

The Δ resonance at the physical point

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The $\Delta(1232)$ resonance

- The $\Delta(1232)$ is the lowest-lying excitation of the nucleon.
- At sufficiently heavy pion masses, it behaves as a stable state.
- Near the physical point, it appears as a resonance in P -wave $N\pi$ scattering.
- Its properties follow from the finite-volume $N\pi$ spectrum rather than from a single energy level.
- We also study the electromagnetic $N \rightarrow \Delta$ transition form factors.



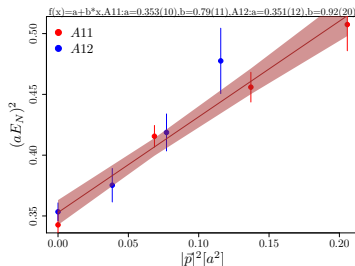
Simulation details

Wilson-Clover $N_f = 2 + 1$

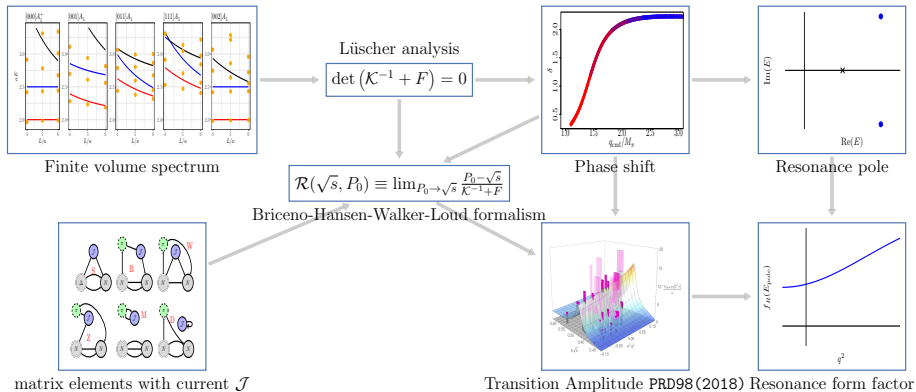
- Two different $M_\pi \sim 200, 260\text{MeV}$
- Two different L spatial size ($L = 2.8, 3.7\text{fm}$)
- Addition: Physical point ensemble

Twisted Clover $N_f = 2, N_f = 2 + 1 + 1$

- physical point ensemble



Plan:



Operator construction

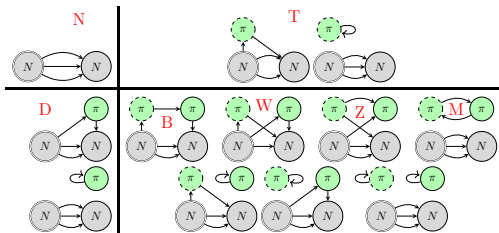
- Local interpolating fields

$$\mathcal{O}_N(t) = \sum_x \epsilon^{abc} \left[u^{aT}(x, t) C \gamma_5 d^b(x, t) \right] u^c(x, t), \quad \mathcal{O}_{\pi^+}(t) = \sum_x \bar{d}(x, t) \gamma_5 u(x, t).$$

$$\mathcal{O}_{\Delta^{++}}^i(t) = \sum_x \epsilon^{abc} \left[u^{aT}(x, t) C \gamma_i u^b(x, t) \right] u^c(x, t)$$

$$\mathcal{O}_{N\pi}(\mathbf{p}, t) = \mathcal{O}_N(\mathbf{p}, t) \mathcal{O}_{\pi}(-\mathbf{p}, t), \quad \mathbf{P} = \mathbf{p} + (-\mathbf{p}) = \mathbf{0}.$$

- Expectation value



- Diagrams

Operator construction

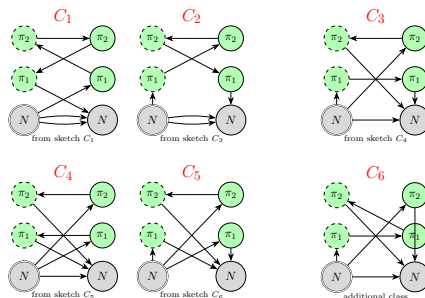
- Local interpolating fields

$$\mathcal{O}_N(t) = \sum_x \epsilon^{abc} \left[u^{aT}(x, t) C \gamma_5 d^b(x, t) \right] u^c(x, t), \mathcal{O}_{\pi^+}(t) = \sum_x \bar{d}(x, t) \gamma_5 u(x, t).$$

