

From 1D to 3D Hadron Structure with lattice QCD simulations

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PDFs are of paramount importance because...

- The uncertainties in PDFs are the **dominant theoretical uncertainties** in Higgs couplings, α_s and the mass of the W boson

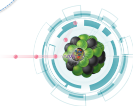
- Beyond the LHC, PDFs play an important role, for instance in astroparticle physics, such as for the accurate predictions for signal and background events at ultra-high energy neutrino telescopes (ANITA, IceCube, Pierre Auger Observatory)



Future Circular Collider
Circumference: 80-100 km
Energy: 100 TeV (pp)
>350 GeV (e^+e^-)

Large Hadron Collider
Circumference: 27 km
Energy: 14 TeV (pp)
209 GeV (e^+e^-)

Tevatron (closed)
Circumference: 6.2 km
Energy: 2 TeV



- PDFs will keep playing an important role for any future high energy collider involving hadrons in the initial state. Therefore improving our understanding of PDFs also strengthens the physics potential of such future colliders

 Gao, Harland-Lang, Rojo (2018)

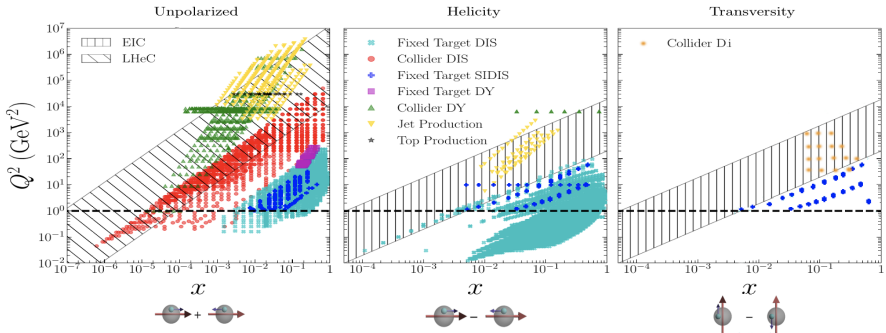
Why are they so challenging?



- non perturbative
- data scarcity

Limited experimental coverage

Going from unpolarized to polarized PDFs, global analyses face the big challenge of data scarcity



We will argue that we can provide an excellent data coverage independent of the PDF polarization

Light cone distributions and lattice QCD



The Ji Idea

- Lattice QCD computes equal time matrix elements
- Displace quarks in space-like interval
- Boost states to “infinite” momentum
- On the frame of the proton displacement becomes lightlike
- But infinite momentum not possible on the lattice
- Use perturbative matching from finite momentum  X. Ji (2013)
- One needs to deal with the divergences

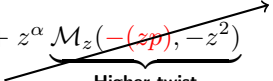
PDFs from the lattice: Pseudo-PDFs Formalism

Starting point: the equal time hadronic matrix element with the quark and anti-quark fields separated by a finite distance  Radyushkin (2017)

$$\mathcal{M}^\alpha(z, p) \equiv \langle p | \bar{\psi}(0) \gamma^\alpha \hat{E}(0, z; A) \tau_3 \psi(z) | p \rangle$$

$z = (0, 0, 0, z_3)$
 $p = (p^0, 0, 0, p)$
 $\alpha = 0$

Lorentz inv. $\rightarrow$

$$\mathcal{M}^\alpha(z, p) = 2p^\alpha \underbrace{\mathcal{M}_p(-\textcolor{red}{(zp)}, -z^2)}_{\text{Leading twist}} + z^\alpha \underbrace{\mathcal{M}_z(-\textcolor{red}{(zp)}, -z^2)}_{\text{Higher twist}}$$


- The Lorentz invariant quantity $\nu = -\textcolor{red}{(zp)}$, is the "loffe time"
- loffe time PDFs $\mathcal{M}(\nu, z_3^2)$ defined at a scale $\mu^2 = 4e^{-2\gamma_E}/z_3^2$ (at leading log level) are the Fourier transform of regular PDFs $f(x, \mu^2)$  Balitsky, Braun (1988),  Braun et al. (1995)

$$\mathcal{M}(\nu, z_3^2) = \int_{-1}^1 dx f(x, 1/z_3^2) e^{i x \nu}$$

Obtaining the loffe time PDF

$$z_3 \rightarrow 0 \Rightarrow \mathcal{M}_p(\nu, z_3^2) = \mathcal{M}(\nu, z_3^2) + \mathcal{O}(z_3^2)$$

But.... large $\mathcal{O}(z_3^2)$ corrections **prohibit** the extraction.

