Temporal Evolution and Spatial Distribution of a Dynamically Generated Nucleon Resonance

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Y. Zhuge and Z.-W. Liu, arXiv:2606.07411

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CONTENTS

Gamow Representation of a resonance in complex momentum space

Wavefunctions for bound states are very essential

The deuteron is a typical bound state of a proton and a neutron.

- Schrödinger equation provides
  - mass of bound state
  - wavefunction of bound state
- One can extract everything from the wavefunction.
- The wavefunction has a direct probability interpretation.

A difference between resonances and bound states

- The $\Lambda(1405)$ is located just below the $\bar{K}N$ threshold.
- It was observed mainly through the $\pi\Sigma$ channel.
- It is widely interpreted approximated as the $\pi\Sigma$-$\bar{K}N$ resonance though other interpretations exist.

finite lifetime $1/\Gamma \longrightarrow$ complex mass $m - i\Gamma/2$

How to describe the constituents of a resonance like $\Lambda(1405)$?

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$$\psi:$$

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ψ :

$$\hat{H}\psi = E\psi?$$

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Hermit $\hat{H}$ vs complex E

Gamow vector with complex scaling method

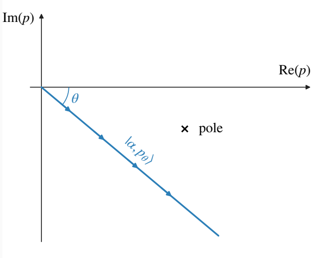
- introduce the two-particle basis with complex-scaling momentum $p \to p_\theta \equiv pe^{-i\theta}$

$$|\alpha, p_\theta\rangle = |\pi\Sigma, p_\theta\rangle, |\bar{K}N, p_\theta\rangle$$

- solve the Hamiltonian equation in the above basis

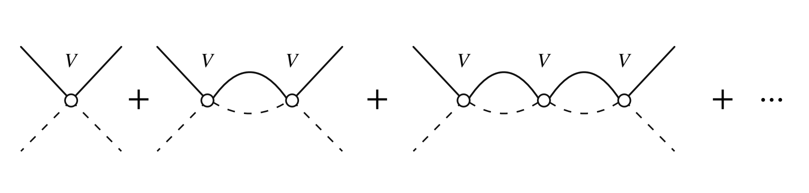
$$H\left|\psi^{\text{Gamow}}\right\rangle = (H_0 + V)\left|\psi^{\text{Gamow}}\right\rangle = E_{\text{pole}}\left|\psi^{\text{Gamow}}\right\rangle .$$

- then obtain resonance pole E_{pole} and corresponding Gamow wavefunction $\left|\psi^{\text{Gamow}}\right\rangle$.



The Gamow vector is spanned in
the complex-momentum space
which cannot be measured directly.

Alternatively Gamow wavefunction can be obtained by residues of T matrix



- obtain the T matrix by solving the Lippmann-Schwinger equation,
  equivalent to the resummation of the above diagrams.
- search for the poles of T matrix on the unphysical Riemann sheet.
- extract the residues $\gamma_\alpha(p_\theta)$ of T matrix

$$T_{\alpha,\beta}(p_\theta, p'_\theta; E) \approx \frac{\gamma_\alpha(p_\theta)\,\gamma_\beta(p'_\theta)}{E - E_{\text{pole}}} + \cdots ,$$

- finally get the relation between the Gamow wavefunction and residues of T matrix

$$\psi_\alpha^{\text{Gamow}}(p_\theta) \equiv \langle \alpha, p_\theta | \psi^{\text{Gamow}} \rangle = \frac{\gamma_\alpha(p_\theta)}{E_{\text{pole}} - \omega_\alpha(p_\theta)} ,$$

Advantage and Disadvantage of Gamow wavefunction

Advantage:

Close correlation to Hamiltonian eigen equation.

$$H \left|\psi^{\text{Gamow}}\right\rangle = (H_0 + V) \left|\psi^{\text{Gamow}}\right\rangle = E_{\text{pole}} \left|\psi^{\text{Gamow}}\right\rangle .$$

