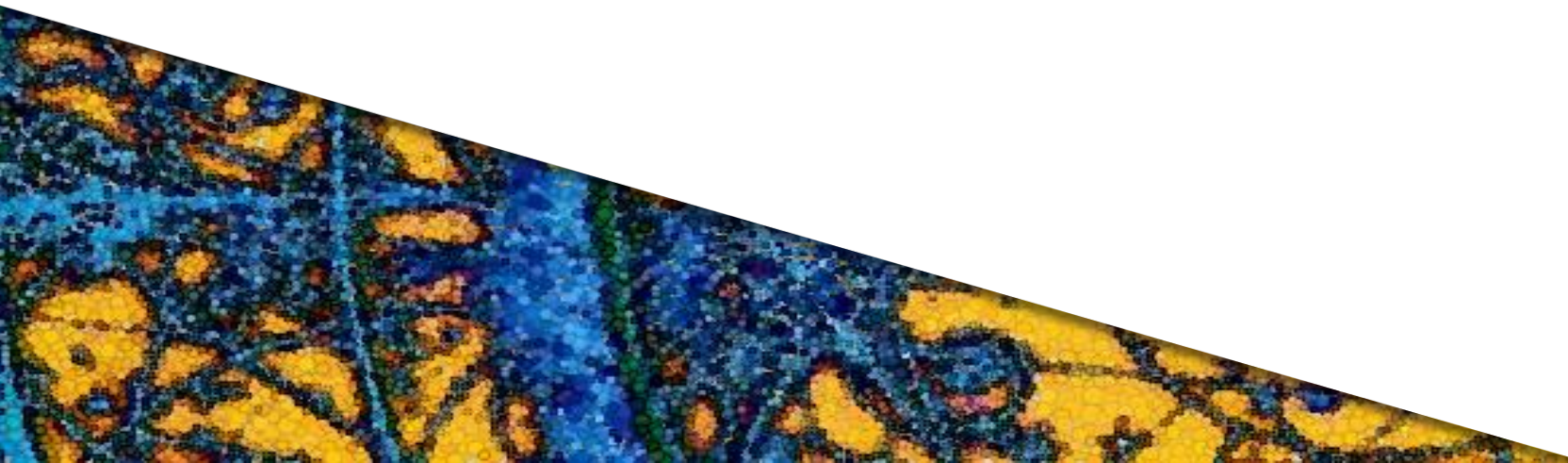


Kinematic fitting of data from electromagnetic calorimeters

Nicolas Kolanus for the CBELSA/TAPS collaboration



Kinematic fitting – an overview

Kinematic Fitting

Measured data

- Measured particles $E_1, \phi_1, \theta_1, \dots$
- Errors in covariance matrix G^{-1}

Impose physical **Conditions**

- e.g. Energy conservation
$$0 = f(E_1, \dots) = E_{\text{total}} - E_1 - \dots$$

Find values $E_{\text{fit}}, \phi_{\text{fit}}, \theta_{\text{fit}}$ which obey the conditions but lie closest to measured data

- Minimize $\chi^2 = \frac{(E_{1,\text{fit}} - E_1)^2}{\sigma_{E_1}^2} + \dots$

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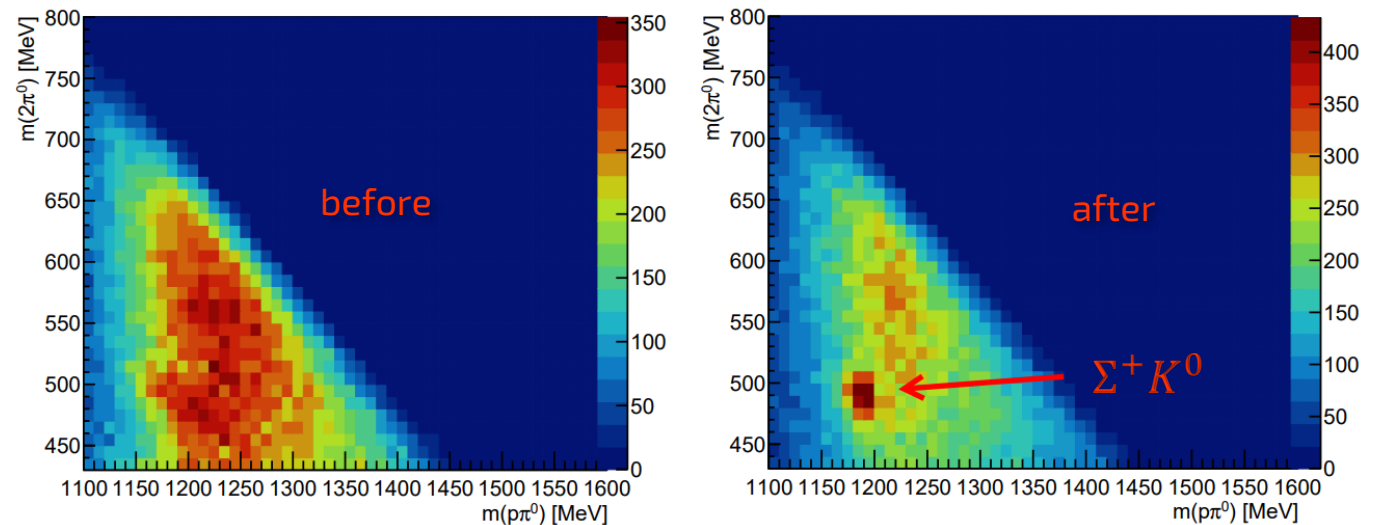
- Minimize $\chi^2 = \frac{(E_{1,\text{fit}} - E_1)^2}{\sigma_{E_1}^2} + \dots$

CBELSA/TAPS data for $\gamma p \rightarrow p 3\pi^0$ reaction
before/after kinematic fit

