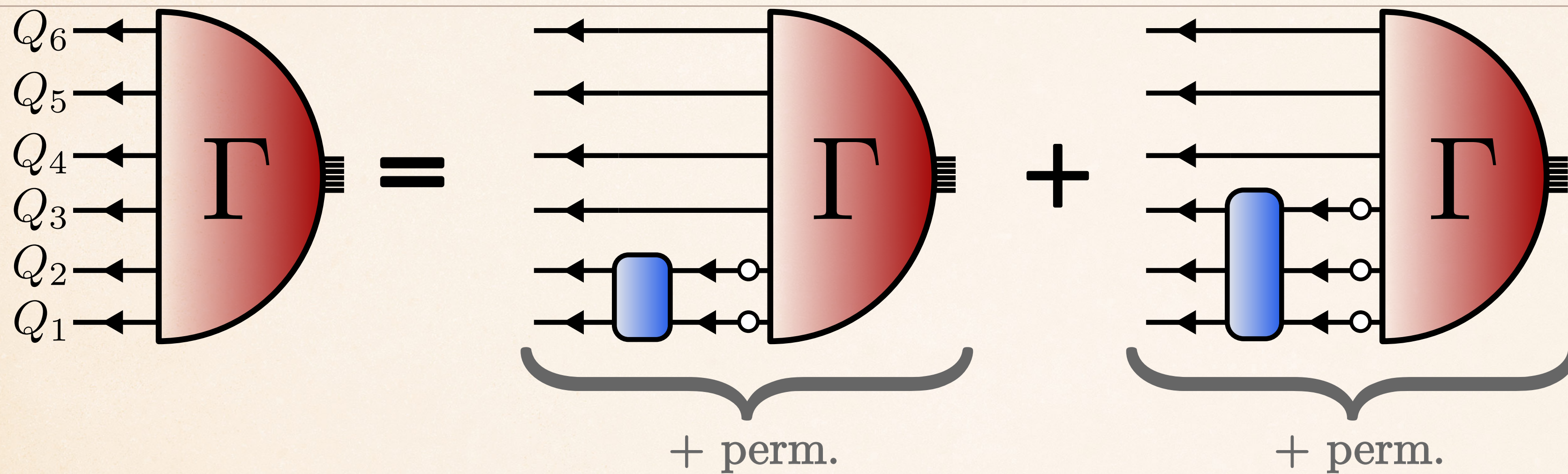


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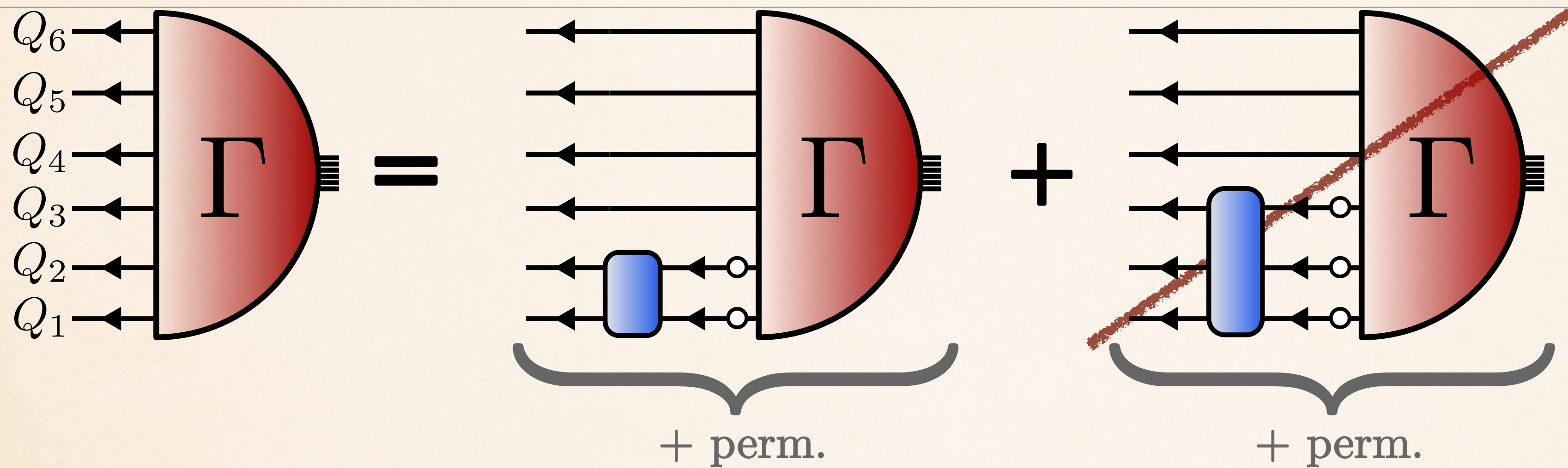
Baryon-Baryon dynamics



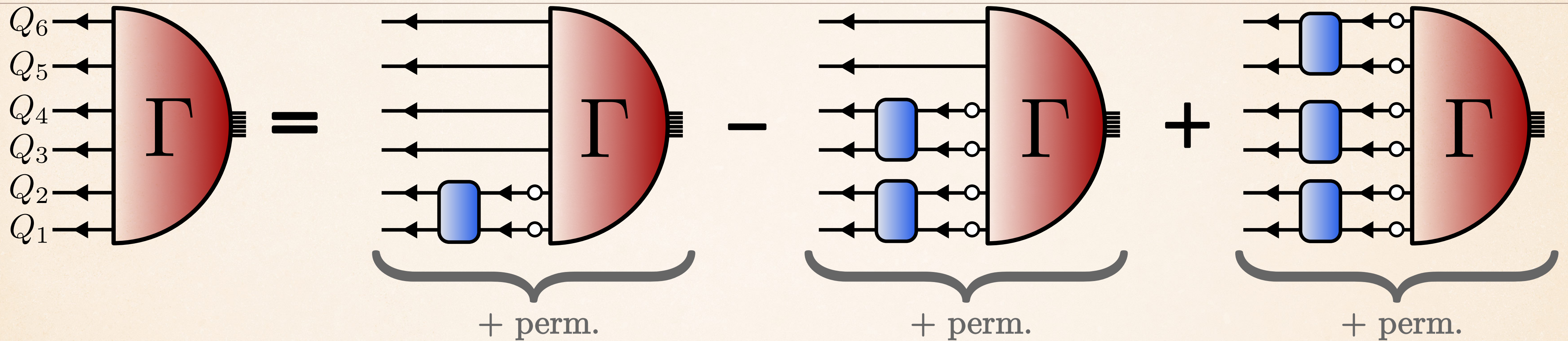
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HEXAQUARK BSE



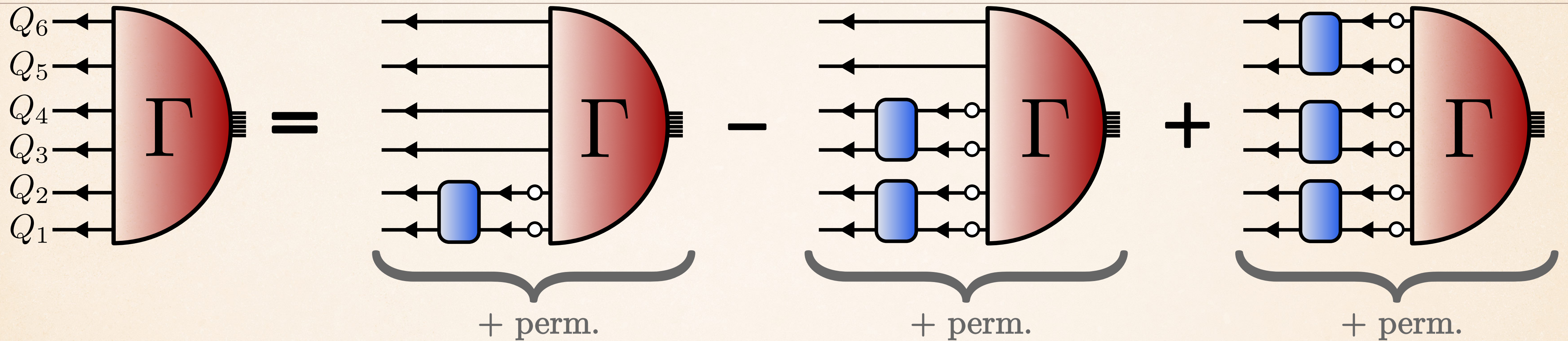
HEXAQUARK BSE



Huang, Weldon, PRD 11 (1975)

Kvinikhidze, Khvedelidze, Theor.Math.Phys. 90 (1992)

HEXAQUARK BSE



Huang, Weldon, PRD 11 (1975)
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10 Baryon-Baryon combinations:

(123)(456)	(125)(346)	(134)(256)	(136)(245)	(146)(235)
(124)(356)	(126)(345)	(135)(246)	(145)(236)	(156)(234)

SIX-QUARK BSA

$$\Gamma^{(\mu)}(k, q, p, r, l, P) = \sum_{i=1}^N f_i(\Omega) \tau_i^{(\mu)}(k, q, p, r, l, P) \otimes \Gamma_C \otimes \Gamma_F$$

SIX-QUARK BSA

$$\Gamma^{(\mu)}(k, q, p, r, l, P) = \sum_{i=1}^N f_i(\Omega) \tau_i^{(\mu)}(k, q, p, r, l, P) \otimes \Gamma_C \otimes \Gamma_F$$

- ❖ Overall BSA needs to be antisymmetric under quark exchange
→ Express in terms of irreducible representations of S_6 →
antisinglet $\bar{\mathbf{1}}$

PERMUTATION GROUP S_6

PERMUTATION GROUP S_6

❖ 720 elements: f_{123456} , ...

