

Systematic account of bound-state motion, missing part of Weinberg's EFT

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- with H. Sazdjian and B. Blankleider
- Motivation:
 - (i) **Necessity** of matching theoretical precision to high precision achieved in experiments
 - (i) Hope some questions to be answered
- High-precision experiments require matching theoretical precision **to test fundamental physics**

Nuclear forces from chiral Lagrangians

- Method is general, **extremely relevant currently** as the only missing part of (although developed in ~ 35 years) extensively used **systematic** approaches to EFT proposed by S. Weinberg, "Nuclear forces from chiral Lagrangians" Phys. Lett. B251, 288 (1990)
- **NN system** is fully described by Schroedinger equation where **effective potentials** nowadays are calculated to very high order of ChPT.
- **Presented PT for systematic account of bound-state motion** is necessary for ab initio high accuracy studies of scattering processes where **bound states** are involved. Simplest example of such processes is electron deuteron scattering, deuteron em form-factor.

On moving bound states

- In the study of amplitudes involving bound states (e.g. em and other currents) it is important to express them in terms of **the bound state wave functions at rest**, because they are much more convenient to model/calculate, than those for the moving bound state. Wave functions just at rest are also important for the study of the bound states' structure (size, shape etc.).
- In the covariant approaches the wave functions in the different reference frames are related to each other by kinematic Lorentz transformations.

- Unfortunately in the most convenient approaches to the extensively used Weinberg's EFT, the TOPT and so called "Unitary transformation method" (MUT), the boost transformations are dynamical, i.e. they **depend on the interaction Hamiltonian**.
- Our PT is just to account for this interaction in the boost transformation.

Examples of "high-accuracy" calculations

- Such PT is especially timely in view of recent studies of nuclei form factors in chiral EFT, where **all but boost** transformation are calculated with high-accuracy, **boost transformation is approximate**

High-accuracy calculation of deuteron charge and quadrupole form factors..., Filin, Moeller, Baru, Epelbaum, Krebs,

Phys.Rev.C 103, 2, 024313 (2021)

Few Body Syst. 65, 2, 39 (2024)

Ulf-G. Meissner, J.Phys.Conf.Ser. 2586, 1, 012006 (2023):

- (i) deuteron structure radius, $r = 1.9729 + 0.0015 - 0.0012$ fm,
- (ii) deuteron quadrupole moment,
 $Q_d = 0.2854 + 0.00380.1700 fm^2$, at N^4LO
in agreement with very precise experimental datum
 $Q_d = 0.285699 fm^2$
- (iii) deuteron structure radius was used to extract neutron charge radius $r_n^2 = -0.105 + 0.005 - 0.006 fm^2$.

PT for quantised boost operator

- Apparently **overlooked history**:
First systematic **PT for quantum boost operator**
A.K., A. Khvedelidze, B. Magradze, V. Matveev,
Theor.Math.Phys. 72, 710 (1988); 78 162 (1989);
78 227 (1989); 78, 252 (1989)
That time PT was applied to high energy physics using
asymptotic freedom.
- The **same PT is just what is needed** in the low energy physics
based on the **chiral EFT**,
often called "the modern theory of nuclear forces"
- Here we present such PT for practitioners of **two most
extensively** used approaches to "chiral effective field theory",
based on (i) **TOPT** and on (ii) **MUT**

PT for boost operator in QFT

- ****REMOVE "bar" in $\bar{M}^{i0}$ ****
- Boost generator: $\bar{M}^{i0} = M_0^{i0} + \int d^4x x^i H_I(x) \delta(x^0)$,
 M_0^{i0} and $M_I^{i0} = \int d^4x x^i H_I(x)$ are its free and interaction parts.
- Boost transformation operator:

$$\bar{T}_\lambda = \exp(i\omega \vec{n}^i \bar{M}^{i0}) = e^{i\omega \vec{n}^i (M_0^{i0} + \int d^4x x^i H_I(x) \delta(x^0))}$$

where $\vec{n} = \frac{\vec{\lambda}}{|\vec{\lambda}|}$, $\lambda = \frac{P}{M}$ is 4-velocity, $\lambda^2 = 1$, $\omega = \text{arcsinh}|\vec{\lambda}|$.

