Three-body unitary coupled-channel analysis on $\eta(1405/1475)$ and other systems

Phys. Rev. D 107, L091505 (2023); 109, 014021 (2024)

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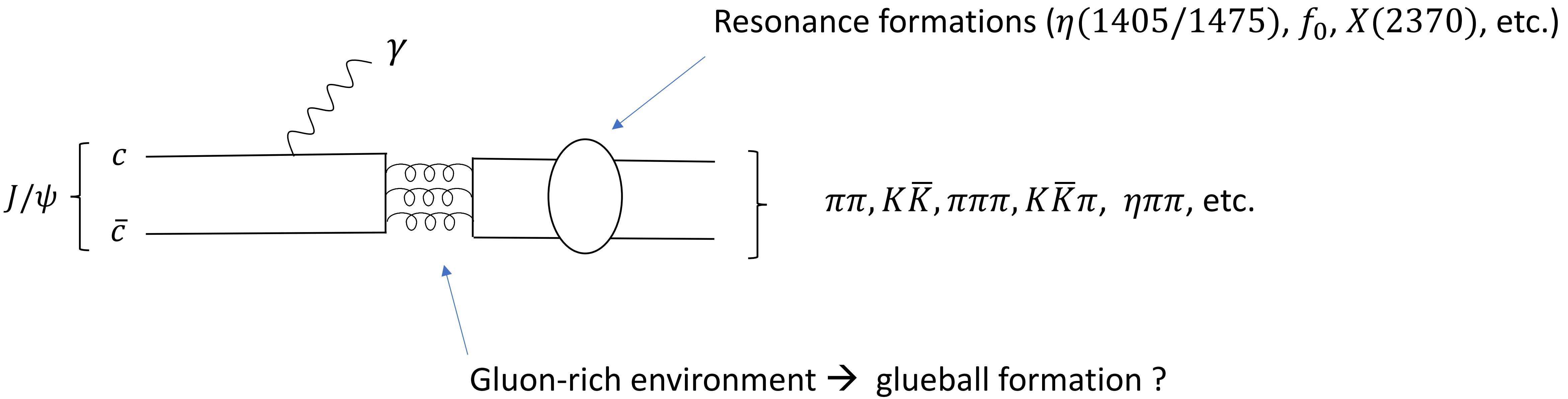
Introduction

J/ψ (charmonia) decays offer great playground to study light (exotic) hadrons



We will study $\eta(1405/1475)$ by analyzing radiative J/ψ decays

Interesting application of three-body unitary dynamical coupled-channel model

$\boldsymbol{\eta(1405/1475)}$: isoscalar pseudoscalar light meson(s) in 1.4– 1.5 GeV region

interesting, controversial, and open questions

- One state or two different states ?

Two peaks in $K\bar{K}\pi$ spectrum data for $\pi^- p \to K^+K^-\pi^0 n,\ p\bar{p} \to K\bar{K}\pi\pi\pi,\ J/\psi \to \gamma(K\bar{K}\pi)$

One peak in $\eta\pi\pi$ spectrum data for $p\bar{p} \to \eta\pi\pi\pi\pi,\ J/\psi \to \gamma(\eta\pi\pi),\ \gamma\gamma \to \eta\pi\pi;\ \ \gamma\gamma \to K\bar{K}\pi$

- Quark model : $\eta(1405/1475)$ → Radial excitation of η' (only one state available in quark model)

How about the other state ? Glueball ? But LQCD predicts mass of 2 - 2.5 GeV

- Large isospin violating decay rate : $\eta(1405/1475) \to \pi\pi\pi$ BESIII, PRL 108, 182001 (2008)

statistically limited data → allow various theoretical interpretations

BESIII amplitude analysis on $J/\psi \to \gamma K_S K_S \pi^0$

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Based on a sample of $\sim 10^{10}$ J/ψ decays → currently the most precise data on $\eta(1405/1475)$

→ quantitative discussions on $\eta(1405/1475)$ at the previously unreachable level

Conclusions

- $\eta(1405/1475)$ has two states → ruling out single-state solution
- Main decay modes

$$\eta(1405) \to a_0(980)\pi$$

$$\eta(1475) \to K^*\bar{K}$$

Theoretical issues still remain

- In the BESIII analysis, $\eta(1405/1475)$ mass and width values are Breit-Wigner parameters (not pole)

Breit-Wigner amplitude does not respect unitarity and not suitable when:

-- more than one resonances are overlapping $\leftarrow$ $\eta(1405)$ and $\eta(1475)$ are close

-- resonance is near its decay channel threshold $\leftarrow$ $\eta(1405)$ is near $K^*\bar{K}$ threshold

Theoretically sound approach $\rightarrow$ analyze BESIII data with unitary coupled-channel model

$\rightarrow$ $\eta(1405/1475)$ poles are searched in analytically continued amplitude

- Same unitary coupled-channel model should also explain:

Different lineshapes for different final states, large isospin violating decay rates, etc.

This work

Three-body unitary coupled-channel analysis of radiative J/ψ decays

Data : Dalitz plots from BESIII energy dependent solution for $J/\psi \to \gamma\eta(1405/1475) \to \gamma(K_S K_S \pi^0)$

Branching ratios for $\eta\pi\pi$ and $\pi\pi\gamma$ final states relative to $K\bar{K}\pi$

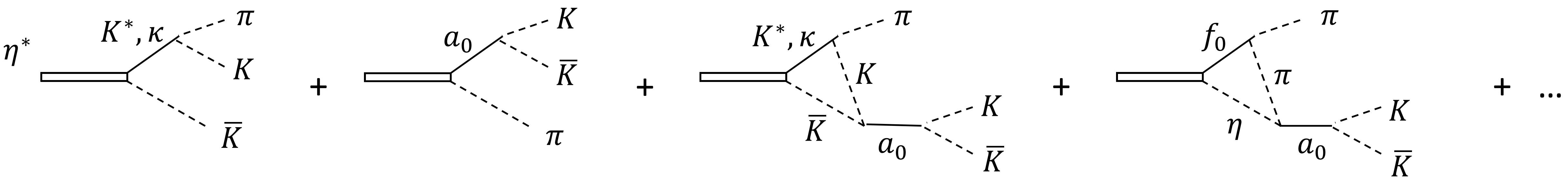
→ On the basis of unitary coupled-channel model, we will address the $\eta(1405/1475)$ puzzles

- Determine $\eta(1405/1475)$ pole locations for the first time (practical issue; no longer academic)
- Predict $\eta(1405/1475) \to \eta\pi\pi$ lineshape → understanding process-dependent lineshape
- Predict $\eta(1405/1475) \to \pi\pi\pi$ branching fractions, lineshapes

MODEL

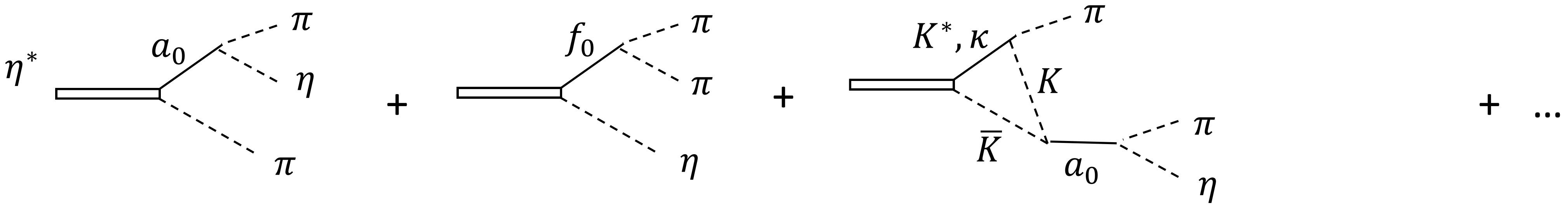
η^* decay processes

$\eta^* \to K\bar{K}\pi$



(until infinite loops)

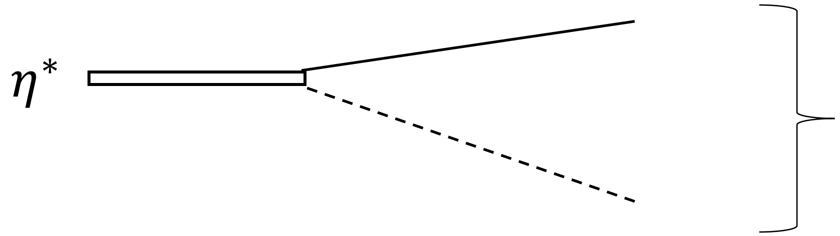
$\eta^* \to \eta\pi\pi$



(until infinite loops)

Final state interactions from Faddeev equation $\rightarrow$ three-body unitarity

Coupled-channels included



$K^*(892)\bar{K}$, $K_0^*(700)\bar{K}$, $a_0(980)\pi$, $a_2(1320)\pi$, $f_0\eta$, $\rho\rho$

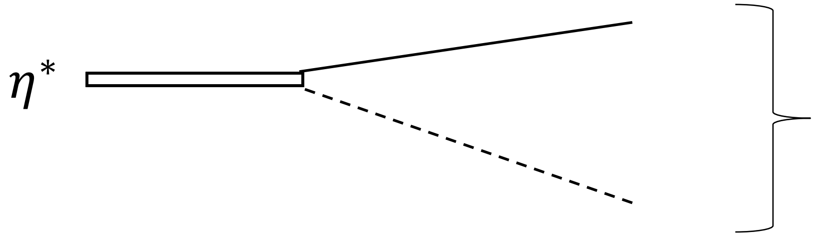
(κ)

$\rightarrow f_0(500)$, $f_0(980)$

$f_0\eta$, $\rho\rho$ $\rightarrow$ $K\bar{K}\pi$

via loop

BESIII model [JHEP 03 (2023) 121]



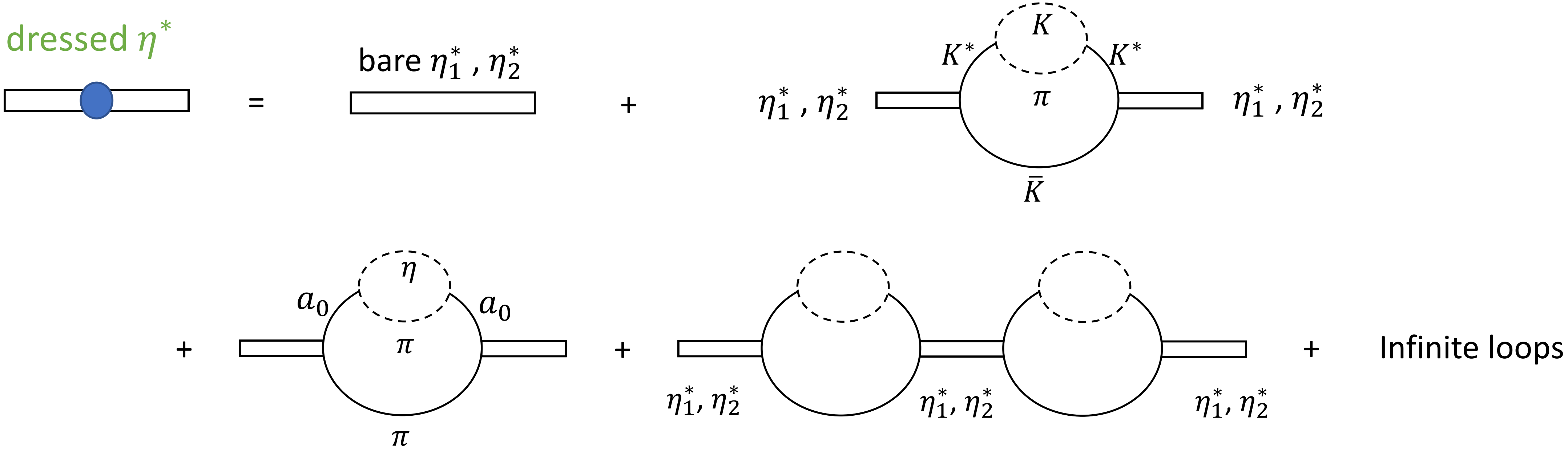
$K^*(892)\bar{K}$, $a_0(980)\pi$, $a_2(1320)\pi$

Why our model needs more channels ? → requirement from unitarity

→ unified description of $K\bar{K}\pi$, $\eta\pi\pi$, $\rho^0\gamma$, $\pi\pi\pi$ final states

η^* propagator

Bare state: seed of resonance;
resonance without meson-meson continuum



$\eta(1405/1475)$ poles are formed by non-perturbative couplings between bare η^* and $K^*\bar{K}$, $a_0(980)\pi$, ...

