四川大學 SICHUAN UNIVERSITY

Structures of N^* and Δ Resonances from Dynamical Coupled-Channel Analyses

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08/09/2026

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Part I: Introduction

Hadron structures

Part I: Introduction

- Structures of matters $\rightarrow$ fundamental task of physics
- Structures of hadrons $\rightarrow$ extremely difficult
  - Not direct observables
  - Non-perturbative nature of QCD at low & middle energies
  - Uncertainties of the hadron spectroscopy
- A bridge between the data and the structures?
  World data $\rightarrow$??? $\rightarrow$ Spectra $\rightarrow$ Indications of the structures
- One possible option: comprehensive data-driven models
  $\rightarrow$ Jülich-Bonn/Jülich-Bonn-Washington Model
- Two studies:
  - Baryon transition form factors from the proton
    [Y.F. Wang *et al.* (Jülich-Bonn-Washington Collaboration), Phys. Rev. Lett. 133, 101901 (2024)]
  - Analyses of Compositeness
    [Y.F. Wang *et al.*, PRC 109, 015202 (2024)] [Y.F. Wang *et al.*, PRD 112, 074010 (2025)]

The Jülich-Bonn Model

Part I: Introduction

Dynamical Coupled-Channel (DCC) model [Döring *et al.*, Prog. Part. Nucl. Phys. 146, 104213 (2026)]

Talks by M. Mai, M. Döring, & D. Rönchen

Hadron dynamics

Lippmann-Schwinger-like equation

$$T_{\mu\nu}(p'', p', z) = V_{\mu\nu}(p'', p', z) +$$
$$\sum_\kappa \int_0^\infty p^2 dp V_{\mu\kappa}(p'', p, z) G_\kappa(p, z) T_{\kappa\nu}(p, p', z)$$

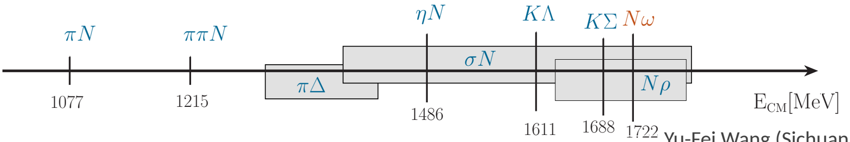
- One-dimensional: time-ordered perturbation theory $+$ JLS basis [Jacob & Wick, Annals Phys. 7, 404 (1959)]
- Effective Lagrangians $\to$ SU(3), chiral, CP...
- Effective three-body channels: ρN, σN, $\pi\Delta$

Photo- & electroproduction

Construction from final state interactions

$$M_{\mu\gamma^*}(Q^2) = V_{\mu\gamma^*}(Q^2) + \sum_\kappa \int p^2 dp T_{\mu\kappa} G_\kappa V_{\kappa\gamma^*}(Q^2)$$

- γ^*: the $\gamma^* N$ channel for electroproduction
- Q^2: photon virtuality
- $V_{\kappa\gamma^*}$ $\to$ phenomenologically parameterized
- Constraints: Siegert's theorem, kinematics, etc.
- Photoproduction $\to$ $Q^2 = 0$

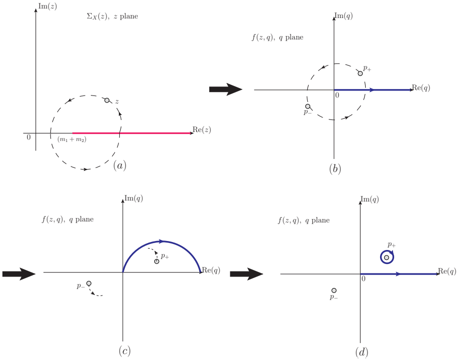


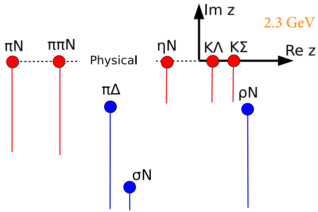
Pole searching

Part I: Introduction

- Resonances $\rightarrow$ poles on the second Riemann sheet
- Analytical continuation $\rightarrow$ contour deformation [Döring *et al.*, NPA 829, 170 (2009)]
- Pole position $z_r = M_r - i\Gamma_r/2$. Coupling strengths $\rightarrow$ normalized residues [PDG, RPP]

  $\tau^{II}_{\mu\nu} \sim \frac{R_\mu R_\nu}{z_r - z} + \cdots$, $NR_\mu \equiv \frac{2R_{\pi N}}{\Gamma_r} \times R_\mu$ [Indication of the structures!]