- Operator $\bar{T}_\lambda$ relates the bound state vector $|P, i\rangle$ with 4-momentum P to that at rest, $|\vec{0}, i\rangle$, with spin variables i ,
 $|P, i\rangle = \sum_j \bar{T}_\lambda |\vec{0}, j\rangle \mathcal{D}(\lambda, \lambda_0)_i^j$,
where $\lambda_0 = (1, \vec{0})$, $\mathcal{D}(\lambda, \lambda_0)_i^j$ is Wigner rotation matrix.

PT for boost operator in QFT

To construct PT for boost operator $\bar{T}_\lambda$ we proceed in direct analogy with well known construction of S-matrix, i.e. consider "evolution equation" for the combination $\bar{A}(\lambda) = T_{\lambda 0}^+ \bar{T}_\lambda$, where $T_{\lambda 0} = \bar{T}_\lambda|_{H_I=0}$ is boost operator with NO interaction.

$$\frac{\partial \bar{A}(\lambda)}{\partial \omega} = T_{\lambda 0}^+ \left(-i \int d^4x \vec{n} \vec{x} H_I(x) \delta(x^0) \right) \bar{T}_\lambda = i \bar{B}(\lambda) \bar{A}(\lambda)$$

where

$$\bar{B}(\lambda) = T_{\lambda 0}^+ \left(- \int d^4x \vec{n} \vec{x} H_I(x) \delta(x^0) \right) T_{\lambda 0} = - \frac{\partial}{\partial \omega} \int d^4x H_I(x) \theta(\lambda x)$$

Note similarity to transition to the interaction representation

The solution to this eq can be written in three different ordered exponent forms, applicable for perturbative calculations,

PT for boost operator in QFT

$$(i) A(\lambda) = T_{\omega} e^{i \int_0^{\omega} d\omega' B(\omega')} \quad T_{\omega} - \text{ordering in velocity } \frac{|\vec{P}|}{M} = \text{sh } \omega$$

velocity for boost transformations is analog of **time** for translations in time

$$(ii) \quad = T_{\frac{x^0}{x^n}} e^{i \int_{\Omega} d^4 x \epsilon(x^0) H_I(x)} \quad T_{\frac{x^0}{x^n}} - \text{ordering in } \frac{x^0}{x^n}$$

T_{ω} and $T_{\frac{x^0}{x^n}}$ orderings are defined as

$$T_{\omega} B(\omega_1) B(\omega_2) = \theta(\omega_1 - \omega_2) B(\omega_1) B(\omega_2) + \theta(\omega_2 - \omega_1) B(\omega_2) B(\omega_1),$$

$$T_{\frac{x^0}{x^n}} H_I(x') H_I(x) = \theta\left(\frac{x'_0}{x'_n} - \frac{x_0}{x_n}\right) H_I(x') H_I(x) + (x' \leftrightarrow x)$$

PT for boost operator in QFT

$$(iii) A(\lambda) = T e^{i \int_{\Omega_1} d^4 x H_I(x)} \left\{ T e^{i \int_{\Omega_2} d^4 x H_I(x)} \right\}^\dagger \\ = T e^{i \int d x \theta(x^0) \theta(-\lambda x) H_I(x)} \left\{ T e^{i \int d x \theta(-x^0) \theta(\lambda x) H_I(x)} \right\}^\dagger =$$

"T" stands for ordinary time-ordering!

This is evolution operator between spacelike hyperplanes
 $x^0 = 0$ and $Px = 0$

Notice S-matrix form of the boost operator, $T e^{i \int d x H_I(x) \theta(x^0) \theta(-\lambda x)}$,
with external field $A(x) = \theta(x^0) \theta(-\lambda x)$

Particular case of generalized Schroedinger equation where wave function is a functional of an arbitrary spacelike hypersurface is known as the Tomonaga-Schwinger equation.

