Extracting the nucleon axial form factor from recent LQCD data using NNLO ChPT

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Extraction of the nucleon axial form factor from Lattice QCD using NNLO chiral perturbation theory

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(Dated: July 28, 2026)

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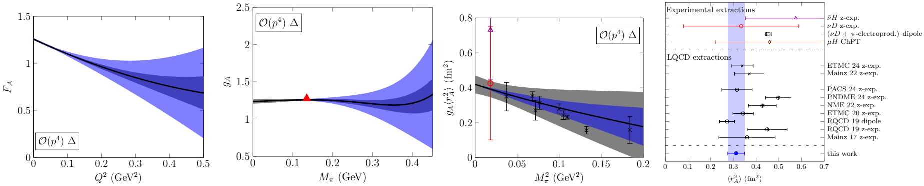


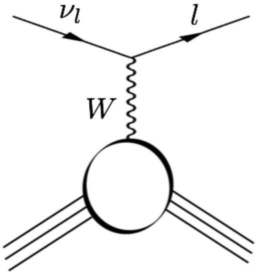


Figure 1: νQES [PRD103(2021)]

- Weak interaction is $V - A$
  - $\langle N' | V | N \rangle$: corresponds to precisely determined EM ff
  - $\langle N' | A^\mu | N \rangle = \bar{u}' \left\{ \gamma_\mu F_A(q^2) + \frac{q_\mu}{2m_N} \tilde{F}_P(q^2) \right\} \gamma_5 u$

    - axial ff, $F_A(Q^2)$, and induced pseudoscalar ff, $\tilde{F}_P$, with $A^\mu = \bar{q}\gamma^\mu\gamma_5 q$
    - $\tilde{F}_P$ pion-pole-dominated

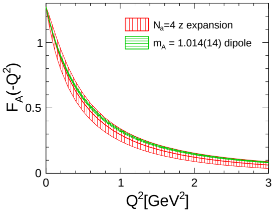
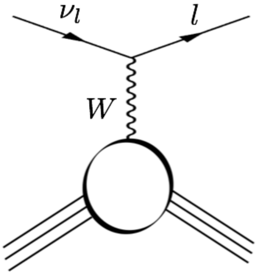


Figure 2: νD extract. [Meyer et al. 1603.03048]



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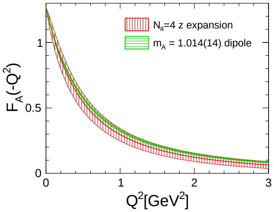

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- Axial form factor, $F_A(Q^2)$
  - fundamental property, e.g. $F_A(0)$ related to spin distribution
  - shapes the νN elastic and quasielastic scattering cross section, upon which neutrino oscillation experimets critically rely

    F_A is extracted from different ν scattering experiments with significant uncertainty (old bubble chamber ANL, BNL & FNL)

    $\Longrightarrow$ F_A is a key input to ν oscillation experiments

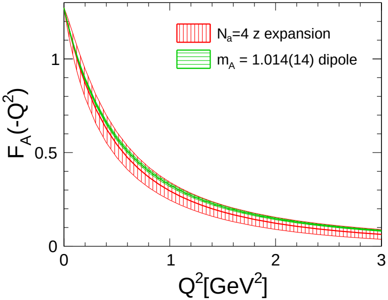


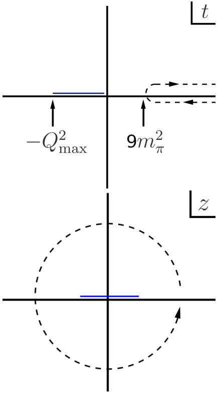
Figure 3: νD extract. [Meyer et al. 1603.03048]

- F_A **Experimental determinations**
  - old bubble chamber νD (ANL, BNL, FNL) data
  - Recent remarkable measurement:
    $\bar{\nu} p \to \mu n$ in MINERvA
    subtracting carbon from plastic [Cai et al Nature 614 (2023)]
- $F_A(q^2) = g_A \left[1 + \frac{1}{6}\langle r_A^2 \rangle q^2 + \mathcal{O}(q^4)\right]$
- $\langle r_A^2 \rangle$ is a benchmark quantity for exp. and th.
  - interesting extraction from weak muon capture in muonic hydrogen (μH): $\mu^- p \to \nu_\mu n$
    chiral Ward id.: $\bar{g}_P = \bar{g}_P(\langle r_A^2 \rangle) + \mathcal{O}(t^2)$, no z-exp.
    (MuCap)[Andreev PRC 91(2015)] [Hill, Rept. Prog. Phys. 81 (2018)]
  - also related to pion-electroproduction in ChPT

