NANJING UNIVERSITY

Transverse-Momentum Structure of Light Pseudoscalar Mesons in Continuum Schwinger Function Methods

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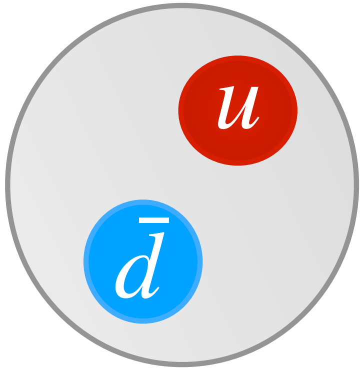
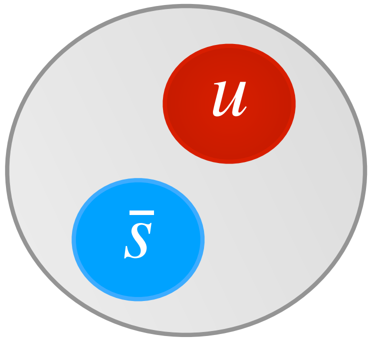
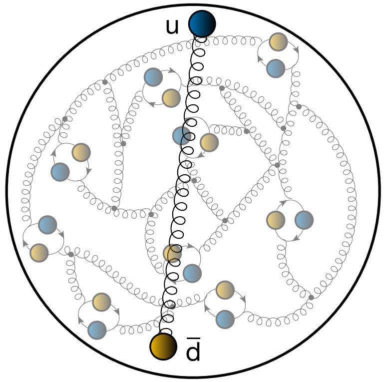
The pion and kaon

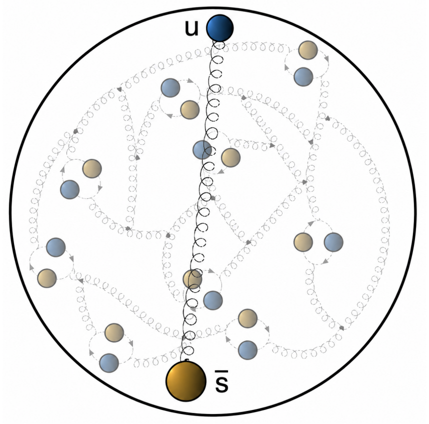
➢ Naive picture: quark + antiquark:





➢ Realistic picture: valence quark + antiquark + sea quarks + gluons





Nambu-Goldstone boson

➢ Pion does not fit naturally into the mass pattern of constituent quark model:

Maris, Roberts, Tandy, Phys. Lett. B 420 (1998) 267-273

$$f_\pi m_\pi^2 = 2 m_u \rho_\pi$$

with f_π, leptonic decay constant; m_π, pion mass; m_u, current quark mass; ρ_π, pseudoscalar projection of the pion wave function onto the origin in configuration space.

- ✓ Gell-Mann-Oakes-Renner (GMOR) relation
- ✓ Pion mass m_π vanishes in the absence of current quark mass m_u- Nambu-Goldstone boson of chiral symmetry breaking (when current quark mass is zero, QCD Lagrangian possesses a chiral symmetry)
- ✓ Mass square of pion rises linearly with the current quark mass, $m_\pi^2 \propto m_u$, whereas in constituent quark model $m_{\text{meson}} \propto m_{\text{quark}}$.

Hadron structure

➢ Form factor: the closest thing we have to a snapshot, the size, shape and makeup of the hadron

✓ Elastic electromagnetic form factor

✓ Two-photon transition form factor

✓ Gravitational form factor

➢ 1D picture of how quarks move within the hadron

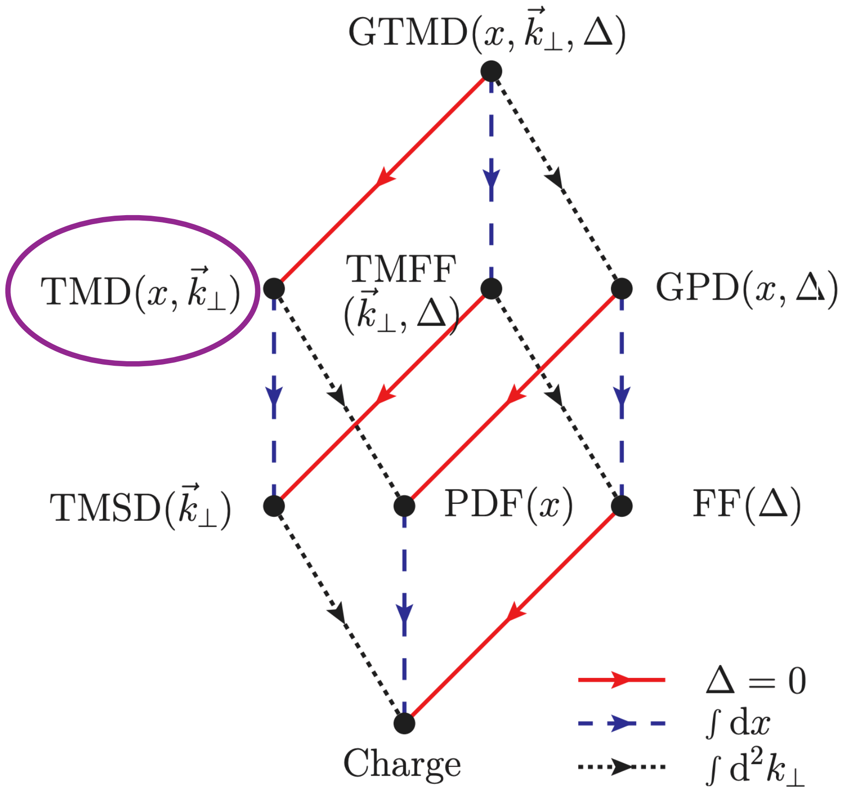
✓ Parton distribution amplitude (PDA)

✓ Parton distribution function (PDF)

➢ A multidimensional view of the hadron structure

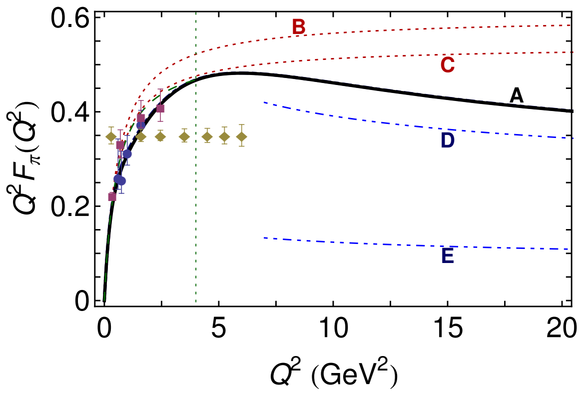
✓ **Transverse momentum dependent distribution function (TMD)**

✓ Generalized parton distribution (GPD)



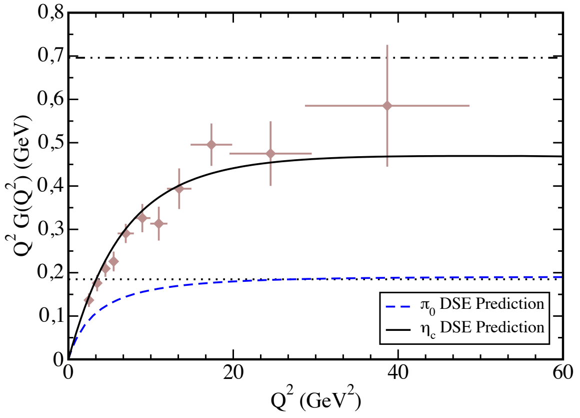
Lorce, Pasquini, Vanderhaeghen, JHEP 05 (2011) 041.

Pion structure



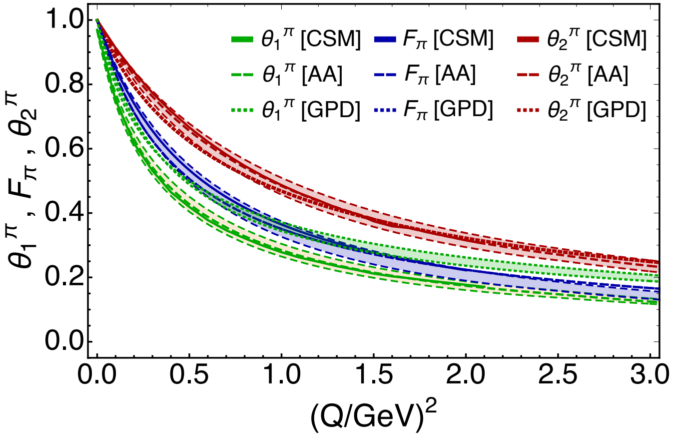
✓ Elastic electromagnetic form factor

Phys. Rev. Lett. 111 (2013) 14, 141802



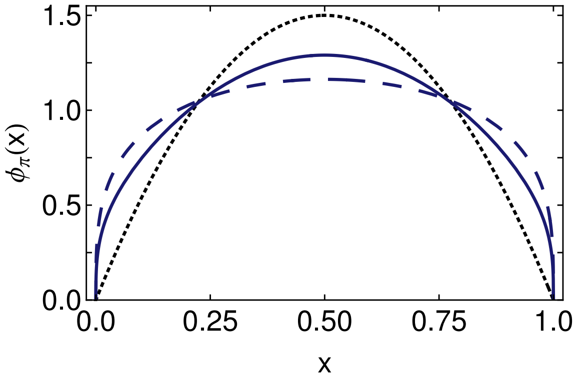
✓ Two-photon transition form factor

Phys. Rev. D 95 (2017) 7, 074014



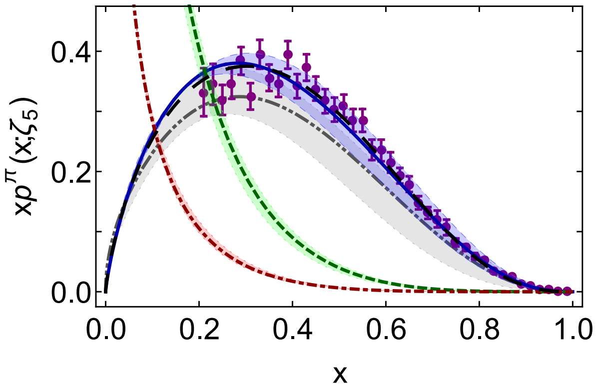
✓ Gravitational form factor

Eur. Phys. J. C 84 (2024) 2, 191



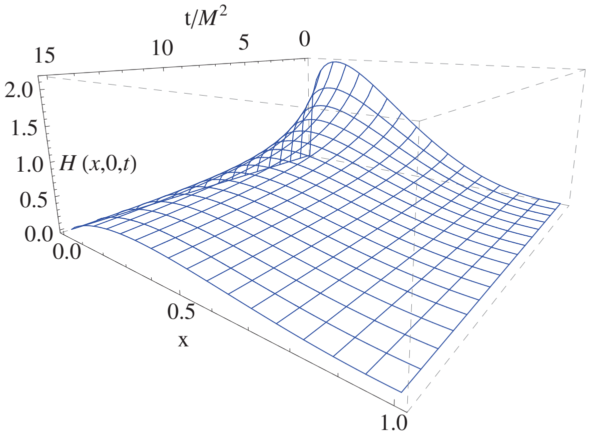
✓ Parton distribution amplitude (PDA)

Phys. Rev. Lett. 110 (2013) 13, 132001



✓ Parton distribution function (PDF)