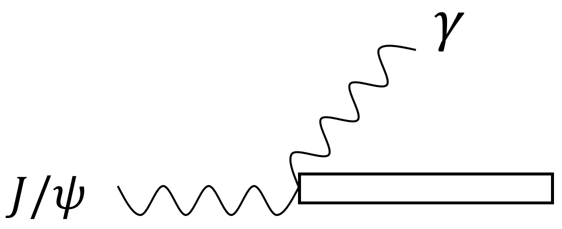
(= poles of dressed η^*propagator)

Unitary coupled-channel model : resonance pole (mass, width) and decay dynamics are explicitly related.

→ unitarity requirement

Breit-Wigner model : decay dynamics are simulated by mass and width parameters

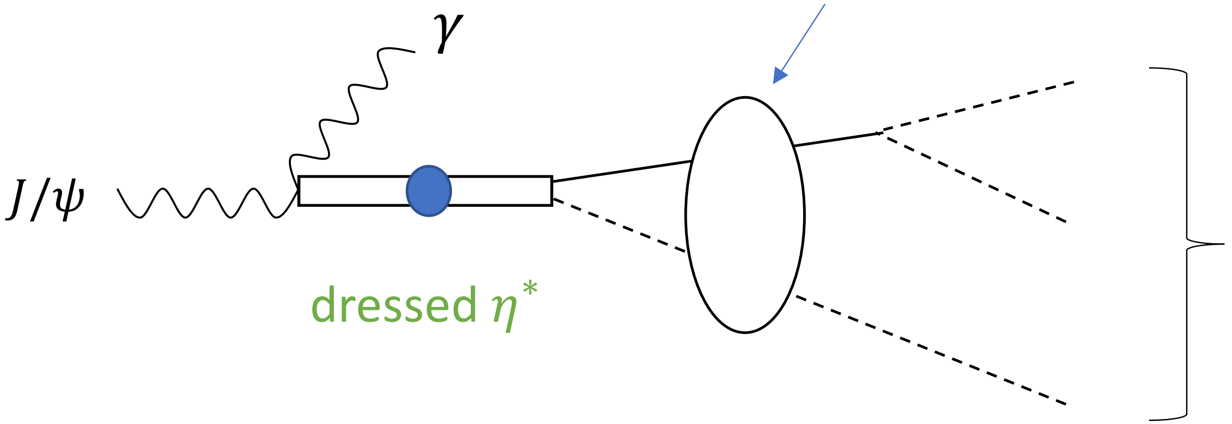
Initial radiative J/ψ decay vertex



bare η_1^* and η_2^*

Full amplitude for $J/\psi \to \gamma\eta(1405/1475) \to \gamma K\bar{K}\pi$, $\gamma\eta\pi\pi, \gamma\pi\pi\gamma, \gamma\pi\pi\pi$

tree, one-loop, two-loop, … (nonresonant Faddeev amplitude)



$K\bar{K}\pi$, $\eta\pi\pi, \pi\pi\gamma, \pi\pi\pi$

Results

Data to fit

- Dalitz plot distribution (pseudodata) for $J/\psi \to \gamma\eta(1405/1475) \to \gamma(K_S K_S \pi^0)$ generated from energy-dependent MC solution of BESIII analysis [JHEP 03 (2023) 121]

$$1300 \leq M_{K_S K_S \pi^0} \leq 1600 \text{ MeV (30 points, 10 MeV interval)}$$

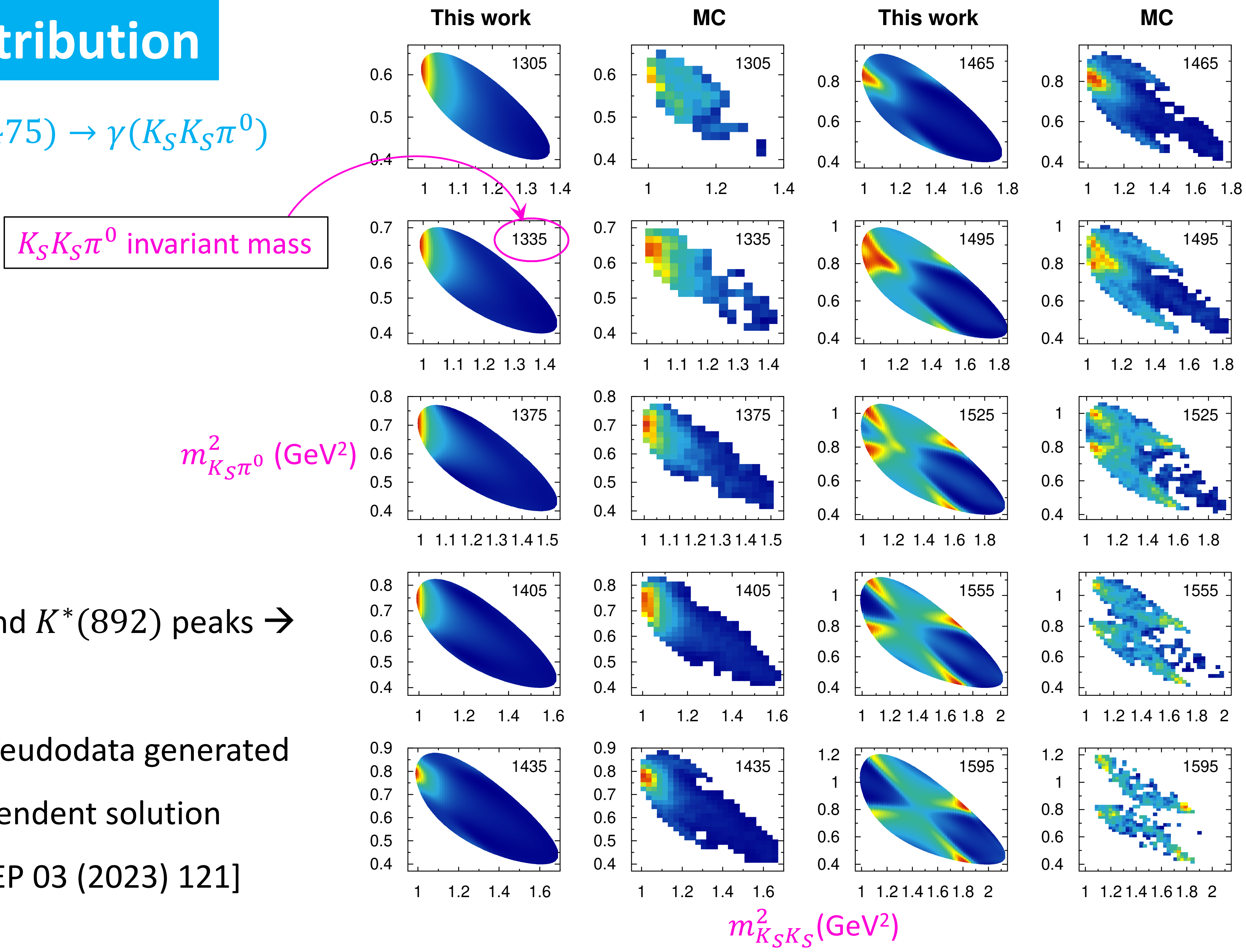
- $$\frac{\mathrm{Br}[\, J/\psi \to \gamma\eta(1405/1475) \to \gamma(K\bar{K}\pi)\,]}{\mathrm{Br}[\, J/\psi \to \gamma\eta(1405/1475) \to \gamma(\eta\pi^+\pi^-)\,]} = \frac{(2.8\pm0.6)\times10^{-3}}{(3.0\pm0.5)\times10^{-4}} \sim 6-13 \quad \text{(PDG)}$$

- $$\frac{\mathrm{Br}[\, J/\psi \to \gamma\eta(1405/1475) \to \gamma(\rho^0\gamma)\,]}{\mathrm{Br}[\, J/\psi \to \gamma\eta(1405/1475) \to \gamma(K\bar{K}\pi)\,]} \sim 0.015-0.043 \quad \text{(MARKIII, BESII combined)}$$

$K_S K_S \pi^0$ final state

Dalitz plot distribution

$J/\psi \to \gamma\eta(1405/1475) \to \gamma(K_S K_S \pi^0)$



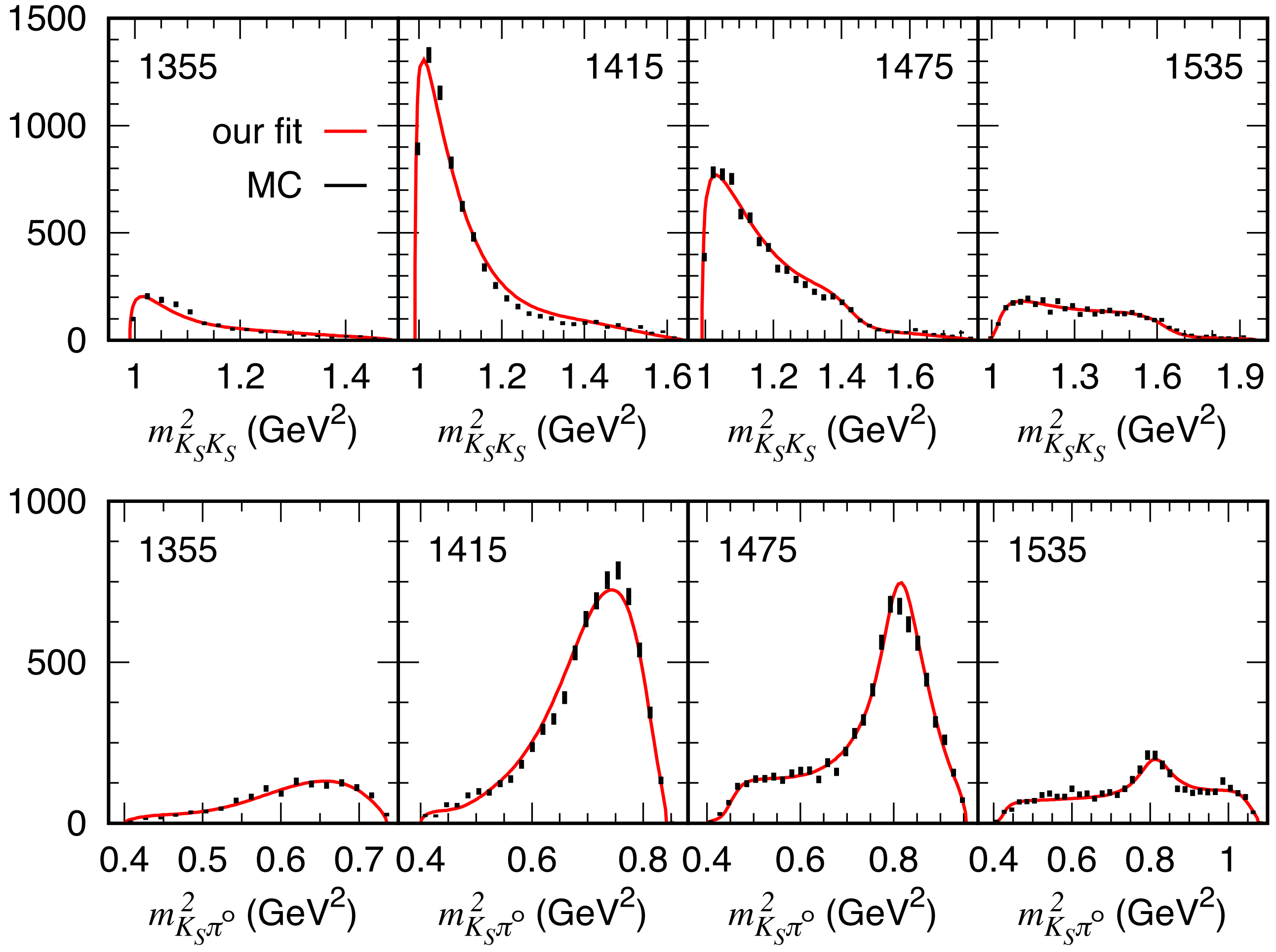
Clear $a_0(980)$-like and $K^*(892)$ peaks →

