

Nuclear matter in a light-front Hamiltonian framework: QMC-BLFQ

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Colaborators

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- Ref. 2605.31030 [hep-ph] Wu, Nair, Kaur, Mondal, Lan, Zhao, de Melo, Frederico

Introduction & Motivation

- **Goal:** Understand strongly interacting matter and nuclear forces directly from quark and gluon degrees of freedom of QCD
- **Quark-Meson Coupling (QMC) Model:** Describes nucleons as composite quark systems interacting self-consistently with scalar (σ) and vector (ω^μ) mean fields in nuclear matter
- **Light-Front (LF) Hamiltonian Dynamics:** Provides a relativistic framework where hadronic structure is described via Light-Front Wave Functions (LFWFs), connecting directly to Parton Distribution Functions (PDFs)
- **Basis Light-Front Quantization (BLFQ):** A nonperturbative Hamiltonian approach to compute hadronic bound states
- **This Work:** Formulate the Equation of State (EOS) of symmetric nuclear matter (SNM) and in-medium parton distributions using an in-medium LF Hamiltonian including valence ($|qqq\rangle$) and one dynamical gluon ($|qqqg\rangle$) Fock sectors

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- **After Integrate in k^- : Bethe-Salpeter Amplitude (Wave Function)**

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- After Integrate in k^- : Bethe-Salpeter Amplitude (Wave Function)
- LF Lorentz Invariant Hamiltonian: $P^2 = P^+P^- - P_\perp^2$

• Light-Front remarks

Four-Vector $\implies x^\mu = (x^0, x^1, x^2, x^3) = (x^+, x^-, x_\perp)$

$$x^+ = t + z \quad x^+ = x^0 + x^3 \quad \implies \text{Time}$$

$$x^- = t - z \quad x^- = x^0 - x^3 \quad \implies \text{Position}$$

Scalar product

$$x \cdot y = x^\mu y_\mu = x^+ y_- + x^- y_+ + x^1 y_1 + x^2 y_2 = \frac{x^+ y^- + x^- y^+}{2} - \vec{x}_\perp \vec{y}_\perp$$

Metric Tensor:

$$g^{\mu\nu} = \begin{pmatrix} 2 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \text{and} \quad g_{\mu\nu} = \begin{pmatrix} 0 & 1/2 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Dirac Matrix and Electromagnetic Current

$$\begin{aligned}\gamma^+ &= \gamma^0 + \gamma^3 \implies \text{Electr. Current } J^+ = J^0 + J^3 \\ \gamma^- &= \gamma^0 - \gamma^3 \implies \text{Electr. Current } J^- = J^0 - J^3 \\ \gamma^\perp &= (\gamma^1, \gamma^2) \implies \text{Electr. Current } J^\perp = (J^1, J^2)\end{aligned}$$

$$p^\mu x_\mu = \frac{p^+ x^- + p^- x^+}{2} - \vec{p}_\perp \vec{x}_\perp$$

$$x^+, x^-, \vec{x}_\perp \implies p^+, p^-, \vec{p}_\perp$$

LF Hamiltonian & BLFQ Framework

- Fock Space Expansion**

$$|P, \Lambda\rangle = \psi_{|qqq\rangle}|qqq\rangle + \psi_{|qqqg\rangle}|qqqg\rangle + \dots$$

- Effective LF Hamiltonian (P_{eff}^-):** Separated into kinetic, interaction, and confining components in the LF gauge $A^+ = 0$

$$P_{\text{eff}}^- = P_{\text{kin}}^- + P_{\text{int}}^- + P_{\text{conf}}^-$$

- Kinetic terms:** Quarks and gluons

$$P_{\text{kin}}^- = \int dx^- d^2x_{\perp} \left[\bar{\varphi} \gamma^+ \frac{m_q^2 + (i\partial_{\perp})^2}{2i\partial^+} \varphi + A_a^{\mu} \frac{m_g^2 + (i\partial_{\perp})^2}{2} A_{\mu}^a \right]$$

- φ, A_a^{μ} : quark and gluon fields, respectively; m_q is the effective quark mass, m_g phenomenological gluon mass

