

Towards the Determination of the Gluon Helicity Distribution from Lattice QCD

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Outline

- Whence the proton spin?
- Pseudo-PDF approach for light-cone matrix elements
- From isovector to flavor-separated hadron structure
- Unpolarized Gluon Distribution in LQCD

A.M.Sturzu *et al.*, arXiv:2602.14260 (PRD, to appear)

T.Khan *et al*, Phys.Rev.D 104 (2021) 9, 094516

Helicity distributions

T.Khan *et al*, Phys.Rev.D 106 (2022) 9, 094511

Image: B&Q

Whence the proton spin

Since the EMC effect of the 80's, there has been an intense focus on the origin of the proton spin. These are encapsulated in two sum rules

Ji's decomposition

X.Ji, PRL, 78, 610 (1997)

$$\frac{1}{2} = J_q + J_g = \frac{1}{2} \Delta \Sigma + L_q^{\text{kin}} + J_g$$

Lattice: *Moments of Generalized Parton Distributions*

S.Zafeiropoulos, this session

Calculations performed in the pseudo-PDF framework

Jaffe-Manohar sum rule

R.Jaffe, A Manohar, NPB 337, 509 (1990).

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_q^{\text{can}} + L_g^{\text{can}}$$



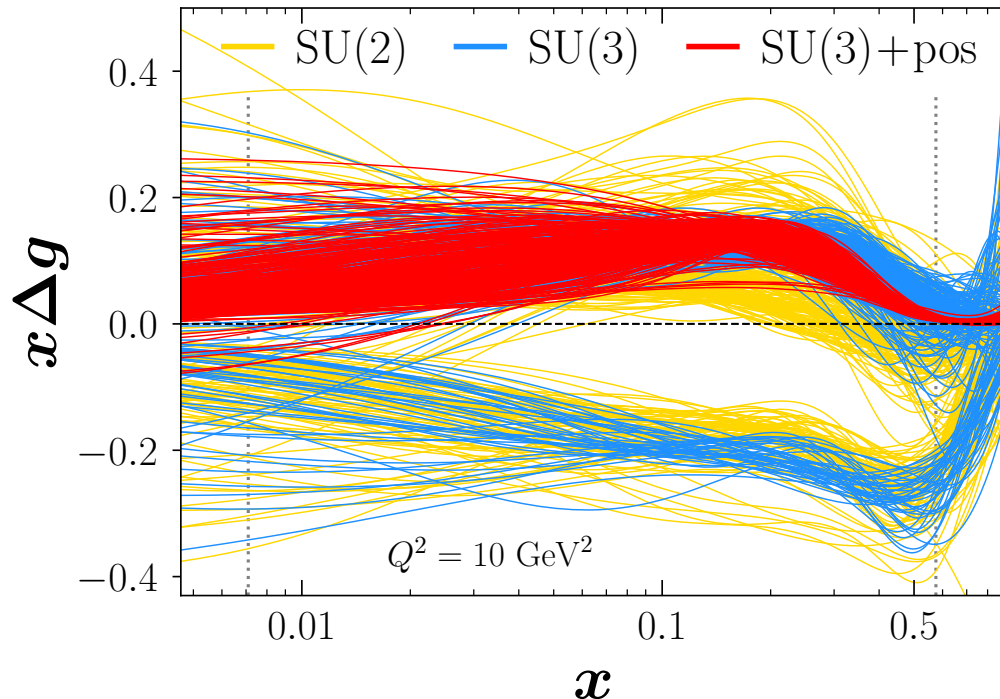
$$\Delta G(\mu) = \int_0^1 dx \Delta g(x, \mu)$$

Gluon Helicity Distribution

- Crucial questions in global analysis - do we need to apply positivity constraint:

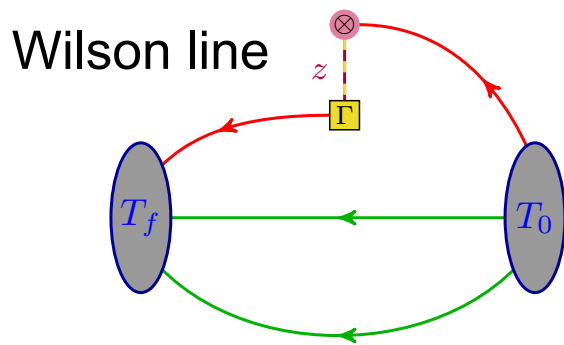
$$| \Delta g(x) | \leq g(x) \forall x$$

Relaxing constraint leads to new “replicas” in global analysis:



Zhou, Sato and Melnitchouk, Phys. Rev. D 105, 074022 (2022)

