Extracting the nucleon axial form factor from recent LQCD data using NNLO ChPT

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RUHR UNIVERSITÄT BOCHUM RUB

Extraction of the nucleon axial form factor from Lattice QCD using NNLO chiral perturbation theory

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(Dated: July 28, 2026)

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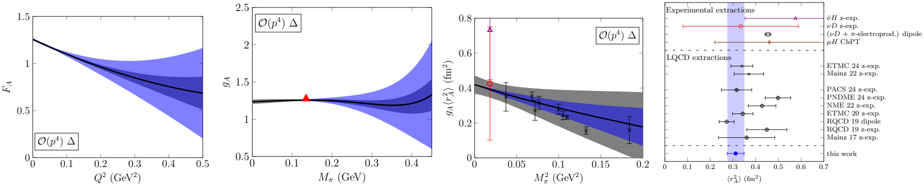


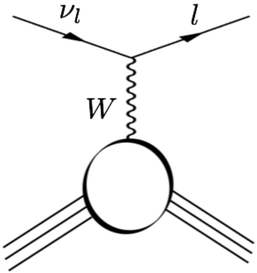


Figure 1: νQES [PRD103(2021)]

- Weak interaction is $V - A$
  - $\langle N'| V |N\rangle$: corresponds to precisely determined EM ff
  - $\langle N'| A^\mu |N\rangle = \bar{u}' \left\{\gamma_\mu F_A(q^2) + \frac{q_\mu}{2m_N}\tilde{F}_P(q^2)\right\}\gamma_5 u$

    - axial ff, $F_A(Q^2)$, and induced pseudoscalar ff, $\tilde{F}_P$, with $A^\mu = \bar{q}\gamma^\mu\gamma_5 q$
    - $\tilde{F}_P$ pion-pole-dominated

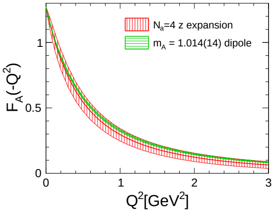
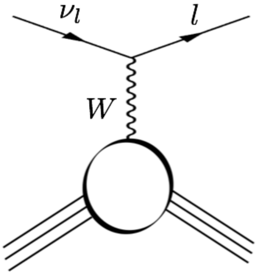


Figure 2: νD extract. [Meyer et al. 1603.03048]



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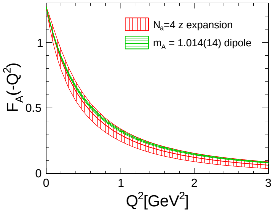

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- Axial form factor, $F_A(Q^2)$
  - fundamental property, e.g. $F_A(0)$ related to spin distribution
  - shapes the νN elastic and quasielastic scattering cross section, upon which neutrino oscillation experimets critically rely

    F_A is extracted from different ν scattering experiments with significant uncertainty (old bubble chamber ANL, BNL & FNL)

    $\Longrightarrow$ F_A is a key input to ν oscillation experiments

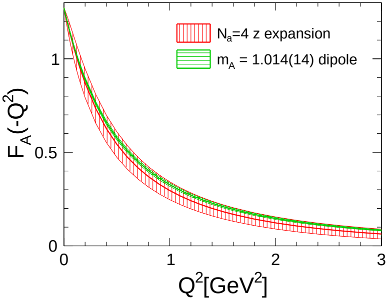


Figure 3: νD extract. [Meyer et al. 1603.03048]

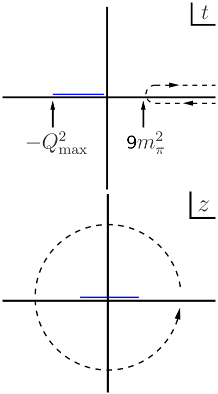
- F_A **Experimental determinations**
  - old bubble chamber νD (ANL, BNL, FNL) data
  - Recent remarkable measurement:
    $\bar{\nu} p \to \mu n$ in MINERvA
    subtracting carbon from plastic [Cai et al Nature 614 (2023)]
- $F_A(q^2) = g_A \left[1 + \frac{1}{6}\langle r_A^2 \rangle q^2 + \mathcal{O}(q^4)\right]$
- $\langle r_A^2 \rangle$ is a benchmark quantity for exp. and th.
  - interesting extraction from weak muon capture in muonic hydrogen (μH): $\mu^- p \to \nu_\mu n$
    chiral Ward id.: $\bar{g}_P = \bar{g}_P(\langle r_A^2 \rangle) + \mathcal{O}(t^2)$, no z-exp.
    (MuCap)[Andreev PRC 91(2015)] [Hill, Rept. Prog. Phys. 81 (2018)]
  - also related to pion-electroproduction in ChPT

- **LQCD systematics**
  1. lattice finite volume, $M_\pi L$
  2. discretization spacing, a
  3. $M_\pi^{\mathrm{LQCD}} > M_\pi^{\mathrm{phys}}$

  - chiral continuum extrapolation, relies on a parameterisation of the dependencies

- **ChPT**
  - EFT of QCD at low E
  - Parameterisation of M_π based on symmetries of QCD
  - Also used in the context of $M_\pi L$ and a effects
  - Useful to deal with excited state contamination

Axial form factor at low q^2



- Experimental and lattice q^2 parametrisation is delicate:
  - dipole ansatz
  - z-expansion at given order (sizable uncertainty)
  - ...

  $\Rightarrow$ different $\langle r_A^2 \rangle$, and F_A

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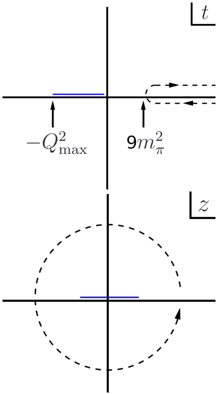

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- We parameterise M_π and q^2 dep. with **ChPT**:
  model independency, uncertainty estimation and can connect to other processes
- Our work on F_A:
  1. compute it in ChPT
  2. fit to a combined set of recent LQCD data
  3. extract F_A & $\langle r_A^2 \rangle$ model-independently and estimating the systematics

Nucleon axial form factor, F_A

- We compute NNLO $\mathcal{O}(p^4)$ F_A in relativistic ChPT
- Relativistic EOMS has been shown to improve HB results
  - e.g. μ_B [Geng et al, PRL 101 (2008)]
- Explicit $\Delta(1232)$ sizable and beneficial:
  - polarizabilities, couplings and form factors , e.g. [Hacker et al PRC 72 (2005)],[Yao et al JHEP 5 (2016)],[Hiller-Blin PRD 96 (2017)],[Thürmann et al PRC 103 (2021)]
  - Δ counting: small scale expansion $\delta = m_\Delta - m_N \sim \mathcal{O}(p)$
- $F_A(q^2) = g_A \left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \mathcal{O}(q^4)\right]$
- We start by focusing on the M_π dep. of $F_A(0) = g_A$

$F_A(0) = g_A$ Calculation

[FA & Alvarez-Ruso PRD 105 (2022)]

- d_{16}: key source of uncertainty in m_q dependence of nuclear properties:
  - ground-state and binding energies [Beane et al. Nucl. Phys. A 717 (2003), Berengut et al. PRD 87 (2013), Epelbaum et al Eur. Phys. J. A 49 (2013)]
  - $\mathcal{L}_{\pi N}^{(1)} \Longrightarrow \mathring{g}_A, \quad \mathcal{L}_{\pi N}^{(2)} \Longrightarrow c_1, \, \tilde{c}_4$,
    $\mathcal{L}_{\pi N}^{(3)} = \bar{\Psi} \frac{d_{16}}{2} \gamma^\mu \gamma_5 \langle \chi_+ \rangle u_\mu \Psi + \ldots$
  - $\mathcal{L}_{\pi N \Delta}^{(1)} \Longrightarrow h_A, \, g_1, \quad \mathcal{L}_{\pi \Delta}^{(2)} \Longrightarrow a_1, \quad \mathcal{L}_{\pi N \Delta}^{(2)} \Longrightarrow \tilde{b}_4$

$$g_A = \mathring{g}_A + 4 d_{16} M_\pi^2 + g_{A\mathrm{loop}}^{(3)\not{\Delta}}(\mathring{g}_A; M_\pi) + g_{A\mathrm{loop}}^{(3)\Delta}(\mathring{g}_A, h_A, g_1; M_\pi)$$
$$+ g_{A\mathrm{loop}}^{(4)\not{\Delta}}(\mathring{g}_A, c_1, \tilde{c}_4; M_\pi) + g_{A\mathrm{loop}}^{(4)\Delta}(\mathring{g}_A, h_A, g_1, c_1, a_1, \tilde{b}_4; M_\pi) + \mathcal{O}(p^5),$$

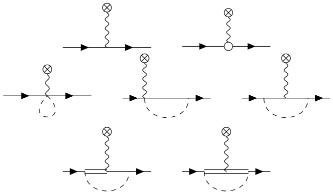


Figure 4: $\mathcal{O}(p)$ and $\mathcal{O}(p^3)$ (w. f. renormalisation not shown)