Overall good fit to pseudodata generated from the energy dependent solution of BESIII analysis [JHEP 03 (2023) 121]

K_SK_S and $K_S\pi^0$ invariant mass distributions

K_SK_S

$K_S\pi^0$

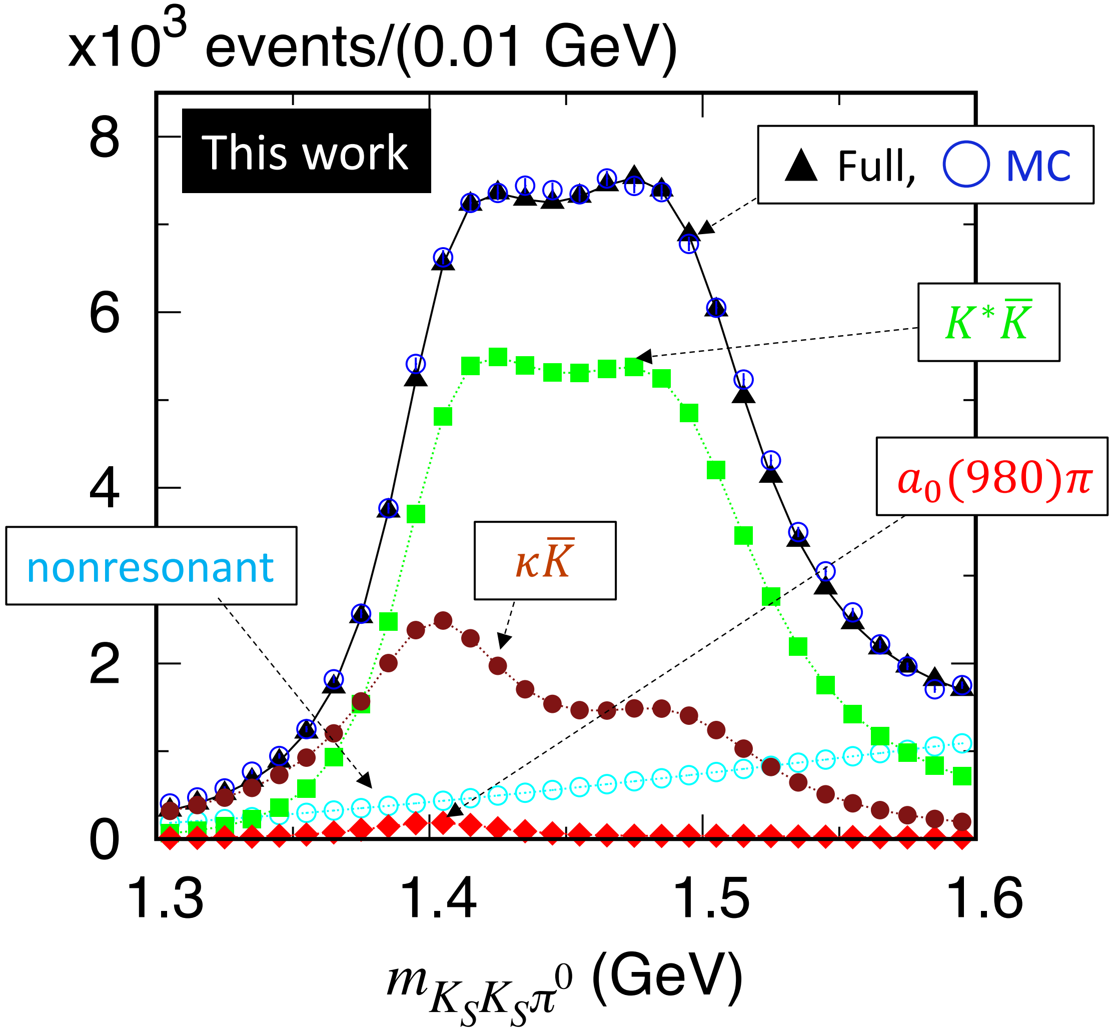


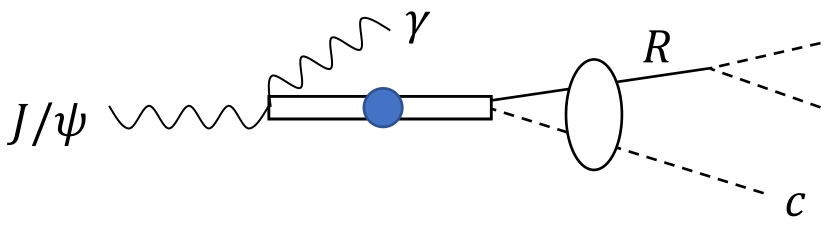
← $a_0(980)$-like peak near K_SK_S threshold

← $K^*(892)$ peak

Overall good fit

$K_S K_S \pi^0$ invariant mass distribution



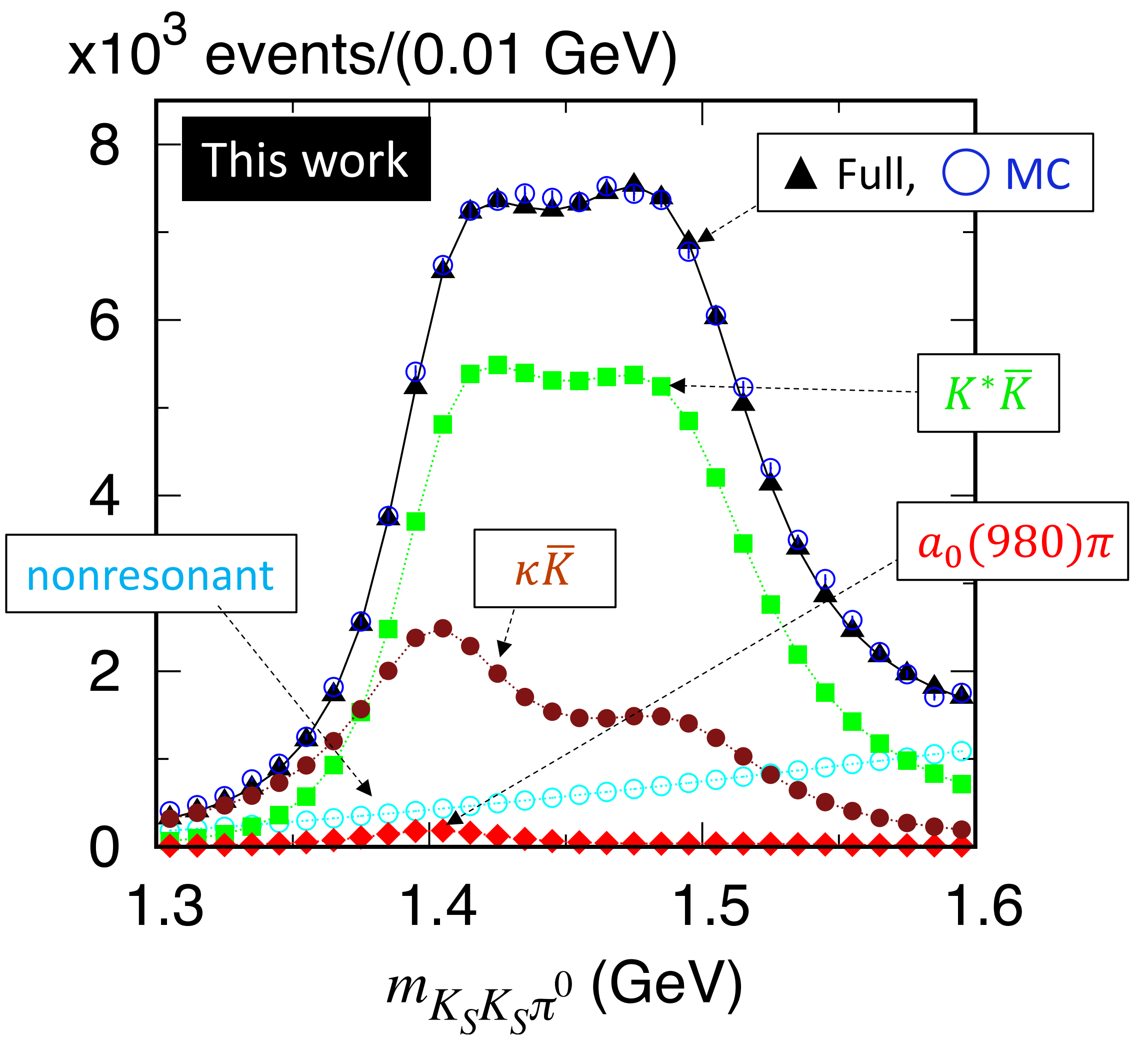


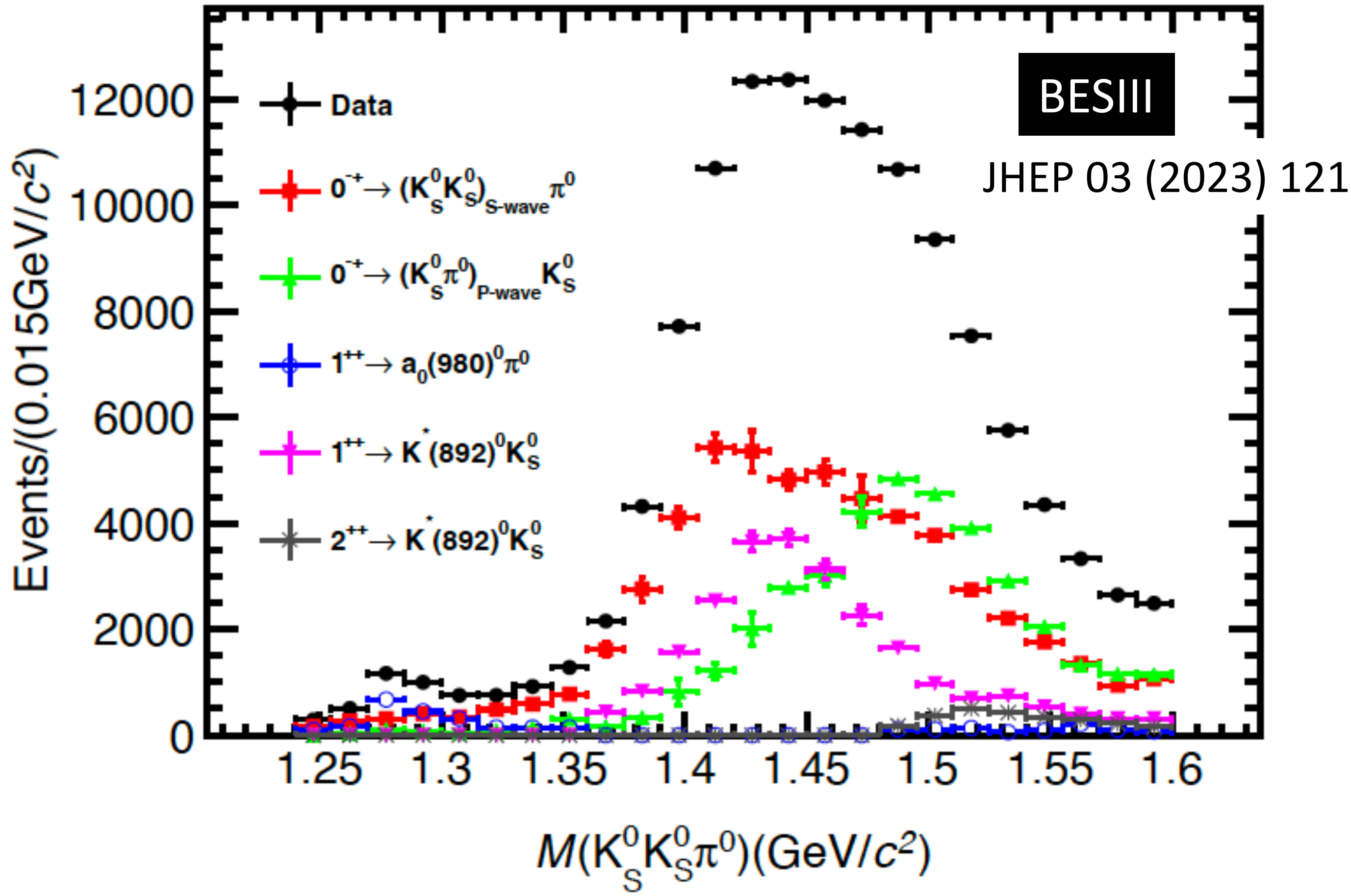
Rc = $K^*\overline{K}$, $a_0\pi$, $\kappa\overline{K}$ contributions are separately shown