Advantage and Disadvantage of Gamow wavefunction

Disadvantage:

(1) Divergence of Gamow wavefunctions at infinite distance.

$$\chi_p(r) \cong \frac{1}{S(p)} e^{-ipr} - e^{+ipr} \xrightarrow{p \to p_{\text{pole}}} 0 - e^{+ip_R r} e^{p_I r} \to \infty$$

Advantage and Disadvantage of Gamow wavefunction

Disadvantage:

(2) Complex compositeness with its real part sometimes even negative.

for the higher pole of $\Lambda(1405)$ using our model parameters

$$X^{\mathrm{H}}_{\pi\Sigma} = -0.20 - 0.17i, \qquad X^{\mathrm{H}}_{\bar{K}N} = 1.20 + 0.17i$$

For how to calculate the channel compositeness X_α derived from the Gamow state, one can

see *[Sekihara:2014kya,Garcia-Recio:2015jsa,Kamiya:2015aea,Miyahara:2018onh]* or more references in our work 2606.07411.

Advantage and Disadvantage of Gamow wavefunction

Disadvantage:

(3) Strong tensions between

- Gamow vector in Complex momentum space;
- experimental measurements in Real momentum space.

Physical Representation of a resonance in real momentum space

We need a new physical representation of resonance.

We should look for a new physical representation for resonance $|\psi^{\text{phys}}\rangle$ if it is

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(1) spanned in real momentum space

$\rightarrow$ ordinary Hillbert space

$\rightarrow$ natural probability interpretation.

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& automatically confined within a finite range.

We need a new physical representation of resonance.

We should look for a new physical representation for resonance $|\psi^{\text{phys}}\rangle$ if it is

(2) able to transform into and back from Gamow wavefunction

$$|\psi^{\text{phys}}\rangle \leftrightarrow |\psi^{\text{Gamow}}\rangle.$$

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$|\psi^{\text{phys}}\rangle$ and $|\psi^{\text{Gamow}}\rangle$ shall contain the same physical information.

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We should look for a new physical representation for resonance $|\psi^{\text{phys}}\rangle$ if it is

(3) constrained by Hamiltonian eigen equation.

Eigenvalue $m - i\Gamma/2$ is the very essential for the resonance.

We need a new physical representation of resonance.

We should look for a new physical representation for resonance $|\psi^{\text{phys}}\rangle$ if it

(4) describes the behaviors that a resonance not only decays but also emits.

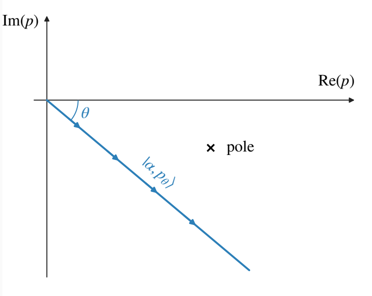
$$\exp(-iHt)\,|\Lambda(1405)^{\text{phys}}\rangle \quad \longrightarrow \quad \begin{cases} \Lambda(1405) & \text{decreases} \\ \pi\Sigma\ldots & \text{increase} \end{cases}$$

We need a new physical representation of resonance.

We should look for a new physical representation for resonance $|\psi^{\text{phys}}\rangle$ if it is

(5) consistent with experimental measurements.

A natural transition to the representation in the real world

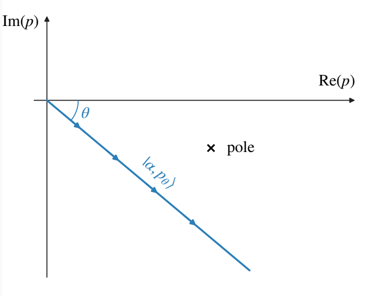


$$\psi_\alpha^{\text{Gamow}}(p_\theta) = \frac{\gamma_\alpha(p_\theta)}{E_{\text{pole}} - \omega_\alpha(p_\theta)},$$

By analytical continuation of the residue $\gamma_\alpha(p_\theta) \to \gamma_\alpha(p)$,

$$|\psi^{\text{phys}}\rangle = \sum_\alpha \int dp\, p^2 \frac{\gamma_\alpha(p)}{E_{\text{pole}} - \omega_\alpha(p)} |\alpha, p\rangle .$$

A natural transition to the representation in the real world



$$\psi_\alpha^{\text{Gamow}}(p_\theta) = \frac{\gamma_\alpha(p_\theta)}{E_{\text{pole}} - \omega_\alpha(p_\theta)}\,,$$

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But how to analytically continue?