$p_{\pm}$: singularities in $G = (z - E_1 - E_2)^{-1}$

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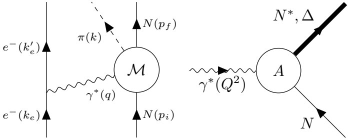
Part II: Baryon Transition Form Factors

Backgrounds

Part II: Baryon Transition Form Factors

Electromagnetic Probes

- EM interactions $\rightarrow$ clean probes of structures
- γN $\rightarrow$ finding states coupling weakly to πN
  [Ireland, Pasyuk, Strakovsky, Prog. Part. Nucl. Phys. 111, 103752 (2020)]
- Electroproduction ($\gamma^* N$) $\rightarrow$ energy scale $Q^2 \equiv -q^2$
- Transition Form Factors (TFFs) $\rightarrow$ "pictures of hadrons"
  [Ramalho, Peña, Prog. Part. Nucl. Phys. 136, 104097 (2024)]
  - Lower Q^2: meson clouds etc.
  - Higher Q^2: the quark core
  - Related to transverse charge densities
    [Tiator & Vanderhaeghen, PLB 672, 344 (2009)]



Towards the TFFs

- Predictions at quark level
  - Quark models & Dyson-Schwinger methods
    [Burkert, Roberts RMP 91, 011003 (2019)]
    [Eichmann *et al.*, Prog. Part. Nucl. Phys. 91, 1 (2016)]
  - Lattice QCD [Agadjanov *et al.*, NPB 886, 1199 (2014)]
- Extraction from data
  - Huge data accumulation
    [Mokeev *et al.*, PRC 93, 025206 (2016)]
    Talks by V. Mokeev & D. Carman on Monday
  - Unitary isobar models $\rightarrow$ real-valued, depending on Breit-Wigner parameters
    [Drechsel, Kamalov, Tiator, EPJA 34, 69 (2007)]
    [Tiator *et al.*, EPJST 198, 141 (2011)]
  - DCC models $\rightarrow$ complex TFFs from the residues
    [Kamano, Few Body Syst. 59, 24 (2018)]

This study: [Y.F. Wang *et al.*, PRL 133, 101901 (2024)]

Formulae & Extraction

Part II: Baryon Transition Form Factors

Origianl definition

[Ramalho, Peña, Prog. Part. Nucl. Phys. 136, 104097 (2024)]

$$A_h = \sqrt{\frac{2\pi\alpha}{K}}\left\langle R, h \middle| \epsilon_+ \cdot J \middle| N, h-1 \right\rangle$$

$$S_{\frac{1}{2}} = \frac{|\mathbf{q}|}{Q}\sqrt{\frac{2\pi\alpha}{K}}\left\langle R, \tfrac{1}{2} \middle| \epsilon_0 \cdot J \middle| N, \tfrac{1}{2} \right\rangle$$

- A, S: helicity transition amplitudes
- $h = 1/2, 3/2$: the helicity
- α: fine structure constant
- $\epsilon(J)$: virtual photon polarization vector (current)
- $\mathbf{q}$: 3-momentum of the virtual photon
- $M_R(m_N)$: mass of the excitation state R (nucleon)
- $K = (M_R^2 - m_N^2)/(2M_R)$

At the pole

[Workman, Tiator, Sarantsev, PRC 87, 068201 (2013)]

$$H_h = C_I\sqrt{\frac{p_{\pi N}}{\omega_0}\frac{2\pi(2J+1)z_p}{m_N\widetilde{R}}}\widetilde{\mathcal{H}}_h$$