It is remarkable that this formalism has been overlooked, although the boost problem has attracted interest for at least 25 years!

Approximate Boost of the Relativistic Two-Body Problem in the Instantaneous Formalism, Stephen J. Wallace, Phys.Rev.Lett. 87, 180401 (2001)

In Theor.Math.Phys. 72, 710 (1988); 78 162 (1989);
78 227 (1989); 78, 252 (1989) the representation

$A(\lambda) = T e^{i \int d^4 x \theta(\lambda x) H_I(x)} \left\{ T e^{i \int d^4 x \theta(x^0) H_I(x)} \right\}^\dagger$ is **wrong** because

the boundary condition $A(\vec{\lambda} = 0) = 1$ is not satisfied,

$A(\vec{\lambda} = 0) = T e^{i \int d^4 x \theta(x^0) H_I(x)} \left\{ T e^{i \int d^4 x \theta(x^0) H_I(x)} \right\}^\dagger = \Omega \Omega^\dagger \neq 1,$

although $\Omega^\dagger \Omega = 1$, because **Moeler operator Ω is not unitary**, if
Hamiltonian possess a bound state!

Boost operator in TOPT

- All three forms given above imply expansion in powers of the product of velocity $\vec{\lambda}$ by couplings involved in the chiral interaction Lagrangian therefore it is interesting how do they interplay in practical calculations
- $FIX **|P\rangle = U_\lambda |\vec{0}\rangle$ can be written in terms of the two nucleon wave functions, $\Psi_P(\vec{p}_1, \vec{p}_2) = \langle p_1, p_2 | P \rangle$
 $\Psi_P(\vec{p}_1, \vec{p}_2) = \int \langle p_1, p_2 | A(\lambda) U_{\lambda 0} (1 + G_M \mathcal{Q} T_M^{ir}) | k_1, k_2 \rangle (dk) \Psi_0(k_1, k_2) .,$
 T^{ir} encodes all but two-nucleon intermediate states contributions
- In the LO $A(\lambda) \sim 1$, $T_M^{ir} = 0$ therefore the boost transformation in the LO approximation is $\langle p_1, p_2 | T_\lambda | k_1, k_2 \rangle \sim \langle p_1, p_2 | T_{\lambda 0} | k_1, k_2 \rangle = \delta^3(\vec{\Lambda}_\lambda^{-1} \vec{p}_1 - \vec{k}_1) \delta^3(\vec{\Lambda}_\lambda^{-1} \vec{p}_2 - \vec{k}_2)$, which corresponds to the kinematic Lorentz transformation given by $\Psi_P(\vec{p}_1, \vec{p}_2) \sim \Psi_{\vec{0}}(\vec{\Lambda}_\lambda^{-1} \vec{p}_1, \vec{\Lambda}_\lambda^{-1} \vec{p}_2),$

- Unitary transformation U diagonalises all Poincare generators $\bar{P}_\alpha$, **not only Hamiltonian**, $P_\alpha = U\bar{P}_\alpha U^+$ is diagonal in number of nucleons
Unitary Transformed boost operator:
 $T_\lambda = U\bar{T}_\lambda U^+ = UT_{\lambda 0}\bar{A}(\lambda)U^+ = (UT_{\lambda 0}U^+)(U\bar{A}(\lambda)U^+) \text{ (A)}$
- $|\vec{P}\rangle^u = U|\vec{P}\rangle$ is bound state vector in UT method
Then bound state current: $\langle \vec{P}' | J^\mu | \vec{P} \rangle = \langle \vec{P}' |^u U J^\mu U^+ | \vec{P} \rangle^u$
- Boosting bound state $|\vec{P}\rangle^u = T_{\vec{v}}|\vec{0}\rangle^u$, explicitly, in variables:
 $\Psi_{\vec{P}}(\vec{p}_1, \vec{p}_2) = \int \langle \vec{p}_1, \vec{p}_2 | T_{\vec{v}} | \vec{k}_1, \vec{k}_2 \rangle \frac{d^3 \vec{k}_1}{2\omega_{k_1}} \frac{d^3 \vec{k}_2}{2\omega_{k_2}} \Psi_{\vec{0}}(\vec{k}_1, \vec{k}_2)$
 $\langle \vec{p}_1, \vec{p}_2 | T_{\vec{v}} | \vec{k}_1, \vec{k}_2 \rangle$ is calculable **perturbatively to any precision** with the help of Eq.(A).