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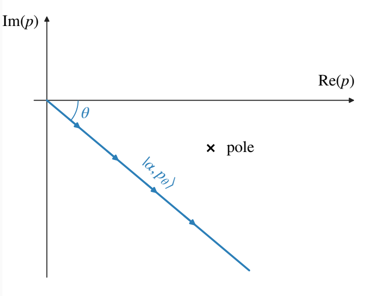

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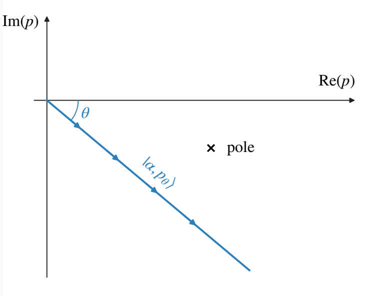
But how to analytically continue?

One can

- fit the Gamow wavefunction in complex momentum space with a complete analytic function set;
- and then extrapolate to the real mometum axis.

However, such a back-rotation is an ill-posed inverse problem and may amplify numerical noise *[Kruppa:2013ala]*

A natural transition to the representation in the real world



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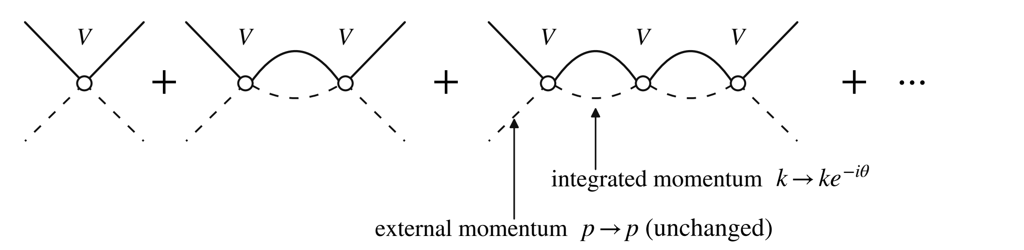
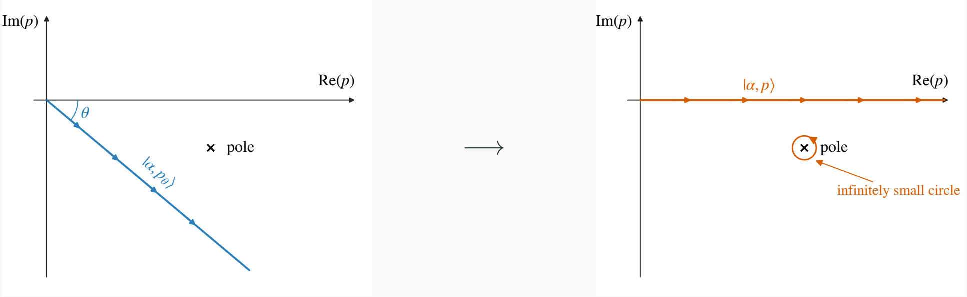
$$|\psi^{\text{phys}}\rangle = \sum_\alpha \int dp\, p^2 \frac{\gamma_\alpha(p)}{E_{\text{pole}} - \omega_\alpha(p)} |\alpha, p\rangle .$$

But how to analytically continue?