Full is coherent sum of all decay channels

- $K^*\overline{K}$ is dominant
- $a_0\pi$ is small

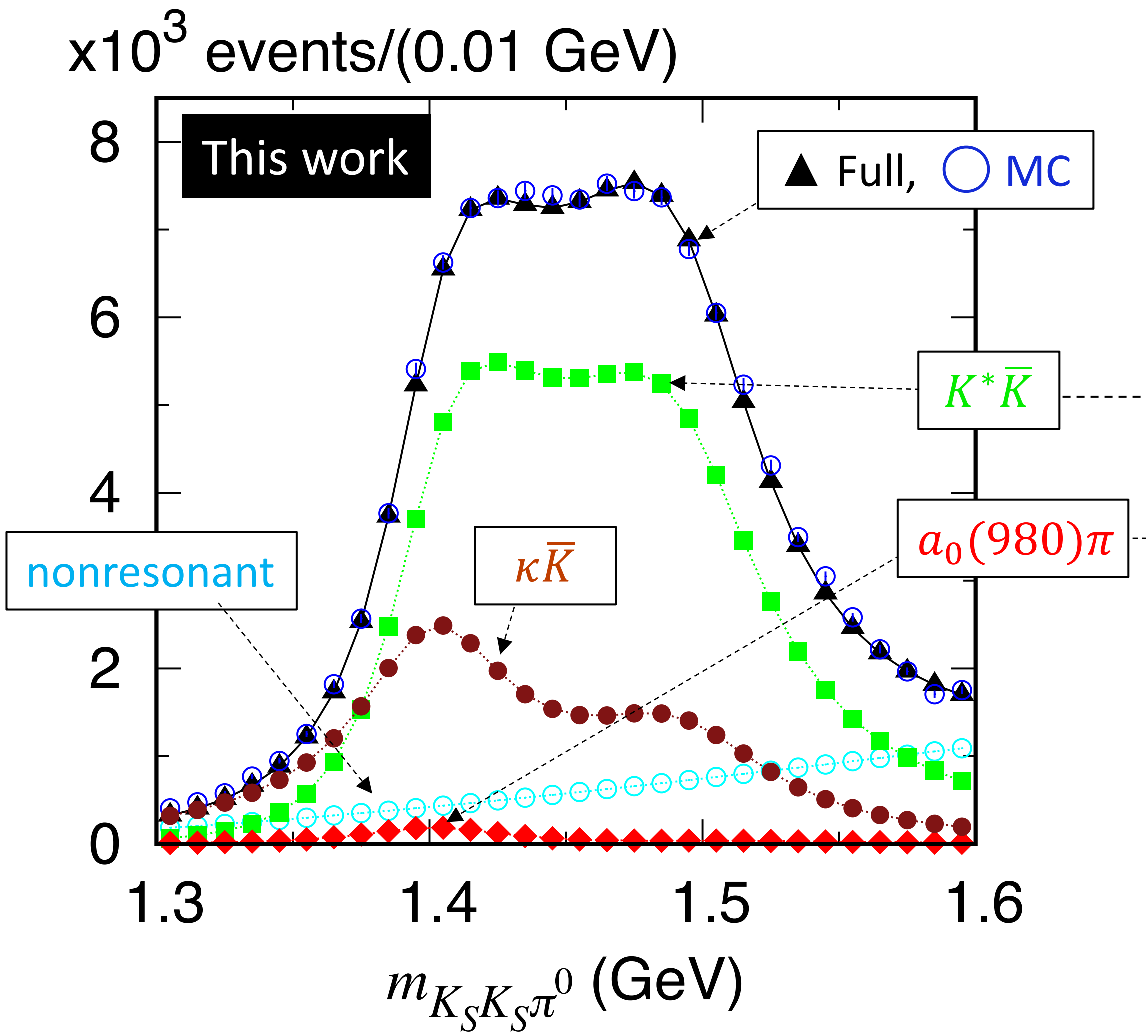
$K_S K_S \pi^0$ invariant mass distribution

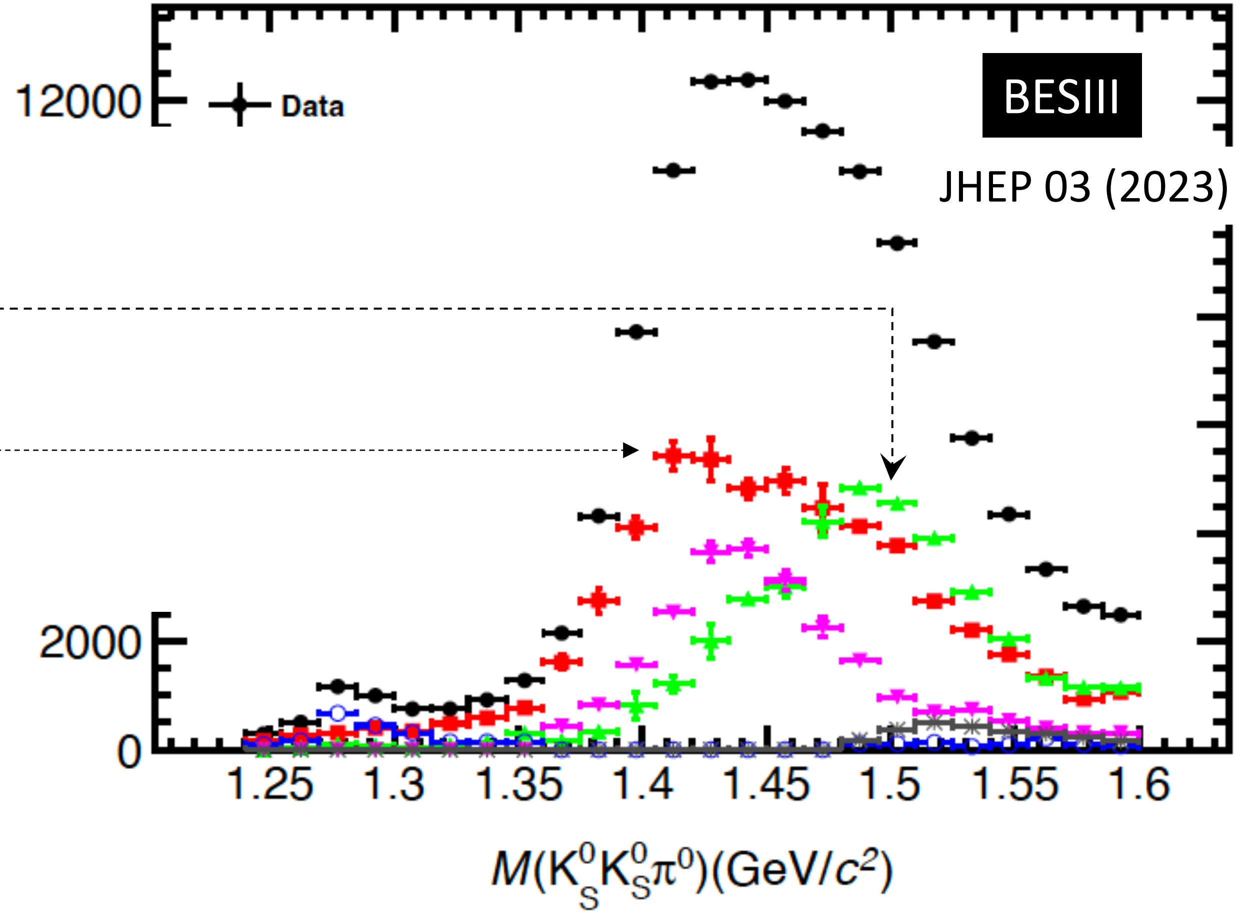




Comparison with BESIII amplitude analysis

$K_S K_S \pi^0$ invariant mass distribution





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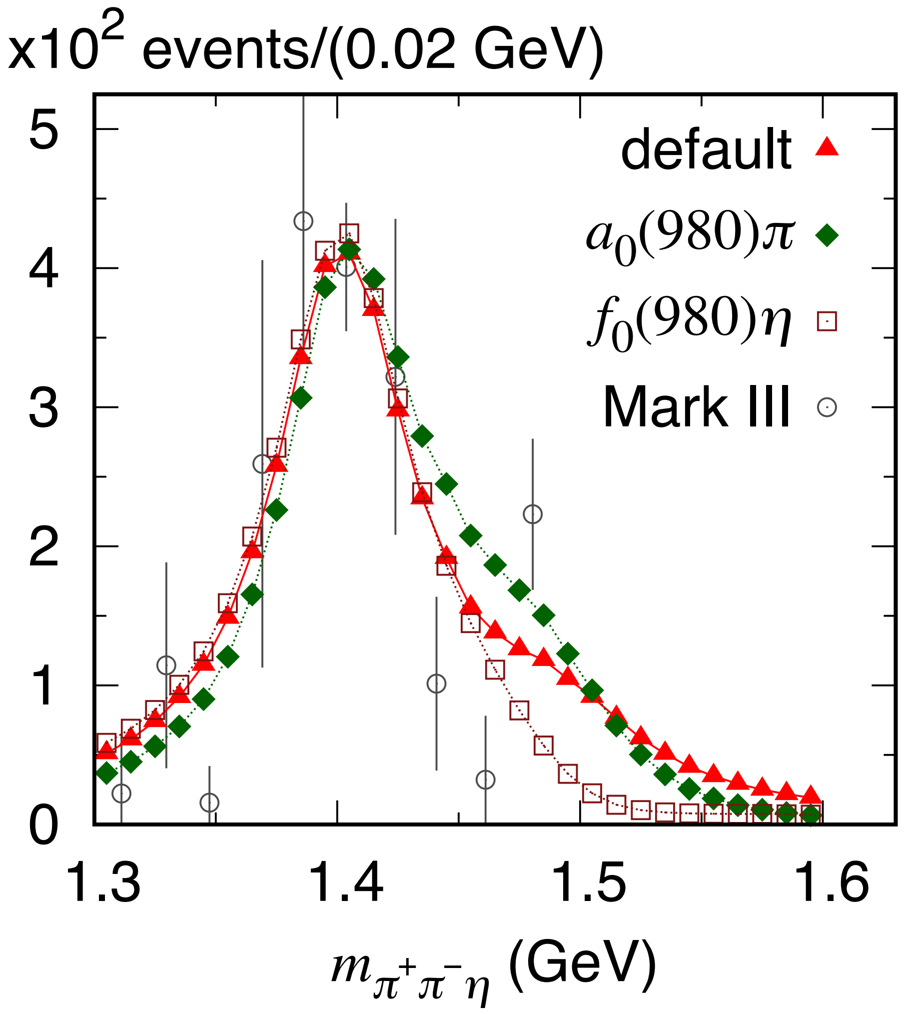
- Qualitative differences in decay dynamics between our and BESIII models
- Our (BESIII) model has small (large) $a_0\pi$ contribution; we considered ratio → $\frac{\mathrm{Br}[\eta(1405/1475) \to K\bar{K}\pi\,]}{\mathrm{Br}[\,\eta(1405/1475)\ \to \eta\pi^+\pi^-\,]}$
- Our model has substantial $\kappa\bar{K}$ contribution (coupled-channel effect)

$\eta\pi\pi$ final state

Prediction of $\pi^+\pi^-\eta$ lineshape



- Consistent with MARK III
- $a_0(980)\pi$ and $f_0(980)\pi$ comparably contribute

Q. Why lineshapes are different between

$\eta(1405/1475) \to K\bar{K}\pi$ and $\eta(1405/1475) \to \eta\pi\pi$?