PT derived from the two-body Schroedinger equation

- AK, H. Sazdjian, B. Blankleider, Phys.Rev., C112, 3, 034001
- This PT is formulated in terms of **NN potentials developed in great details** and extensively used for the analysis of the scattering amplitudes etc. in both, **TOPT and MUT** approaches,

- It is based on **relativistic** Schroedinger eq for moving bound state with momentum P,

$$\left(P^0 - \sqrt{m_1^2 + \vec{p}_1^2} - \sqrt{m_2^2 + \vec{p}_2^2} \right) \Psi_{\vec{P}}(\vec{p}) \\ = \int d^3k V(P, \vec{p}, \vec{k}) \Psi_{\vec{P}}(\vec{k})$$

- represents expansion of the solution to eq for moving bound state, $\Psi_{\vec{P}}(\vec{k})$ around its solution at the rest frame, $\Psi_{\vec{0}}(\vec{k})$,

$$\left(M - 2\sqrt{m^2 + \vec{p}^2} \right) \Psi_{\vec{0}}(\vec{p}) = \int d\vec{k} V(M, \vec{0}, \vec{p}, \vec{k}) \Psi_{\vec{0}}(\vec{k})$$

Unusual application of the generalised standard PT

- Straightforward leading order (LO) approximation would be the wave function, $\Psi_P^{LO}(\vec{p}) = \Psi_{\vec{0}}(\vec{p})$, where $\vec{p} = \frac{1}{2}(\vec{p}_1 - \vec{p}_2)$ is non-relativistic relative momentum. $\Psi_{\vec{0}}(\vec{p})$ describes a system with relativistic relative motion of constituents, because it is the solution of relativistic eq at rest, but moving non-relativistically as a whole.

Boost directly in terms of nuclear forces

- Extension of standard PT to handle E-dependence of potentials,

$$\Psi_P(\vec{p}) = \int \langle \vec{p} | [1 - G_u^b(M) \Delta(P)]^{-1} | \vec{k} \rangle d\vec{k} \Psi_{\vec{0}}(\vec{k}) =$$

$$\Psi_{\vec{0}}(\vec{p}) + \int \langle \vec{p} | G_u^b [\Delta + \Delta G_u^b \Delta + \mathcal{O}(\Delta^3)] | \vec{k} \rangle d\vec{k} \Psi_{\vec{0}}(\vec{k}), \quad (PT)$$

No further expansion analogous to one of bound state energy inherent to standard PT

- Δ is perturbation involving difference between Hamiltonians and energies of the system at move and at rest,

$$\langle \vec{p}' | \Delta(P) | \vec{p} \rangle = V(P, \vec{p}', \vec{p}) - V(M, 0, \vec{p}', \vec{p}) +$$

$$\left(M - 2\sqrt{m^2 + \vec{p}^2} - P^0 + \sqrt{m^2 + \vec{p}_1^2} + \sqrt{m^2 + \vec{p}_2^2} \right) \delta(\vec{p}' - \vec{p})$$

$$= \vec{P} \frac{\partial}{\partial \vec{P}} V(P, \vec{p}', \vec{p})_{\vec{P}=0} - \frac{v^2 p^2 + 2(\vec{v} \cdot \vec{p})^2}{M} \delta(\vec{p}' - \vec{p}) + \dots$$

$$\frac{\partial}{\partial \vec{P}} V(P, \vec{p}', \vec{p})_{\vec{P}=0} = A(\vec{p}', \vec{p}) \vec{p}' + B(\vec{p}', \vec{p}) \vec{p},$$