- **LQCD systematics**
    1. lattice finite volume, $M_\pi L$
    2. discretization spacing, a
    3. $M_\pi^{\mathrm{LQCD}} > M_\pi^{\mathrm{phys}}$
    - chiral continuum extrapolation, relies on a parameterisation of the dependencies

- **ChPT**
    - EFT of QCD at low E
    - Parameterisation of M_π based on symmetries of QCD
    - Also used in the context of $M_\pi L$ and a effects
    - Useful to deal with excited state contamination

Axial form factor at low q^2



- Experimental and lattice q^2 parametrisation is delicate:

  - dipole ansatz
  - z-expansion at given order (sizable uncertainty)
  - ...

  $\Rightarrow$ different $\langle r_A^2 \rangle$, and F_A

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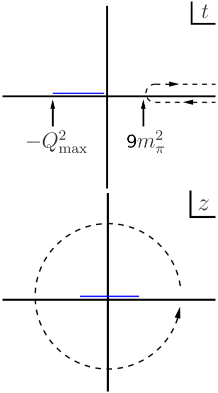

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- We parameterise M_π and q^2 dep. with **ChPT**:
  model independency, uncertainty estimation and can connect to other processes
- Our work on F_A:
  1. compute it in ChPT (informed by πN data)
  2. analyse a combined set of recent LQCD data and see if we obtain a good description (some LECs are fitted)
  3. extract F_A & $\langle r_A^2 \rangle$ model-independently and estimating the systematics

Nucleon axial form factor, F_A

- We compute NNLO $\mathcal{O}(p^4)$ F_A in relativistic ChPT
- Relativistic EOMS has been shown to improve HB results
  - e.g. μ_B [Geng et al, PRL 101 (2008)]
- Explicit $\Delta(1232)$ sizable and beneficial:
  - polarizabilities, couplings and form factors , e.g. [Hacker et al PRC 72 (2005)],[Yao et al JHEP 5 (2016)],[Hiller-Blin PRD 96 (2017)],[Thürmann et al PRC 103 (2021)]
  - Δ counting: small scale expansion $\delta = m_\Delta - m_N \sim \mathcal{O}(p)$
- $F_A(q^2) = g_A \left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \mathcal{O}(q^4)\right]$
- We start by focusing on the M_π dep. of $F_A(0) = g_A$

$F_A(0) = g_A$ Calculation

[FA & Alvarez-Ruso PRD 105 (2022)]

- d_{16}: key source of uncertainty in m_q dependence of nuclear properties:
  - ground-state and binding energies [Beane et al. Nucl. Phys. A 717 (2003), Berengut et al. PRD 87 (2013), Epelbaum et al Eur. Phys. J. A 49 (2013)]
  - $\mathcal{L}^{(1)}_{\pi N} \Longrightarrow \mathring{g}_A$, $\quad \mathcal{L}^{(2)}_{\pi N} \Longrightarrow c_1,\, \tilde{c}_4$,
    $\mathcal{L}^{(3)}_{\pi N} = \bar{\Psi}\frac{d_{16}}{2}\gamma^\mu\gamma_5\langle\chi_+\rangle u_\mu \Psi + \ldots$
  - $\mathcal{L}^{(1)}_{\pi N\Delta} \Longrightarrow h_A,\, g_1$, $\quad \mathcal{L}^{(2)}_{\pi\Delta} \Longrightarrow a_1$, $\quad \mathcal{L}^{(2)}_{\pi N\Delta} \Longrightarrow \tilde{b}_4$

$$g_A = \mathring{g}_A + 4d_{16}M_\pi^2 + g^{(3)\not\Delta}_{A\text{loop}}(\mathring{g}_A; M_\pi) + g^{(3)\Delta}_{A\text{loop}}(\mathring{g}_A, h_A, g_1; M_\pi)$$
$$+ g^{(4)\not\Delta}_{A\text{loop}}(\mathring{g}_A, c_1, \tilde{c}_4; M_\pi) + g^{(4)\Delta}_{A\text{loop}}(\mathring{g}_A, h_A, g_1, c_1, a_1, \tilde{b}_4; M_\pi) + \mathcal{O}(p^5),$$