In a **ratio** z_3^2 corrections might cancel  Radyushkin (2017)

$$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}_p(\nu, z_3^2)}{\mathcal{M}_p(0, z_3^2)}$$

- Much **smaller** $\mathcal{O}(z_3^2)$ corrections and therefore this ratio could be used to extract the loffe time PDFs
- All UV singularities are **exactly cancelled** and when computed in lattice QCD it can be extrapolated to the continuum limit

■

$$\mathfrak{M}(\nu, z^2) = \int_0^1 d\alpha \mathcal{C}(\alpha, z^2 \mu^2, \alpha_s(\mu)) \mathcal{Q}(\alpha \nu, \mu) + \sum_{k=1}^{\infty} \mathcal{B}_k(\nu) (z^2)^k,$$

μ is the factorization scale and $\mathcal{Q}(\nu, \mu)$ is the loffe time PDF

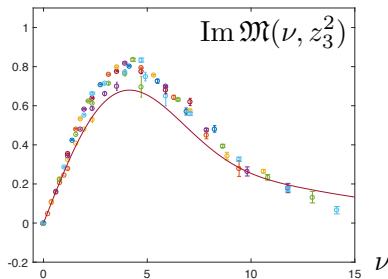
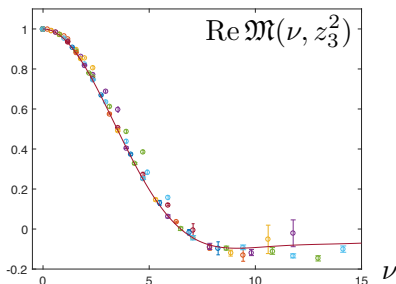
Access to moments beyond the x — dependence

$$Q(\nu, \mu) = \int_{-1}^1 dx q(x, \mu) e^{ix\nu}.$$

$$(-i)^n \left. \frac{\partial^n Q(\nu, \mu)}{\partial \nu^n} \right|_{\nu=0} = \int_{-1}^1 dx x^n q(x, \mu) = a_{n+1}(\mu).$$

you don't have to reconstruct the entire PDF to get its moments. You only need the n -th derivative around $\nu = 0$  Karpie, Orginos and SZ JHEP 11 (2018) 178.

Results for the Re and Im parts of $\mathfrak{M}(\nu, z_3^2)$



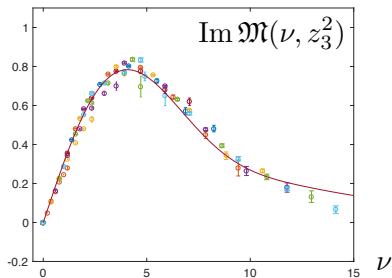
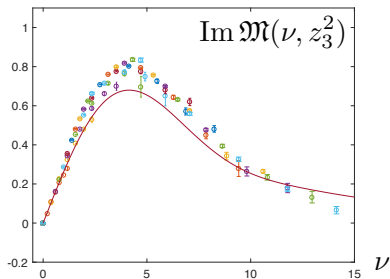
- First case study in an unphysical setup Karpie, Orginos, Radyushkin SZ, Phys.Rev. D96 (2017) no.9, 094503

Curves represent Re and Im Fourier transforms of $q_v(x) = \frac{315}{32} \sqrt{x}(1-x)^3$.

- Considering CP even and odd combinations

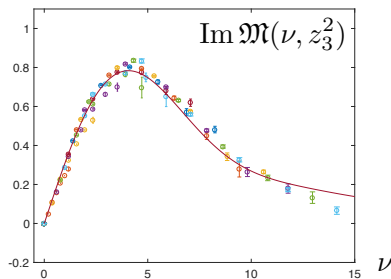
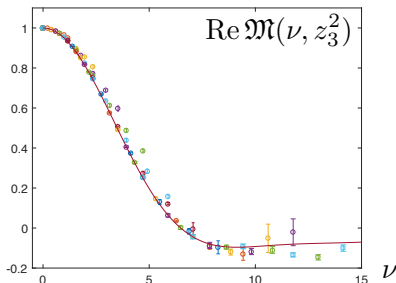
- ▶ even: $q_-(x) = f(x) + f(-x) = q(x) - \bar{q}(x) = q_v(x)$
- ▶ odd: $q_+(x) = f(x) - f(-x) = q(x) + \bar{q}(x) = q_v(x) + 2\bar{q}(x)$