Ans. Main decay mechanisms are different

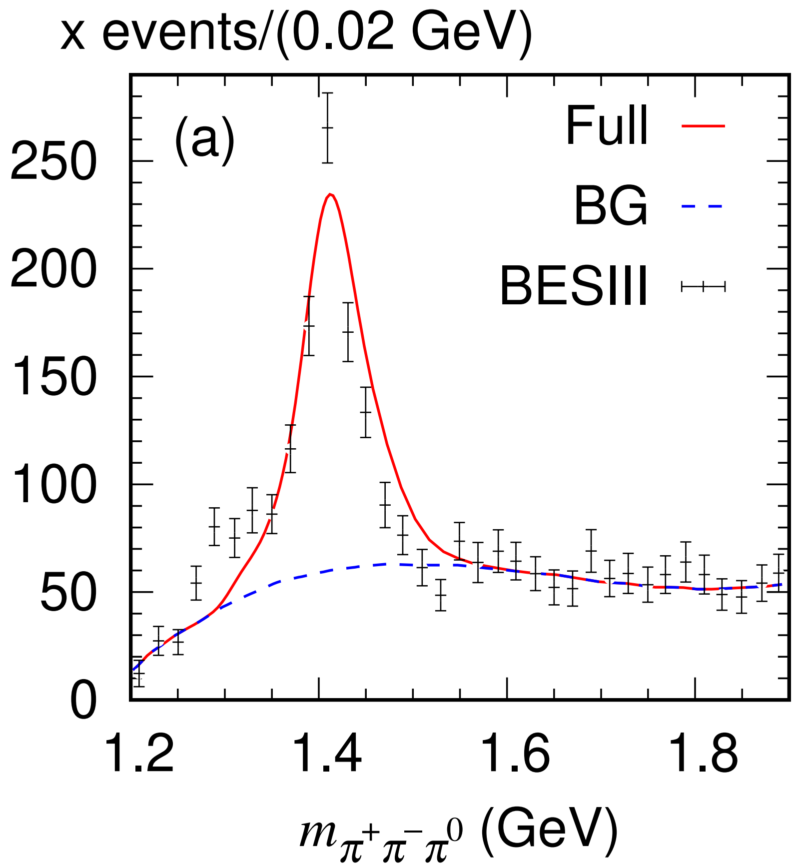
$$\eta(1405/1475) \to K^*\bar{K} \to K\bar{K}\pi$$

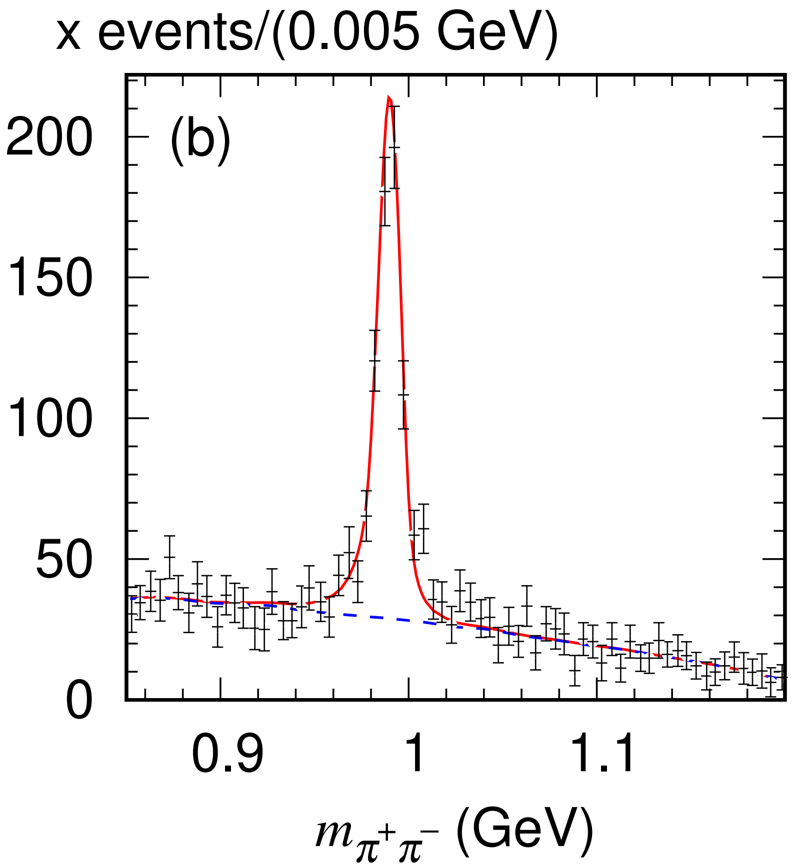
$$\eta(1405/1475) \to a_0(980)\pi, f_0(980)\eta \to \eta\pi\pi$$

and $K^*\bar{K}$ and $a_0(980)\pi$ couple differently with $\eta(1405)$ and $\eta(1475)$

Isospin-violating $\pi\pi\pi$ final state

Prediction of $\pi\pi\pi$ lineshape



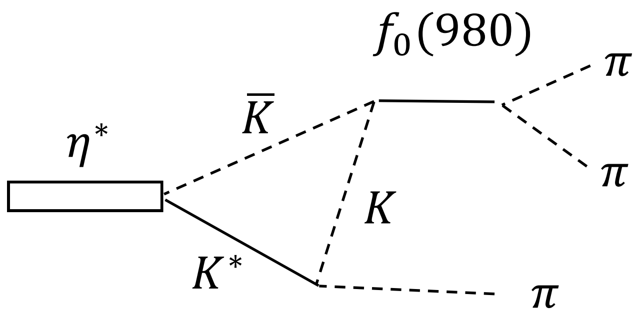


Good agreement with

BESIII data [PRL 108, 182001 (2012)]

Main mechanism

[Wu et al. PRL 108 (2012)]



- Isospin violation is caused by incomplete cancellation between $K^*K^+K^-$ and $K^*K^0\bar{K}^0$ triangle loops ($m_{K^\pm} \neq m_{K^0}$)
- Only in small kinematical window of $m_{\pi\pi}$ near $K\bar{K}$ threshold, significant isospin violation occurs
- Triangle singularity further enhances the isospin violation at $m_{\pi\pi\pi} \sim 1.4$ GeV

Prediction of $\eta(1405/1475) \to \pi\pi\pi$ branching fractions

DATA

$$R \equiv \frac{\mathrm{Br}[J/\psi \to \gamma\eta(1405/1475) \to \gamma(\pi\pi\pi)]}{\mathrm{Br}[J/\psi \to \gamma\eta(1405/1475) \to \gamma(K\bar{K}\pi)]}$$

Our prediction

BESIII, PRL 108, 182001 (2008)

$(\pi^+\pi^-\pi^0) \quad = \dfrac{(1.50\pm0.11\pm0.11)\times10^{-5}}{(2.8\pm0.6)\times10^{-3}} \sim 0.004 - 0.007$ $\qquad R = 0.002$

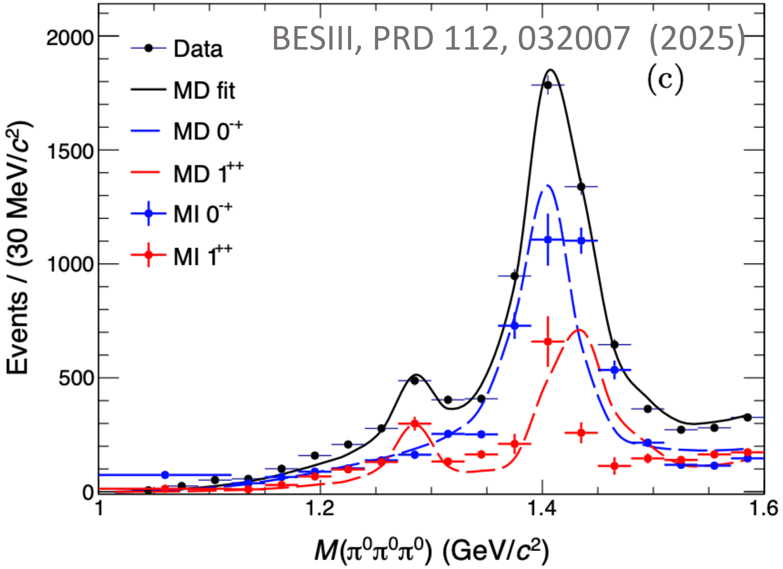
Underestimate?

$(\pi^0\pi^0\pi^0) \quad = \dfrac{(7.10\pm0.82\pm0.72)\times10^{-6}}{(2.8\pm0.6)\times10^{-3}} \sim 0.002 - 0.003$ $\qquad R = 0.001$

Recent development at BESIII PRD 112, 032007 (2025)

$$\frac{\mathrm{Br}[J/\psi \to \gamma f_1 \to \gamma(\pi\pi\pi)]}{\mathrm{Br}[J/\psi \to \gamma\eta(1405/1475) \to \gamma(\pi\pi\pi)]} \sim 0.8$$

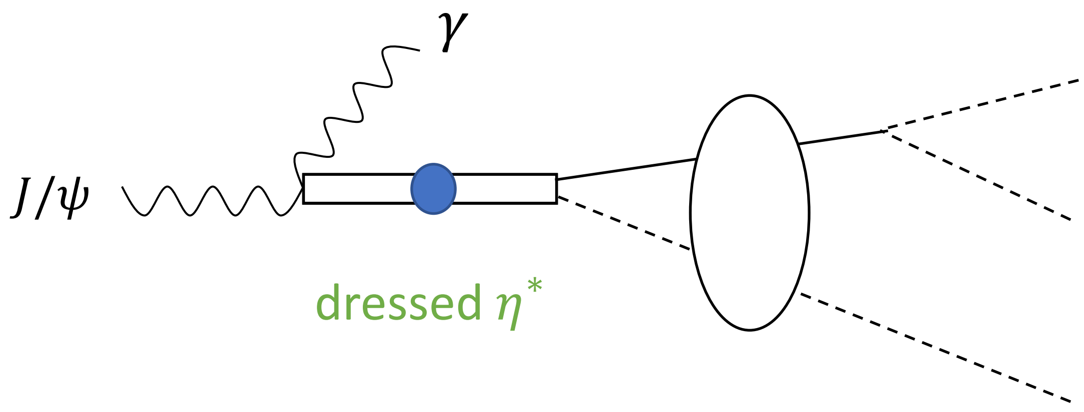
→ Data and our prediction agree fairly

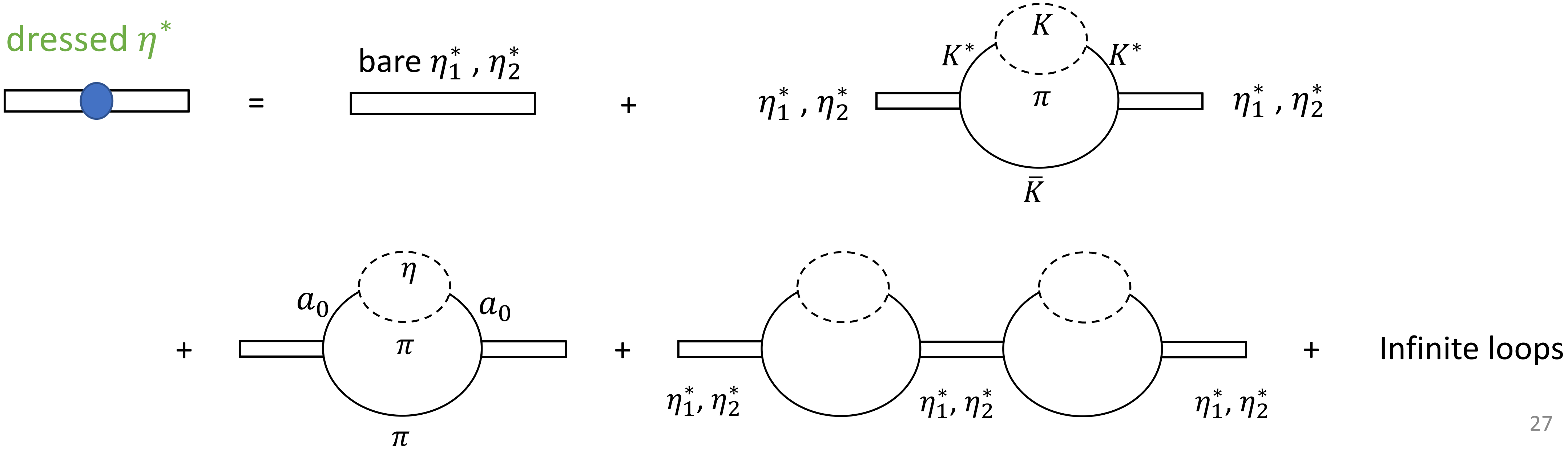


$\eta(1405/1475)$ poles

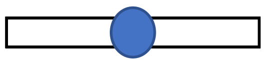
$\eta(1405/1475)$ poles → poles of dressed η^* propagator

