Cusp Effects and Near-Threshold Structures: $\Lambda\Lambda$ Interactions Simulation Study with CBM

G. Appagere on behalf of the *CBM Collaboration* and *QCD at FAIR*
Stockholm University

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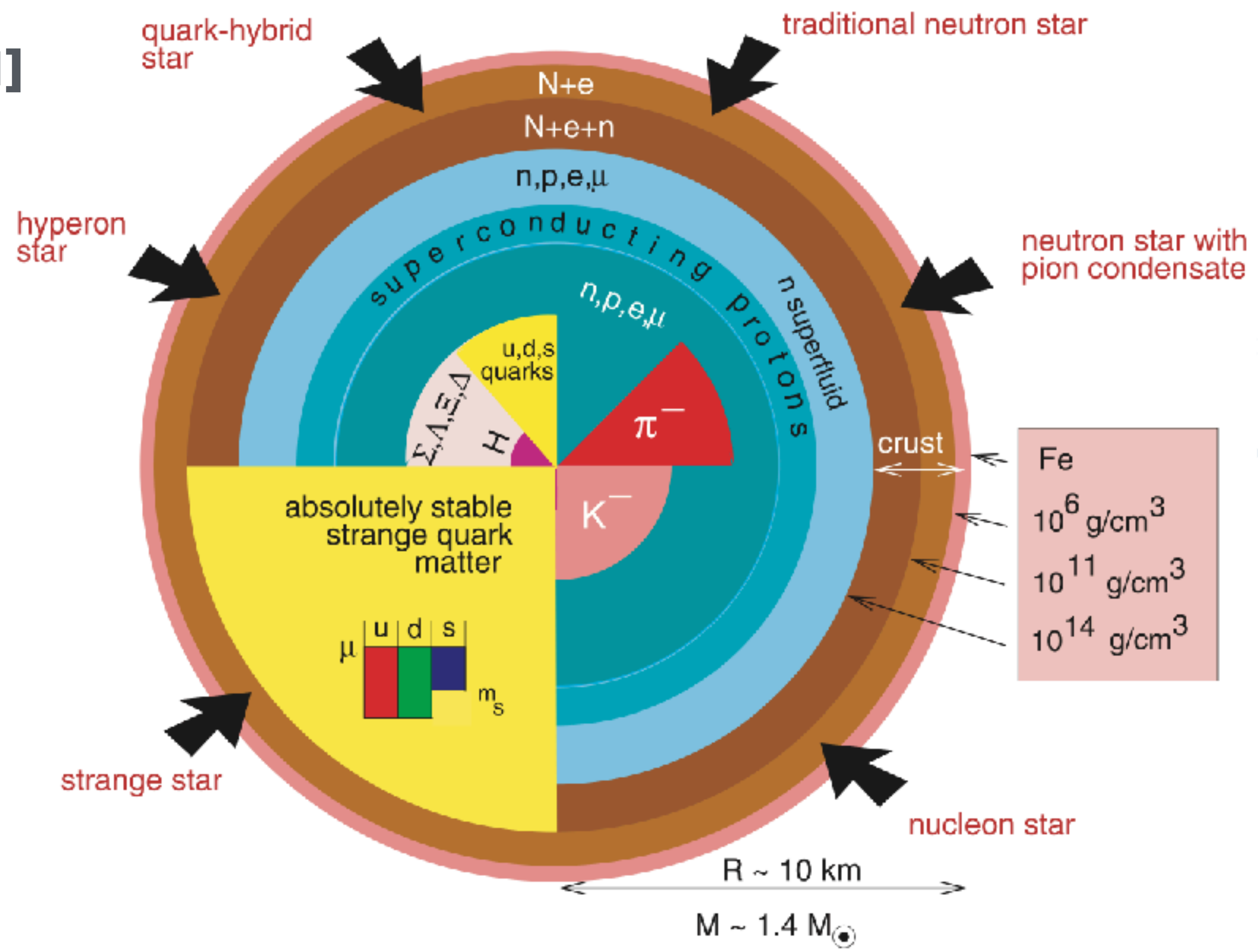
NSTAR 2026 Conference, Seville

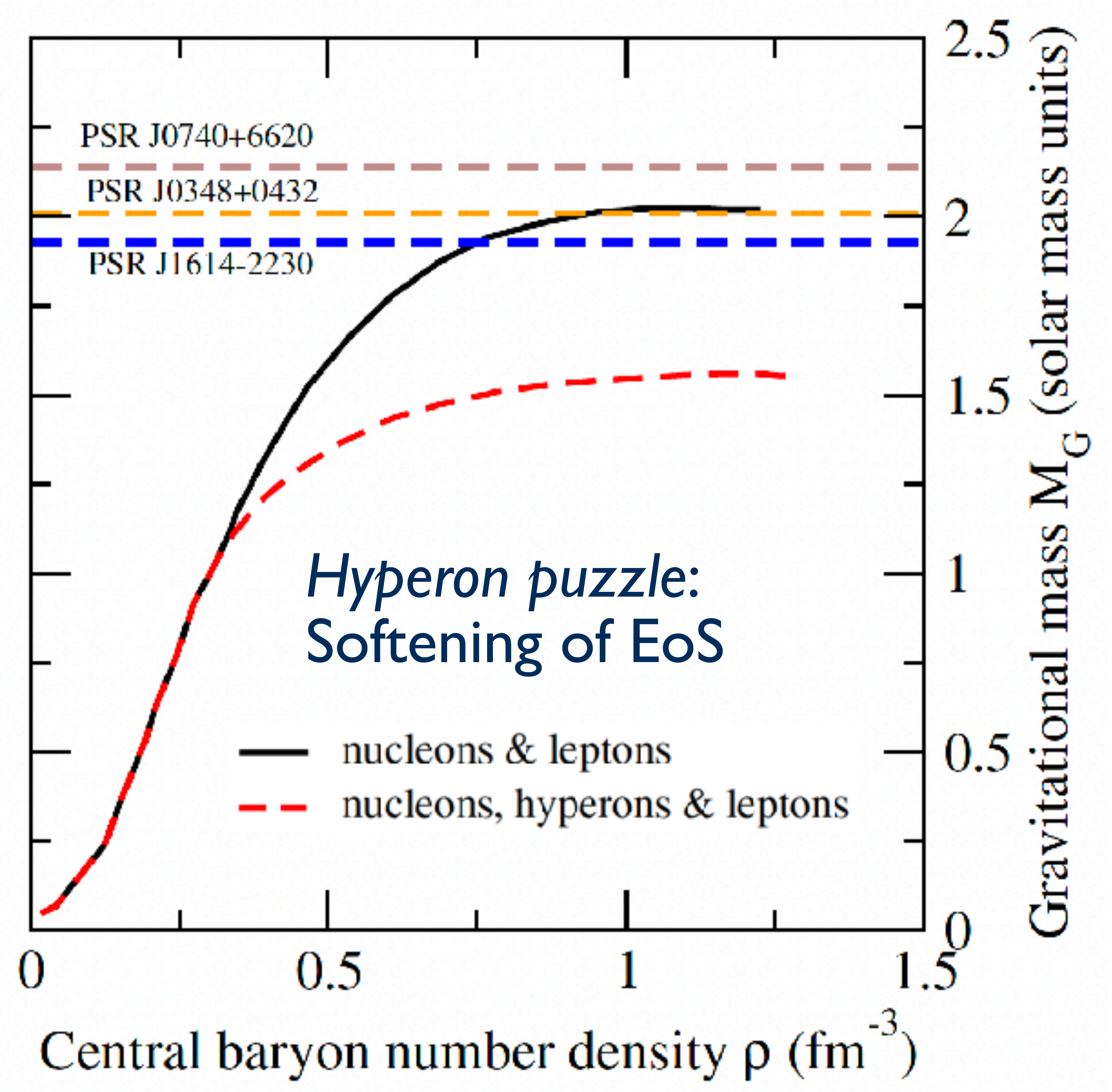
Neutron Stars and Λ-Λ Studies

Conditions (energy, pressure, temperature) inside *neutron stars* are favourable for *strangeness* production: *Hyperonization*

[1]



[2]



Λ-Λ interaction studies at low rel. momentum may help solve the Hyperon puzzle

[1] F. Weber *et al.*, arXiv:0705.2708 [astro-ph] (2007)
[2] J. G. Messchendorp, F. Nerling (eds.) *et al.*, arXiv:2512.15986 [hep-ex] (2026)

A laboratory YY force without a Λ Beam

- Λ lives $c\tau$= 7.89 cm. There is no Λ beam and no $\Lambda\Lambda$ scattering experiment
- Produce the pair exclusively in $pp \to \Lambda\Lambda K^+K^+$. The leftover interaction sits in $M(\Lambda\Lambda)$ near threshold
- That leftover force is $a_{\Lambda\Lambda}$: the missing laboratory YY number for the EoS
- Scale split: short-ranged production Φ, long-ranged FSI. Large momentum transfer, Φ varies slowly with m^2
- This is production FSI, not a source-function or correlation analysis

