

PDF Moments from Flowed Local Operators

A Lattice QCD Approach to Hadron Structure

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PDF and Lattice QCD

Direct lattice calculations of higher PDF moments are notoriously difficult

Curci, Furmanski, Petronzio, Nucl. Phys. B 175 (1980) 27
Collins, Soper, Nucl. Phys. B 194 (1982) 445

$$\langle x^{n-1} \rangle_q^h(\mu) = \int_0^1 dx x^{n-1} [q^h(x, \mu) + (-1)^n \bar{q}^h(x, \mu)]$$

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$$\langle h(p) | \hat{O}_{\{\mu_1 \dots \mu_n\}}^{qq} | h(p) \rangle = 2 \langle x^{n-1} \rangle_q^h(\mu) (p_{\mu_1} \cdots p_{\mu_n} - \text{traces})$$

$$\hat{O}_{\{\mu_1 \dots \mu_n\}}^{qq}(x) = \bar{\psi}^q(x) \gamma_{\{\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n\}} \psi^q(x) - \text{traces}$$

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- Continuum limit fails for $\langle x^n \rangle$ for $n > 3$
- For $n=2,3$: need of non-zero spatial momenta $\rightarrow$ destroys signal-to-noise
- Power divergences prevent renormalization of higher moments
- As a result, only $\langle x \rangle$ and at best (very poorly) $\langle x^2 \rangle$ and $\langle x^3 \rangle$ were accessible

Kronfeld, Photiadis (1985)
Martinelli, Sachrajda (1987 - 1988)

Moment	Status (pre-flow)
$\langle x \rangle$	✓ reliable
$\langle x^2 \rangle$	✓ barely usable
$\langle x^3 \rangle$	✗ extremely noisy
$\langle x^4 \rangle, \langle x^5 \rangle, \dots$	✗ impossible

PDF and Lattice QCD

Approaches have been developed to determine the x -dependence of the PDFs

- Hadronic tensor Liu, Dong (1994)
- Auxiliary scalar field Aglietti et al. (1998)
- Fictitious heavy quark Detmold, Lin (2005)
- Auxiliary fermion quark Braun, Müller (2008)
- quasi-PDF (LaMET) Ji (2013)
- Smeared OPE Monahan, Orginos (2015)
- pseudo-PDF Radyushkin (2017)
- Compton amplitude + OPE Chambers et al. (2017)
- Good Lattice Cross Sections Ma, Qiu (2018)
- PDF without Wilson line Zhao (2024)

short distance ($z \rightarrow 0$) $\longleftrightarrow$ large Ioffe time ($\nu = P_z z$)

- **small $z \rightarrow$ control UV but need large P_z**
Current lattices reach only $P_z \lesssim 3 \text{ GeV} \Rightarrow$
limited ν window
- **large $P_z \rightarrow$ noise/excited states $\rightarrow$ precision saturates**
Lattices must get much finer or new strategies developed

These approaches allow in principle an indirect determination of the moments of PDF of nucleons and pions

Gao et al.: (2020-2023)

Egerer et al. (2022)

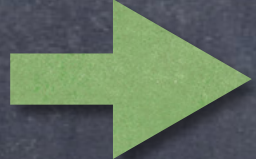
Bhattacharya et al.: (2023)

Moments of the PDF: standard method

$$O_n^{rs}(x) = O_{\mu_1 \dots \mu_n}^{rs}(x) = \bar{\psi}^r(x) \gamma_{\{\mu_1} \overleftrightarrow{D}_{\mu_2} \dots \overleftrightarrow{D}_{\mu_n\}} \psi^s(x)$$

- Calculate matrix elements using lattice QCD
 - Rotational group symmetry is broken into the hypercubic group H(4)
- Irreducible representations of O(4) generally become reducible representations of H(4) inducing unwanted mixings under renormalization
 - Irreps of H(4) allow mixing with lower dimensional operators and complicate mixings with operators of the same dimension
- Operators with different index combinations belong to different irreps of H(4)

Beccarini et al.,
Nucl. Phys. B 456 (1995) 271
Göckeler et al.,
Phys. Rev. D (1996) 5705

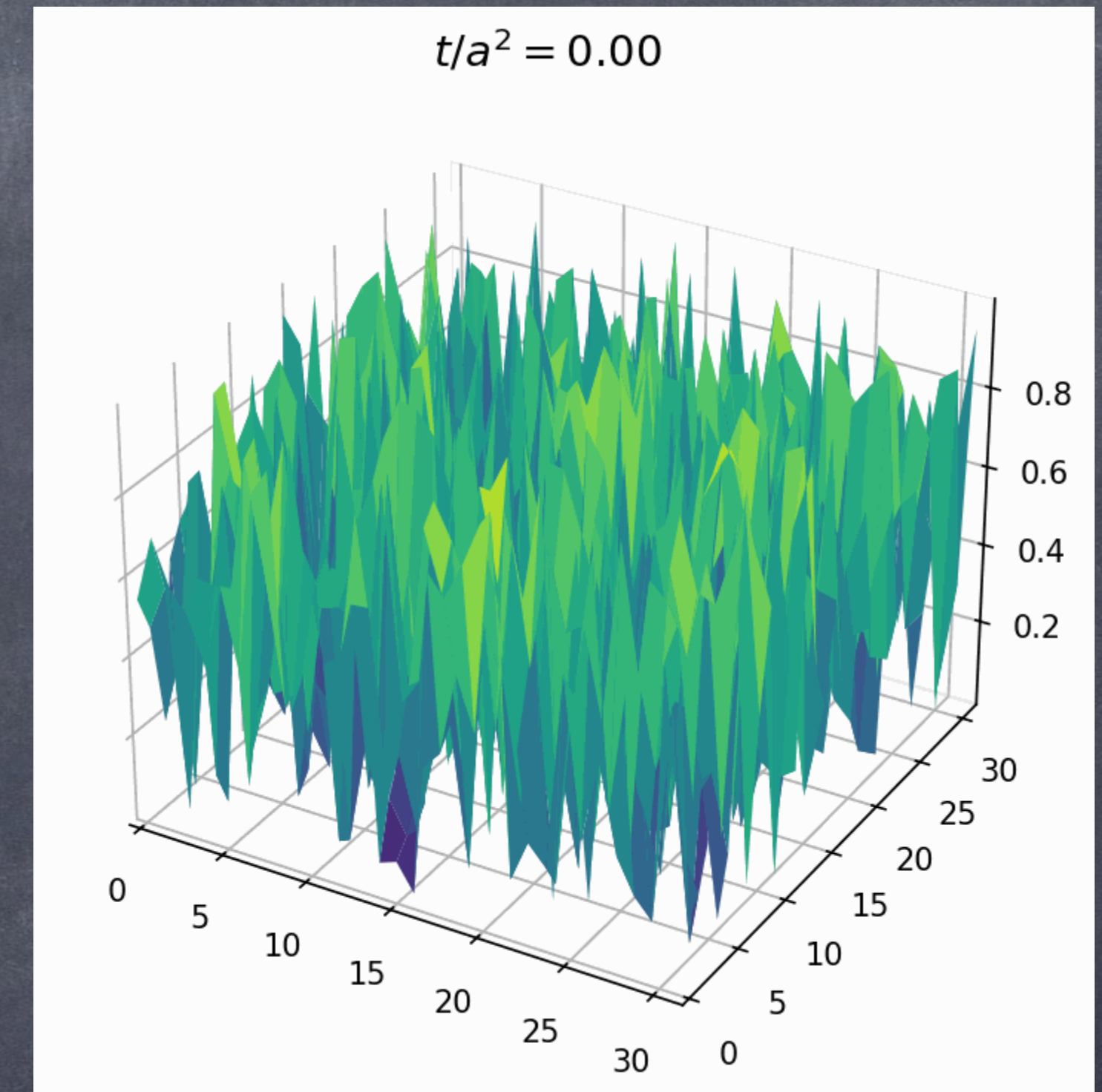
O_3	$\mu_1 = \mu_2 = \mu_3$		$1/a^2 \delta_{\mu_i \mu_j} \cos(ap_{\mu_j})$	Kronfeld, Photiadis: 1985
	$\mu_1 \neq \mu_2 = \mu_3$		$O_{411} - O_{433}$	Martinelli, Sachrajda: 1987
	$\mu_1 \neq \mu_2 \neq \mu_3$		$\langle h(p) O_n h(p) \rangle = 2 (p_{\mu_1} \dots p_{\mu_n} - \text{trace terms}) \langle x^{n-1} \rangle_h$	

Method that addresses both the theoretical and numerical challenges faced in the past, which hindered the direct calculation of moments of any order from lattice QCD

Gradient flow

Lüscher JHEP 08 (2010) 071
Lüscher, Weisz JHEP 02 (2011) 051

$$A_\mu(x) \xrightarrow{\text{gradient flow}} B_\mu(x, t)$$



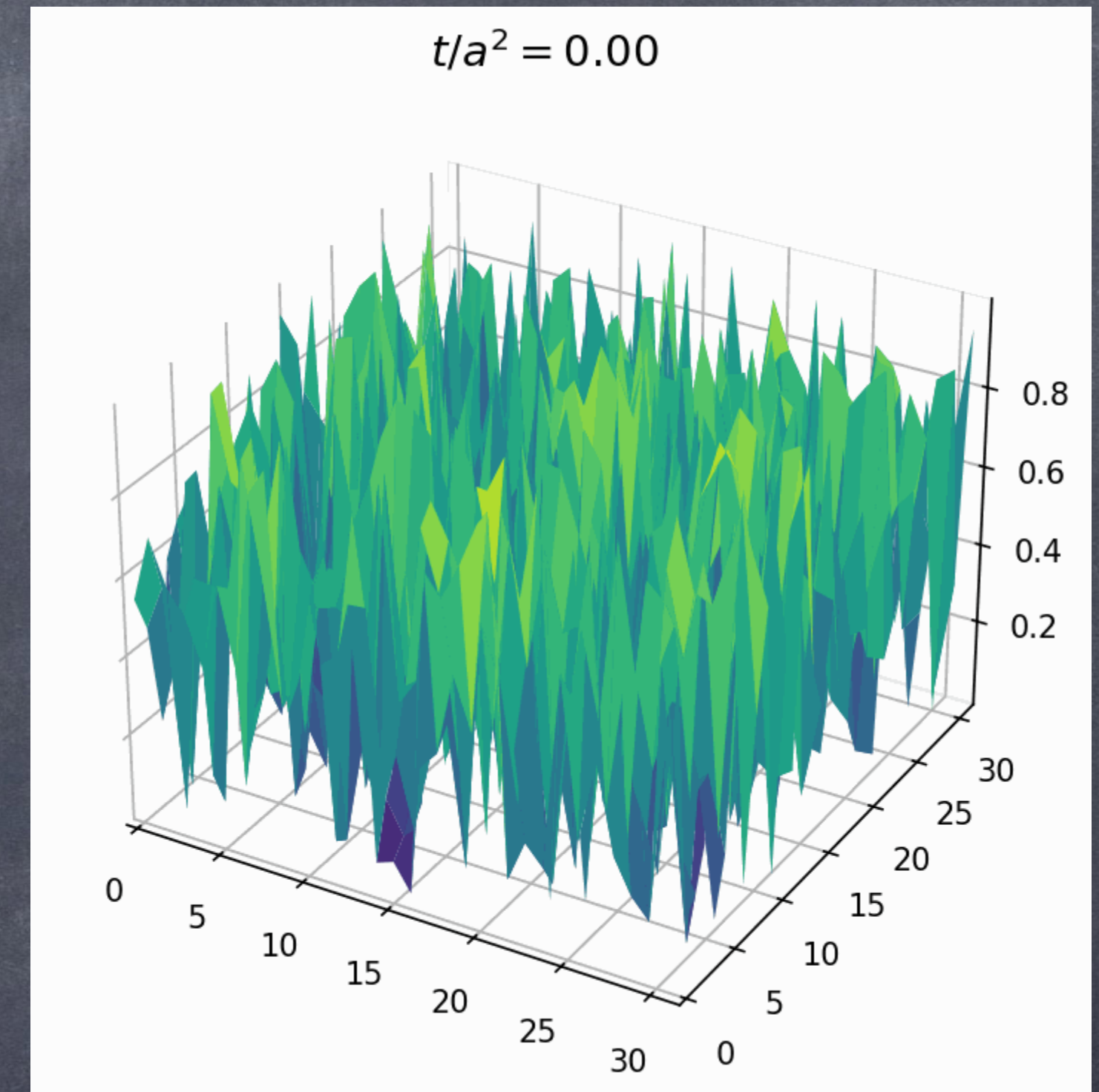
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$$\partial_t B_\mu(x, t) = D_\nu G_{\nu\mu}(x, t)$$

$$B_\mu(x, t)|_{t=0} = A_\mu(x)$$



Gradient flow

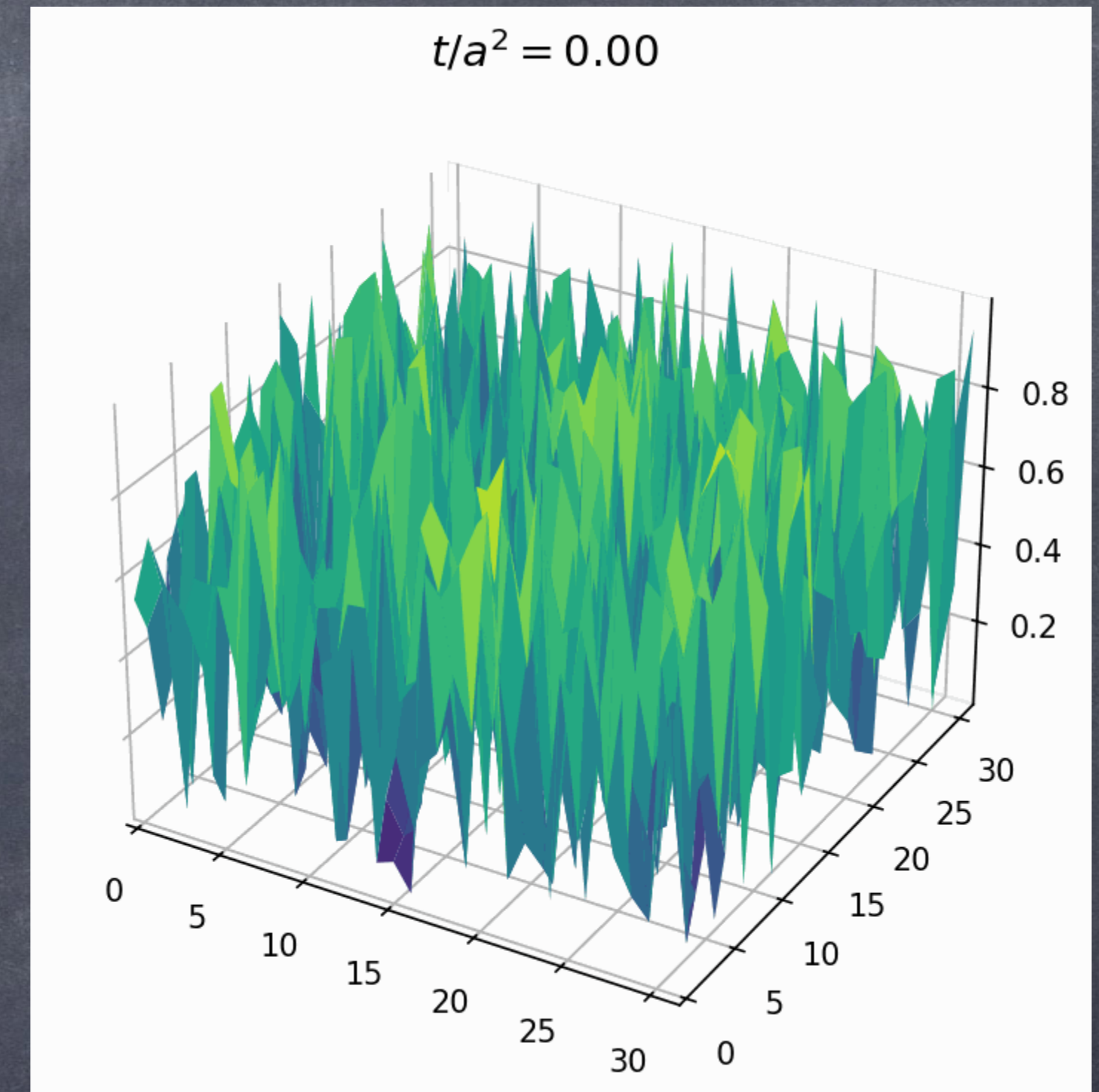
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- Gaussian damping at large momenta
- Smoothing at short distance over a range $\sqrt{8t}$