$$\mathcal{O}_{\Delta^{++}}^i(t) = \sum_x \epsilon^{abc} \left[u^{aT}(x, t) C \gamma_i u^b(x, t) \right] u^c(x, t)$$

$$\mathcal{O}_{N\pi\pi}(\mathbf{p}_N, \mathbf{p}_1, \mathbf{p}_2, t) = \mathcal{O}_N(\mathbf{p}_N, t) \mathcal{O}_{\pi}(\mathbf{p}_1, t) \mathcal{O}_{\pi}(\mathbf{p}_2, t), \mathbf{p}_N + \mathbf{p}_1 + \mathbf{p}_2 = \mathbf{0}.$$

- Expectation value



Factorization of the $N\pi$ box diagram

V_2 reduction — nucleon side

$$(V_{2,\Gamma})_n^{\alpha_0\alpha_1\alpha_2} = \epsilon_{abc}\epsilon_{lmn}\Gamma^{\beta_0\beta_1}_{\beta_0\beta_1}\phi_c^{\beta_0}S_{1bm}^{\beta_1\alpha_0}S_{2al}^{\alpha_1\alpha_2}$$

Fourier transform at $\vec{p}_N$

Three open spin indices and one open color index.

V_3 reduction — pion side

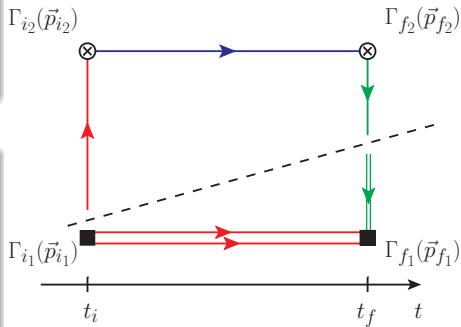
$$(V_{3,\Gamma})_b^\beta = \sum_{a,\alpha_0,\alpha_1} \phi_a^{\alpha_0*} \Gamma_{\alpha_0\alpha_1} S_{ab}^{\alpha_1\beta}$$

Fourier transform at $\vec{p}_\pi$

One open spin index and one open color index.

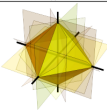
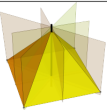
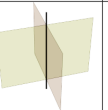
Final B-diagram contraction

$$C_{N\pi}^{(B)} = \text{Tr}_{s,c} [V_2(\vec{p}_N) \Gamma_{\text{source}} V_3(\vec{p}_\pi)]$$



Finite-volume projections

- In finite volume, continuous rotational symmetry reduces to cubic symmetry.
- Each lattice irrep receives contributions from infinitely many continuum angular momenta.
- In the center-of-mass frame, the relevant symmetry group for baryons is the double cover of the octahedral group, O_h^D .
- At rest, the Δ channel subduces primarily into the H_g irrep.

$\frac{L}{2\pi}\vec{P}$	(0,0,0)	(0,0,1)	(0,1,1)	(1,1,1)
Group LG	$O_h^{(D)}$	$C_{4v}^{(D)}$	$C_{2v}^{(D)}$	$C_{3v}^{(D)}$
Axis and planes of symmetry				
g_{LG}	96	16	8	12
$\Lambda(J^P) : \pi(0^-)$	$A_{1u}(0^-, 4^-, \dots)$	$A_2(0, 1, \dots)$	$A_2(0, 1, \dots)$	$A_2(0, 1, \dots)$
$\Lambda(J^P) : N(\frac{1}{2}^+)$	$G_{1g}(\frac{1}{2}^+, \frac{7}{2}^+, \dots)$	$G_1(\frac{1}{2}, \frac{3}{2}, \dots)$	$G(\frac{1}{2}, \frac{3}{2}, \dots)$	$G(\frac{1}{2}, \frac{3}{2}, \dots)$
$\Lambda(J^P) : \Delta(\frac{3}{2}^+)$	$H_g(\frac{3}{2}^+, \frac{5}{2}^+, \dots)$	$G_1(\frac{1}{2}, \frac{3}{2}, \dots) \oplus G_2(\frac{3}{2}, \frac{5}{2}, \dots)$	$(2)G(\frac{1}{2}, \frac{3}{2}, \dots)$	$G(\frac{1}{2}, \frac{3}{2}, \dots) \oplus F_1(\frac{3}{2}, \frac{5}{2}, \dots) \oplus F_2(\frac{3}{2}, \frac{5}{2}, \dots)$