- H is either A or S
- C_I: isospin factor, $C_{1/2} = -\sqrt{3}$ and $C_{3/2} = \sqrt{2/3}$
- $p_{\pi N}$: πN c.m. momentum
- ω_0: photon energy at $Q^2 = 0$
- $z_p = M_R - i\Gamma_R/2$ the pole position
- $\widetilde{R}$, $\widetilde{\mathcal{H}}$: the residues of πN, $\gamma^* N$ channels
- Understanding: the $|R\rangle \to |R)$ defined from the Gamow state
- Complex-valued

Numerical details

Part II: Baryon Transition Form Factors

The latest JBW results

- $\gamma^* p$ initial state
- Coupled-channel study of πN, ηN, and $K\Lambda$
  [Mai *et al.*, EPJA 59, 286 (2023)]
- Based on the JüBo2017 solution
  [Rönchen *et al.*, EPJA 54, 110 (2018)]
- C.M. energy range $z \in [1.13, 1.8]$ GeV
- Virtuality $Q^2 \in [0, 8]$ GeV^2
- Orbital angular momentum $L \leq 3$

Database & errors

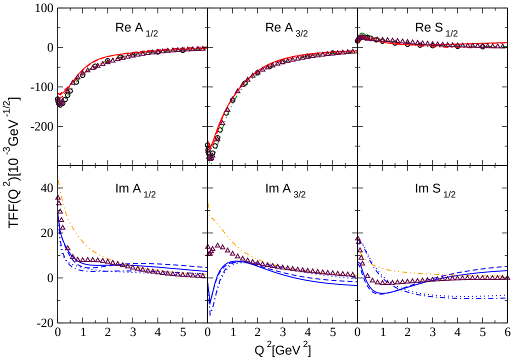
- Database [Mai *et al.*, EPJA 59, 286 (2023)]
  - 10^5 data points vs 533 fit parameters
  - 5×10^4 from photoproduction/hadronic
- Four solutions $\to$ fully explored parameter space
  - weighted vs unweighted χ^2
  - different local minima
- Fit $\to$ supercomputers

	$\chi^2_{\rm dof}$	$\chi^2_{\rm pp}(\pi^0 p)$	$\chi^2_{\rm pp}(\pi^+ n)$	$\chi^2_{\rm pp}(\eta p)$	$\chi^2_{\rm pp}(K^+\Lambda)$
FIT$_1$	1.42	1.40	1.47	1.49	0.70
FIT$_2$	1.35	1.38	1.35	1.40	0.58
	$\chi^2_{\rm wt,dof}$	$\chi^2_{\rm pp}(\pi^0 p)$	$\chi^2_{\rm pp}(\pi^+ n)$	$\chi^2_{\rm pp}(\eta p)$	$\chi^2_{\rm pp}(K^+\Lambda)$
FIT$_3$	1.12	1.44	1.61	1.08	0.33
FIT$_4$	1.06	1.42	1.44	1.09	0.32

Result: $\Delta(1232)$

Part II: Baryon Transition Form Factors

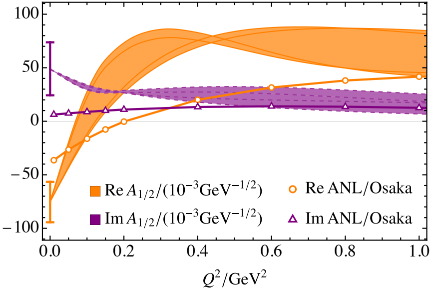
- Solid, dashed, dotted, dash-dotted curves: four fit solutions
- Dash-double-dotted curves: "L+P" extraction from MAID analyses [Workman, Tiator, Sarantsev, PRC 87, 068201 (2013)]
- Triangles: ANL-Osaka[Kamano, Few Body Syst. 59, 24 (2018)]