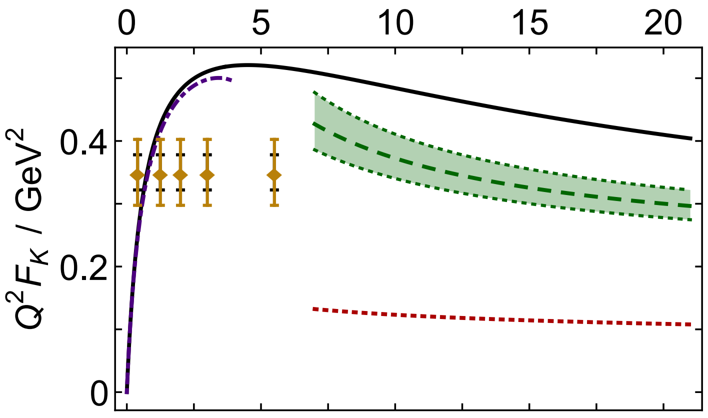
Phys. Rev. D 101 (2020) 5, 054014



✓ Generalized parton distribution (GPD)

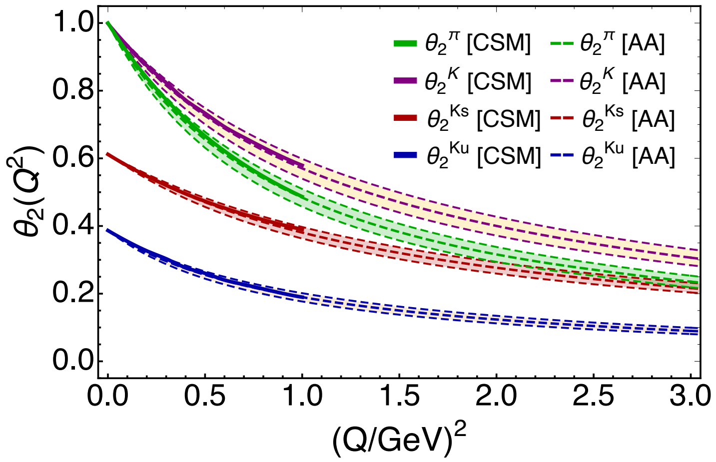
Phys. Lett. B 741 (2015) 190-196

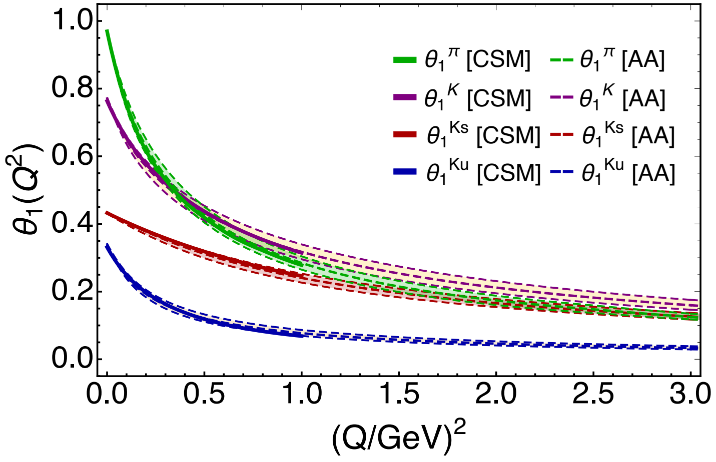
Kaon structure



✓ Elastic electromagnetic form factor

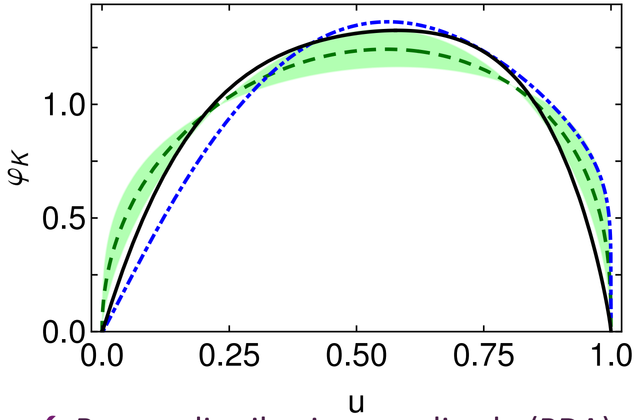
Phys. Rev. D 96 (2017) 3, 034024





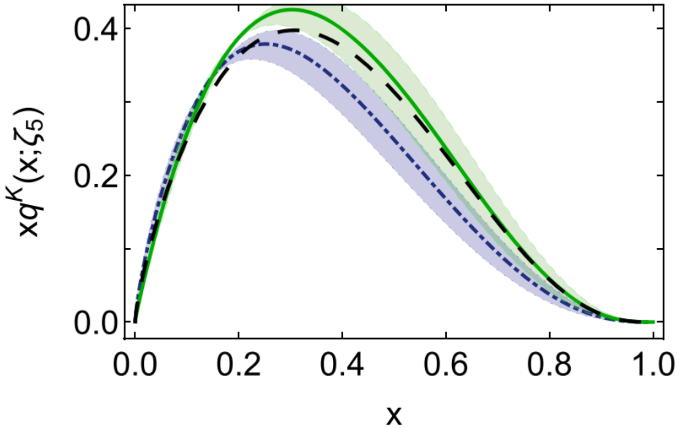
✓ Gravitational form factors

Eur. Phys. J. C 84 (2024) 2, 191



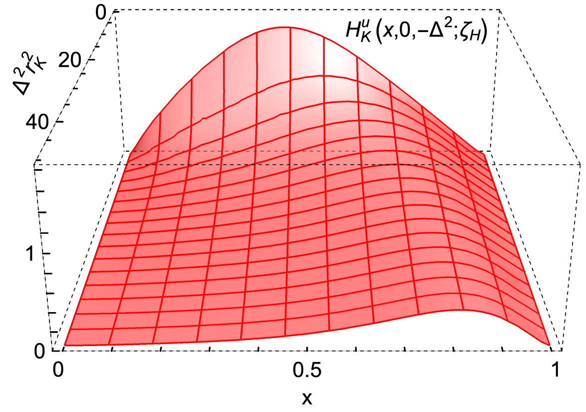
✓ Parton distribution amplitude (PDA)

Phys. Lett. B 738 (2014) 512-518



✓ Parton distribution function (PDF)

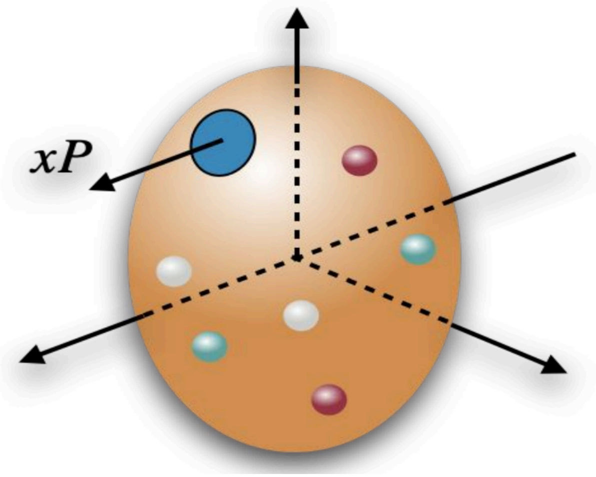
Eur. Phys. J. C 80 (2020) 11, 1064



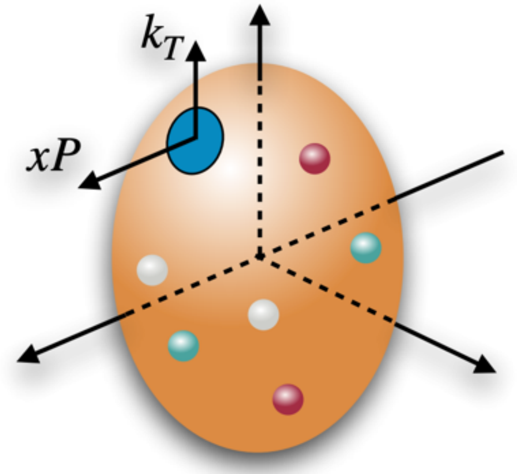
✓ Generalized parton distribution (GPD)

Chin. Phys. C 46 (2022) 1, 013105

Intrinsic longitudinal/transverse momentum of quarks/gluons in hadrons



- Parton Distribution functions (PDFs) $q(x)$:
  - The probability density for finding a particle with a certain *longitudinal momentum* fraction $x = k^+/P^+$ at resolution scale.



- Transverse momentum dependent distribution functions (TMDs) $f(x, k_\perp)$:
  - It gives the joint probability density to find a parton inside a fast-moving hadron carrying longitudinal momentum fraction x and *transverse momentum* $k_\perp$ (defined perpendicular to the hadron's light-cone direction).

Intrinsic transverse momentum of quarks/gluons in hadrons

➢ 49 years ago

Correlations among particles and jets produced with large transverse momenta,
R.P. Feynman, R.D. Field, G.C. Fox, Nucl. Phys. B128, 1, 1977.

– Drell-Yan process: $A + B \to h_1 + h_2 + X$

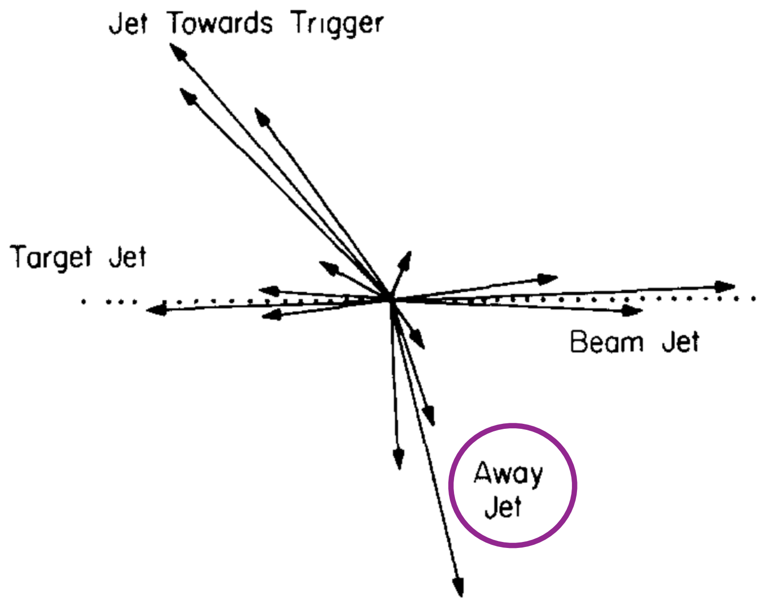
– Abstract: "…If we allow the quarks within the initial hadrons to have <u>*a large mean transverse momentum*</u>, $\langle k_\perp \rangle_{h\to q}$, then good agreement with data is found…".

– An assumption: the transverse momentum distribution is independent of x, $f(x, k_\perp) = q(x)\mathscr{F}(k_\perp)$.

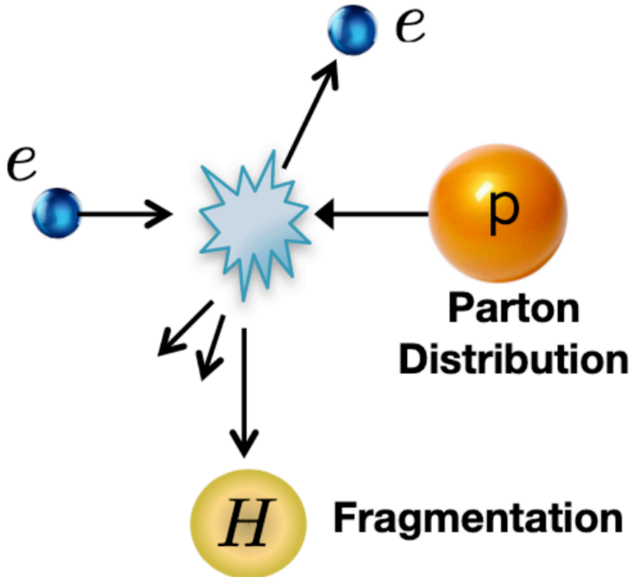
– An exponential form was chosen $\mathscr{F}(k_\perp) = b^2 \exp(-b|k_\perp|)/2\pi$.