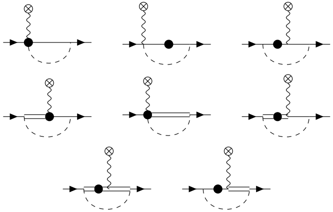
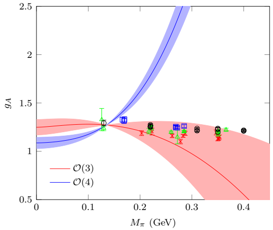
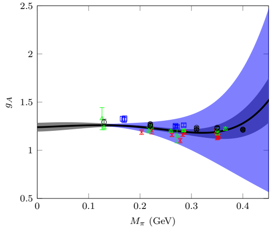


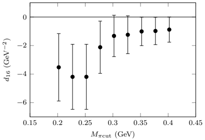
Figure 5: $\mathcal{O}(p^4)$

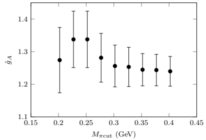
What we learn from $g_A(M_\pi)$ in ChPT

1. πN elast. & inelast scatt ($\not{\Delta}$) LECs relativistic $\mathcal{O}(p^4)$ not compatible with $g_A^{\mathrm{LQCD}}(M_\pi)$
2. Δ necessary
3. Slow convergence
   - **Difference between orders $\simeq$ theoretical uncertainty** [Epelbaum et al EPJA 53 (2015)]
     $\Delta g_{A\chi}^{(4)} = \max\left\{\left(\frac{M_\pi}{\Lambda}\right)^4 |\mathring{g}_A|, \left(\frac{M_\pi}{\Lambda}\right)^2 \left|g_A^{(3)}\right|, \frac{M_\pi}{\Lambda}|g_A^{(4)}|\right\}$



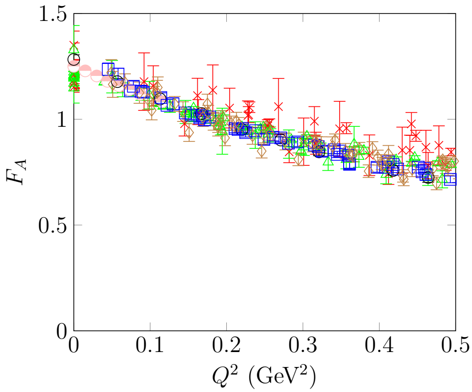






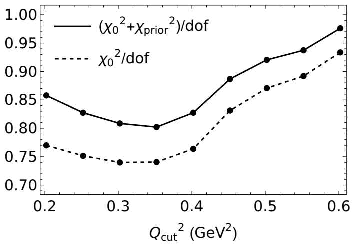
Axial form factor

- We perform a **meta-analysis of large set of recent LQCD results**
  - Many recent works $\Rightarrow$ substantial improvements
  - "Mainz"[1] + ETMC[2]+ NME[3] + RQCD[4] +PNDME[5] + PACS[6]
  - lattice spacing corrections are implemented
  - only large vols., $M_\pi L \geq 3.5$



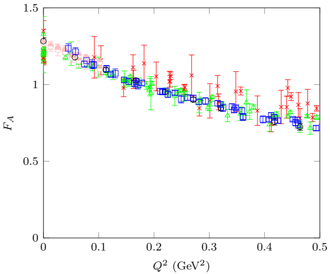
[1] Capitani et al. Modern Phys. A 34 (2019)
[2] Alexandrou et al. PRD 103 (2021)
[3] Park et al. PRD 105 (2022)
[4] Bali et al. JHEP 05 (2020)
[5] Jang et al PRD 109 (2024)
[6] Tsuji et al PRD 109 (2024)

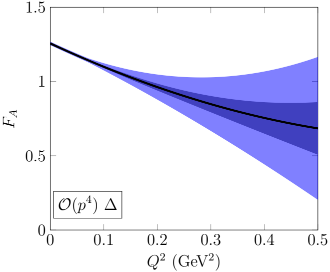
F_A fit



- $\mathbf{F_A}$ **fit procedure:**
  - $\Delta F_{A\chi}$ is added to LQCD errors in the χ^2
  - LECs have naturalness priors
  - fit range: χ_0^2 plateau $\Longrightarrow M_\pi^{\mathrm{cut}} \simeq 400$ MeV, $\sqrt{Q^2_{\mathrm{cut}}} \simeq 600$ MeV
  - χ^2 accounts for theo uncertainty (difference between orders)

F_A Results





- **F_A results: good description**
  - Δ baryon is a necessary d.o.f. in g_A & $\langle r_A^2 \rangle$
  - accurate description at M_π^{phys}
  - $\mathcal{O}(p^5)$ still needed for full convergence at high M_π, Q^2
  - Evident convergent behaviour
    χ_0^2/dof: $5.22_{3\not\Delta} < 2.87_{3\Delta} < 1.31_{4\Delta}$

Axial charge results

Axial charge, $F_A(0) \equiv g_A$

- Historically LQCD was considered to underpredict g_A
- $g_A = 1.2754(13)_{\mathrm{exp}}(2)_{\mathrm{RC}}$ disp. β decay [Gorchtein et al JHEP 10 (2021)]
- $g_A^{\text{FLAG 2+1+1}} = 1.263 \pm 0.010$ [FLAG(2024)] almost there, after technical improvements & reduction of exc. state contam.
- However, *the plot thickens...*

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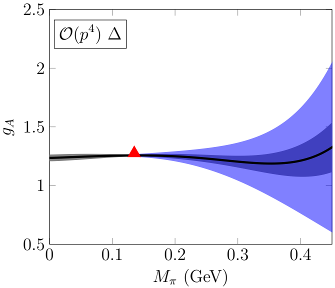
- However, *the plot thickens...*
- Unaccounted percent-level isospin breaking radiative corrections [Cirigliano et al PRL 129 (2022)]
- $\Rightarrow$ LQCD (typically isosymm) "should get" a low $g_A^{\text{QCD}} = 1.240(9)$, pure QCD (isosymm) data-driven pred. [Tomalak et al, Universe 12 (2026)]

g_A from F_A analysis



- $\boxed{g_A(M_{\pi(\text{phys})}) = 1.257 \pm 0.011}$ ($\simeq$[FA & LAR, PRD 105 (2022)])
  compat. g_A^{QCD} and FLAG,
  $\sim 1.5\sigma$ lower than "classic" β decay [Gorchtein]

	This work	FLAG	Exp [Gorchtein]	Exp g_A^{QCD} [Tomalak]
g_A	1.257 ± 0.011	1.263 ± 0.010	$1.2754 \pm 0.0013_{\text{exp}} \pm 0.0002_{\text{RC}}$	1.240 ± 0.009

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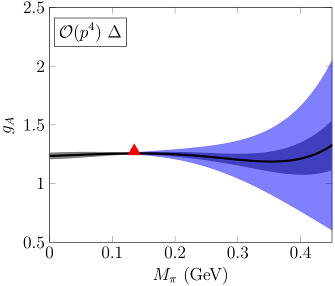

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- $d_{16} = -0.82 \pm 0.72\ \text{GeV}^{-2}$

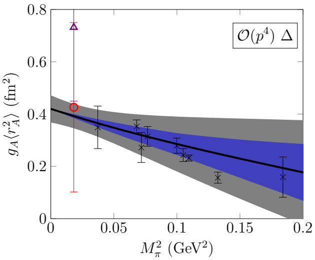
  $\longrightarrow$ M_π dependence of long range nuclear forces
  - Can not be extracted from πN elastic scattering
  - In line with $d_{16} = -1.0 \pm 1.0\ \text{GeV}^{-2}$ from $\pi N \to \pi\pi N$
    [Siemens et al. PRC 96 (2017)] (EOMS corrected)

Axial radius results

$F_A = g_A \left(1 + \frac{1}{6} \boxed{\langle r_A^2 \rangle} q^2\right)$ axial radius

- loop with Δ also influences M_π^2 slope
  - LEC d_{22} compatible with $\mathcal{O}(p^3)$ π electroprod. [Guerrero et al. PRD, 102 (2020)]

- $\boxed{\langle r_A^2 \rangle (M_{\text{phys}}) = 0.312 \pm 0.037 \text{ fm}^2}$



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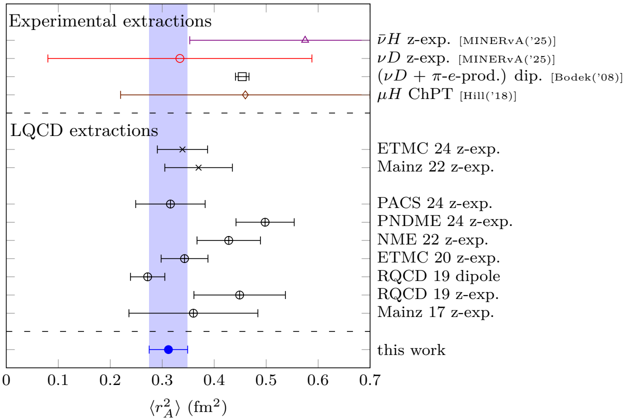