- Interaction:**

$$P_{\text{int}}^- = g_s \int dx^- d^2x_{\perp} \bar{\varphi} \gamma_{\mu} T^a A_a^{\mu} \varphi + \frac{g_s^2}{2} \int dx^- d^2x_{\perp} \bar{\varphi} \gamma^+ T^a \varphi \frac{1}{(i\partial^+)^2} \bar{\varphi} \gamma^+ T^a \varphi$$

- g_s strong coupling constant, $T^a = \lambda^a/2$ **SU(3)** generators of color
- Confinement:** Implemented via a two-body transverse harmonic oscillator potential restricted to the valence sector:

$$P^+ P_{\text{conf}}^- = \frac{\kappa^4}{2} \sum_{i \neq j} \left[r_{ij\perp}^2 - \frac{\partial_{x_i} (x_i x_j \partial_{x_j})}{(m_i + m_j)^2} \right]$$

- Transverse separation between partons:** $r_{ij\perp} = \sqrt{x_i x_j} (r_{i\perp} - r_{j\perp})$
- $\partial_{r_{ij\perp}}$ denotes differentiation with respect to the relative transverse coordinate

- **Proton wave function:**

$$\Psi_{\mathcal{N},\{\lambda_i\}}^{\Lambda}(\{x_i, p_{i\perp}\}) = \sum_{\{n_i, m_i\}} \psi_{\mathcal{N}}^{\Lambda}(\{\alpha_i\}) \prod_i^{\mathcal{N}} \phi_{n_i, m_i}(p_{i\perp}; b),$$

- $\mathcal{N}$ -th / Particle Fock-component
- ϕ_{n_i, m_i} / Transverse HO basis functions
- $\psi_{\mathcal{N}}^{\Lambda}(\{\alpha_i\})$ / Longitudinal and spin-dependent components of the eigenvectors

BLFQ Parameters:

- **Truncations** $\{N_{\max}, K\} = (9, 16.5)$
- **HO basis scale** $b = 0.7 \text{ GeV}$
- **Instantaneous interaction is regulated with a ultraviolet cutoff** $b_{\text{inst}} = 3 \text{ GeV}$
- **Constituent quark masses, effective gluon mass, confinement strength, couplings:**
 $\{m_u, m_d, m_g, \kappa, m_f, g_s\} = \{0.31, 0.25, 0.50, 0.54, 1.80, 2.40\}$ in units of **GeV** (except for g_s)

Vacuum: Proton is $\approx 42\%$ valence and 58% quark-gluon ($|qqqg\rangle$)

Ref.

- Vary, et al, *Phys. Rev. C*81, 035205 (2010)
- X. Zhao, H. Honkanen, P. Maris, J. P. Vary, S. J. Brodsky
Phys. Lett. B 737 (2014) 65
- Lan, Mondal, Jia, Zhao, Vary, *Phys. Rev Lett.*122 (2019)172001
- S. Xu, C. Mondal, X. Zhao, Y. Li, J. P. Vary,
Phys. Rev. D 108 (9) (2023) 094002

► **See talk by Chandan Mondal - Nucleon Structure from Basis Light-Front Quantization: Recent Developments and Perspectives**

In-Medium Effective LF Hamiltonian

- **Medium Modification:** Quarks couple to classical scalar (σ) and vector (ω_0) mean fields generated by Symmetric Nuclear Matter (SNM)
- **Effective Quark Mass:** Shifted by the scalar field

$$m_q^* = m_q - g_\sigma^q \sigma$$

- **In-Medium LF Eigenvalue Problem**

$$P_{\text{eff}}^{-*} = P_{\text{eff}}^- + \int dx^- d^2x_\perp \bar{\varphi} \gamma^+ \frac{(m_q - g_\sigma^q \sigma)^2 - m_q^2}{2i\partial^+} \varphi + 3g_\omega^q \omega_0$$

$$(P_{\text{eff}}^{-*} - g_\omega \omega_0) |\Psi_N^*\rangle = \frac{[M_N^*(\sigma)]^2 + P_\perp^2}{P_+} |\Psi_N^*\rangle$$

- **Yields a self-consistent, density-dependent nucleon mass $M_N^*(\sigma)$ and medium-modified LFWFs**

Nucleon-sigma coupling: $g_\sigma^N = -\frac{\partial M^*}{\partial \sigma}$

Ref. 2605.31030 [hep-ph]: Xiaoyi Wu et al.