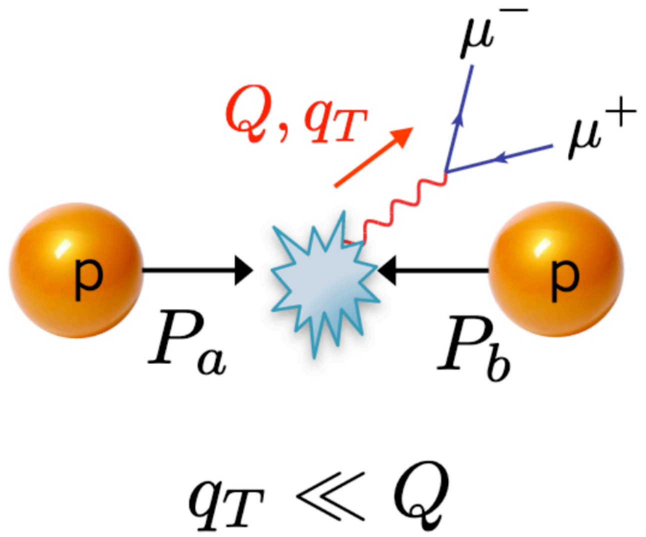
– The mean momentum $\langle k_\perp \rangle = \int dk_\perp k_\perp \mathscr{F}(x, k_\perp) = 2/b.$

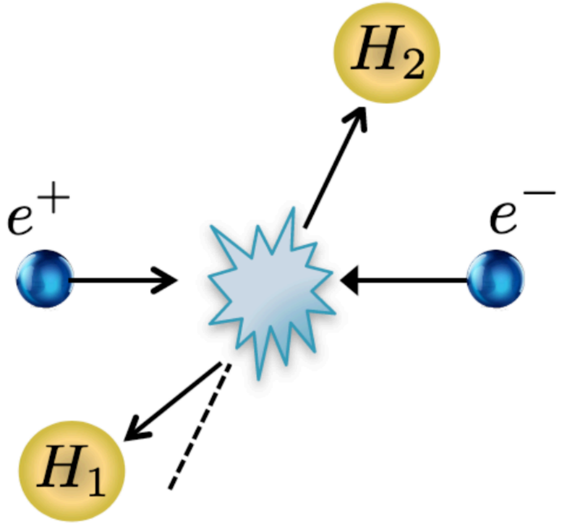
– Typical choices are $\langle k_\perp \rangle = 300$ MeV and $\langle k_\perp \rangle = 500$ MeV.



Intrinsic transverse momentum of quarks/gluons in hadrons





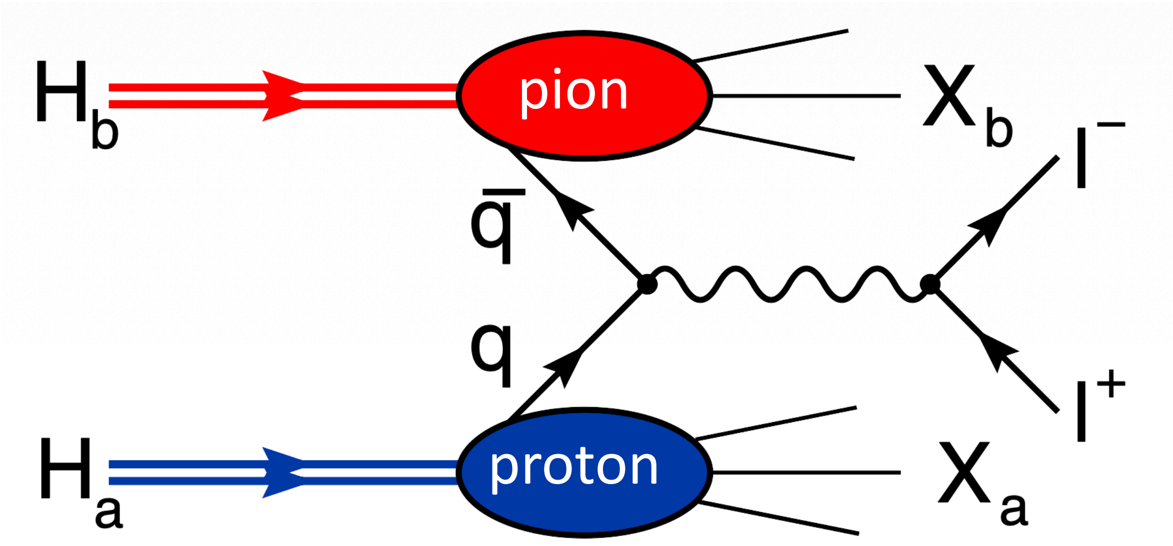


- ➢ Semi-Inclusive Deep Inelastic Scattering (SIDIS)
- ➢ Drell-Yan Process
- ➢ Dihadron production in e^+e^-

– The *transverse momentum of the observed hadron or lepton pair is measured* in the $q_T \ll Q$ region, where it cannot arise from hard scattering alone and is instead dominated by intrinsic parton transverse momentum and soft radiation.

Drell-Yan process with pions and polarized nucleons $\pi^- + p^\uparrow \to \mu^+\mu^- + X$

➢ DY cross section with pions and **polarized proton** at leading twist is described in terms of *six structure functions* Bastami, Gamberg, Parsamyan, Pasquini, Prokudinb, and Schweitzer, JHEP 02, (2021), 166.



$$f_{\bar{u}|\pi} \otimes f_{u|p}$$

pion proton

$$F_{UU}^{1} \sim f_{1,\pi} \otimes f_{1,p}$$

$$F_{UU}^{\cos 2\phi} \sim h_{1,\pi}^{\perp} \otimes h_{1,p}^{\perp}$$

$$F_{UL}^{\sin 2\phi} \sim h_{1,\pi}^{\perp} \otimes h_{1L,p}^{\perp}$$

$$F_{UT}^{\sin \phi_S} \sim f_{1,\pi} \otimes f_{1T,p}^{\perp}$$

$$F_{UT}^{\sin(2\phi-\phi_S)} \sim h_{1,\pi}^{\perp} \otimes h_{1,p}$$

$$F_{UT}^{\sin(2\phi+\phi_S)} \sim h_{1,\pi}^{\perp} \otimes h_{1T,p}^{\perp}$$

Drell-Yan process with pions and polarized nucleons $\pi^- + p^\uparrow \to \mu^+\mu^- + X$

➢ DY cross section with pions and **polarized proton** at leading twist is described in terms of <u>*six structure functions*</u>

pion		proton		
unpolarized TMD	$\otimes$	unpolarized TMD	F_{UU}^{1}	$\sim f_{1,\pi} \otimes f_{1,p}$
Boer–Mulders function	$\otimes$	Boer–Mulders function	$F_{UU}^{\cos 2\phi}$	$\sim h_{1,\pi}^{\perp} \otimes h_{1,p}^{\perp}$
Boer–Mulders function	$\otimes$	worm-gear L	$F_{UL}^{\sin 2\phi}$	$\sim h_{1,\pi}^{\perp} \otimes h_{1L,p}^{\perp}$
unpolarized TMD	$\otimes$	Sivers function	$F_{UT}^{\sin \phi_S}$	$\sim f_{1,\pi} \otimes f_{1T,p}^{\perp}$
Boer–Mulders function	$\otimes$	transversity	$F_{UT}^{\sin(2\phi-\phi_S)}$	$\sim h_{1,\pi}^{\perp} \otimes h_{1,p}$
Boer–Mulders function	$\otimes$	pretzelosity	$F_{UT}^{\sin(2\phi+\phi_S)}$	$\sim h_{1,\pi}^{\perp} \otimes h_{1T,p}^{\perp}$

Drell-Yan process with pions and polarized nucleons $\pi^- + p^\uparrow \to \mu^+\mu^- + X$

➢ DY cross section with pions and **unpolarized proton** at leading twist

pion		proton
unpolarized TMD	$\otimes$	unpolarized TMD
Boer–Mulders function	$\otimes$	Boer–Mulders function

$$F_{UU}^{1} \sim f_{1,\pi} \otimes f_{1,p}$$

$$F_{UU}^{\cos 2\phi} \sim h_{1,\pi}^{\perp} \otimes h_{1,p}^{\perp}$$

➢ Proton Boer-Mulders function in DY process $h_{1,p}^{\perp}$ is expected to be positive.

Peng, Liu, and Xu, Phys. Lett. B 876 (2026) 140415.

➢ Therefore, the sign of pion Boer-Mulders function in DY process $h_{1,\pi}^{\perp}$ can be determined by the sign of $F_{UU}^{\cos 2\phi}$.

Drell-Yan process with pions and polarized nucleons $\pi^- + p^\uparrow \to \mu^+\mu^- + X$

➢ DY cross section with pions and **unpolarized proton** at leading twist

pion		proton
unpolarized TMD	$\otimes$	unpolarized TMD
Boer–Mulders function	$\otimes$	Boer–Mulders function

$$F_{UU}^{1} \sim f_{1,\pi} \otimes f_{1,p}$$

$$F_{UU}^{\cos 2\phi} \sim h_{1,\pi}^{\perp} \otimes h_{1,p}^{\perp}$$

➢ Angular distributions (including sub-leading twist/power F_{UU}^{2} and $F_{UU}^{\cos\phi}$)

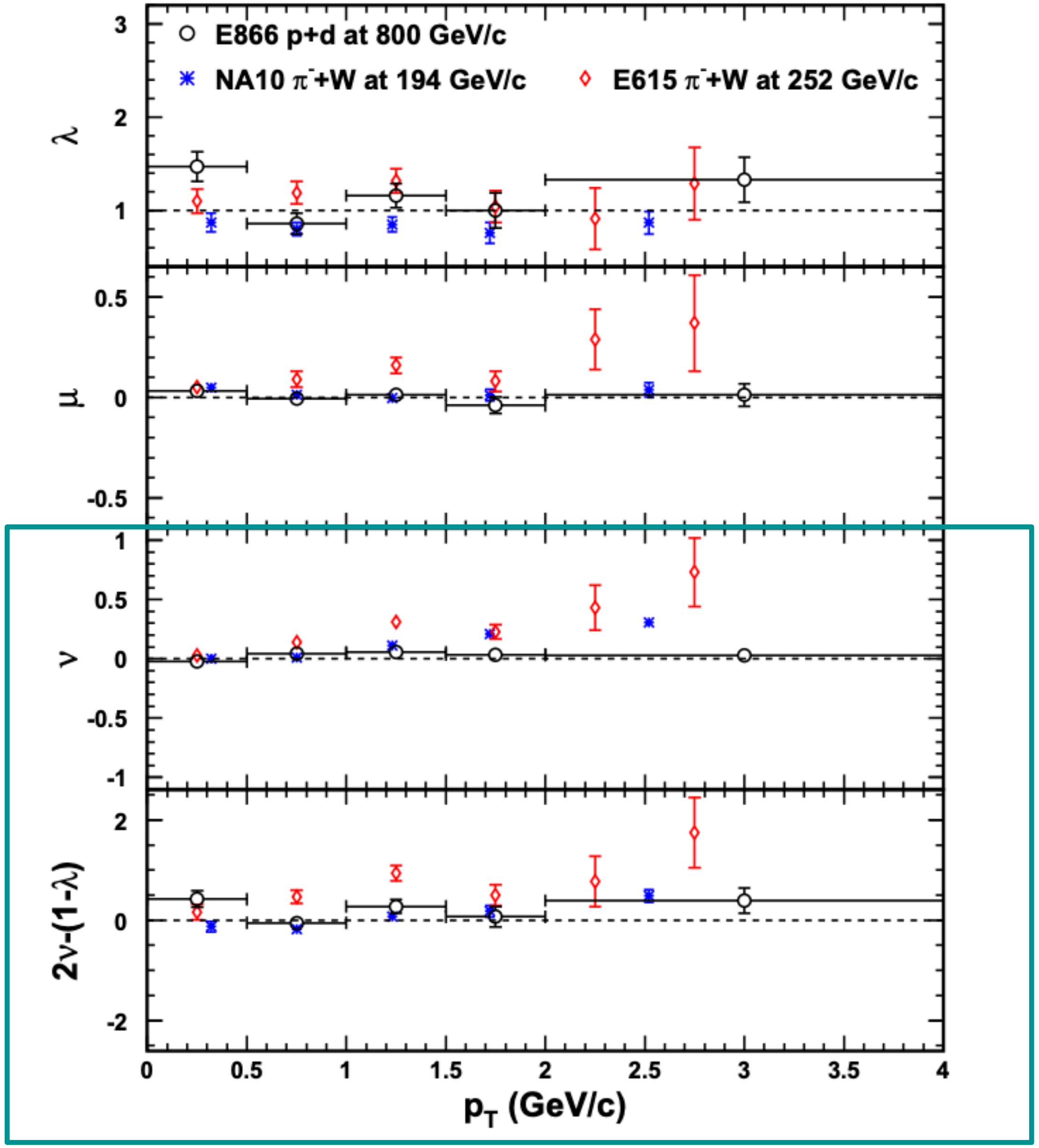
$$\lambda = \frac{F_{UU}^{1} - F_{UU}^{2}}{F_{UU}^{1} + F_{UU}^{2}}, \qquad \mu = \frac{F_{UU}^{\cos\phi}}{F_{UU}^{1} + F_{UU}^{2}}, \qquad \nu = \frac{2F_{UU}^{\cos 2\phi}}{F_{UU}^{1} + F_{UU}^{2}}$$