- recent MINERvA measurements from $\bar{\nu}H$ and νD
  - large error (z-exp)
- z-exp value varies significantly depending on the order
  - LQCD z-exp avg $(k_{\max} = 6)$ $\langle r_A^2 \rangle^{\text{phys}} = 0.369 \pm 0.030\ \text{fm}^2$ [Meyer 2601.02676]
- $\Rightarrow Q^2$ parametrization is critical
- mismatch between LQCD and empirical determinations (model dependent)
- Our parameterisation is based on QCD and estimates uncertainties
  - $\langle r_A^2 \rangle (M_{\text{phys}}) = 0.312 \pm 0.037\ \text{fm}^2$ in line with most individual LQCD and mild tension with $\bar{\nu}H$

- Successful description of LQCD $F_A(q^2)$ using $\mathcal{O}(p^4)$ relativistic ChPT
  - describes data in $M_\pi \leq 400$ MeV, $\sqrt{Q^2} \leq 600$ MeV
  - $\Delta(1232)$ is a necessary dof to describe F_A and $\pi N \to \pi N/\pi\pi N$

- We provide a model-independent and systematically improvable parametrization of M_π & Q^2, offering a framework for extrapolating LQCD results and for improving predictions of low-energy weak interactions involving nucleons

- We extract $g_A^{\text{phys}} = 1.257 \pm 0.011$ in mild tension with "classic" β decay [Gorchtein (2021)] & compatible with recent pure QCD data-driven [Tomalak (2026)]

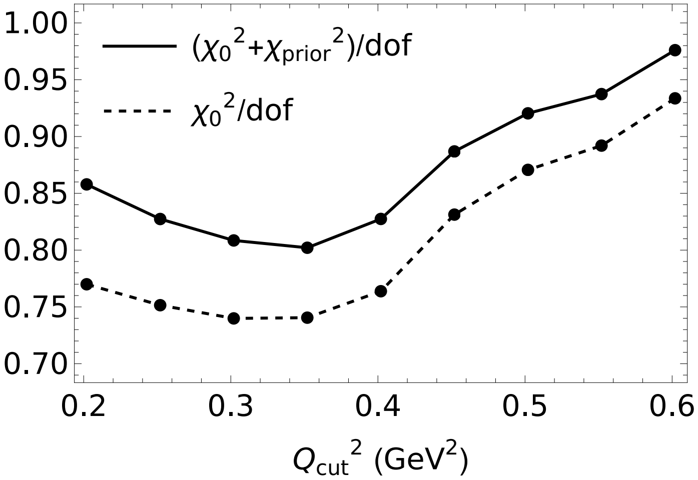
- We extract $\langle r_A^2 \rangle^{\text{phys}} = 0.312 \pm 0.037\ \text{fm}^2$ in mild tension with $\bar{\nu}H$
  - Existing mismatch between the experimental and lattice extraction of $\langle r_A^2 \rangle$
  - New experiments on light targets are most needed for F_A, and $\langle r_A^2 \rangle$ in particular

- LECs d_{16} and d_{22} extracted in agreement with phenomenology
  - Combined analysis with πN scatt and π-electroprod would fully exploit the ChPT potential

Thank you!

(want more? I discuss meson-baryon scattering on the lattice on Thursday)

When talking about F_A explain chi2 definition and emphasise that Δ-full reduces chi2 order by order (sign of convergence), particularly convergence in Q^2, like commented in paper



- EM form factors of the nucleon
- Advent of the proton radius puzzle

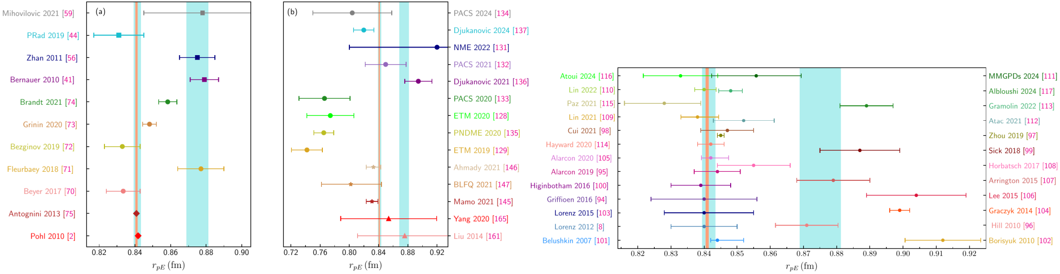
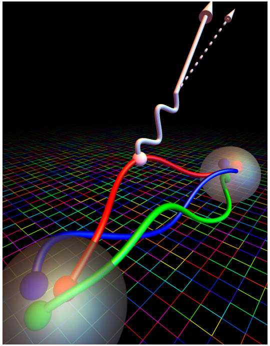


Figure 6: Fig. from [Goharipour et al., Nucl.Phys.B 1017 (2025)].

- Active QCD subject
  - Lattice: see Fig. (b)
  - Dispersive methods, e.g. [Lin et al. PRL 128 (2022) 052002, Alarcon et al. PRC 99 (2019) 044303]

Electromagnetic form factor at low q^2



- **Lattice QCD**
  - Nucleon f.f. is a benchmark for LQCD
  - Uncertainties reduced for unphysical large M_π
  - Technical difficulties $\to$ recent progress
  - Experimental and lattice q^2 parametrisation:

    - dipole ansatz
    - z-expansion of different orders
    - ...

    $\Longrightarrow$ different $\langle r_1^2 \rangle$, and $F_{\rm EM}$ in general
- Theoretical input needed

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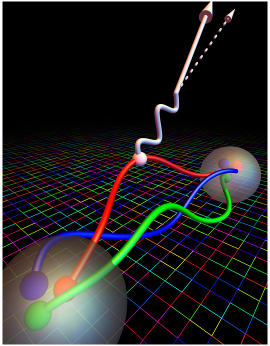

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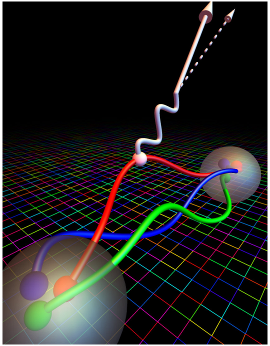
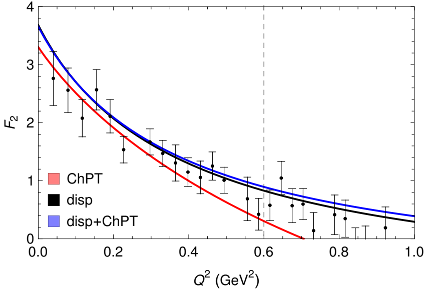
    $\Longrightarrow$ different $\langle r_1^2 \rangle$, and F_{EM} in general
- Theoretical input needed
  - **Chiral Perturbation Theory** (χPT) $\rightarrow$ parametrise M_π and q^2 dep.
  - **Dispersion theory** $\rightarrow$ enlarge q^2 range
  - **Goal**: Disp+χPT = good q^2 and M_π description

Electromagnetic form factor at low q^2



- **Lattice QCD** parametrisation question
- **Chiral Perturbation Theory** (ChPT)
  - QCD based parametrisation of q^2 and M_π dependencies $\Longrightarrow$ extrapolate lattice results to the phys. point and extract $\langle r_i^2 \rangle$ and κ from the lattice simulations
- **Modern Baryon ChPT**
  - Relativistic EOMS:
    terms that break power counting are absorbed in LECs,
    e.g. κ [Geng et al PRL 101 (2008)], πN [Alarcon et al Annals Phys. 336 (2013)]
  - Relativistic+Dynamical Δ is necessary,
    e.g. πN [Yao et al JHEP 5 (2016)], ffs.[Hiller Blin PRD 96 (2017), Thürmann et al. PRC 103 (2021)]

EM Form Factors



(FA, Di An, Luis Alvarez-Ruso & Stefan Leupold)

- **ChPT** $\to$ parameterise M_π and q^2 dep.
- **Dispersion theory** $\to$ enlarge q^2 range (ρ resonance)
- **Goal**: Disp+χPT
  - $=$ good q^2 and M_π description of the LQCD EM form factors
  - accurate extrapolation to M_π^{phys}

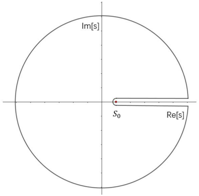
- Enlarge the q^2 range of χPT (ρ dynamics)
  1. Analiticity $\Rightarrow$ Disp. rel. (Cauchy)

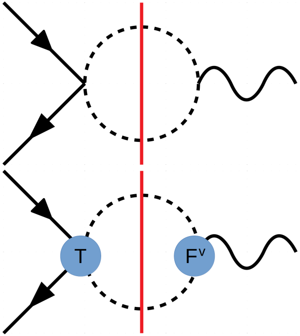
$$F(q^2) = \int_{4M_\pi^2}^{\infty} \frac{ds}{\pi} \frac{\Im F(s)}{s - q^2 - i\epsilon}$$