We resolve the dynamical equation as illustrated below



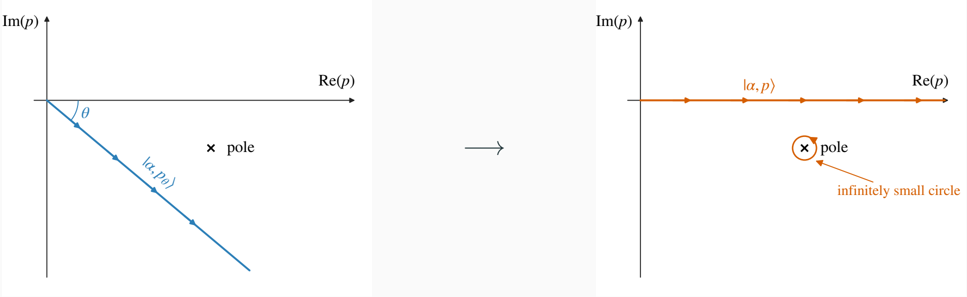
A New Relations between Gamow and Physical Representations



- By introducing a auxiliary vector state spanned around the basis states with the kinetic energy approaching the pole

$$|\psi^{\text{auxiliary}}\rangle = \sum_{\{\alpha \,|\, \theta_\alpha \neq 0\}} \oint_{\mathcal{C}_\epsilon^\alpha \to p_\alpha^{\text{on}}} dp\, p^2 \frac{\gamma_\alpha(p)}{E_{\text{pole}} - \omega_\alpha(p)} |\alpha, p\rangle,$$

A New Relations between Gamow and Physical Representations

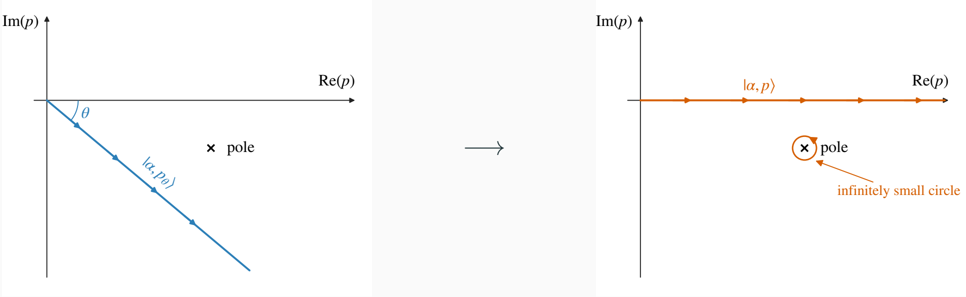


- we can prove

$$H\left(|\psi^{\text{phys}}\rangle + |\psi^{\text{auxiliary}}\rangle\right) = E_{\text{pole}}\left(|\psi^{\text{phys}}\rangle + |\psi^{\text{auxiliary}}\rangle\right)$$

  - $|\psi^{\text{phys}}\rangle$ in the ordinary Hilbert space
  - $|\psi^{\text{auxiliary}}\rangle$ in the special complex space closely around the pole

A New Relations between Gamow and Physical Representations



- With respect to the Hamiltonian equation,

$$|\psi^{\text{Gamow}}\rangle \cong |\psi^{\text{phys}}\rangle + |\psi^{\text{auxiliary}}\rangle \,,$$

which is a new essential relation between $|\psi^{\text{Gamow}}\rangle$ and $|\psi^{\text{phys}}\rangle$ in addition to their analytic connection.

Application to the $\Lambda(1405)$ Dynamically Generated by $\pi\Sigma$-$\bar{K}N$ interaction

Our framework for the dynamics of $\Lambda(1405)$

In this work,

- we assume that $\Lambda(1405)$ is mainly a $\bar{K}N$-$\pi\Sigma$ dynamically generated state.

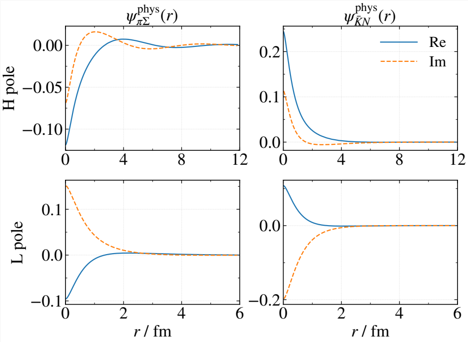
- we use a simple separable interaction in the $\pi\Sigma$ and $\bar{K}N$ channels

$$V_{\alpha\beta}(p,p') = \frac{3v_{\alpha\beta}}{4\pi^2 f_\pi^2}\, u_\alpha(p)\, u_\beta(p')\,, \quad u_\alpha(p) = \left(1 + \frac{p^2}{\Lambda_\alpha^2}\right)^{-2} .$$

- by adjusting the coupling parameters, we obtain
  - the higher-mass (H) pole $E^{\mathrm{H}}_{\mathrm{pole}} = 1430 - 22\,i$ MeV,
  - the lower-mass (L) pole $E^{\mathrm{L}}_{\mathrm{pole}} = 1338 - 89\,i$ MeV.