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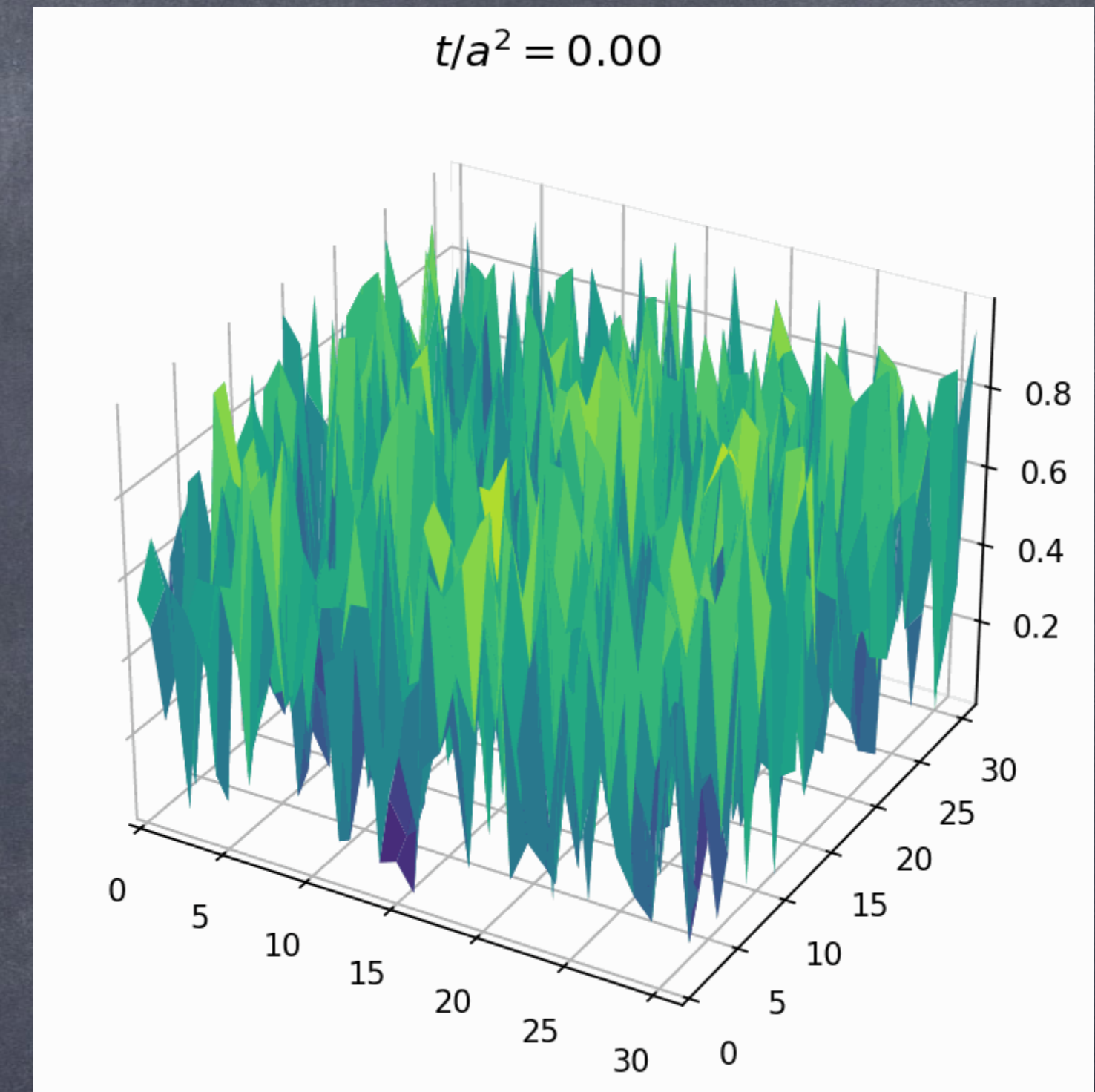
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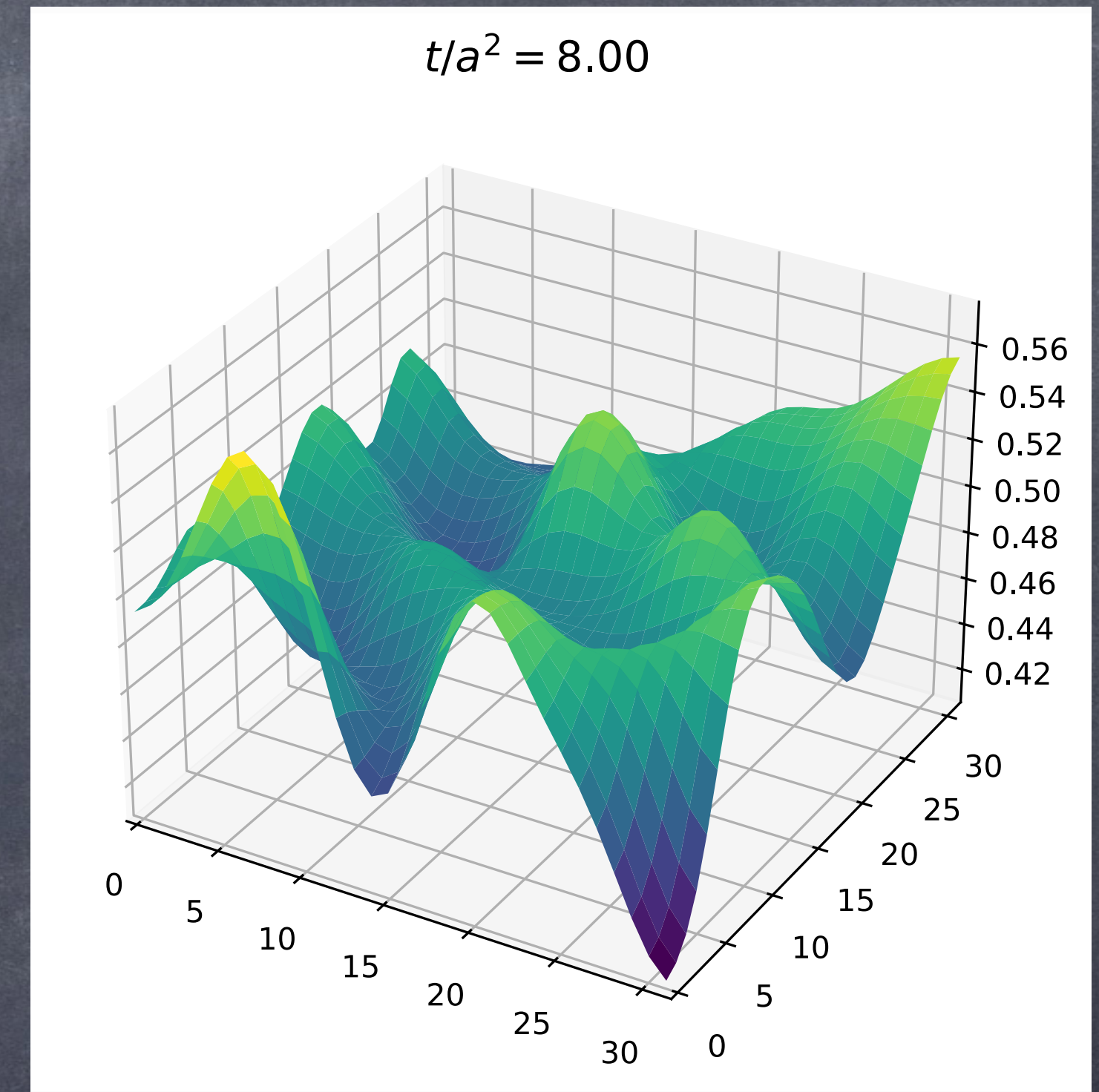
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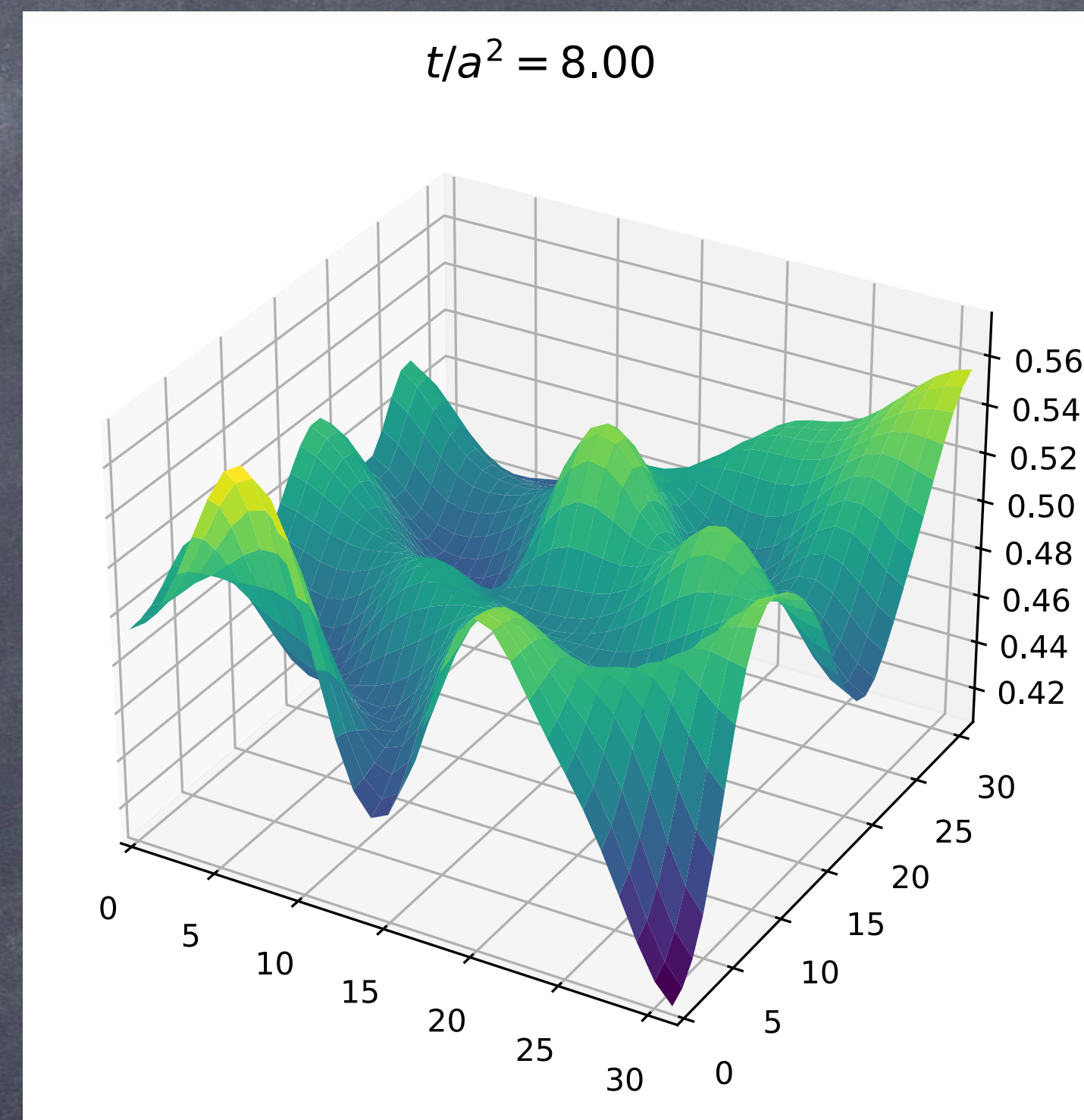
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$$\langle \cdots B_\mu(x, t) \cdots \rangle \quad t > 0 \quad \text{finite}$$

Gradient flow

Lüscher JHEP 04 (2013) 123

$$\psi(x) \xrightarrow{\text{gradient flow}} \chi(x, t)$$

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$$\chi_R(x, t) = Z_\chi^{1/2} \chi(x, t) \quad \mathcal{O}_R = Z_\chi \mathcal{O}$$