PERMUTATION GROUP S_6

- ❖ 720 elements: f_{123456} , ...
- ❖ Transposition T : $1 \leftrightarrow 2$

PERMUTATION GROUP S_6

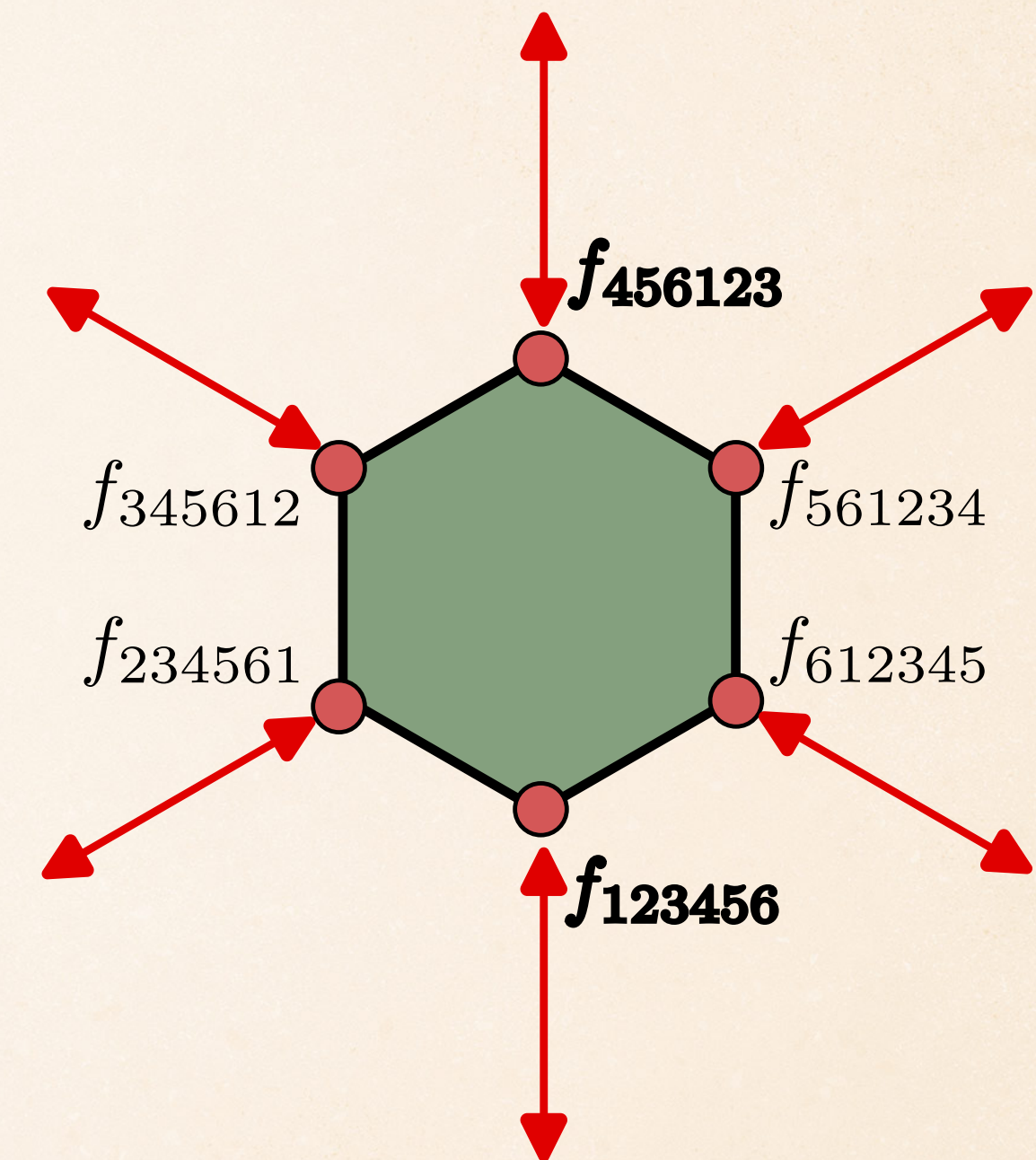
- ❖ 720 elements: f_{123456} , ...
- ❖ Transposition T : $1 \leftrightarrow 2$
- ❖ Cyclic permutation C :
 $1 \rightarrow 2 \rightarrow \dots \rightarrow 6 \rightarrow 1$

PERMUTATION GROUP S_6

- ❖ 720 elements: f_{123456} , ...
- ❖ Transposition T : $1 \leftrightarrow 2$
- ❖ Cyclic permutation C :
 $1 \rightarrow 2 \rightarrow \dots \rightarrow 6 \rightarrow 1$
- ❖ Generate group $S_6 = \langle T, C \rangle$

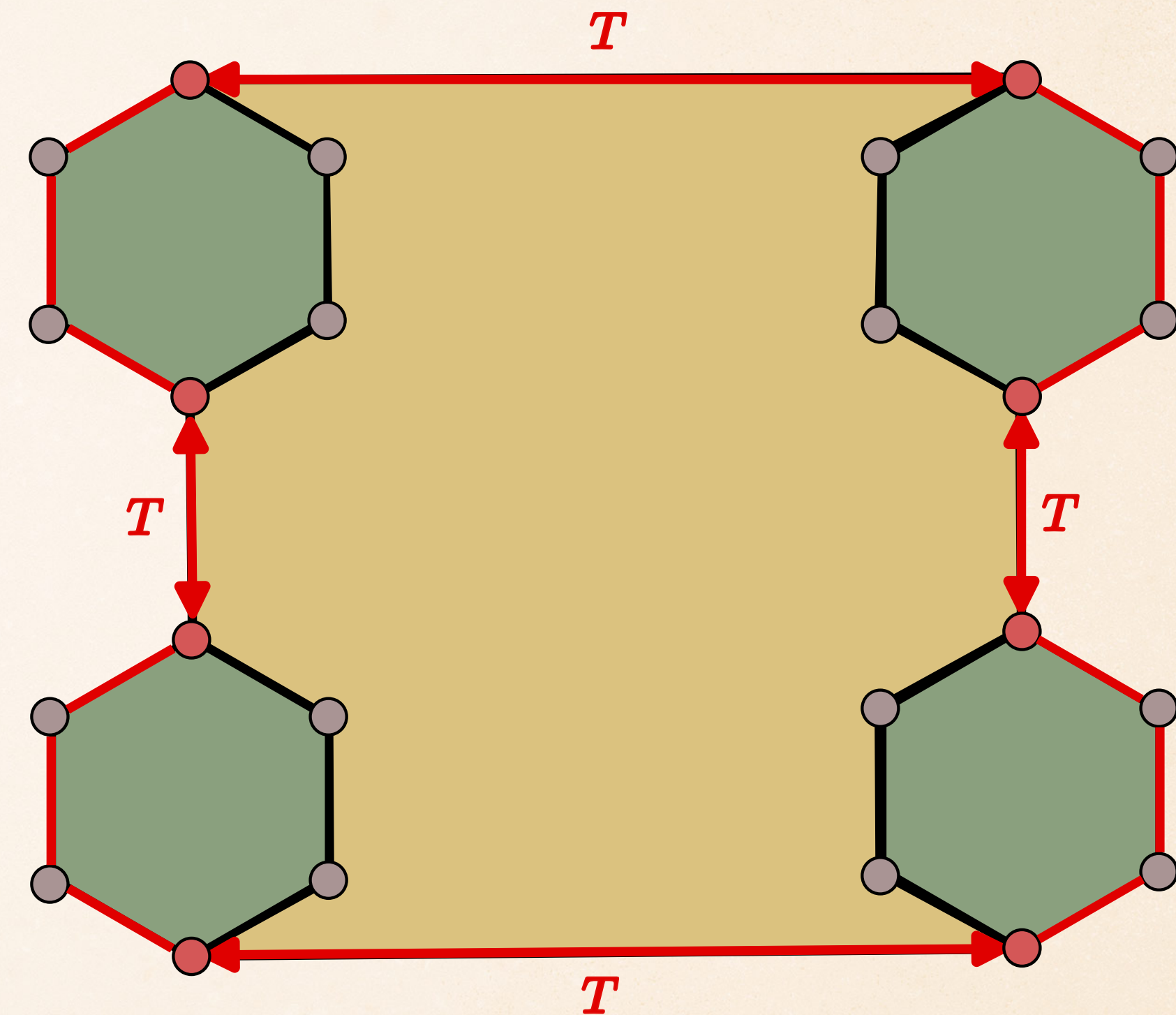
PERMUTATION GROUP S_6

- ❖ 720 elements: f_{123456} , ...
- ❖ Transposition T : $1 \leftrightarrow 2$
- ❖ Cyclic permutation C :
 $1 \rightarrow 2 \rightarrow \dots \rightarrow 6 \rightarrow 1$
- ❖ Generate group $S_6 = \langle T, C \rangle$
- ❖ Use $C^6 \rightarrow$ **120 Hexagons** connected by T