Results for the Im part of $\mathfrak{M}(\nu, z_3^2)$



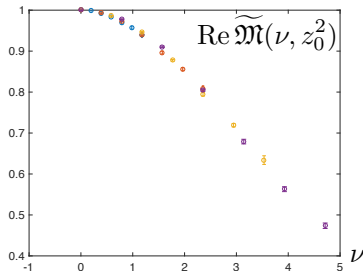
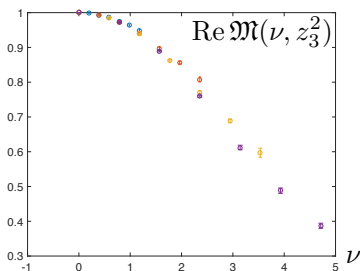
- Curves represent the Im Fourier transforms of $q_v(x) = q(x) - \bar{q}(x)$ and $q_+(x) = q(x) + \bar{q}(x) = q_v(x) + 2\bar{q}(x)$ respectively.
- The agreement with the data is strongly improved if we use a non-vanishing antiquark contribution, namely $\bar{q}(x) = \bar{u}(x) + \bar{d}(x) = 0.07[20x(1-x)^3]$.

Results for the Re and Im parts of $\mathfrak{M}(\nu, z_3^2)$



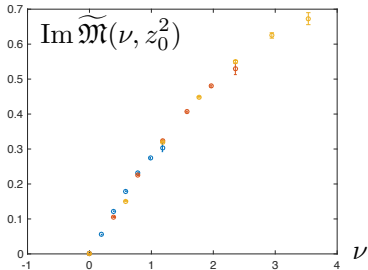
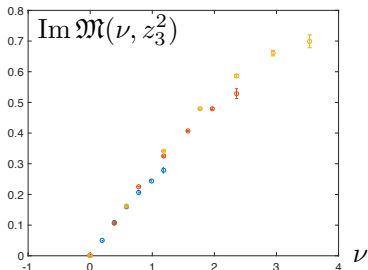
- Data as function of the loffe time. A **residual** z_3 -dependence can be seen.
- This is more visible when, for a **particular** ν we have several data points corresponding to **different values of** z_3 .
- Different values of z_3^2 for the same ν correspond to the loffe time distribution at different scales.

Before and after evolution



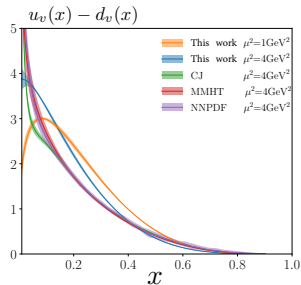
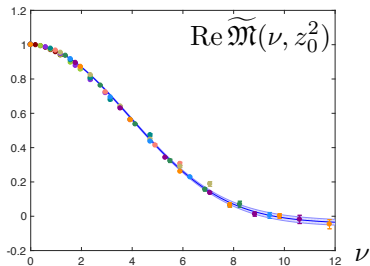
The ratio $\mathfrak{M}(\nu, z_3^2)$ for $z_3/a = 1, 2, 3$, and 4 . **LHS:** Data **before evolution**. **RHS:** Data **after evolution**. The reduction in scatter indicates that evolution collapses all data to the same universal curve.

Before and after evolution



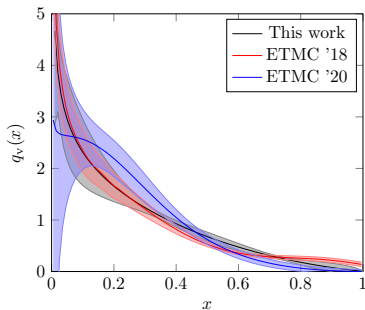
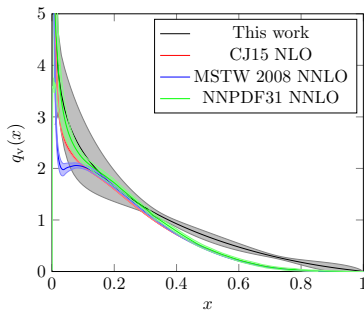
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Comparisons with global fits



Left: Data points for $\text{Re } \widetilde{\mathcal{M}}(\nu, z_3^2)$ with $z_3 \leq 10a$ evolved to $z_3 = 2a$ as described in the text. Right: Curve for $u_v(x) - d_v(x)$ built from the evolved data shown in the left panel and treated as corresponding to the $\mu^2 = 1 \text{ GeV}^2$ scale; then evolved to the reference point $\mu^2 = 4 \text{ GeV}^2$ of the global fits.