- ➢ Main experimental data (E866, NA10, E615)
  - ✓ ν is nonzero and positive
  - ✓ Boer-Mulders functions for proton and pion are nonzero and pion Boer-Mulders function in DY process $h_{1,\pi}^{\perp}$ is positive.
- ➢ Lam-Tung relation

$$\lambda + 2\nu = 1$$

  - ✓ The relation is violated indicating perhaps nonzero Boer-Mulders function

FNAL E866/NuSea Collaboration, Phys. Rev. Lett. 99, 082301 (2007)



Drell-Yan process with pions and polarized nucleons $\pi^- + p^\uparrow \to \mu^+\mu^- + X$

➢ DY cross section with pions and **longitudinally polarized proton** at leading twist

pion		proton	
unpolarized TMD	$\otimes$	unpolarized TMD	$F_{UU}^{1} \sim f_{1,\pi} \otimes f_{1,p}$
Boer–Mulders function	$\otimes$	Boer–Mulders function	$F_{UU}^{\cos 2\phi} \sim h_{1,\pi}^{\perp} \otimes h_{1,p}^{\perp}$
Boer–Mulders function	$\otimes$	worm-gear L	$F_{UL}^{\sin 2\phi} \sim h_{1,\pi}^{\perp} \otimes h_{1L,p}^{\perp}$

➢ At present, no experiments have been conducted with longitudinal proton polarization, so no data are available for $F_{UL}^{\sin 2\phi}$.

Drell-Yan process with pions and polarized nucleons $\pi^- + p^\uparrow \to \mu^+\mu^- + X$

➢ DY cross section with pions and **transversely polarized proton at leading twist**

pion		proton	
unpolarized TMD	$\otimes$	unpolarized TMD	$F_{UU}^{1} \sim f_{1,\pi} \otimes f_{1,p}$
Boer–Mulders function	$\otimes$	Boer–Mulders function	$F_{UU}^{\cos 2\phi} \sim h_{1,\pi}^{\perp} \otimes h_{1,p}^{\perp}$
Boer–Mulders function	$\otimes$	worm-gear L	$F_{UL}^{\sin 2\phi} \sim h_{1,\pi}^{\perp} \otimes h_{1L,p}^{\perp}$
unpolarized TMD	$\otimes$	Sivers function	$F_{UT}^{\sin \phi_S} \sim f_{1,\pi} \otimes f_{1T,p}^{\perp}$
Boer–Mulders function	$\otimes$	transversity	$F_{UT}^{\sin(2\phi-\phi_S)} \sim h_{1,\pi}^{\perp} \otimes h_{1,p}$
Boer–Mulders function	$\otimes$	pretzelosity	$F_{UT}^{\sin(2\phi+\phi_S)} \sim h_{1,\pi}^{\perp} \otimes h_{1T,p}^{\perp}$

COMPASS Drell-Yan experiment for single-spin asymmetries in 2015

$$A^{\sin\phi_S} \sim f_{1,\pi} \otimes f_{1T,p}^{\perp}$$

pion unpolarized TMD $\otimes$ proton Sivers function

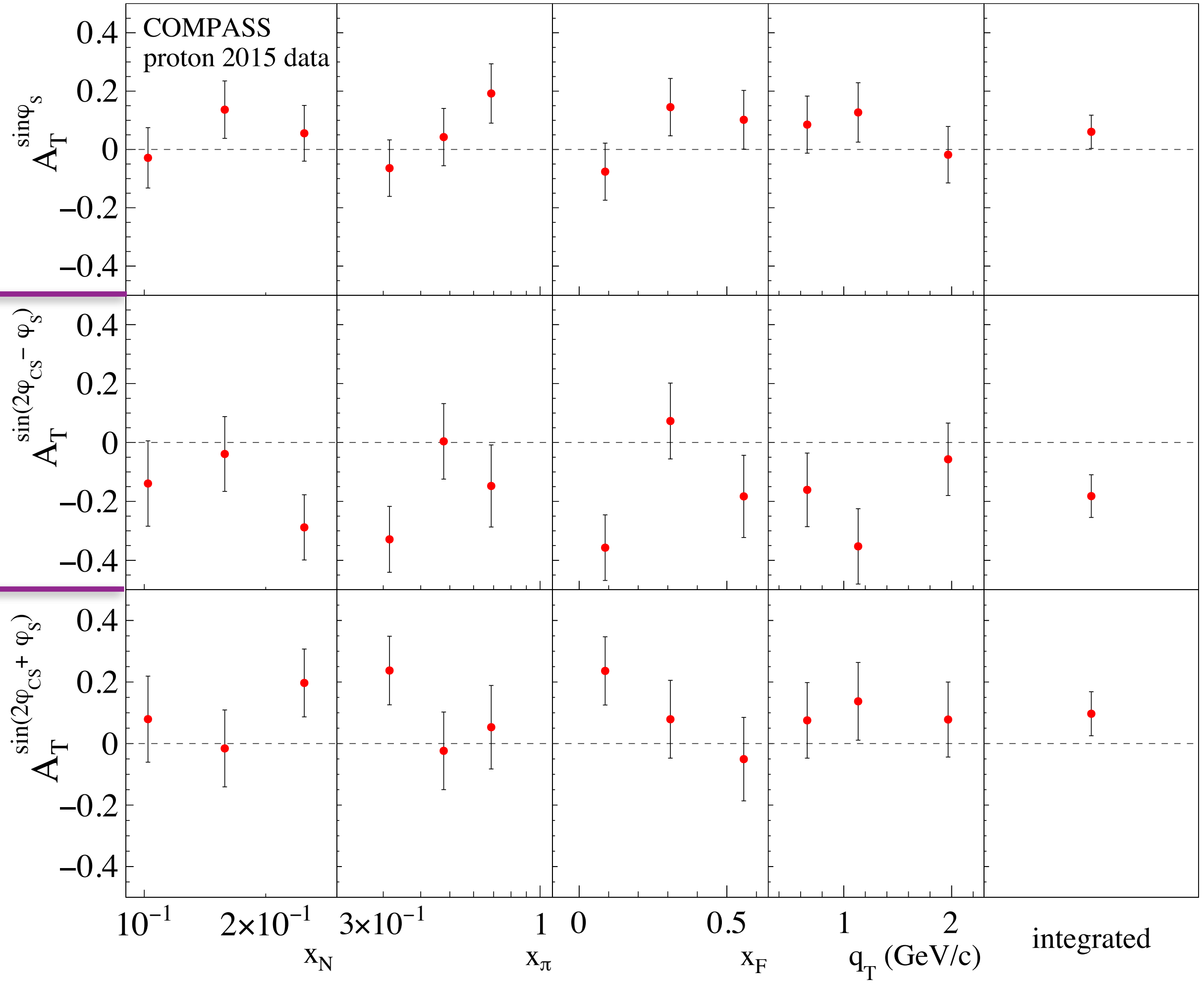
$$A^{\sin(2\phi_{CS}-\phi_S)} \sim h_{1,\pi}^{\perp} \otimes h_{1,p}$$

pion Boer–Mulders function $\otimes$ proton transversity

$$A^{\sin(2\phi_{CS}+\phi_S)} \sim h_{1,\pi}^{\perp} \otimes h_{1T,p}^{\perp}$$

pion Boer–Mulders function $\otimes$ proton pretzelosity

$$A_{XY}^{\text{weight}} = \frac{F_{XY}^{\text{weight}}}{F_{UU}^{1}}$$

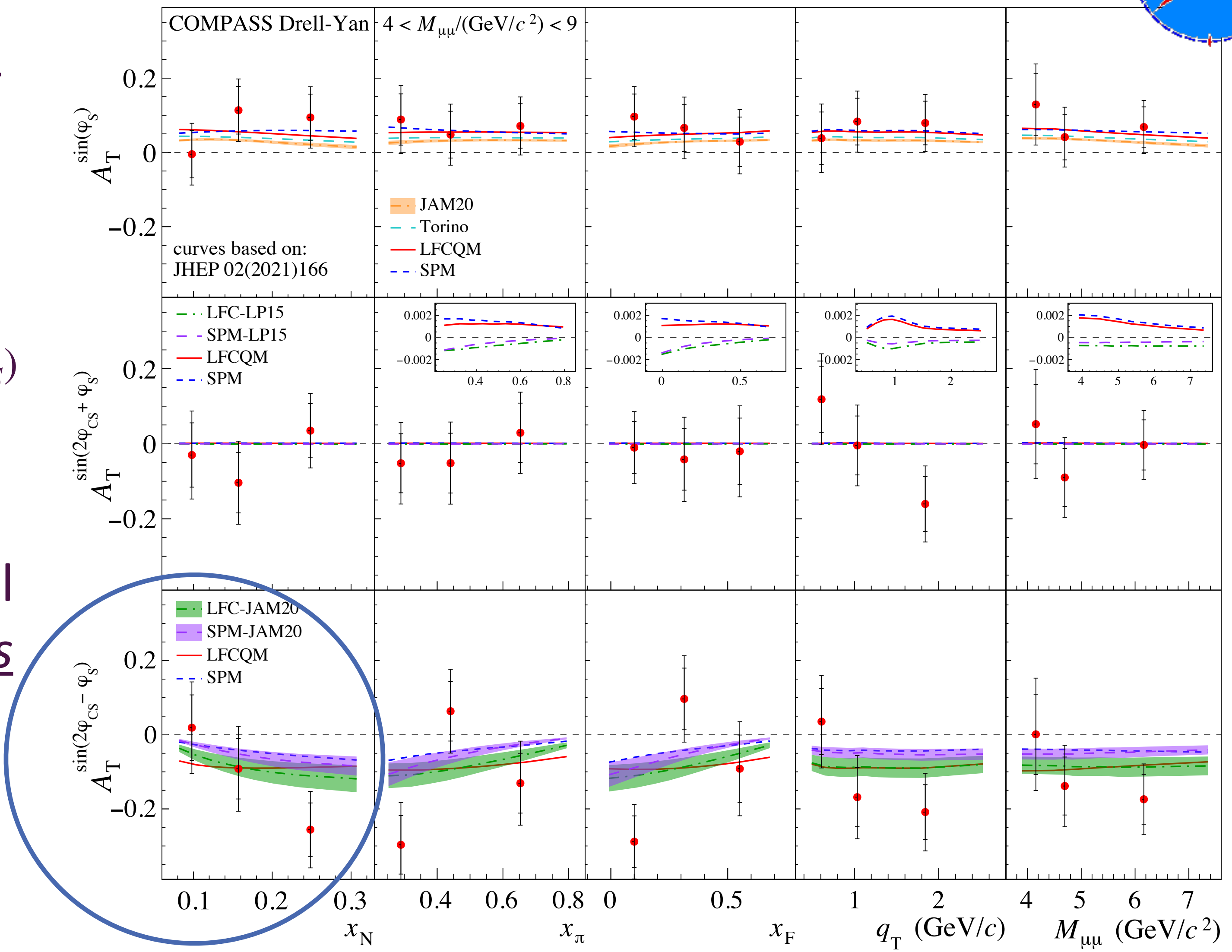


First measurement of transverse-spin-dependent azimuthal asymmetries in the Drell-Yan process, COMPASS Collaboration, Phys. Rev. Lett. 119, 112002 (2017)