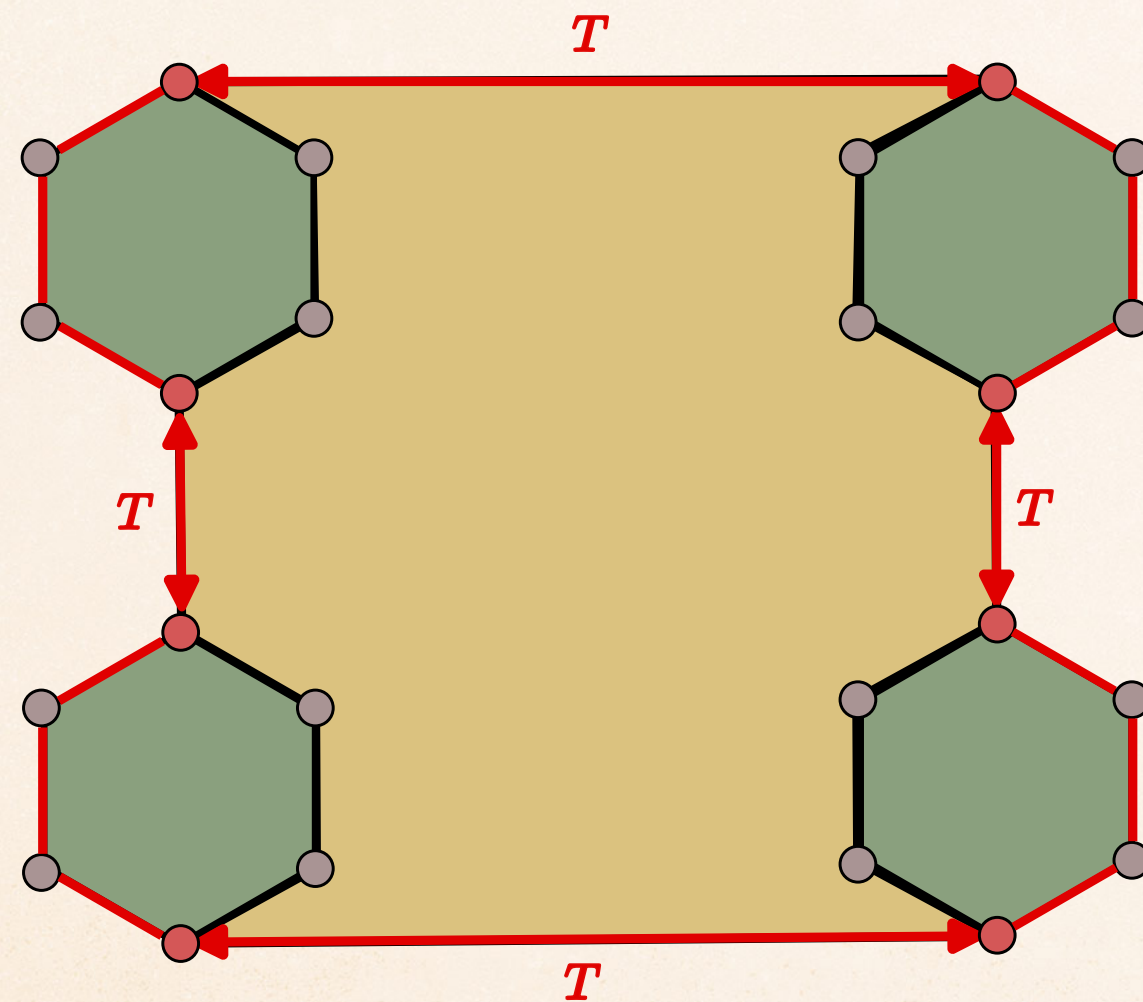


PERMUTATION GROUP S_6

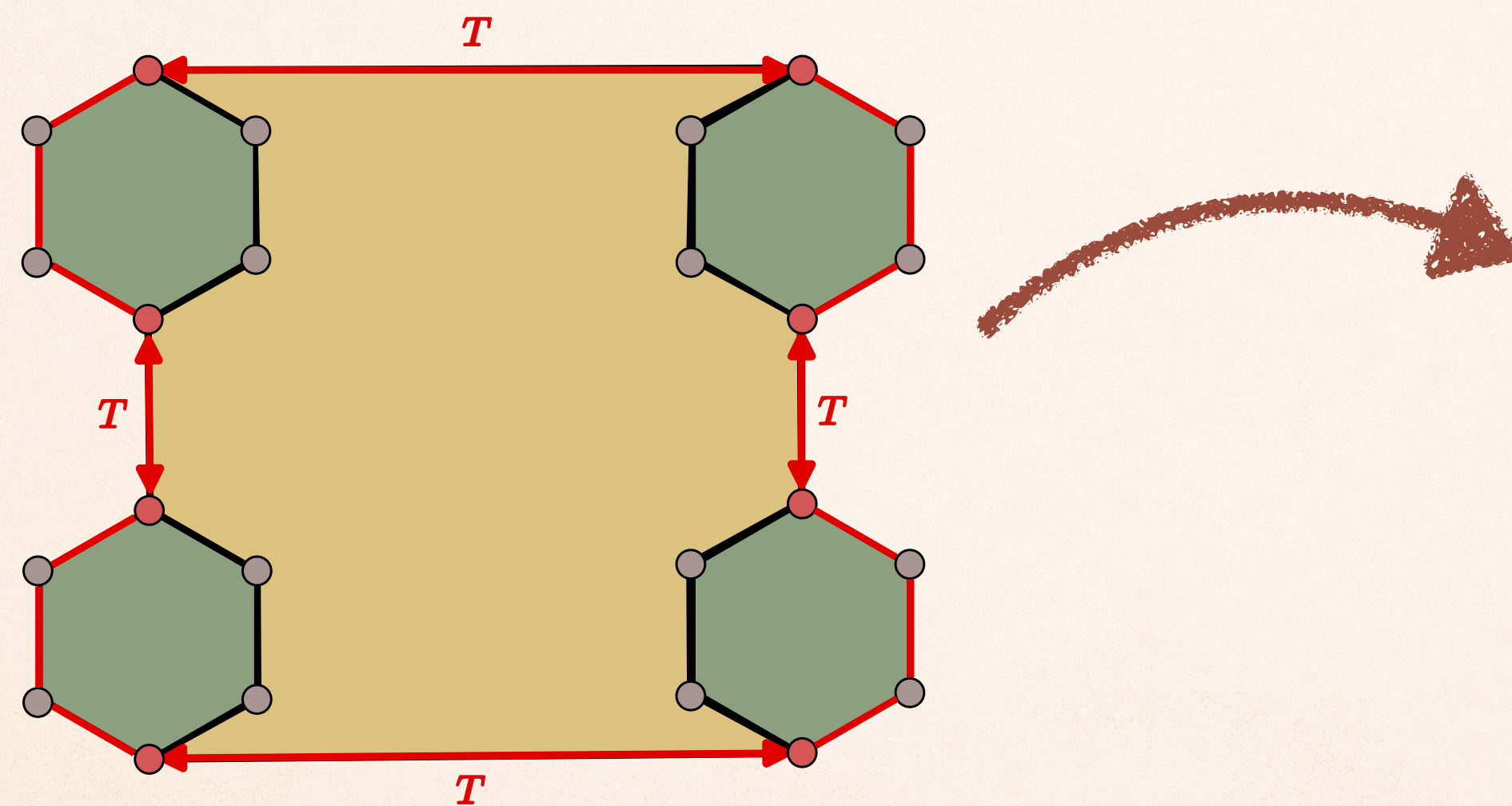
- ❖ 720 elements: $f_{123456}, \dots$
- ❖ Transposition $T: 1 \leftrightarrow 2$
- ❖ Cyclic permutation C :
 $1 \rightarrow 2 \rightarrow \dots \rightarrow 6 \rightarrow 1$
- ❖ Generate group $S_6 = \langle T, C \rangle$
- ❖ 120 Hexagons connected by T
- ❖ Repeat $C^3 T$
 $\rightarrow$ 90 “plaquettes” with 8 elements



CHESSEBOARD PLANES FOR S_6



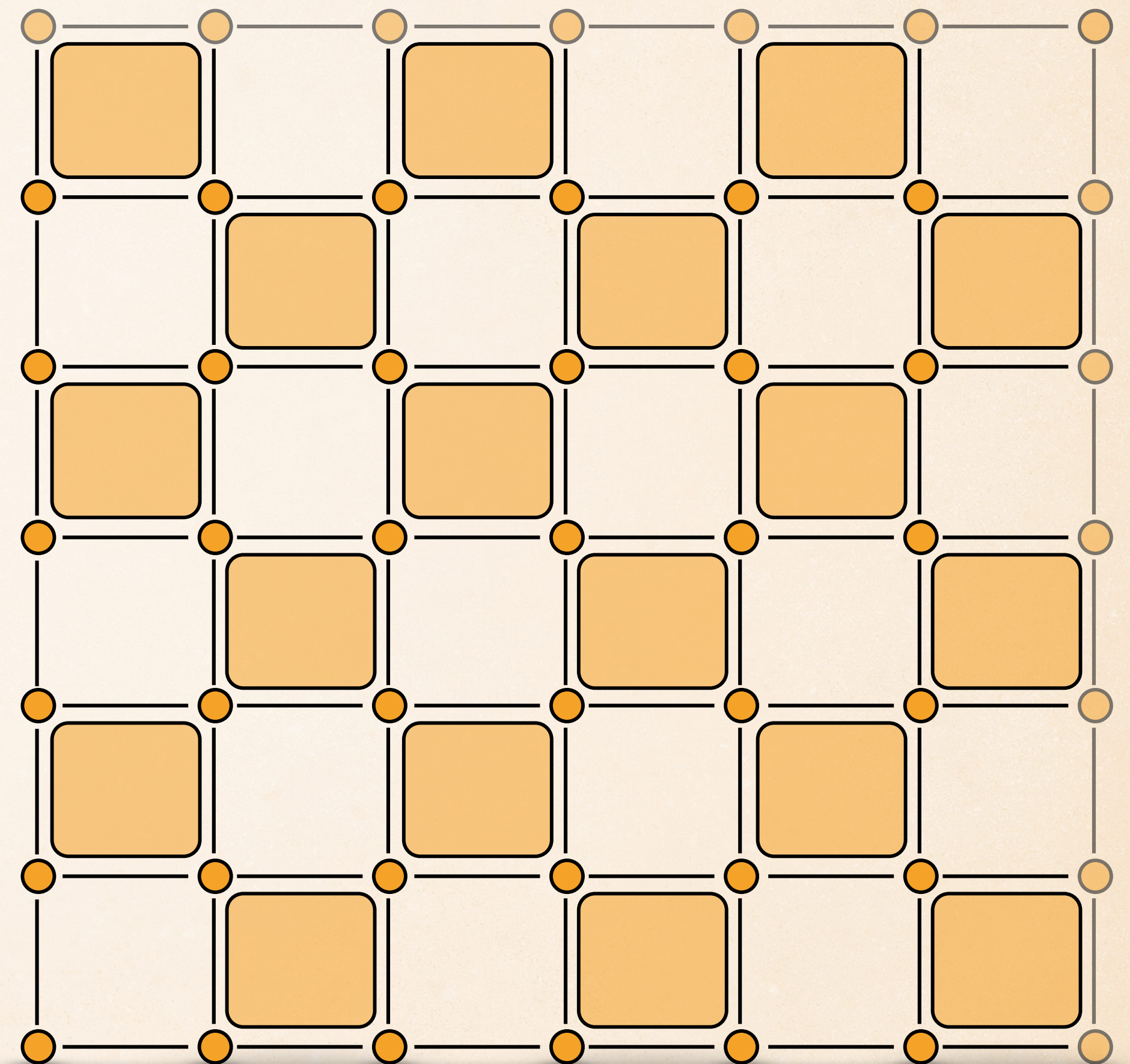
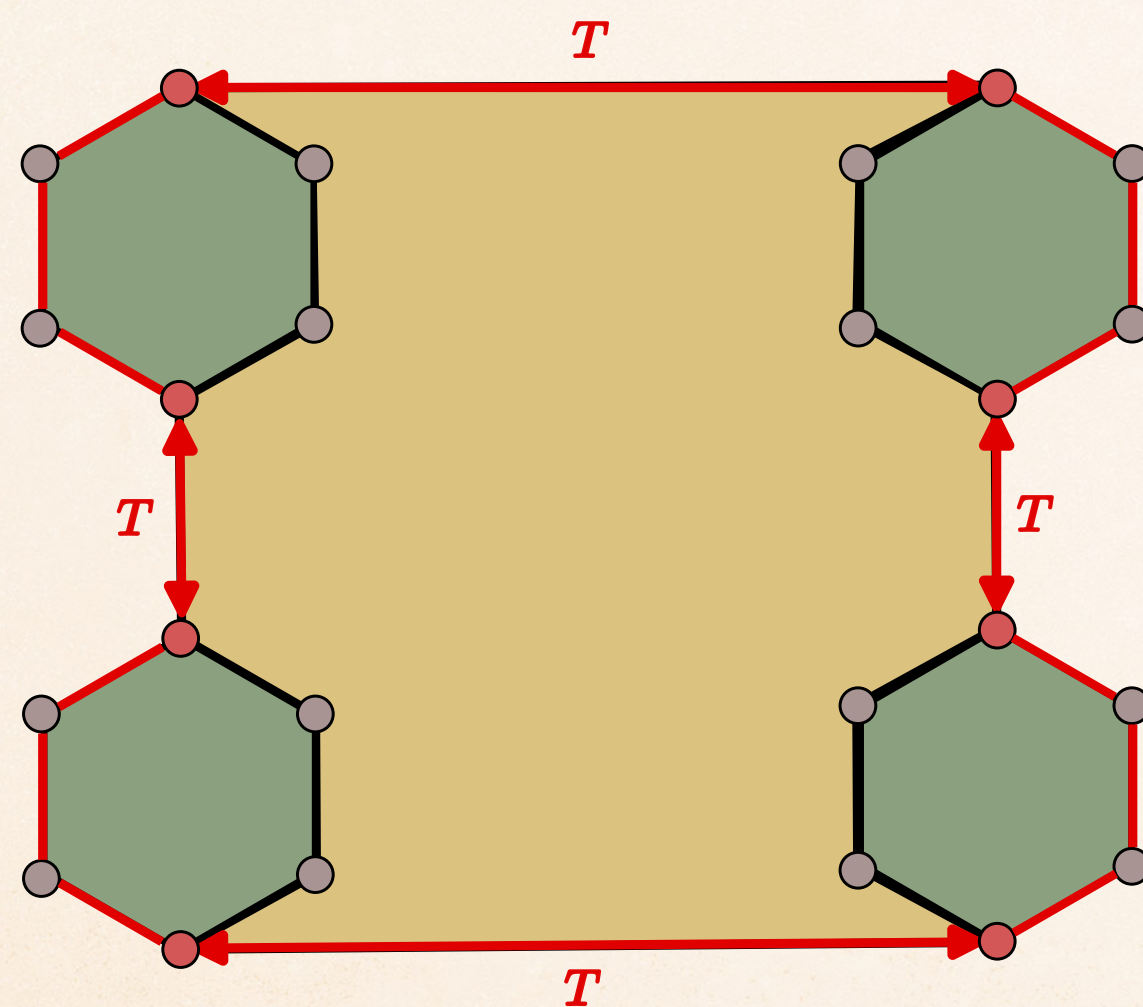
CHESSEBOARD PLANES FOR S_6