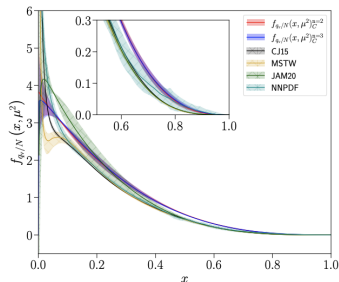
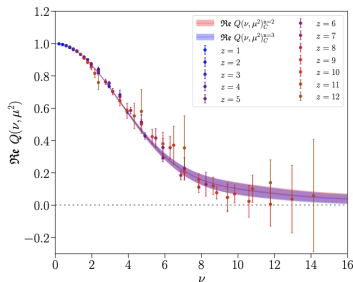
New results with $N_f = 2 + 1$ fermions for the nucleon



Our determination of the phys. pion mass nucleon valence PDF compared to pheno and other lattice determinations. [Joo, Karpie, Orginos, Radyushkin, Richards, S.Z.](#)

Phys.Rev.Lett. 125 (2020) 23, 232003

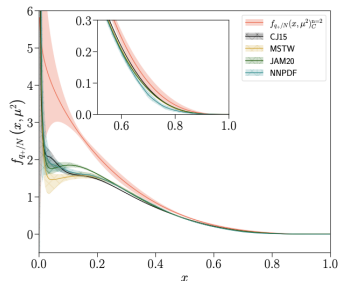
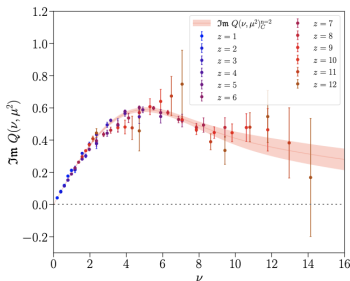
Towards High-Precision Parton Distributions From Lattice QCD via Distillation



Real part of the matched ITD, compared with cosine-transform fits using two- and three-parameter PDFs and NLO/NNLO global analyses. [Egerer,](#)

Edwards, Kallidonis, Orginos, Radyushkin, Richards, Romero, S. Z. JHEP11(2021)148

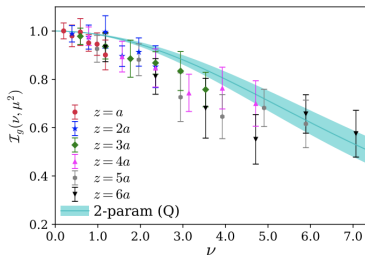
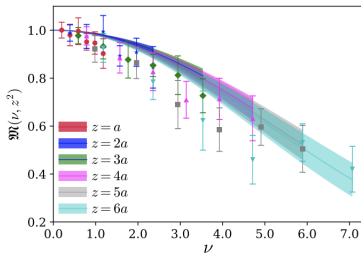
Towards High-Precision Parton Distributions From Lattice QCD via Distillation



Imaginary component of the matched ITD, fitted using the sine transform of a two-parameter PDF model. Compared with NLO CJ15 and JAM20, and NNLO MSTW and NNPDF global analyses at the same scale.  Egerer, Edwards,

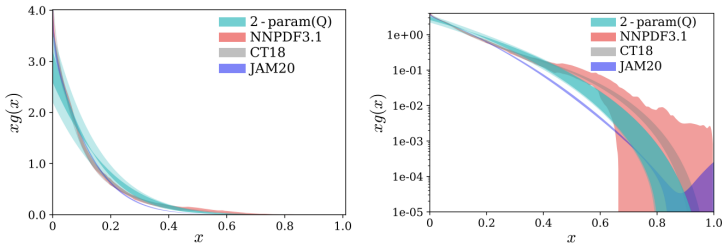
Kallidonis, Orginos, Radyushkin, Richards, Romero, S. Z. JHEP11(2021)148