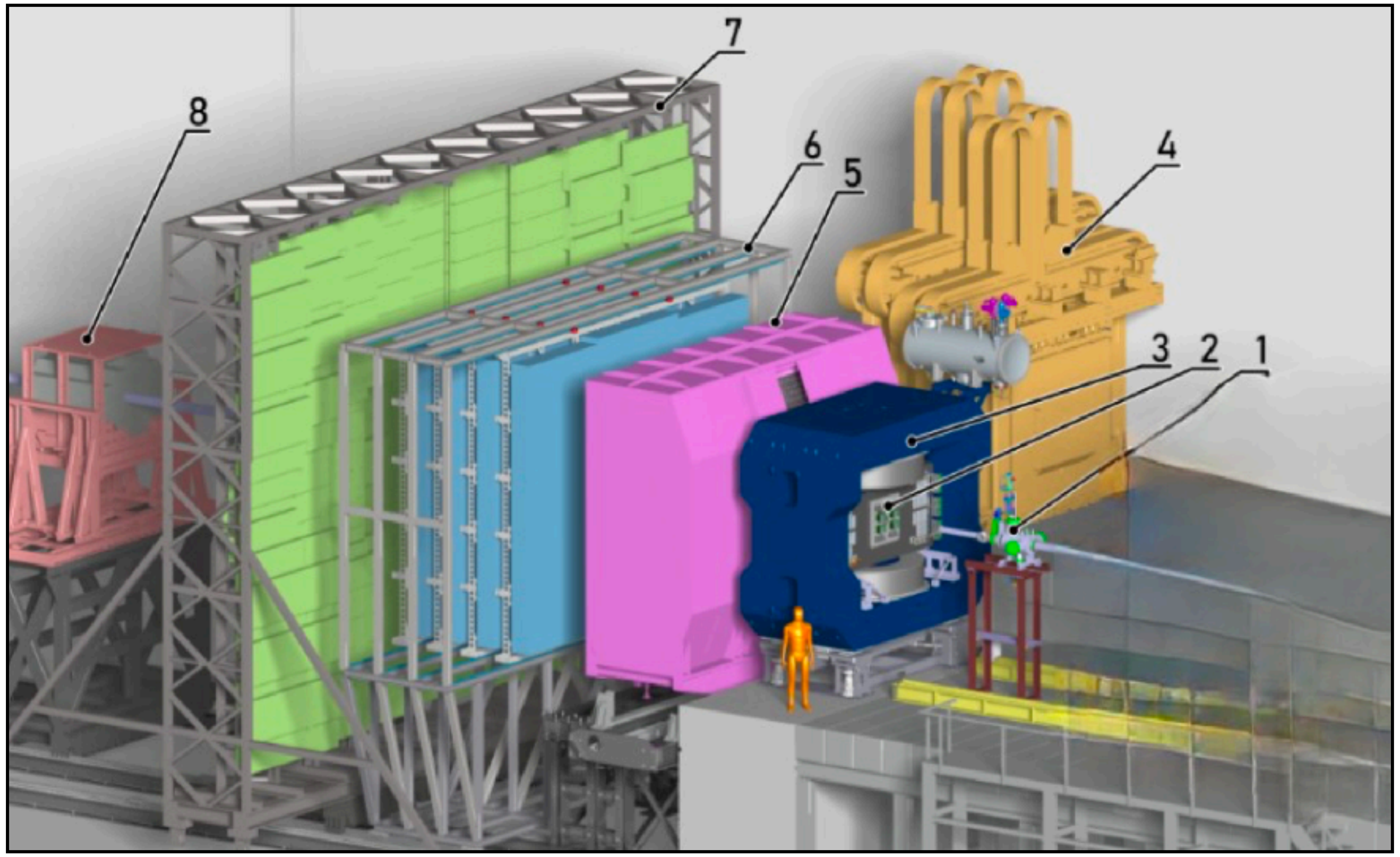
Determination	Reported	Implied sign
${}^{6}_{\Lambda\Lambda}$He (Nagara)	$\Delta B_{\Lambda\Lambda} = 0.67 \pm 0.17\,\mathrm{MeV}$	attractive, no a
^{12}C$(K^-, K^+\Lambda\Lambda X)$	$a_{\Lambda\Lambda} = -1.2 \pm 0.6 \pm 0.4\,\mathrm{fm}$	$a < 0$, attractive
STAR, Au+Au 200 GeV	$f_0^{-1} = -0.91 \pm 0.31\,\mathrm{fm}^{-1}$	$a > 0$, repulsive
Re-analysis, same data	$1/a_0 < -0.8\,\mathrm{fm}^{-1}$	$a < 0$, attractive
ALICE, pp and p–Pb	region in (f_0^{-1}, d_0)	$a < 0$, shallow
HAL QCD (lattice)	$f_0^{-1} = 1.45 \pm 0.25\,\mathrm{fm}^{-1}$	$a \simeq -0.69\,\mathrm{fm}$

FAIR: Facility for Antiproton and Ion Research

CBM Detector

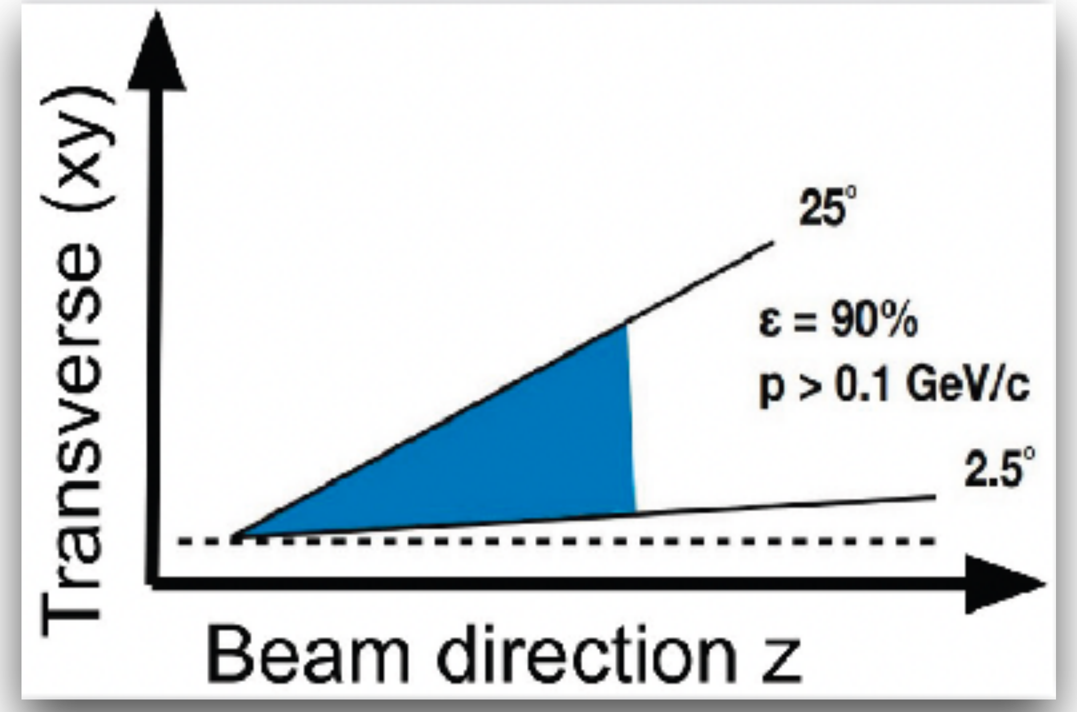
Compressed **B**aryonic **M**atter

Fixed target Magnetic spectrometer for SIS100 energies (FAIR, Germany)



1. T0 Detector
2. Superconducting Dipole Magnet
3. Silicon Tracking System / MVD
4. Muon Chamber
5. RICH
6. Transition Radiation Detector
7. ToF Detector
8. Forward Spectator Detector

- Interaction rates: 1-10 MHz
- High-rate capabilities with advanced readout systems
- For pp interactions: **CH_2 or LH_2** target



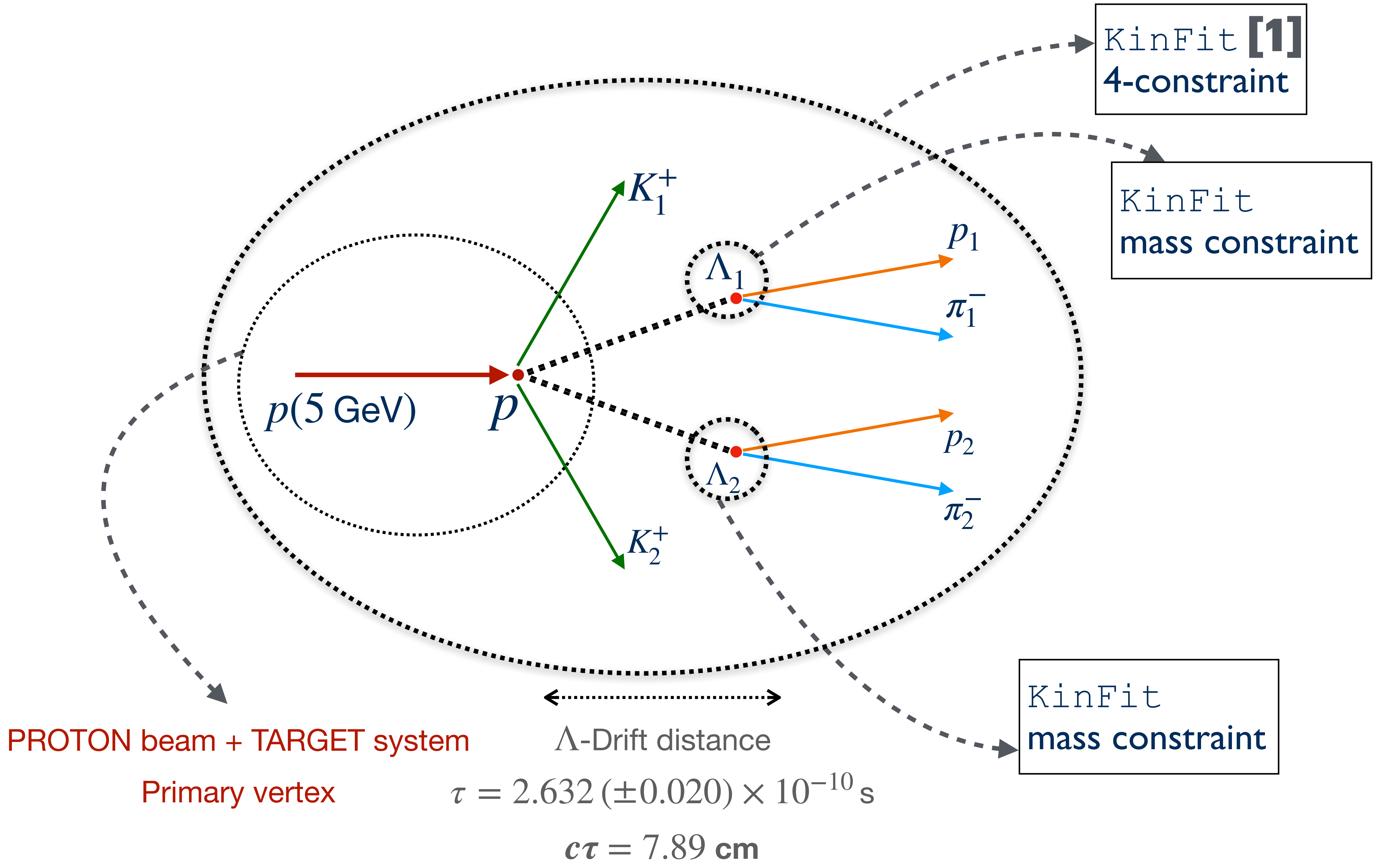
Further Information

Plenary Talk: F. Nerling (09/11/2026, 09:30 AM)