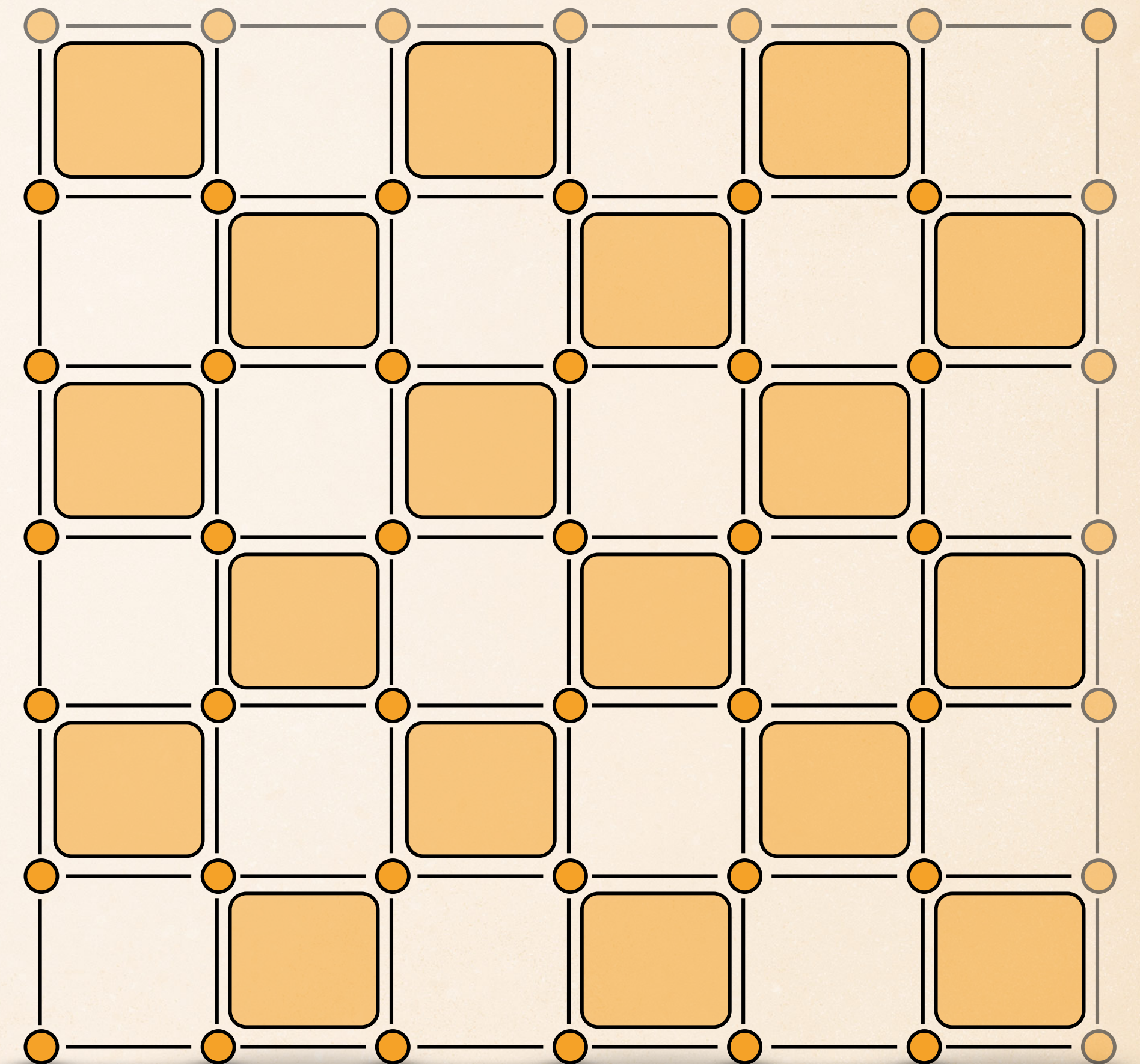
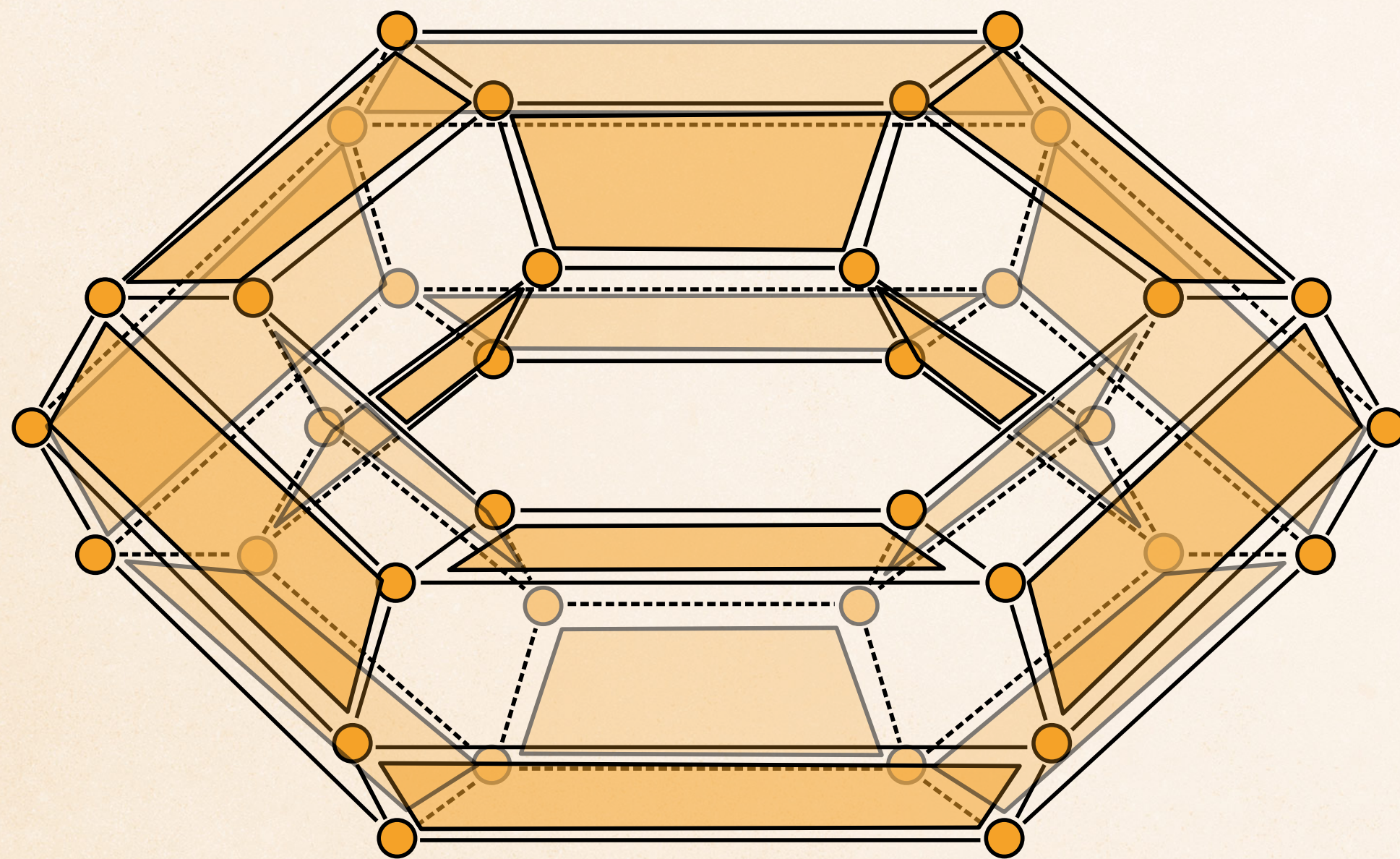
CHESSBOARD PLANES FOR S_6

❖ Chessboard Planes

❖ 18 plaquettes $\rightarrow$ 144 elements



CHESSBOARD PLANES FOR S_6



CHESSEBOARD PLANES FOR S_6

- ❖ 10 unique Chessboard Planes with 18 “plaquettes”
- ❖ Pick 9 “plaquettes” per Chessboard Plane → get all 90 once

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CHESSBOARD PLANES FOR S_6

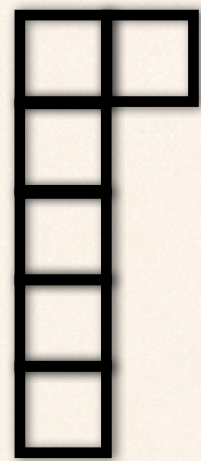
- ❖ 10 unique Chessboard Planes with 18 “plaquettes”
- ❖ Pick 9 “plaquettes” per Chessboard Plane $\rightarrow$ get all 90 once
- ❖ Sum elements to get explicit expressions of irreducible representations of S_6

IRREDUCIBLE REPRESENTATIONS OF S_6



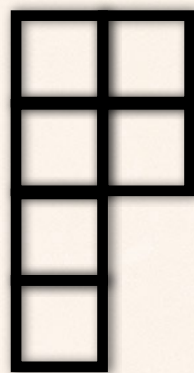
$\bar{1}$

$\mathcal{A}$



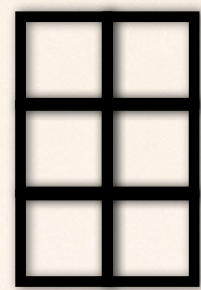
$\bar{5}$

$\mathcal{V}^-$



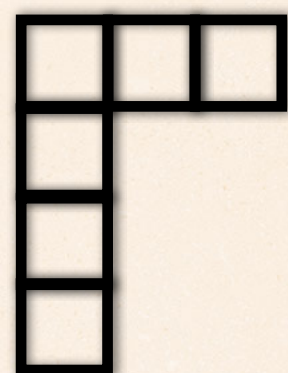
$\bar{9}$

$\mathcal{N}^-$



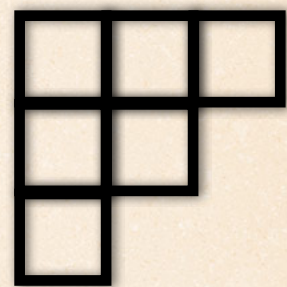
$\bar{5}'$

$\mathcal{Q}^-$



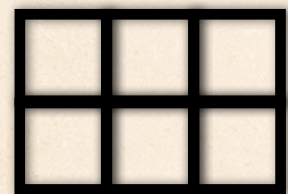
$\bar{10}$

$\mathcal{D}^-$



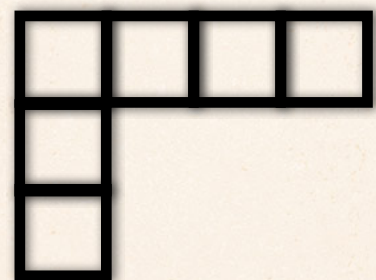
16

$\mathcal{X}$



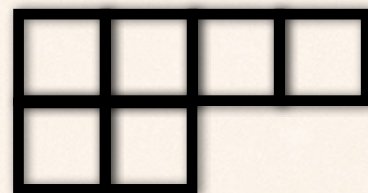
$5'$

$\mathcal{Q}^+$



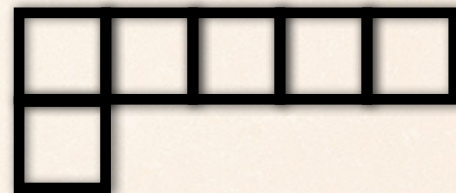
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$\mathcal{D}^+$