COMPASS Drell-Yan experiment for single-spin asymmetries in 2018

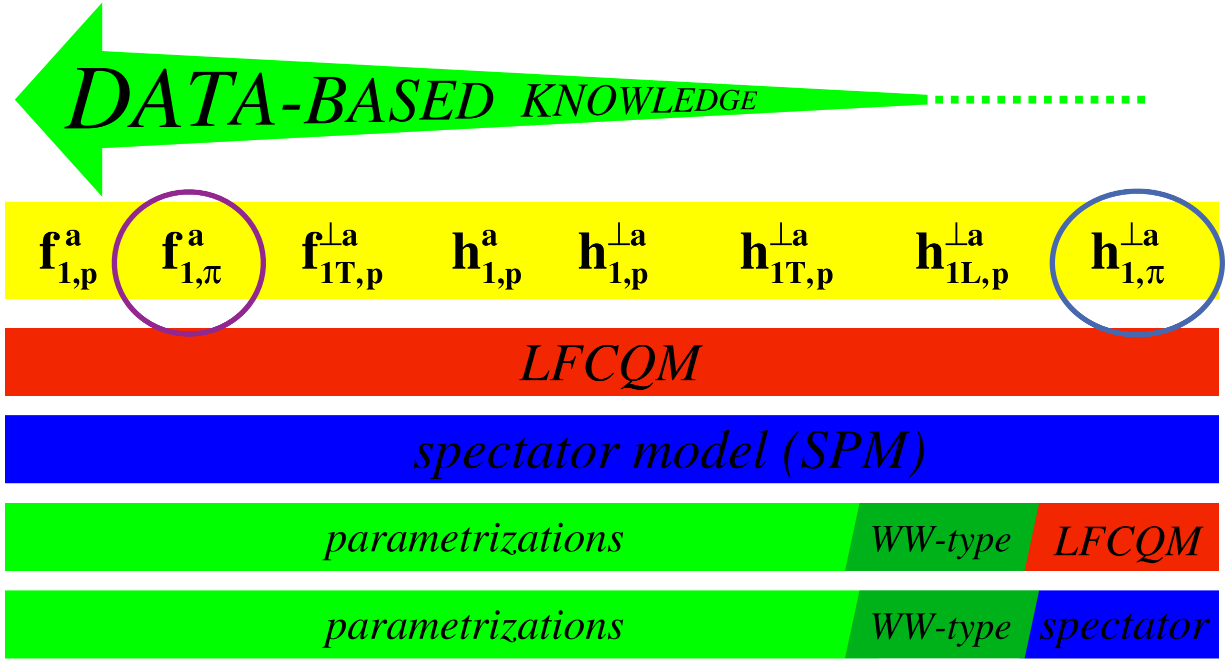
- Nucleon's <u>*prezelocity distribution*</u> $h_{1T,p}^{\perp}$ is yet unknown.
- Since, the proton's u-quark <u>*transversity distribution*</u> $h_{1,p}$ is found to be positive, $A^{\sin(2\phi_{CS}-\phi_S)}$ is negative, however, the coordinate system adopted by COMPASS is opposite to the usual convention, so <u>*pion Boer-Mulders function in DY process $h_{1,\pi}^{\perp}$ is positive.*</u>

Peng, Liu, Xu, Phys. Lett. B 876 (2026) 140415.



Final COMPASS Results on the Transverse-Spin-Dependent Azimuthal Asymmetries in the Pion-Induced Drell-Yan Process, COMPASS Collaboration, Phys. Rev. Lett. 133 (2024) 071902

Pion and Proton TMDs: From Best to Least Constrained



1. LFCQM all

2. spectator all

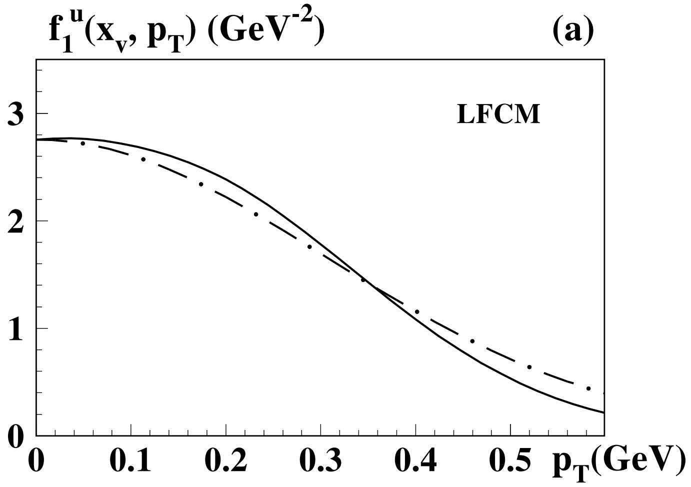
3. LFCQM hybrid

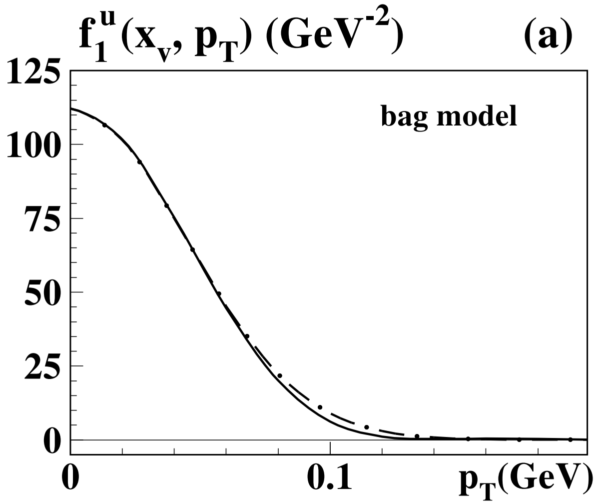
4. spectator hybrid

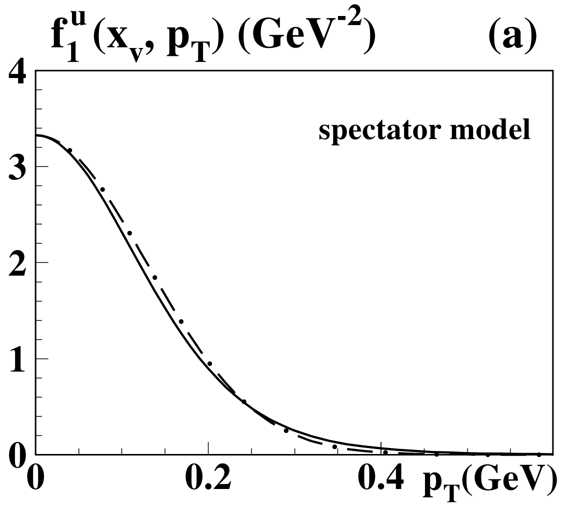
Bastami, Gamberg, Parsamyan, Pasquini, Prokudinb, and Schweitzer, JHEP 02, (2021), 166.

- The pion unpolarized TMDs $f_{1,\pi}(x, k_\perp)$ have been studied and much progress was achieved in incorporating effects of QCD evolution.
- The pion Boer-Mulders function is the *least known of the TMDs* needed to describe the pion-proton DY process at leading twist. No extractions are currently available for $h^{\perp}_{1,\pi}(x, k_\perp)$.

Unpolarized Pion TMD: in models







(a) Pion	LFCM		
TMD	$\langle p_T \rangle$	$\langle p_T^2 \rangle^{1/2}$	R_G
f_1^q	0.28	0.32	0.99
(b) Pion	LFCM	Bag	Spectator
TMD	$\langle p_{T,v}^2 \rangle^{1/2}$	$\langle p_{T,v}^2 \rangle^{1/2}$	$\langle p_{T,v}^2 \rangle^{1/2}$
f_1^q	0.420	0.063	0.180

$$R_G = \frac{2}{\sqrt{\pi}} \frac{\langle p_T \rangle}{\langle p_T^2 \rangle^{1/2}}$$

Gaussian approximations

- light-front constituent model (LFCM),
- bag model,
- spectator model.

Lorcé, Pasquini, Schweitzer, Transverse pion structure beyond leading twist in constituent models, Eur. Phys. J. C 76 (2016) 7, 415.

Twist-2 TMDs of pion

unpolarized TMD   Boer–Mulders function

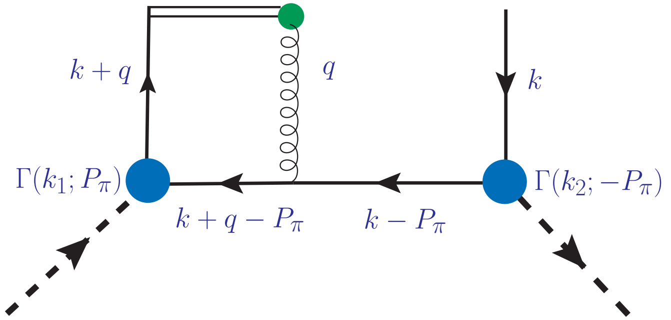
$$\Phi_{ij}(x,\boldsymbol{k}_\perp) = \int dk^- \Phi_{ij}(k,P,S)\Big|_{k^+=xP^+} = f_1(x,\boldsymbol{k}_\perp)\gamma^+ + h_1^\perp(x,\boldsymbol{k}_\perp)\sigma^{+i}$$

$$f_1(x,\boldsymbol{k}_\perp) = \int dk^- \mathrm{Tr}\left[\gamma^+\Phi(k)\right]\Big|_{k^+=xP^+} = \int d^4q \mathrm{Tr}\left[\gamma^+\Phi(q)\right]\delta(q^+ - xP^+)\delta(k_\perp - q_\perp)$$

$$h_1^\perp(x,\boldsymbol{k}_\perp) = \int dk^- \mathrm{Tr}\left[\sigma^{+i}\Phi(k)\right]\Big|_{k^+=xP^+} = \int d^4q \mathrm{Tr}\left[\sigma^{+i}\Phi(k)\right]\delta(q^+ - xP^+)\delta(k_\perp - q_\perp) = 0$$

- Wilson line contribution in

$$\Phi_{ij}(k,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi e^{ik\cdot\xi}\langle P,S|\bar{\psi}_j(0)\mathcal{U}(0,\xi)\psi_i(\xi)|P,S\rangle$$



- Interaction between the eikonalised quark and the spectator ⇒ nonzero $h_1^\perp(x,\boldsymbol{k}_\perp)$

Cheng, Cui, Ding, Roberts, Schmidt, Eur. Phys. J. C 85 (2025) 1, 115

Cheng, Ding, Binosi, Roberts, arXiv: 2603.25941 [hep-ph]

Twist-2 unpolarised TMD of pion

➢ TMD $$f_1(x,\boldsymbol{k}_\perp) = \int d^4q \mathrm{Tr}\left[\gamma^+\Phi(q)\right]\delta(q^+ - xP^+)\delta(k_\perp - q_\perp)$$