"QCD at FAIR" -- A hadron physics driven compaign towards FAIR completion

ΛΛ Reaction Topology

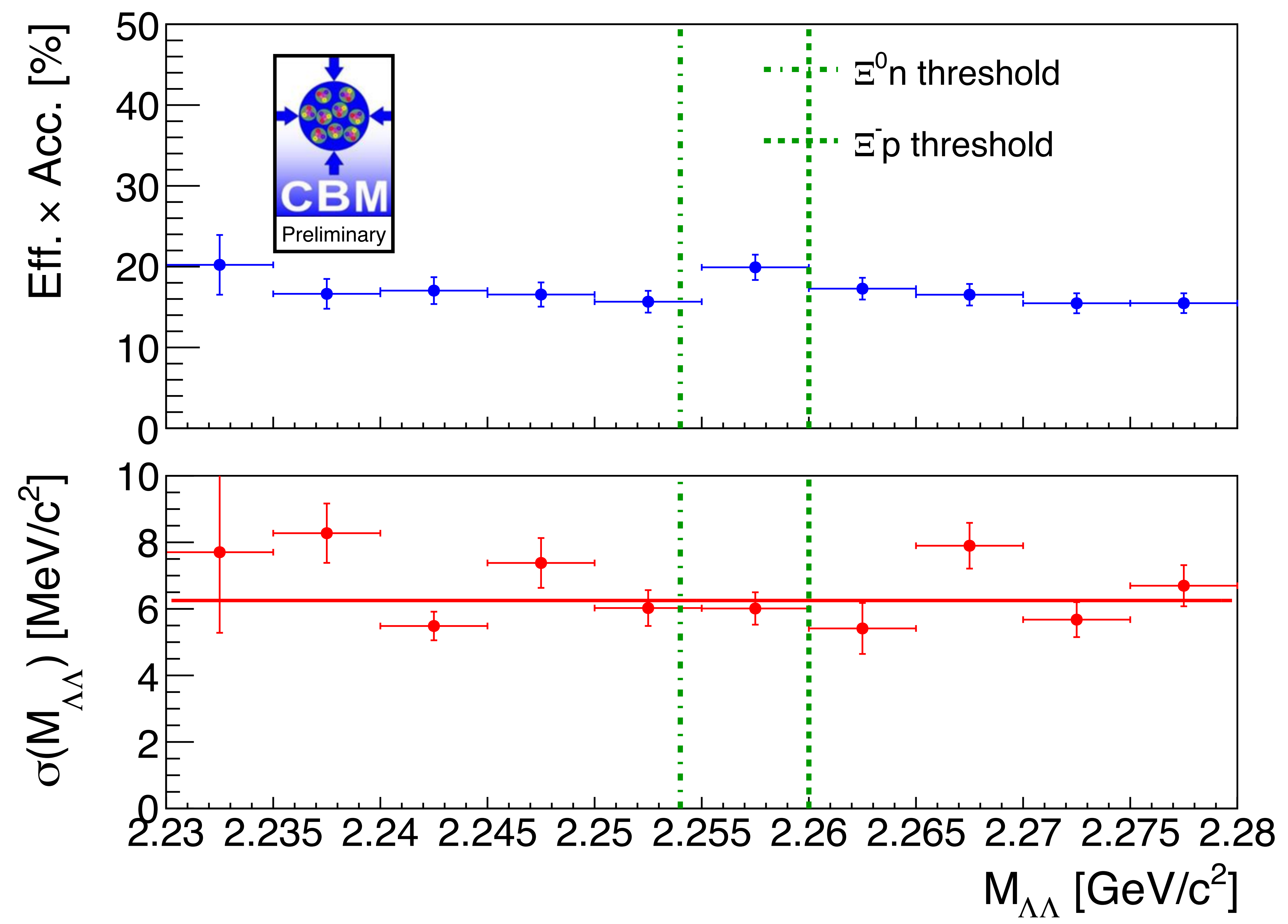
$$\boldsymbol{p}\ (5.0\ \text{GeV})\ \boldsymbol{p} \rightarrow \boldsymbol{K^+K^+} + \boldsymbol{\Lambda}\,[p\,\pi^-]\ \boldsymbol{\Lambda}\,[p\,\pi^-]$$



T = 5.0 GeV: $\sqrt{s_{pp}} = 3.59$ GeV → Excess energy of **0.37** GeV

CBM can handle T from 5.0 GeV, up to 29 GeV

Efficiency and Resolution



CBM offers nearly 20% efficiency and a resolution of ~6.0 MeV for the ΛΛ invariant mass, $M_{\Lambda\Lambda}$, reconstruction near threshold

[1] W. Esmail *et al.*, Comput. Softw. Big Sci. 8 (2024)

Dispersion Formalism, Scattering Length

Idea: Fit $\frac{d^2\sigma}{dM^2_{\Lambda\Lambda}dt} \propto p|A(M_{\Lambda\Lambda})|^2$ → insert into dispersion relation → $\Lambda\Lambda$ scattering length

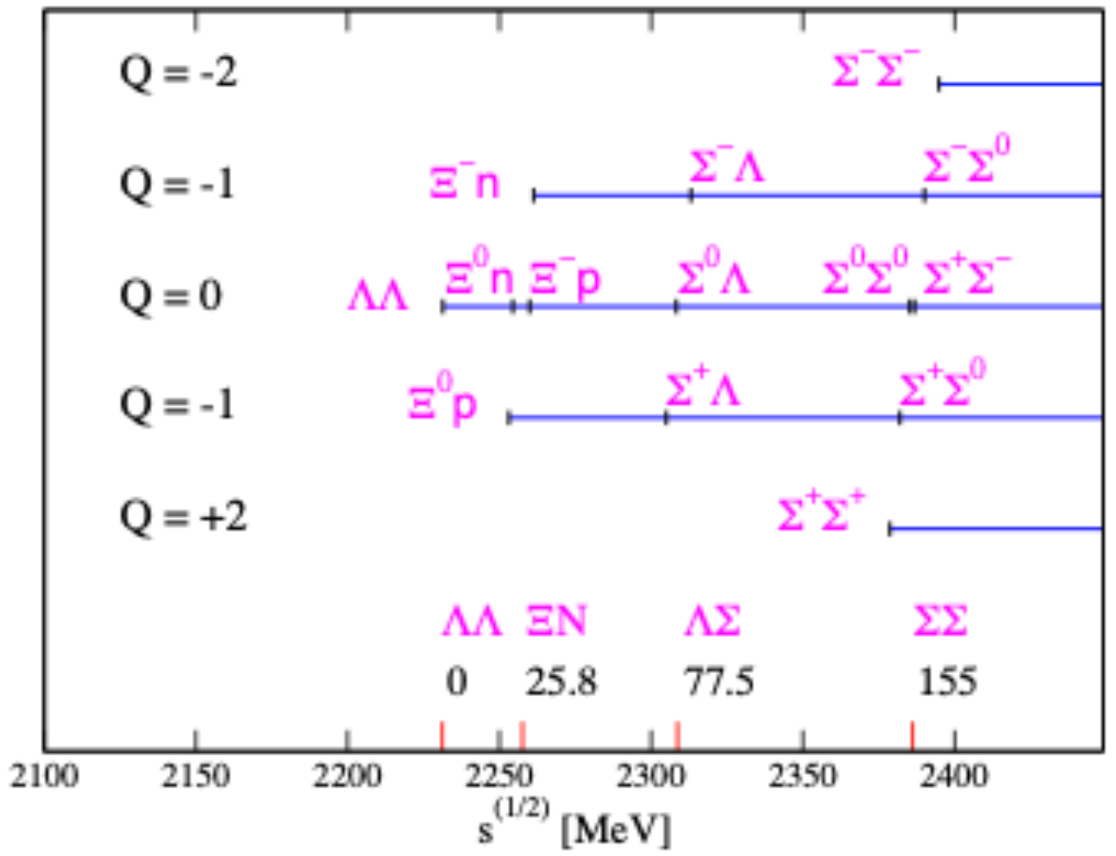
- Systematic model independent of source function
- FSI energy dependence directly shapes the invariant-mass spectrum. Parametrization is as follows:

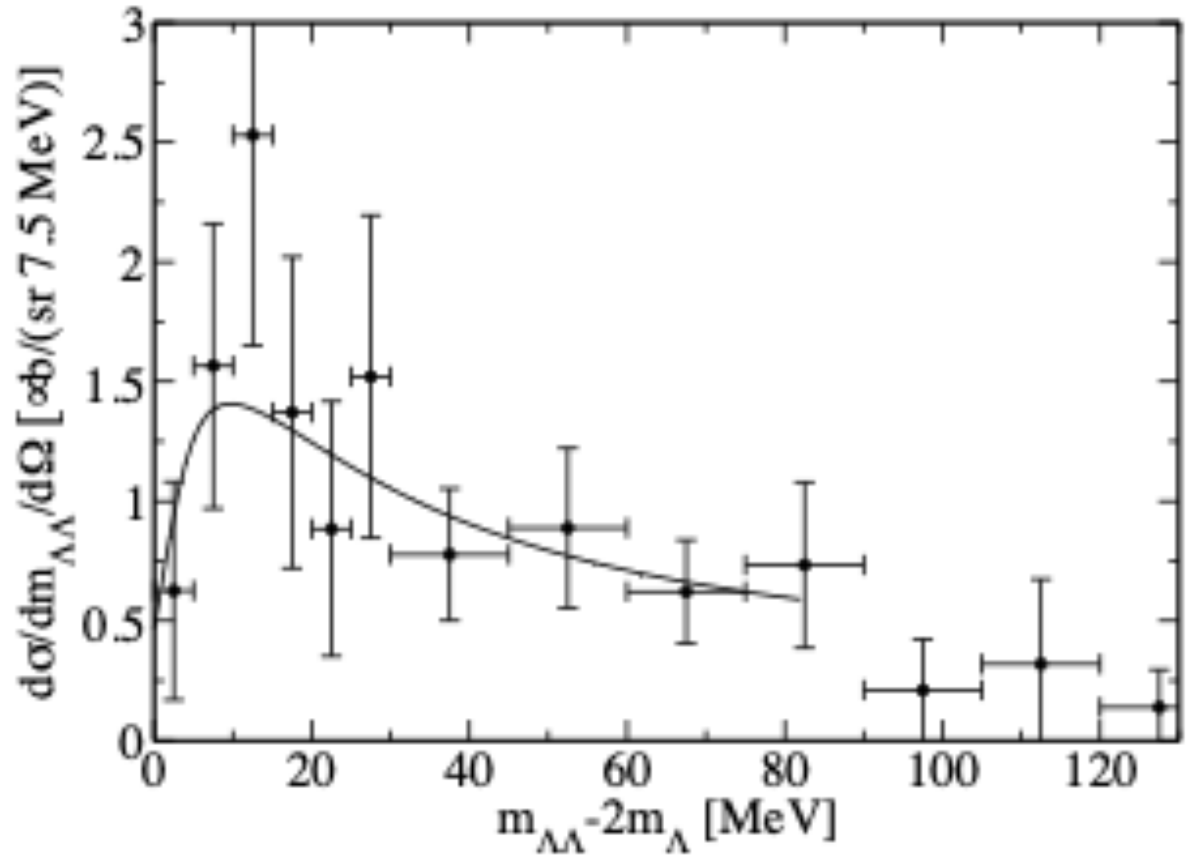
$$|A(M_{\Lambda\Lambda})|^2 \sim \exp\left[C_0 + \frac{C_1^2}{M^2_{\Lambda\Lambda} - C_2^2}\right]$$

$$a_{\Lambda\Lambda} = -C_1^2\sqrt{\frac{(m^2_{\max} - 4m^2_\Lambda)}{(m^2_{\max} - C_2)(4m^2_\Lambda - C_2)^3}}$$