Gluon PDF



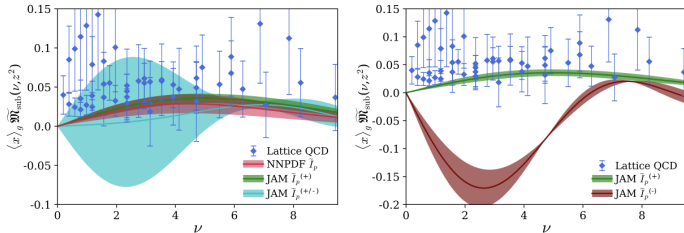
🔗 Khan, Sufian, Karpie, Monahan, Egerer, Joo, Morris, Orginos, Radyushkin, Richards, Romero, S. Z. Phys.Rev.D 104 (2021) 9, 094516 Lattice reduced pseudo-ITD shown along with their reconstructed fitted bands and the $\overline{\text{MS}}$ matched ITD at 2GeV.

Gluon PDF



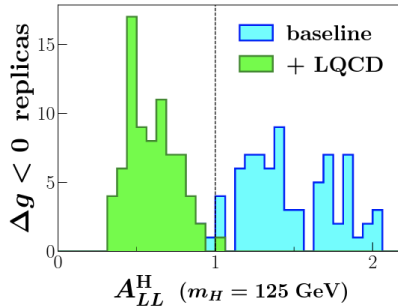
 Khan, Sufian, Karpie, Monahan, Egerer, Joo, Morris, Orginos, Radyushkin, Richards, Romero, S. Z. Phys.Rev.D 104 (2021) 9, 094516 Unpolarized gluon PDF (cyan band) from lattice data using the 2-param. (Q) model, compared with global fits. Normalized using $\langle x \rangle_g^{\overline{\text{MS}}}(2, \text{GeV}) = 0.427(92)$ from  Alexandrou et al, Phys.Rev.D 101 (2020) 9, 094513 .
Right/left panels rescale $xg(x)$ to highlight the large-(x) region.

Gluon helicity distribution



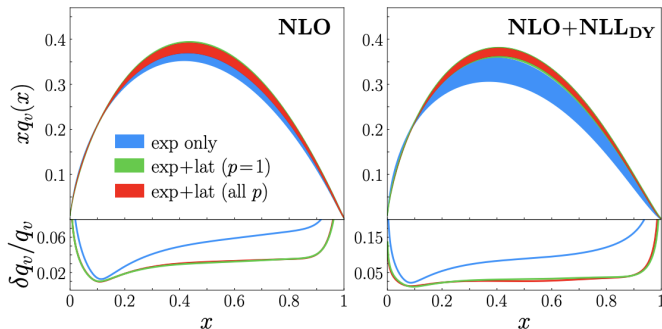
[Egerer, Joo, Karpie, Karthik, Khan, Monahan, Morris, Orginos, Radyushkin, Richards, Romero, Sufian and S.Z., Phys.Rev.D 106 \(2022\) 9, 094511](#)
[De Florian, Forte and Vogelsang Phys. Rev. D 109, 074007](#)
[JAM Collaboration N.T. Hunt-Smith et al 2403.08117 \[hep-ph\]](#)
 The lattice-reduced pITD compared with gluon helicity ITDs from global fits. The red band shows the NNPDF result, while the green and cyan bands show the JAM results with and without the positivity constraint. The green and maroon bands correspond to the positive and negative gluon-helicity PDF solutions from JAM.

Gluon helicity distribution



 JAM Collaboration N.T. Hunt-Smith et al 2403.08117 [hep-ph] Lattice data added to the “baseline” don’t lead to negative cross-sections for physical Higgs mass, so still one can not exclude them. Finally, the combination of baseline, lqcd data and high- x polarized DIS data resolved the debate.