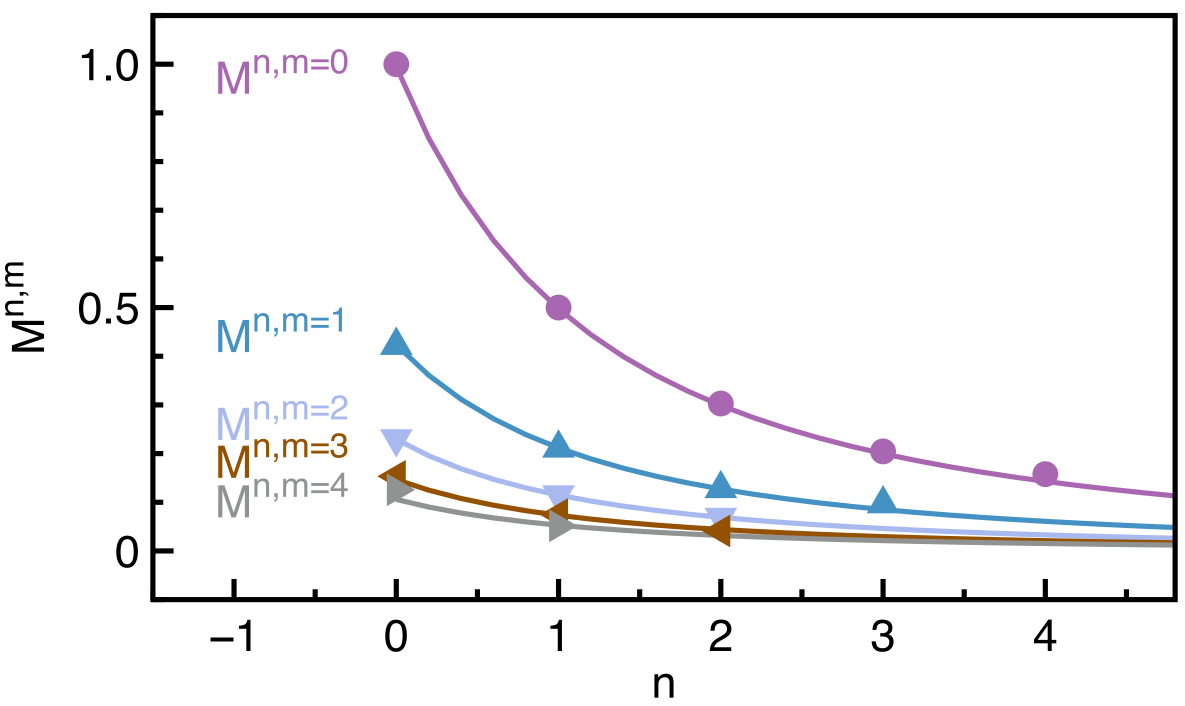
➢ Mellin-transverse moments

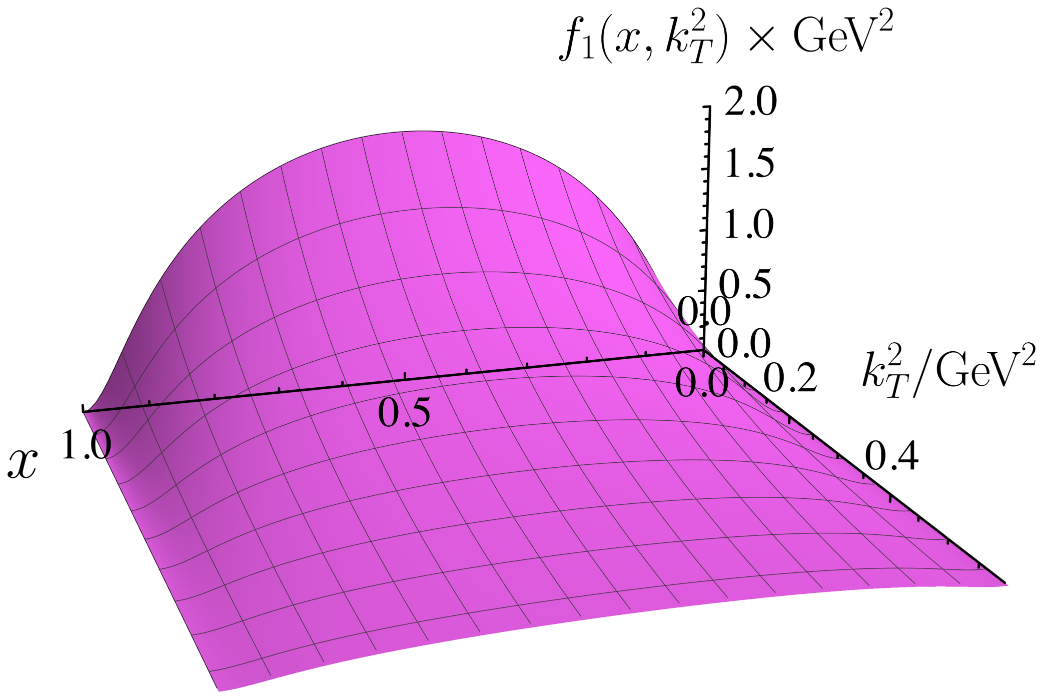
$$\mathscr{M}^{n,m} = \int_0^1 dx\, x^n \int d^2k_T\, k_T^m\, f_1(x,k_T^2)$$

$$\sim \int d^4k \mathrm{Tr}\left[\gamma^+\Phi(k,P)\right](k^+)^n/(P^+)^n(q_{Tn})^m$$

where $q_{Tn} = |\boldsymbol{q}_T|$ is the magnitude of the transverse momentum, defined using the transverse basis vectors n_{T_1} and n_{T_2} as $q_{Tn} = \sqrt{(n_{T_1}\cdot q)^2 + (n_{T_2}\cdot q)^2}$, $n_{T_1} = (0,1,0,0), n_{T_2} = (0,0,1,0)$.

Twist-2 unpolarised TMD of pion





- ➢ Lowest seventeen moments Mellin-transverse moments of the pion unpolarised valence-quark TMD. Ding, Phys. Lett. B 879 (2026) 140677

$$f_1(x, k_T^2) = \frac{1}{2} \ln\left[1 + \frac{x^2(1-x)^2}{\rho_{TMD}^2}\right] \frac{e^{-k_T^2/\langle k_T^2\rangle}}{\pi\langle k_T^2\rangle},$$

with $\langle k_T^2\rangle = 0.231$ GeV^2 and $\rho_{\mathrm{TMD}} = \rho_{\mathrm{DF}}$.

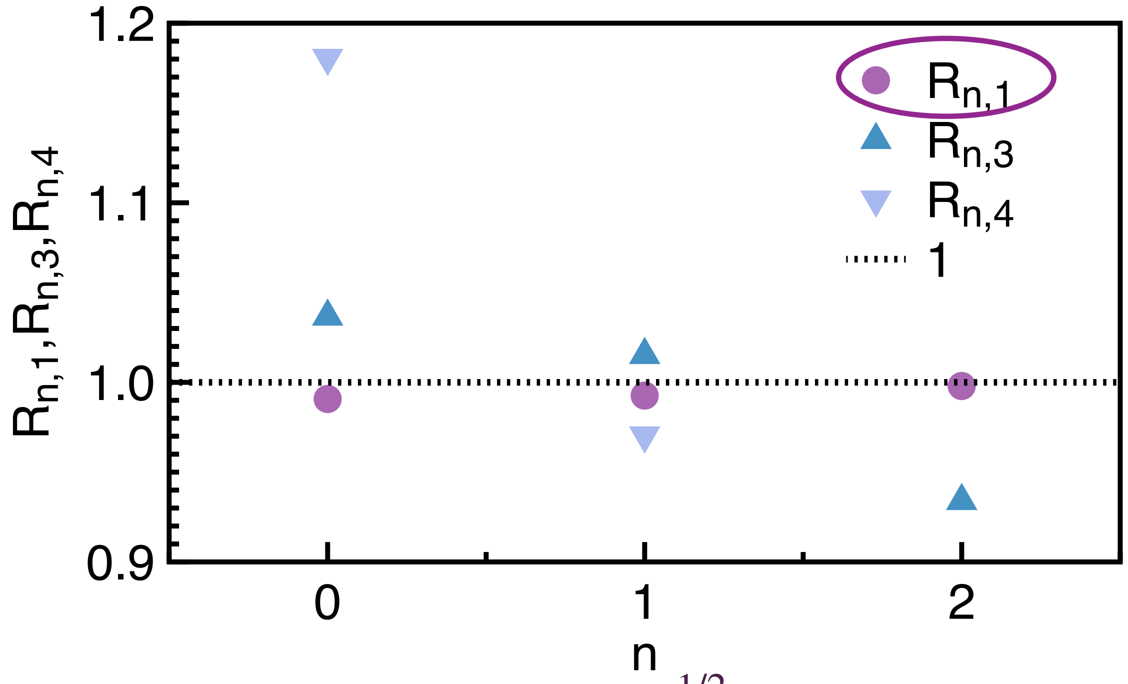
$\langle \boldsymbol{k}_T\rangle = 0.422$ GeV

Gaussianity of the transverse-momentum distribution

➢ To quantify deviations from a Gaussian transverse profile, we construct dimensionless ratios

$$R_{n,m} \equiv \frac{1}{\Gamma\left(1+\frac{m}{2}\right)} \frac{\mathscr{M}^{n,m}/\mathscr{M}^{n,0}}{\left(\mathscr{M}^{n,2}/\mathscr{M}^{n,0}\right)^{m/2}},$$

➢ which satisfy $R_{n,m}=1$ for all m if the transverse-momentum dependence is purely Gaussian.



➢ $\mathscr{M}^{0,1}=\langle k_T\rangle = 0.422$ GeV, $\mathscr{M}^{0,2}=\langle k_T^2\rangle = 0.231$ GeV^2. Therefore $\langle k_T\rangle \simeq \left(\frac{\pi}{4}\langle k_T^2\rangle\right)^{1/2}$ confirming ***near-Gaussian behavior of the transverse-momentum distribution at the hadron scale.***

➢ The intrinsic transverse width $\langle k_\perp^2\rangle = 0.267$ GeV^2 ($\omega = 0.4$ GeV), and $\langle k_\perp^2\rangle = 0.193$ GeV^2 ($\omega=0.6$ GeV), In all cases, $\langle k_T\rangle \simeq \left(\frac{\pi}{4}\langle k_T^2\rangle\right)^{1/2}$ is satisfied to high precision.

Large k_T behavior of $f_1^\pi(x, \boldsymbol{k}_\perp)$

- It is possible to determine the power behavior at large $k_\perp$ for the different distributions, for example Bacchetta, Boer, Diehl, Mulders, JHEP 08 (2008) 023

$$f_1(x, k_\perp^2) \sim \frac{1}{k_\perp^2}\alpha_s \sum_{i=q,\bar{q},g} q^i(x) \otimes K_1^i$$
$$h_1^\perp(x, k_\perp^2) \sim \frac{M^2}{k_\perp^4}\alpha_s \sum_{i=q,\bar{q},g} [\text{collinear twist-three distributions}] \otimes K_3^i$$

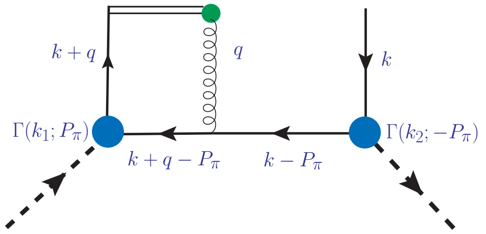
- If this is true, then

$$\langle \boldsymbol{k}_\perp^m \rangle = \int dk_\perp^2 (|\boldsymbol{k}_\perp|)^m f_1(x, k_\perp) \quad (\text{power divergence for } m \geq 1)$$

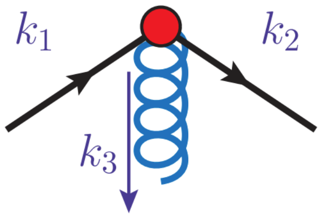
- The calculated TMD primarily describes the soft ***nonperturbative component of the transverse-momentum distribution*** and does not reproduce the perturbative large- k_T tail expected in full QCD.