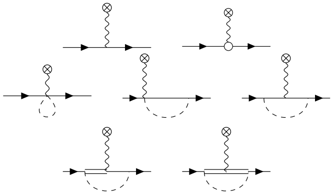


Figure 4: $\mathcal{O}(p)$ and $\mathcal{O}(p^3)$ (w. f. renormalisation not shown)

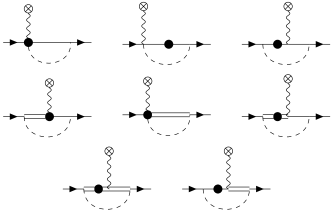
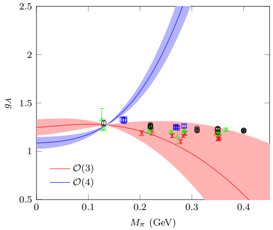
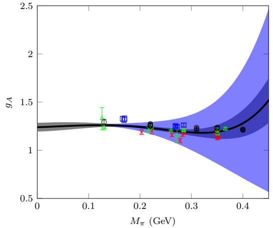


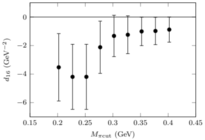
Figure 5: $\mathcal{O}(p^4)$

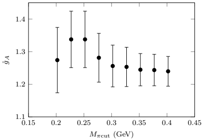
What we learn from $g_A(M_\pi)$ in ChPT

1. πN elast. & inelast scatt ($\not\Delta$) LECs relativistic $\mathcal{O}(p^4)$ not compatible with $g_A^{\rm LQCD}(M_\pi)$
2. Δ necessary
3. Slow convergence
   - **Difference between orders $\simeq$ theoretical uncertainty** [Epelbaum et al EPJA 53 (2015)]
     $\Delta g_{A\chi}^{(4)} = \max\left\{\left(\frac{M_\pi}{\Lambda}\right)^4 |\mathring{g}_A|, \left(\frac{M_\pi}{\Lambda}\right)^2 \left|g_A^{(3)}\right|, \frac{M_\pi}{\Lambda}|g_A^{(4)}|\right\}$



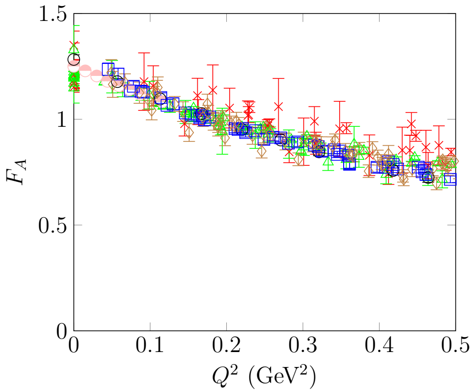






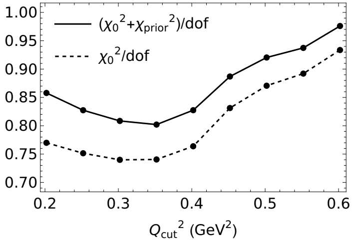
Axial form factor

- We perform a **meta-analysis of large set of recent LQCD results**
  - Many recent works $\Rightarrow$ substantial improvements
  - "Mainz"[1] + ETMC[2]+ NME[3] + RQCD[4] +PNDME[5] + PACS[6]
  - lattice spacing corrections are implemented
  - only large vols., $M_\pi L \geq 3.5$



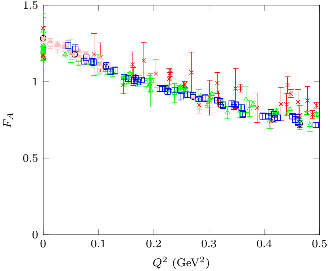
[1] Capitani et al. Modern Phys. A 34 (2019)
[2] Alexandrou et al. PRD 103 (2021)
[3] Park et al. PRD 105 (2022)
[4] Bali et al. JHEP 05 (2020)
[5] Jang et al PRD 109 (2024)
[6] Tsuji et al PRD 109 (2024)