Fit Conditions

- Masses of pions
- 4-Momentum conservation

Overview CBELSA/TAPS experiment:
• Plenary talk on Tuesday by Prof. R. Beck



Signal from decay $K^0 \Sigma^+ \rightarrow (2\pi^0)(p\pi^0)$

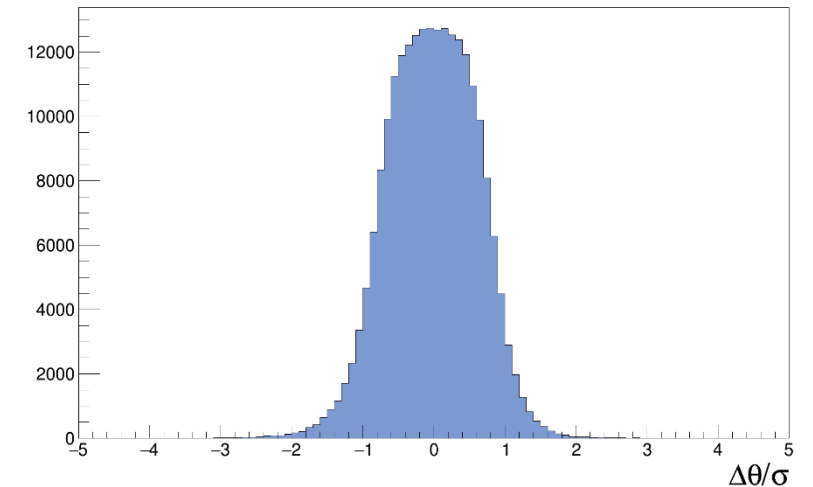
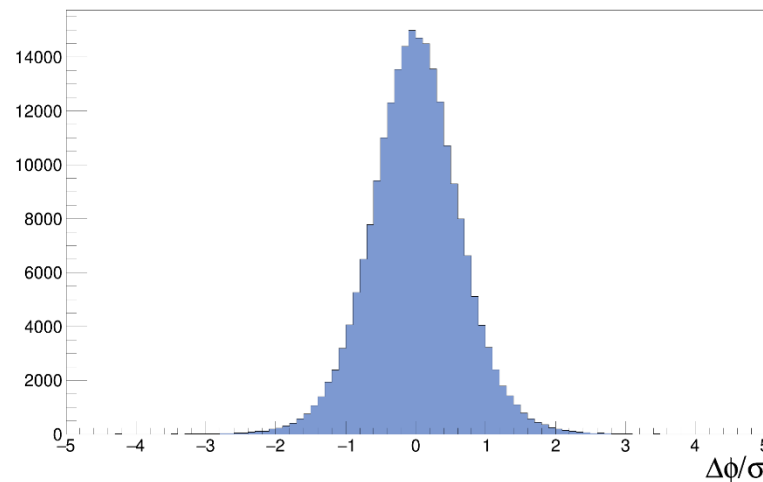
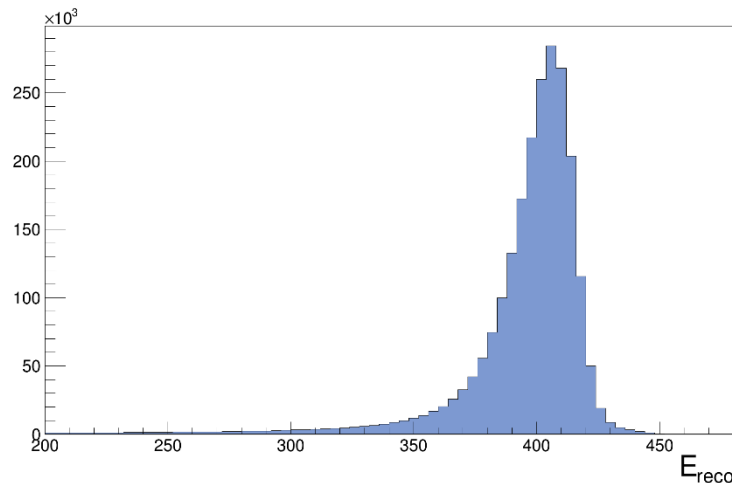
Problems of the basic fitting approach

Minimization of χ^2 assumes that measured quantities are:

- At least symmetrical
- Ideally Gaussian

This is not generally the case!

- ϕ distributions are almost Gaussian and symmetrical
- θ still quite symmetrical but non Gaussian (dominated by target length)
- Energy displays strong asymmetry – generally true for calorimetric measurements



Energy Distributions

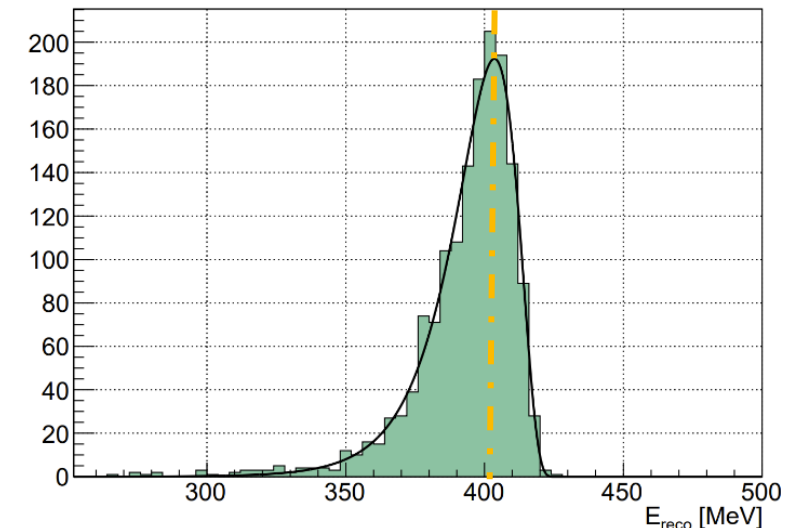
Energy distributions in calorimeters exhibit a long tail towards lower energies

- Losing energy is far more likely than gaining additional energy
- Can be described by so-called *Novosibirsk* function with an additional 'tail' parameter η
- Parametrization is generally applicable to calorimeters

$$f(x) = A \exp \left(-\frac{1}{2\eta^2} \log^2 \left(1 + \frac{(x - \mu) \sinh(\eta \sqrt{\log 4})}{\sigma \sqrt{\log 4}} \right) - \eta^2 \right)$$

A = Area μ = MPV (not mean) σ = Width η = Tail

MPV: Most Probable
Value



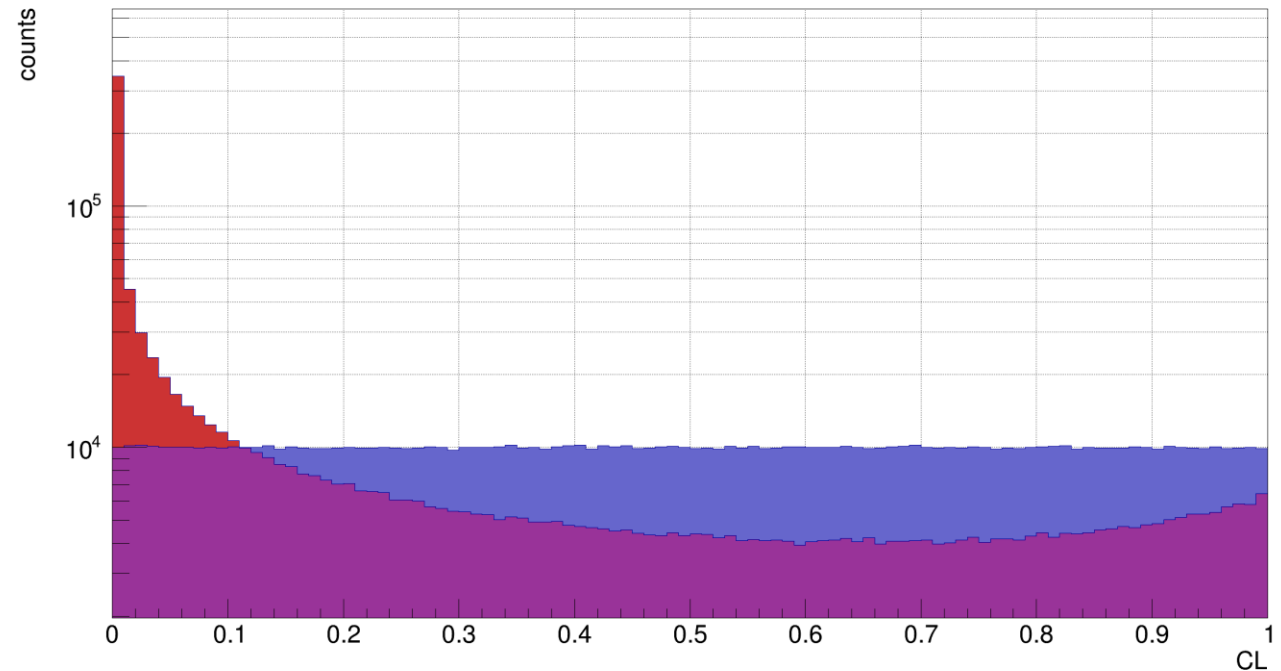
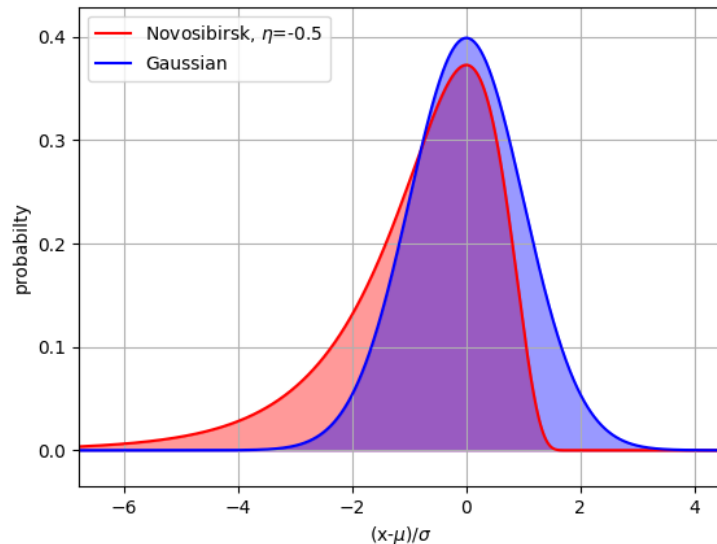
Statistical metrics

