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Topic:

Spectroscopy of All-Heavy Pentaquark States

Presented by

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Outline

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2. Theoretical methodology
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   - Hypertadial Schrodinger equation

3. Results and discussion

4. Conclusion

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1.

Introduction and motivation

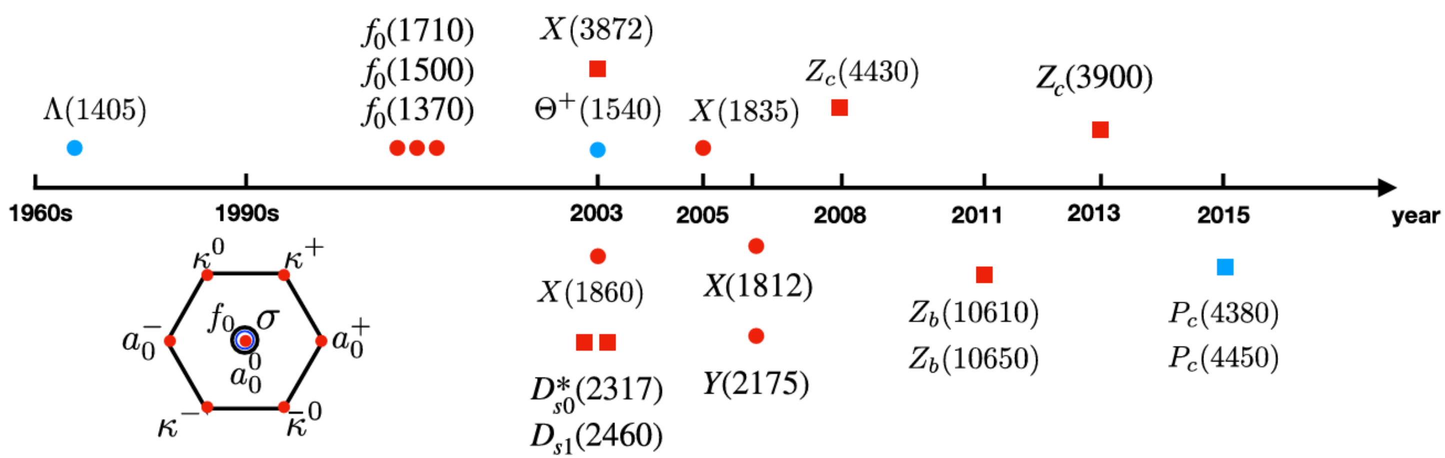
Introduction to the Exotic particles

What Are Exotic Particles?

- According to quark model, conventional mesons are quark- antiquark states and baryons are three-quark states.
- Quark model successfully explain the spectrum of mesons and baryons.
- The discovery of multiquark states has open the new era of particle physics. These unconventional hadrons are called Exotic hadrons.
- Quark model extension was first proposed by Gell-Mann and Zweig.[1-3]

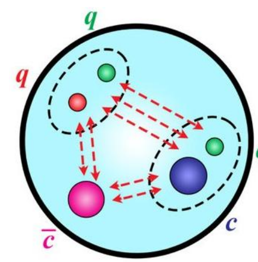
- The quantum chromodynamics (QCD) which is the theory of strong inter actions, allows those unconventional hadrons such as,
  - Tetraquark: (qq$\bar{q}\bar{q}$)
  - Pentaquarks: (qqqq$\bar{q}$)
  - Hexaquarks: (qqq$\bar{q}\bar{q}\bar{q}$, qqqqqq)
  - Hybrids: Quarks + excited gluons
  - Glueballs: Bound states of gluons only

- Observation of some typical exotic states given below [4]



Pentaquarks:

Pentaquark State	Experimental Facility	Year	Remark
$P_c(4380)$, $P_c(4450)$	LHCb	2015	First observation of structures consistent with hidden-charm pentaquarks
$P_c(4312)$, $P_c(4440)$, $P_c(4457)$	LHCb	2019	Higher-statistics data revealed the narrow $P_c(4312)$and resolved the previous$P_c(4450)$ structure into$P_c(4440)$ and $P_c(4457)$
$P_{cs}(4459)$	LHCb	2020	Observation of the first prominent strange hidden-charm pentaquark candidate



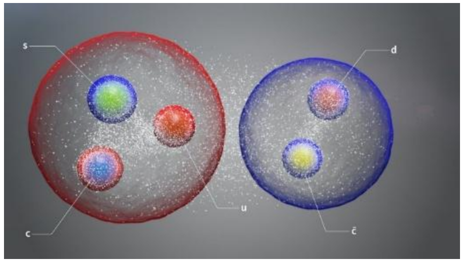
Pentaquark systems (qqqq$\bar{q}$) can be studied using different internal structure configurations, based on how the quarks are dynamically clustered. The three most widely adopted models are: [5,6]

1. Molecular Model:

- Treats the pentaquark as a **loosely bound hadronic molecule**, composed of a **baryon and a meson.**

2. Diquark–Diquark–Antiquark Model:

- The pentaquark is modeled as **two tightly bound diquarks (qq) + one antiquark ($\bar{q}$)**.
- This compact configuration is suitable for describing **color–spin correlations** and **short- range interactions**.