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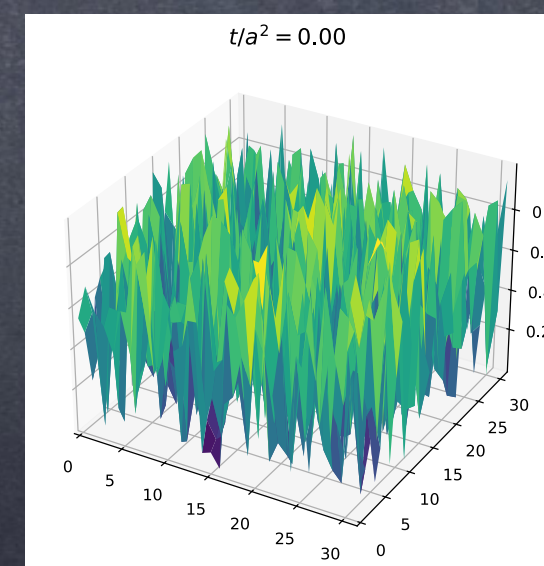
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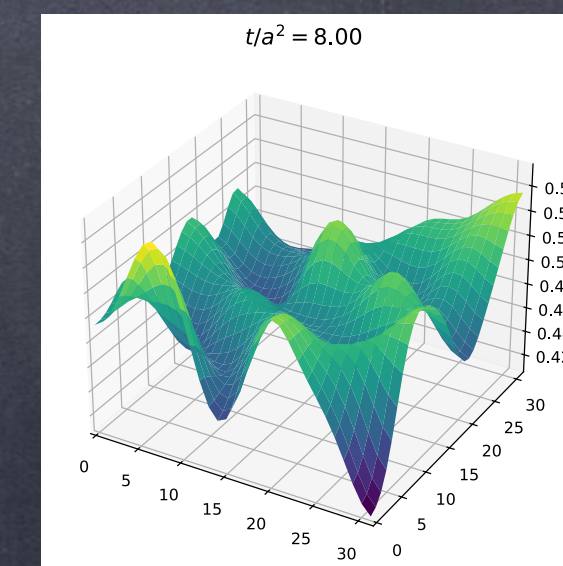
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A.Sh.: lattice 2026 review



$$\partial_t B_\mu(x, t) = D_\nu G_{\nu\mu}(x, t)$$

$$\partial_t \chi(x, t) = \Delta \chi(x, t)$$



Short-flow-time expansion

Lüscher, Weisz JHEP 02 (2011) 051
Suzuki PTEP 2013 (2013) 083B03
Lüscher PoS LATTICE2013 (2014)

Flowed operators admit an OPE-like expansion for small flow-time t

$$\mathring{O}_i(t) = \sum_j \mathring{\zeta}_{ij}(t, \mu) [O_j(t=0, \mu)]_{\text{R}} + O(t)$$

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LQCD

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PT - LQCD

$$\zeta_{ij}(t, \mu) = \delta_{ij} + \frac{\alpha_s(\mu)}{4\pi} \zeta_{ij}^{(1)}(t, \mu) + O(\alpha_s^2)$$

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Lüscher, Weisz JHEP 02 (2011) 051
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- **New UV regulator** that preserves continuum symmetries
- **Simplifies renormalization** and improves removal of the hard lattice cutoff
- **Enables new strategies** for hadronic matrix elements

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PT - LQCD

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- **Enables new strategies** for hadronic matrix elements
- “Multiplicative” matching — logarithmic divergence on the lattice
- “Multiplicative” matching — power divergences on the lattice
- Power divergences in flow time — power divergences on the lattice

Strategy

A.Sh.
Phys.Rev.D 110 (2024) 5, L051503

- Consider flowed operators $\mathcal{O}_n^{rs}(x, t) = \bar{\chi}^r(x, t) \gamma_{\{\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n\}} \chi^s(x, t)$

- Renormalize flowed operators $\rightarrow$ renormalization is ALWAYS multiplicative

$$\hat{\mathcal{O}}_n^{rs}(t) = \lim_{a \rightarrow 0} Z_n \mathcal{O}_n^{rs}(t), \quad Z_n = Z_\chi$$

Continuum limit is finite for any n

- Perform a short flow time expansion $\rightarrow$ consider $O(4)$ irreps $\rightarrow$ traceless operators

$$\hat{\mathcal{O}}_n^{rs}(x, t) = \bar{\chi}^r(x, t) \gamma_{\{\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n\}} \chi^s(x, t) - \text{terms with } \delta_{\mu_i \mu_j}$$

$$\langle h(p) | \hat{\mathcal{O}}_n(t) | h(p) \rangle = 2 (p_{\mu_1} \cdots p_{\mu_n} - \text{trace terms}) \langle x^{n-1} \rangle_h(t)$$

No boost needed for any n

- Matching is multiplicative for traceless operators for any n and any choice of Lorentz indices

$$\hat{\mathcal{O}}_n^{rs}(t) = \zeta_n(t, \mu) \hat{\mathcal{O}}_n^{rs, \overline{\text{MS}}}(\mu) + O(t)$$

Matching coefficients

A.Sh.
Phys.Rev.D 110 (2024) 5, L051503

Matching equations $\left\langle \psi^r \hat{\mathcal{O}}_n^{rs}(t) \bar{\psi}^s \right\rangle = \zeta_n(t, \mu) \left\langle \psi^r \hat{\mathcal{O}}_n^{rs, \overline{\text{MS}}}(t=0, \mu) \bar{\psi}^s \right\rangle$

- Expand integrands of loop integrals in all scales excluding $\dagger$
 - Analytic structure altered $\rightarrow$ distortion of IR structure
 - in matching equation the IR modification drops out in the difference
 - Expanding loop integrals in the RHS vanish in DR $\rightarrow$ UV and IR are identical
 - The LHS is UV-finite, beside the renormalization of the bare parameters and flowed fermion fields
 - The IR singularities on the LHS exactly match the UV MS counterterms

$$\zeta_n(t, \mu) = 1 + \frac{\bar{g}^2(\mu)}{(4\pi)^2} \zeta_n^{(1)}(t, \mu) + O(\bar{g}^4) \quad \zeta_n^{(1)}(t, \mu) = C_F [\gamma_n \log(8\pi\mu^2 t) + B_n] \quad \gamma_n = 1 + 4 \sum_{j=2}^n \frac{1}{j} - \frac{2}{n(n+1)}$$

Gross and Wilczek, Phys.
Rev. D 9 (1974) 980

$$B_n = \frac{4}{n(n+1)} + 4 \frac{n-1}{n} \log 2 + \frac{2-4n^2}{n(n+1)} \gamma_E - \frac{2}{n(n+1)} \psi(n+2) + \frac{4}{n} \psi(n+1) - 4\psi(2) - 4 \sum_{j=2}^n \frac{1}{j(j-1)} \frac{1}{2^j} \phi(1/2, 1, j) - \log(432)$$

$n=2$ Makino and Suzuki,
PTEP 2014 (2014) 063B02

$$\phi(z, s, a) = \sum_{k=0}^{\infty} \frac{z^k}{(k+a)^s}$$

What is OpenLat



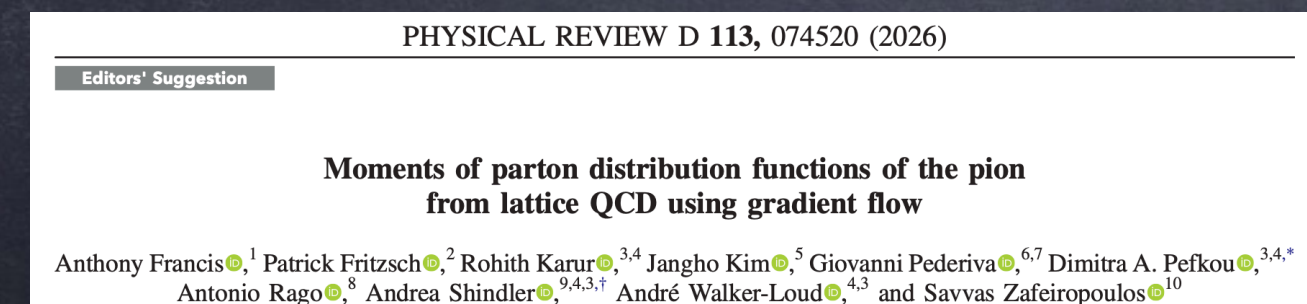
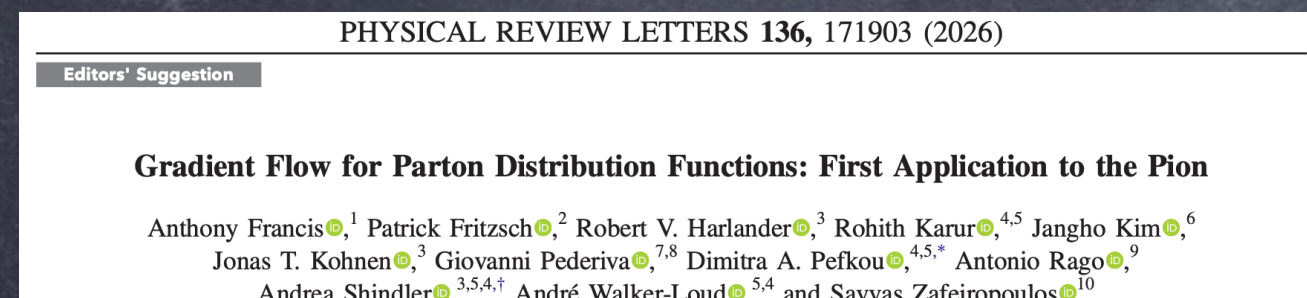
<https://openlat1.gitlab.io>

- OpenLat provides open access to cutting-edge (2+1)-flavor QCD gauge field ensembles, enabling the study of strong interaction phenomena with state-of-the-art simulations.
- The initiative bridges the gap for researchers without access to large-scale collaborations, fostering innovation and inclusivity in lattice QCD.
- Strategic choice of stabilized Wilson fermion framework for precision and stability in QCD simulations.



Test case: Heavy pion at SU(3) flavor-symmetric point

- SU(3)-flavor symmetric point: $m_u = m_d = m_s$ ($m_\pi \simeq 410\text{MeV}$)
- Stabilized Wilson fermions + Lüscher–Weisz gauge action
Francis, Fritsch, Lüscher, Rago
Comput. Phys. Commun. 255 (2020)
- 4 lattice spacings (0.12 $\rightarrow$ 0.064 fm)
- 400–800 configurations per ensemble
- NLO calculation $\forall n$ A.Sh. Phys.Rev.D 110 (2024) 5, L051503
- NNLO calculation $n = 2, \dots, 6$ Harlander, Kohnen, A.Sh.
Eur.Phys.J.C 86 (2026) 4, 439