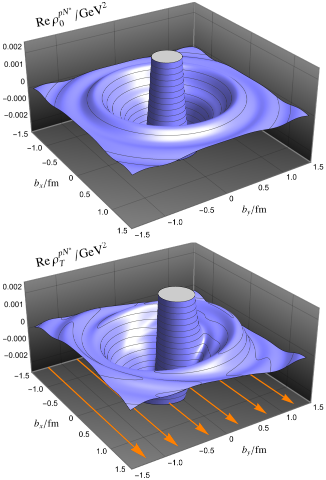


Result: $N^*(1440)$

Part II: Baryon Transition Form Factors

- A zero crossing!!
- ρ_0, ρ_T [Tiator & Vanderhaeghen, PLB 672, 344 (2009)]

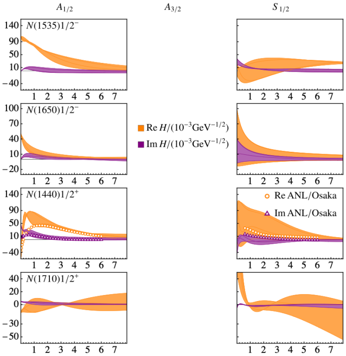


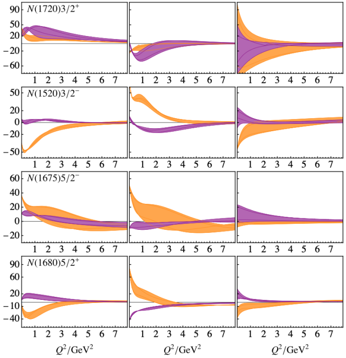


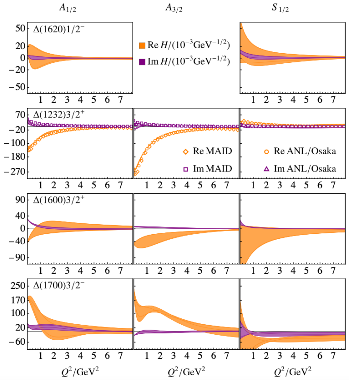
Summary of the results

Part II: Baryon Transition Form Factors

First combined analyses of the TFFs at the poles, based on multi-channel data







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Part III: Analyses of Compositeness

Studies of Compositeness

- Compact quark/gluon states v.s. molecules
- Weinberg's criterion on deuteron $|d\rangle$ (1960s)
  [Weinberg, PR 137, B672 (1965)]

$$a = -\frac{2(1-Z)}{2-Z}R + \mathcal{O}(L)$$
$$r = -\frac{Z}{1-Z}R + \mathcal{O}(L)$$

  Z: "elementariness". $R \sim 4.3$ fm: deuteron radius. $L \sim m_\pi^{-1}$ interaction range.
  a: scattering length. r: effective range.

- Unstable resonances?

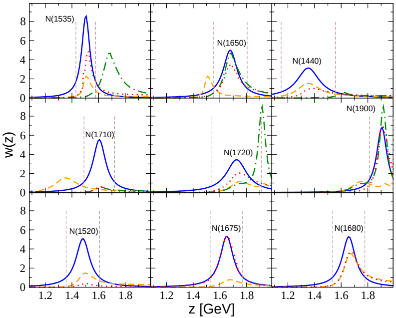
Criteria

- Pole-counting rule [Morgan, NPA 543, 632 (1992)]
  S-wave near-threshold molecule
  $\to$ only one pole
- Spectral density functions [Baru *et al.*, PLB 586, 53 (2004)]
  Weinberg's Z
  $\to$ spectral density function $w(E)$
- Gamow wave functions [Sekihara, PRC 95, 025206 (2017)]
  Weinberg's Z
  $\to$ complex for Gamow states (resonances)
- Jülich-Bonn model
  $\to$ pole parameters etc. for the criteria!