Our decay amplitude





Pole search

With analytic continuation, find complex energy E where $G(E) = \frac{1}{0}$ $G(E) =$ 

dressed η^*

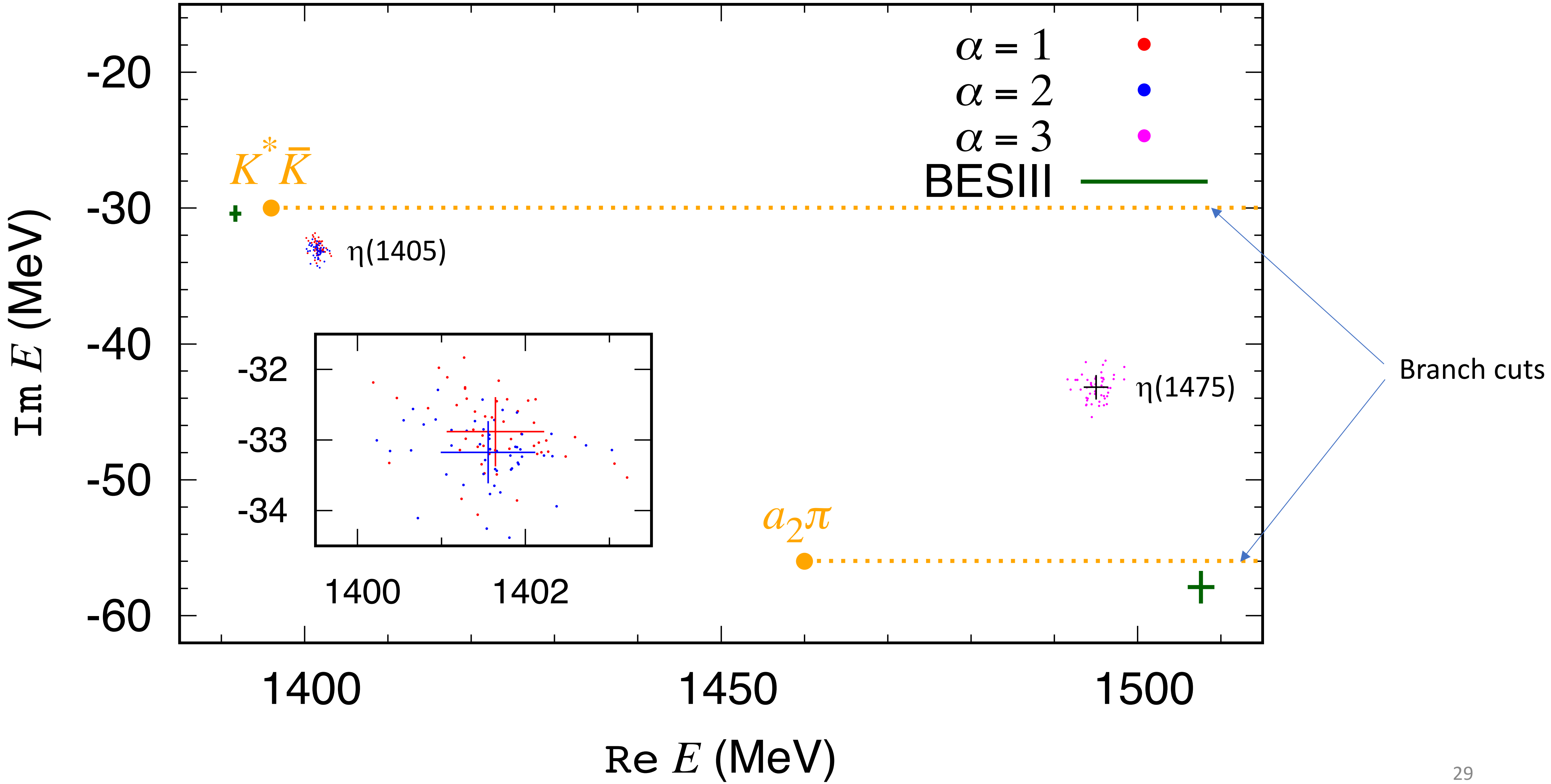
	M (MeV)	Γ (MeV)	Riemann sheet for $K^*\bar{K}, a_2\pi$ channels (physical or unphysical)	
$\alpha = 1$	1401.6 ± 0.6	65.8 ± 1.0	(up)	← η(1405)
$\alpha = 2$	1401.6 ± 0.6	66.3 ± 0.9	(pp)	
$\alpha = 3$	1495.0 ± 1.5	86.4 ± 1.8	(up)	← η(1475)
BESIII [JHEP 03 (2023) 121]	1391.7 ± 0.7	60.8 ± 1.2		← η(1405)
	1507.6 ± 1.6	115.8 ± 2.4		← η(1475)

- Statistical errors are estimated with bootstrap method (50 pseudodata sets)
- Three poles, corresponding to two states

  η(1405) pole is close to $K^*\bar{K}$ branch point → pole is split into two poles on different Riemann sheet
- Pole positions are somewhat different from Breit-Wigner parameters of BESIII analysis

  → possibly artifact of using Breit-Wigner parameters to define M and Γ

Pole locations from 50 bootstrap fits



Summary

Three-body unitary coupled-channel analysis of radiative J/ψ decays

Data : Dalitz plots from BESIII energy dependent solution on $J/\psi \to \gamma\eta(1405/1475) \to \gamma(K_S K_S \pi^0)$

Additional data : Branching ratios for $\eta\pi\pi$ and $\pi\pi\gamma$ final states relative to $K\bar{K}\pi$ final state

- The data are well fitted
- Isospin-violating $\pi\pi\pi$ final states are well predicted (lineshape, branching fractions)
- $\eta(1405/1475)$ poles are determined for the first time

First-ever pole determination based on a manifestly three-body unitary coupled-channel framework

applied to experimental Dalitz plot distributions

Previous Dalitz plot analyses → isobar models, two-body unitary models

Possible future developments

- Extend coupled-channel analysis to include more quantum numbers (e.g., f_1, f_2 for $J/\psi \to \gamma(K_S K_S \pi^0)$)

→ directly fit experimental data (not MC outputs), consistently determine $\eta(1405/1475)$ and $f_1(1420)$ poles

- High-precision data of various processes involving $\eta(1405/1475)$ are expected in the near future

Radiative J/ψ decays to $K\bar{K}\pi$, $\eta\pi\pi$, $\rho^0\gamma$, 4π, etc. via $\eta(1405/1475)$

χ_{cJ} decays involving $\eta(1405/1475)$ and more…

→ Combined analysis is important to fully understand $\eta(1405/1475)$ (standard in N* physics)

Three-body unitary coupled-channel analysis for N* physics

J-PARC E45 experiment aims to perform Dalitz-plot analyses of $\pi N \to \pi\pi N$

ANL–Osaka DCC model (approximately three-body unitary) is a suitable framework Kamano and Lee, arXiv:2601.17389

backup

$a_0(980)$ model

Relative coupling strengths of $a_0(980) \to \pi\eta$ and $a_0(980) \to K\bar{K}$ in our model

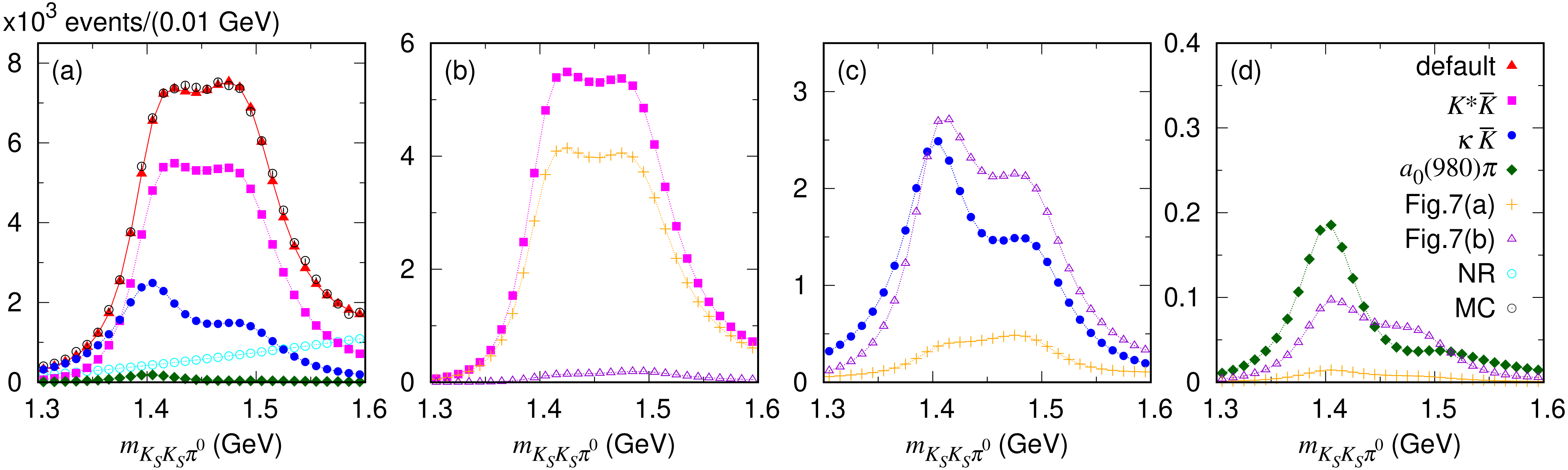
$|g_{a_0\to\pi\eta}| \sim |g_{a_0\to K\bar{K}}|$ based on Abele et al. PRD 57, 3860 (1998) analyzing $p\bar{p} \to K\bar{K}\pi, \eta\pi\pi$

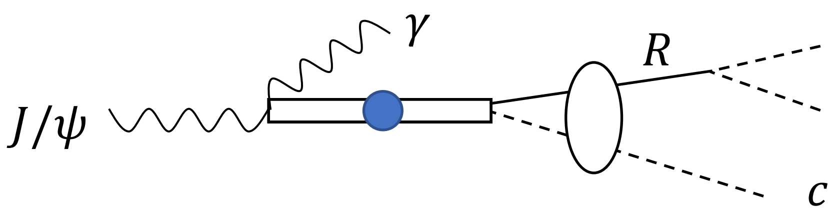
Alternative choice:

$2 \times |g_{a_0\to\pi\eta}| \sim |g_{a_0\to K\bar{K}}|$ based on Lu and Moussallam EPJC 80, 436 (2020) analyzing $\gamma\gamma \to K\bar{K}, \eta\pi$

Model-dependence should be checked → future work

Somewhat larger $a_0\pi$ contribution is expected in $J/\psi \to \gamma\eta(1405/1475) \to \gamma(K_S K_S \pi^0)$





Final Rc = $K^*\bar{K}$, $a_0\pi$, $\kappa\bar{K}$ contributions are separately shown

default is coherent sum of all final Rc channels

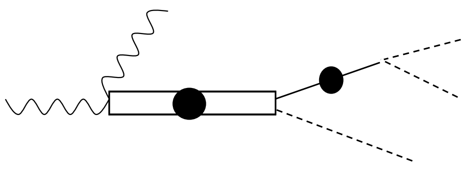


Fig.7 (a)