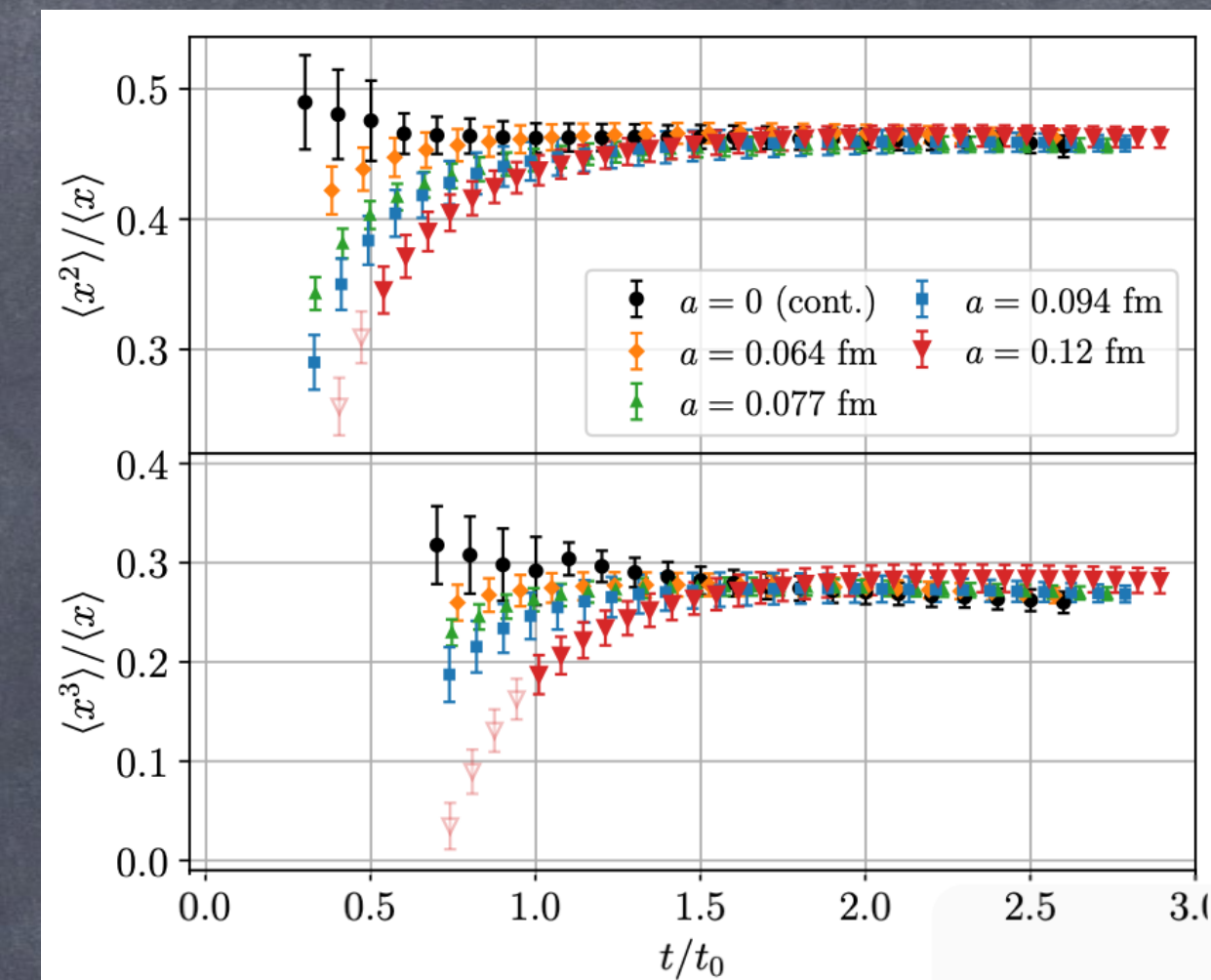


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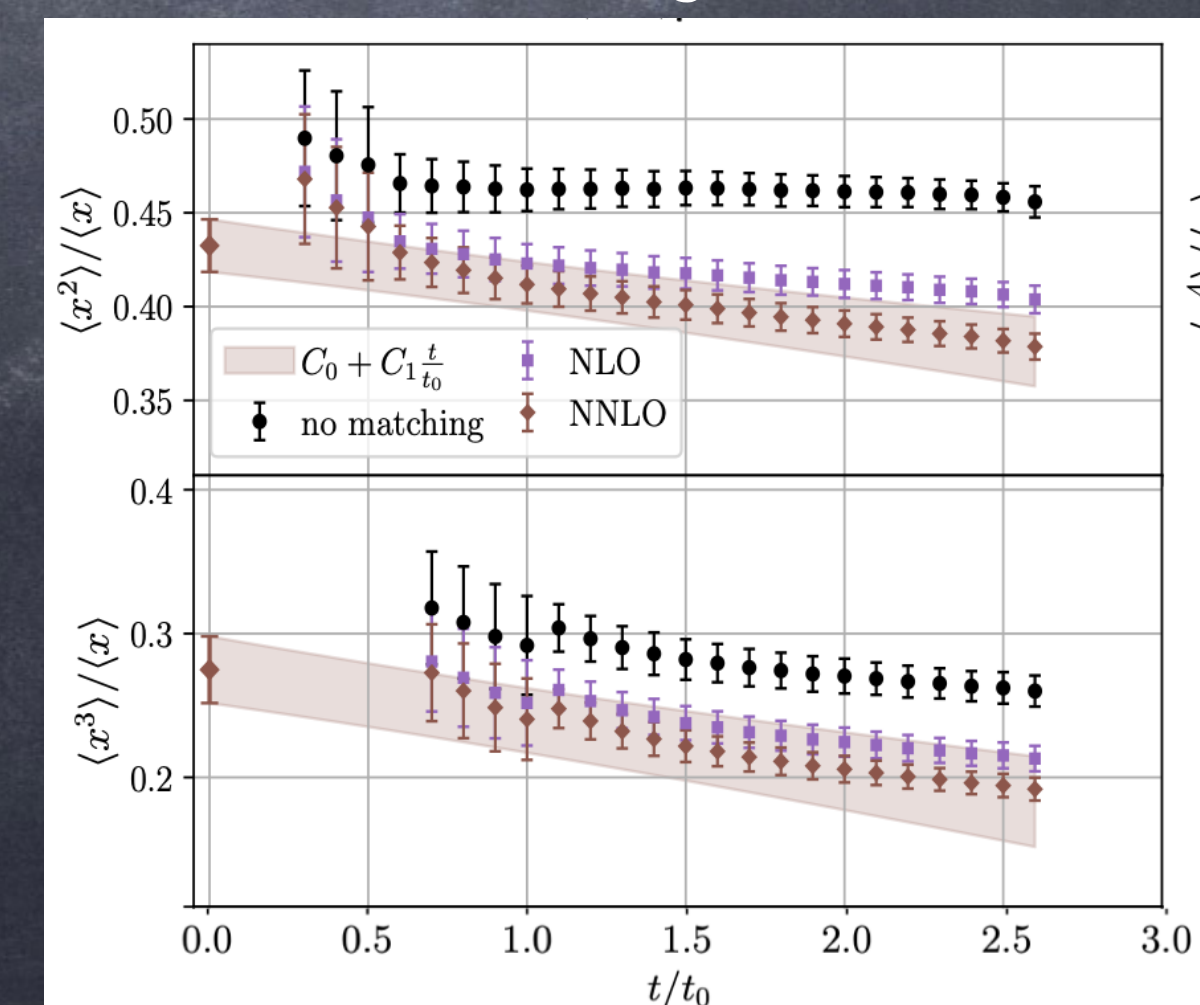
Phys.Rev.Lett. 136 (2026) 17, 171903
Phys.Rev.D 113 (2026) 7, 074520

The method shows excellent scaling and stable $t \rightarrow 0$ behavior, validating it for precision applications

Flowed moments



$t \rightarrow 0$ matching (NLO/NNLO)

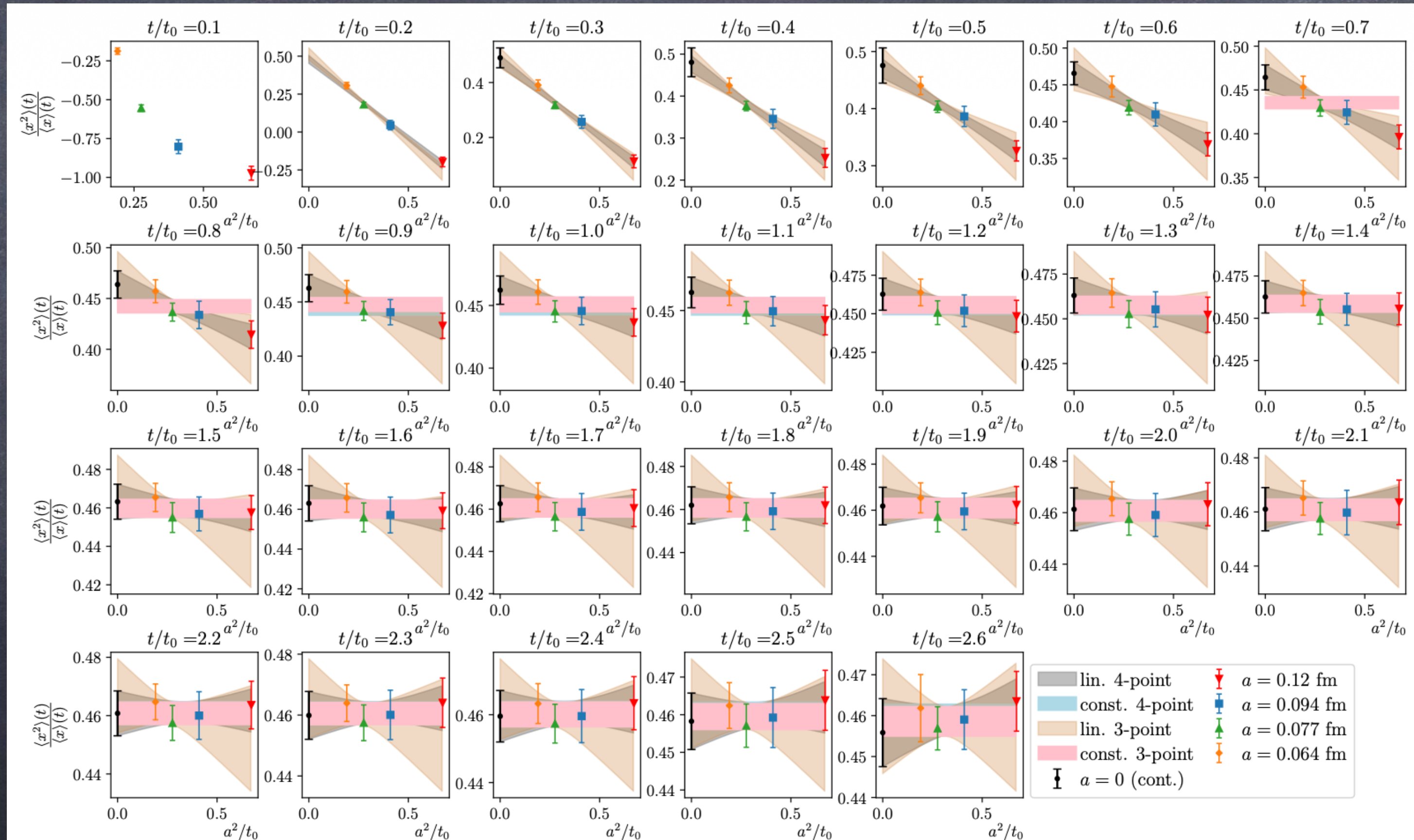


Continuum limit

Moments of parton distribution functions of the pion from lattice QCD using gradient flow

Anthony Francis¹, Patrick Fritzsch², Rohith Karur^{3,4}, Jangho Kim⁵, Giovanni Pederiva^{6,7}, Dimitra A. Pefkou^{3,4,*},
Antonio Rago⁸, Andrea Shindler^{9,4,3,†}, André Walker-Loud^{4,3} and Savvas Zafeiropoulos¹⁰

$$r_n(a^2/t_0, t/t_0) = \frac{\langle x^{n-1} \rangle}{\langle x \rangle} (t/t_0) + \delta_{n,2}(t/t_0) \frac{a^2}{t_0} + \dots$$

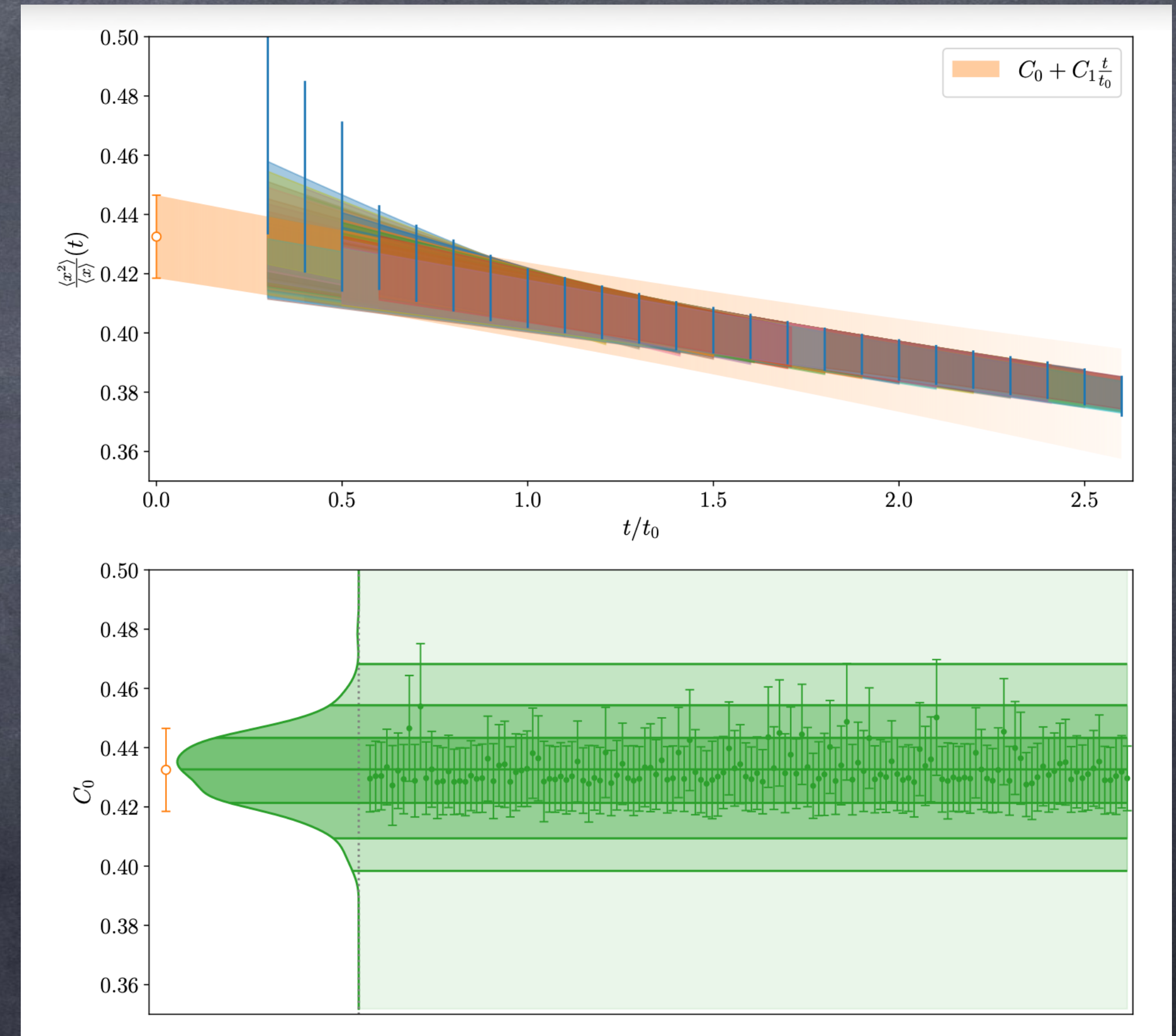


Extrapolation to $t \rightarrow 0$

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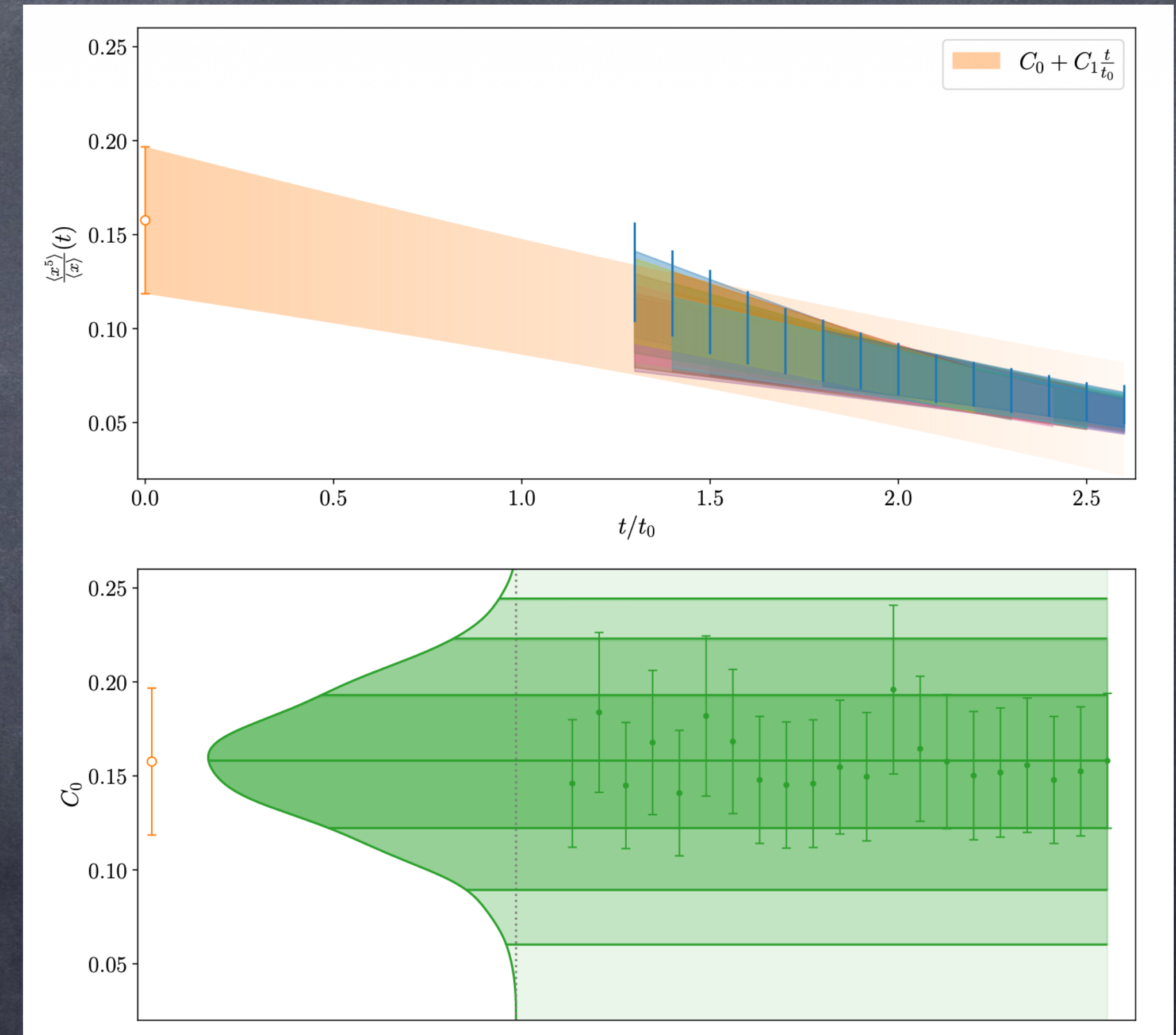
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- Residual t -dependence
- Within the current statistical precision, no clear logs observed
- Simple linear ansatz $C_0 + C_1 t/t_0$
- The minimum values used are:
 $(t/t_0)_{\min} = [0.3, 0.7, 0.9, 1.3]$
 for $n = [3, 4, 5, 6]$
- We perform fits over all flow-time intervals with width bigger than 0.8
- Only fits with $p > 0.1$ are retained
- The final central value is from a flat average over all accepted fits
- The systematic uncertainty from the spread of the accepted fit results
- Varying the minimum and maximum flow-time cuts within reasonable ranges gives stable results