Statistical quantities:

$$\text{pull} = \frac{x - x_0}{(\sigma_0^2 - \sigma^2)^{1/2}}$$

Confidence Level (p-Value):

$$\text{CL} = \int_{\chi^2}^{\infty} f_{d.o.f.}(\tilde{\chi}^2) d\tilde{\chi}^2$$



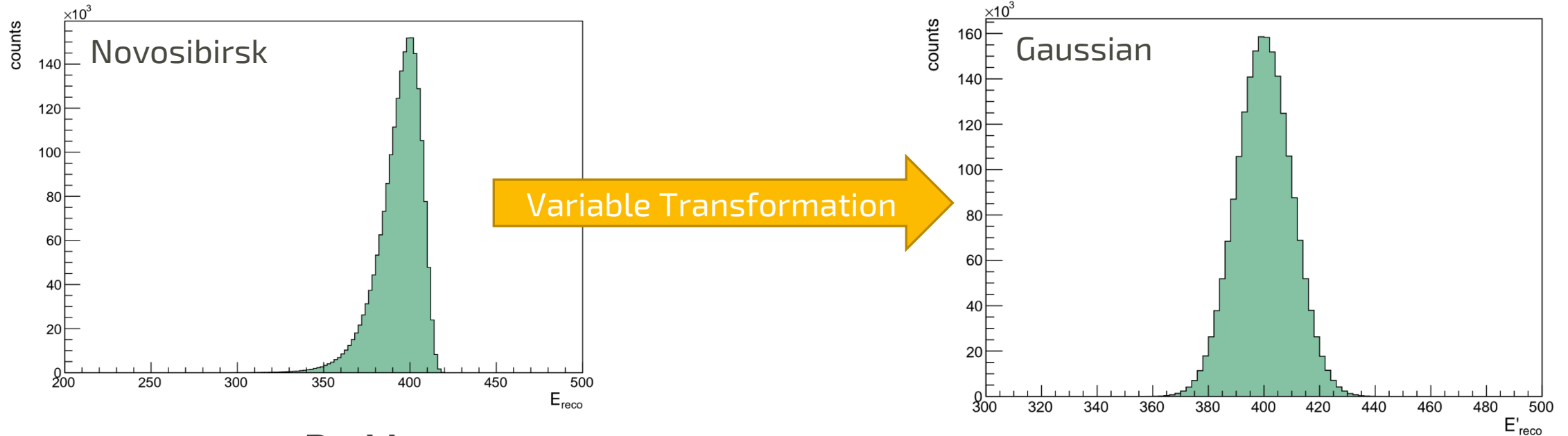
Toy model

- Sample points from different distributions with known width σ and compute CL
- Gaussian distributions (blue)
- Novosibirsk distributions (red)

When cutting on CL many 'good' signal events would be **lost** because of very low CL!

Variable transformation

Transformation to variable with Gaussian probability distribution (PDF)



Problems

- Every photon measurement is sampled from different PDF
- From which PDF was the measured value sampled?
- **Knowledge of true energy necessary!**

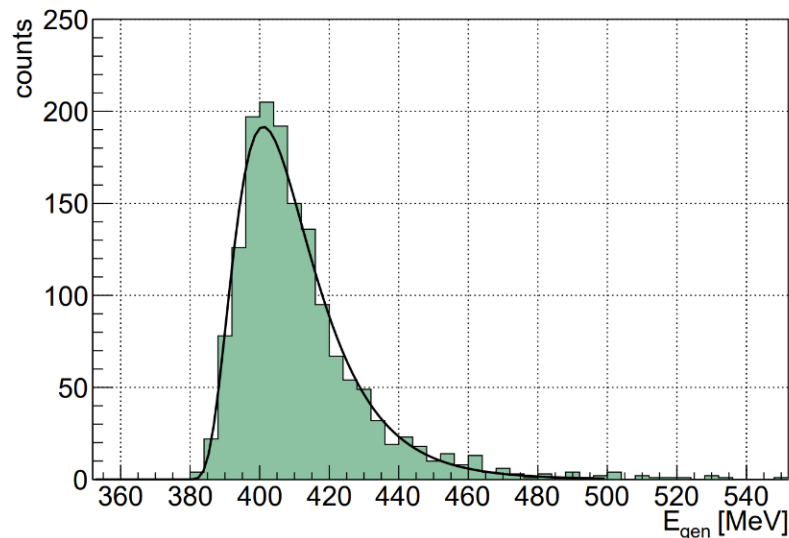
As the true energy is not known a **different approach** is needed

Likelihood distribution

Use likelihood distribution:

- Quantify of how likely a **true energy** value is, given a **measured value**
- More likely for the **true value** to be larger than **measured value** (asymmetry)
- Also described by Novosibirsk

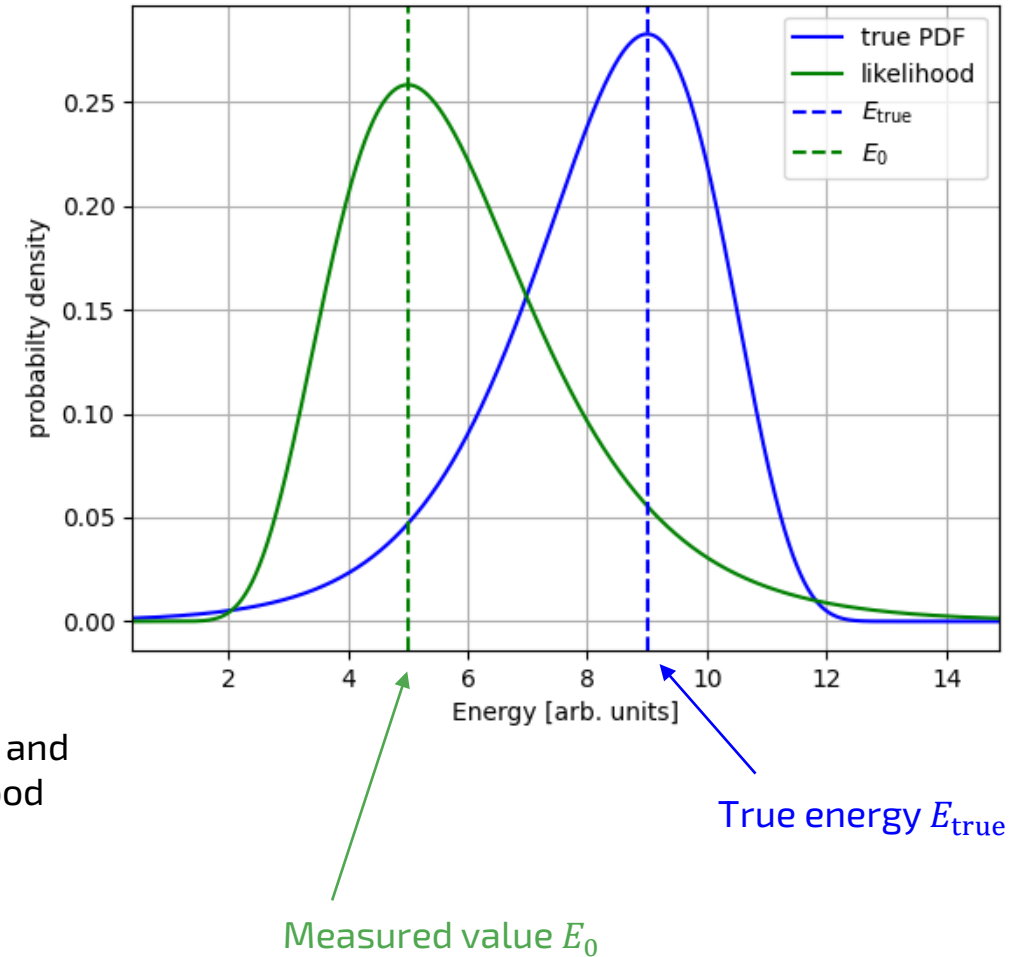
In general – magnitude of width σ and tail η are not the same between **PDF** and **likelihood**