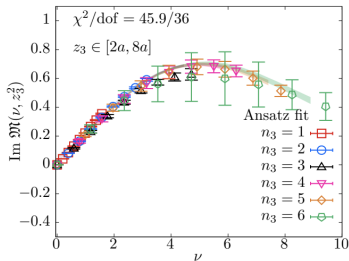
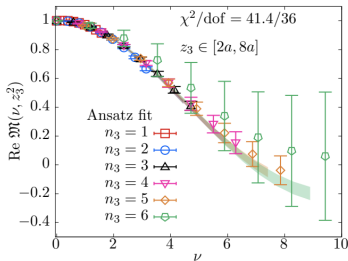
Complementarity of experimental and lattice QCD data on pion parton distributions



Valence quark distributions (**top**) when extracted from experimental data alone (blue), combined with the $p = 1$ lattice data (green), and combined with all the lattice data (red) for the NLO (**left**) and NLO+NLL_{DY} (**right**) cases, along with the relative uncertainties (**bottom**). The bands represent a 1σ uncertainty level. [Barry, Egerer, Karpie, Melnitchouk, Orginos, Richards, Sato, Sufian, Qiu, S.Z.](#)

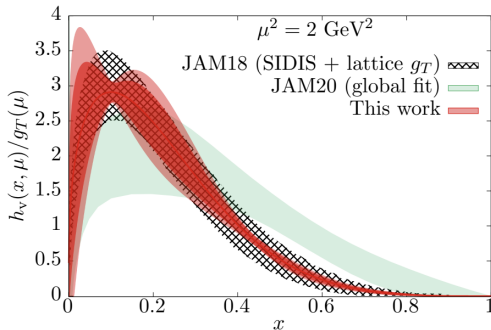
Phys.Rev.D 105 (2022) 11, 114051

Transversity PDF



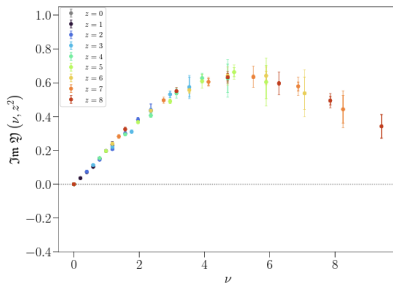
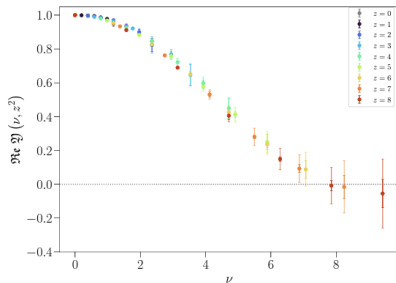
 Egerer, Kallidonis, Karpie, Karthik, Morris, Orginos, Radyushkin, Romero, Sufian, S. Z. Phys.Rev.D 105 (2022) 3, 034507 The real and imaginary parts of $\mathfrak{M}$ are shown as a function of ν .

Transversity PDF



 Egerer, Kallidonis, Karpie, Karthik, Morris, Orginos, Radyushkin, Romero, Sufian, S. Z., Phys. Rev. D 105 (2022) 3, 034507. Valence transversity distribution $h_v(x, \mu)/g_T(\mu)$, compared with previous SIDIS + lattice results (JAM18, patterned band) and the updated SIDIS global fit (JAM20, green band), without lattice g_T .

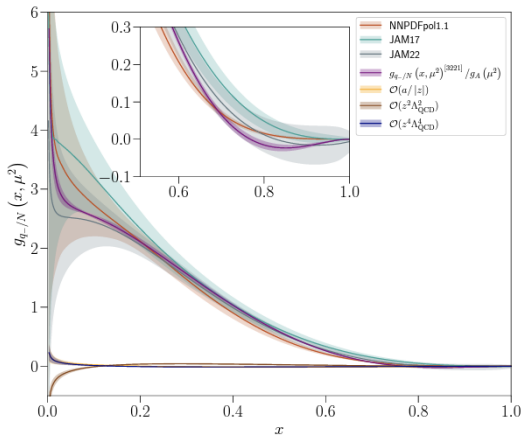
Helicity PDF



Edwards, Egerer, Karpie, Karthik, Monahan, Morris, Orginos, Radyushkin, Romero, Sufian, S. Z. JHEP 03 (2023)

086 Real and Imaginary components of the reduced pseudo-ITD

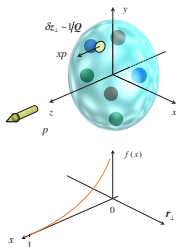
Helicity PDF



[Edwards, Egerer, Karpie, Karthik, Morris, Orginos, Radyushkin, Romero, Sufian, S. Z. Phys.Rev.D 105 \(2022\) 3, 034507](#) The leading-twist valence helicity quark PDF (purple) and x -space contaminations compared with the recent global analyses