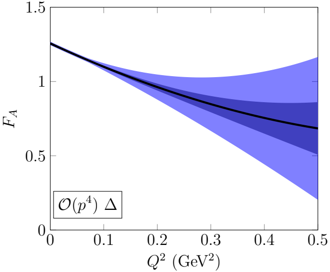
F_A fit



- **$\mathbf{F_A}$ fit procedure:**
  - $\Delta F_{A\chi}$ is added to LQCD errors in the χ^2
  - Some LECs fixed by πN, others have naturalness priors
  - fit range: χ_0^2 plateau $\Longrightarrow M_\pi^{\text{cut}} \simeq 400$ MeV, $\sqrt{Q^2_{\text{cut}}} \simeq 600$ MeV
  - χ^2 accounts for theo uncertainty (difference between orders)

F_A Results





- **F_A results: good description**
  - Δ baryon is a necessary d.o.f. in g_A & $\langle r_A^2 \rangle$
  - accurate description at M_π^{phys}
  - $\mathcal{O}(p^5)$ still needed for full convergence at high M_π, Q^2
  - Evident convergent behaviour
    χ_0^2/dof: $5.22_{3\not\Delta} < 2.87_{3\Delta} < 1.31_{4\Delta}$

Axial charge results

Axial charge, $F_A(0) \equiv g_A$

- Historically LQCD was considered to underpredict g_A
- $g_A = 1.2754(13)_{\rm exp}(2)_{\rm RC}$ disp. β decay [Gorchtein et al JHEP 10 (2021)]
- $g_A^{\text{FLAG 2+1+1}} = 1.263 \pm 0.010$ [FLAG(2024)] almost there, after technical improvements & reduction of exc. state contam.
- However, *the plot thickens...*

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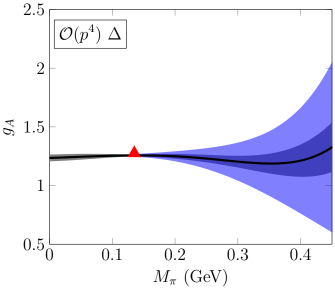
- However, *the plot thickens...*
- Unaccounted percent-level isospin breaking radiative corrections [Cirigliano et al PRL 129 (2022)]
- $\Rightarrow$ LQCD (typically isosymm) "should get" a low $g_A^{\rm QCD} = 1.240(9)$, pure QCD (isosymm) data-driven pred. [Tomalak et al, Universe 12 (2026)]

g_A from F_A analysis



- $\boxed{g_A(M_{\pi(\mathrm{phys})}) = 1.257 \pm 0.011}$ ($\simeq$[FA & LAR, PRD 105 (2022)])
  compat. g_A^{QCD} and FLAG,
  $\sim 1.5\sigma$ lower than "classic" β decay [Gorchtein]

	This work	FLAG	Exp [Gorchtein]	Exp g_A^{QCD} [Tomalak]
g_A	1.257 ± 0.011	1.263 ± 0.010	$1.2754 \pm 0.0013_{\mathrm{exp}} \pm 0.0002_{\mathrm{RC}}$	1.240 ± 0.009

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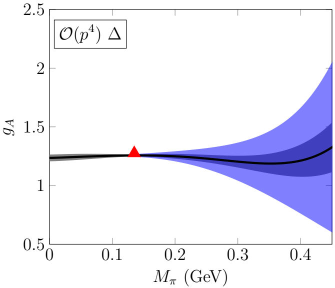

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- $d_{16} = -0.82 \pm 0.72\ \mathrm{GeV}^{-2}$

  $\longrightarrow$ M_π dependence of long range nuclear forces
  - Can not be extracted from πN elastic scattering
  - In line with $d_{16} = -1.0 \pm 1.0\ \mathrm{GeV}^{-2}$ from $\pi N \to \pi\pi N$

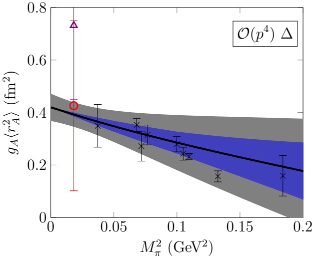
    [Siemens et al. PRC 96 (2017)] (EOMS corrected)

Axial radius results

$F_A = g_A\left(1 + \frac{1}{6}\boxed{\langle r_A^2\rangle}\, q^2\right)$ axial radius

- loop with Δ also influences M_π^2 slope
  - LEC d_{22} compatible with $\mathcal{O}(p^3)$ π electroprod. [Guerrero et al. PRD, 102 (2020)]