- Thus connecting the fit parameters directly to the scattering length

- Previous measurement from $K^-d \mapsto K^0\Lambda\Lambda$ at COSY **[1]**
- $\Lambda\Lambda$ scattering length was extracted using dispersion relation



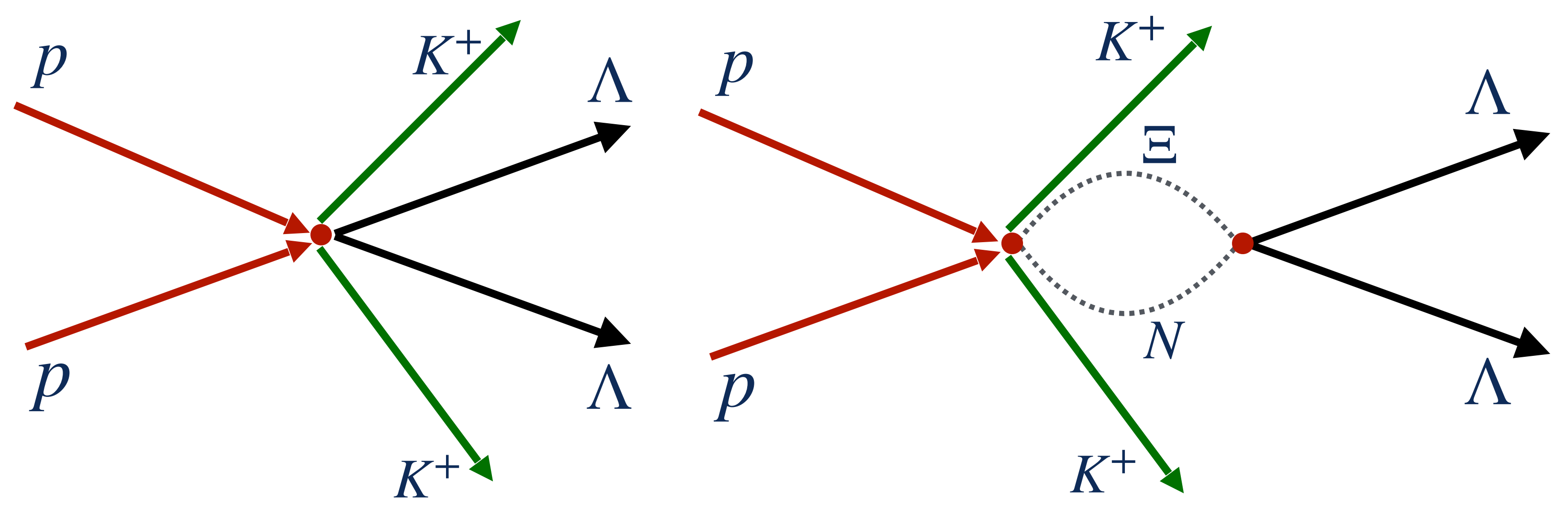


$$a_{\Lambda\Lambda} = -1.2 \pm 0.6 \pm 0.4 \text{ fm}$$

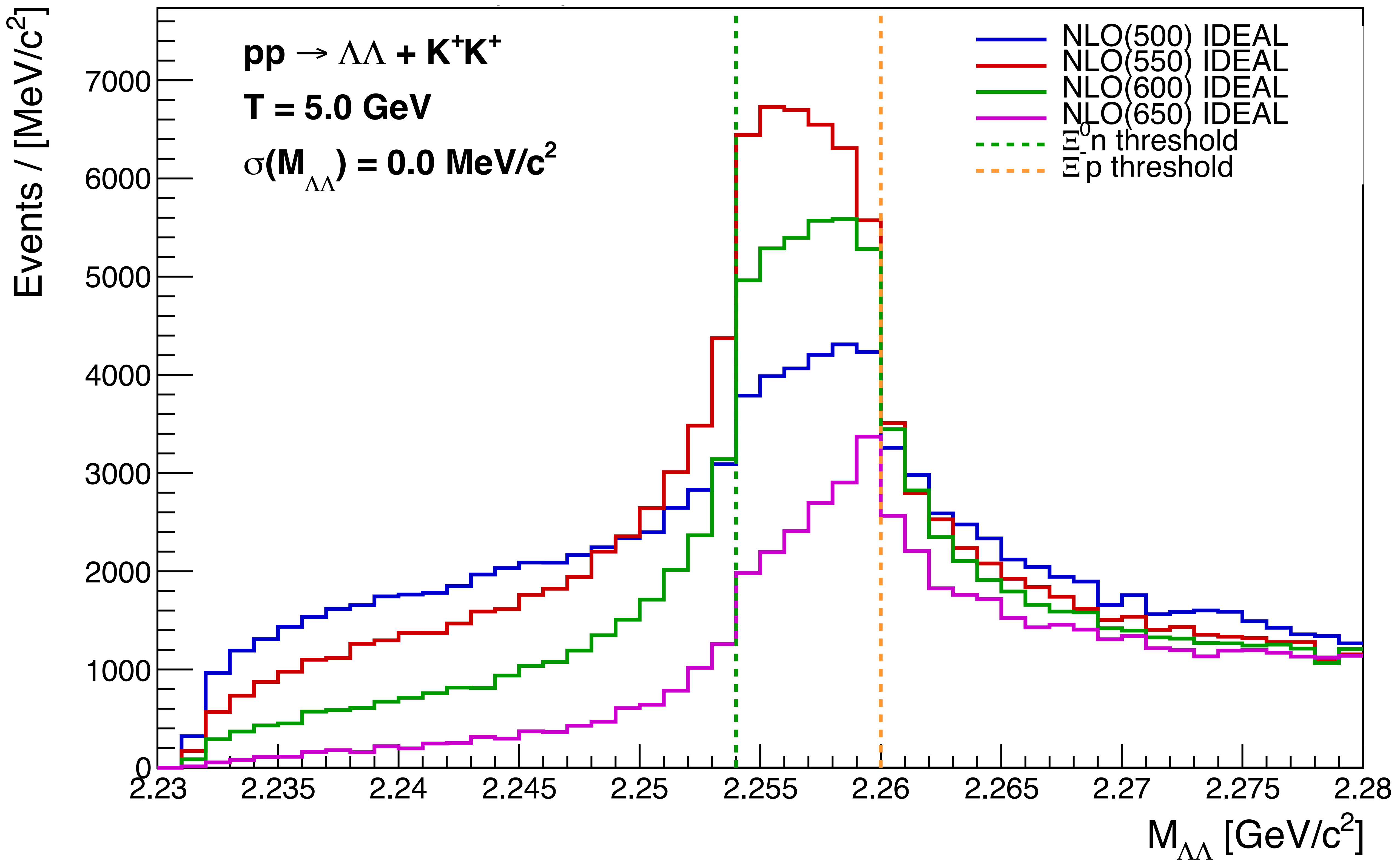
ΛΛ is the lightest S = −2 two-baryon system, but the ΞN channel opens around ~25 MeV above that mass. Those in-elastic channels are not observed in the from K^-d COSY data. **More measurements are needed.**

[1] A. Gasparyan *et al.*, Phys. Rev. C 85, 015204 (2012)

$M_{\Lambda\Lambda}$ Distributions with Coupled Channels

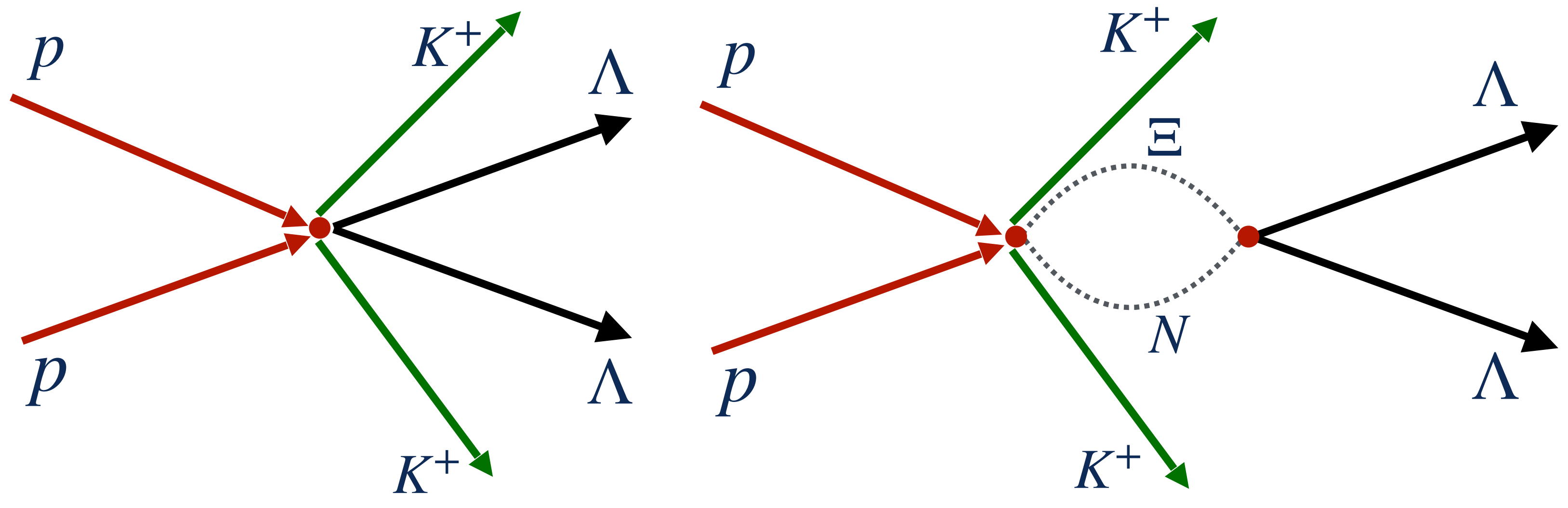


- $M_{\Lambda\Lambda}$ distributions generated with amplitude folded into the sampled events at different cutoffs
- Cusp signature is observed at ΞN channel threshold coupled to $\Lambda\Lambda$ channel
- The structures are dependent on the NLO cut-off used to model the amplitude
- One can extract the $a_{\Lambda\Lambda}$ by fitting up-to the first cusp ($\Xi^0 n$ threshold)
- But, simple dispersion relation technique with elastic phase-shift may not be sufficient to fit the invariant mass distribution with coupled channel signatures