  2. Unitarity$\Rightarrow$ $\Im F = \frac{1}{2}\sum_n T_{\gamma^* n} T^\dagger_{n\bar{N}N}$, $n = \pi^+\pi^-, \ldots$
     - $\ell = 1$, $\pi\pi$ must be iso-vector (ρ channel)
       $\Rightarrow F_i = \frac{1}{2}F_i^{(s)} + F_i^{(v)}\frac{\sigma^3}{2}$, $F_i^{(v)} = F_i^p - F_i^n$
  3. Using full $N\bar{N}\pi\pi$ and $\gamma^*\pi\pi$ vertices with M_π dep.

$$F(q^2) = \frac{1}{12\pi} \int_{4M_\pi^2}^{\infty} \frac{ds}{\pi} \frac{T\, p_{\rm cm}^3\, F_\pi^{V*}}{s^{1/2}(s - q^2 - i\epsilon)} ,$$

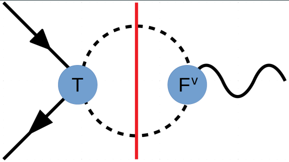
[Granados et al, EPJ A 53 (2017)]

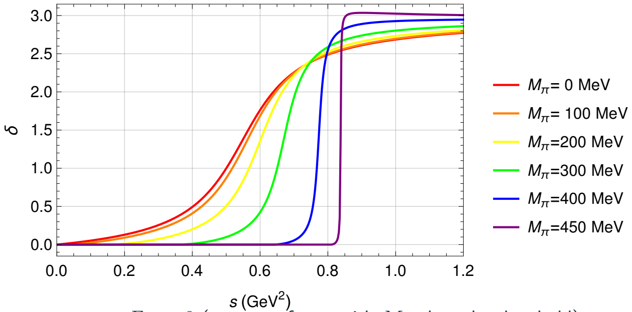




Dispersion theory

- $\pi\pi$ scattering included in F_π^V & T via Omnès with IAM δ (nonperturbative)
- M_π dep. taken into account
  - $\Omega(s) = \exp\left\{ s \int_{4M_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\delta(s')}{s'(s'-s-i\epsilon)} \right\}$
  - We fit NLO IAM δ to physical results [Garcia-Martin PRD 83(2011)]
  - We check that the M_π dependence is realistic
  - Our prediction of $m_\rho(M_\pi)$ agrees well with LQCD [Andersen et al, 1808.05007]





$\rho \nrightarrow \pi\pi$, $\Gamma_\rho \to 0$ (m_ρ runs faster with M_π than the threshold)

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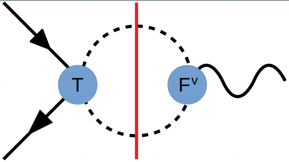
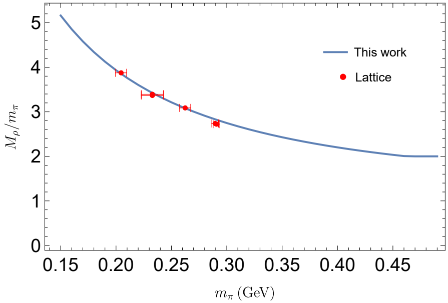




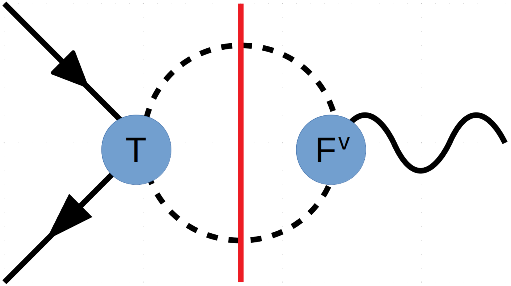
$\rho \nrightarrow \pi\pi$, $\Gamma_\rho \to 0$ (m_ρ runs faster with M_π than the threshold)

Dispersion theory

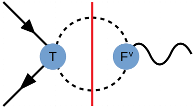
- Our two vertices, T and F_π^V, account for M_π dep.
  - $\boxed{F_\pi^V(s) = [1 + \alpha_v s]\ \Omega(s)}$
  - $\mathrm{Im} F_{2\pi}(s) = F_v^*(s)\ \frac{p_{\mathrm{cm}}^3}{12\pi\sqrt{s}}\ T(s) \in \mathbb{R}$
  - $\boxed{T_i(s) = K_i(s) + \Omega(s)\, P_i + I_i(s)}$
    - P and K determined from p-wave projected πN in ChPT

$$\Rightarrow \boxed{F(q^2) = \frac{1}{12\pi} \int_{4M_\pi^2}^{\infty} \frac{ds}{\pi} \frac{T\ p_{\mathrm{cm}}^3\ F_\pi^{V*}}{s^{1/2}(s - q^2 - i\epsilon)}}$$

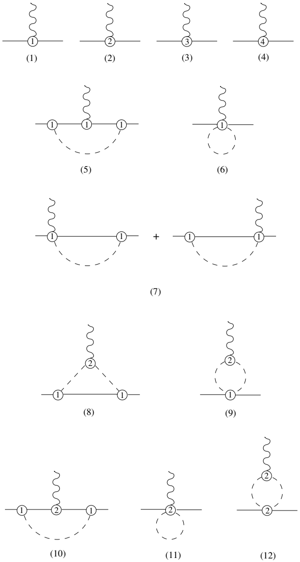
more in backup slides



- $\mathbf{F}_{\mathrm{EM}} = \mathbf{F}^{\mathrm{disp}}_{\mathrm{EM}} +$ **diagrams w/o 2π cut from χPT**
  - $F(q^2) = \int\limits_{4M_\pi^2} \frac{\mathrm{d}s}{\pi} \frac{\mathrm{Im}F_{2\pi}(s)}{s-q^2} + F_{\chi\mathrm{PT\ no\ }2\pi}$
  - $F^{\mathrm{disp}}_{\mathrm{EM}}$



  - $+\chi$PT diagrams



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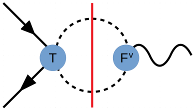
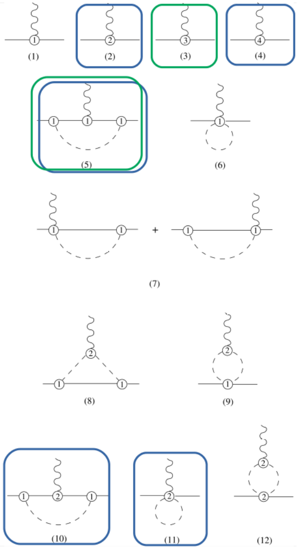


  - $+\chi$PT diagrams
    - Relativistic and with explicit $\Delta(1232)$ [Bauer et al., PRC 86 (2012)]
    - green: $F_1 - F_1(0)$
    - blue: F_2



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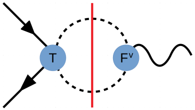
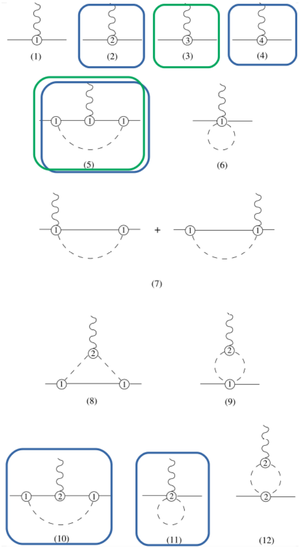


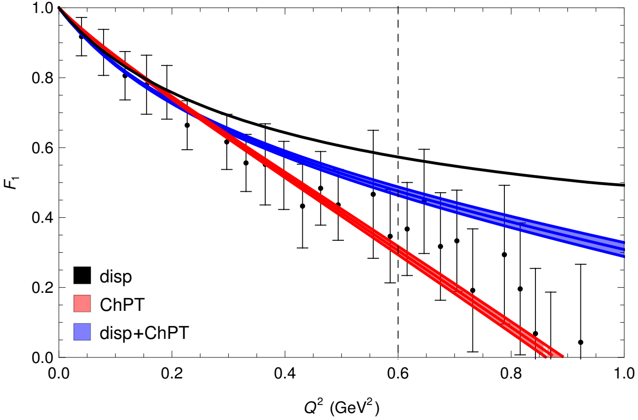
  - $+\chi$PT diagrams
    - Relativistic and with explicit $\Delta(1232)$ [Bauer et al., PRC 86 (2012)]
    - green: $F_1 - F_1(0)$
    - blue: F_2
  - Disp and χPT renormalizations
    - At $\mathcal{O}(p^3)$ disp and χPT agree on the M_π nonanalyticities
    - $F_1^{\mathrm{point}} \sim F_1^{(9)\mathrm{ChPT}} \sim q^2 \log M_\pi$
    - differences in the renorm. are absorbed in LECs



- **Dirac f.f., F_1**

FA, An, Alvarez-Ruso & Leupold PRD 108 (2023)

  - $\langle N | V^\mu | N \rangle = \bar{u}' \left[\gamma^\mu F_1 + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2 \right] u$
  - $F_1 = 1 + \frac{q^2}{6} \left[-12 d_6 + \langle r_1^{2(\log M_\pi)} \rangle \log M_\pi \right] + \mathcal{O}(p^4)$
  - Comparison with LQCD:
    10 CLS ensembles [Djukanovic et al. PRD 103 (2021)] $\longleftarrow$ controlled FV and discret. effects
  - In the χPT and disp+χPT F_1, d_6 is fitted to LQCD
  - Disp $\longrightarrow q^2$ curvature