Pseudo-PDFs



A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time*. $\nu = p \cdot z$

B.Ioffe, PL39B, 123 (1969); V.Braun *et al*, PRD51, 6036 (1995)

$$M^\alpha(p, z) = \langle p | \bar{\psi} \gamma^\alpha U(z; 0) \psi(0) | p \rangle$$

$$p = (p^+, m^2/2p^+, 0_T) \quad \Downarrow \quad z = (0, z_-, 0_T)$$

$$M^\alpha(z, p) = 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

Ioffe-time pseudo-Distribution (**pseudo-ITD**) generalization to *space-like z*

Inverse problem

$$Q(\nu) = \int_{-1}^1 dx q(x) e^{i\nu x}$$

$$q(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu e^{-i\nu x} Q(\nu)$$

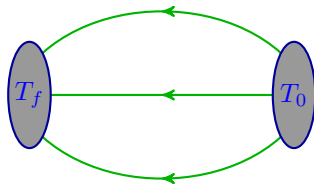
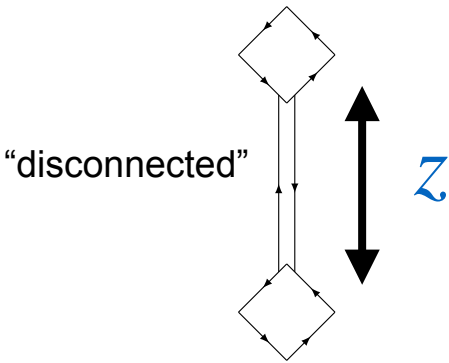
Gluon Contribution to unpolarized PDF

c.f. Z.Fan, H-W-Lin, arXiv:2104.06372, arXiv:2007.16113
T.Khan *et al.* (Hadstruc), *Phys.Rev.D* 104 (2021) 9, 094516

$$M_{\mu\alpha;\lambda\beta}(z, p) \equiv \langle p | G_{\mu\alpha}(z) W[z, 0] G_{\lambda\beta}(0) | p \rangle$$



$$O_g(z) = G_{ji}(z) U(z, 0) G_{ij}(0) U(0, z) - G_{ti}(z) U(z, 0) G_{it}(0) U(0, z).$$



Two-point functions as in isovector case

Reduced matrix element: $\mathfrak{M}(\nu, z^2) = \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu, 0)|_{z=0}} \right) / \left(\frac{\mathcal{M}(0, z^2)|_{p=0}}{\mathcal{M}(0, 0)|_{p=0, z=0}} \right)$

Flavor-singlet quantities are subject to severe signal-to-noise problems compared with isovector measures:

- Use **distillation** and many more measurements per configuration - *sampling of lattice*
- Use of summed **Generalized Eigenvalue Problem (sGEVP)** - *better control over excited state contributions*
- Use of **Gradient Flow** - *smoothing of short-distance fluctuations*

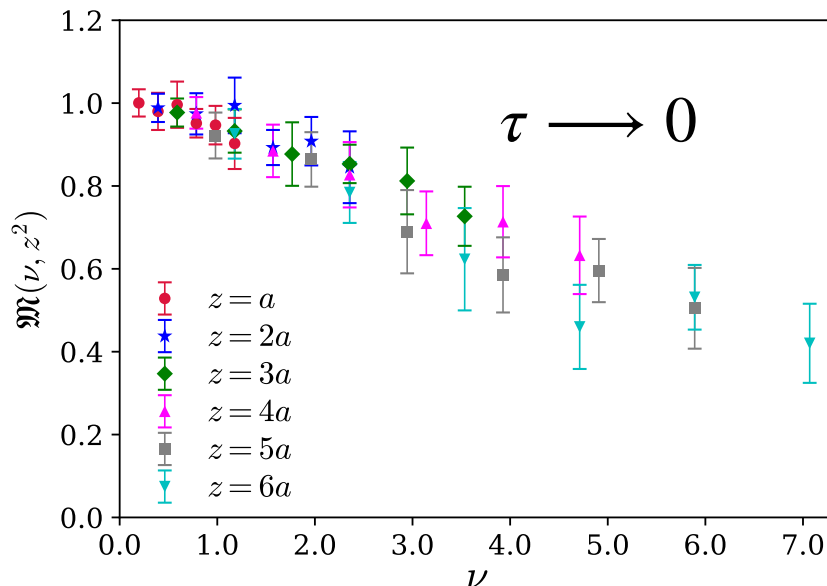
loffe-time distributions

Introduce **gradient flow**, implemented on lattice through

$$\partial_\tau B_\mu(\tau, x) = -g_0^2 \{ \partial_{x,\mu} S(B_\mu) \} B_\mu(\tau, x)$$

Smooths fluctuations at large τ , and as ultra-violet regulator; *relate operator to renormalized operator at some scale μ*

M.Luscher; M.Luscher and P.Weisz; Harlander, Kuth, Lange;
R. V. Harlander, J. T. Kohnen, and A. Shindler, arXiv:2511.17145 - **NNLO**



ID	a_s (fm)	m_π (MeV)	$L_s^3 \times N_t$	N_{cfg}	N_{srcs}	$R_{\mathcal{D}}$
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	349	4	64