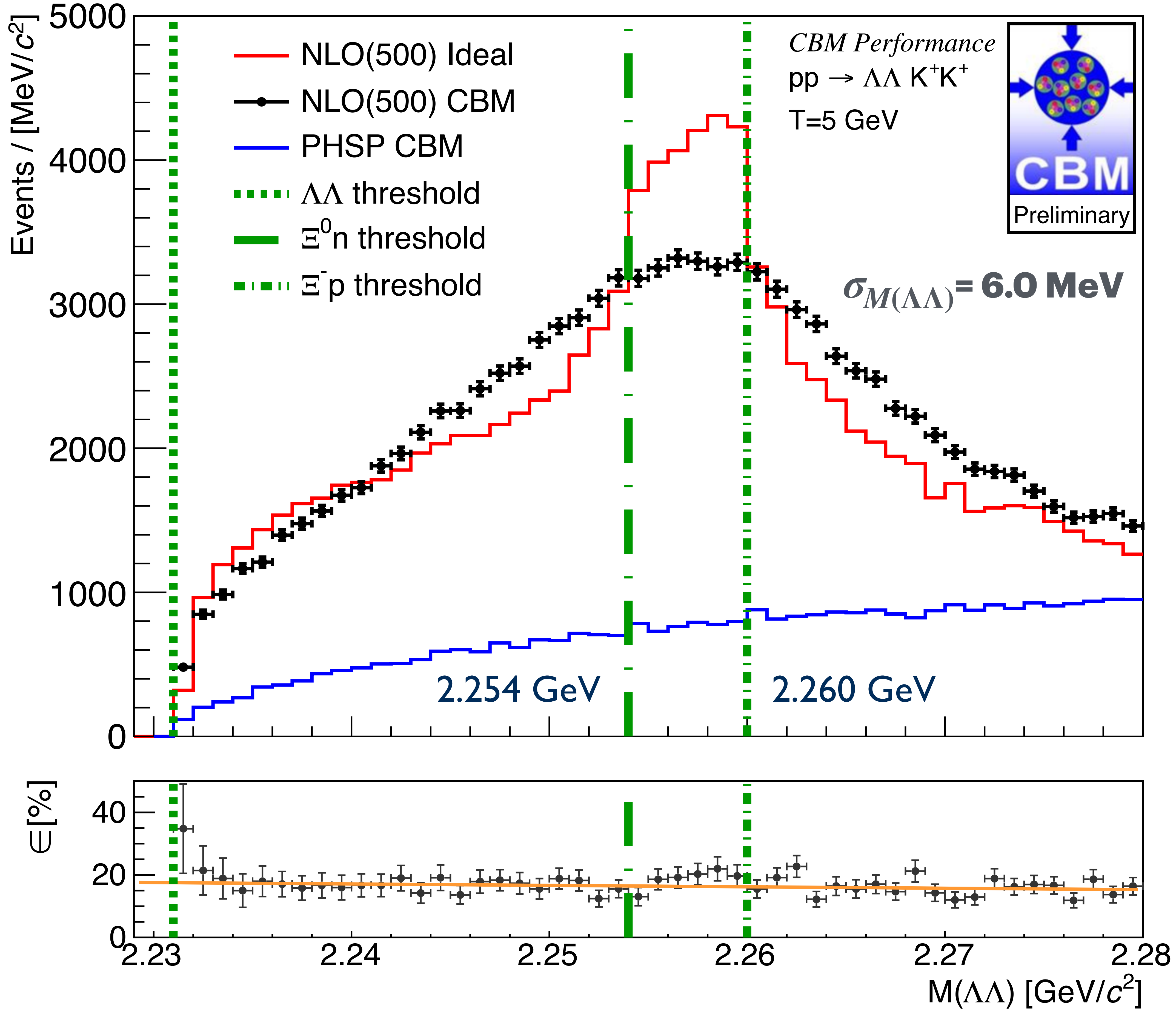


Amplitude model provided by Johann Haidenbauer

Feasibility: Scattering Length from Cusps



- Shape of response including scattering amplitude is distinguishable from phase-space
- Fitting cusp sensitive to scattering lengths of $\Xi^0 n$ and $\Xi^- p$
- The observable cusp structure is fitted by varying *single-channel* scattering lengths a_{ii} and *channel coupling* a_{ij}, whose poles determine the invariant-mass line shape Z. Zhang and F. Guo, PLB 863 (2025)



10 days of data taking assuming a cross-section of 0.35 μb **[1]**

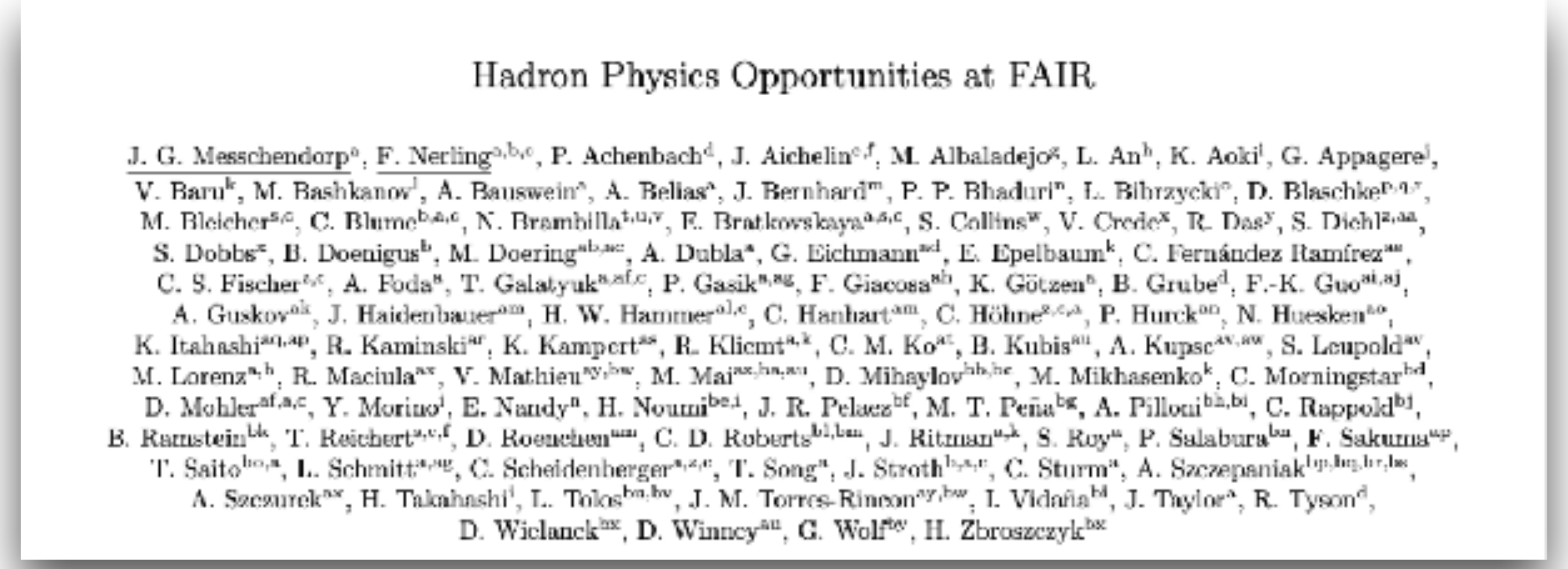
[1] HADES Collaboration, EPJ A 57 (2021)

Summary and Outlook

- Demonstrate feasibility of hadron-hadron interaction studies with cusp and threshold signatures at CBM
- White paper "**Hadron Physics Opportunities at FAIR**" is on arXiv: 2512.15986
- Flagship studies: **Hadron-hadron interactions**, Hadron structure & spectroscopy, Exotics, and Hadrons and di-leptons