Analyses of N^* and Δ

Part III: Analyses of Compositeness

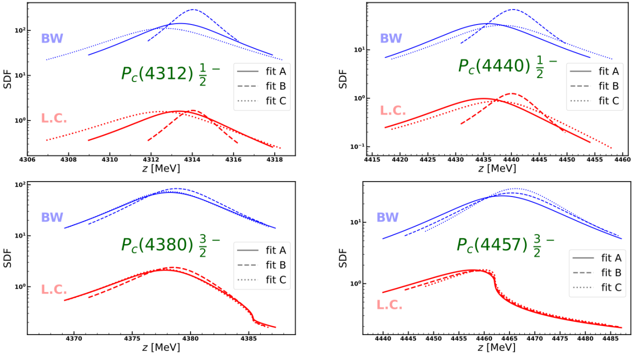
- Publication:
  [Y.F. Wang *et al.*, PRC 109, 015202 (2024)]
- Spectral density functions + Gamow wave functions
- Parameters from JüBo2022 solution
  [Rönchen *et. al.*, EPJA 58, 229 (2022)]
- High compositeness
  $N(1535)\frac{1}{2}^-, N(1440)\frac{1}{2}^+, N(1710)\frac{1}{2}^+, N(1520)\frac{3}{2}^-$
- High elementariness
  $N(1650)\frac{1}{2}^-, N(1900)\frac{3}{2}^+, N(1680)\frac{5}{2}^+,$
  $\Delta(1600)\frac{3}{2}^+$



Analyses of P_c exotic states

Part III: Analyses of Compositeness

- Publication: [Y.F. Wang *et al.*, PRD 112, 074010 (2025)]
- Near-threshold S-wave states $\to$ three criteria
- Four P_c states in this model
  $P_c(4312)\frac{1}{2}^-$, $P_c(4440)\frac{1}{2}^-$, $P_c(4380)\frac{3}{2}^-$, $P_c(4457)\frac{3}{2}^-$
- Molecular interpretation favoured!



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Part IV: Conclusion & Outlook

Conclusion & Outlook

Part IV: Conclusion & Outlook

Conclusions

- The Jülich-Bonn Model
  - Comprehensive dynamical coupled-channel approaches
  - Data driven PWA $\rightarrow$ resonance spectra
  - Connecting experimental observations to hadron structures!
- Baryon transition form factors
  - Determined by multi-channel data
  - Defined at the poles
  - Realistic uncertainties
  - Outputs for twelve states
- Compositeness:

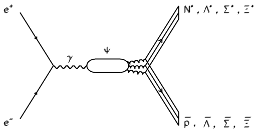
  the model provides information for compositeness criteria

Outlook

- The hyperon spectroscopy in the near future
- JBW model $\rightarrow$ energy extension to 1.95 GeV
- ωN photoproduction underway $\rightarrow$ more modern data!
- Improvement of the statistics
- Inclusion of $\pi\pi N$ data
- Broader applications $\rightarrow$ BES: $J/\psi(\psi') \rightarrow \bar{N}N^*$

  [BES, PRL 97, 062001 (2006)] [BES, PRL 110, 022001 (2013)]

  [BESIII, CPC 44, 040001 (2020)]




Thank you

Backups

The Jülich-Bonn Model

Backups

A comprehensive coupled-channel model, fitting to a worldwide collection of data

Hadronic part

- The πN elastic scatterings [Schütz *et al.*, PRC 51, 1374 (1995)] [Schütz *et al.*, PRC 49, 2671 (1994)]
- Extended to $\pi\pi N$ and ηN [Schütz *et al.*, PRC 57, 1464 (1998)] [Krehl *et al.*, PRC 62, 025207 (2000)] [Gasparyan *et al.*, PRC 68, 045207 (2003)]
- Extended to $K\Lambda$ and $K\Sigma$ [Döring *et al.*, NPA 851, 58 (2011)] [Rönchen *et al.*, EPJA 49, 44 (2013)]
- Extended to ωN [Wang *et al.*, PRD 106, 094031 (2022)]
- Analytical continuation for searching poles [Döring *et al.*, NPA 829, 170 (2009)]

Photo- & Electroproduction

- Photoproduction
  [Rönchen *et al.*, EPJA 50, 101 (2014)] [Rönchen *et al.*, EPJA 51, 70 (2015)][Rönchen *et al.*, EPJA 54, 110 (2018)] [Rönchen *et al.*, EPJA 558, 229 (2022)]
- Electroproduction (Jülich-Bonn-Washington Model)
  [Mai *et al.*, PRC 103, 065204 (2021)] [Mai *et al.*, PRC 106, 015201 (2022)] [Mai *et al.*, EPJA 59, 286 (2023)]
- P_c states [Wang *et al.*, EPJC 82, 497 (2022)] [Shen *et al.*, EPJC 84, 764 (2024)] [Wang *et al.*, PRD 112, 074010 (2025)]