9

$\mathcal{N}^+$



5

$\mathcal{V}^+$



1

$\mathcal{S}$

SIX-QUARK BSA

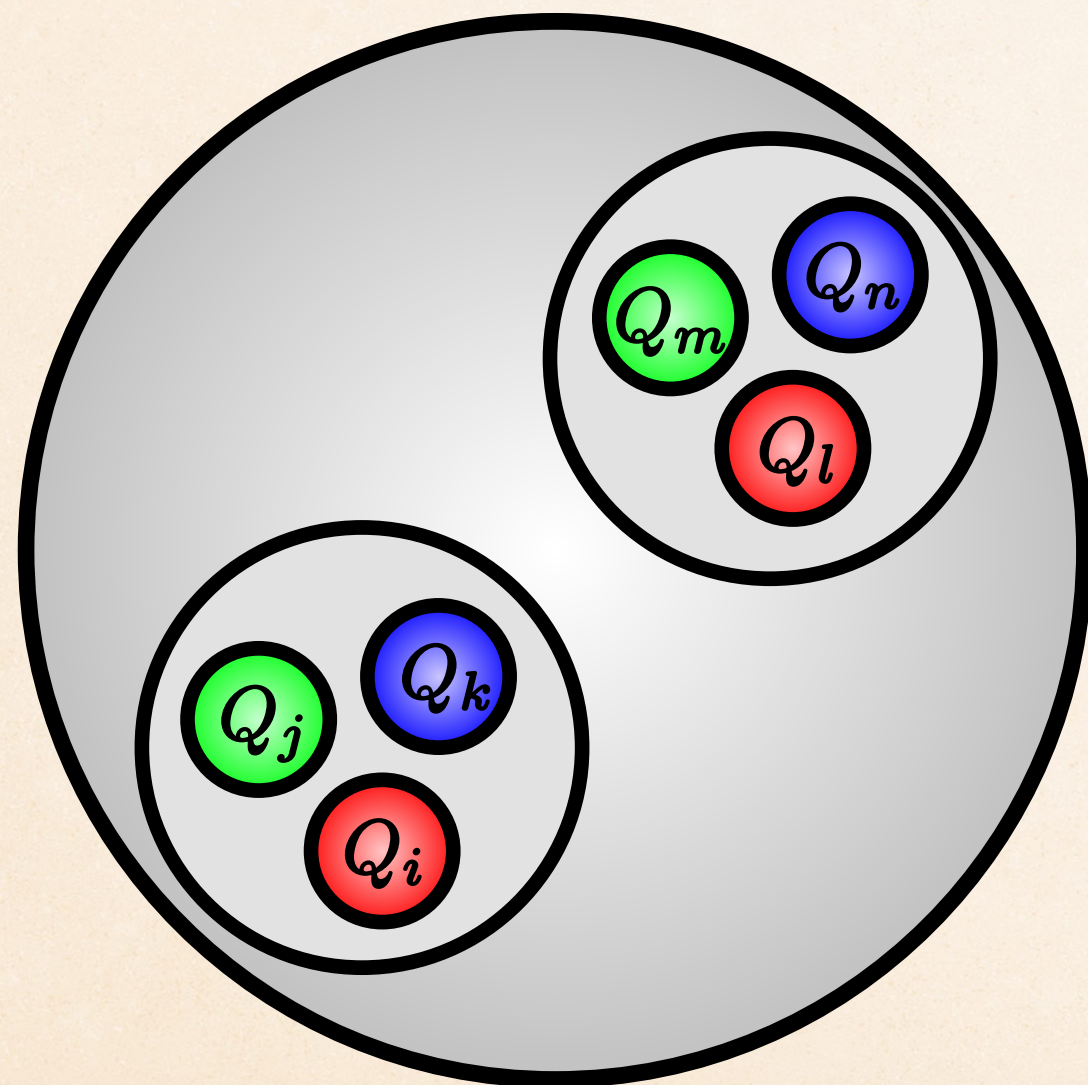
$$\Gamma^{(\mu)}(k, q, p, r, l, P) = \sum_{i=1}^N f_i(\Omega) \tau_i^{(\mu)}(k, q, p, r, l, P) \otimes \Gamma_C \otimes \Gamma_F$$

- ❖ Overall BSA needs to be antisymmetric under quark exchange
→ Express in terms of irreducible representations of S_6 →
antisinglet $\bar{\mathbf{1}}$
- ❖ $\mathbf{M}_D \otimes \mathbf{M}_C \otimes \mathbf{M}_F = \bar{\mathbf{1}}$

SIX-QUARK BSA

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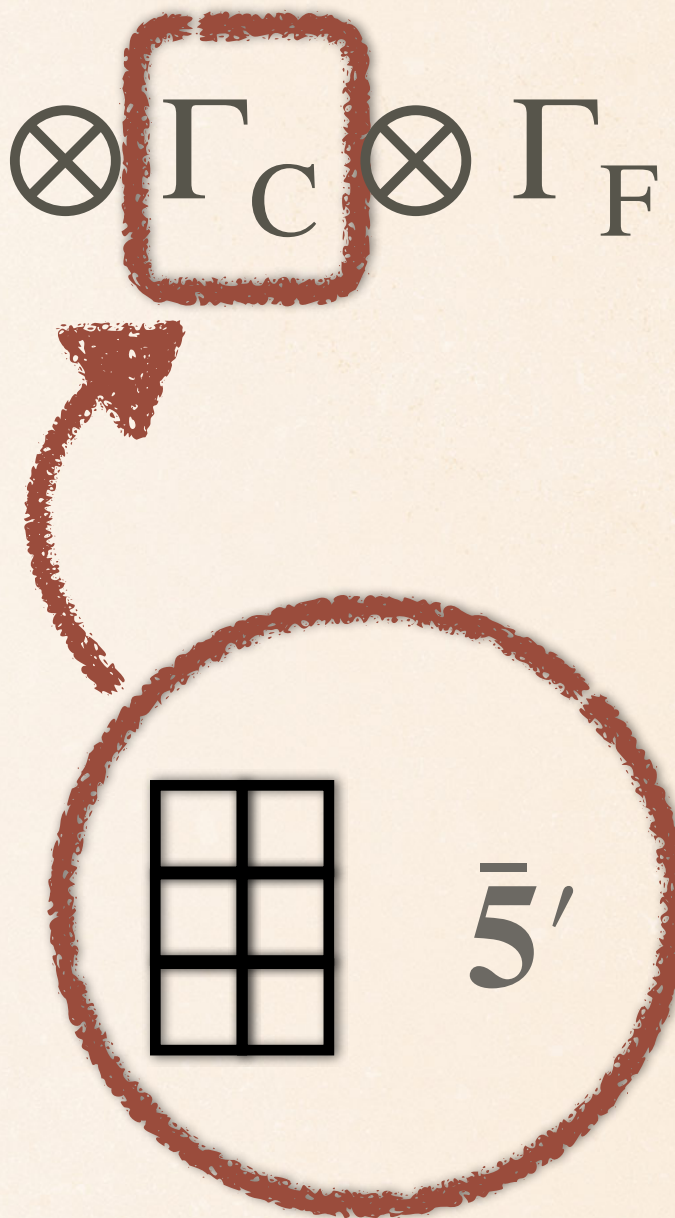
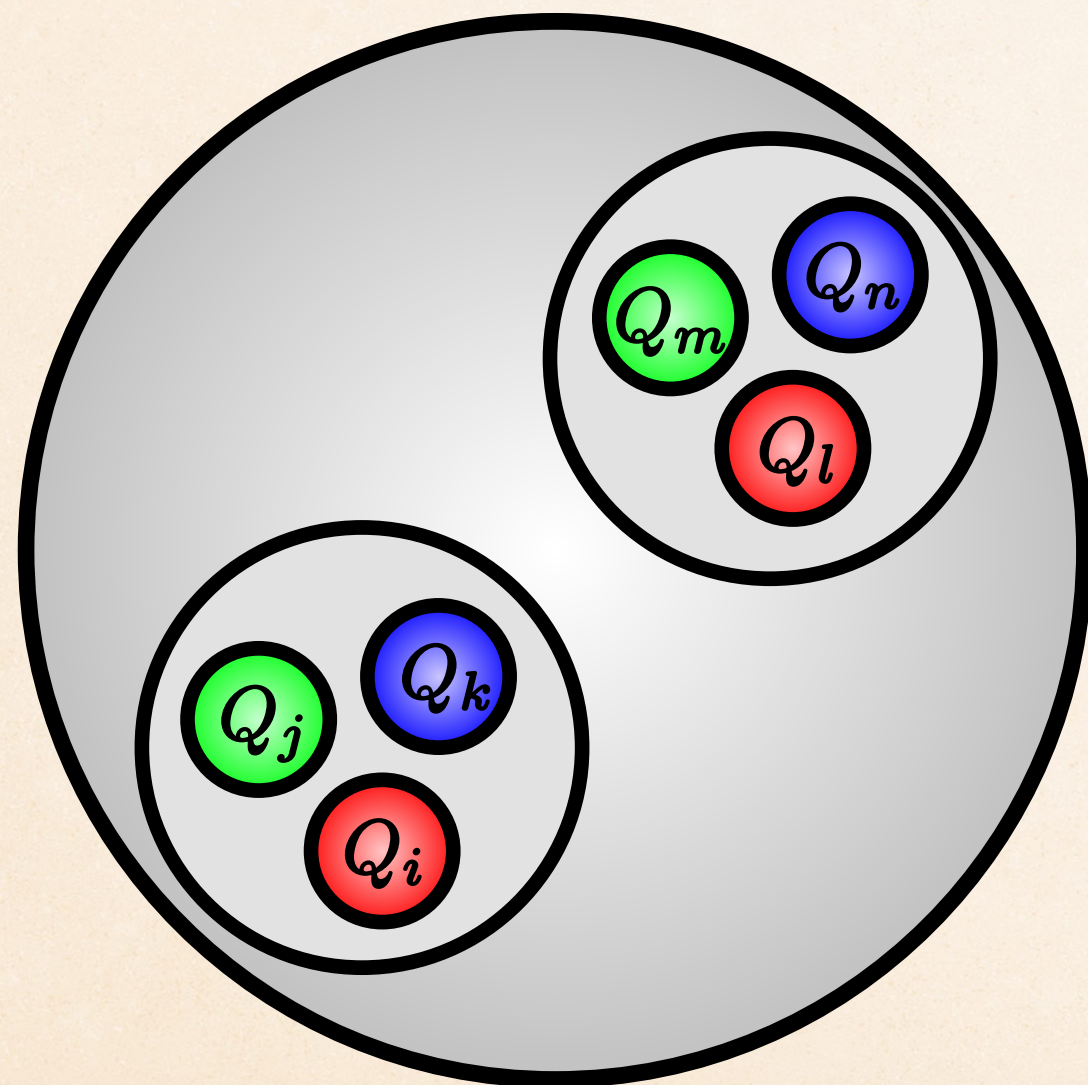
$$\Gamma_C = \varepsilon^{A_i A_j A_k} \varepsilon^{A_l A_m A_n}$$