To be done:

Simulate photons for all energies and directions and extract the likelihood spectra

Energies of photons reconstructed with an energy of $E_{\text{reco}} = 400$ MeV at $\theta_{\text{reco}} = 46^\circ$



Likelihood parameters of Crystal Barrel Calorimeter

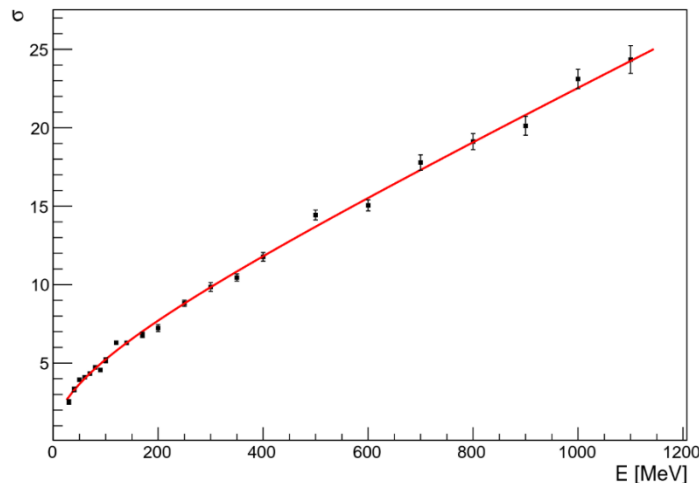
Likelihood distributions may be different for every measured energy and position in the detector

Method tested with Crystal Barrel calorimeter of the CBELSA/TAPS experiment

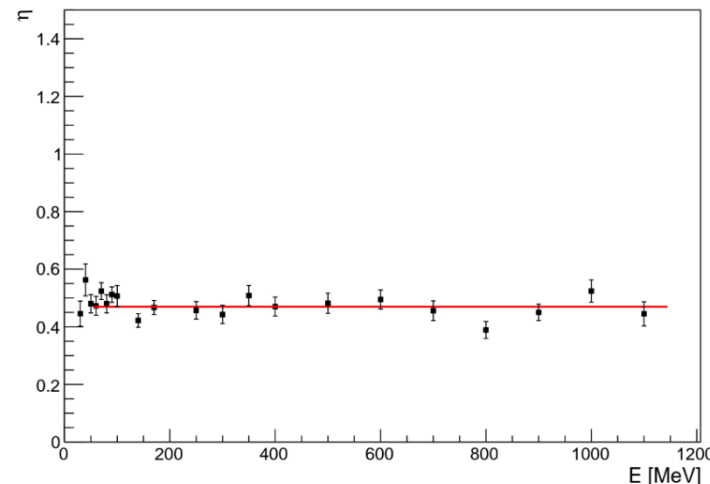
- ϕ -symmetric setup
- Only need to consider θ and Energy dependence

Energy dependence of σ is well described as

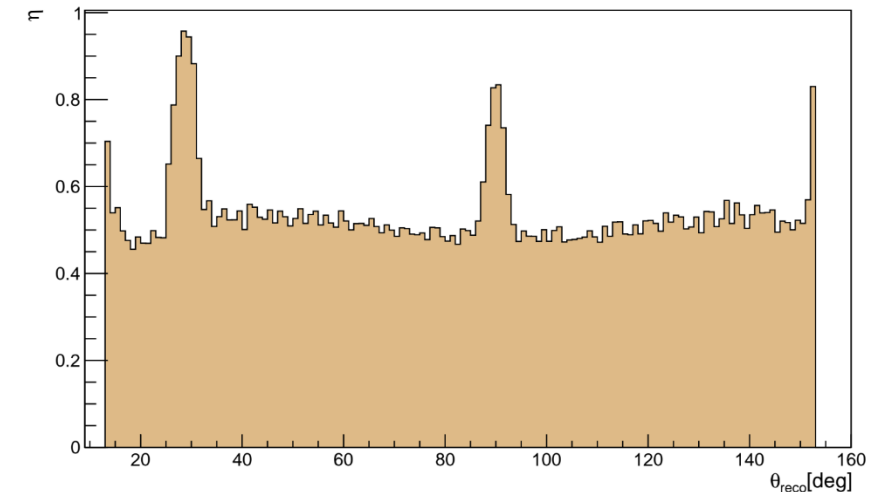
$$(\sigma)^2 = (A\sqrt{E})^2 + (BE)^2 + C^2$$