Coordinate Wavefunctions of $\Lambda(1405)$

The wavefunctions $\psi_\alpha^{\mathrm{phys}}(r) \equiv \langle \alpha, r | \psi^{\mathrm{phys}} \rangle$ in coordinate space can be obtained by Fourier transformation.



In $\Lambda(1405)$, $\pi\Sigma$ and $\bar{K}N$ are confined within finite ranges.

Compositeness from $|\psi^{\text{Gamow}}\rangle$ vs probability from $|\psi^{\text{phys}}\rangle$

$|\psi^{\text{Gamow}}\rangle \longrightarrow$ Compositeness

$$X_\alpha \equiv \int dp_\theta\, p_\theta^2 \,\langle \alpha, p_\theta | \psi^{\text{Gamow}} \rangle^2$$

$\longrightarrow$ reflect constituents of a resonance.

$|\psi_{\text{H}}^{\text{phys}}\rangle$:

ordinary Hilbert-space vector

$\longrightarrow$ natural probability interpretation.

In the higher pole of $\Lambda(1405)$:

$X_{\pi\Sigma}^{\text{H}} = -0.20 - 0.17i$;

$X_{\bar{K}N}^{\text{H}} = 1.20 + 0.17i$.

In the higher pole of $\Lambda(1405)$:

$\pi\Sigma$ constituents occupy 46%;

$\bar{K}N$ constituents occupy 54%.

imaginary part!

negative real part!

Same parameters in our model are used for both compositeness from $|\psi^{\text{Gamow}}\rangle$ and probability from $|\psi^{\text{phys}}\rangle$.

Temporal evolution for the two $\Lambda(1405)$ poles

With

$$|\psi^{\text{phys}}\rangle = \sum_{\alpha} \int dp\, p^2 \frac{\gamma_\alpha(p)}{E_{\text{pole}} - \omega_\alpha(p)} |\alpha, p\rangle \,,$$

we can calculate the evolution of the states

$$|\psi^{\text{phys}}, t\rangle = e^{-iHt} |\psi^{\text{phys}}\rangle \,.$$

In detail

- insert the completeness relation with $|\alpha, p\rangle$
- discretize the resulting momentum integrals using Gaussian quadrature
- thus reduce time evolution to a finite-dimensional matrix-vector operation for numerical solutions.

Resonances decay with the scattering stated emitted

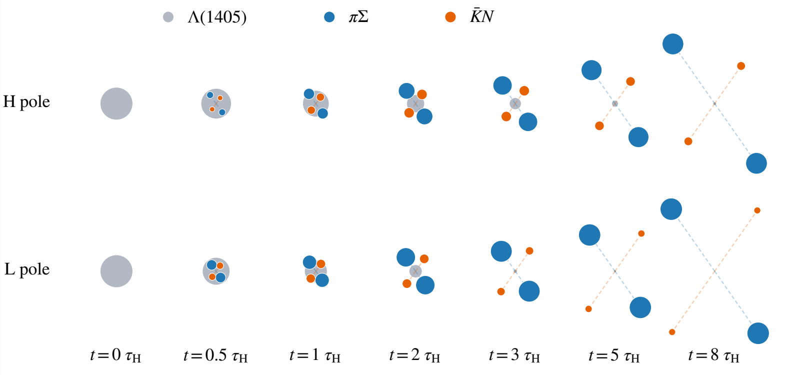


Illustration of the time evolution $|\psi^{\text{phys}}, t\rangle = e^{-iHt}\,|\psi^{\text{phys}}\rangle$ of the resonance components