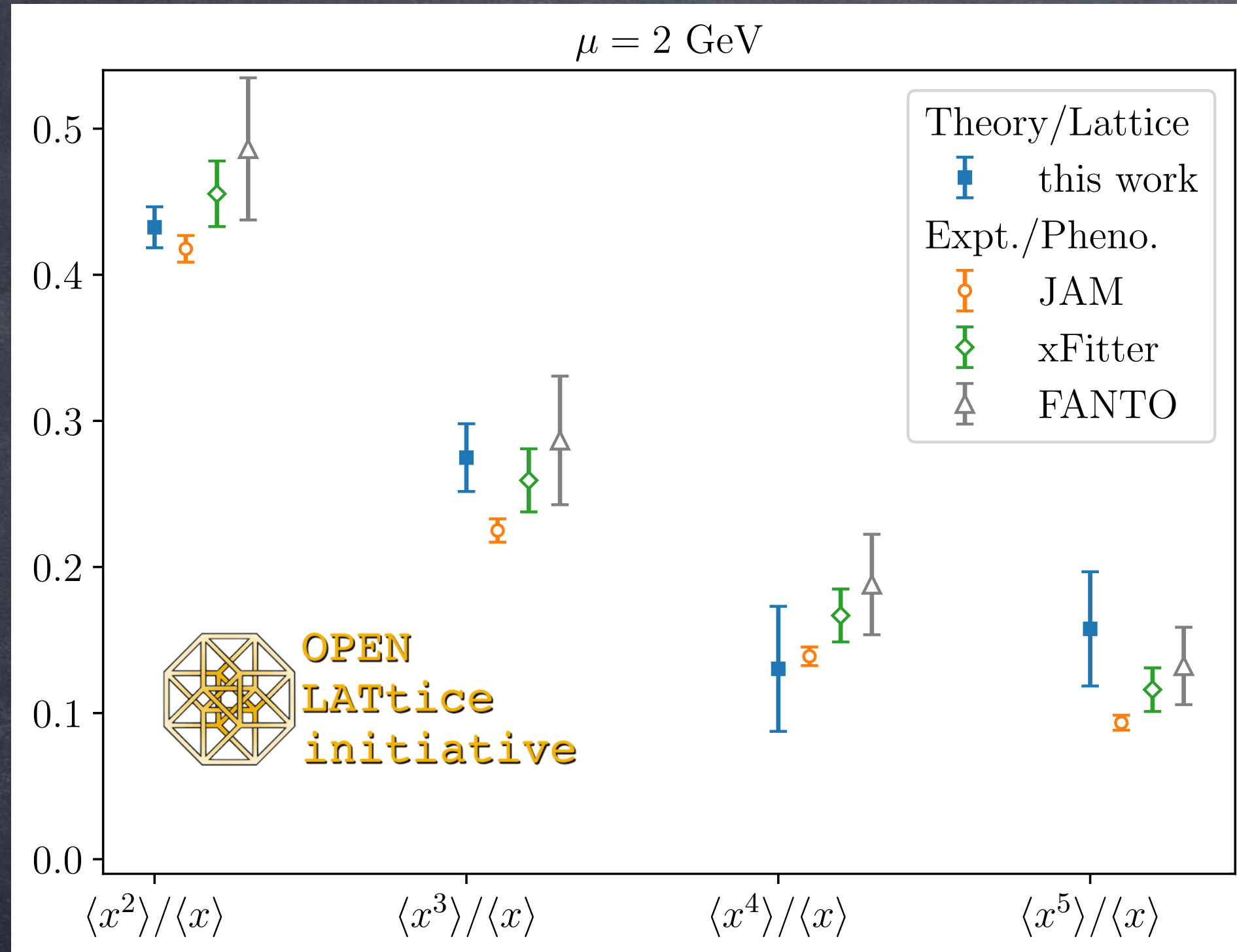
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Comparison with experiment

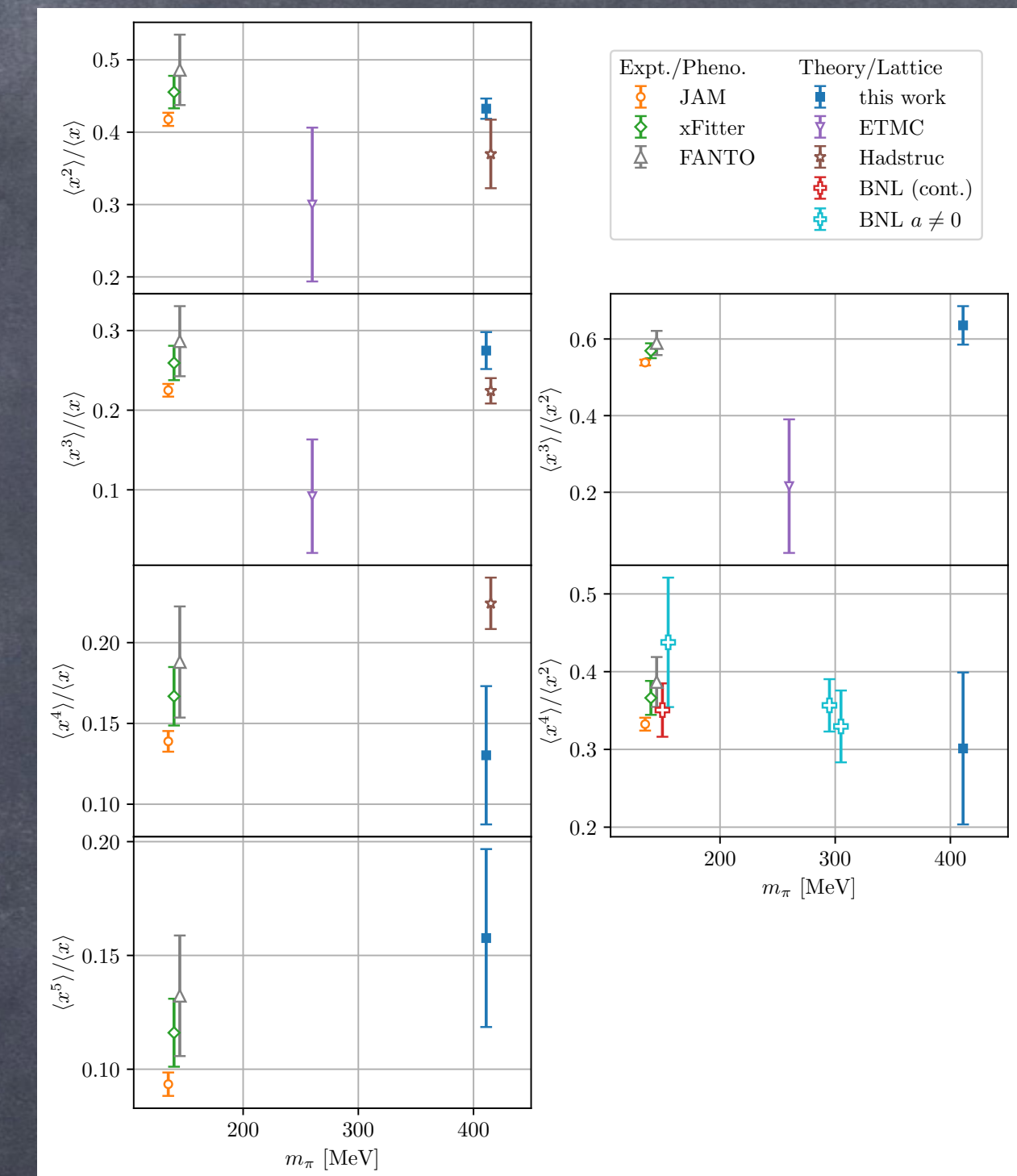
Comparison with exp/pheno



Phys.Rev.Lett. 136
(2026) 17, 171903
Phys.Rev.D 113
(2026) 7, 074520

JAM: 2108.05822
xFitter: 2002.02902
FANTO: 2505.13594

Comparison with lattice calculations

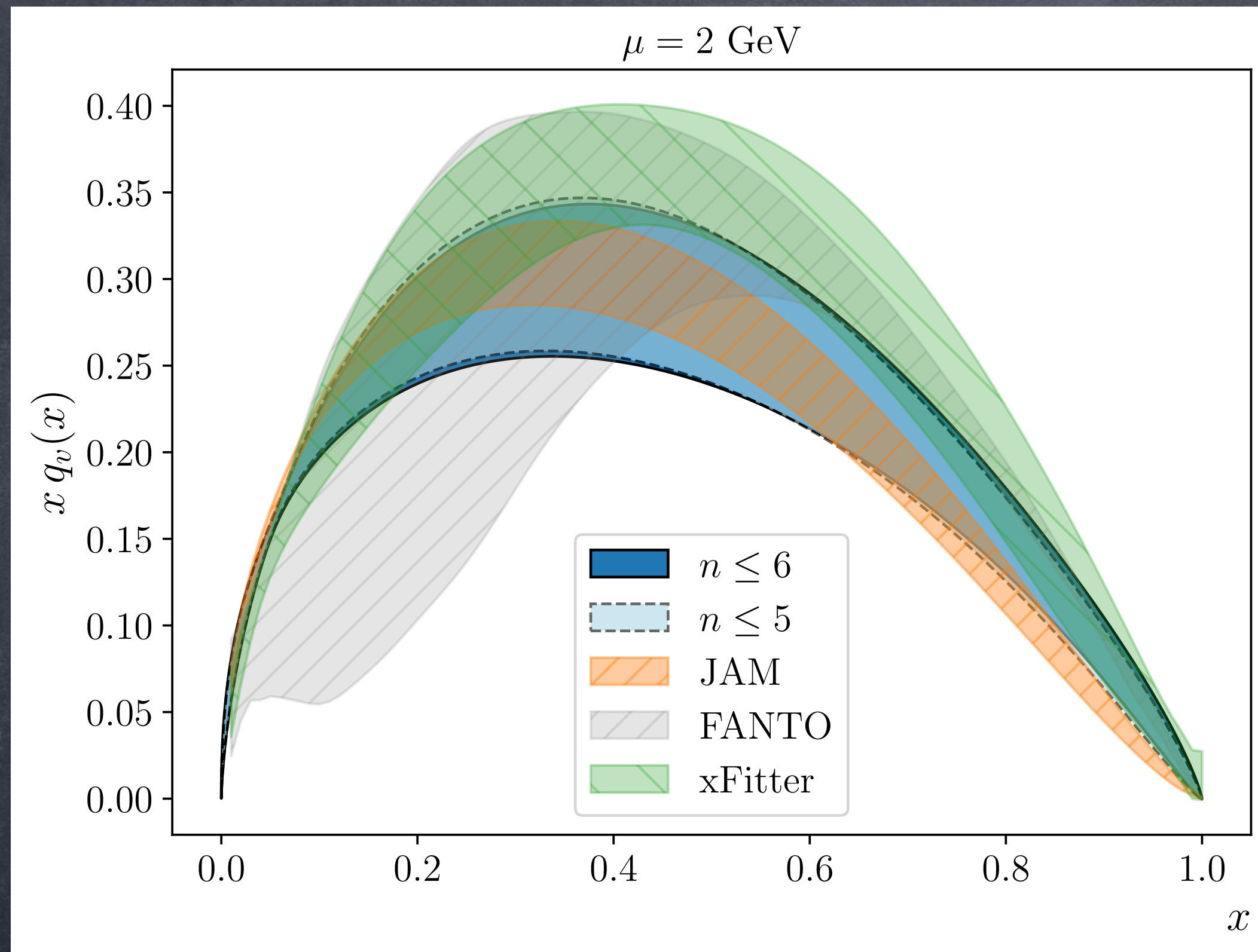


Joó et al. Phys.Rev.D 100 (2019) 11, 114512
Gao et al. Phys.Rev.D 106 (2022) 11, 114510

ETMC: Alexandrou et al.
Phys.Rev.D 103 (2021) 1, 014508
Phys.Rev.D 104 (2021) 5, 054504

Even with limited statistics, our results match global fits and reach competitive or leading precision for many lattice moments

PDF reconstruction



$$q_v(x) = \mathcal{N} x^\alpha (1-x)^\beta p(x)$$

Detmold, Melnitchouk, Thomas
Eur. Phys. J. direct 3, 13 (2001)

- Non-trivial use of priors to stabilize fit
- Small- x exponent α hard to constrain
- Large x parameter is robust ($\beta \sim 1$)
- Gaussian Processes reconstruction underway (UC Berkeley PhD student Rohith Karur)