Details of the scattering equation

Backups

The Lippmann-Schwinger-like equation

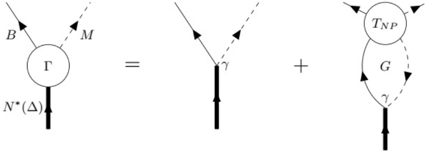
$$T_{\mu\nu}(p'', p', z) = V_{\mu\nu}(p'', p', z) + \sum_\kappa \int_0^\infty p^2 dp V_{\mu\kappa}(p'', p, z) G_\kappa(p, z) T_{\kappa\nu}(p, p', z)$$

- Reaction channels $\nu \to \mu$ (after PW and isospin projection, JLS basis [Jacob & Wick, Annals Phys. 7, 404 (1959)],$J \leq 9/2$)
- Intermediate channel: κ
- CM initial (final) momentum: $p'(p'')$. CM energy: z
- Potential (kernel): V. Amplitude: T ($\to$ observables)
- Propagator: G ($\pi\pi N$ channel: effective channels $\rho N, \sigma N, \pi\Delta$. E/ω - energy of the baryon/meson.)

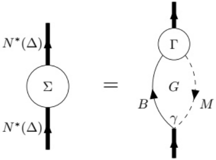
$$G_\kappa(z, p) = \begin{cases} (z - E_\kappa - \omega_\kappa + i0^+)^{-1} & \text{(if } \kappa \text{ is a two-body channel)} , \\ \left[z - E_\kappa - \omega_\kappa - \Sigma_\kappa(z, p) + i0^+\right]^{-1} & \text{(if } \kappa \text{ is an effective channel)} . \end{cases}$$

Details of the scattering equation
Backups

- Separating the amplitude $\rightarrow$ with/without s-channel poles $T = T^P + T^{NP}$
- Reconstruction of the amplitude $\rightarrow$ $T^{NP} = V^{NP} + \sum \int p^2 dp V^{NP} G T^{NP}$,
  $T^P_{\mu\nu}(p'', p', z) = \sum_{i,j} \Gamma^a_{\mu,i}(p'') D_{ij}(z) \Gamma^c_{\nu,j}(p')$, $(D^{-1})_{ij} = \delta_{ij}(z - m_i^b) - \Sigma_{ij}(z)$
  - $\Gamma(\gamma)$: the dressed (bare) vertices (a - annihilation, c - creation)
  - Σ: coupled-channel self-energy functions of the s-channel states



(a) The vertex.



(b) The self energy.

Details of the scattering equation

Backups

Potentials $\longrightarrow$ field-theoretical construction

Parameters $\longrightarrow$ determined by fits

The NP part

- Tree-level potentials
  - t-channel $+$ u-channel $+$ contact
  - Stemming from effective Lagrangians $\longrightarrow$ SU(3) flavour symmetry, CP conservation, chiral symmetry
  - Established by time-ordered perturbation theory (TOPT) $\longrightarrow$ stationary perturbation in Schrödinger picture
  - TOPT$+$partial wave $\longrightarrow$ one-dimensional integral $\int p^2 dp$
  - Regulators for every vertex $\longrightarrow$ to make the integral converge: $F(q) \sim \left(\frac{\Lambda^2 - m^2}{\Lambda^2 + q^2}\right)^n$

    m: the mass of the exchanged particle. Λ: cut-off (fit parameter)
- Beyond tree-level $\longrightarrow$ correlated two-pion exchanges [Schütz *et al.*, PRC 49, 2671 (1994)] [Schütz *et al.*, PRC 51, 1374 (1995)]

The P part

- Stemming from effective Lagrangians with CP conservation (tree-level bare vertices)
- Phenomenological contact terms $\longrightarrow D \sim (1 - \Sigma)^{-1}$ [Rönchen *et al.*, EPJA 51, 70 (2015)]
- Renormalization of the nucleon mass