↓ ↓
~1100 ~64

ITD to PDF

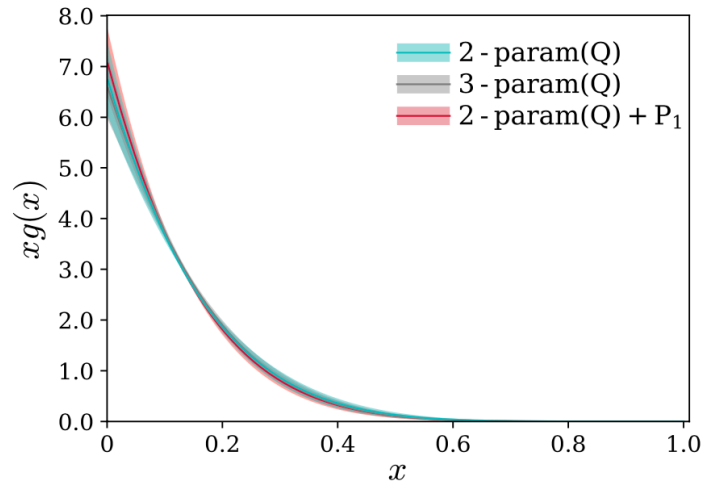
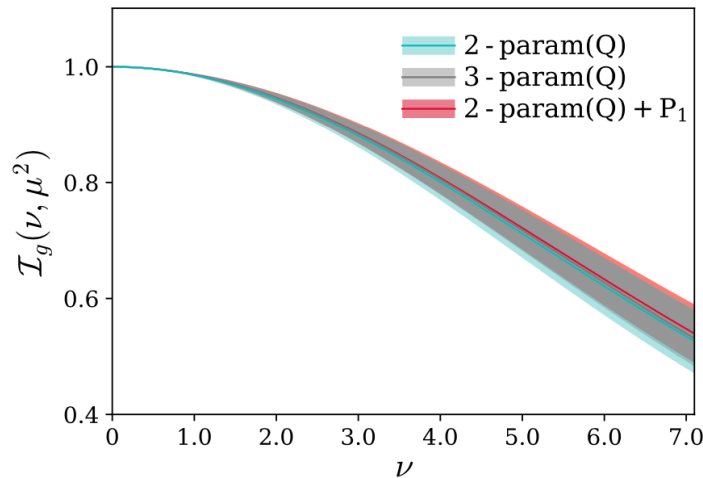
Matching:

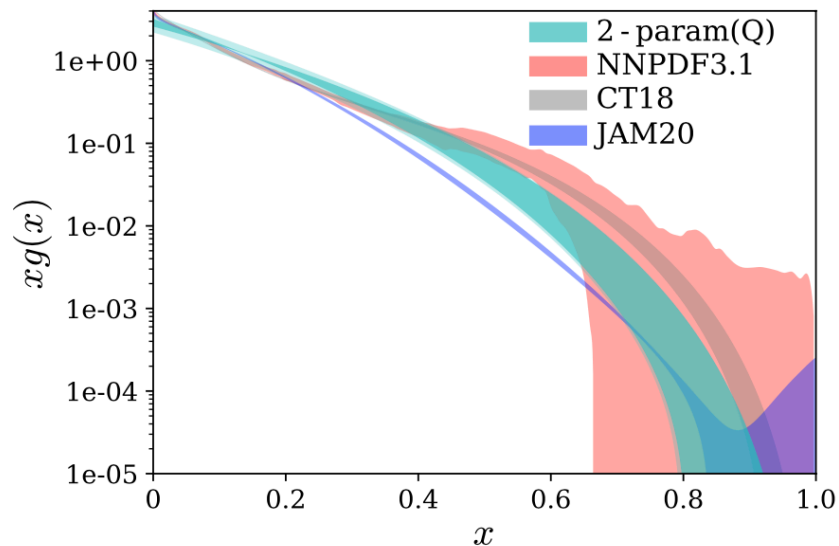
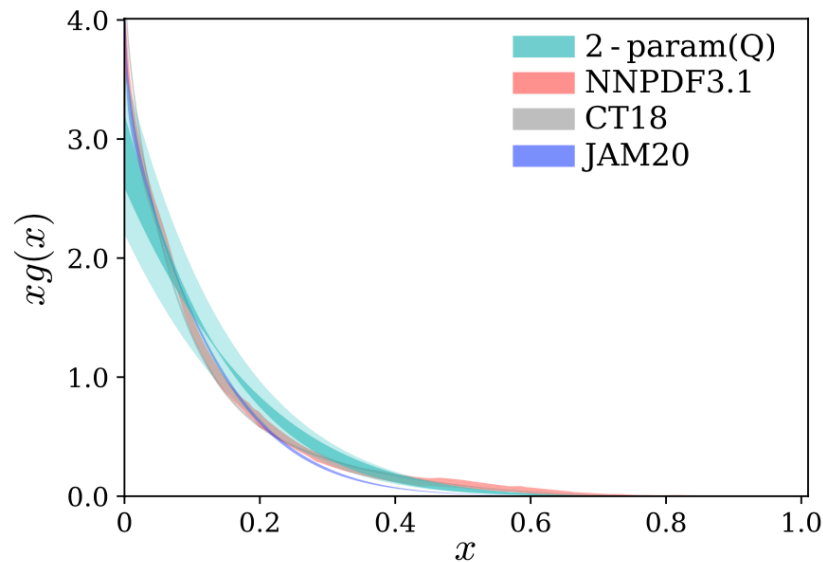
I.Balitsky,W.Morris,A.Radyushkin,Phys.Lett.B 808 (2020) 135621