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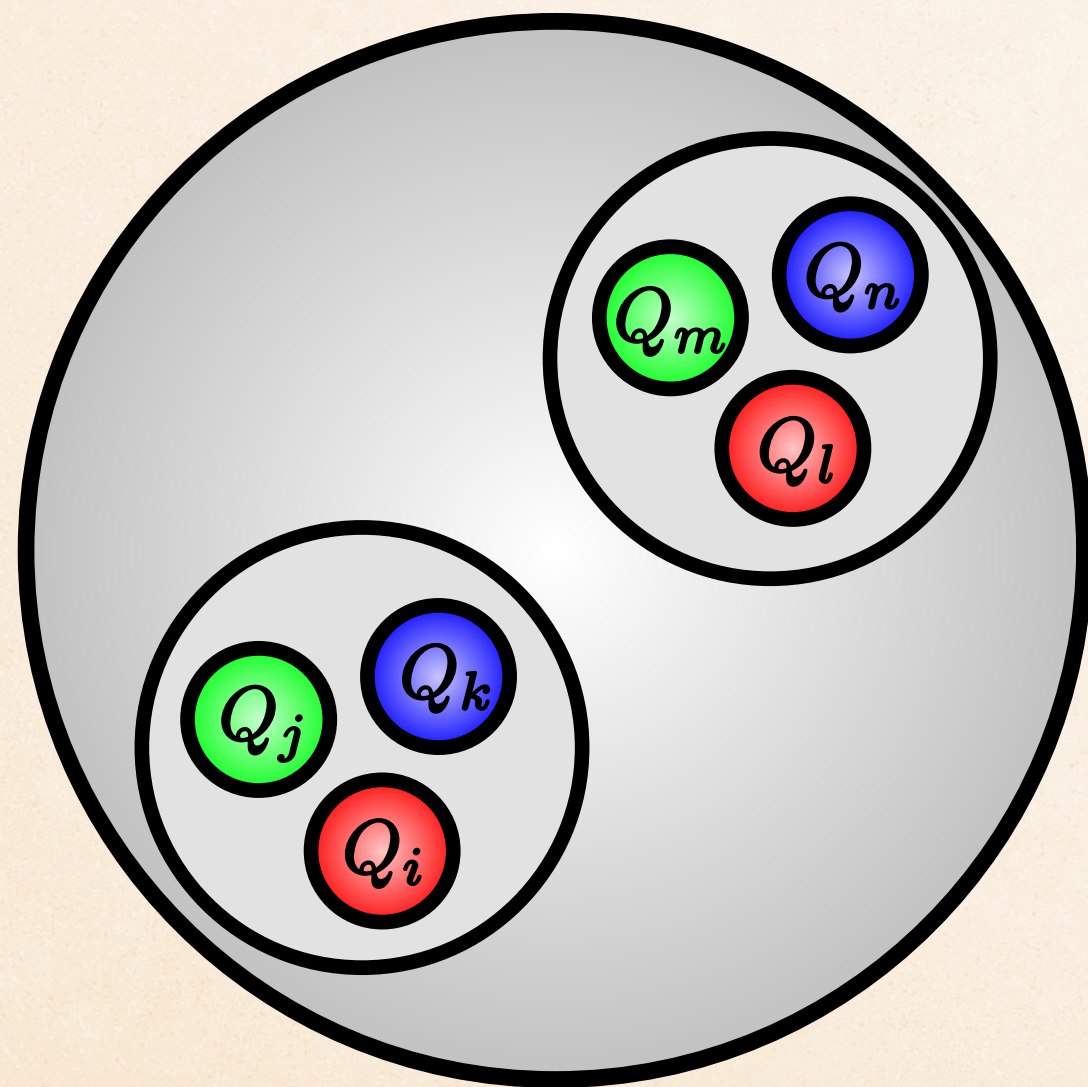
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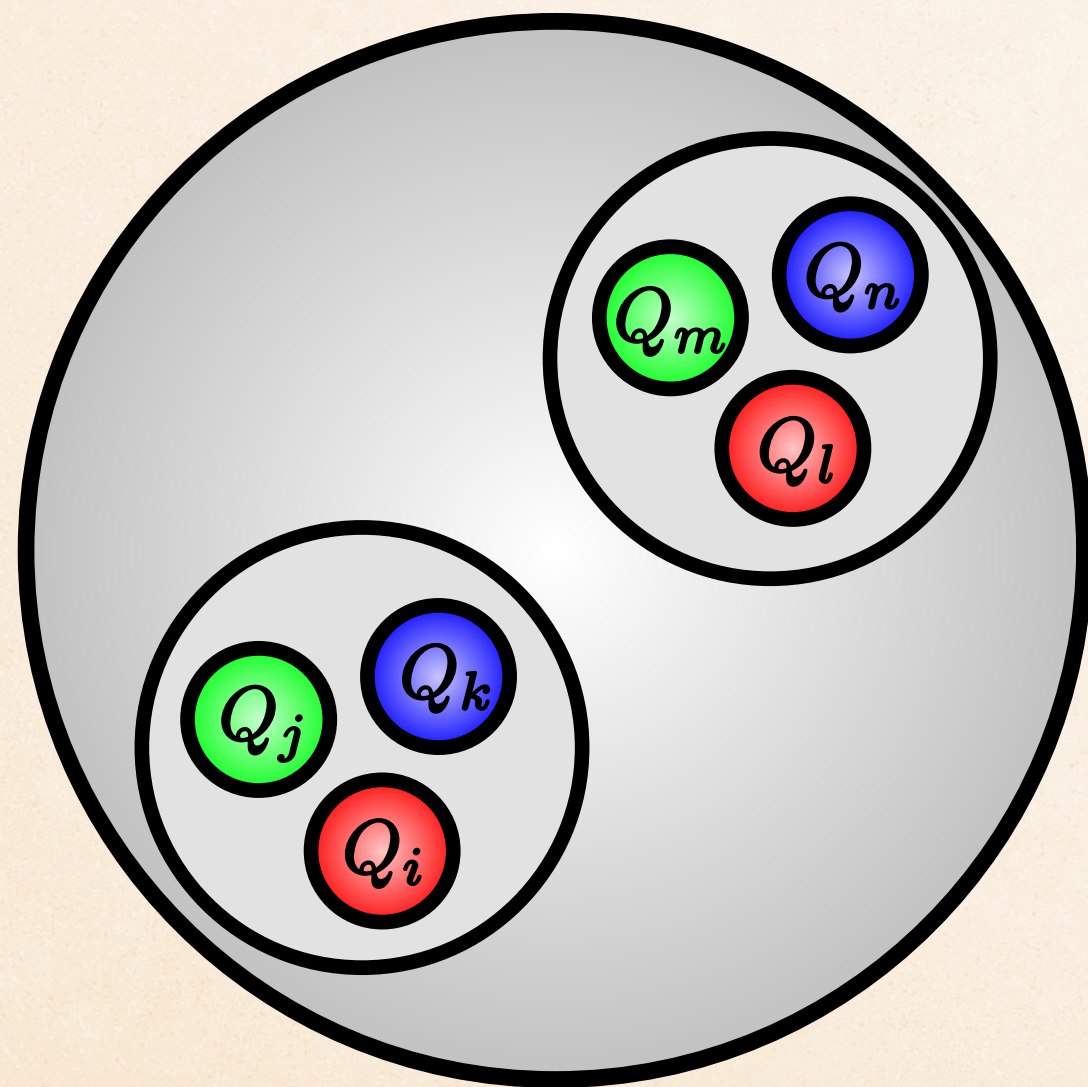
$$Q_i Q_j Q_k Q_l Q_m Q_m = u u d u d d$$



SIX-QUARK BSA

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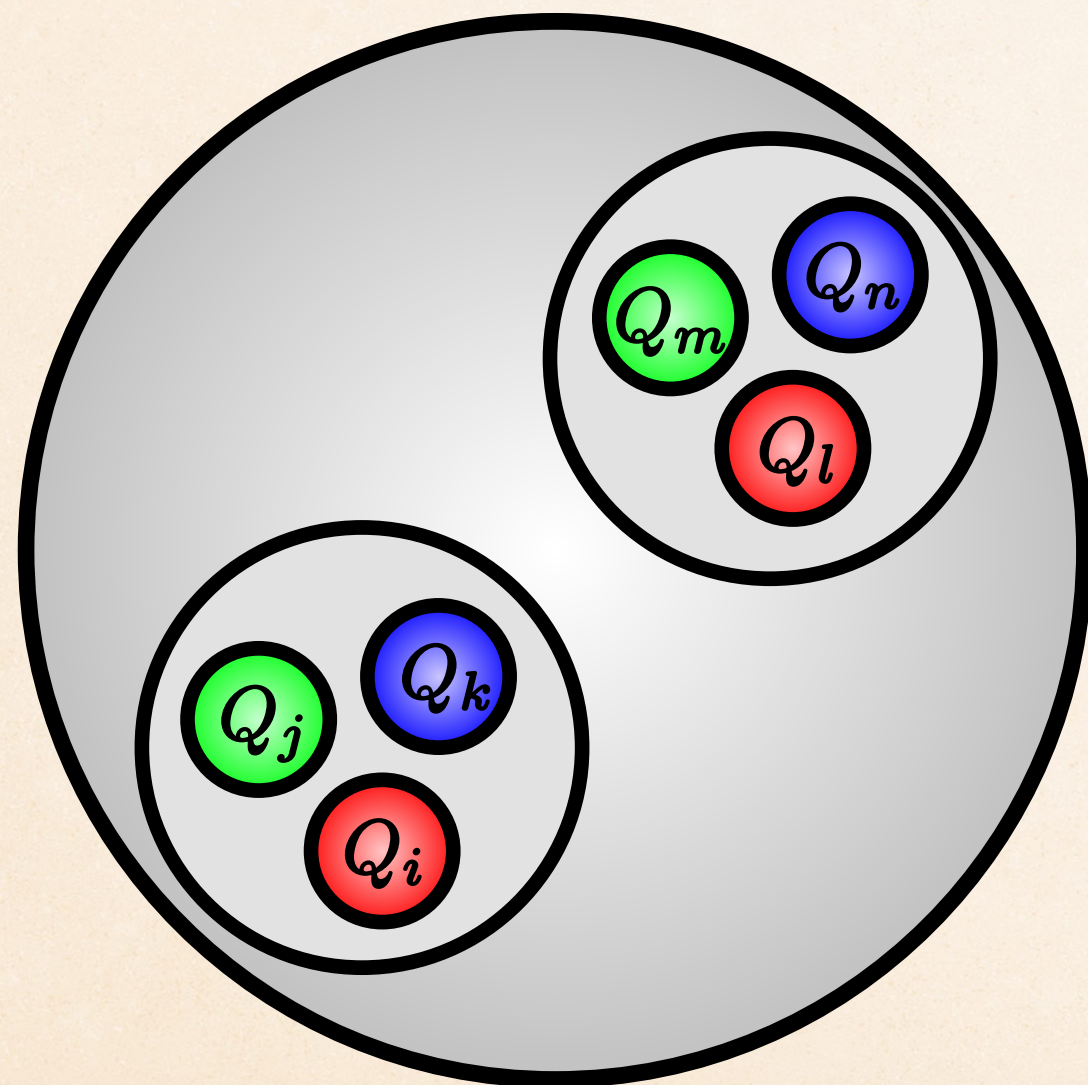


$$\begin{array}{c} 5' \\ \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline \end{array} \\ I = 0 \end{array}$$

SIX-QUARK BSA

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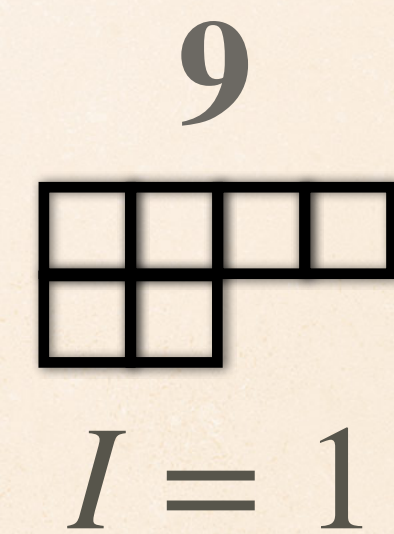
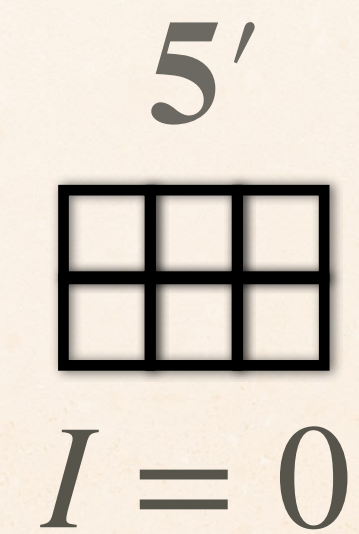
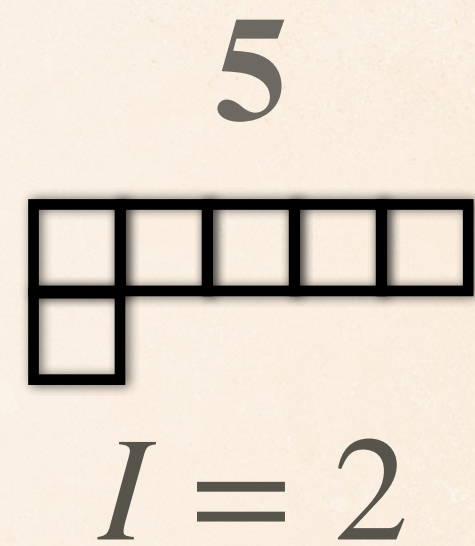
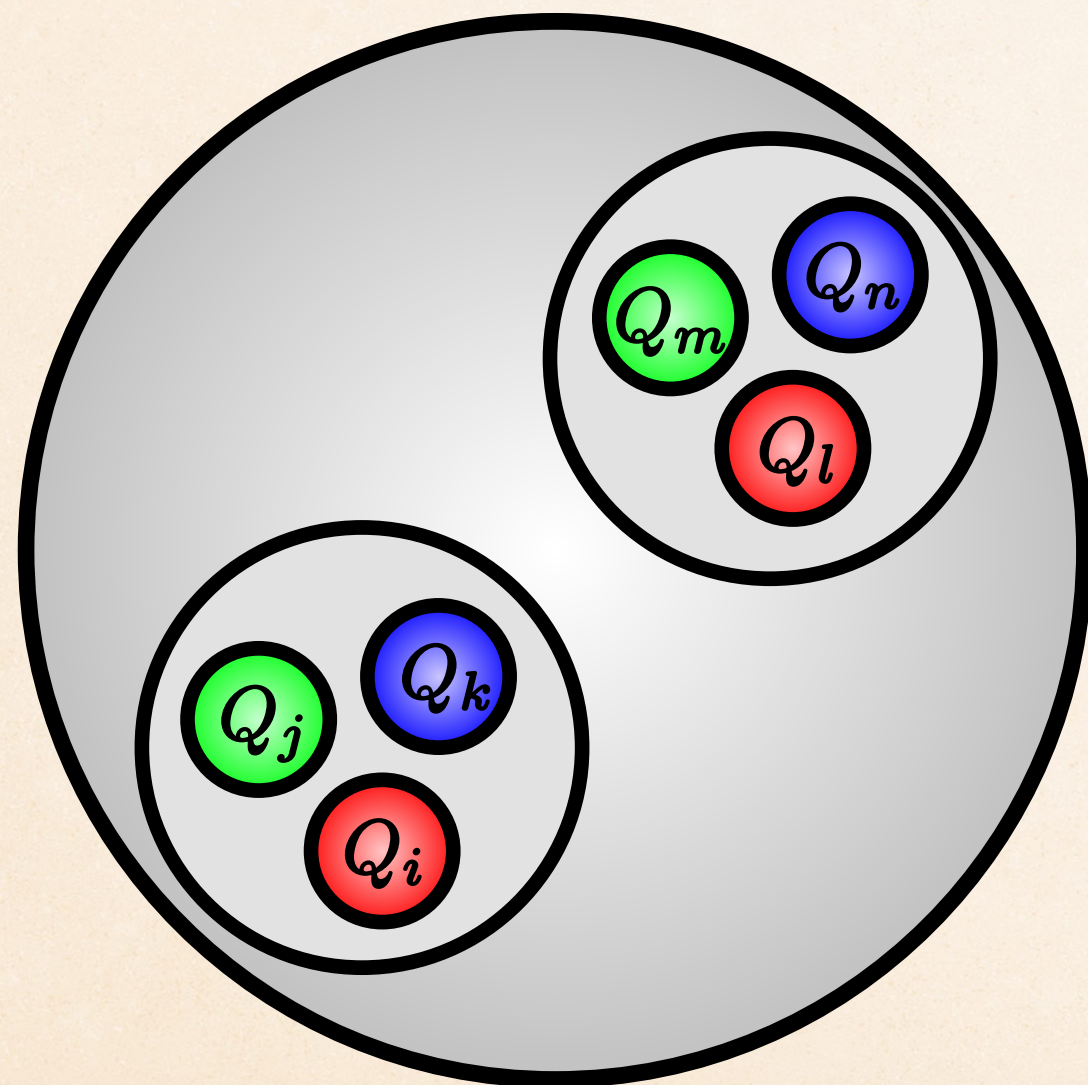
$$\begin{array}{c} 5' \\ \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline \end{array} \\ I = 0 \end{array}$$