Phys.Rev.Lett. 136 (2026) 17, 171903
Phys.Rev.D 113 (2026) 7, 074520

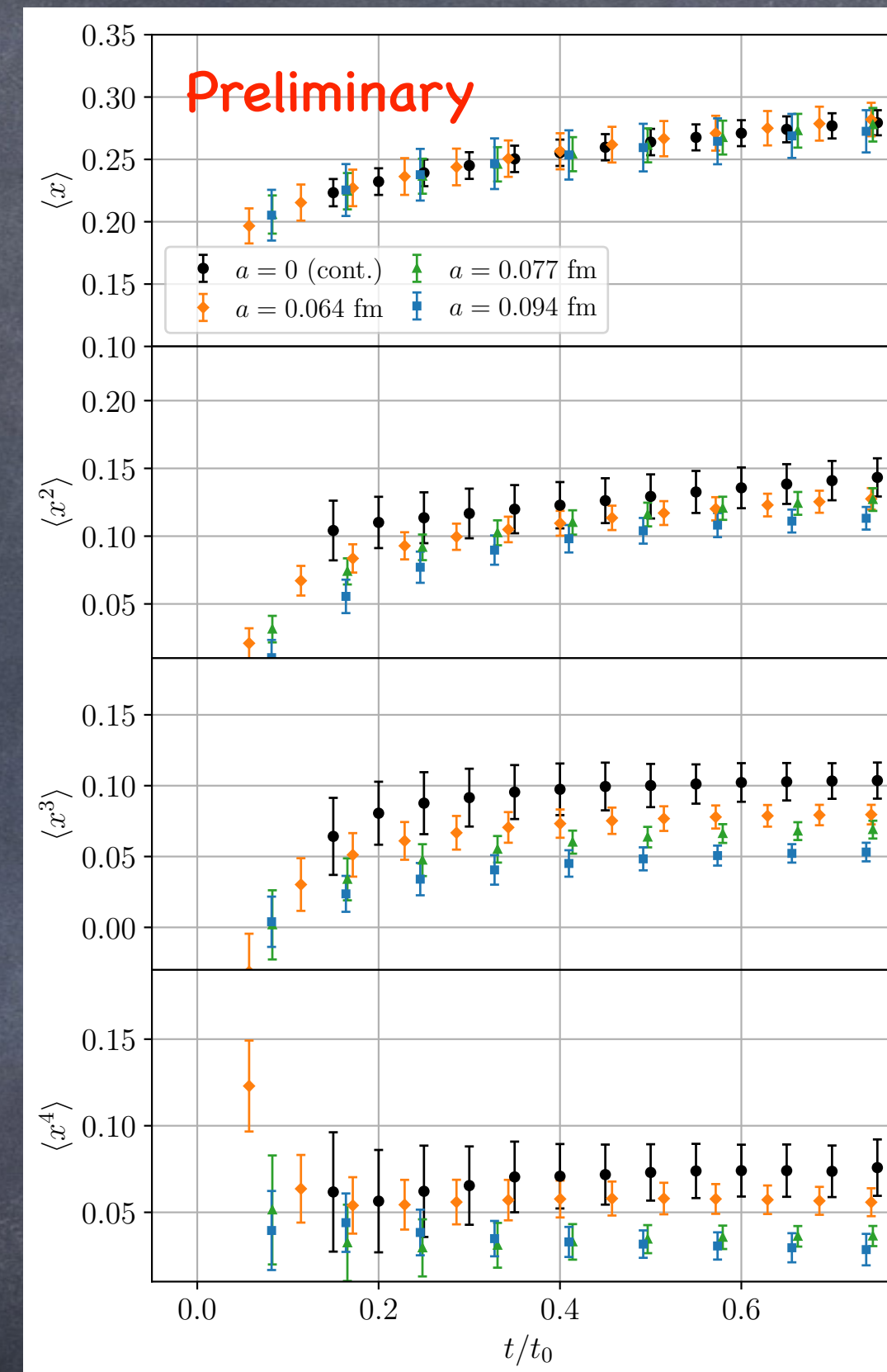
Reconstruction is consistent with global fits and already competitive in precision

Nucleon valence moments (u-d)

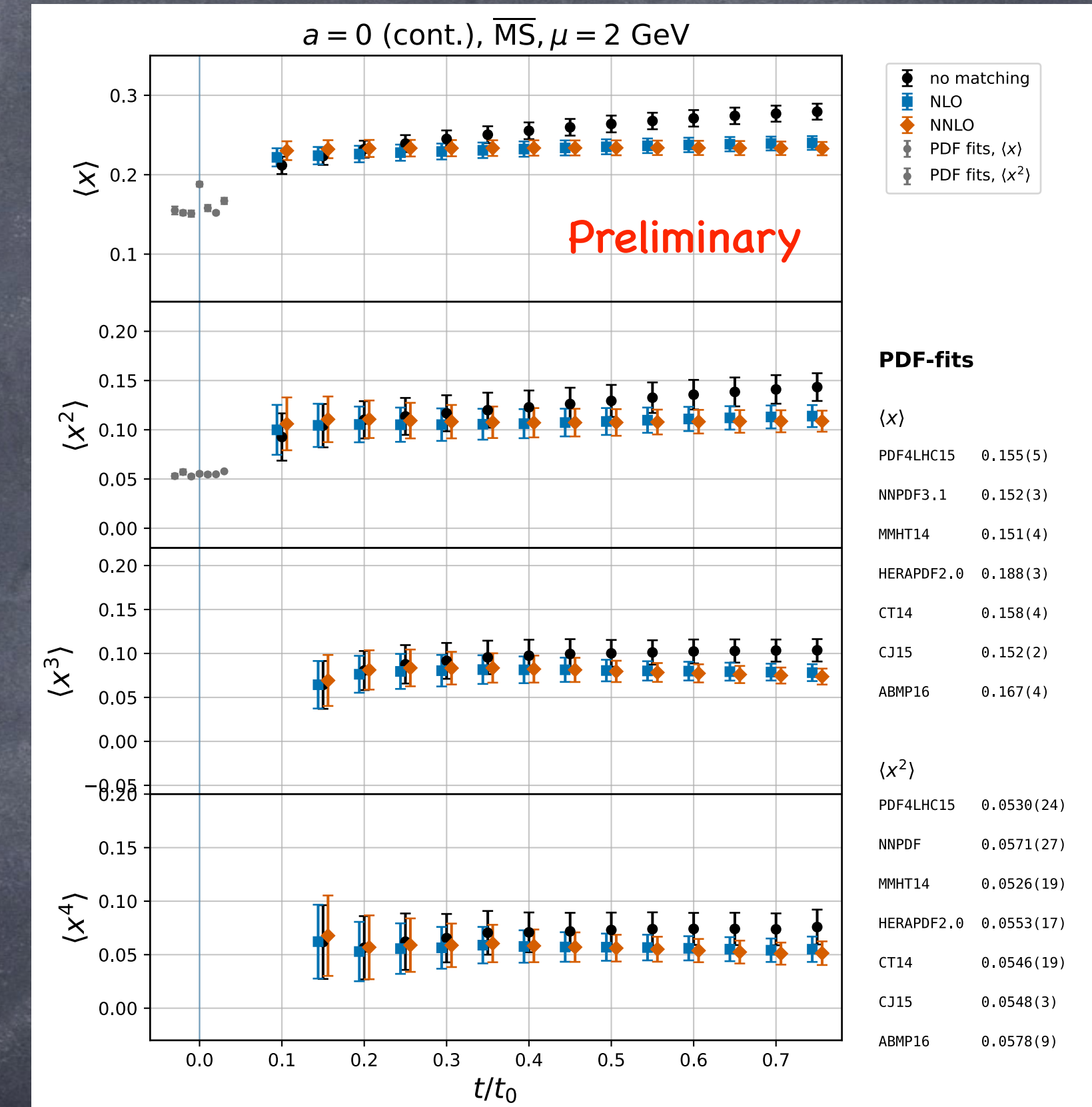
Kim, Pefkou, A.Sh.,
(OpenLat)

- Away from SU(3)-flavor symmetric point $m_\pi = 300$ MeV
- Stabilized Wilson fermions + Lüscher-Weisz gauge action
- 3 lattice spacings ($0.094 \rightarrow 0.064$ fm)
- 100–200 configurations per ensemble
- 10 sources
- Source-Sink $\rightarrow 1.5$ fm
- State-of-the-art statistics (10^5)
reach global-fit errors at $m_\pi \approx 300$ MeV
- Preserve this precision and keep matching systematics at the same level
- From gA:
physical-point precision may require only $\sim 2-4$ more statistics,
not orders of magnitude

Flowed moments



$t \rightarrow 0$ matching (NLO/NNLO)



Nucleon valence moments (u-d)

Kim, Pefkou, A.Sh.,
(OpenLat)

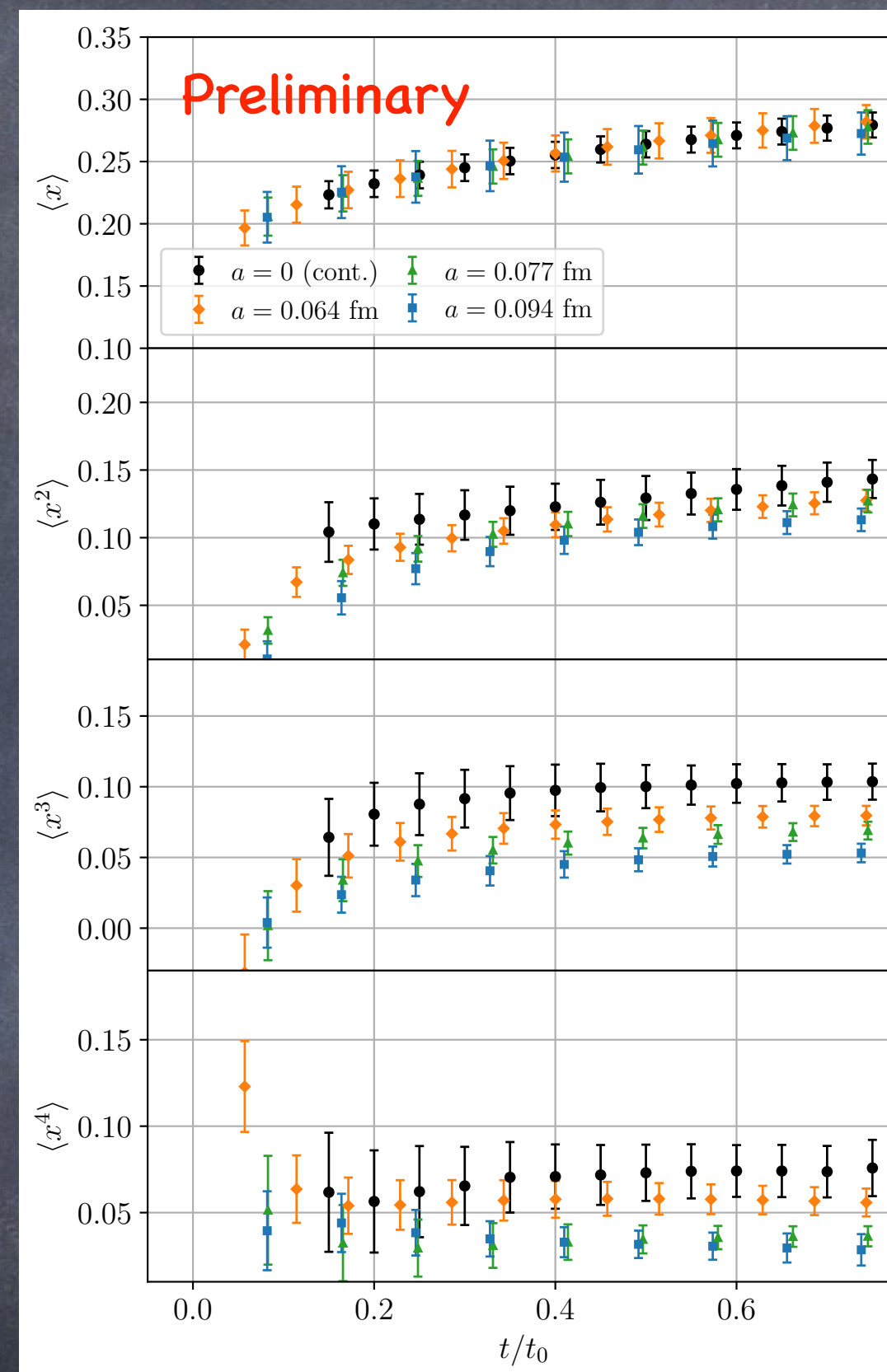
Community White Paper
Prog.Part.Nucl.Phys. 100 (2018) 107-160

Source	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$	$\langle x^4 \rangle$
This work, NNLO, $t/t_0 = 0.3$, errors / 10	0.44%	1.57%	2.23%	3.46%
PDF4LHC15	3.23%	4.53%	-	-
NNPDF / NNPDF3.1	1.97%	4.73%	-	-
MMHT14	2.65%	3.61%	-	-
HERAPDF2.0	1.60%	3.07%	-	-
CT14	2.53%	3.48%	-	-
CJ15	1.32%	0.55%	-	-
ABMP16	2.40%	1.56%	-	-