3. Triquark–Diquark Model:

- One triquark cluster (qq$\bar{q}$) is bound to a diquark (qq).

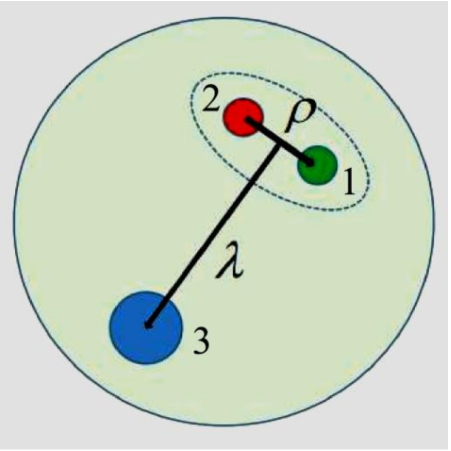
2.

Theoretical Methodology

The Hypercentral Constituent Quark Model

The hyperspherical coordinates:

- For a three body system, we define **Jacobi vectors,** ρ and λ [6,7]



$$\mathbf{x} = (\boldsymbol{\rho}, \boldsymbol{\lambda})$$

$$\boldsymbol{\rho} = \frac{1}{\sqrt{2}}(\mathrm{r}_1 - \mathrm{r}_2)$$

$$\boldsymbol{\lambda} = \frac{(\mathrm{m}_1\mathrm{r}_1 + \mathrm{m}_2\mathrm{r}_2) - (\mathrm{m}_1 + \mathrm{m}_2)\mathrm{r}_3}{\sqrt{\mathrm{m}_1{}^2 + \mathrm{m}_2{}^2 + (\mathrm{m}_1 + \mathrm{m}_2)^2}}$$

$$\mathbf{R} = \frac{1}{\mathrm{M}}(\mathrm{m}_1\mathrm{r}_1 + \mathrm{m}_2\mathrm{r}_2 + \mathrm{m}_3\mathrm{r}_3)$$

$$\mathrm{M} = \mathrm{m}_1 + \mathrm{m}_2 + \mathrm{m}_3$$

- From these, we define hyper radius and hyper angle as:

$$\mathrm{X} = \sqrt{\rho^2 + \lambda^2}, \qquad \xi = \arctan(\rho/\lambda)$$

- ➢ X - Overall size of the system (hyperradius)
- ➢ ξ -Shape of configuration

- ➢ $(\theta_\rho, \emptyset_\rho)$ –Direction of $\boldsymbol{\rho}$ (Between 1 and 2)
- ➢ $(\theta_\lambda, \emptyset_\lambda)$ –Direction of $\boldsymbol{\lambda}$ (between 3 and CM of 1 & 2)

- Lagrangian of the three-body system is written as,

$$\mathrm{L} = \sum_{\mathrm{i}=1}^{3} \frac{1}{2} \mathrm{m_i} \dot{\mathrm{r}}_{\mathrm{i}}^{2} - \mathrm{V}(\mathrm{r}_1, \mathrm{r}_2, \mathrm{r}_3)$$

- Transform into Jacobi coordinates → separates center-of-mass and internal motion.

$$\mathrm{T} = \mathrm{T_{CM}} + \mathrm{T_{int}}$$

$$\sum_{\mathrm{i}=1}^{3} \frac{1}{2} \mathrm{m_i} \dot{\mathrm{r}}_{\mathrm{i}}^{2} = \frac{1}{2} \mathrm{M}\, \dot{\mathrm{R}}^2 + \frac{1}{2} \mathrm{m_\rho} \dot{\rho}^2 + \frac{1}{2} \mathrm{m_\lambda} \dot{\lambda}^2$$

- Reduced masses are,

$$\mathrm{m_\rho} = \frac{2\mathrm{m_1 m_2}}{\mathrm{m_1} + \mathrm{m_2}}$$

$$\mathrm{m_\lambda} = \frac{2\mathrm{m_3}(\mathrm{m_1^2} + \mathrm{m_2^2} + \mathrm{m_1 m_2})}{(\mathrm{m_1}+\mathrm{m_2})(\mathrm{m_1}+\mathrm{m_2} + \mathrm{m_3})}$$