- $\boxed{\langle r_A^2\rangle(M_{\text{phys}}) = 0.312 \pm 0.037\ \text{fm}^2}$



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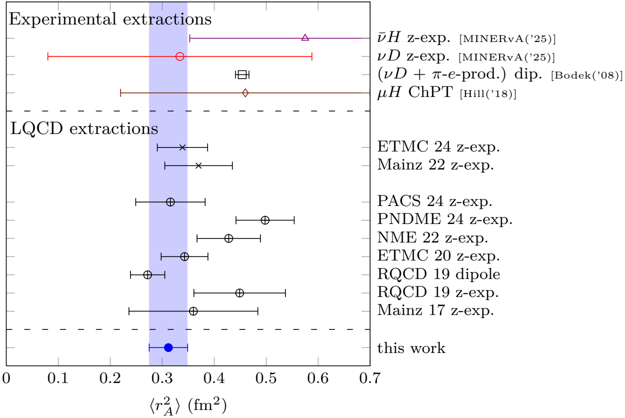


- recent MINERvA measurements from $\bar{\nu}H$ and νD
  - large error (z-exp)
- z-exp value varies significantly depending on the order
  - LQCD z-exp avg $(k_{\max} = 6)$ $\langle r_A^2 \rangle^{\text{phys}} = 0.369 \pm 0.030$ fm^2 [Meyer 2601.02676]
- $\Rightarrow Q^2$ parametrization is critical
- mismatch between LQCD and empirical determinations (model dependent)
- Our parameterisation is based on QCD and estimates uncertainties
  - $\langle r_A^2 \rangle (M_{\text{phys}}) = 0.312 \pm 0.037$ fm^2 in line with most individual LQCD and mild tension with $\bar{\nu}H$

- Successful description of LQCD $F_A(q^2)$ using $\mathcal{O}(p^4)$ relativistic ChPT
  - describes data in $M_\pi \leq 400$ MeV, $\sqrt{Q^2} \leq 600$ MeV
  - $\Delta(1232)$ is a necessary dof to describe F_A and $\pi N \to \pi N / \pi\pi N$

- We provide a model-independent and systematically improvable parametrization of M_π & Q^2, offering a framework for extrapolating LQCD results and for improving predictions of low-energy weak interactions involving nucleons

- We extract $g_A^{\text{phys}} = 1.257 \pm 0.011$ in mild tension with "classic" β decay [Gorchtein (2021)] & compatible with recent pure QCD data-driven [Tomalak (2026)]

- We extract $\langle r_A^2 \rangle^{\text{phys}} = 0.312 \pm 0.037\ \text{fm}^2$ in mild tension with $\bar{\nu} H$
  - Existing mismatch between the experimental and lattice extraction of $\langle r_A^2 \rangle$
  - New experiments on light targets are most needed for F_A, and $\langle r_A^2 \rangle$ in particular

- LECs d_{16} and d_{22} extracted in agreement with phenomenology
  - Combined analysis with πN scatt and π-electroprod would fully exploit the ChPT potential

Thank you!

$$G_P(t) = \frac{4mg_{\pi N}F_\pi}{M_\pi^2 - t} - \frac{2}{3}\,g_A\,m^2\,r_A^2\, + \mathcal{O}(t, M_\pi^2)$$

[Bernard et al PRD 50 (1994)]