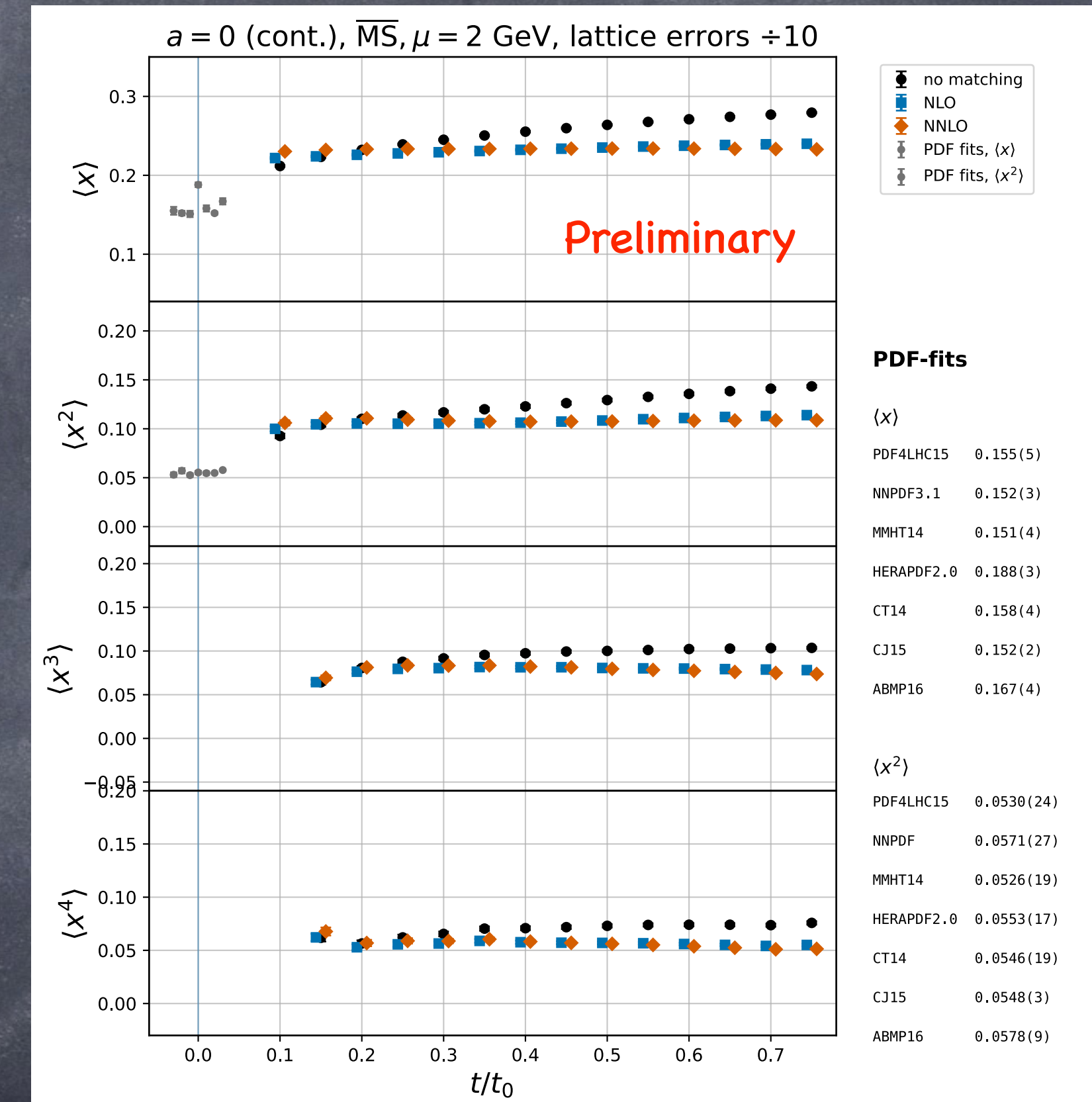
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Hall et al., Phys. Rev. C 113 (2026) 055206
Gupta et al., Phys. Rev. D 98 (2018) 034503
Djukanovic et al., Phys. Rev. D 109 (2024) 094511

Flowed moments



$t \rightarrow 0$ matching (NLO/NNLO)

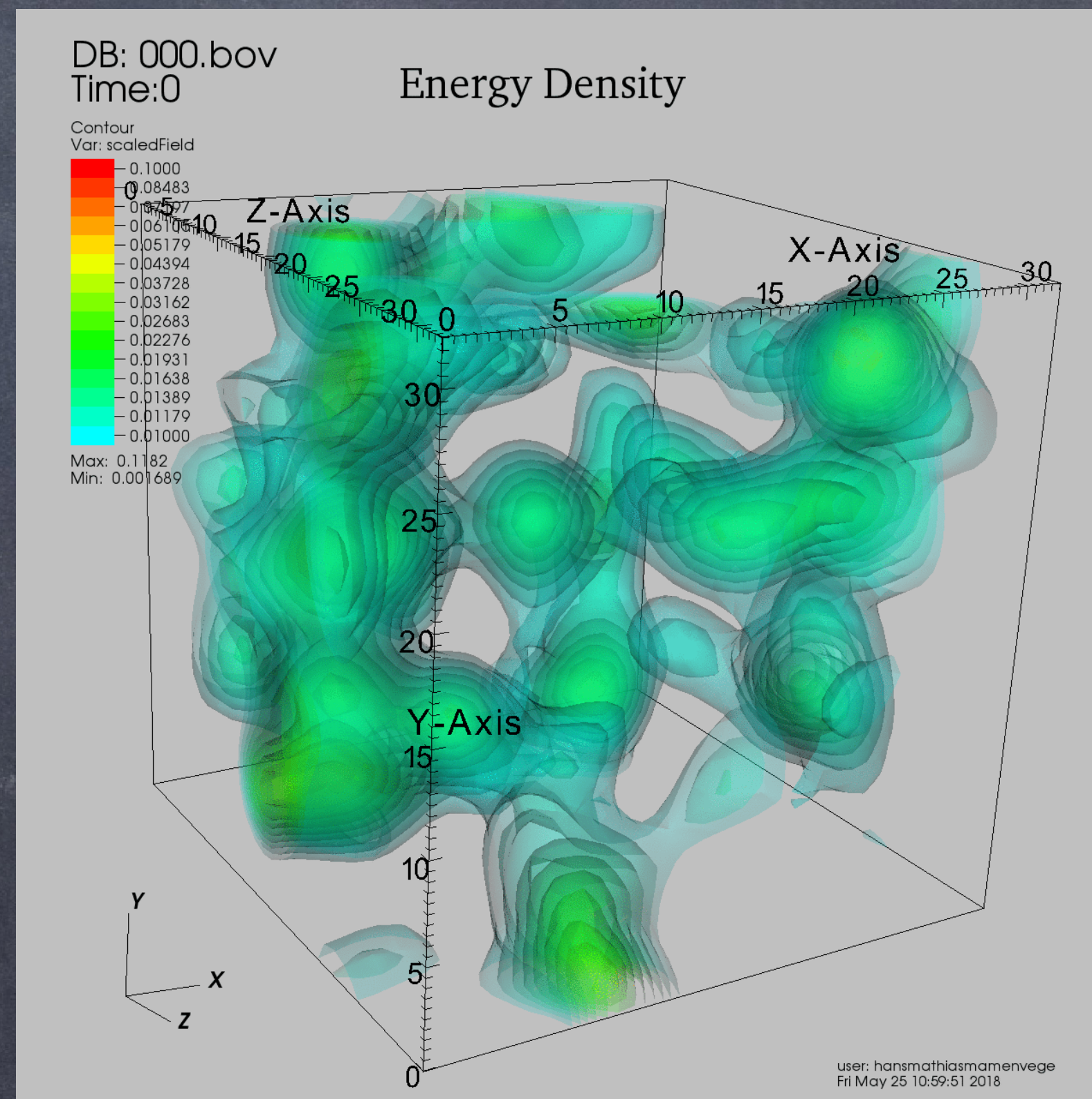


10^5 measurements bring lattice errors to global-fit precision

Summary

- **First direct QCD extraction** of flavor non-singlet pion and nucleon PDF moment ratios using GF
- Resolves the **power-divergence problem** for higher moments
- General method to compute PDF moments of any order, valid for any lattice action
- **Gradient-flow regulator**, simplifies the **continuum limit** and restores $O(4)$ before **perturbative matching**
- **Zero-momentum ratios** further improve S/N and control continuum extrapolation
- Moments up to $\langle x^5 \rangle$ for pion show **very good agreement** with phenomenology even with modest statistics
- Moments up to $\langle x^4 \rangle$ for nucleon with moderate numerical effort
- Could provide **high-precision inputs** valuable for global PDF analyses
- **Future extensions:** physical-pion, singlet+gluon, helicity/transversity, GPD form factors, higher twist

Thanks!



Backup slides

Parton Distribution Functions

PDFs are central inputs to many of the most important precision predictions at current and future colliders

Higgs boson production

- PDF uncertainty remains one of the largest theory uncertainties in Higgs predictions

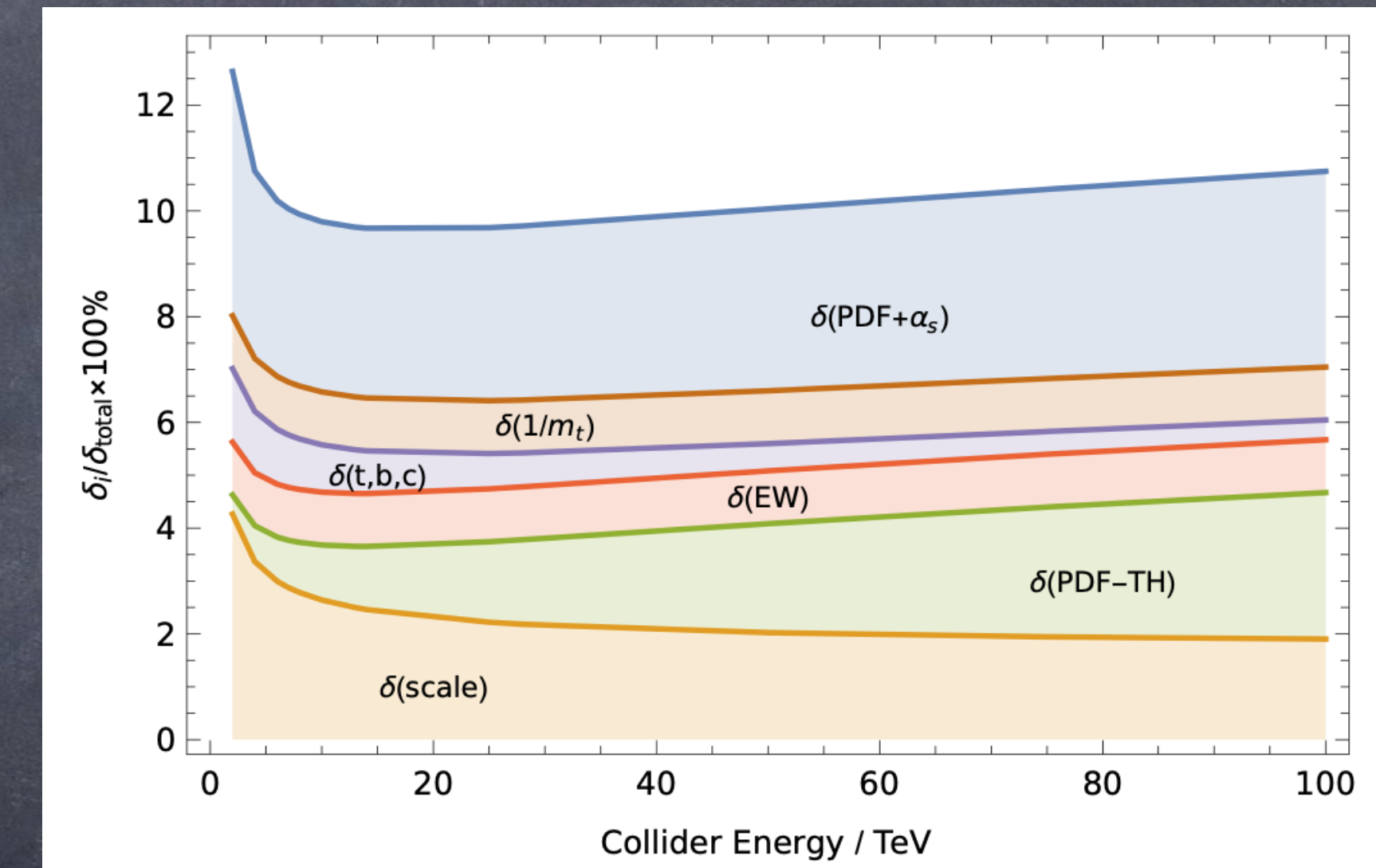
SM parameters

PDFs enter precision extractions of →

- strong coupling constant
- W boson mass
- ...

New physics searches

- At large x PDF uncertainties grow due to limited constraints
- This affects predictions for the high-energy tails → indirect BSM searches



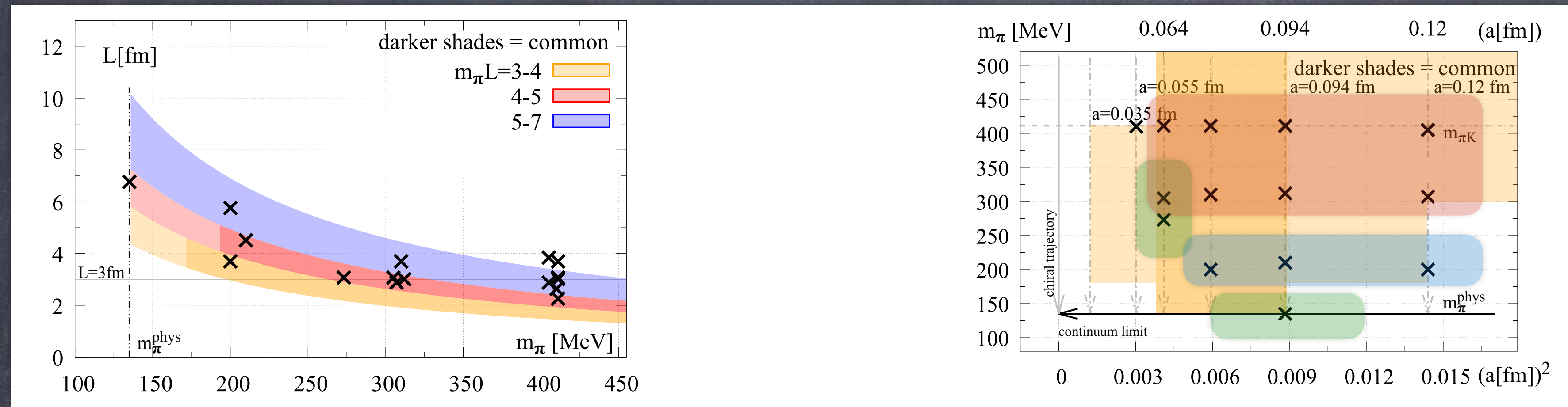
LHC Higgs Cross section Working Group (2016)
Snowmass 2021 White paper

Ball, Candido, Forte, Hekhorn, Nocera, Rojo, Schwan (2022)

OpenLat: Current Status & Key Research Projects

OpenLat has made significant progress in producing and sharing high-quality lattice QCD ensembles.

Our two main research projects focus on precision studies of the thetaEDM parameter and advancements in lattice PDF calculations.