Small k_T behavior of $f_1^\pi(x, \boldsymbol{k}_\perp)$

- Within the present *impulse approximation* framework, the TMD is determined solely by the dressed-quark propagator and pion Bethe–Salpeter amplitude, whose ultraviolet behavior leads to a substantially faster falloff.
- *Lower transverse moments* remain numerically stable and provide reliable information on the *infrared structure*, whereas *higher moments* become increasingly sensitive to *ultraviolet details* and should therefore be interpreted with appropriate caution.
- A more complete description of the ultraviolet region will require
  - ✓ explicit incorporation of gauge-link contributions,
  - ✓ exchange-current effects beyond impulse approximation,
  - ✓ transverse components of the dressed quark–photon vertex.



Cheng, Cui, Ding, Roberts, Schmidt, Eur. Phys. J. C 85 (2025) 1, 115



Costa et al, Phys. Rev. C 104 (2021) 4, 045201.

$$\Gamma_\mu(k,p) = \Gamma_\mu^L(k,p) + \Gamma_\mu^T(k,p)\,,$$

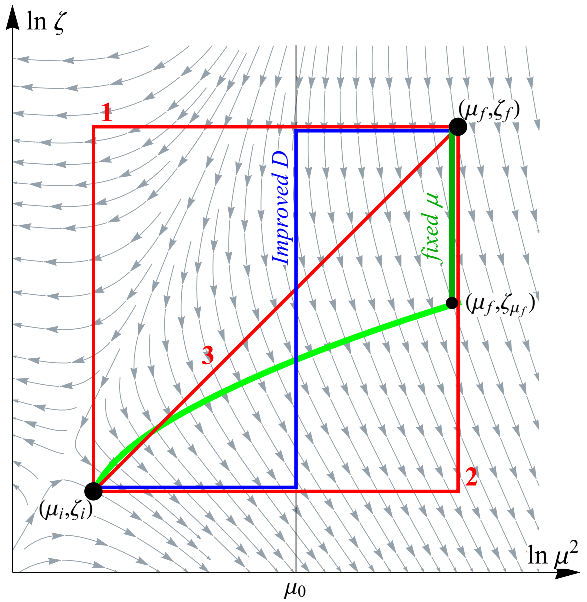
$$\Gamma_\mu^L(k,p) = \sum_{j=1}^{4} \lambda_j(k,p)\, L_\mu^j(k,p)\,,$$

$$\Gamma_\mu^T(k,p) = \sum_{j=1}^{8} \tau_j(k,p)\, T_\mu^j(k,p)\,,$$

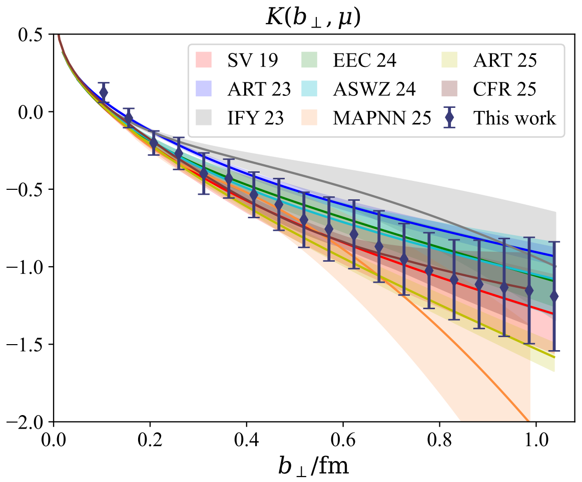
Qin, Chang, Liu, Roberts, Schmidt, Phys. Lett. B 722 (2013) 384-388.

QCD evolution

- TMDs generally depend on a renormalization scale μ and rapidity scale ζ, the latter governing *Collins–Soper evolution*.
- The present calculation is performed at the hadron scale μ_H, where the pion is described solely in terms of dressed valence quark and antiquark. The x-dependent functions correspond to collinear distributions and the exponential factors are *"primordial shapes" of TMDs at the initial scale*. This particular dependence is *often used in phenomenology*, corresponds to the Gaussian Ansatz and is supported in models.
- Consequently, the resulting TMD may be defined at a fixed hadron-scale rapidity parameter ζ_H, naturally of order μ_H^2.



Scimemi and Vladimirov, J. High Energ. Phys. 2018, 3 (2018)



Jin-Xin Tan et al.,.Phys.Rev.D 113 (2026) 5, 054505

TMDs of pion: Boer-Mulders function

- Working at leading twist one may in general rewrite quark + quark correlation function as:

$$\Phi_{q/\pi}(x,\mathbf{k}_\perp) = \frac{1}{2}\left[f_{1\pi}^q(x,k_\perp^2)i\gamma\cdot n + h_{1\pi}^{q\perp}(x,k_\perp^2)\frac{1}{M}\sigma_{\mu\nu}k_{\perp\mu}n_\nu\right],$$

- $f_{1\pi}^q(x,k_\perp^2)$ is helicity-independent TMD, $h_{1\pi}^{q\perp}(x,k_\perp^2)$ is Boer-Mulders function.
- M is a dynamically generated mass scale that is characteristic of EHM. A sensible choice for pion is $M \approx 4f_\pi$ where f_π is the pion leptonic decay constant.
- It is plain that

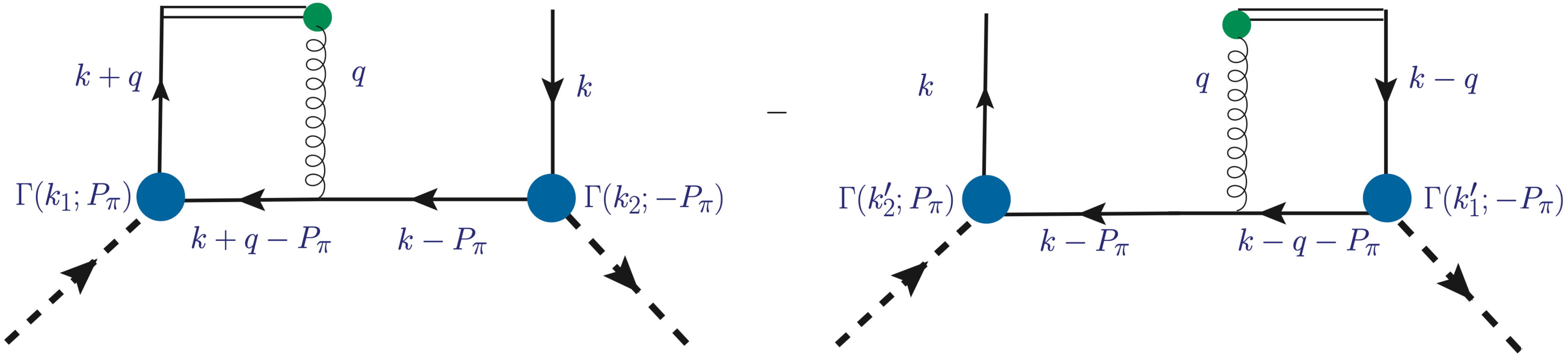
$$f_{1\pi}^q(x,k_\perp^2) = \mathrm{tr}\frac{1}{2}i\gamma\cdot\bar{n}\Phi_{q/\pi}(x,\mathbf{k}_T),$$

$$\frac{k_\perp^2}{M^2}h_{1\pi}^{q\perp}(x,k_\perp^2) = \mathrm{tr}\frac{1}{2M}\sigma_{\mu\nu}k_{\perp\mu}\bar{n}_\nu\Phi_{q/\pi}(x,\mathbf{k}_T).$$

- It is plain that many studies use $M \to m_\pi$, i.e., the pion mass; however, we judge this to be ill-advised because $m_\pi \to 0$ in the chiral limit whereas f_π does not.

TMDs of pion: Boer-Mulders function

- The helicity-independent TMD, $f_{1\pi}^{q}(x, k_\perp^2)$, is always nonzero; but if the gauge links are omitted, then the Boer-Mulders function vanishes.
- This is the signal that the interaction between the eikonalised quark and the spectator is crucial to obtaining $h_{1\pi}^{q\perp}(x, k_\perp^2) \neq 0$. Double line – leading eikonal approximation to quark propagation under the influence of the gauge link.

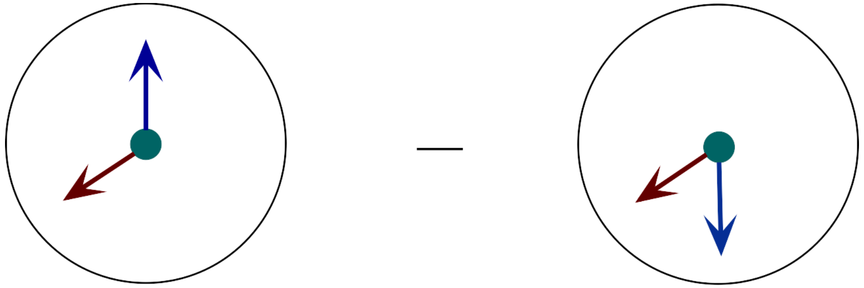


TMDs of pion: Boer-Mulders function

➢ The number density asymmetry corresponds to the following difference:

$$f_{q^{\uparrow}/\pi}(x,\vec{k}_{\perp}) - f_{q^{\downarrow}/\pi}(x,\vec{k}_{\perp}) = -\, h_{1\pi}^{q\perp}(x,k_{\perp}^{2})\frac{|\vec{k}_{\perp}|}{M}\sin(\phi_{k_{\perp}} - \phi_{S})\,,$$

✓ where the azimuthal angles are measured between the indicated transverse vector and $\hat{P}$.



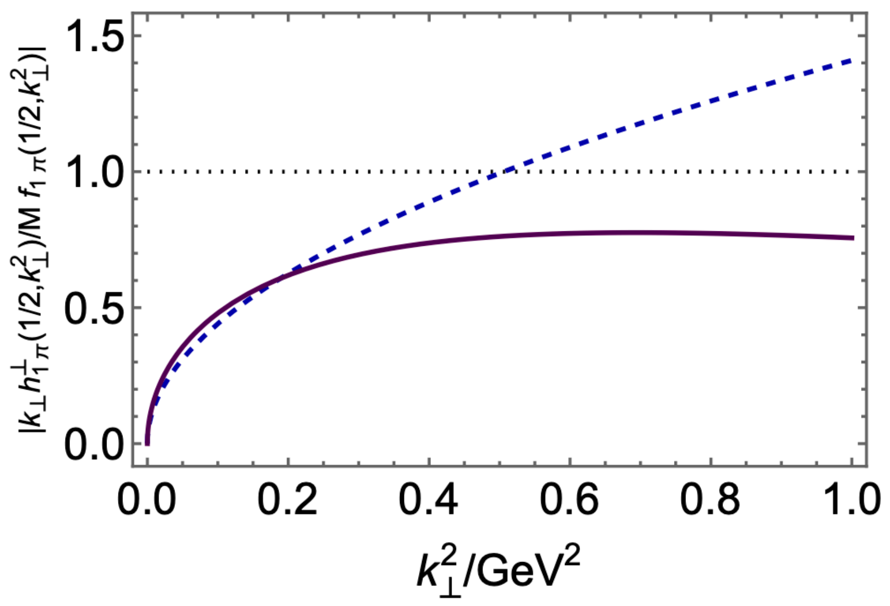
✓ Number density interpretation of the Boer Mulders function. Vertical blue arrows - transverse polarisation of the quark; oblique red vectors - quark transverse momentum vector.