QHD Mean-Field Equations & SNM Properties

- **Energy Density ($\mathcal{E}$) and Pressure ($\mathcal{P}$)**

$$\mathcal{E} = \frac{\gamma}{(2\pi)^3} \int_{k < k_F} d^3k \sqrt{k^2 + M_N^{*2}} + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2$$

$$\mathcal{P} = \frac{\gamma}{3(2\pi)^3} \int_{k < k_F} d^3k \frac{k^2}{\sqrt{k^2 + M_N^{*2}}} - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2$$

- k_F Fermi momentum and $\gamma = 4$ isopin-isospin degeneracy factor for SNM (symmetric nuclear matter)

- Self-consistent equation for the scalar field // minimizing the energy density with respect to σ

$$\sigma = -\frac{\partial M_N^*}{\partial \sigma} \frac{1}{m_\sigma^2} \frac{\gamma}{(2\pi)^3} \int_{|k| < k_F} d^3k \frac{M_N^*}{\sqrt{k^2 + M_N^{*2}}}$$

- The above self-consistent equation for the scalar field and the one for the vector field can be written compactly as:

$$\sigma = \frac{g_\sigma}{m_\sigma^2} \rho_s \quad \text{and} \quad \omega_0 = \frac{g_\omega}{m_\omega^2} \rho$$

where

$$\rho_s = \frac{\gamma}{(2\pi)^3} \int_{|k| < k_F} d^3k \frac{M_N^*}{\sqrt{k^2 + M_N^{*2}}} \quad \text{and} \quad \rho = \frac{\gamma}{6\pi^2} k_F^3$$

- **The incompressibility:** $K = 9 \frac{\partial P}{\partial \rho}$

- **Parameters Fitted to Saturation:**

$\rho_0 = 0.15 \text{ fm}^{-3}$, $B_0 = -15.8 \text{ MeV}$, yielding
 $g_\sigma^q = 2.6445$ and $g_\omega^q/m_\omega = 0.0039 \text{ MeV}^{-1}$

- **Incompressibility:** $K_0 = 313.7 \text{ MeV}$

(consistent with empirical constraints $220 \pm 40 \text{ MeV}$ compared to linear Walecka model's 548 MeV)

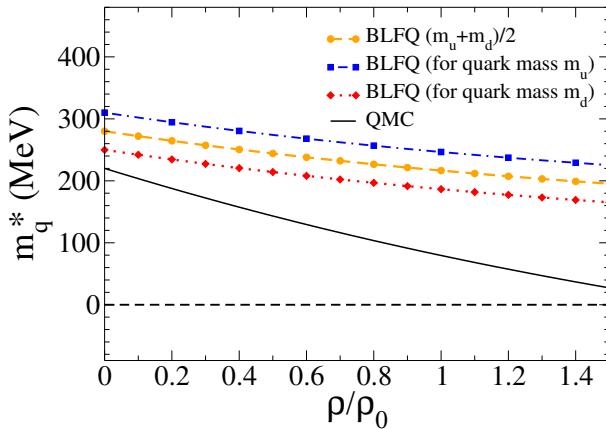
Refs.

- P. G. Reinhard, Rept. Prog. Phys. 52 (1989) 439
- Y. Wang, C. Guo, Q. Li, A. Le Fèvre, Y. Leifels, W. Trautmann Phys. Lett. B 778 (2018) 207

Ref. 2605.31030 [hep-ph], Xiaoyi Wu et al.