Criterion I

Backups

Weinberg's criterion

- Weinberg's criterion on deuteron $|d\rangle$

$$a = -\frac{2(1-Z)}{2-Z}R + \mathcal{O}(L)$$
$$r = -\frac{Z}{1-Z}R + \mathcal{O}(L)$$

  Z: "elementariness". $R \sim 4.3$ fm: deuteron radius. $L \sim m_\pi^{-1}$ interaction range. a: scattering length. r: effective range.
- $Z = 1 - \int d\alpha |\langle \alpha | d \rangle|^2$. α: two-nucleon continuum
- Experimental results $\rightarrow Z \simeq 0$
- Conditions: S-wave, near-threshold, stable
- Model-independent [van Kolck, Symmetry 14, 1884 (2022)]

Pole-counting rule

- Near-threshold single-channel amplitudes: $T \sim (rp^2/2 - ip + 1/a)^{-1}$
- One bound state p_- and one virtual state p_+: $|p_+| + |p_-| \geq \frac{2}{|r|}$
- A molecule $\rightarrow$ quantum mechanical potentials
  $\rightarrow |r|$ being the interaction range
  $\rightarrow$ only one pole inside $1/|r|$
- A genuine state $\rightarrow |r|$ extremely large,
  $|p_+| \simeq |p_-|$ [Castillejo *et al.*, PR 101, 453 (1956)]
  $\rightarrow$ two nearby poles
- Can be extended to coupled-channel scatterings
- Only for S-wave near threshold states
- Applications [Meng *et al.*, PRD 92, 034020 (2015)]
  [Gong *et al.*, PRD 94, 114019 (2016)][Cao *et al.*, CPC 45, 103102 (2021)]

Criterion II

Backups

Spectral Density Functions

- Weinberg's Z for bound states: $Z = |\langle\psi_0|\Psi_B\rangle|^2$
- Lower decay channels: $Z \to w(E) = -\mathrm{Im}D(E)/\pi$ $\to$ distribution on energy E
- Narrow resonance $E = E_R - i\Gamma_R/2$:
  $w(E) \simeq \frac{Z_B}{\pi} \frac{\Gamma_R/2}{(E-E_R)^2+(\Gamma_R/2)^2}$
- S-wave nearby channels:
  $w(E) \simeq -\frac{1}{\pi}\mathrm{Im}(E - E_0 - \sum_j i g_j^2 p_j/2)^{-1}$
  j: channels. E_0 and g_j's: from residues (couplings) and pole positions
- Elementariness $\to$ deviation of $w(E)$ from "Breit-Wigner":
  $BW(E) = \frac{1}{\pi} \frac{\Gamma_R/2}{(E-E_R)^2+(\Gamma_R/2)^2}$
- Applications [Kalashnikova and Nefediev, PRD 80, 074004 (2009)]
  [Baru *et al.*, EPJA 44, 93 (2010)][Hanhart *et al.*, EPJA 47, 101 (2011)]

Complex Compositeness

- Resonances
  - Unphysical Gamow states
    [Gamow, Zeitschrift für Physik 51, 204 (1928)]
    [Civitarese and Gadella, Physics Reports 396, 41(2004)]
  - Complex eigenstates of the Hamiltonian
    $\hat{H}|\Psi_R) = E_R|\Psi_R)$
  - Non-normalizable [Xiao and Zhou, PRD 94, 076006 (2016), J. Math. Phys. 58, 062110 (2017)]
- Complex elementariness: $Z_R = (\Psi_R^*|\psi_0\rangle\langle\psi_0|\Psi_R)$, complex-valued
- Complex compositeness $\to$ off-shell residues $r(k)$
  $T(E, p_i, p_f) \sim \frac{r(p_i)r(p_f)}{E-E_R}$
  $X = 1 - Z_R = \int_C \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{r^2(k)}{[\mathcal{E}_R - k^2/(2\mu)]^2}$
  [Sekihara, PRC 95, 025206 (2017)]
- Naive measure, e.g. $\tilde{X}_j = \frac{|X_j|}{\sum_n |X_n| + |Z_R|}$