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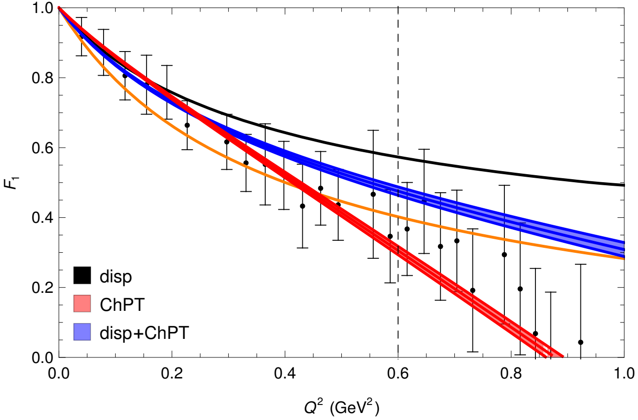
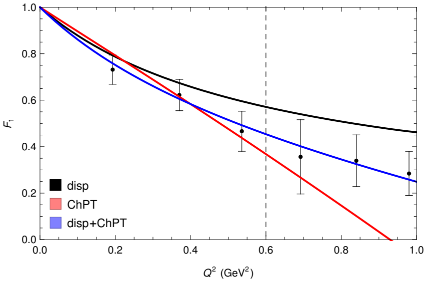
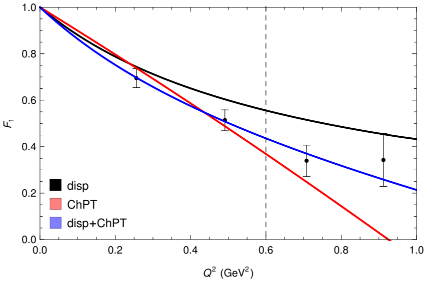




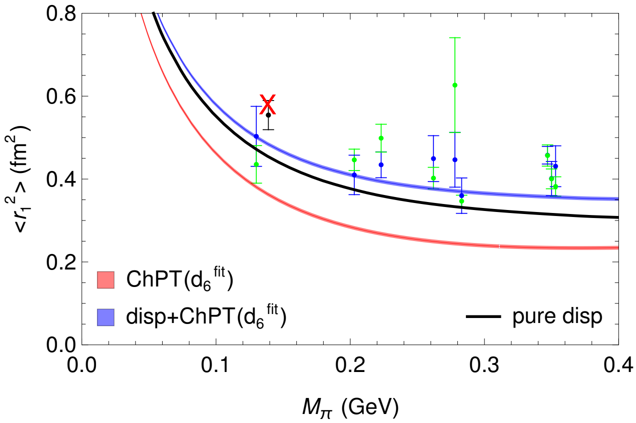
(h) H105 $M_\pi = 0.278$ GeV



(i) N302 $M_\pi = 0.353$ GeV

- **Disp+χPT describes well the M_π dep.**
  - Only one parameter fit, d_6
  - Describes data at Q^2 as high as $Q^2 = 0.6$ GeV^2 ($\sim m_\rho^2$) and all M_π ($M_\pi \lesssim 350$ MeV)
  - outperforms the pure dispersive and plain χPT descriptions

	Disp (prediction)	χPT	**Disp+χPT**
$d_6(\mu = m_\rho)$ (GeV^{-2})	-	0.074 ± 0.010	0.416 ± 0.010
$d_6(\mu = m_N)$ (GeV^{-2})	-	-0.422 ± 0.010	0.155 ± 0.010
χ^2/dof	$108.9/47 = 2.32$	$73.9/(47-1) = 1.61$	$24.6/(47-1) = 0.53$
$\langle r_1^2 \rangle_\text{phys}$ (fm^2)	0.4541	0.3626 ± 0.0047	$\mathbf{0.4838} \pm \mathbf{0.0047}$



- $F_1^{(v)} = 1 + \frac{1}{6}\langle r_1^2\rangle^{(v)} q^2 + \mathcal{O}(q^4)$
- disp. has the $\log M_\pi$ predicted by chiral symmetry breaking
- $\langle r_1^2\rangle^{\mathrm{PDG}} = 0.577$ fm^2, heavy baryon fit to LQCD from [Djukanovic PRD 103(2021)]: $\langle r_1^2\rangle^{\mathrm{HB}} = 0.554 \pm 0.035$ fm^2

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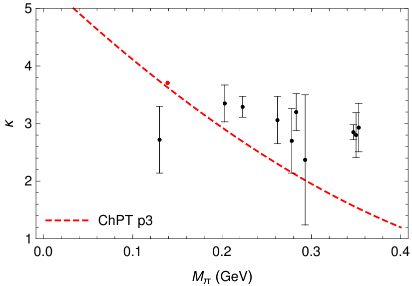
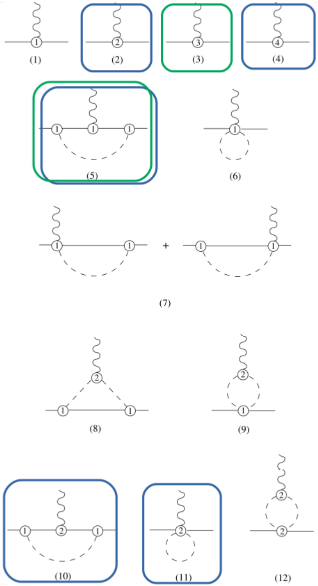
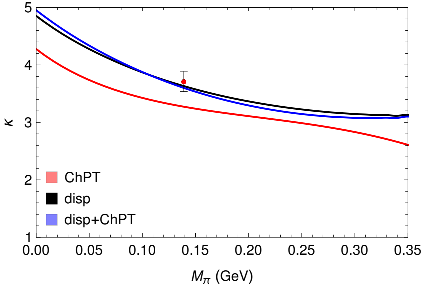
- **Pauli f.f., F_2**
  - $\langle N | V^\mu | N \rangle = \bar{u}' \left[\gamma^\mu F_1 + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2 \right] u,$
    $F_2 = \frac{F_M - F_E}{(1+Q^2/(4m^2))}$
  - $F_2^{(v)} = \kappa^{(v)} \left[1 + \frac{1}{6} \langle r_2^2 \rangle^{(v)} q^2 + \mathcal{O}(q^4) \right]$
  - $\mathcal{O}(p^3)$ χPT is not enough$F_2(0)^{o(3)} = c_6 - \left(\frac{\pi m_N g_A^2}{4\pi^2 F^2} \right) M_\pi$

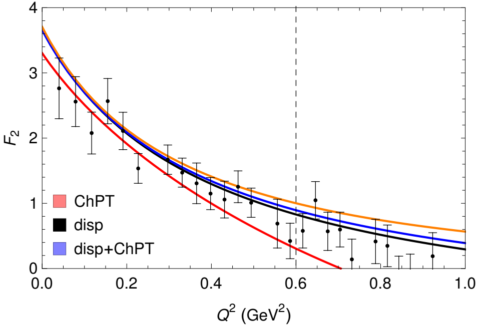


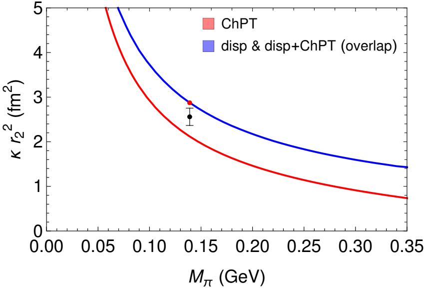
  - We implement the necessary $\mathcal{O}(p^4)$ contributions ($\not{\Delta}$)



- **Pauli f.f., F_2, fit to LQCD**
  - Constraints from πN scattering simplify the calculation
  - We are able to describe the F_2 and extract κ and $\langle r_2^2 \rangle$ (close to the PDG values)
  - When there are several orders and LECs involved it is interesting to perform an analysis of the weight of each order, i.e. a specific study of the convergence of the chiral series and the systematic error (including the systematic error of the LECs themsleves)
    - for the moment we have performed such a study of the chiral convergence only for F_A







Nucleon axial form factor at low q^2

$g_A \equiv F_A(q^2 = 0)$ analysis

$\boldsymbol{g_A(M_\pi)}$ study

FA & Alvarez-Ruso PRD 105 (2021)

Motivations:

1. **test** the LECs extracted from $\boldsymbol{\pi N}$ **scattering**
2. **determine** $\boldsymbol{d_{16}}$ from fit to lattice:
   - source of uncertainty in m_q dependence of nuclear properties (ground-state and binding energies)
3. **study** more deeply the **convergence of** $\boldsymbol{g_A}$ in χPT

$g_A \equiv F_A(q^2 = 0)$ analysis

- $\left.\begin{array}{l} \pi N \to \pi\pi N \Rightarrow d_{16}, c_i \\ \pi N \to \pi N \Rightarrow c_i \end{array}\right\} \to$ combined fit Refs. [*]
- $g_A^{\text{phys}} = g_A(M_\pi^{\text{phys}}) \Rightarrow \mathring{g}_A$