Ratio method for $N\pi$ levels

For a principal correlator $\lambda_n(t, t_0)$, we define

$$R_n^q(t, t_0) = \frac{\lambda_n(t, t_0)}{C_N(\mathbf{p}, t - t_0) C_\pi(\mathbf{q} - \mathbf{p}, t - t_0)}. \quad (1)$$

At sufficiently large Euclidean time,

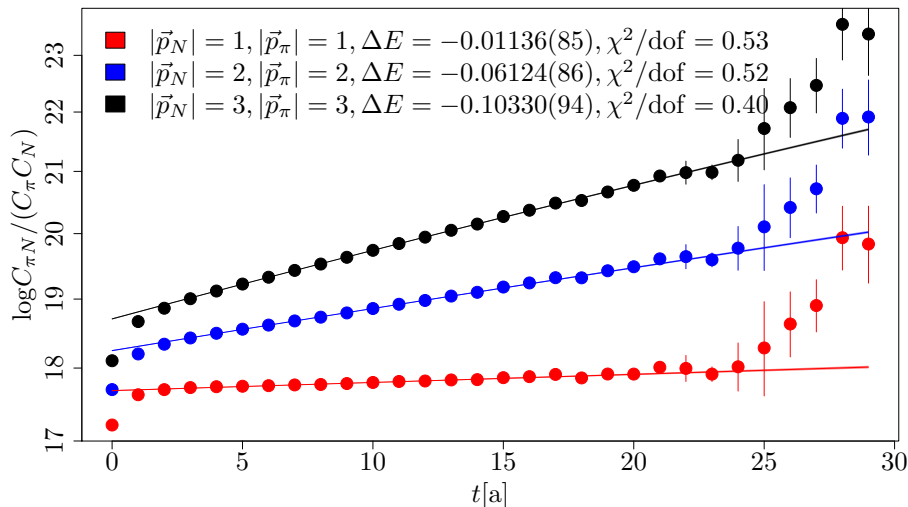
$$R_n^q(t, t_0) \longrightarrow A_n \exp[-\Delta E_n(t - t_0)], \quad (2)$$

with

$$\Delta E_n = E_n - E_N(\mathbf{p}) - E_\pi(\mathbf{q} - \mathbf{p}), E_h(\mathbf{p}) = \sqrt{m_h^2 + \mathbf{p}^2}, \mathbf{p} = \frac{2\pi}{L} \mathbf{n}. \quad (3)$$

- The lowest $N\pi$ contribution has relative P -wave momentum and therefore $\mathbf{p} \neq 0$.

Ratio method example:



Truncated Hankel correlator method

Starting from the spectral decomposition

$$C(t) = \sum_{\ell} Z_{\ell} e^{-E_{\ell} t}, \quad (4)$$

construct the Hankel matrix from all selected time slices,

$$H_{ij}^{(0)} = C(t_0 + (i+j)\Delta), \quad H_{ij}^{(1)} = C(t_0 + \delta t + (i+j)\Delta). \quad (5)$$

Diagonalize the full Hankel matrix and retain the k dominant modes,

$$H^{(0)} = U \text{diag}(s_1, \dots, s_N) U^{\dagger}, \quad H_k^{(0)} = U_k \text{diag}(s_1, \dots, s_k) U_k^{\dagger}. \quad (6)$$

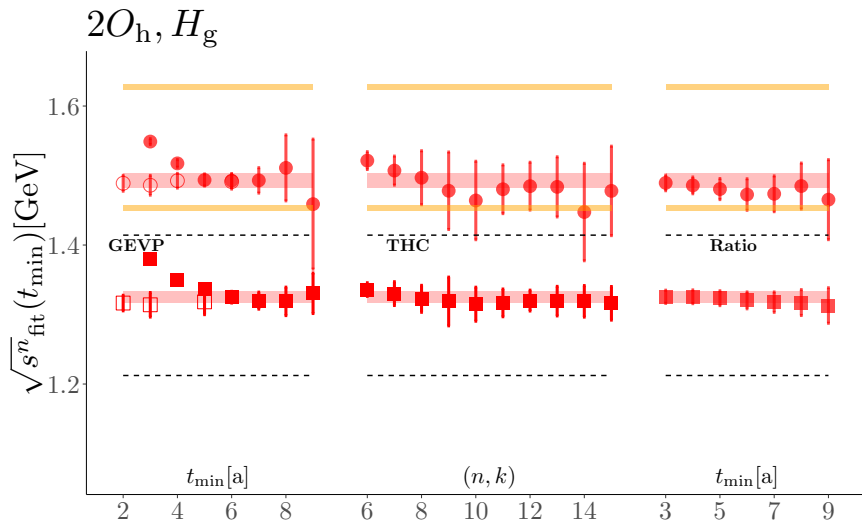
The energies follow from the truncated Prony GEVP,