For power counting analysis $|\vec{P}| \sim |\vec{p}|$,

Boost directly in terms of nuclear forces

- $p^0 = \sqrt{\vec{p}^2 + M^2}$

$$\langle \vec{p}' | G_u^b(E) | \vec{p} \rangle = G_u(E, \vec{p}', \vec{p}) - \frac{\Psi_{\vec{0}}(\vec{p}') \bar{\Psi}_{\vec{0}}(\vec{p})}{E - M},$$

$G_u(E, \vec{p}', \vec{p})$ is Green function of NN system at rest,

$$\begin{aligned} & \left(E - 2\sqrt{m^2 + \vec{p}'^2} \right) G_u(E, \vec{p}', \vec{p}) \\ & - \int d\vec{k} V(E, \vec{0}, \vec{p}', \vec{k}) G_u(E, \vec{k}, \vec{p}) = \delta^3(\vec{p}' - \vec{p}), \end{aligned}$$

- Crucial! (PT) is expressed in terms of the NN potential V , just one, studied in more than three decades in great details, to high precision, in both approaches (TOPT and MUT) and is being extensively used for the systematic analysis of NN scattering.
- The first version also provides PT for given $H_I(x)$, but it is NOT expressed in terms of NN potential

Relativistically modified leading order approximation

- Modifying total momentum dependence of LO wave function, $\Psi_P^{LO}(\vec{p}) = \Psi_{\vec{0}}(\vec{p}) \rightarrow \Psi_{\vec{0}}(\overrightarrow{\Lambda^{-1}p})$, improves convergence of the expansion.
 $\overrightarrow{\Lambda^{-1}p}$ is the relativistic relative momentum **if** the constituents **would be** on-mass-shell, where p is "on-mass-shell" 4D relative momentum of constituents,
 $p = \left(\frac{\vec{p}\vec{p}}{P^0}, \vec{p} \right)$ (because $(\frac{P}{2} \pm p)^2 = m^2 \rightarrow Pp = 0$), Λ^{-1} is boost transformation matrix reducing the 4-vector P to its rest frame, $\Lambda^{-1}P = (M, \vec{0})$,
- Just $\Psi_{\vec{0}}(\overrightarrow{\Lambda^{-1}p})$ has been used as a moving deuteron wf in previous approximate approaches, which in our approach is only LO approximation!

Thanks

THANK YOU!

Shortest derivation of extended time independent PT, A.K. Blankleider, Am.J.Phys. 93, 8, 652 (2025)

- ****CONT**** $[G_0^{-1}(E_n) - K_0(E_n) - K_1(E_n)]|\psi_n\rangle = 0$ where $K_1(E)$ is the perturbation potential **with arbitrary E dependence, NOT just linear.**
- Green function of the unperturbed eq, $G_u(E)$, is known $[G_0^{-1}(E) - K_0(E)]G_u(E) = 1$
- Unperturbed potential $K_0(E)$ gives rise to a bound state at energy $\mathcal{E}_n$, then $G_u(E)$ has a pole at $\mathcal{E}_n$,
 $\langle x|G_u(E)|y\rangle = \frac{\phi_n(x)\bar{\phi}_n(y)}{E - \mathcal{E}_n} + G_u^b(E)$ ****Include proof of pole contribution to G****
- Use the factorised form of the pole contribution, $|\phi_n\rangle\langle\phi_n|$, to solve the perturbed eq algebraically,
 $|\psi_n\rangle = [1 - G_u^b(E_n)K_1(E_n)]^{-1}|\phi_n\rangle$