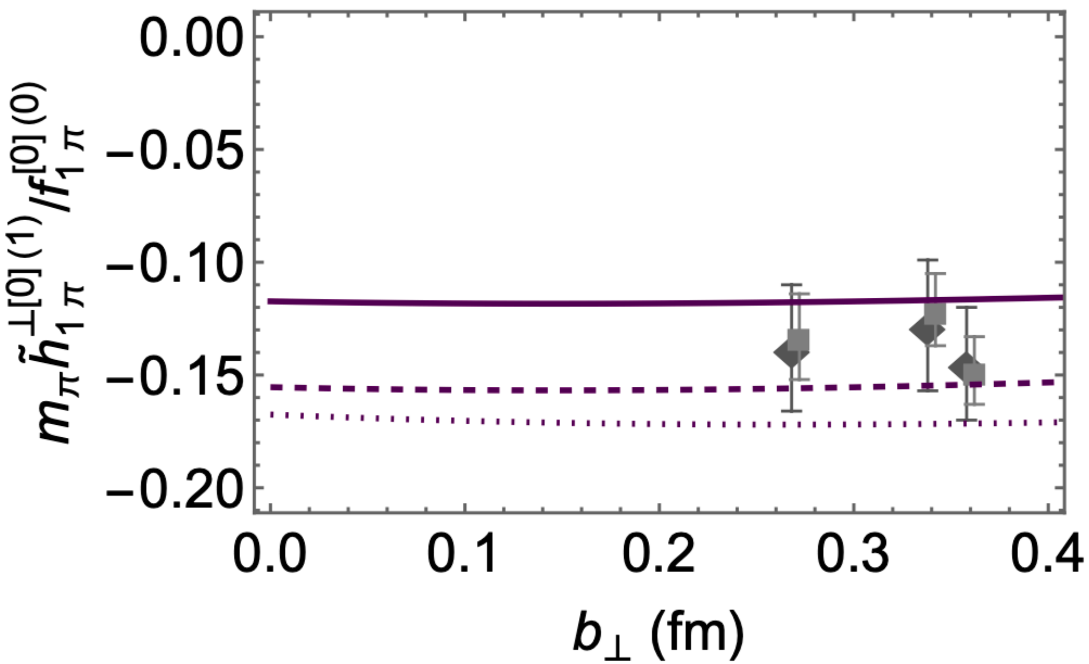
➢ Exploiting the requirements imposed by positivity of the defining matrix element, one arrives at the pointwise positivity bound:

$$|k_{\perp} h_{1\pi}^{q\perp}(x,k_{\perp}^{2})/M| \leq f_{1\pi}^{q}(x,k_{\perp}^{2})$$

TMDs of pion: Boer-Mulders function

Cheng et al, Eur. Phys. J. C 85 (2025) 1, 115



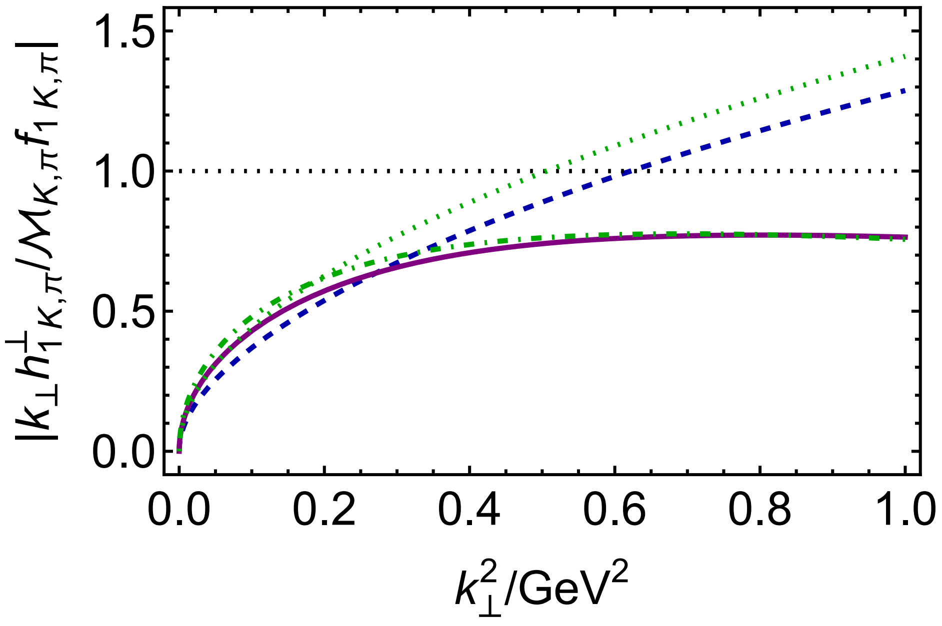


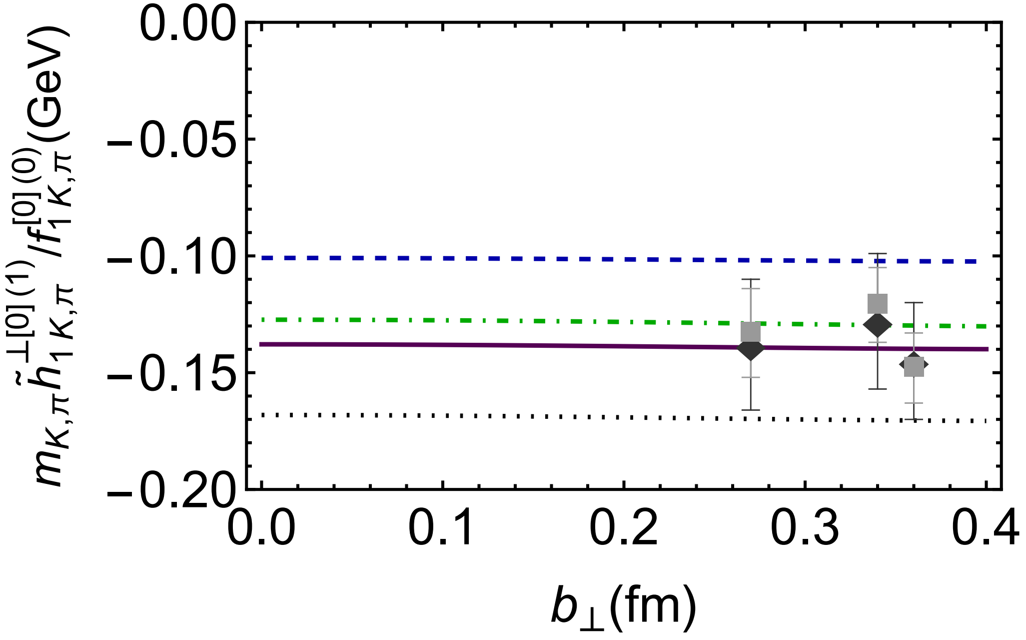
- **Positivity constraint**: the bound is violated if the curve crosses the horizontal dotted line. $|k_\perp h_{1\pi}^{\perp \mathrm{MI}}(x=1/2,k_\perp^2)/Mf_{1\pi}(x=1/2,k_\perp^2)|$ -- dashed blue cf. $|k_\perp h_{1\pi}^{\perp}(x=1/2,k_\perp^2)/Mf_{1\pi}(x=1/2,k_\perp^2)|$ solid purple.

- **Generalized Boer-Mulders shift.** Dotted purple curve -- hadron scale physical pion mass result; dashed purple curve -- hadron scale heavy pion ($m_\pi = 0.518$ GeV); and solid purple curve -- heavy pion result evolved $\zeta_{\mathscr{H}} \to \zeta_2$. LQCD results, obtained with $m_\pi = 0.518$ GeV at a resolving scale $\zeta = \zeta_2 = 2$ GeV.

TMDs of kaon: Boer-Mulders function

Cheng et al, Eur. Phys. J. C 86 (2026) 9, 1029





- **Positivity constraint**: the bound is violated by any result that crosses the horizontal dotted line. solid purple curve, momentum-dependent gauge link completion; and dashed blue curve, pure-SCI result. Green curves, SCI pion results – dotted, momentum-independent gluon; dot-dashed, momentum-dependent gauge link completion.

- **Generalized Boer-Mulders shift.** Hadron-scale values for π, u-in-K, s-in-K -- dotted black curve. After evolution $\zeta_{\mathcal{H}} \to \zeta_2$: u-in-K-- solid purple curve; s-in-K -- dashed blue curve; u=d-in-π-- dot-dashed green curve; points -- available lQCD results, computed with $m_\pi = 0.518$ GeV at a resolving scale $\zeta = \zeta_2 = 2$ GeV. N.B. 0.2 GeV $\approx 1\mathrm{fm}^{-1}$.

Future facilities and experiments on pion and kaon PDFs: COMPASS++/AMBER at CERN

➢ AMBER at CERN - Apparatus for Meson and Baryon Experimental Research. Three phase-1 experiments:

✓ Proton charge-radius measurement using muon-proton elastic scattering

✓ Drell-Yan and J/ψ production experiments using the conventional M2 hadron beam

- The structure of the pion: determination of the pion valence, sea-quark and gluon distributions

✓ Measurement of proton-induced antiproton production cross sections for dark matter searches

Program	Physics Goals	Beam Energy [GeV]	Beam Intensity [s^{-1}]	Trigger Rate [kHz]	Beam Type	Target	Earliest start time, duration	Hardware additions
muon-proton elastic scattering	Precision proton-radius measurement	100	$4 \cdot 10^6$	100	$\mu^\pm$	high-pressure H2	2022 1 year	active TPC, SciFi trigger, silicon veto,
Hard exclusive reactions	GPD E	160	$2 \cdot 10^7$	10	$\mu^\pm$	$NH_3^\uparrow$	2022 2 years	recoil silicon, modified polarised target magnet
Input for Dark Matter Search	$\bar{p}$ production cross section	20-280	$5 \cdot 10^5$	25	p	LH2, LHe	2022 1 month	liquid helium target
$\bar{p}$-induced spectroscopy	Heavy quark exotics	12, 20	$5 \cdot 10^7$	25	$\bar{p}$	LH2	2022 2 years	target spectrometer: tracking, calorimetry
Drell-Yan	Pion PDFs	190	$7 \cdot 10^7$	25	$\pi^\pm$	C/W	2022 1-2 years	
Drell-Yan (RF)	Kaon PDFs & Nucleon TMDs	$\sim$100	10^8	25-50	$K^\pm, \bar{p}$	$NH_3^\uparrow$, C/W	2026 2-3 years	"active absorber", vertex detector
Primakoff (RF)	Kaon polarisability & pion life time	$\sim$100	$5 \cdot 10^6$	> 10	K^-	Ni	non-exclusive 2026 1 year	
Prompt Photons (RF)	Meson gluon PDFs	≥ 100	$5 \cdot 10^6$	10-100	$K^\pm$ $\pi^\pm$	LH2, Ni	non-exclusive 2026 1-2 years	hodoscope
K-induced Spectroscopy (RF)	High-precision strange-meson spectrum	50-100	$5 \cdot 10^6$	25	K^-	LH2	2026 1 year	recoil TOF, forward PID
Vector mesons (RF)	Spin Density Matrix Elements	50-100	$5 \cdot 10^6$	10-100	$K^\pm, \pi^\pm$	from H to Pb	2026 1 year	

Table 2: Requirements for future programmes at the M2 beam line after 2021. Muon beams are in blue, conventional hadron beams in green, and RF-separated hadron beams in red.

Letter of Intent: A New QCD facility at the M2 beam line of the CERN SPS (COMPASS++/AMBER), arXiv:1808.00848 [hep-ex]. Proposal for Phase-1: COMPASS++/AMBER: Proposal for Measurements at the M2 beam line of the CERN SPS Phase-1: 2022-2024.

Summary

- ➢ Determination of the pion and kaon leading-twist unpolarized valence-quark TMDs and Boer–Mulders functions.
- ➢ A transverse-projection method was developed to compute transverse-momentum moments of the TMDs, while the Boer–Mulders functions were studied within a contact-interaction framework.
- ➢ The transverse-projection method is general and readily extendable to other hadrons and transverse observables, providing a practical pathway toward a broader understanding of three-dimensional hadron structure.

Outlook

- ➢ Boer-Mulders function with realistic interaction, proton.
- ➢ Collins-Soper kernel, QCD evolution.
- ➢ Comparison with COMPASS 2018 Pion-Induced Polarized Drell–Yan Data.

Thank you!