$$H_k^{(1)} v_{\ell} = \lambda_{\ell}^{(k)} H_k^{(0)} v_{\ell}, \quad E_{\ell}^{(k)} = -\frac{1}{\delta t} \log \lambda_{\ell}^{(k)}. \quad (7)$$

Interpretation

Stability of $E_{\ell}^{(k)}$ for $k \geq k_{\min}$ indicates that the retained rank is sufficient.

Comparison



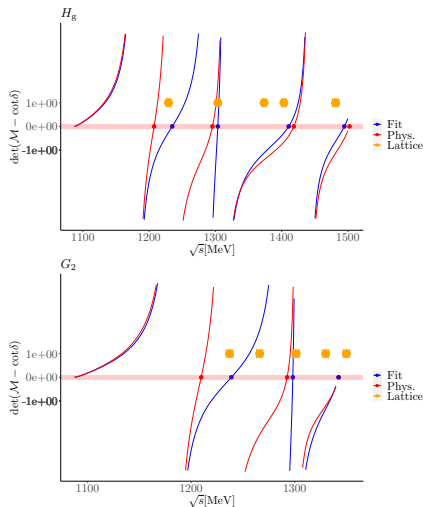
Getting the phase shift

Lüscher-method

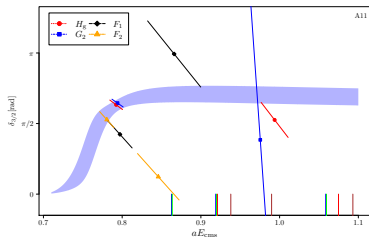
- Two particle energy levels in a finite box with size L
- Volume dependence of the energy shift related to scattering observables at $L = \infty$

$$\det \left(\mathcal{M}_{J\ell\mu, J'\ell'\mu'}^{\vec{P}} - \delta_{JJ'}\delta_{\ell\ell'}\delta_{\mu\mu'}\cot\delta_{J\ell} \right) = 0$$

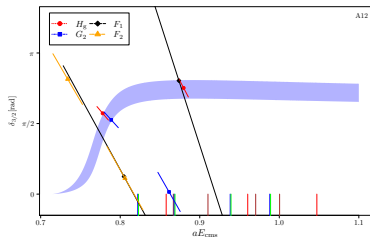
- Determinant is taken in angular momentum space
- Important: For $\ell = 1$ dominant irreps there is a one-to-one correspondence between phase-shift and finite volume energy levels (ignoring contributions from higher partial waves)



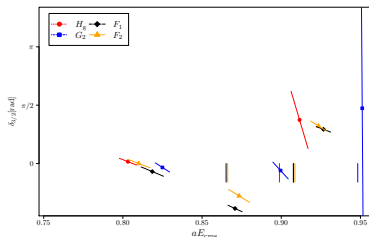
Phase-shift results (preliminary)



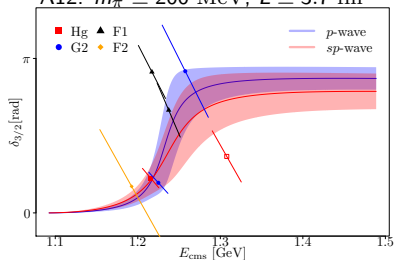
A11: $m_\pi \simeq 200$ MeV, $L \simeq 2.8$ fm



A12: $m_\pi \simeq 200$ MeV, $L \simeq 3.7$ fm



A8: $m_\pi \simeq 260$ MeV, $L \simeq 3.7$ fm



A15: $m_\pi \simeq 140$ MeV (physical point)

Conclusions and outlook

- We extract the finite-volume Δ and $N\pi$ spectrum at several pion masses and volumes.
- The resulting energies determine the P -wave $N\pi$ phase shift.
- Production of the required three-point functions is under way.
- Our next goal is the extraction of the electromagnetic transition form factors G_{M1} , G_{E2} , and G_{C2} .

Thank you for your attention