Equation of State (EOS) Results

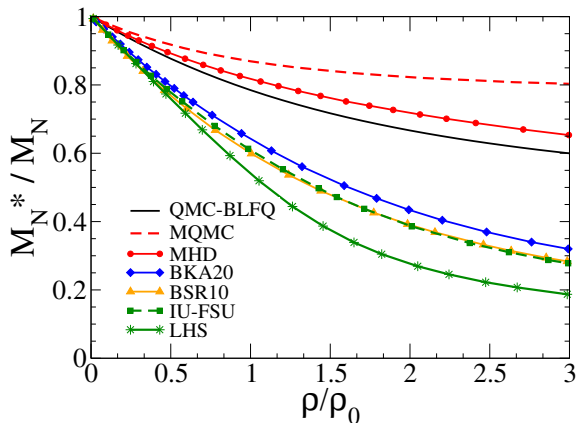
- **Energy per Nucleon and Pressure:** The QMC-BLFQ model successfully reproduces empirical saturation curves and heavy-ion collision constraints.
- **Comparison:** The internal quark substructure results in a softer density dependence of M_N^*/M_N and a realistic EOS compared to point-like nucleon models.
- **Exp. References:**
 - S. Huth et al., Nature 606, 276 (2022).
 - A. Le Fèvre et al., Nucl. Phys. A 945, 112 (2016).
 - P. Russotto et al., Phys. Rev. C 94, 034608 (2016).
 - W. G. Lynch et al., Prog. Part. Nucl. Phys. 62, 427 (2009).
 - P. Danielewicz et al., Science 298, 1592 (2002).



Quark masses as a function of ρ/ρ_0 , and nucleon mass in symmetric nuclear matter (SNM) as a function of the relative density

• G. H. S. O. Yabusaki, J. P. B. C. de Melo, K. Tsushima, T. Frederico, W. de Paula Phys. Rev. D 109 (5) (2024) 054024.

Ref. 2605.31030 [hep-ph], Xiaoyi Wu et al.



[MQMC] E. F. Batista, B. V. Carlson, T. Frederico, Nucl. Phys. A 697 (2002) 469.

[LHS] P. G. Reinhard, Rept. Prog. Phys. 52 (1989) 439.

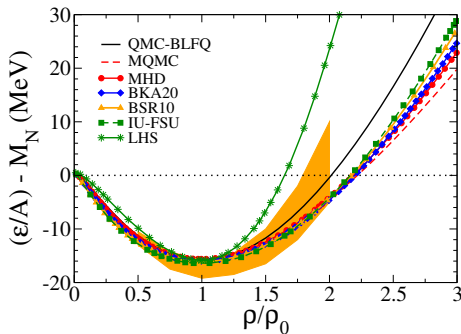
[MHD] W. de Paula, C.-R. Ji, J. P. B. C. de Melo, T. Frederico, O. Lourenço, Phys. Lett. B 803 (2020) 135339.

[BKA20] B. K. Agrawal, Phys. Rev. C 81 (2010) 034323.

[BSR10] S. K. Dhiman, R. Kumar, B. K. Agrawal, Phys. Rev. C 76 (2007) 045801.

[IU-FSU] F. J. Fattoyev, C. J. Horowitz, J. Piekarewicz, G. Shen, Phys. Rev. C 82 (2010) 055803.

Present work: 2605.31030 [hep-ph], Xiaoyi Wu et al.



Energy per nucleon and pressure vs. ρ/ρ_0 for the present QMC-BLFQ model and other models/parametrizations

Ref.: [MQMC] E. F. Batista, B. V. Carlson, T. Frederico, Nucl. Phys. A 697 (2002) 469

[MHD] de Paula, Ji, de Melo, Frederico, Lourenço, Phys. Lett. B 803 (2020) 135339.