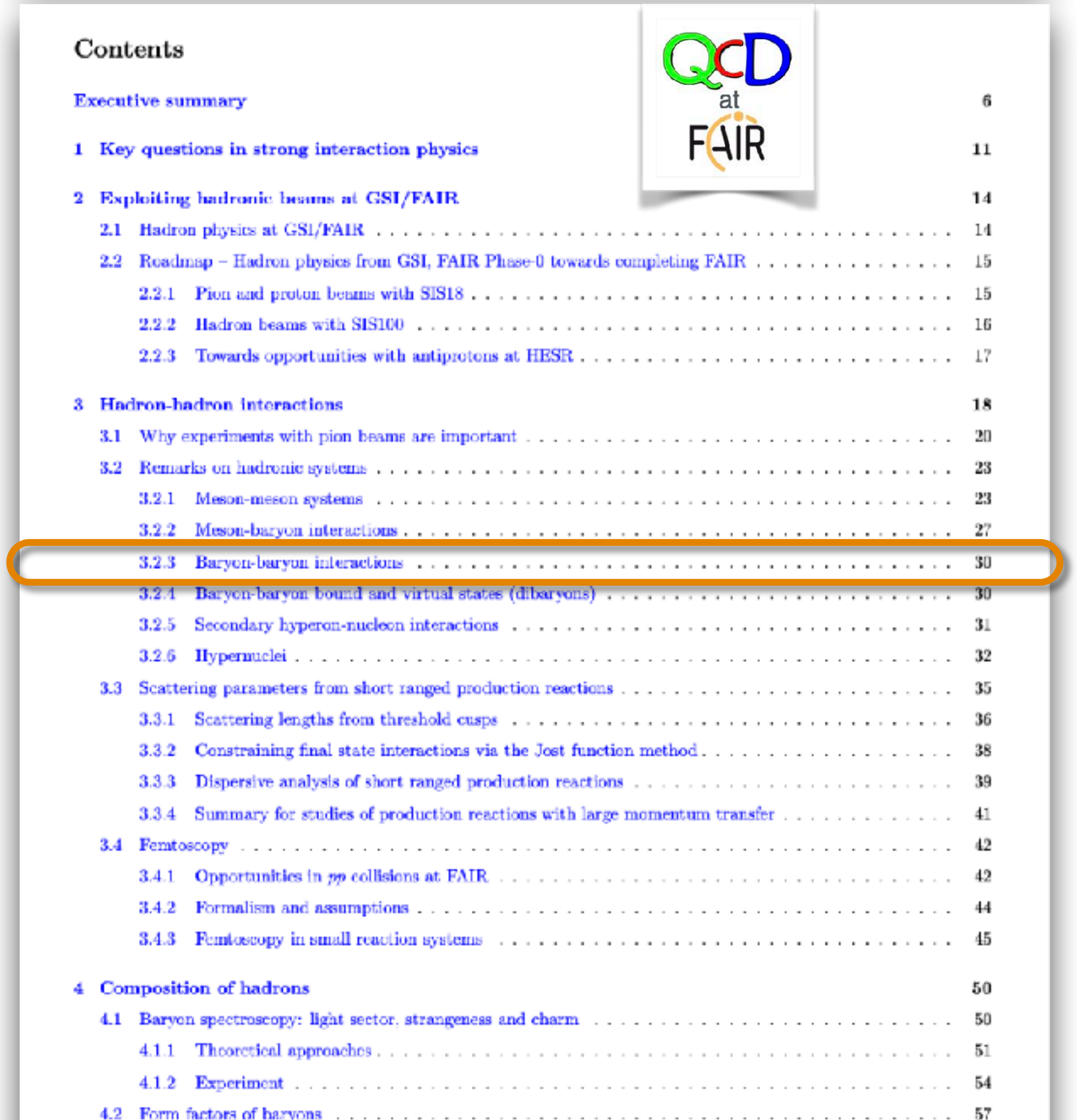
Plenary Talk: F. Nerling (09/11/2026, 09:30 AM)
"QCD at FAIR" -- A hadron physics driven compaign towards FAIR completion

- **Ongoing**: Fit using two methods — exponential interpolant and generated amplitude itself, $|\Omega(M; a, r_e)|^2 \times \rho_4(M)$, with ρ_4 the four-body phase space, floating $a_{\Lambda\Lambda}$ at fixed r_e
- **Closure**: Inject a known $a_{\Lambda\Lambda}$ and check that the fit returns it
- **Sensitivity**: Study the sensitivity in the extraction of $a_{\Lambda\Lambda}$ against the fit window, the dispersion cutoff, and the sample size
- **Next:** Add background channels to study to obtain an understanding of the detector capability to isolate the channel



Hadron Physics Opportunities at FAIR

J. G. Messchendorp, F. Nerling, P. Achenbach, J. Aichelin, M. Albaladejo, L. An, K. Aoki, G. Appagere, V. Baru, M. Bashkanov, A. Bauswein, A. Belias, J. Bernhard, P. P. Bhaduri, L. Bibrzycki, D. Blaschke, M. Bleicher, C. Blume, N. Brambilla, E. Bratkovskaya, S. Collins, V. Crede, R. Das, S. Diehl, S. Dobbs, B. Doenigus, M. Doering, A. Dubla, G. Eichmann, E. Epelbaum, C. Fernández Ramírez, C. S. Fischer, A. Foda, T. Galatyuk, P. Gasik, F. Giacosa, K. Götzen, B. Grube, F.-K. Guo, A. Guskov, J. Haidenbauer, H. W. Hammer, C. Hanhart, C. Höhne, P. Hurck, N. Huesken, K. Itahashi, R. Kaminski, K. Kampert, R. Klicmt, C. M. Ko, B. Kubis, A. Kupsc, S. Leupold, M. Lorenz, R. Maciula, V. Mathieu, M. Mai, D. Mihaylov, M. Mikhasenko, C. Morningstar, D. Mohler, Y. Morino, E. Nandy, H. Noumi, J. R. Pelaez, M. T. Peña, A. Pilloni, C. Rappold, B. Ramstein, T. Reichert, D. Roenchen, C. D. Roberts, J. Ritman, S. Roy, P. Salabura, F. Sakuma, T. Saito, L. Schmitt, C. Scheidenberger, T. Song, J. Stroth, C. Sturm, A. Szczepaniak, A. Szczurek, H. Takahashi, L. Tolos, J. M. Torres-Rincon, I. Vidaña, J. Taylor, R. Tyson, D. Wielanck, D. Winney, G. Wolf, H. Zbroszczyk



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APPENDIX

$\Lambda\Lambda$ Interaction, Current status and References

Determination	Production / system	$\Lambda\Lambda$ region probed	Reported	Implied sign	Reference
${}^{6}_{\Lambda\Lambda}$He, Nagara event (KEK-E373)	Ξ^- captured at rest in emulsion, K^- beam at $1.66\,\mathrm{GeV}/c$	bound hypernucleus, no asymptotic k^*	$\Delta B_{\Lambda\Lambda} = 0.67 \pm 0.17\,\mathrm{MeV}$	attractive in medium, no free-space a	Ahn *et al.*, PRC **88** (2013) 014003
^{12}C$(K^-, K^+\Lambda\Lambda X)$, KEK-PS E522, + dispersion analysis	K^- at $1.67\,\mathrm{GeV}/c$ on ^{12}C, inclusive $\Lambda\Lambda$ spectrum	$\varepsilon \lesssim 60\,\mathrm{MeV}$ above $2m_\Lambda$, $k^* \lesssim 260\,\mathrm{MeV}/c$	$a_{\Lambda\Lambda} = -1.2 \pm 0.6\,_{\mathrm{exp}} \pm 0.4\,_{\mathrm{th}}\,\mathrm{fm}$	$a < 0$, attractive	Yoon *et al.*, PRC **75** (2007) 022201; Gasparyan *et al.*, PRC **85** (2012) 015204
STAR femtoscopy	Au+Au at $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$	$k^* \lesssim 300\,\mathrm{MeV}/c$	$f_0^{-1} = -0.91 \pm 0.31\,\mathrm{fm}^{-1}$, $d_0 = 8.52 \pm 2.56\,\mathrm{fm}$	$a > 0$, repulsive	STAR, PRL **114** (2015) 022301
Re-analysis of the same STAR data	identical data, residual correlations and feed-down retreated	$k^* \lesssim 300\,\mathrm{MeV}/c$	$1/a_0 < -0.8\,\mathrm{fm}^{-1}$	$a < 0$, attractive	Morita, Furumoto, Ohnishi, arXiv:1512.08444
ALICE femtoscopy	pp at 7 and $13\,\mathrm{TeV}$, p–Pb at $\sqrt{s_{NN}} = 5.02\,\mathrm{TeV}$	Lednický fit for $k^* < 460\,\mathrm{MeV}/c$	allowed region in (f_0^{-1}, d_0); if bound, $B_{\Lambda\Lambda} = 3.2^{+1.6}_{-2.4}\,(\mathrm{stat})^{+1.8}_{-1.0}\,(\mathrm{syst})\,\mathrm{MeV}$	$a < 0$, shallow attraction, no deep bound state	ALICE, PLB **797** (2019) 134822, arXiv:1905.07209
HAL QCD, lattice	$2+1$ flavour lattice, near-physical $m_\pi \simeq 146\,\mathrm{MeV}$	near threshold	$f_0^{-1} = 1.45 \pm 0.25\,\mathrm{fm}^{-1}$, $d_0 = 5.16 \pm 0.82\,\mathrm{fm}$ (preliminary, quoted by ALICE)	$a \simeq -0.69\,\mathrm{fm}$, attractive	Hatsuda, Front. Phys. **13** (2018) 132105, $\Lambda\Lambda$ values priv. comm.; cf. Sasaki *et al.*, PoS LATTICE2016, 116
χEFT at NLO	$SU(3)$ chiral EFT, $S = -2$ sector, cutoff $550\,\mathrm{MeV}$	near threshold, $\Lambda\Lambda$–ΞN coupled	$a_{\Lambda\Lambda} = -0.61\,\mathrm{fm}$, $r_{\mathrm{eff}} = 6.06\,\mathrm{fm}$ (NLO550)	$a < 0$, attractive	Haidenbauer *et al.*, NPA **954** (2016) 273
CBM, this work (simulation)	$pp \to K^+K^+\Lambda\Lambda$ at $T = 5\,\mathrm{GeV}$, $\sqrt{s_{pp}} = 3.59\,\mathrm{GeV}$	$\varepsilon \lesssim 49\,\mathrm{MeV}$ above $2m_\Lambda$, $k^* \lesssim 234\,\mathrm{MeV}/c$	exclusive $M(\Lambda\Lambda)$, dispersive extraction of $a_{\Lambda\Lambda}$, no source model	to be determined	this work

Invariant-Mass Fit to Scattering Length

Formalism

- Factorization: Short-range production × long-range FSI.
  - Production operator Φ is **short ranged** and slowly varying in m^2
  - Exponential encodes unitarity + analyticity

$$A_S(s,t,m^2) = \exp\left[\frac{1}{\pi}\int_{m_0^2}^{m_{max}^2} \frac{\delta_S(m'^2)}{m'^2 - m^2 - i0}\, dm'^2\right] \Phi(s,t,m^2)$$

$$\boxed{q_t \gg p} \qquad \boxed{A = \Phi \times A_{FSI}}$$

- FSI energy dependence directly shapes the invariant-mass spectrum

$$\frac{d^2\sigma}{dm^2 dt} \propto p|A(m)|^2$$

- Near-threshold FSI: effective-range expansion
  - Using dispersion relations. Leading to the elastic amplitude

$$p\cot\delta = -\frac{1}{a} + \frac{r_e}{2}p^2 \qquad A(m) \propto \frac{1}{-\frac{1}{a} + \frac{r_e}{2}p^2 - ip}$$

- Practical parametrization for fitting data: Since $\Phi(m^2)$ varies slowly and the FSI dominates

$$A(m) \sim \exp\left[C_0 + \frac{C_1^2}{m^2 - C_2^2}\right]$$

This analytic form is used to fit the invariant-mass spectrum, from which the scattering length is extracted through the dispersion relation.