$$\begin{array}{c} 9 \\ \begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline \end{array} \\ I = 1 \end{array}$$

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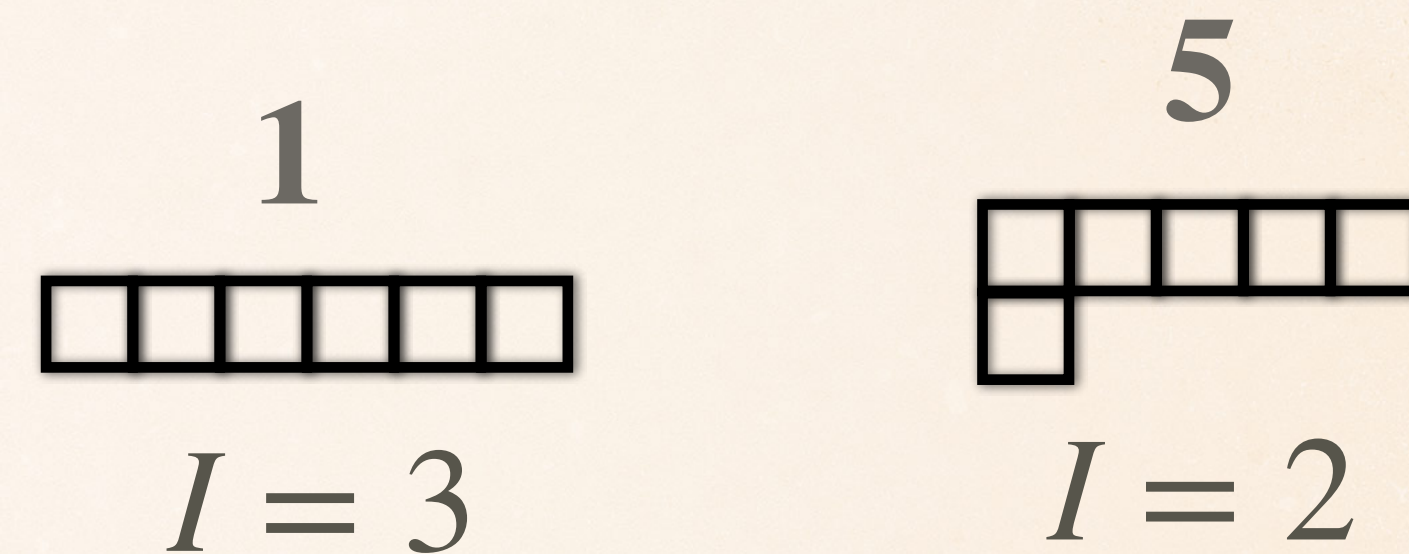
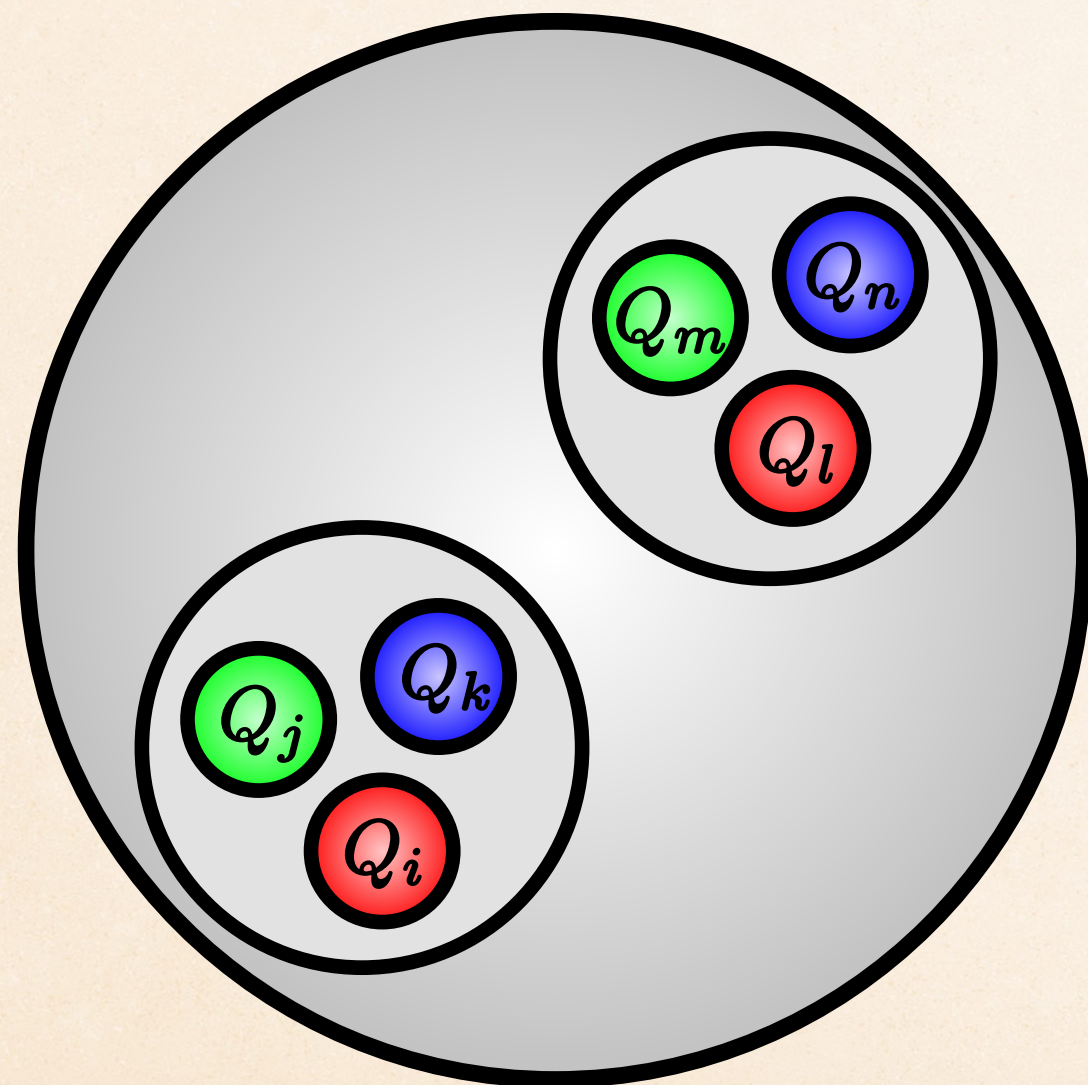
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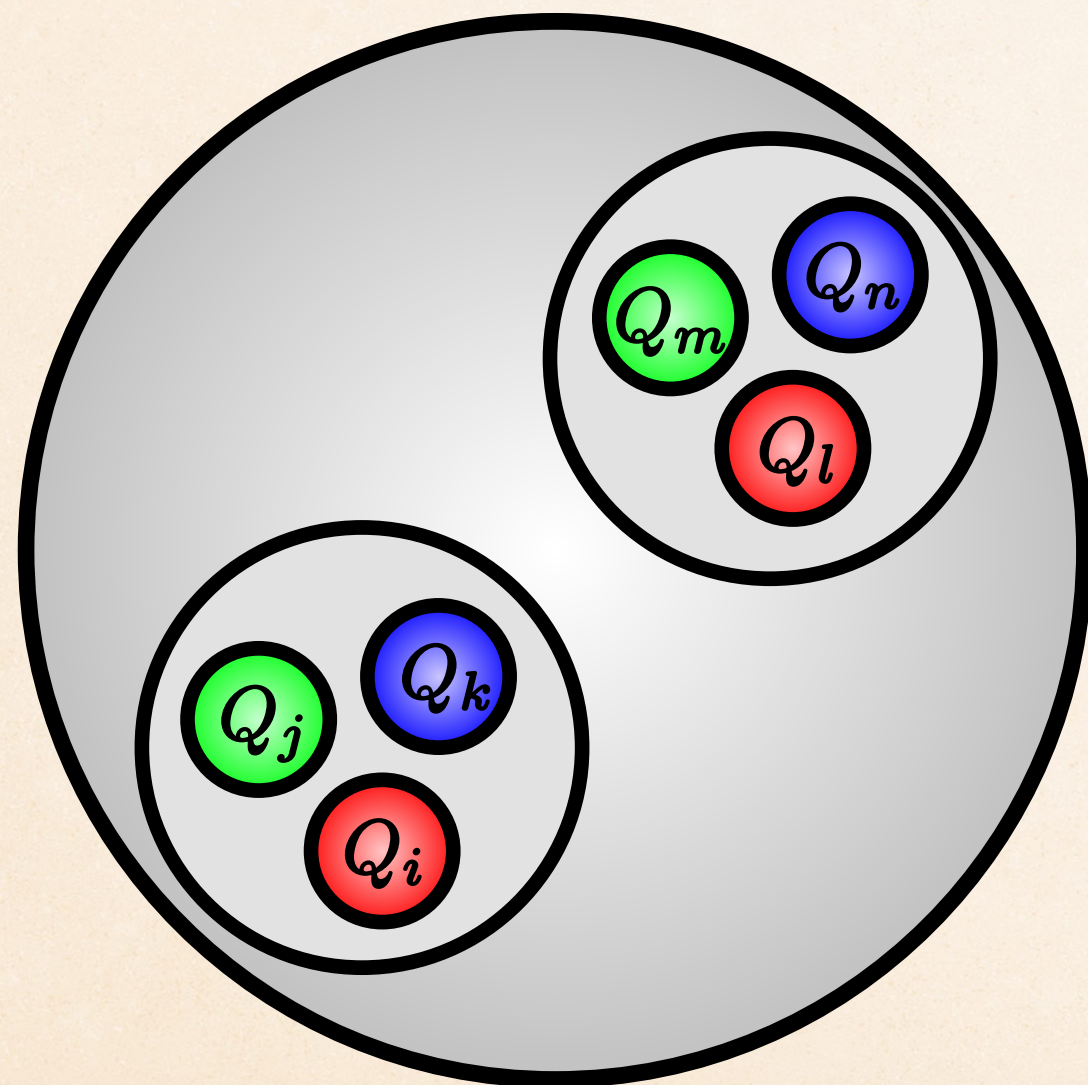
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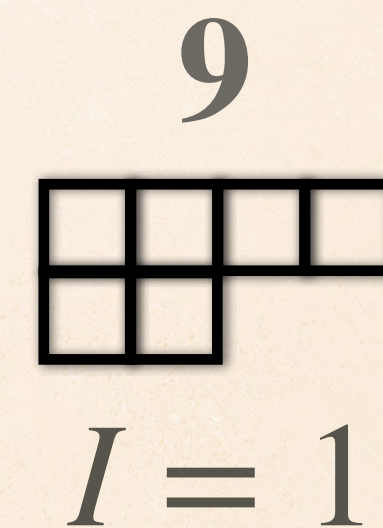
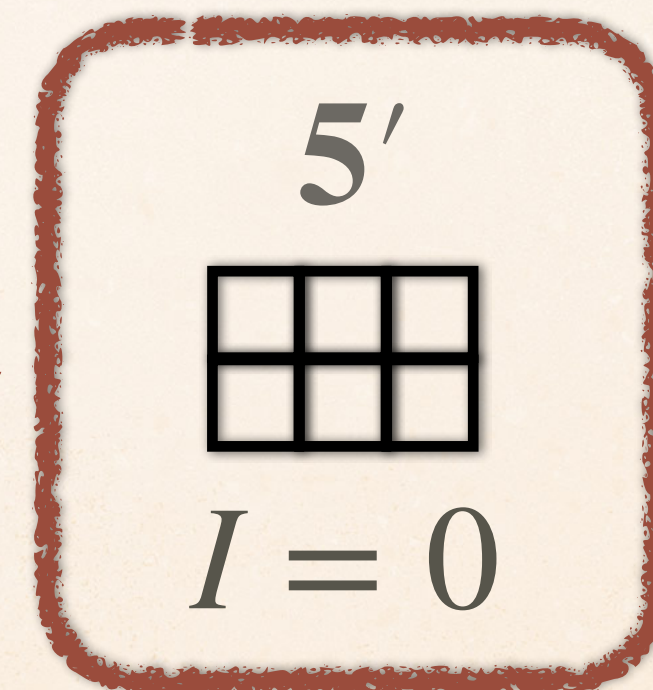
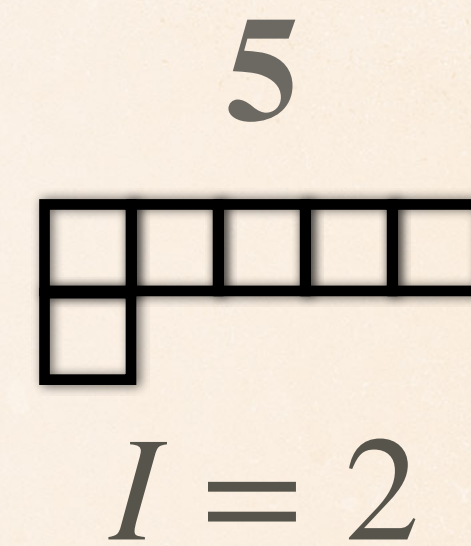
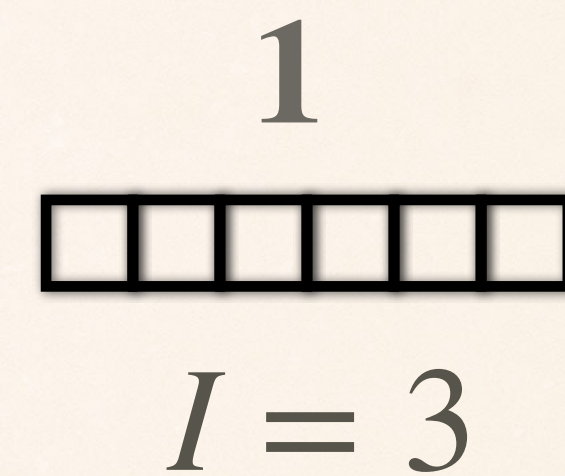
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Deuteron