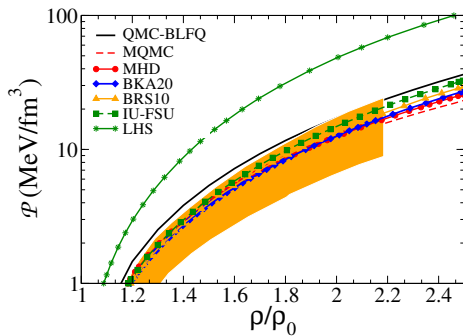
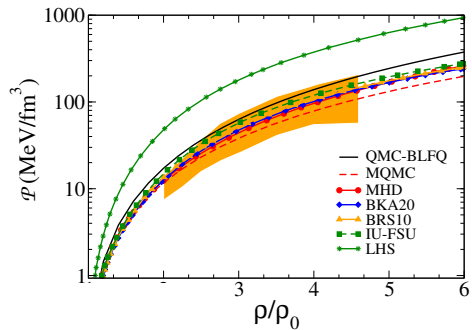
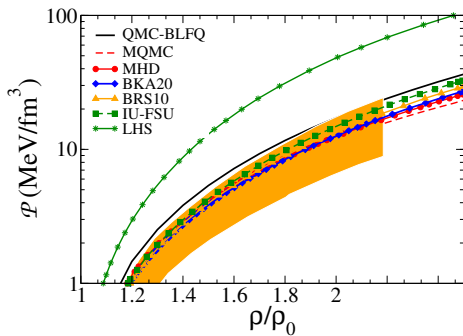
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Parton distributions in the nuclear medium

- QMC-BLFQ: Quark level Density dependence
- Medium nucleon wave function

$$\Psi_{\mathcal{N},\{\lambda_i\}}^{\Lambda*}(\{x_i, p_{i\perp}\}) = \sum_{\{n_i, m_i\}} \psi_{\mathcal{N}}^{\Lambda*}(\{\alpha_i\}) \prod_i^{\mathcal{N}} \phi_{n_i, m_i}(p_{i\perp}; b)$$

- Medium quark and gluon PDFs

$$f_{\rho/\rho_0}^{q,g}(x) = \sum_{\mathcal{N},\{\lambda_i\}} \int [d\Gamma_{\mathcal{N}}] \left| \Psi_{\mathcal{N},\{\lambda_i\}}^{\Lambda*}(\{x_i, p_{i\perp}\}) \right|^2 \delta(x - x_j)$$

- Integration measure

$$[d\Gamma_{\mathcal{N}}] = \prod_{i=1}^{\mathcal{N}} dx_i d^2p_{i\perp} \delta\left(1 - \sum_{i=1}^{\mathcal{N}} x_i\right) \delta^{(2)}\left(\sum_{i=1}^{\mathcal{N}} p_{i\perp}\right)$$

- i denoting the active parton (q : quark and g : gluon)

Nuclear Medium PDF Moments

ρ/ρ_0	u quark			d quark			gluon
	$\langle x^n \rangle_u^{\text{tot}}$	$\langle x^n \rangle_u^{3q}$	$\langle x^n \rangle_u^{3qg}$	$\langle x^n \rangle_d^{\text{tot}}$	$\langle x^n \rangle_d^{3q}$	$\langle x^n \rangle_d^{3qg}$	$\langle x^n \rangle_g$
$n = 0$							
0	2	0.837387	1.16261	1	0.418693	0.581307	0.581307
0.662	2	0.775632	1.22437	1	0.387816	0.612184	0.612184
1.583	2	0.710601	1.28940	1	0.355301	0.644699	0.644699
$n = 1$							
0	0.595429	0.288621	0.306808	0.255481	0.130073	0.125409	0.149090
0.662	0.585543	0.266902	0.318641	0.250662	0.120914	0.129748	0.163795
1.583	0.571845	0.243345	0.328501	0.245929	0.111956	0.133973	0.182225

Moments $\langle x^n \rangle_{q,g}$ with $n = 0, 1$ in the nuclear medium as a function of ρ/ρ_0 for the proton at the nucleon scale $\mu_0^2 = 0.291 \text{ GeV}^2$.

Present work: 2605.31030 [hep-ph], Xiaoyi Wu et al.