Dispersion inversion, Scattering Length

Fit $|A(m)|^2$ → insert into dispersion relation → $a_S(C_1, C_2)$

- Start from dispersion relation (inversion)

$$a_S = \lim_{m^2 \to m_0^2} \frac{1}{2\pi} \left(\frac{m_a + m_b}{\sqrt{m_a m_b}} \right) P \int_{m_0^2}^{m_{max}^2} \frac{dm'^2}{\sqrt{m'^2 - m_0^2}(m'^2 - m^2)} \log\left(\frac{1}{p'} \frac{d^2\sigma_S}{dm'^2 dt} \right)$$

Knowing,

$$\frac{d^2\sigma}{dm^2 dt} \propto p\,|A(m)|^2 \qquad A(m) \sim \exp\left[C_0 + \frac{C_1^2}{m^2 - C_2^2} \right]$$

- Analytic evaluation of the integral: Substituting the fit into the dispersion relation yields

$$a_s(C_1, C_2) = -\frac{1}{2} C_1^2 \sqrt{\frac{m_0^2}{m_N m_\Lambda} \frac{(m_{max}^2 - m_0^2)}{(m_{max}^2 - C_2)^3 (m_0^2 - C_2)^3}}$$

- Thus connects the fit parameters directly to the scattering length

Scattering lengths of strangeness $S = -2$ baryon-baryon interactions

A.M. Gasparyan[1,2], J. Haidenbauer[3,4], and C. Hanhart[3,4]
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[2] SSC RF ITEP, Bolshaya Cheremushkinskaya 25, 117218 Moscow, Russia
[3] Institute for Advanced Simulation, Forschungszentrum Jülich, D-52425 Jülich, Germany
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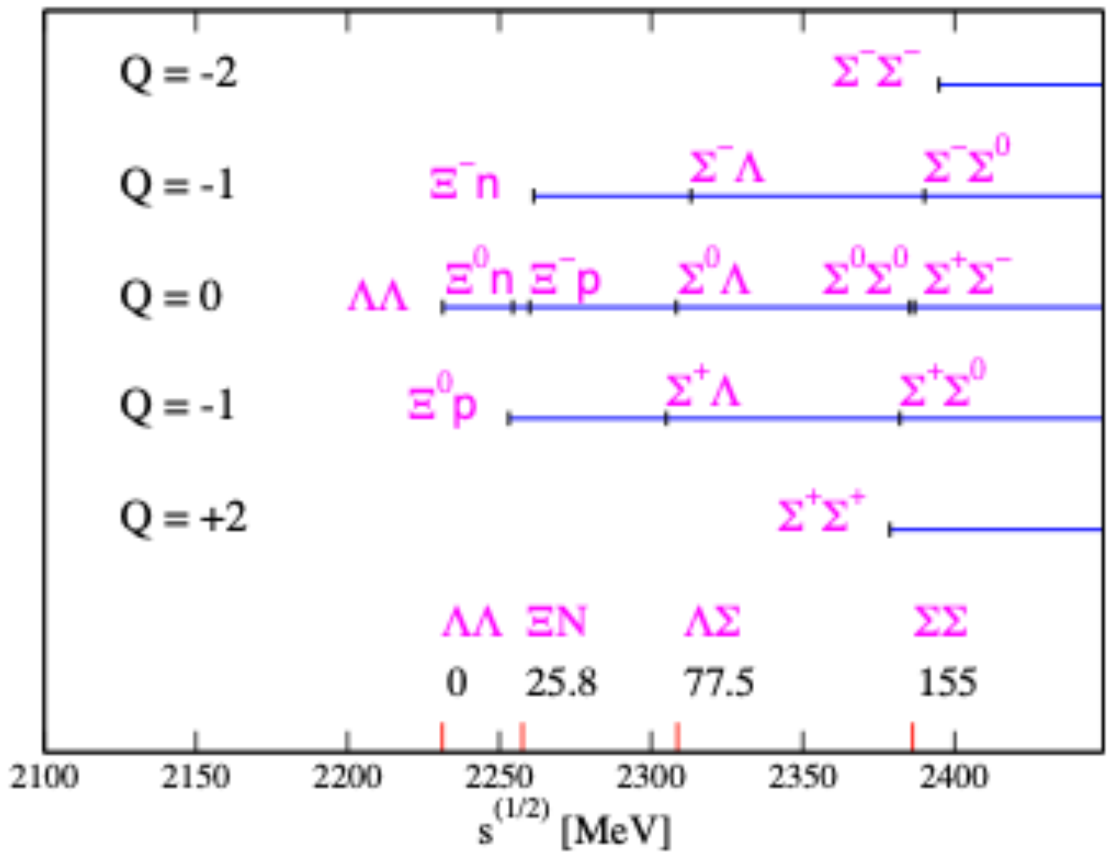
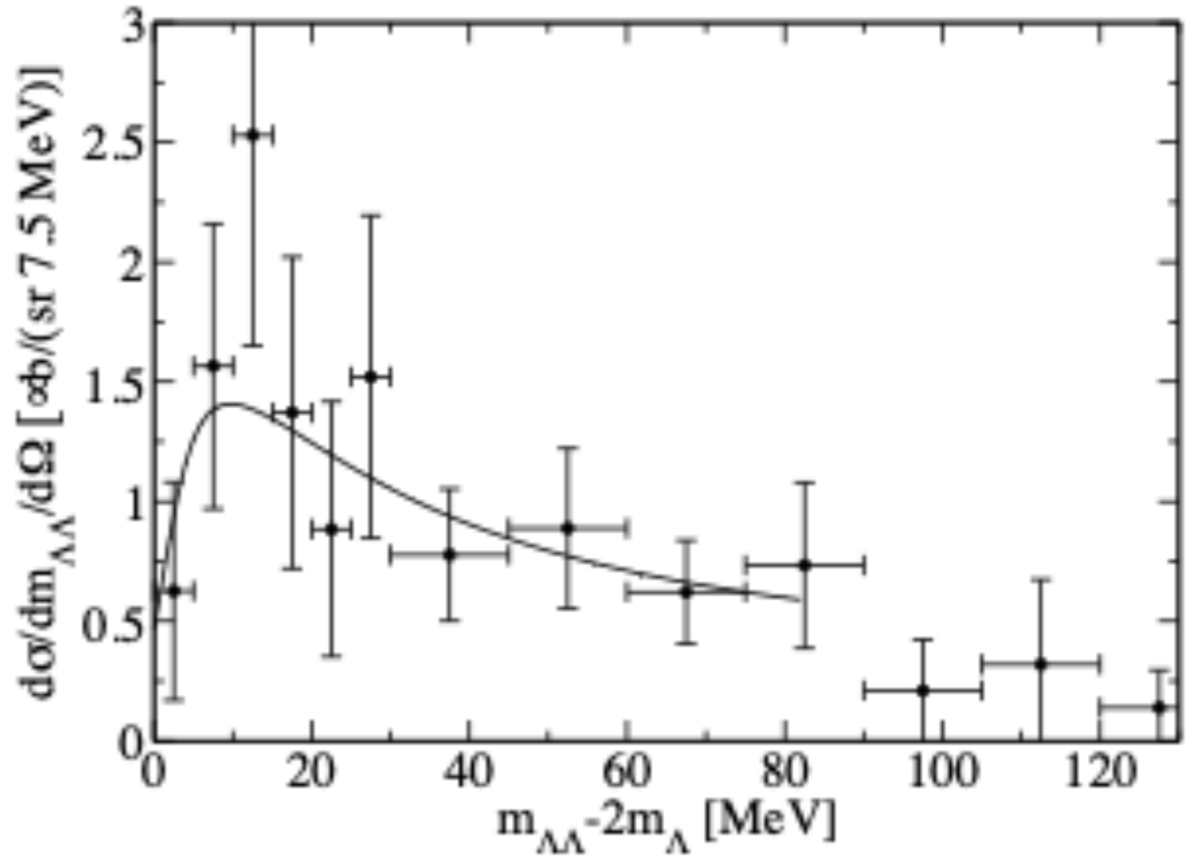
We reconsider a method based on dispersion theory, that allows one to extract the scattering length of any two-baryon system from corresponding final-state interactions in production reactions. The application of the method to baryon-baryon systems with strangeness $S = -2$ and $S = -3$ systems is discussed. Theoretical uncertainties due to the presence of inelastic channels with near-by thresholds are examined for the specific situation of the reaction $K^- d \to K^0 \Lambda\Lambda$ and the coupling of $\Lambda\Lambda$ to the ΞN channel. The possibility to disentangle spin-triplet and spin-singlet scattering lengths by means of various polarization measurements is demonstrated for several production reactions in $K^- d$ and γd scattering. Employing the method to available data on the $\Lambda\Lambda$ invariant mass from the reaction $^{12}C(K^-, K^+\Lambda\Lambda X)$, a $^1S_0$ scattering length of $a_{\Lambda\Lambda} = -1.2 \pm 0.6$ fm is deduced.