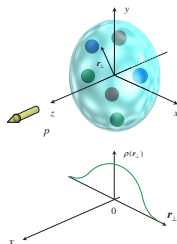
Measures of hadronic structure

Our knowledge of hadron structure is encapsulated in a variety of measures



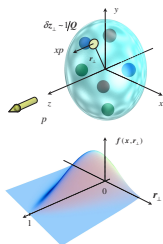
Unpolarized/polarized
parton distribution func-
tions (PDFs)

- 1D, longitudinal



Electric/magnetic form
factors (FFs)

- transverse distribution
of charge and currents

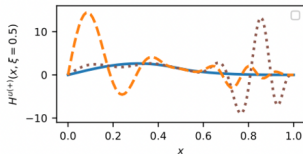
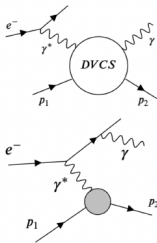


Generalized parton dis-
tributions (GPDs)

- correlating longitudinal
and transverse
structures

Deconvolution problem and GPDs

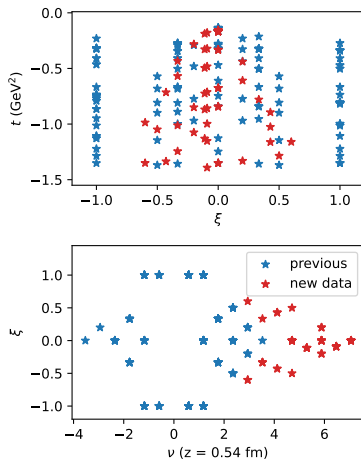
$$\text{CFF}(\xi, t, Q^2) = \int_{-1}^1 \frac{dx}{\xi} K\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) \text{GPD}(x, \xi, t, \mu^2)$$



These three GPDs give the same NLO contribution to DVCS up to 10^{-5} absolute precision in a range of $Q^2 \in [1, 100] \text{GeV}^2$.  Bertone,

Dutrieux, Mezrag, Moutarde, Snajder Phys.Rev.D 103 (2021) 11, 114019

Kinematic coverage



Kinematic coverage of our simulations. A milder inverse problem than deconvolution, extensively studied and addressed by our team with diverse techniques. [HadStruc Collaboration, Dutrieux et al, \(2024\)](#) [HadStruc Collaboration, Dutrieux et al, \(2026\)](#)

Moments of GPDs

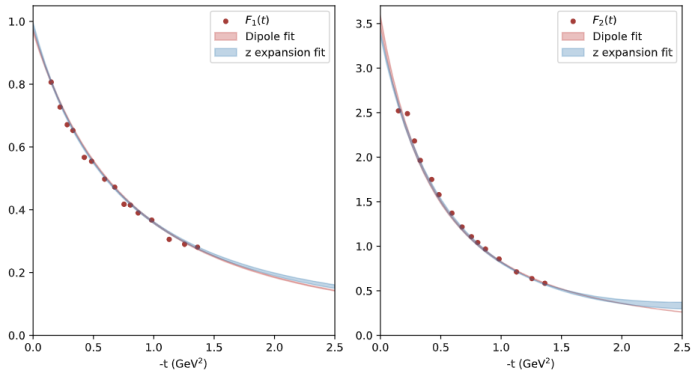
Value at $t = 0$

Dipole mass (GeV)