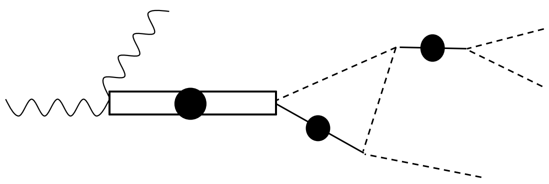
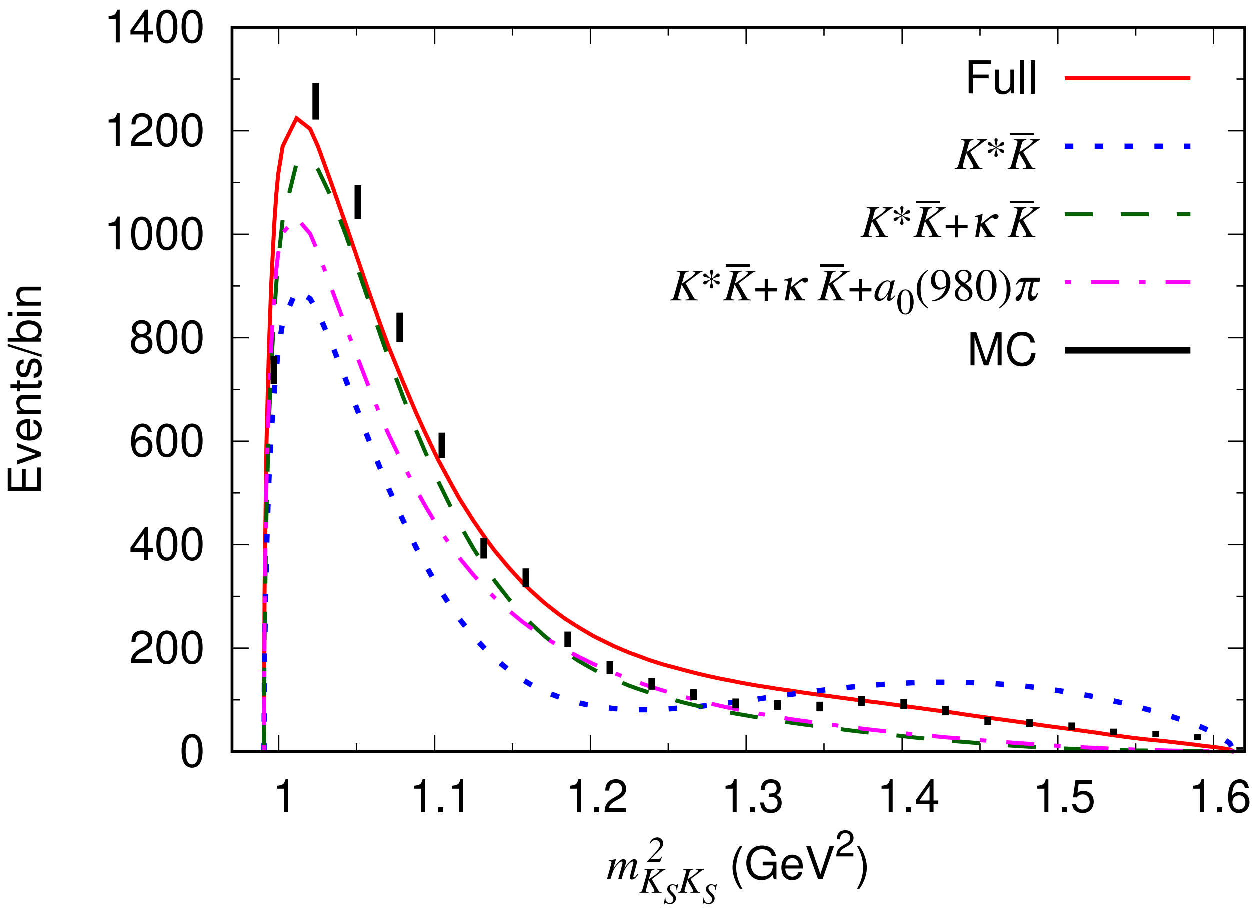


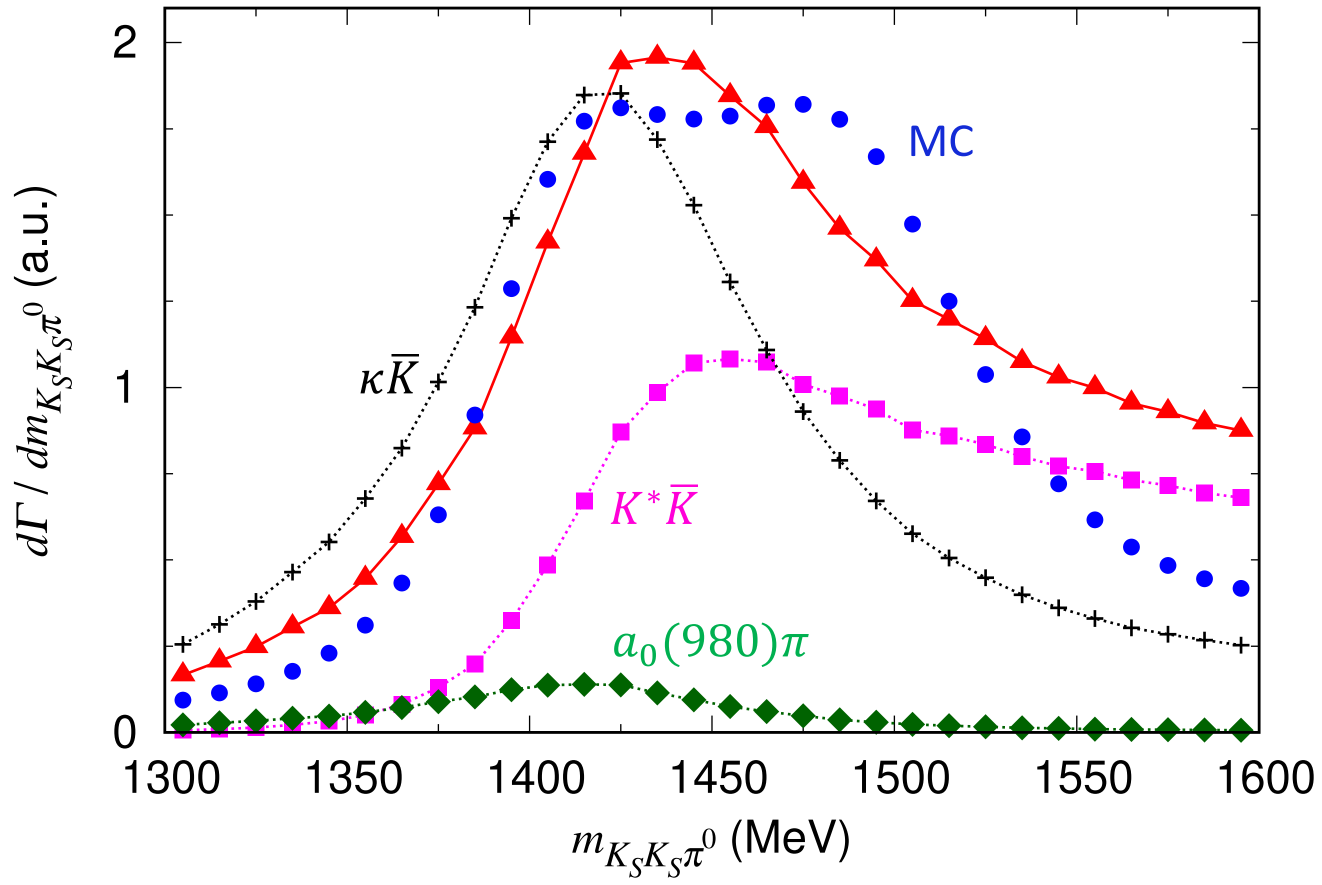
Fig.7 (b)

We show contributions from single triangle diagrams



We discuss how the $a_0(980)$-like peak is created from $K^*\overline{K}$ and $\overline{K}^*K$ interference

Fit with one bare η^* model



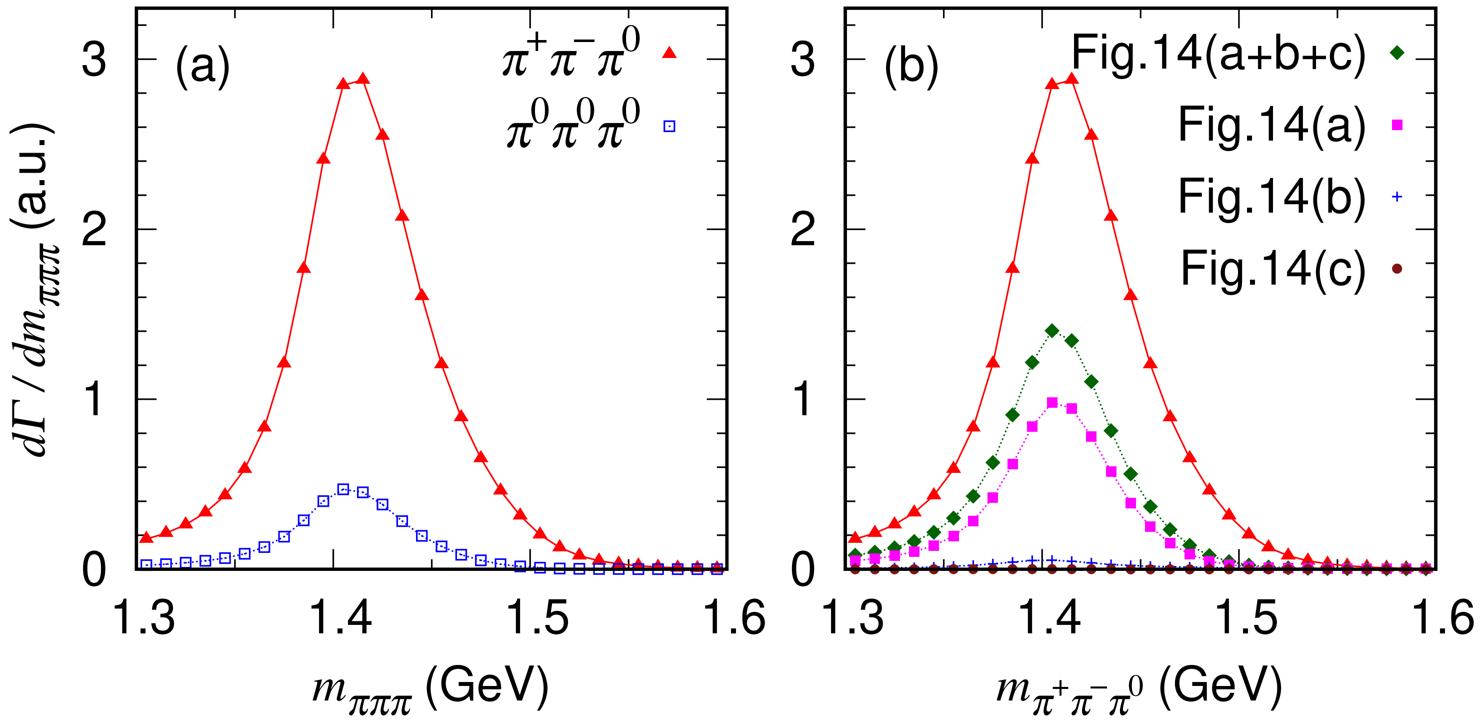
Single-pole solution was proposed for $\eta(1405/1475)$; should be checked

One-bare model is fitted to $K_S K_S \pi^0$ invariant mass distribution

Flat peak structure (with shallow dip) is not well reproduced by the one-bare model

→ two bare states necessary

Prediction of $\pi\pi\pi$ lineshape in $J/\psi \to \gamma\eta(1405/1475) \to \gamma(\pi\pi\pi)$



We discuss main mechanisms for the isospin-violating process

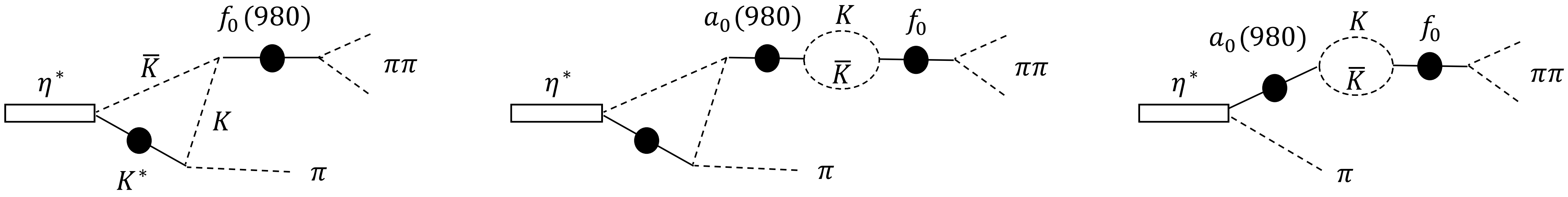


Fig.14 (a) Fig.14 (b) Fig.14 (c)

Our unitary coupled-channel model

bare state 1 (~1.6 GeV)

$s\bar{s}$ (1.4–1.7 GeV)

bare state 2 (2–2.4 GeV)

excited $\eta^{(\prime)}$ (1.7–2.5 GeV)
hybrid (2–2.5 GeV)
Glueball (2.4–2.6 GeV)

Strong coupling from
coupled-channel scattering equation

meson-meson continuum states
$K_J^* \bar{K}$, $a_0\,\pi$, $\rho\rho$, etc.