Energy dependence of tail parameter is $\sim$ constant



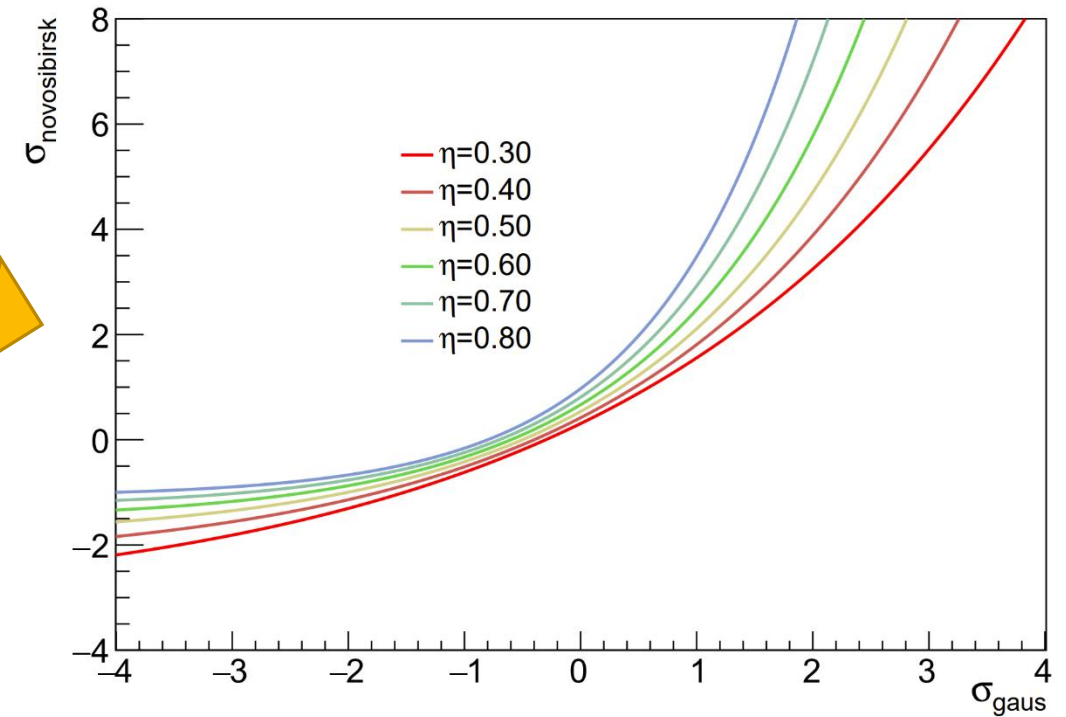
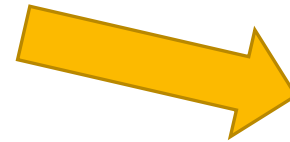
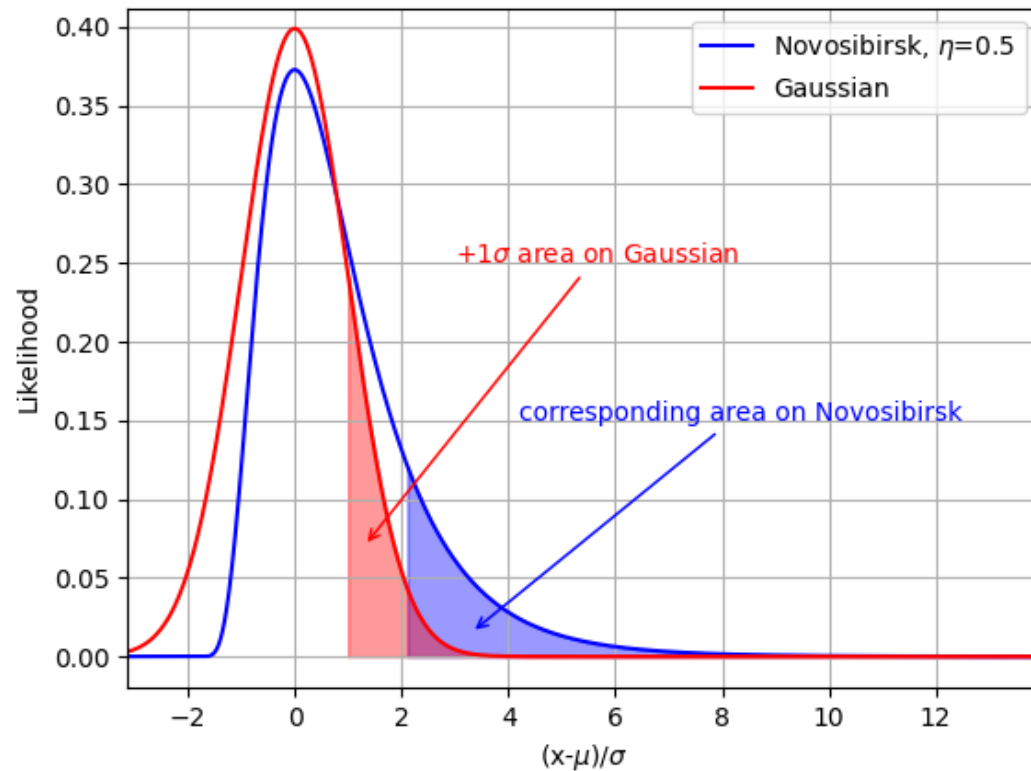
Tail parameter still varies with polar angle (insensitive material)



Transformation function

Function maps n -th quantile of Novosibirsk function on n -th quantile of Gaussian

Example:



Fit procedure (simulated data)

Kinematic Fit is especially crucial in final states with multiple neutral meson decays

- Resolve combinatorics
- Improve mass resolution of intermediate particles

Test performed for multiple channels

- $\gamma p \rightarrow p 3\pi^0$ ($\pi^0 \rightarrow 2\gamma$)
- $\gamma p \rightarrow p \eta$ ($\eta \rightarrow 3\pi^0$, $\pi^0 \rightarrow 2\gamma$)
- $\gamma p \rightarrow p \pi^0 \pi^0 \eta$ ($\pi^0 \rightarrow 2\gamma$, $\eta \rightarrow 2\gamma$)
- ...

Fits performed as 4C-Fit

Conditions/ Free parameters	Number of constraints
4-Momentum Conservation	4
Intermediate particle masses	3
Momentum of missing proton	-3
Σ	4

Proton is treated as missing particle to isolate effects of the photon measurement – mass is set to known PDG value

The initial state γp is taken from the simulation as exact values!

Results (Simulation)

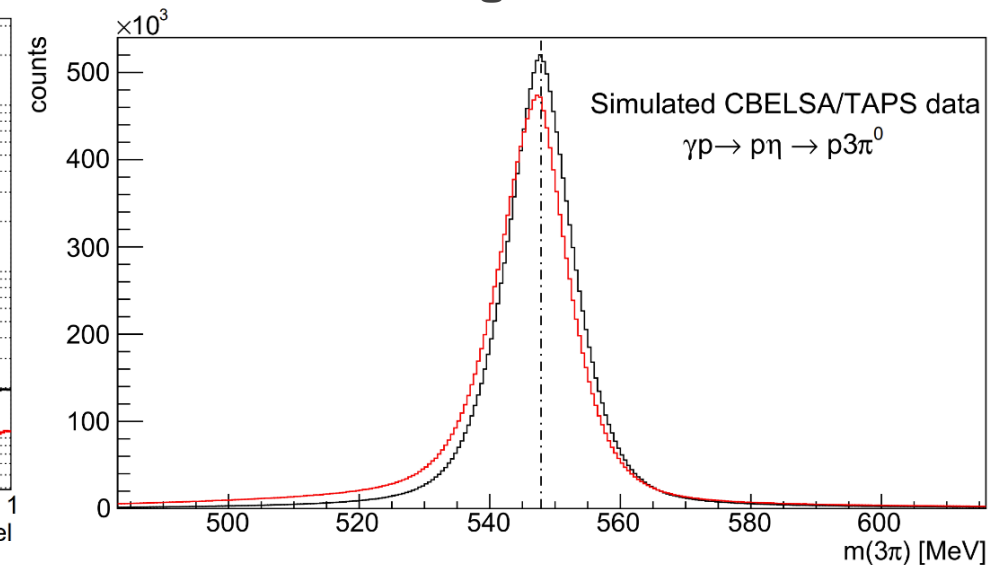
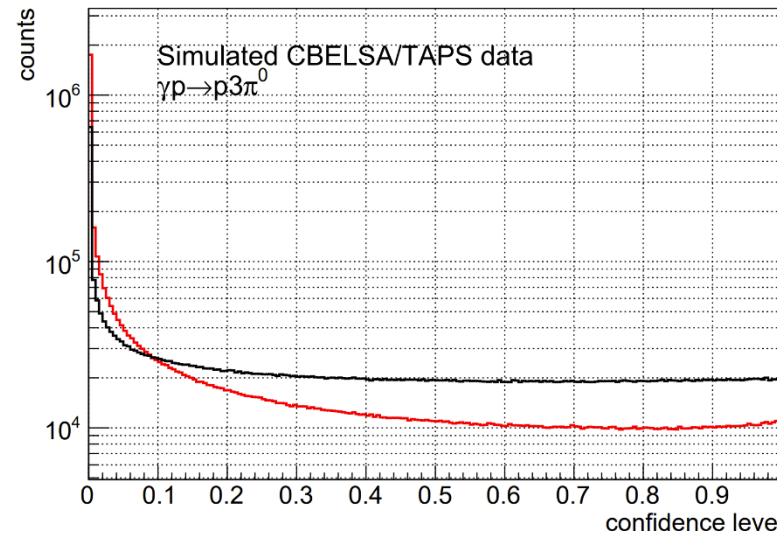
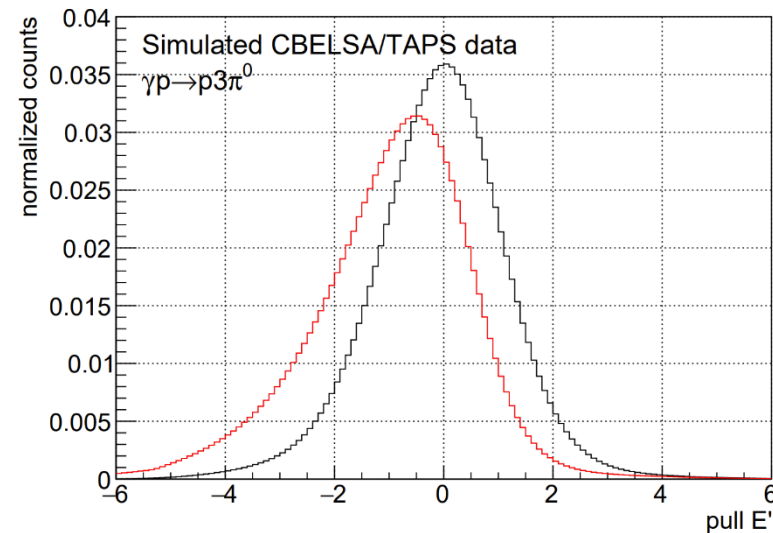
Results show clear improvement