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{I}_g(\nu, \mu^2)}{\mathcal{I}_g(0, \mu^2)} - \frac{\alpha_s N_c}{2\pi} \int_0^1 du \frac{\mathcal{I}_g(u\nu, \mu^2)}{\mathcal{I}_g(0, \mu^2)} \left\{ \ln\left(\frac{z^2 \mu^2 e^{2\gamma_E}}{4}\right) B_{gg}(u) + 4 \left[\frac{u + \ln(\bar{u})}{\bar{u}} \right]_+ + \frac{2}{3} [1 - u^3]_+ \right\} - \frac{\alpha_s C_F}{2\pi} \ln\left(\frac{z^2 \mu^2 e^{2\gamma_E}}{4}\right) \int_0^1 dw \frac{\mathcal{I}_S(w\nu, \mu^2)}{\mathcal{I}_g(0, \mu^2)} \mathfrak{B}_{gq}(w).$$

Implementation for obtaining the PDFs follows that of the isovector distribution
– *Expand in Jacobi Polynomials*

$$x^\alpha (1-x)^\beta + J_1^{\alpha, \beta} + a/|z|$$





Require normalization of $xg(x)$

$$\langle x \rangle_q^{\text{MS}}(\mu = 2 \text{ GeV}) = 0.427(92)$$

C.Alexandrou et al., Phys. Rev. Lett. 119, 142002 (2017)

Cf *Isovector distribution through quark-counting rule*

Gluon Momentum Fraction

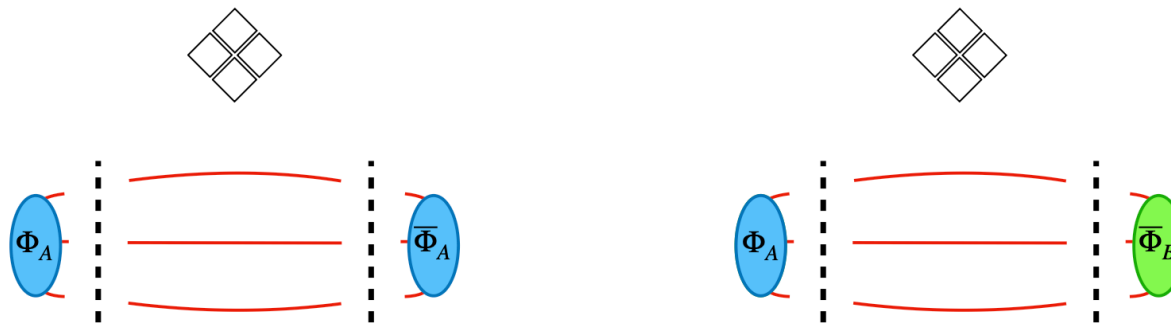
A.M.Sturzu *et al.*, Phys.Rev.D 114 (2026) 1, 014516

Defined through $\langle x \rangle_g(\mu) = \frac{1}{2} \int_{-1}^1 dx \, x \, g(x, \mu)$

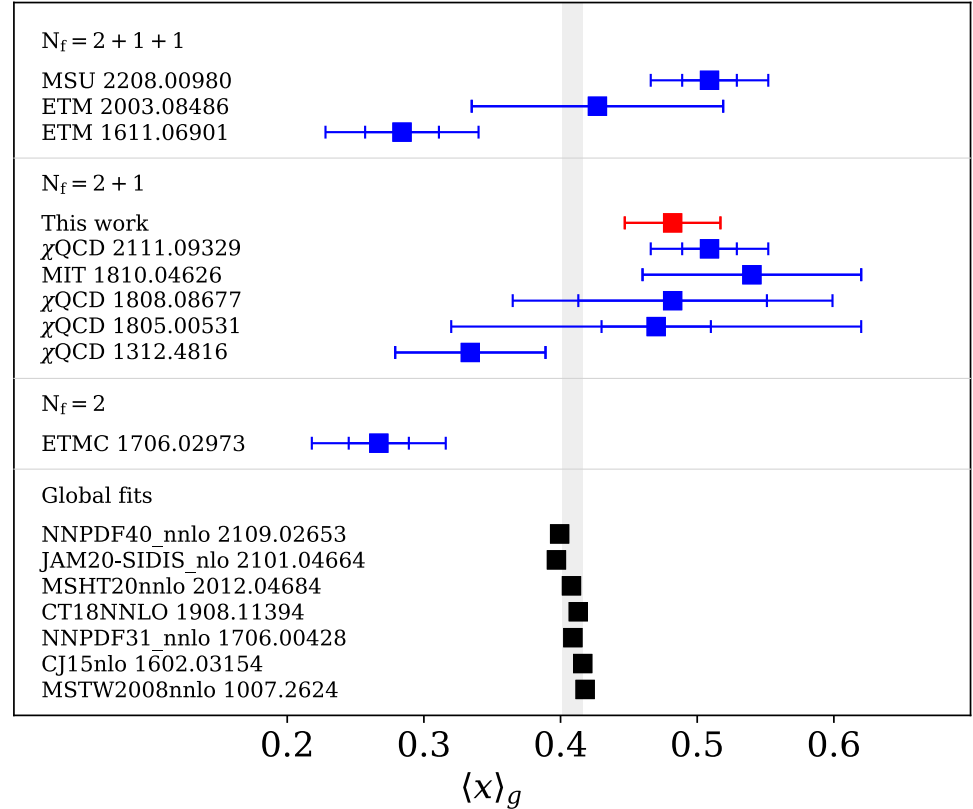
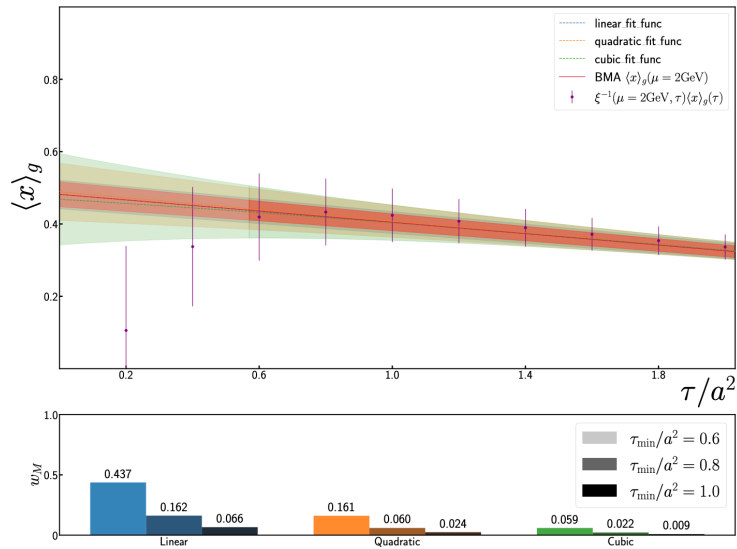
Gluon Momentum fraction is matrix element of component of energy-momentum tensor:

$$T_g^{\mu\nu} = \frac{1}{g_0^2} \text{Tr} \left\{ G^{\mu\rho} G_\rho^\nu - \frac{1}{4} g^{\mu\nu} G^{\rho\sigma} G_{\rho\sigma} \right\} \longrightarrow \langle P | T_g^{\{\mu\nu\}} | P \rangle = 2 \langle x \rangle_g P^{\{\mu} P^{\nu\}}$$

Require appropriate representation of gluon field-strength tensor



Gluon matrix elements



Residual extrapolation to $\tau = 0$

$$\langle x \rangle_g(\mu = 2\text{GeV}) = 0.482(35)$$

Helicity Gluon PDF

Gluon Helicity PDF

Matrix elements of spatially separated gluon fields

$$\tilde{m}_{\mu\alpha;\lambda\beta} = \langle p, s | G_{\mu\alpha}(x) W[z,0] \tilde{G}_{\alpha\beta}(0) | p, s \rangle$$

Combination corresponding to polarized gluon distribution

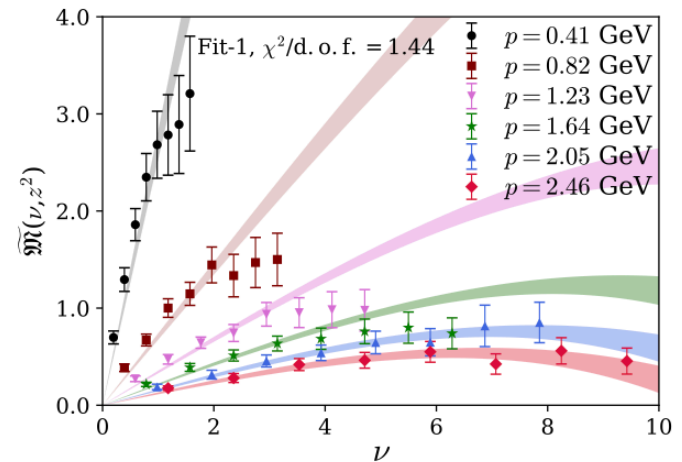
$$\tilde{M}_{\mu\alpha;\lambda\beta}(z, p, s) = \tilde{m}_{\mu\alpha;\lambda\beta}(z, p, s) - \tilde{m}_{\mu\alpha;\lambda\beta}(-z, p, s)$$

loffe-time distribution is related to gluon distribution through inverse problem

$$\tilde{\mathcal{F}}(\nu) = \frac{i}{2} \int_{-1}^1 e^{-ix\nu} x \Delta g(x)$$

$$\tilde{\mathcal{M}}(\nu, z^2) = \left[\tilde{\mathcal{M}}_{sp}^{(+)}(\nu, z^2) - \nu \tilde{\mathcal{M}}_{pp}(\nu, z^2) \right] - \frac{m_p^2}{p_z^2} \nu \tilde{\mathcal{M}}_{pp}(\nu, z^2)$$