$\Rightarrow$ $\boxed{\text{Bad } \Delta\text{-less } g_A(M_\pi) \text{ prediction}}$

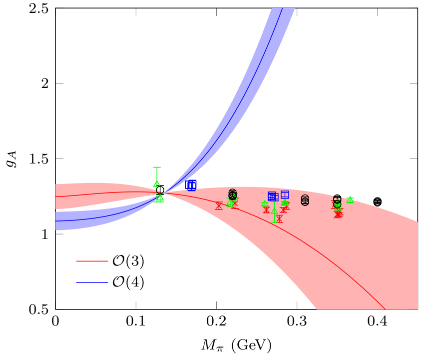


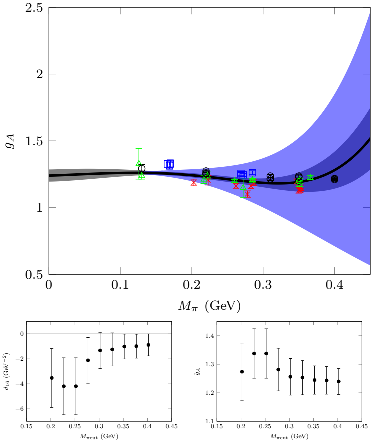
Table 1: $\mathcal{O}(p^4)$: steep rise (OK with [**])

	$\mathcal{O}(p^3)$	$\mathcal{O}(p^4)$
d_{16}	-2.2 ± 1.1	-1.86 ± 0.80
c_1	-	-0.89 ± 0.06
c_2	-	3.38 ± 0.15
c_3	-	-4.59 ± 0.09
c_4	-	3.31 ± 0.13

- $\Rightarrow$ **We include explicit $\Delta(1232)$ and fit d_{16} to lattice**
- $\Rightarrow$ **This reconciles g_A^{LQCD} with the πN LECs, c_i** (with the caveat of a slow convergence)
- we will discuss the whole $F_A(q^2)$ directly

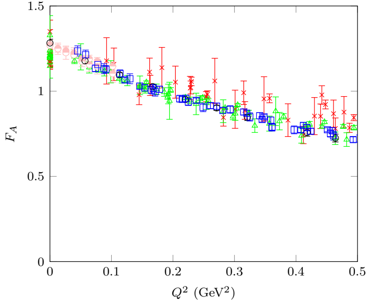
[*] Siemens et al. PRC 94 (2016), Siemens et al. PRC 96 (2017), (value converted to standard EOMS)
[**] Bernard et al PLD 639 (2006)

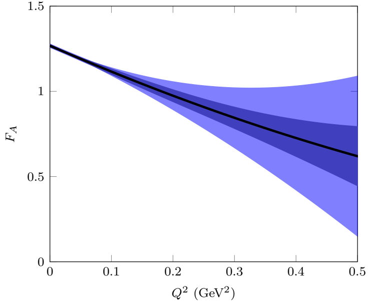
F_A fit



- **$\mathbf{F_A}$ fit procedure:**
  - **Difference between orders $\simeq$ theoretical uncertainty** [Epelbaum EPJA 53 (2015)]
    $\Delta g_{A\chi}^{(4)} = \max\left\{\left(\frac{M_\pi}{\Lambda}\right)^4 |\mathring{g}_A|, \left(\frac{M_\pi}{\Lambda}\right)^2 \left|g_A^{(3)}\right|, \frac{M_\pi}{\Lambda}|g_A^{(4)}|\right\}$
  - $\Delta F_{A\chi}$ is added to LQCD errors in the χ^2
  - LECs have naturalness priors
  - fit range: χ^2 plateau $\Longrightarrow M_\pi^{\mathrm{cut}} \simeq 400$ MeV, $Q^2_{\mathrm{cut}} = 0.36$ GeV^2

F_A fit





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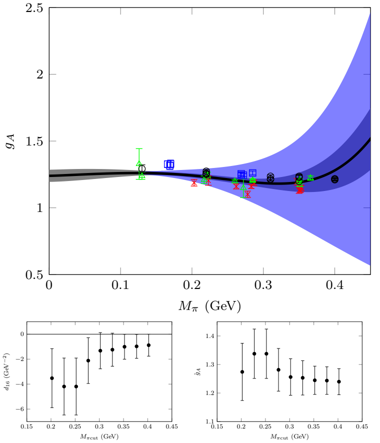
  - LECs have naturalness priors
  - fit range: χ^2 plateau $\Longrightarrow M_\pi^{\rm cut} \simeq 400$ MeV, $Q^2_{\rm cut} = 0.36$ GeV^2
  - lattice spacing corrections are implemented
  - only large vols., $M_\pi L \geq 3.5$
- **Fit results: good description**
  - accurate description at the physical point
  - Δ baryon is a necessary d.o.f.
  - $\mathcal{O}(p^5)$ still needed for full convergence

$F_A(q^2 = 0) = \boxed{g_A}$



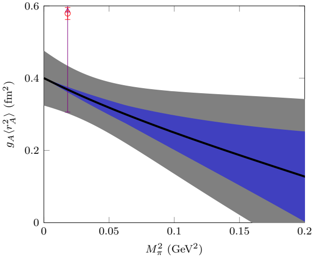
- **Axial charge results** from $F_A(q^2)$ fit
  - $g_A(M_{\pi\text{phys}}) = 1.273 \pm 0.014$ vs $g_A^{\text{exp}} = 1.2754(13)_{\text{exp}}(2)_{\text{RC}}$
    $\Rightarrow$ excellent agreement with exp.
    vs $g_A^{\text{FLAG}} = 1.246 \pm 0.028$

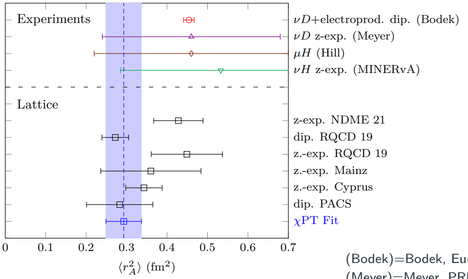
- $g_A(M_\pi) = \mathring{g}_A + 4d_{16}M_\pi^2 + \text{loop}(M_\pi)$
- $\boxed{d_{16} = -1.46 \pm 1.00 \text{ GeV}^{-2}}$

$\longrightarrow$ M_π dependence of long range nuclear forces

  - Can not be extracted from πN elastic scattering
  - In line with $d_{16} = -1.0 \pm 1.0 \text{ GeV}^{-2}$ from $\pi N \to \pi\pi N$
    [Siemens et al. PRC 96 (2017)] (EOMS corrected)

$F_A = g_A\left(1 + \frac{1}{6}\boxed{\langle r_A^2\rangle}\, q^2\right)$ axial radius





(Bodek)=Bodek, Eur. Phys. J. C 53, 349 (2008)
(Meyer)=Meyer, PRD 93, 113015 (2016)
(Hill)=Hill, Rept. Prog. Phys. 81 (2018)
(MINERvA)=Cai, Nature 614 (2023)

- **Our $\mathcal{O}(p^4)$ χPT extraction:**
  - M_π slope driven by loops with Δ
  - $d_{22} = 0.29 \pm 1.69$ GeV^{-2} (no assumptions on $\Delta\Delta\pi$ coupling enlarges error)
    - d_{22} compatible with $\mathcal{O}(p^3)$ π electroprod. Guerrero et al. PRD, 102 (2020)
  - $\boxed{\langle r_A^2\rangle(M_{\text{phys}}) = 0.293 \pm 0.044\ \text{fm}^2}$
  - Empirical determinations (model dependent) are in tension with ours and with most of LQCD extractions
  - Tipically the extracted $\langle r_A^2\rangle^{\text{phys}}$ value varies depending on the parametrisation
  - **Our QCD-based parametrisation leads to a value in line with most individual LQCD extractions**

Axial form factor

- Our ChPT analysis will be updated with new LQCD data
  - [Djukanovic PRD 106 (2022) 074503, Jang PRD 109 (2024) 014503, Alexandrou PRD 109 (2024) 034503, Tsuji PRD 109 (2024) 094505]

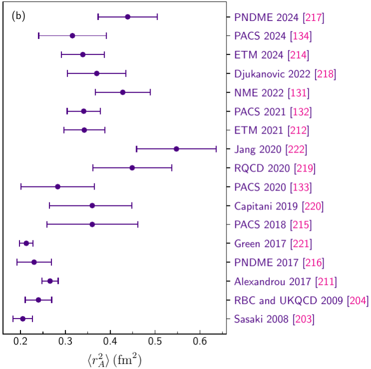


Figure 8: Fig. from [Goharipour et al., Nucl.Phys.B 1017 (2025)].