without $\leftrightarrow$ with transformation

- Pulls now strongly resemble normal distributions
- Distribution centered around zero

- Excess at very low confidence level is greatly reduced
- Confidence is flatter at the high end

- Peak of intermediate η -meson corrected towards PDG value (dashed-dotted line)
- More symmetric peak shape without long tail



Results using the transformation method are shown in black
Results obtained from the previous approach in red

(Mass of the intermediate η -meson is **not** included in fit!)

Results (Simulation)

Cut on Confidence level can be used to reject background

Example: Measurement of $\gamma p \rightarrow p\pi^0\pi^0\eta$

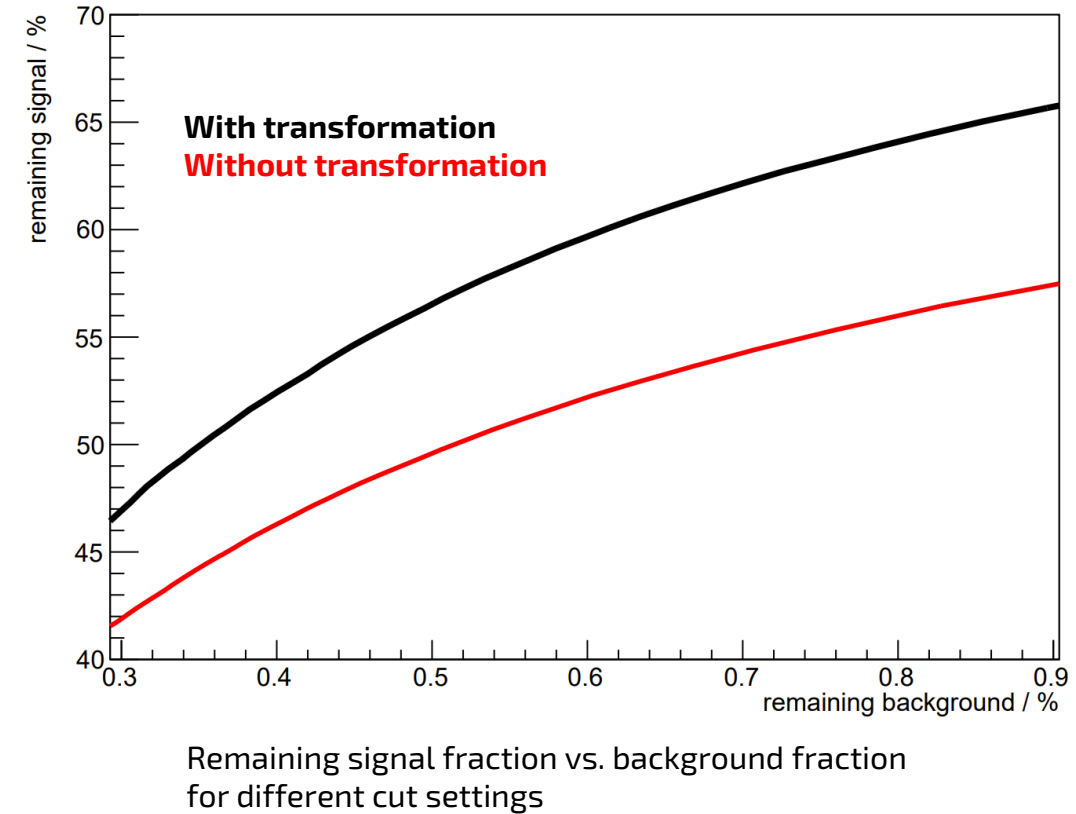
- Dominant background from $\gamma p \rightarrow p\pi^0\pi^0\pi^0$

Perform two fits with hypotheses:

- $\gamma p \rightarrow p\pi^0\pi^0\eta$ (hypothesis)
- $\gamma p \rightarrow p\pi^0\pi^0\pi^0$ (anti-hypothesis)

To reduce background:

- Reject the anti-hypothesis $CL_{3\pi^0} < p_{\text{anti}}$
- Not reject hypothesis $CL_{2\pi^0\eta} > p_0$



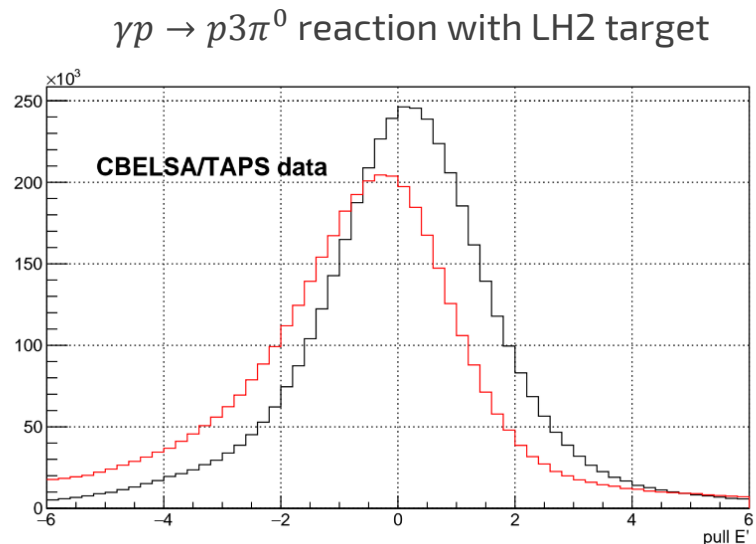
For a given cut less signal events are lost, as there are fewer signal events at very low confidence levels!

Summary

Transformation method based on likelihood distributions significantly improves fit performance:

- Statistical metrics improved (pulls and confidence level)
- Masses of intermediate particles reproduced more precisely
- Improved signal to background when cutting on CL

Method was also tested on real CBELSA/TAPS data and is already included in ongoing analyses!