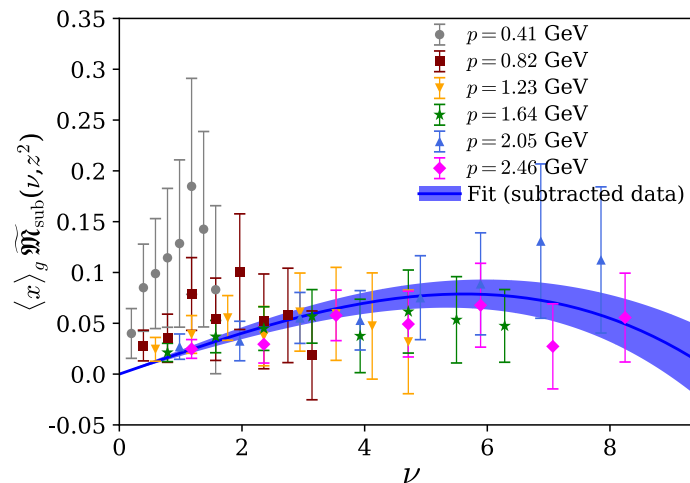
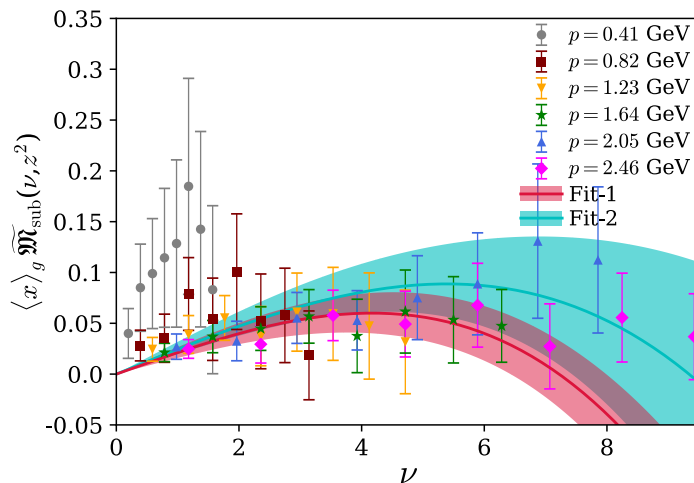
“Nuisance term”



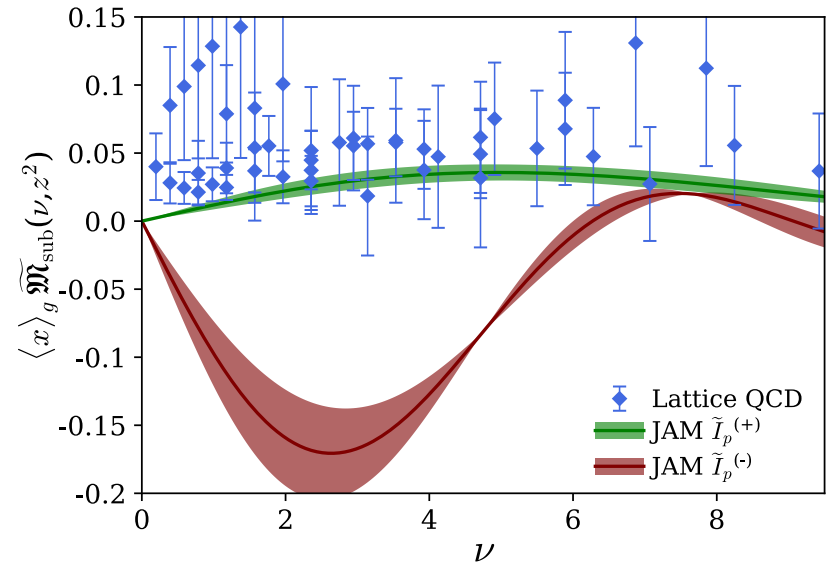
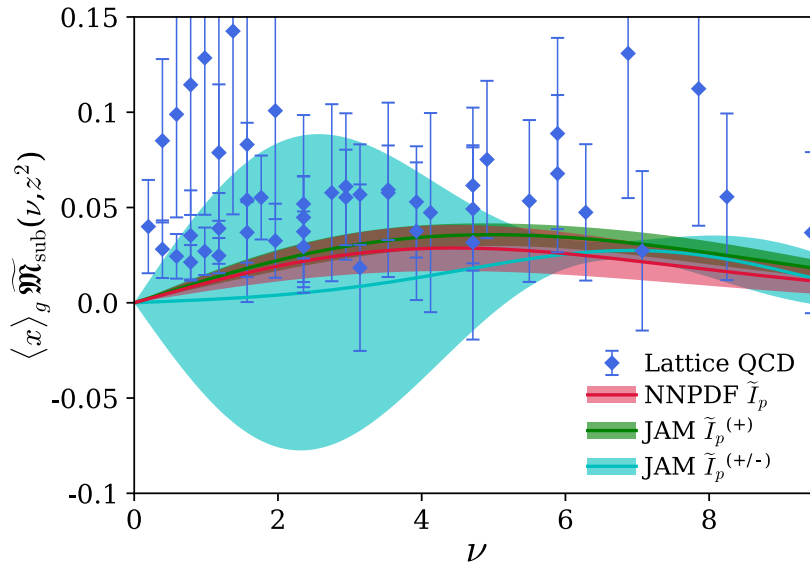
Rather than fitting to $\tilde{\mathcal{M}}$ directly define subtracted matrix element

$$\tilde{\mathcal{M}}_{\text{sub}}(z, p_z) = \tilde{\mathcal{M}}_{sp}^{(+)}(\nu, z^2) - \nu \tilde{\mathcal{M}}_{pp}(\nu, z^2) - \nu \frac{m_p^2}{p_z^2} \left[\tilde{\mathcal{M}}_{pp}(\nu, z^2) - \tilde{\mathcal{M}}_{pp}(\nu = 0, z^2) \right]$$