Perlmutter (NERSC)



JUWELS (FZJ)



Irene Joliot-Curie

PRACE
EuroHPC
Gauss

Renormalization flowed fermions

$$C_{PP}(x_4, t) = -\frac{G_\pi(t) G_\pi}{2m_\pi} e^{-m_\pi x_4}$$

$$C_{AP}(x_4, t) = -\frac{f_\pi(t) G_\pi}{2} e^{-m_\pi x_4}$$

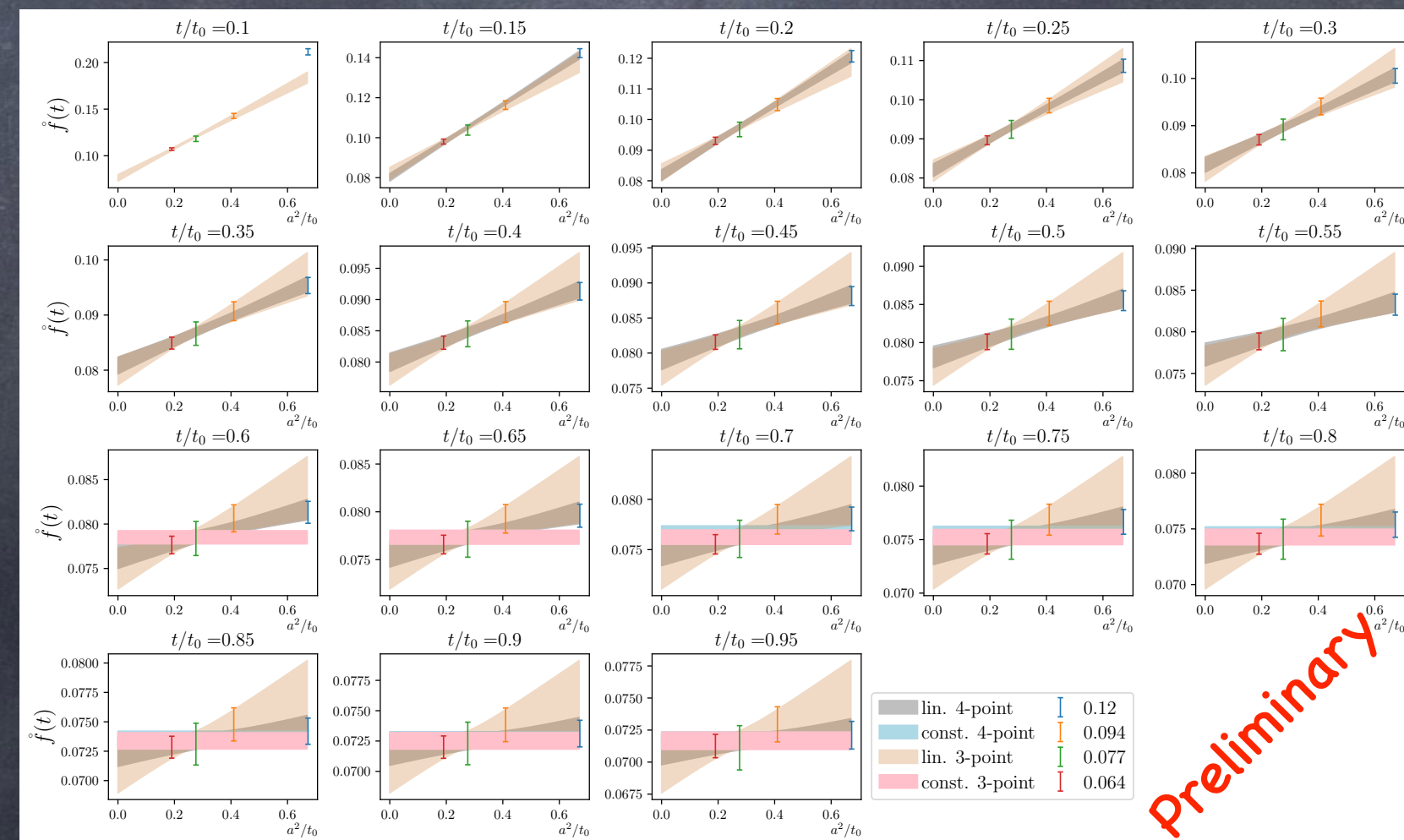
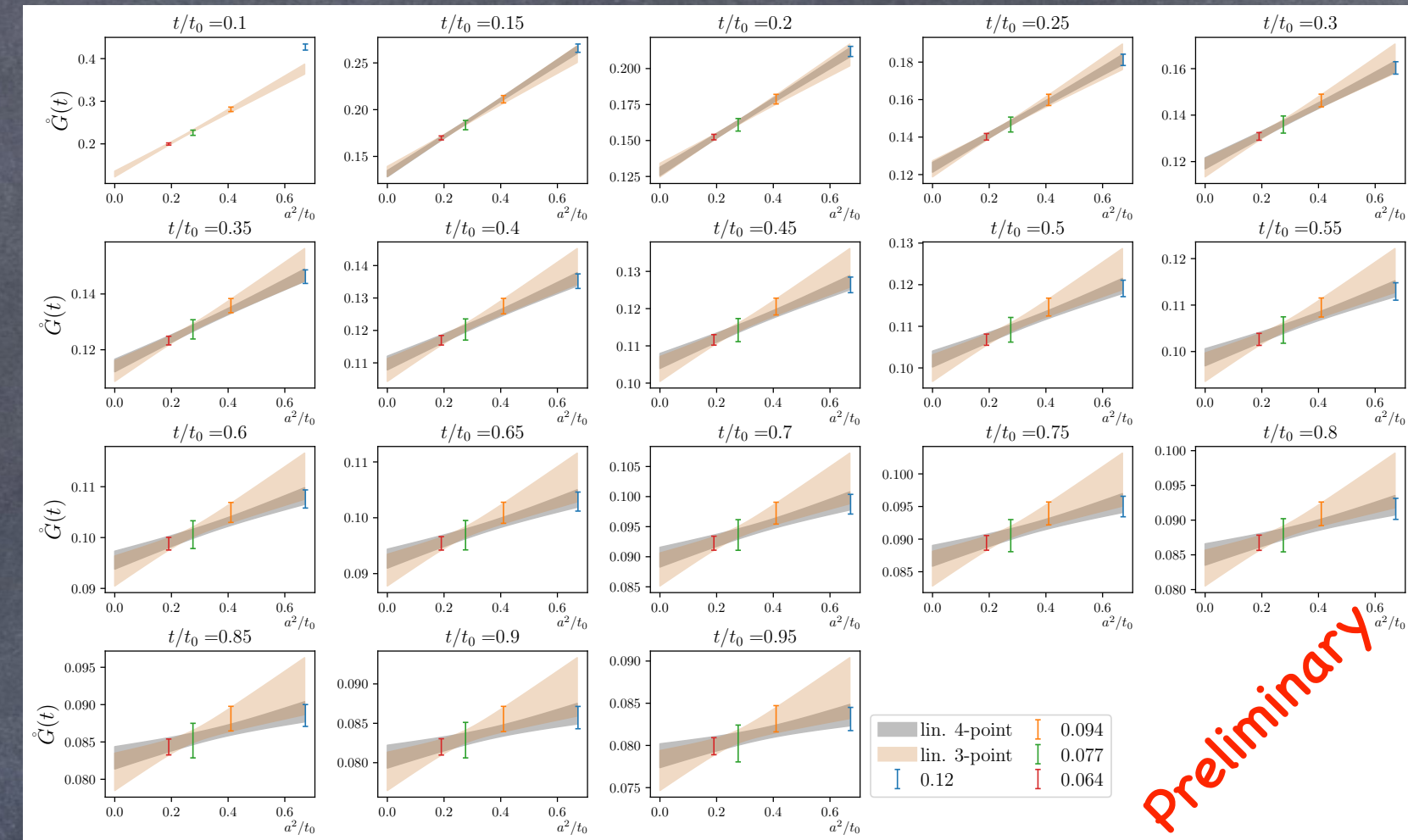
$$\dot{Z}_\chi = -\frac{N_c}{(4\pi t)^2} \left[\left\langle \bar{\chi}(t) \overleftrightarrow{D} \chi(t) \right\rangle \right]^{-1}$$

$$\dot{G}_\pi(t) = \lim_{a \rightarrow 0} \dot{Z}_\chi G_\pi(t)$$

$$\dot{f}_\pi(t) = \lim_{a \rightarrow 0} \dot{Z}_\chi f_\pi(t)$$

Ringed scheme

Makino, Suzuki: 2014



O(4) irreducible representations

GL(4) irrep $T_{\{\mu_1 \cdots \mu_n\}} = \frac{1}{n!} \sum_{\substack{\sigma \in \text{all} \\ \text{permutations}}} T_{\mu_{\sigma(1)} \cdots \mu_{\sigma(n)}}$

In O(4) an additional operation is allowed that commutes with orthogonal trafo: contraction of 2 indices

$$T_{\mu_1 \cdots \mu_n}^{(12)} = T_{\alpha\alpha\mu_3 \cdots \mu_n} = \delta_{\mu_1\mu_2} T_{\mu_1 \cdots \mu_n} \quad \text{rank } n-2 \text{ tensor}$$

Subspace of traceless tensors is invariant under O(4), i.e. the traceless rank n tensors are transformed among themselves under O(4)

Always possible to decompose $T_{\mu_1 \cdots \mu_n} = \hat{T}_{\mu_1 \cdots \mu_n} + \delta_{\mu_1\mu_2} T_{\mu_1 \cdots \mu_n}^{(12)} + \cdots$ Invariant under O(4)
 n(n-1)/2 terms

E.g. $\hat{T}_{\mu_1\mu_2} = T_{\mu_1\mu_2} - \frac{1}{4} \delta_{\mu_1\mu_2} T_{\alpha\alpha}$ $\hat{T}_{\mu_1\mu_2\mu_3} = T_{\mu_1\mu_2\mu_3} - \frac{1}{6} [\delta_{\mu_1\mu_2} T_{\alpha\alpha\mu_3} + \delta_{\mu_1\mu_3} T_{\alpha\mu_2\alpha} + \delta_{\mu_2\mu_3} T_{\mu_1\alpha\alpha}]$

Traceless tensors invariant under vector index permutations → starting point to construct all the irreducible representations of O(4) (Young symmetrizers)

Traceless and symmetrized rank-n tensors are an irreducible representation of O(4)

4+1 Local field theory

$$S = S_G + S_{G,\text{fl}} + S_{F,\text{QCD}} + S_{F,\text{fl}}$$

$$S_{F,\text{fl}} = \int_0^\infty dt \int d^4x \left[\bar{\lambda}(t, x) (\partial_t - \Delta) \chi(t, x) + \bar{\chi}(t, x) \left(\overleftarrow{\partial}_t - \overleftarrow{\Delta} \right) \lambda(t, x) \right]$$

- Wick contractions
- Renormalization. All order proof
- Chiral symmetry and Ward identities
- Wilson twisted mass

Lüscher, Weisz: 2013
Hieda, Makino, Suzuki: 2017

Lüscher: 2013
A.S.: 2013

A.S.: 2013

O(a) improvement

A.Sh.
Phys.Rev.D 110 (2024) 5, L051503

$$O_n^{rs}(x) = O_{\mu_1 \dots \mu_n}^{rs}(x) = \bar{\psi}^r(x) \gamma_{\{\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n\}} \psi^s(x) \quad \left\langle h(p) | \hat{O}_n | h(p) \right\rangle = 2 p_{\mu_1} \cdots p_{\mu_n} \left\langle x^{n-1} \right\rangle_h$$

- Beside the O(a) from the lattice theory twist-2 fields are affected by specific O(a) that depend on n
- Improvement coefficients are known only for n=2 and only in PT

$$\hat{\hat{O}}_n^{rs}(x, t) = \hat{\hat{\chi}}^r(x, t) \gamma_{\{\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n\}} \hat{\hat{\chi}}^s(x, t) - \text{terms with } \delta_{\mu_i \mu_j}$$

$$\left\langle h(p) | \hat{\hat{O}}_n(t) | h(p) \right\rangle = 2 (p_{\mu_1} \cdots p_{\mu_n} - \text{trace terms}) \left\langle x^{n-1} \right\rangle_h(t)$$

- Hadronic matrix elements of flowed operators beside the O(a) from the lattice theory are only affected by O(am)
- Short distance O(a) negligible at large physical distances
- The O(am) are independent on n (depend only on the fermion content)
- With ratios discretization effects are O(a²) → clover fermions are back in the game

$$\frac{\left\langle x^{n-1} \right\rangle_h(t)}{\left\langle x^{m-1} \right\rangle_h(t)} \quad n \neq m$$



Finite continuum limit and O(a) improved

Flowed fermion fields

Lüscher JHEP 04 (2013) 123

$$\chi_R(x, t) = Z_\chi^{1/2} \chi(x, t)$$

$$\mathcal{O}(x, t) = \bar{\chi}(x, t) \Gamma(x, t) \chi(x, t)$$

$$\mathcal{O}_R = Z_\chi \mathcal{O}$$

$$\Sigma(t) = \langle \bar{\chi}(x, t) \chi(x, t) \rangle$$

$$\Sigma_R(t) = Z_\chi \Sigma(t)$$

No additive divergences

Continuum limit finite after normalizing fermion fields

“Ringed” fields

$$\left\langle \overset{\circ}{\bar{\chi}}^r(x, t) \overset{\leftrightarrow}{D} \overset{\circ}{\chi}^r(x, t) \right\rangle = -\frac{N_c}{(4\pi)^2 t^2}$$

Makino, Suzuki PTEP 2014 (2014) 063B02

$$\chi^{r, \text{MS}}(x, t) = (8\pi t)^{\epsilon/2} \zeta_\chi^{1/2} \overset{\circ}{\chi}^r(x, t)$$

$$\zeta_\chi = 1 - \frac{\bar{g}^2}{(4\pi)^2} C_F (3 \log(8\pi \mu^2 t) - \log(432))$$

$$D = 4 - 2\epsilon$$

$$\bar{\chi}^{r, \text{MS}}(x, t) = (8\pi t)^{\epsilon/2} \zeta_\chi^{1/2} \overset{\circ}{\bar{\chi}}_r(x, t)$$

Harlander, Kluth, Lange
Eur.Phys.J.C 78 (2018) 11, 944

NNLO

$$C_F = \frac{N_c^2 - 1}{2N_c}$$

Artz et al. JHEP 06 (2019) 121

$$\log \mu^2 = \log \bar{\mu}^2 + \gamma_E - \log 4\pi$$

Flowed moments

A.Sh.
Phys.Rev.D 110 (2024) 5, L051503

- UV finite at $t > 0$
- multiplicative renormalization
- no power divergences

$$\langle h(p) | \hat{O}_n(t) | h(p) \rangle = 2 (p_{\mu_1} \cdots p_{\mu_n} - \text{trace terms}) \langle x^{n-1} \rangle_h(t)$$



- flowed fermion operators renormalize universally
- ratios are finite for all n
- zero external momentum

$$\frac{\langle x^{n-1} \rangle_h(t)}{\langle x^{m-1} \rangle_h(t)} \quad n \neq m$$



- OPE-like matching
- multiplicative
- known to NNLO

$$\langle x^{n-1} \rangle_h^{\overline{\text{MS}}}(\mu) = \langle x \rangle_h^{\overline{\text{MS}}}(\mu) \frac{\zeta_2(t, \mu)}{\zeta_n(t, \mu)} \frac{\langle x^{n-1} \rangle_h(t)}{\langle x \rangle_h(t)}$$

Result: All Mellin moments computable with finite continuum limit and no power divergences