SIX-QUARK BSA

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❖ Example Deuteron:

SIX-QUARK BSA

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❖ Example Deuteron:

$$\text{❖ } \mathbf{M}_D \otimes \mathbf{M}_C \otimes \mathbf{M}_F = \mathbf{M}_D \otimes \bar{\mathbf{5}}' \otimes \mathbf{5}' \stackrel{!}{=} \bar{\mathbf{1}}$$

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❖ Restrictions on representations $\mathbf{M}_D$ for Dirac part of BSA

SIX-QUARK BSA

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- ❖ Number of needed Dirac tensors N
- ❖ 4096 for $J = 0 \rightarrow 320$ S -wave
- ❖ 12288 for $J = 1 \rightarrow 576$ S -wave

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- ❖ Lorentz invariants Ω :
 - ❖ Singlet: $\mathcal{S}_0$ Quintets: $\mathcal{V}_1^+, \mathcal{V}_2^+$ Nonet: $\mathcal{N}^+$

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❖ Strategy for solution:

SIX-QUARK BSA

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- ❖ Strategy for solution:
- ❖ Neglect “subleading” Lorentz invariants
- ❖ $f_i(\mathcal{S}_0, \mathcal{V}_1^+, \mathcal{V}_2^+, \mathcal{N}^+) \rightarrow f_i(\mathcal{S}_0, \mathcal{N}^+)$

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- ❖ Dynamic three-body pole generation in $\mathcal{N}^+$

SIX-QUARK BSA

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- ❖ “Physical basis” Ansatz:

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 - ❖ $f_i(\mathcal{S}_0, \mathcal{N}^+) \rightarrow f_i(\mathcal{S}_0) \cdot P_1 \cdot P_2$

SIX-QUARK BSA

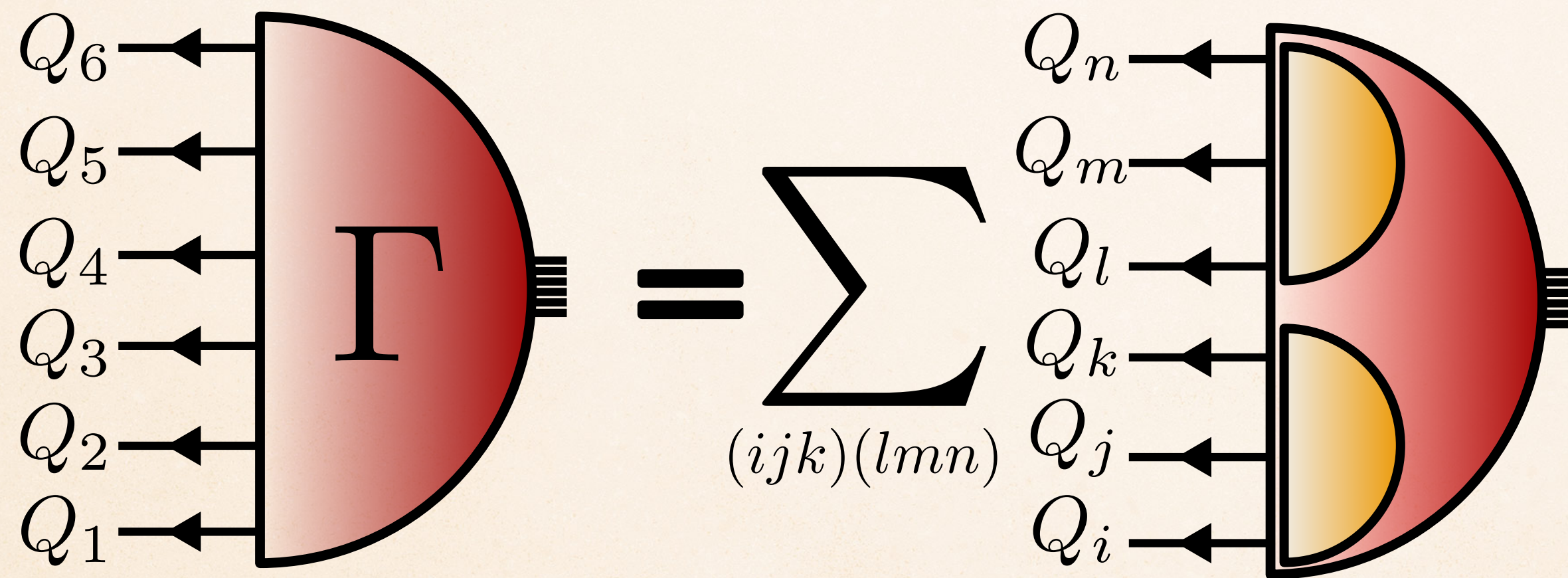
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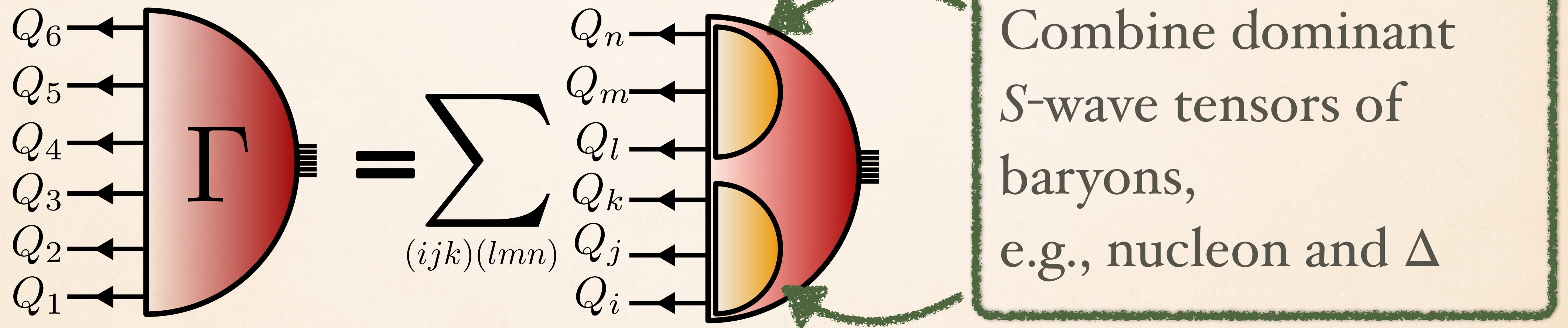


with $i < j < k$ & $l < m < n$

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❖ “Physical” BSA:



with $i < j < k$ & $l < m < n$

CONCLUSION & OUTLOOK

- ❖ Construct hexaquark BSA using the “physical” Ansatz and in terms of the S_6 multiplets
- ❖ Calculate the dibaryon spectrum numerically
 - ❖ E.g., Deuteron and cousins, H-dibaryon, all-charm dibaryons
- ❖ Investigate the internal structure