Still contains nuisance term - but small



ITD $\leftrightarrow$ PDF



C.Egerer *et al.* (*HadStruc*), Phys.Rev.D 106 (2022) 9, 094511

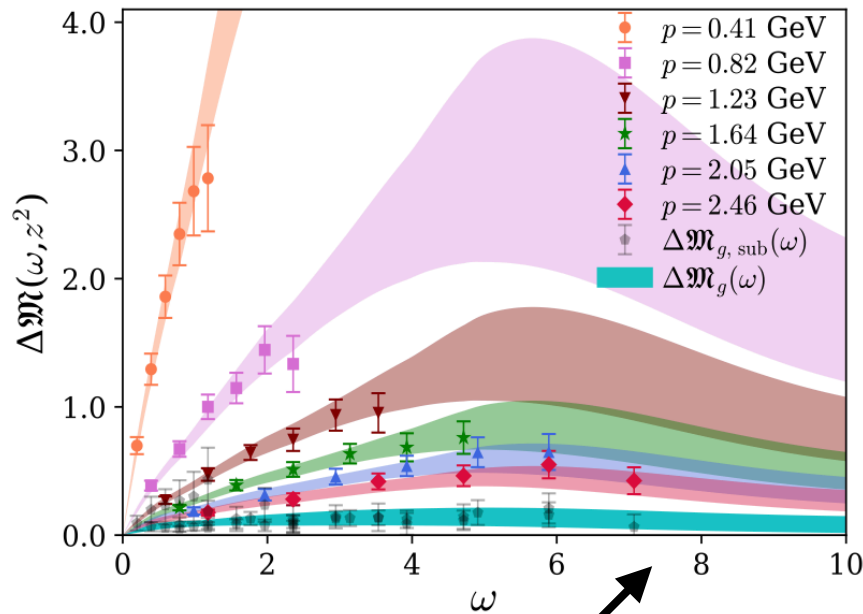
JAM with/without helicity constraint.

LQCD Calculation of gluon helicity distribution compared with global analyses

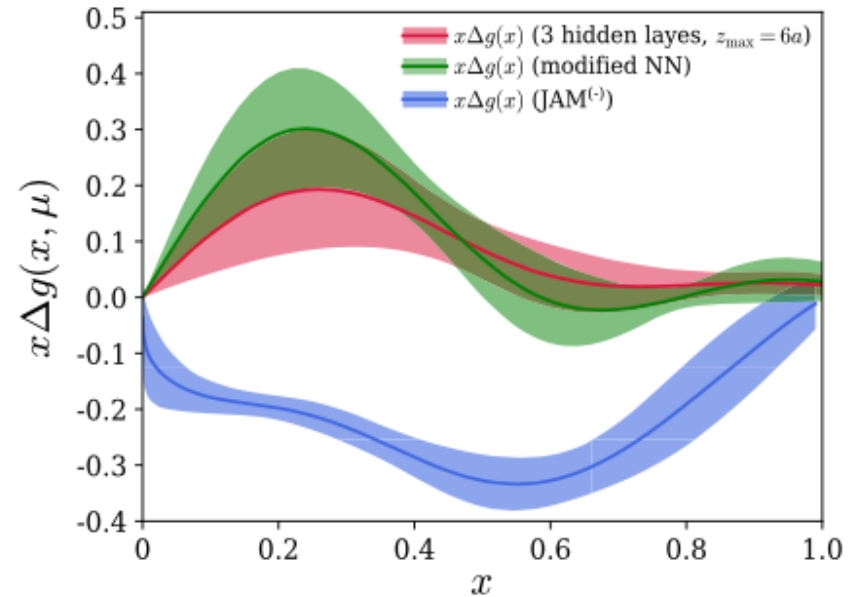
Caveat! *Mixing with sea quarks not yet included*

Re-analysis using NN

T.Khan,T.Liu and R.Sufian, Phys. Rev. D 108, 074502



Physical helicity matrix element



T.Chowdhury et al., PRD 111, 074509 (2025)