- Remarcable work of the LQCD community in the recent years
- Isovector Dirac ff, F_1
  - Dispersive 2π cut necessary to describe curvature
  - The dispersive improvement of ChPT outperforms the separate descriptions
  - single LEC fits well the LQCD F_1 for $Q^2 \lesssim 0.6$ GeV^2 and $M_\pi \lesssim 350$ MeV
- F_A
  - Succesful description of LQCD $F_A(q^2)$ using $\mathcal{O}(p^4)$ relativistic χPT
    - Explicit $\Delta(1232)$ necessary
    - Fit describes data in $M_\pi^{\rm cut} \simeq 400$ MeV, $Q^2_{\rm cut} = 0.36$ GeV^2
  - In general there is tension between the experimental and lattice extractions of $\langle r_A^2 \rangle$
  - $\langle r_A^2 \rangle^{\rm phys} = 0.291 \pm 0.052 \text{ fm}^2$ extracted with systematic chiral error
- Relativisitc ChPT (and EFTs in general) are beneficial to control systematics
  - Moreover useful LECs extracted from these calculations: connection with other processes

Backup: F_{EM}

- **Summary of dispersive nucleon F_{EM}**

1. $F(q^2) \approx \int_{4M_\pi^2}^{\Lambda^2} \frac{\mathrm{d}s}{\pi} \frac{\mathrm{Im}F_{2\pi}(s)}{s-q^2-i\epsilon} + F_{\text{ChPT no }2\pi}$
2. $\mathrm{Im}F_{2\pi}(s) = F_\pi^*(s) \frac{p_{\text{cm}}(s)^3}{12\pi\sqrt{s}} T(s)$, need $F_\pi(s)$ and $T(s) \hookrightarrow$ given by $\pi\pi$ scattering
3. We use t_{IAM} NLO with a Blatt-Weiskopf ff on t_2 to go to π smoothly: $\tilde{t}_2 = t_2/(1+r^2p_{\text{cm}}^2)$.
   - We fit $l_2 - 2l_1$ and r to $\delta(s)$ of [García-Martín] at M_π^{phys}
   - Describe well the $m_\rho(M_\pi)$ from LQCD
4. $F_\pi^V(s, M_\pi) = (1+\alpha_V(M_\pi))\Omega(s, M_\pi)$, α_V determined from $\mathrm{d}F_\pi^V(s)/\mathrm{d}s$ in ChPT and $\Omega \to$ improves F_π^V ρ peak
5. T has to respect Watson th. ($\Im F_{2\pi} \in \mathbb{R}$), therefore Omnes solution
   $$T(s) = K(s) + \Omega(s)\,P + \Omega(s) \int_{4M_\pi^2}^{\Lambda^2} \frac{ds'}{\pi} \frac{\sin\delta_1(s')\,K(s')}{|\Omega(s')|\,(s'-s-i\epsilon)}$$
   - P and K determined from ChPT πN p-wave projected
6. Subtrctions on $T_{1,2}$ and $F_{1,2}$ if M_π-indep
   - T_1 subtracted, $P_1 = P_1^{\text{LO}}$, $T_1 = T_1^{\text{LO}}$ enough for leading 1-loop F_1. (F_1 also trivially subtracted)
   - T_2 and F_2 unsubtracted due to M_π dependences (but still the Λ dep. mild)

Backup

- EM f.f.
  - $V^\mu = \bar{q} Q \gamma^\mu q$,
    $\langle N(p')| V^\mu(0) |N(p)\rangle = \bar{u}' \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2(q^2)\right] u$
  - $G_E^N = F_1^N - \frac{Q^2}{4m_N^2} F_2^N, \quad G_M^N = F_1^N + F_2^N$
  - $F_1 = \; 1 \times \left[1 + \frac{1}{6}\langle r_1^2\rangle q^2 + \mathcal{O}(q^4)\right]$
    $F_1(0) = F_E(0) = 1$ electric charge
  - $F_2 = \kappa \left[1 + \frac{1}{6}\langle r_2^2\rangle q^2 + \mathcal{O}(q^4)\right]$
    $\kappa = F_M(0) = \mu - 1$ anom. magn. mom.
  - $\langle r_E^2\rangle = \langle r_1^2\rangle + \frac{3}{2m_N^2}\kappa$ proton radius puzzle
  - $F_i = \frac{1}{2} F_i^{(s)} + F_i^{(v)} \frac{\sigma^3}{2}$
  - $F_1^{(v)} = \; 1 \times \left[1 + \frac{1}{6}\langle r_1^2\rangle^{(v)} q^2 + \mathcal{O}(q^4)\right]$
    $F_1^{(v)}(0) = F_E^{(v)}(0) = 1$ electric charge
  - $F_2^{(v)} = \kappa^{(v)} \left[1 + \frac{1}{6}\langle r_2^2\rangle^{(v)} q^2 + \mathcal{O}(q^4)\right]$
    $\kappa^{(v)} = F_M^{(v)}(0) = \mu^{(v)} - 1$ magnetic moment
  - $\langle r_E^2\rangle^{(v)} = \langle r_1^2\rangle^{(v)} + \frac{3}{2m_N^2}\kappa^{(v)}$ proton radius puzzle

$$G_P(t) = \frac{4mg_{\pi N}F_\pi}{M_\pi^2 - t} - \frac{2}{3}\, g_A\, m^2\, r_A^2\, + \mathcal{O}(t, M_\pi^2)$$

[Bernard et al PRD 50 (1994)]

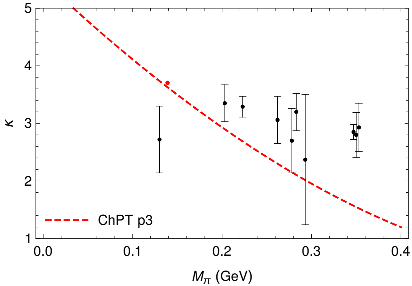
Extra 2

	$\mathcal{O}(p^3)$ $\not\Delta$	$\mathcal{O}(p^3)$ Δ	$\mathcal{O}(p^4)$ Δ
$\mathring{g}_A$ (free)	1.1603 ± 0.0066	1.1860 ± 0.0066	1.236 ± 0.031
d_{16} (GeV^{-2}) (free)	-0.959 ± 0.044	1.043 ± 0.054	-0.82 ± 0.72
d_{22} (GeV^{-2}) (free)	1.273 ± 0.078	5.2 ± 1.8	1.1 ± 1.4
h_A	-	1.35	1.35
g_1 (free)	-	-1.19 ± 0.63	0.40 ± 0.46
c_1 (GeV^{-1})	-	-	-1.15
c_2 (GeV^{-1})	-	-	1.57
c_3 (GeV^{-1})	-	-	-2.54
c_4 (GeV^{-1})	-	-	2.61
a_1 (GeV^{-1})	-	-	0.90
b_1 (GeV^{-2}) (free)	-	-	-0.3 ± 5.0
b_2 (GeV^{-2}) (free)	-	-	2.6 ± 2.0
$\tilde{b}_4$ (GeV^{-2}) (free)	-	-	-13.04 ± 0.92
x_1 (fm^{-2}) (free)	-6.1 ± 5.6	-2.1 ± 5.8	3 ± 14
x_2 (fm^{-2}) (free)	-5.4 ± 2.4	-3.6 ± 2.5	-2.3 ± 3.8
x_3 (fm^{-1}) (free)	-0.06 ± 0.21	0.15 ± 0.22	0.54 ± 0.40
x_4 (fm^{-1}) (free)	-0.33 ± 0.28	-0.10 ± 0.29	0.12 ± 0.49
y_1 (fm^{-2} GeV^{-2}) (free)	-101 ± 39	-63 ± 43	-47 ± 102
y_2 (fm^{-2} GeV^{-2}) (free)	-31 ± 20	-16 ± 21	-12 ± 41
y_3 (fm^{-1} GeV^{-2}) (free)	-0.6 ± 1.4	0.9 ± 1.5	2.9 ± 3.5
y_4 (fm^{-1} GeV^{-2}) (free)	-2.0 ± 1.6	-0.5 ± 1.7	0.7 ± 3.7
$\mathring{m}$ (GeV)	0.874	0.855	0.855
$\mathring{m}_\Delta$ (GeV)	-	1.166	1.166
χ^2/dof	$56.98/(158-11) = 0.388$	$47.84/(158-12) = 0.328$	$20.27/(158-15) = 0.142$
χ_0^2/dof	$767.17/(158-11) = 5.22$	$419.06/(158-12) = 2.87$	$186.63/(158-15) = 1.31$

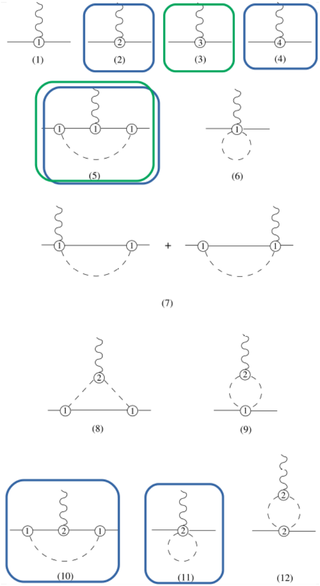
g_A Backup

	$\mathcal{O}(p^4)$ Δ
$\mathring{g}_A$ (free par.)	1.240 ± 0.046
d_{16} (free par.)	-0.88 ± 0.88
h_A	$h_A^{N_c} = 1.35$
g_1	$-\|g_1^{N_c}\| = -2.29$
c_1	-1.15 ± 0.05
c_2	1.57 ± 0.10
c_3	-2.54 ± 0.05
c_4	2.61 ± 0.10
a_1	0.90
$\tilde{b}_4$ (free par.)	-12.3 ± 1.0
$\mathring{m}$	0.855
$\mathring{m}_\Delta$	1.166
χ^2/dof	$11.14/(43-7) = 0.31$