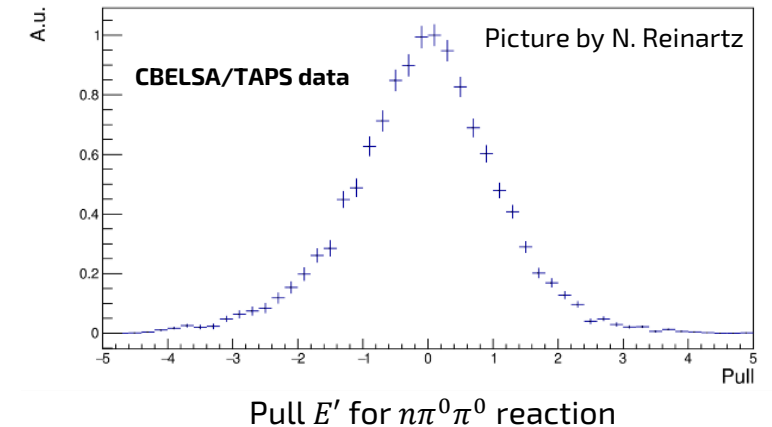
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Used in Analyses by N. Reinartz (Bochum) and L. Reschke (Gießen) for polarized photoproduction off neutron:

$$\begin{aligned}\vec{\gamma}\vec{n} &\rightarrow n\pi^0\pi^0 \\ \vec{\gamma}\vec{n} &\rightarrow n\pi^0\eta\end{aligned}$$

- Initial fermi momentum of neutron also considered in fit routine
- Neutron direction also fitted

Fit performance using trafo is **robust** even in more complex fits





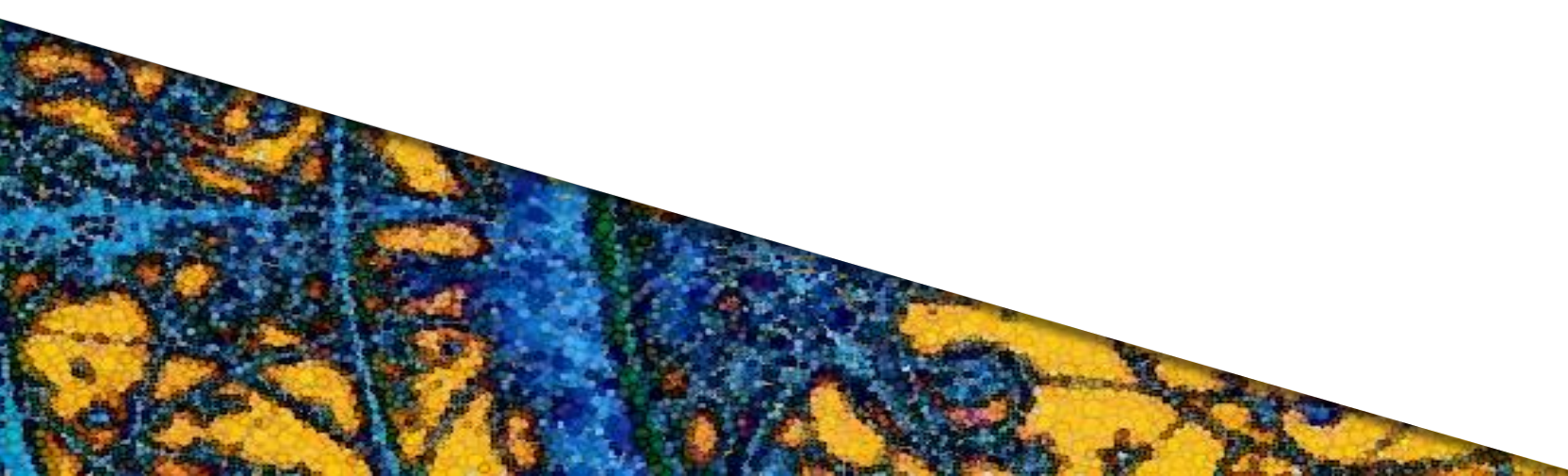
Thank you for your attention!

N. Kolanus, J. Kohlen, U. Thoma

“Kinematic Fitting of Electromagnetic Calorimeter Data - an Improved Method” - Submitted to arXiv