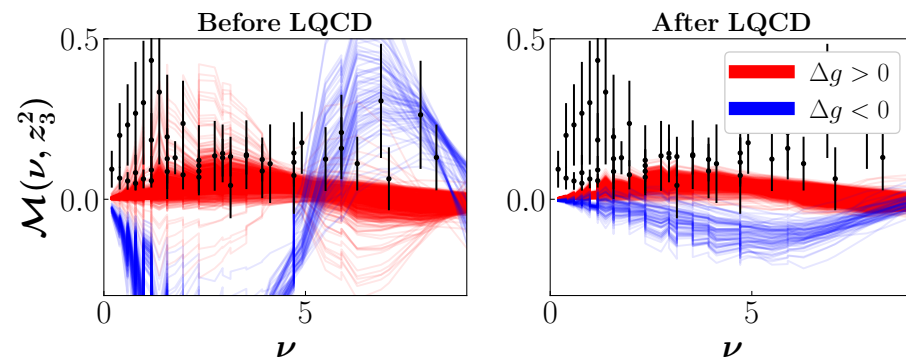
NN fit to Ioffe-time distribution to remove subtraction term

Lattice + Expt

The culmination of QGT is a framework where LQCD + Expt can provide a more faithful description of hadron structure than either alone.

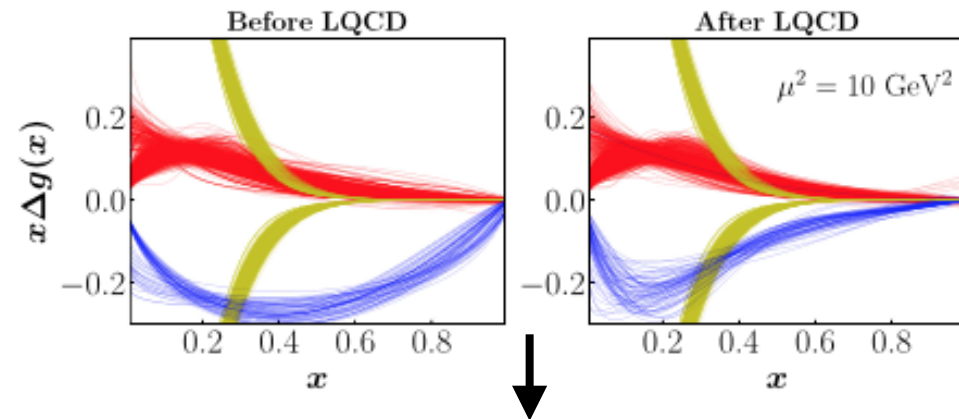
J.Karpie *et al.*, Phys.Rev.D 109 (2024) 3, 036031

Does QCD admit negative solutions $\Delta g(x) < 0$?

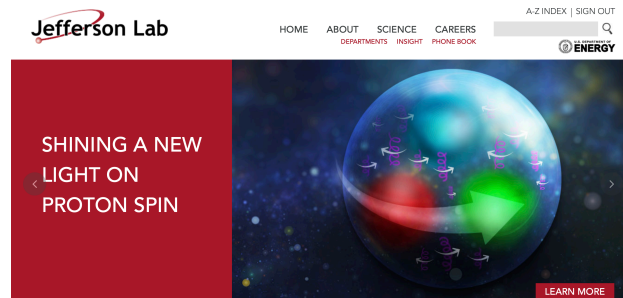


Significant change in parametrization but insufficient to exclude negative solute in global analysis

RHIC polarized jet, large-x JLab + LQCD
eliminate negative solutions: N.Hunt-Smith *et al.*, arXiv:2403.08117



Impact of LQCD on global analysis



Direct Calculation of ΔG

In contrast to the case of the gluon momentum fraction, there is no local, gauge-invariant operator that corresponds to the total helicity.

Can relate to a *gauge-dependent* expression $S_g = \int d^3x \mathbf{E}^a(x) \times \mathbf{A}_{\text{phys}}^a(x)$
Y.B. Yang et al, Physical Review Letters **118**, 102001 (2017)

Results using effective-theory matching:

$$\Delta G^{\text{MS}}(\mu^2 = 10 \text{ GeV}^2) = 0.251(47)(16).$$

Recent proposal to relate total helicity to matrix element of topological charge
Pang, Yao, & Zhang, JHEP, Phys. 2024, 222 (2024).

$$K^\mu(x) = \epsilon^{\mu\nu\rho\sigma} \text{Tr}[A_\nu F_{\rho\sigma} - \frac{2}{3} i g_0 A_\nu A_\rho A_\sigma](x)$$

Recent calculation at three lattice spacings: Zhao et al (CLQCD), arXiv:2512.24315

$$\Delta G^{\text{MS}}(\mu^2 = 10 \text{ GeV}^2) = 0.231(17)_{\text{stat}}(44)_{\text{syst}}$$

Both results are broadly consistent, and employing different methodologies

Summary and Outlook

- Calculation of gluon distributions, and in particular those for helicity, provide the opportunity to gain insights into hadron structure beyond those obtained by experiment.
- Calculation of x -dependent $\Delta g(x)$ provides already provides important insights - *strongly suggests positivity of gluon helicity distributions.*
- Next steps
 - Inclusion of isoscalar quark contributions
 - Control over systematic effects
- Extension to GPDs and GFFs provide a framework for understanding the OAM of the gluons
- GTMDs, and the interpolation between the Ji and Jaffe-Manohar sum rules

e.g. [Englehardt, Lattice 2021](#);