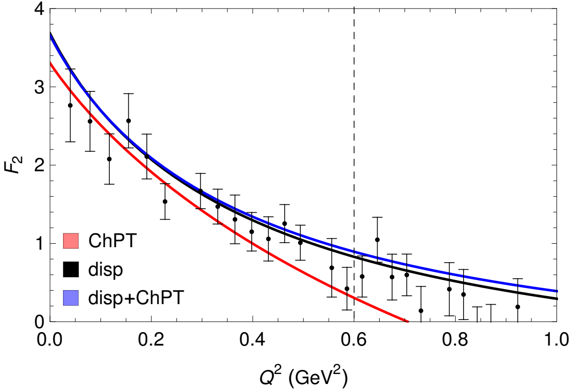
- **Pauli f.f., F_2**
  - $\langle N | V^\mu | N \rangle = \bar{u}' \left[\gamma^\mu F_1 + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2 \right] u,$
    $F_2 = \frac{F_M - F_E}{(1+Q^2/(4m^2))}$
  - $F_2^{(v)} = \kappa^{(v)} \left[1 + \frac{1}{6} \langle r_2^2 \rangle^{(v)} q^2 + \mathcal{O}(q^4) \right]$
  - $\mathcal{O}(p^3)$ χPT is not enough$F_2(0)^{o(3)} = c_6 - \left(\frac{\pi m_N g_A^2}{4\pi^2 F^2} \right) M_\pi$

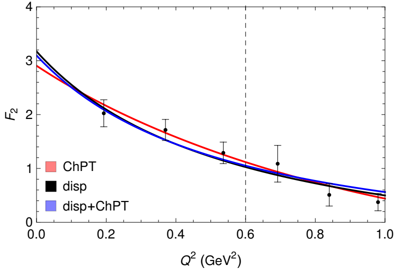


  - $\Longrightarrow$ include Δ $\mathcal{O}(p^4)$
  - disp+χPT $\mathcal{O}(p^4)$: $F_2 = F_2^{\rm disp} + F_2^{\rm tree} + F_2^{\chi \rm PTloop}$,
    $F_2^{\rm tree} = c_6 - 16 e_{106} m_N M_\pi^2 + 2q^2(d_6 + 2e_{74} m_N)$

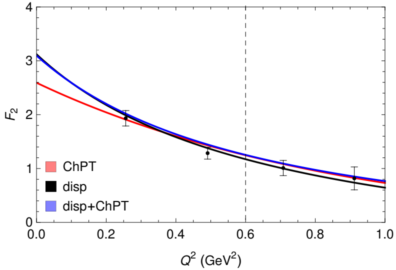


- **F_2 fit to LQCD**
  - In F_2^{disp} free c_6
  - In $F_2^{\chi\text{PT}}$ and $F_2^{disp+\chi\text{PT}}$, free c_6, e_{106}, e_{74}
  - χPT $\mathcal{O}(p^4)$ and disp separately are good enough to describe the data



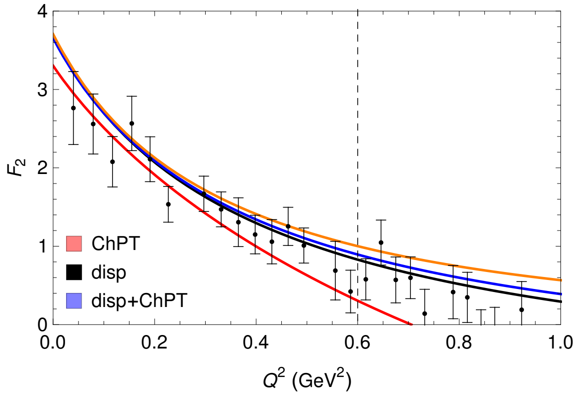


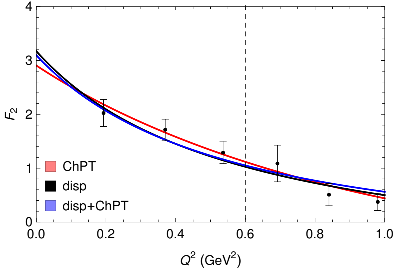
(a) H105 $M_\pi = 0.278$ GeV



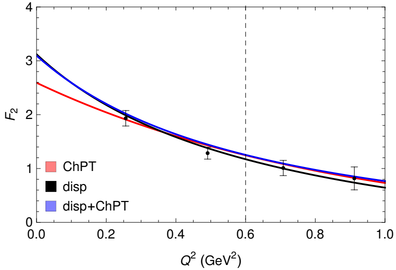
(b) N302 $m_\pi = 0.353$ GeV

- F_2 **fit to LQCD**
  - In $F_2^{\rm disp}$ free c_6
  - In $F_2^{\chi \rm PT}$ and $F_2^{disp+\chi \rm PT}$, free c_6, e_{106}, e_{74}
  - χPT $\mathcal{O}(p^4)$ and disp separately are good enough to describe the data





(a) H105 $M_\pi = 0.278$ GeV



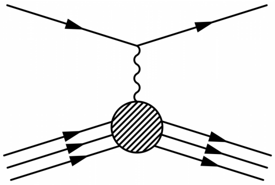
(b) N302 $m_\pi = 0.353$ GeV

Electroweak structure of the nucleon

- Structure of protons and neutrons is encoded in form factors
  - EM current, $V^\mu = \bar{q} Q \gamma^\mu q$,
    $\langle N | V^\mu | N \rangle = \bar{u}' \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2(q^2) \right] u$
    - $F_{1,2}$ are the Dirac and Pauli ffs
    - $G_E = F_1 - \frac{Q^2}{4m_N^2} F_2, \quad G_M = F_1 + F_2$
    - slope=charge radius, $\langle r_E^2 \rangle = 6 \left. \frac{\mathrm{d}G_E}{\mathrm{d}q^2} \right|_0$
  - Axial current, $A^\mu = \bar{q} \gamma^\mu \gamma_5 q$,
    $\langle N | A^\mu | N \rangle = \bar{u}' \left\{ \gamma_\mu F_A(q^2) + \frac{q_\mu}{2m_N} \tilde{G}_P(q^2) \right\} \gamma_5 u$
  - F_A is the axial ff
    - $F_A(0) \to$ related to spin distribution
  - Weak interaction is $V - A$

    $\Rightarrow F_A$ is exrtracted from different ν scattering exp. with significant uncertainty

    - F_A is a key input to ν oscillation experiments



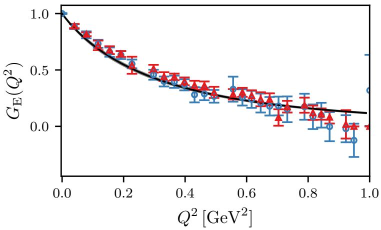
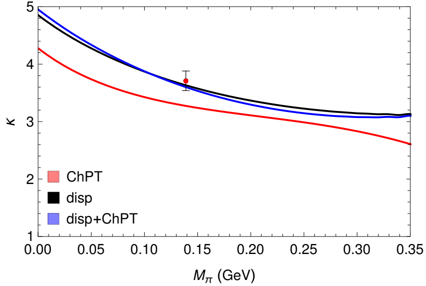
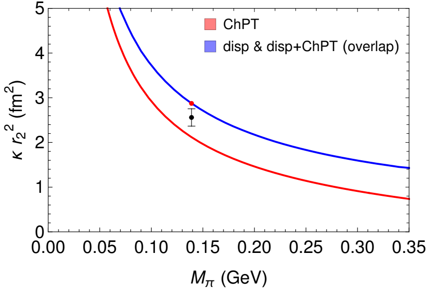


Figure 10: Djukanovic et al. [2102.07460]





- $F_2^{(v)} = \kappa^{(v)} \left[1 + \frac{1}{6}\langle r_2^2\rangle^{(v)} q^2 + \mathcal{O}(q^4)\right]$

	disp+c_6	ChPT	disp+ChPT	HB	exp. (PDG)
χ^2/dof	$\frac{49.95}{47-2} = 1.110$	$\frac{44.18}{47-4} = 1.027$	$\frac{56.08}{47-4} = 1.304$		
χ_0^2/dof	1.09	1.027	1.283		
κ_{phys}	3.632 ± 0.037	3.423 ± 0.059	$\mathbf{3.605 \pm 0.067}$	3.71 ± 0.17	3.706
$\langle r_2^2\rangle_{\text{phys}}$ (fm^2)	0.792 ± 0.011	0.61885 ± 0.0069	$\mathbf{0.788 \pm 0.015}$	0.690 ± 0.042	0.7754 ± 0.0080

- In general **good $F_{1,2}$ description at different M_π values**
- **Extracted $\langle r_{1,2}^2\rangle$ and κ**