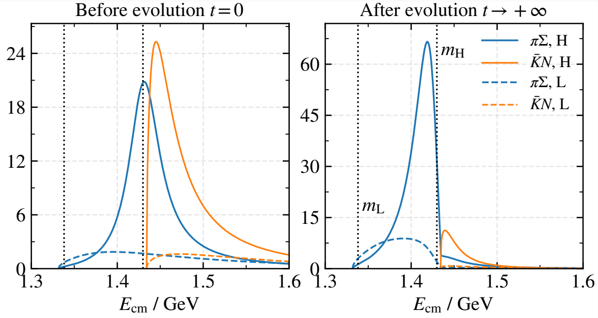
- The gray area is proportional to the survival probability
  and decreases with time in units of $\tau_{\text{H}} = 1\,/\,|\Gamma^{\text{H}}_{\text{pole}}|$
- The emitted $\pi\Sigma$ and $\bar{K}N$ move away from the interaction region.
  Their distances from the center are proportional to their root-mean-square radii.

The asymptotic state at the infinite time

The asymptotic state $|\psi^{\text{phys}}, t \to +\infty\rangle$ in the Schrödinger picture

$$|\psi^{\text{phys}}, t \to +\infty\rangle = \sum_{\alpha} \int dp\, p^2 \psi_{\alpha}^{\text{phys}}(p) \times \left(e^{-i\omega_\alpha(p)t} |\alpha, p\rangle + \sum_{\beta} \int dq\, q^2 \frac{T_{\beta,\alpha}(q, p; E)}{\omega_\beta(q) - \omega_\alpha(p) + i\epsilon} e^{-i\omega_\beta(q)t} |\beta, q\rangle \right) .$$

The resulting distribution $|f_\alpha(E)|^2 = |\langle \alpha, E \,|\, \psi^{\text{phys}}(t = +\infty)\rangle|^2$

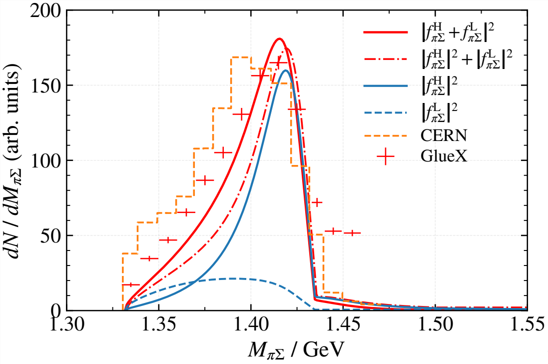


For the high-mass pole

- the $\bar{K}N$ component is significant when initially formed
- most of it turns into the $\pi\Sigma$ rather than $\bar{K}N$ scattering states
  because of
  - energy conservation when $t \to \infty$
  - the larger mass of $\bar{K}N$ than $m^{\mathrm{H}}_{\text{pole}}$

Y. Zhuge and Z. W. Liu, arXiv:2606.07411.

Comparison with the experimental measurements



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We give the full results with both

- the strongest interference scenario $|f^{\mathrm{H}}_{\pi\Sigma}(E) + f^{\mathrm{L}}_{\pi\Sigma}(E)|^2$
- no interference assumption $|f^{\mathrm{H}}_{\pi\Sigma}(E)|^2 + |f^{\mathrm{L}}_{\pi\Sigma}(E)|^2$

The currently observed $\pi\Sigma$ spectrum prefers the strongest interference scenario.

Summary

Summary

For resonances, we have showed a physical description $|\psi^{\text{phys}}\rangle$ which is

- spanned in real momentum space and automatically confined in finite range;
- able to transform into and back from Gamow wavefunction;
- constrained by Hamiltonian eigen equation;
- able to describe the behaviors that a resonance not only decays but also emits;
- consistent with experimental measurements.

Bound state: wavefunction $\longrightarrow$ extract all its properties.
Resonance: wavefunction will play much more important roles in future when other observables in addition to mass and width are focused and measured.

Thank you for your attention!