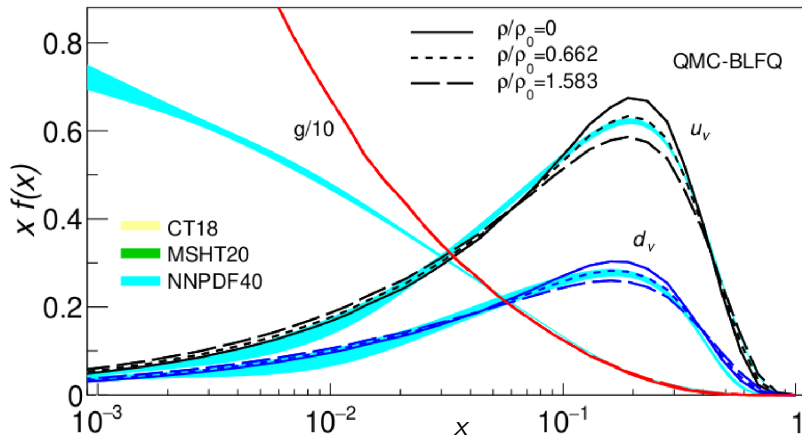
Table considerations

- **Objective:** Study the parton moments $\langle x^n \rangle_{q,g}$ ($n = 0, 1$) for quarks (u, d) and gluons in a proton immersed in a nuclear medium (ρ/ρ_0).
- **Initial Scale:** Calculated at $\mu_0^2 = 0.291 \text{ GeV}^2$, ranging from the free nucleon ($\rho = 0$) to high densities ($\rho = 1.583\rho_0$).
- **Structural Sectors:**
- **Pure Sector ($3q$):** States strictly composed of three valence quarks.
- **Gluon Sector ($3qg$):** Extended Fock states incorporating explicit gluons ($|qqqg\rangle$).

Results: Moments $n = 0$ and $n = 1$

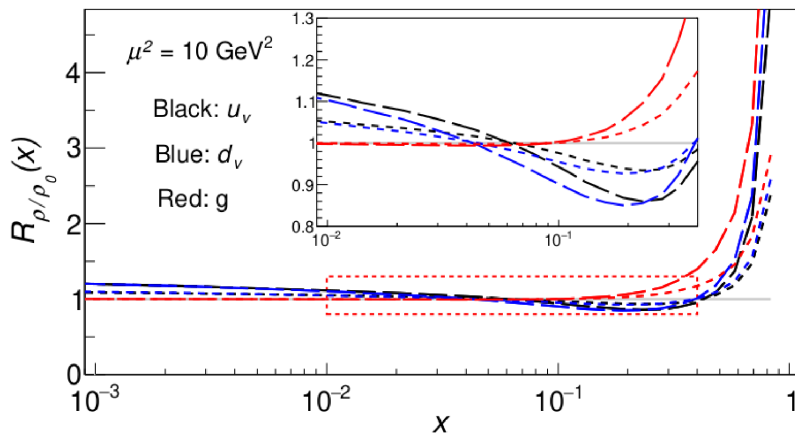
- **Zeroth-Order Moment ($n = 0$):**
 - Associated with valence quark counting: fixed at $\langle x^0 \rangle_u^{\text{tot}} = 2$ and $\langle x^0 \rangle_d^{\text{tot}} = 1$.
 - The nuclear medium induces redistribution: the $3q$ sector decreases while the $3qg$ sector increases with density ρ/ρ_0 .
- **First-Order Moment ($n = 1$):**
 - Represents the fraction of the proton's linear momentum.
 - Sum rule verified for the free nucleon (≈ 1.0).
 - **Medium Effect:** Quarks slightly lose momentum, whereas gluons gain expression (rising from 0.1491 to 0.1822), indicating gluonic polarization in the nuclear medium.

- Quark momentum fraction in the valence component $|qqq\rangle$ decreases with the valence probability
- Quark momentum fraction in the $|qqqg\rangle$ component increases as its probability grows
- The gluon momentum fraction rises up to 20% for $\rho/\rho_0 = 1.583$
 $\langle x^0 \rangle_g$ increases by only 10%, show a different behavior compared to the quark momentum fractions in both Fock components presented