Extra 2

	$\mathcal{O}(p^3)$ $\not\Delta$	$\mathcal{O}(p^3)$ Δ	$\mathcal{O}(p^4)$ Δ
$\mathring{g}_A$ (free)	1.1603 ± 0.0066	1.1860 ± 0.0066	1.236 ± 0.031
d_{16} (GeV^{-2}) (free)	-0.959 ± 0.044	1.043 ± 0.054	-0.82 ± 0.72
d_{22} (GeV^{-2}) (free)	1.273 ± 0.078	5.2 ± 1.8	1.1 ± 1.4
h_A	-	1.35	1.35
g_1 (free)	-	-1.19 ± 0.63	0.40 ± 0.46
c_1 (GeV^{-1})	-	-	-1.15
c_2 (GeV^{-1})	-	-	1.57
c_3 (GeV^{-1})	-	-	-2.54
c_4 (GeV^{-1})	-	-	2.61
a_1 (GeV^{-1})	-	-	0.90
b_1 (GeV^{-2}) (free)	-	-	-0.3 ± 5.0
b_2 (GeV^{-2}) (free)	-	-	2.6 ± 2.0
$\tilde{b}_4$ (GeV^{-2}) (free)	-	-	-13.04 ± 0.92
x_1 (fm^{-2}) (free)	-6.1 ± 5.6	-2.1 ± 5.8	3 ± 14
x_2 (fm^{-2}) (free)	-5.4 ± 2.4	-3.6 ± 2.5	-2.3 ± 3.8
x_3 (fm^{-1}) (free)	-0.06 ± 0.21	0.15 ± 0.22	0.54 ± 0.40
x_4 (fm^{-1}) (free)	-0.33 ± 0.28	-0.10 ± 0.29	0.12 ± 0.49
y_1 (fm^{-2} GeV^{-2}) (free)	-101 ± 39	-63 ± 43	-47 ± 102
y_2 (fm^{-2} GeV^{-2}) (free)	-31 ± 20	-16 ± 21	-12 ± 41
y_3 (fm^{-1} GeV^{-2}) (free)	-0.6 ± 1.4	0.9 ± 1.5	2.9 ± 3.5
y_4 (fm^{-1} GeV^{-2}) (free)	-2.0 ± 1.6	-0.5 ± 1.7	0.7 ± 3.7
$\mathring{m}$ (GeV)	0.874	0.855	0.855
$\mathring{m}_\Delta$ (GeV)	-	1.166	1.166
χ^2/dof	$56.98/(158-11) = 0.388$	$47.84/(158-12) = 0.328$	$20.27/(158-15) = 0.142$
χ_0^2/dof	$767.17/(158-11) = 5.22$	$419.06/(158-12) = 2.87$	$186.63/(158-15) = 1.31$

g_A Backup

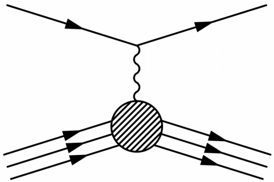
	$\mathcal{O}(p^4)$ Δ
$\mathring{g}_A$ (free par.)	1.240 ± 0.046
d_{16} (free par.)	-0.88 ± 0.88
h_A	$h_A^{N_c} = 1.35$
g_1	$-\lvert g_1^{N_c} \rvert = -2.29$
c_1	-1.15 ± 0.05
c_2	1.57 ± 0.10
c_3	-2.54 ± 0.05
c_4	2.61 ± 0.10
a_1	0.90
$\tilde{b}_4$ (free par.)	-12.3 ± 1.0
$\mathring{m}$	0.855
$\mathring{m}_\Delta$	1.166
χ^2/dof	$11.14/(43-7) = 0.31$

Electroweak structure of the nucleon

- Structure of protons and neutrons is encoded in form factors
  - EM current, $\color{red}{V^\mu} = \bar{q} Q \gamma^\mu q$,
    $\langle N | V^\mu | N \rangle = \bar{u}' \left[\gamma^\mu \color{red}{F_1}(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} \color{red}{F_2}(q^2) \right] u$
    - $F_{1,2}$ are the Dirac and Pauli ffs
    - $G_E = F_1 - \frac{Q^2}{4m_N^2} F_2, \quad G_M = F_1 + F_2$
    - slope=charge radius, $\langle r_E^2 \rangle = 6 \left. \frac{\mathrm{d}G_E}{\mathrm{d}q^2} \right|_0$
  - Axial current, $\color{blue}{A^\mu} = \bar{q} \gamma^\mu \gamma_5 q$,
    $\langle N | A^\mu | N \rangle = \bar{u}' \left\{ \gamma_\mu \color{blue}{F_A}(q^2) + \frac{q_\mu}{2m_N} \tilde{G}_P(q^2) \right\} \gamma_5 u$
  - $\color{blue}{F_A}$ is the axial ff
    - $\color{blue}{F_A(0)} \to$ related to spin distribution
  - Weak interaction is $\color{red}{V} - \color{blue}{A}$

    $\Rightarrow F_A$ is exrtracted from different ν scattering exp. with significant uncertainty

    - F_A is a key input to ν oscillation experiments



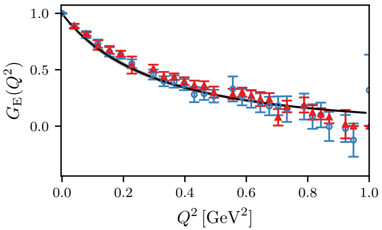


Figure 6: Djukanovic et al. [2102.07460]