PACS numbers: 11.55.Fv,13.75.-n,13.75.Ev,25.40.-h

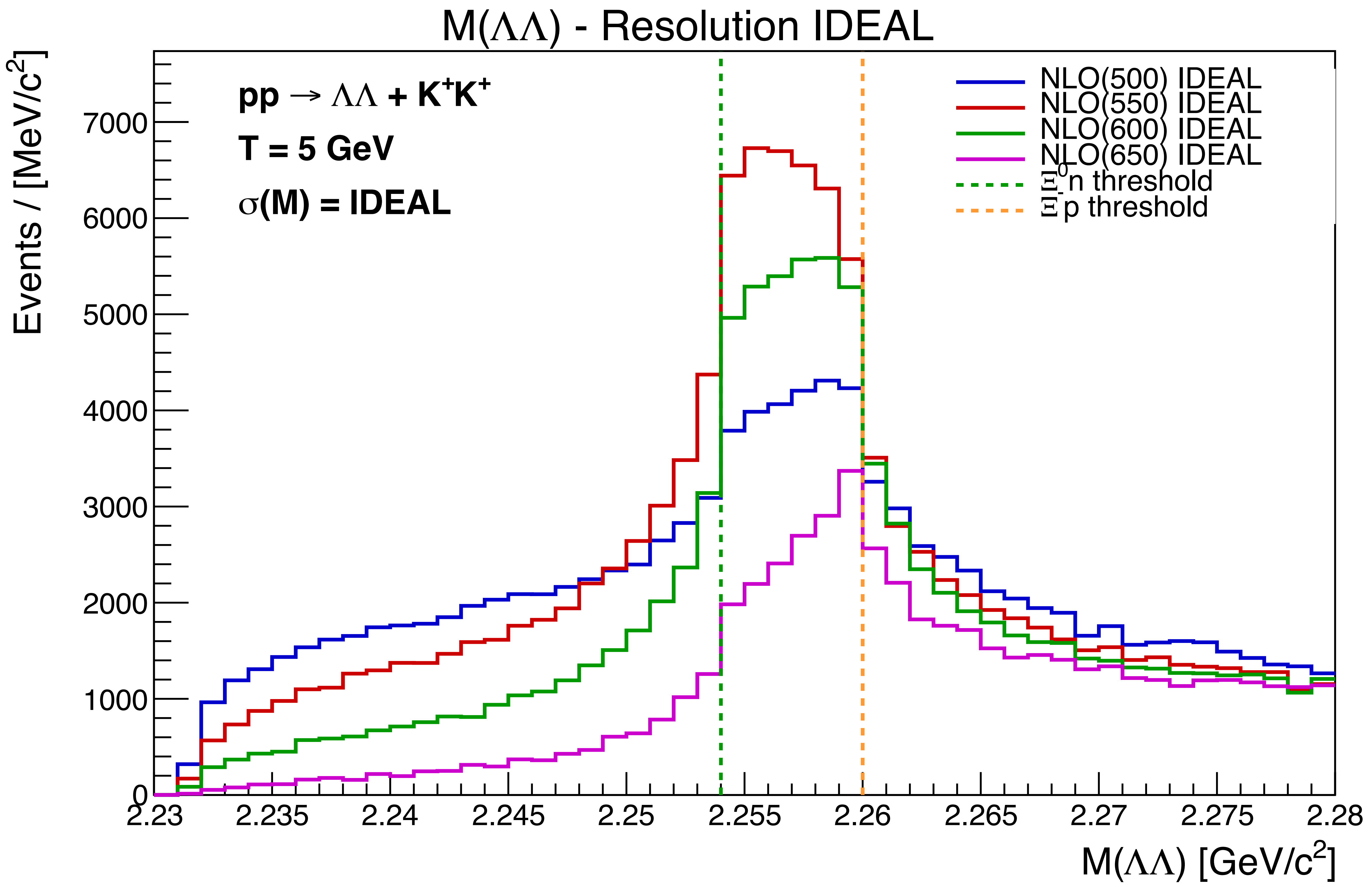


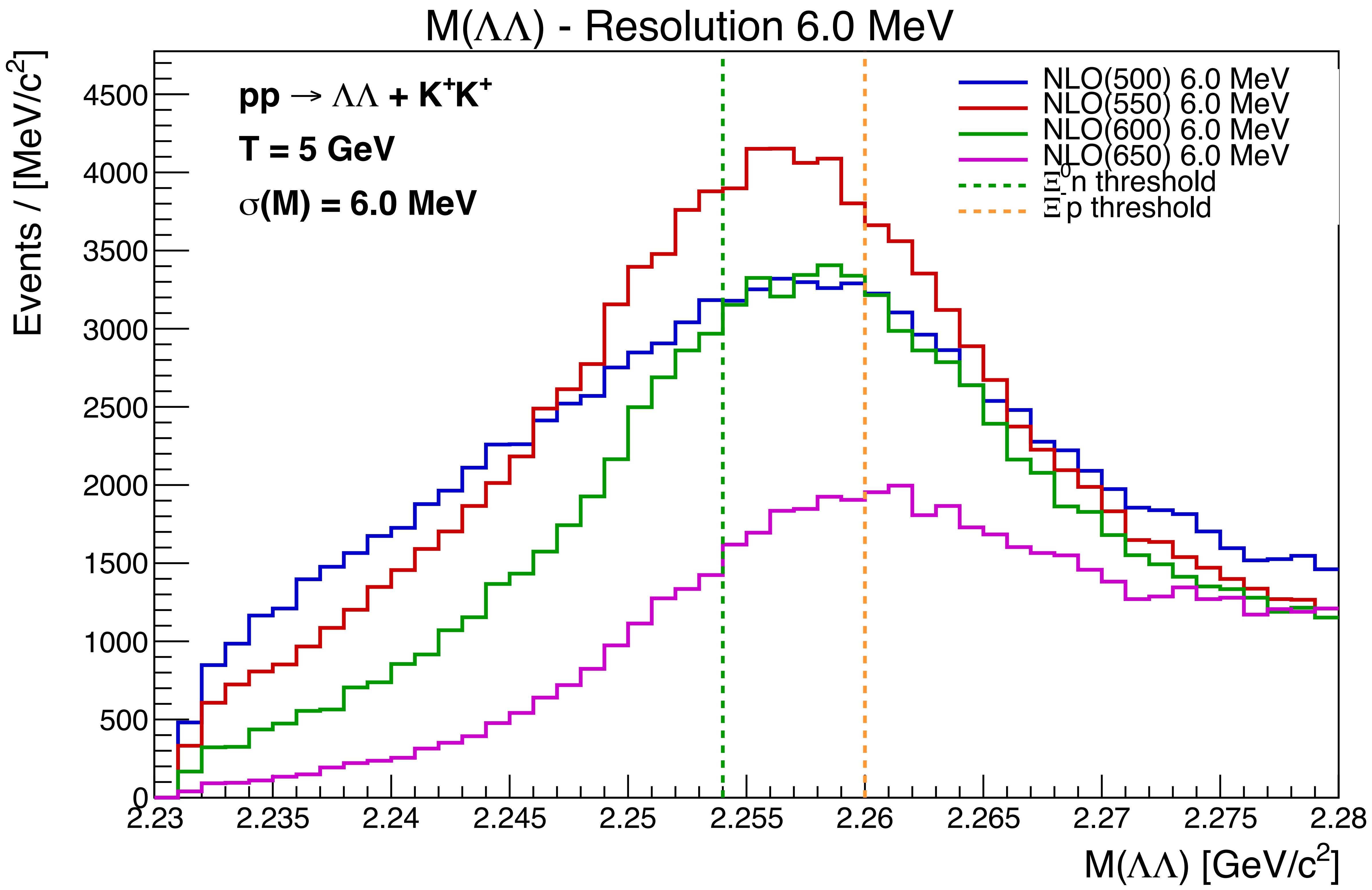


$$a_{\Lambda\Lambda} = -1.2 \pm 0.6 \pm 0.4 \text{ fm}$$

ΛΛ threshold is the lowest S = −2 baryon–baryon threshold, but the ΞN channel opens around ~25 MeV above it, so it has a nearby inelastic channel. They are not observed in the distribution from $K^- d$ data.

Distributions as a function of Cutoffs





$$|A(M_{\Lambda\Lambda})|^2 \sim \exp\left[C_0 + \frac{C_1^2}{M_{\Lambda\Lambda}^2 - C_2^2}\right]$$

$$a_{\Lambda\Lambda} = -C_1^2\sqrt{\frac{(m_{\max}^2 - 4m_\Lambda^2)}{(m_{\max}^2 - C_2)(4m_\Lambda^2 - C_2)^3}}$$

NLO Cutoffs

$$p\cot\delta = -\frac{1}{a} + \frac{r_e}{2}p^2$$

Table 1		NLO				LO			
	Λ [MeV]	500	550	600	650	550	600	650	700
$\Lambda\Lambda$	$a_{^1S_0}$	-0.62	-0.61	-0.66	-0.70	-1.52	-1.52	-1.54	-1.67
	$r_{^1S_0}$	6.95	6.06	5.05	4.56	0.82	0.59	0.31	0.34
$\Xi^0 n$	$a_{^3S_1}$	-0.25	-0.20	-0.26	-0.34	-0.34	-0.25	-0.20	-0.15
	$r_{^3S_1}$	5.35	8.36	5.26	2.93	-5.86	-8.27	-4.36	16.3
$\Xi^0 p$ $(I=1)$	$a_{^1S_0}$	0.37	0.39	0.34	0.31	0.21	0.19	0.17	0.13
	$r_{^1S_0}$	-4.71	-4.86	-7.07	-8.99	-30.7	-37.7	-52.8	-98.5
	$a_{^3S_1}$	-0.20	-0.04	0.021	0.039	0.02	0.00	0.02	0.03
	$r_{^3S_1}$	35.6	575.1	1797	450	968	$> 10^4$	1166	548
ΞN $(I=0)$	$a_{^3S_1}$	-0.33	-0.39	-0.62	-0.85	-0.85	-0.59	-0.43	-0.32
	$r_{^3S_1}$	-6.87	-1.77	1.00	1.42	-1.84	-3.93	-7.16	-12.56

a and r_e in fm. Haidenbauer, Meißner, Petschauer, NPA **954** (2016) 273. Nuclear convention: $a < 0$ attractive.

Table 2	$I=0,\ {}^1S_0$	$I=1,\ {}^1S_0$		$I=0,\ {}^3S_1$		$I=1,\ {}^3S_1$	
potential	a_s	a_s	r_s	a_t	r_t	a_t	r_t
NLO(500)	$-7.71 - i\,2.03$	0.37	-4.80	-0.33	-6.86	-1.17	3.44
NLO(550)	$-7.24 - i\,20.79$	0.39	-4.95	-0.39	-1.77	-1.15	3.80
NLO(600)	$-10.89 - i\,14.91$	0.34	-7.20	-0.62	1.00	-1.13	3.95
NLO(650)	$-8.14 - i\,2.43$	0.31	-9.16	-0.85	1.42	-0.90	4.27

ΞN in the isospin basis, fm. Haidenbauer and Meißner, EPJ A **55** (2019) 23. The $\Lambda\Lambda$ ${}^1S_0$ couples to ΞN $I=0$ ${}^1S_0$; a_s is complex because $m_{\Xi N} > 2m_\Lambda$. That coupling, not $a_{\Lambda\Lambda}$, sets the cusp height in $M(\Lambda\Lambda)$.