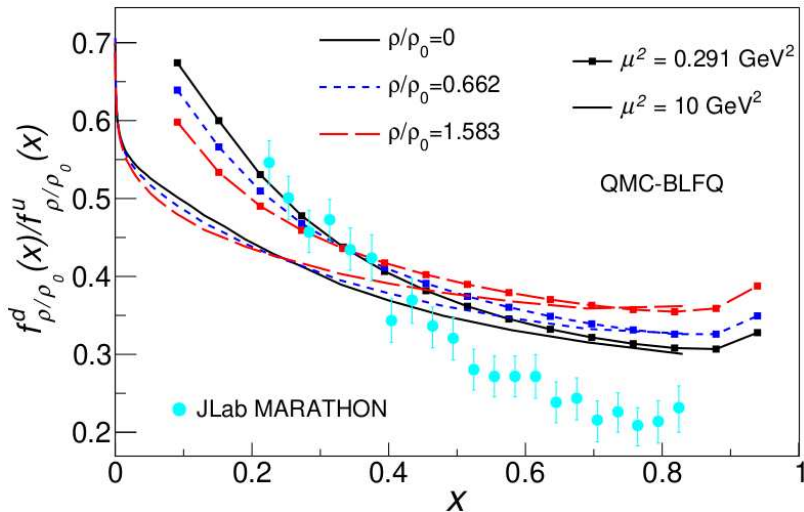
PDFs proton obtained within the QMC-BLFQ framework at the scale $\mu^2 = 10 \text{ GeV}^2$ for different nuclear densities ρ/ρ_0

- A. Buckley et al. LHAPDF6: Parton density access in the LHC precision era, Eur. Phys. J. C 75 (2015) 132



- Ratios defined $R_{\rho/\rho_0}^i(x) = \frac{f_{\rho/\rho_0}^i(x)}{f_0^i(x)}$, $i = q, g$, for $i = u_v, d_v, g$, density dependence of the parton distributions

Present work: 2605.31030 [hep-ph], Xiaoyi Wu et al.



Proton $f_{\rho/\rho_0}^d(x)/f_{\rho/\rho_0}^u(x)$ with $\mu_0^2 = 0.291 \text{ GeV}^2$ (lines with marker) and $\mu^2 = 10 \text{ GeV}^2$ (lines without marker) with different nuclear densities

- Ref. D. Abrams, et al., Measurement of the Nucleon F_2^n/F_2^p Structure Function Ratio. Jefferson Lab MARATHON Tritium/Helium-3 Deep Inelastic Scattering Experiment, Phys. Rev.

The central point this work is the construction of a self-consistent relationship between microscopic structure and nuclear matter

Nuclear mean field $\rightarrow m_q^* \rightarrow H_{\text{BLFQ}}^* \rightarrow \Psi_N^* \rightarrow M_N^* \rightarrow \text{EOS and PDFs}$

- The scalar field reduces the effective quark mass, modifying the internal dynamics of the nucleon.
- The new wave function yields a non-linear effective mass $M_N^*(\sigma)$ and, via the Feynman-Hellmann theorem, a density-dependent scalar coupling.
- This internal response softens the equation of state compared to the linear Walecka model.
- At the same time, the same wave function provides the modified PDFs in the medium.
- This allows the study of effects such as:
 - The suppression around $x \simeq 0.3$,
 - The growth of the gluonic contribution,
 - The alteration of the d/u ratio at large x .

- 1 **Development of an Effective In-Medium Formulation:** A light-front Hamiltonian formalism was developed where nucleon quarks couple to scalar and vector mean fields of the nuclear medium. This structure self-consistently solves the nucleon eigenvalue problem, allowing for a dynamic description of medium-induced nucleon modifications.
- 2 **Consistency with Empirical Data:** The resulting Equation of State (EOS) for symmetric nuclear matter, calculated through this framework, exhibits saturation properties (energy per nucleon, pressure, and incompressibility) that show excellent agreement with experimental constraints and empirical data from heavy-ion collisions.
- 3 **Parton Momentum Redistribution:** A modification in the parton distribution within the nucleon was observed: at saturation density, there is an increase in the gluon probability, accompanied by a decrease in the valence probability and the momentum fraction carried by the quarks.
- 4 **EMC Effect and Quark Structure:** The model reproduces fundamental characteristics of in-medium structure, including the suppression of the quark distribution at intermediate x values (the "EMC effect") and an enhancement of the distribution at large x values.
- 5 **Validation of the d/u Ratio:** The model's predictions for the ratio between the d and u quark parton distribution functions (PDFs) in the in-medium proton are aligned with experimentally observed trends, as demonstrated by the comparison with data from the JLab MARATHON experiment.

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