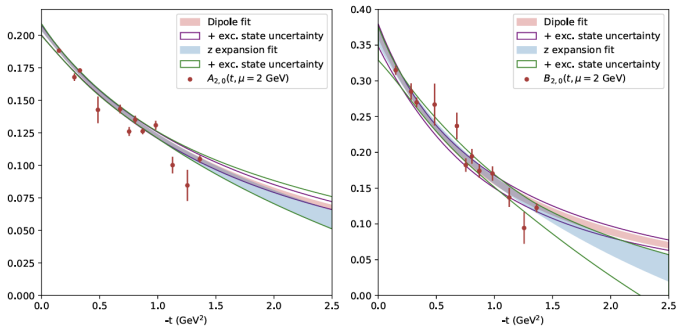
GPD H ^{u-d}			GPD E ^{u-d}			GPD H ^{u-d}			GPD E ^{u-d}		
$A_{1,0}$			$B_{1,0}$			$A_{1,0}$			$B_{1,0}$		
0.943 ± 15			3.17 ± 6			1.37 ± 5			1.0221 ± 8		
$A_{2,0}$			$B_{2,0}$			$A_{2,0}$			$B_{2,0}$		
0.215 ± 10			0.38 ± 2			1.70 ± 3			1.29 ± 10		
$A_{3,0}$	$A_{3,2}$		$B_{3,0}$	$B_{3,2}$		$A_{3,0}$	$A_{3,2}$		$B_{3,0}$	$B_{3,2}$	
0.070 ± 4	0.39 ± 18		0.12 ± 2	0.6 ± 2		2.2 ± 4	1.2 ± 5		1.6 ± 3	1.2 ± 4	
$A_{4,0}$	$A_{4,2}$		$B_{4,0}$	$B_{4,2}$		$A_{4,0}$	$A_{4,2}$		$B_{4,0}$	$B_{4,2}$	
0.025 ± 2	0.06 ± 3		0.035 ± 6	0.08 ± 4		2.1 ± 2	Inconclusive		1.3 ± 2	2.3 ± 10	
$A_{5,0}$	$A_{5,2}$	$A_{5,4}$	$B_{5,0}$	$B_{5,2}$	$B_{5,4}$	$A_{5,0}$	$A_{5,2}$	$A_{5,4}$	$B_{5,0}$	$B_{5,2}$	$B_{5,4}$
0.010 ± 4	0.04 ± 2	0.14 ± 5	0.014 ± 11	0.08 ± 6	0.2 ± 4	2.7 ± 8	1.9 ± 5	1.3 ± 2	Inconclusive	1.4 ± 6	1.3 ± 7
$A_{6,0}$	$A_{6,2}$	$A_{6,4}$	$B_{6,0}$	$B_{6,2}$	$B_{6,4}$	$A_{6,0}$	$A_{6,2}$	$A_{6,4}$	$B_{6,0}$	$B_{6,2}$	$B_{6,4}$
0.0057 ± 14	0.019 ± 8	0.04 ± 3	0.007 ± 3	0.02 ± 1	0.05 ± 3	2.2 ± 6	2.6 ± 9	2.6 ± 11	1.2 ± 4	2.0 ± 9	2.6 ± 14

Result of dipole fits on the GPR extraction of generalized form factors.

Elastic Form Factors

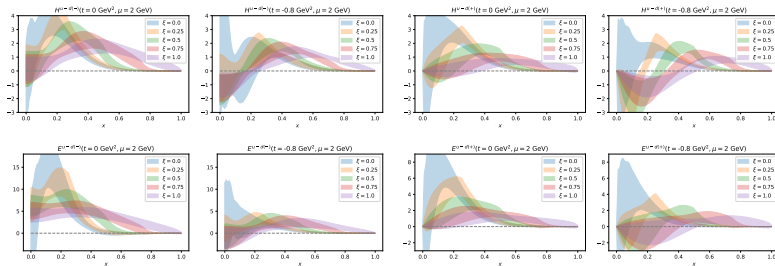


Gravitational Form Factors



results on the GFF

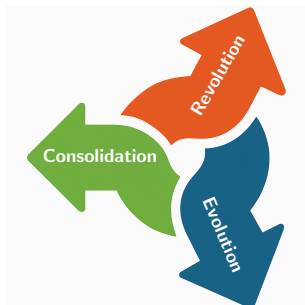
Full GPDs



We develop a new lattice-QCD framework that reconstructs GPDs over the full (x, ξ, t) dependence by extracting their DD representation directly from pseudo-distributions, using a GP-regularized 3D reconstruction that minimizes bias and guarantees positivity by construction and verifies the positivity bounds.  HadStruc Collaboration, Dutrieux et al, (2026)

Conclusions

Synergies with phenomenology,
to fully exploit current & upcoming experimental programs (CERN, EIC, ...)

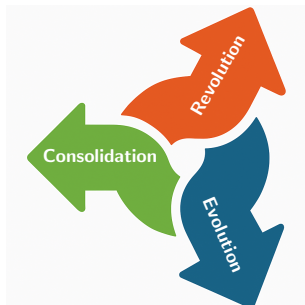


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High-fidelity GPDs,
for the first *ab-initio*
tomographic image
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Thank you!