$\eta(1405/1475)$ as two bare states mixed and dressed by meson-meson continuum

Poles :
$1401 - 33i$, $1495 - 43i$ MeV

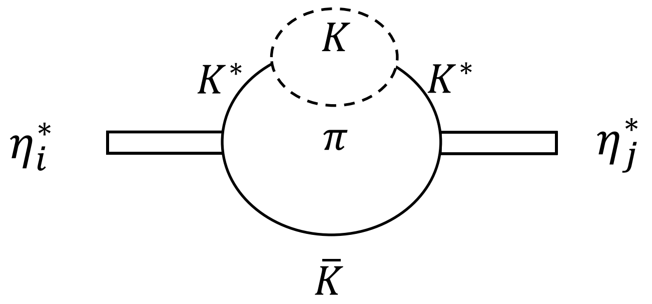
Dressed η^* propagator G_{ij}

$$[G^{-1}(E)]_{ij} = \left(E - m_{\eta_i}\right)\delta_{ij} - \Sigma_{ij}(E) \qquad (E = m_{K_S K_S \pi^0})$$

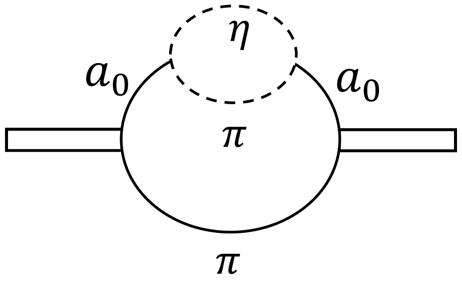
bare mass for η_i^*

self energy of η_1^*, η_2^*

Σ_{ij} =



+



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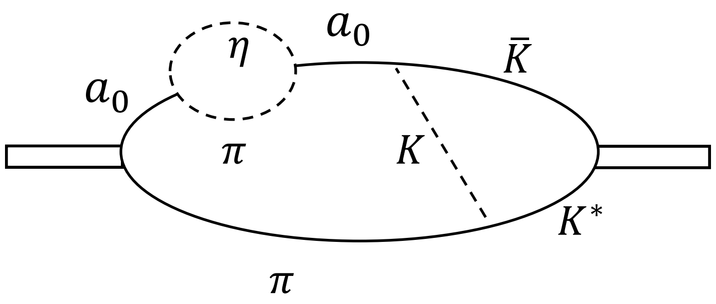
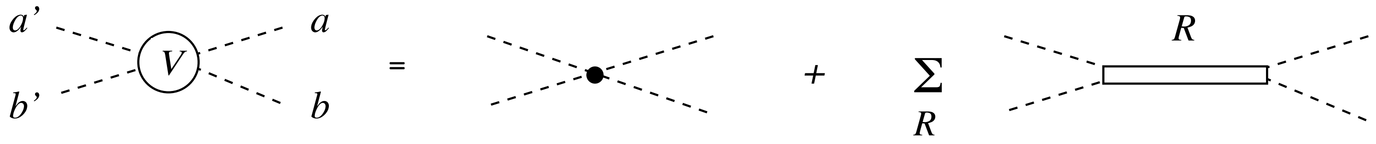


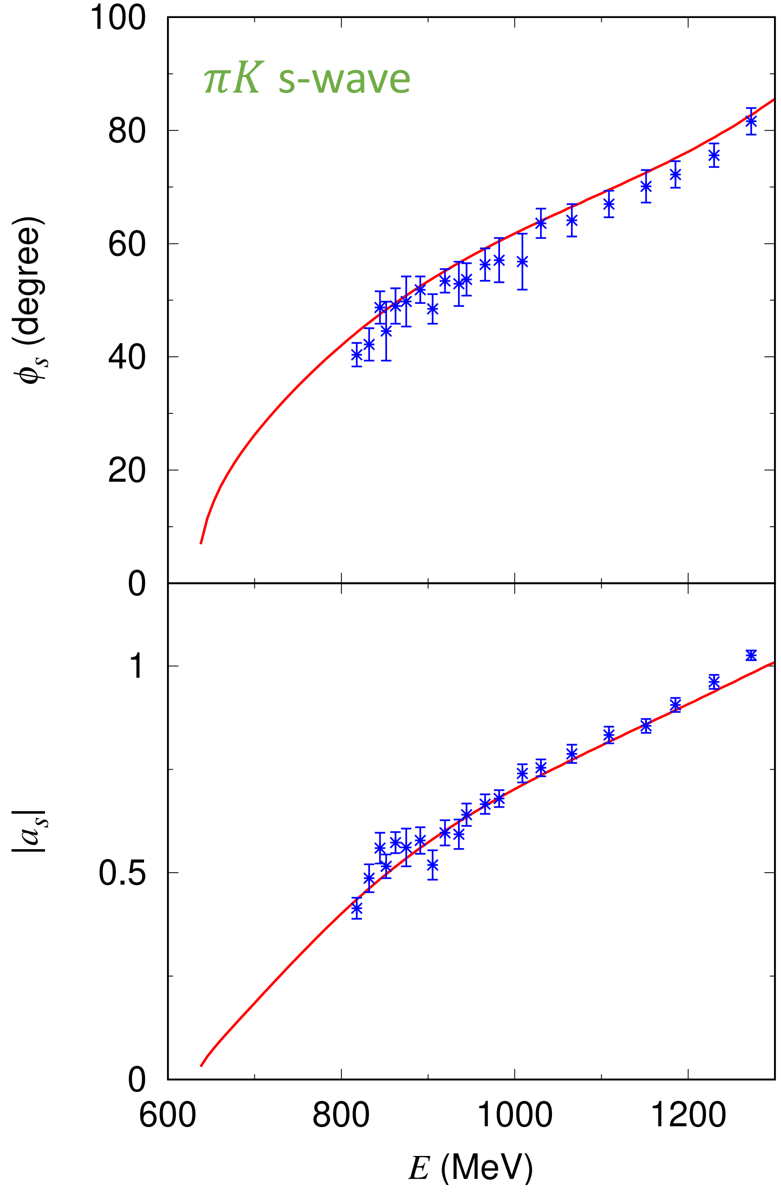
+ Infinite loops (η^* appears at both ends; no intermediate η^*)

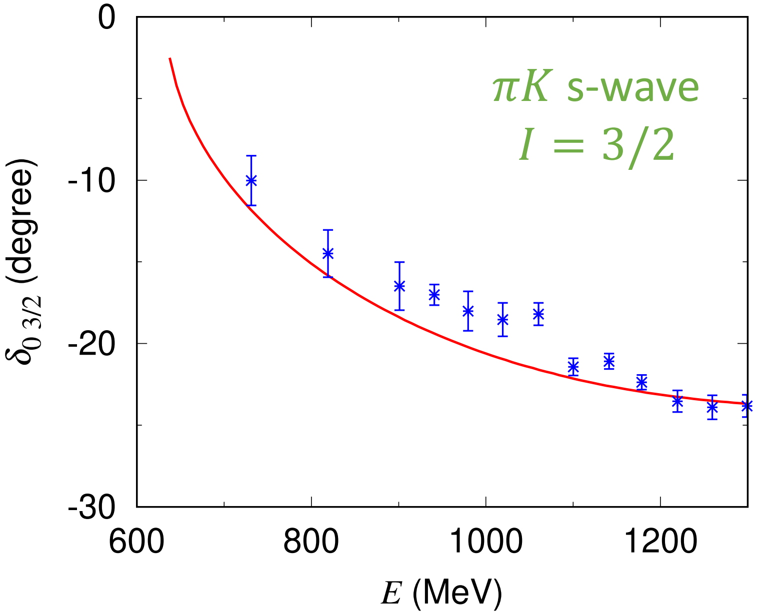
Two-meson scattering models

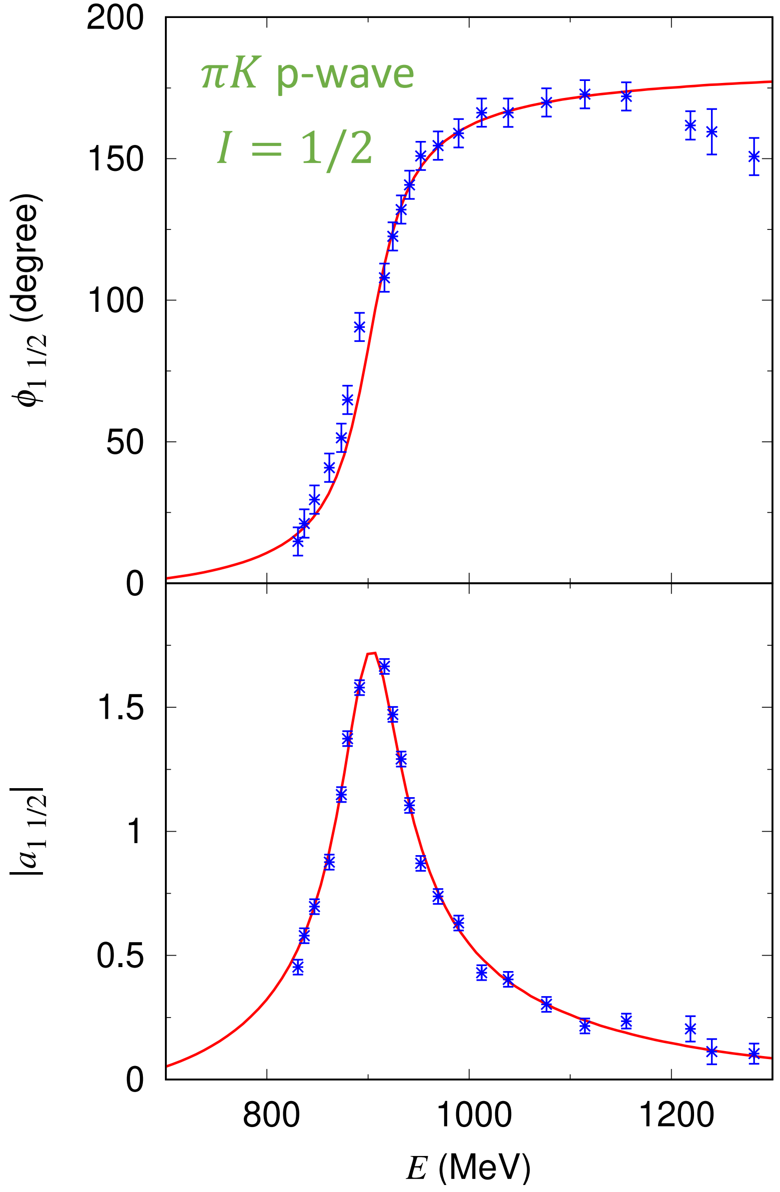


$$\rightarrow\; T = V + VGT$$

Two-meson scattering models







Two-meson scattering models