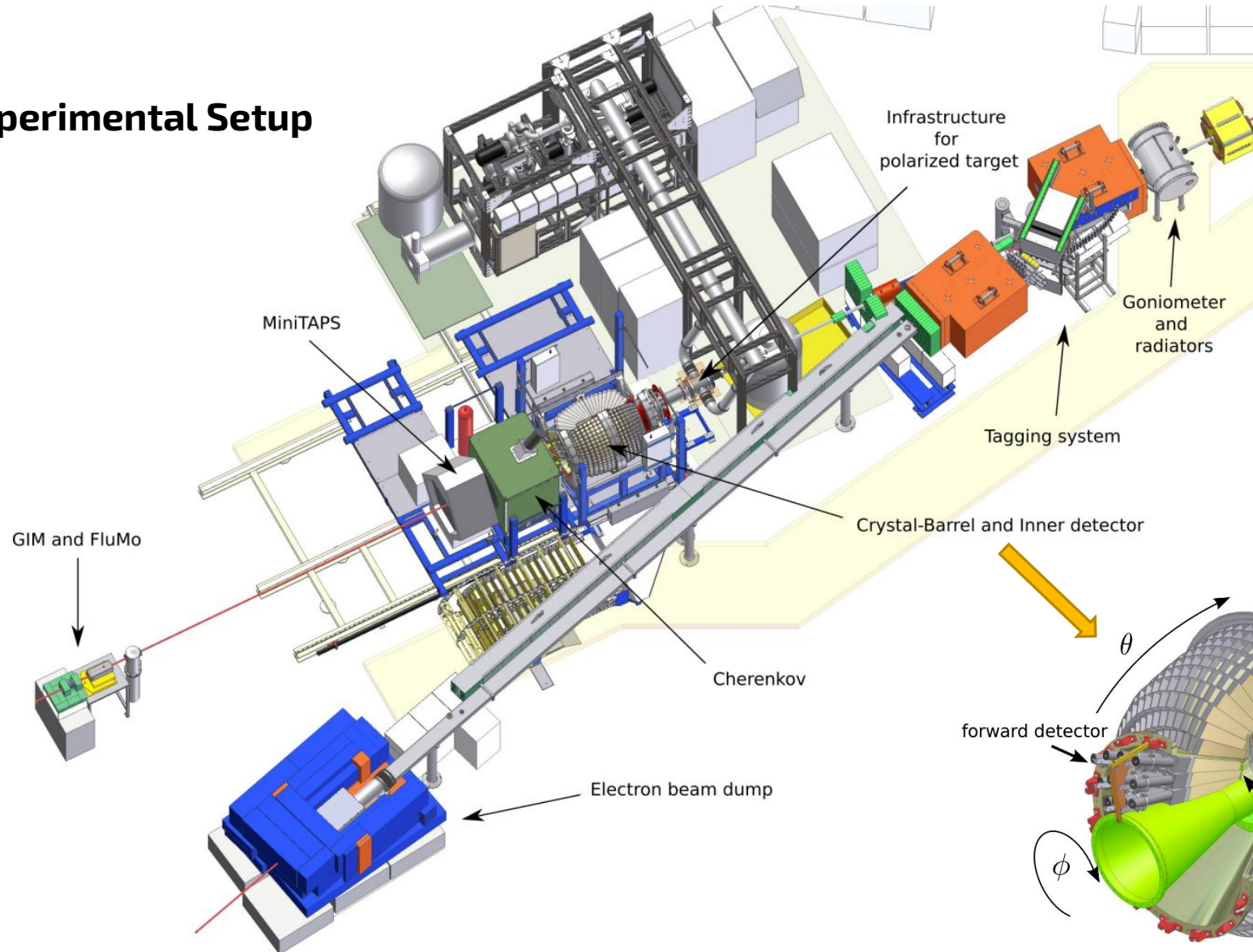
Appendix

Additional Slides



CBELSA/TAPS Experiment

Experimental Setup

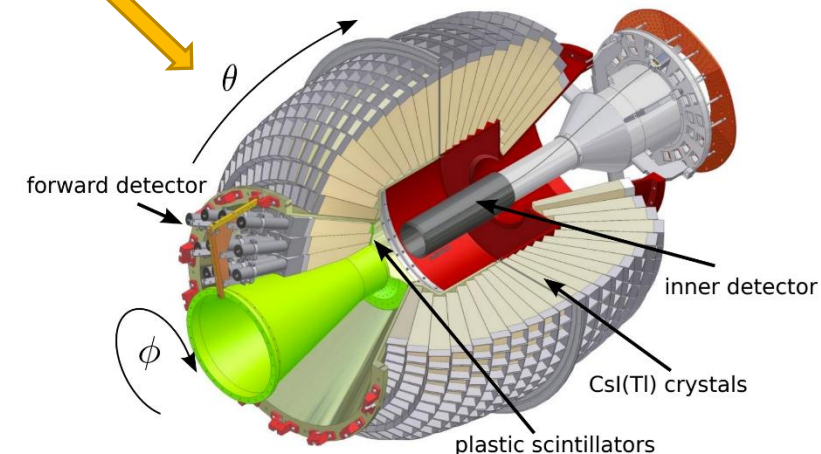


Scientific Program

Study baryon resonances in $\gamma N \rightarrow N^*/\Delta^* \rightarrow NX$ reactions

Experiment is specialized in detection of photons

➤ Focus on neutral meson final states e.g. $p\pi^0\pi^0$

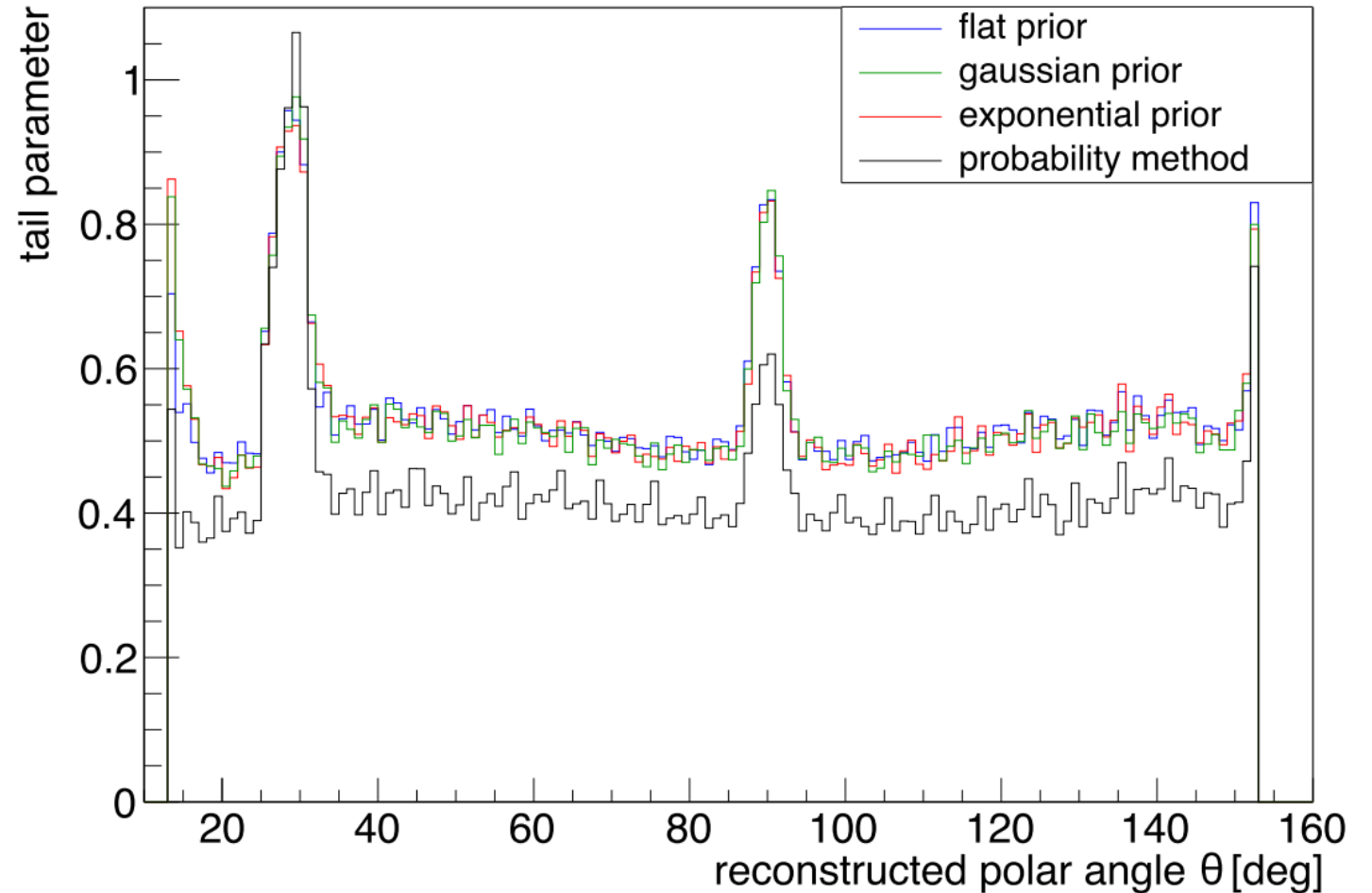
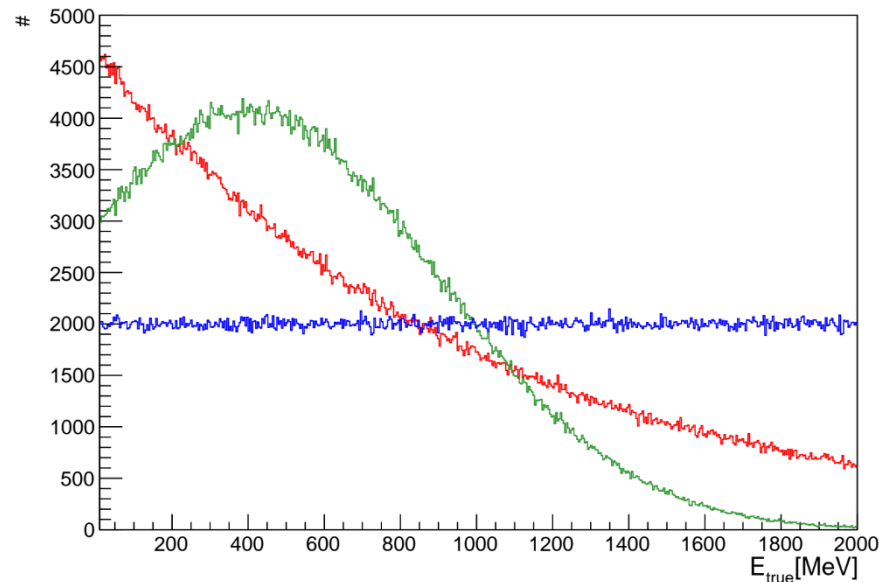


Influence of energy prior distribution

Likelihood distributions depend on overall distribution of true energies

Simulate different distributions to asses influence of priors:

- Flat (minimal information)
- Broad Gaussian
- Exponential



Differences of parameters using these different priors are not significant!