- Kinetic energy operator,

$$\sum_{\mathrm{i}} \frac{\hbar^2}{2\mathrm{m_i}} \nabla_{\mathrm{i}}^2 = -\frac{\hbar^2}{2\mathrm{M}} \nabla_{\mathrm{R}}^2 - \frac{\hbar^2}{2\mathrm{m_\rho}} \nabla_{\rho}^2 - \frac{\hbar^2}{2\mathrm{m_\lambda}} \nabla_{\lambda}^2$$

- Kinetic energy operator in 6 dimension,

$$\mathrm{T_{int}} = -\frac{\hbar^2}{2\mathrm{m}}\left(\nabla^2_{\rho,\lambda}\right) = -\frac{\hbar^2}{2\mathrm{m}}\left(\frac{\partial^2}{\partial X^2} + \frac{5}{X}\frac{\partial}{\partial X} - \frac{L^2(\Omega)}{X^2}\right)$$

- The final Hamiltonian in these coordinates look like,

$$H = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial X^2} + \frac{5}{X}\frac{\partial}{\partial X} - \frac{L^2(\Omega)}{X^2}\right) + V(X)$$

- The potential energy,

$$V(x) = V^{(0)}(x) + V_{SD}(x),$$

$$V^{(0)}(x) = \frac{\tau}{x} + \beta x + V_0,$$

Here, $\tau = k\alpha_s$, where k is the color factor and α_s is running coupling constant, and β is string tension.

- The expression for the color factor is[6],

$$k = \frac{1}{4}\sum_{i<j} \epsilon_{ij} \sum_{a=1}^{8} \lambda_i^a \lambda_j^a,$$

  - λ_j^a are Gell-Mann matrices.
  - For qq pair, $\epsilon_{ij} = 1$
  - For $q\bar{q}$ pair, $\epsilon_{ij} = -1$

- For two quarks, $3_c \otimes 3_c = \overline{3_c} \oplus 6_c$
  - For $\bar{3}$, $k = -2/3$
  - For 6, $k = 1/3$

- Each diquark in the diquark-diquark-antiquark model is typically assumed to be a color antitriplet because attractive interactions enhance binding and the 3 configuration is energetically favorable.

- The total color decomposition of pentaquark made up of diquarks and antiquark as,

$$\overline{3_c} \otimes \overline{3_c} \otimes \overline{3_c} = (3_c \oplus \overline{6_c}) \otimes \overline{3_c} = (1_c \oplus 8_c) \oplus (8_c \oplus \overline{10_c}).$$

- In this work, we concentrate on the color singlet component of the pentaquark wave function, which represents the physically observable compact pentaquark state with an effective color factor,

$$k = -2/3.$$

- The Running Coupling is written as,

$$\alpha_s = \frac{\alpha_s(\mu_0)}{1+\frac{33-2n_f}{12\pi}\alpha_s(\mu_0)\ln(\frac{m_1+m_2+m_3}{\mu_0})}$$

$$V_{SD}(x) = V_{SS}(x)\left(\overrightarrow{S_\rho}.\overrightarrow{S_\lambda}\right) + V_{\gamma S}(x)\left(\vec{\gamma}.\vec{S}\ \right) + V_T(x)(S^2 - 3\frac{\left(\vec{S}.\vec{x}\right)^2}{x^2}).$$

$$\mathrm{V_{linear}} = \mathrm{V_s} = \alpha \mathrm{x}$$
$$\mathrm{V_{coulomb}} = \mathrm{V_V} = \frac{\tau}{\mathrm{X}}$$

- The spin-orbit term,

$$V_{LS}(x) = \frac{1}{2\mathrm{m_\rho m_\lambda}}(3\frac{dV_V}{dx} - \frac{dV_S}{dx})$$

- The spin-spin term,

$$V_{SS}(x) = \frac{1}{3\mathrm{m_\rho m_\lambda}}\nabla^2 V_V = \frac{\mathrm{A}}{6\mathrm{m}_{\boldsymbol{\rho}}\mathrm{m}_{\boldsymbol{\lambda}}}\frac{\mathrm{e}^{-\mathrm{x/x_0}}}{\mathrm{xx_0^2}}$$

- The tensor term,

$$V_T(x) = \frac{1}{6\mathrm{m_\rho m_\lambda}}(\frac{d^2V_V}{dx^2} - \frac{1}{x}\frac{dV_V}{dx})$$

➢ The effective spin-spin operator,

$$\overrightarrow{S_\rho}.\overrightarrow{S_\lambda} = \frac{1}{2}[S(S-1) - \sum_k S_\mathrm{k}(S_\mathrm{k} - 1)],$$

➢ The spin-orbit operator,

$$\vec{\gamma}.\vec{S} = \frac{1}{2}[j(j-1) - \gamma(\gamma - 1) - S(S-1)].$$

For S–wave states, the orbital angular momentum vanishes, and consequently, the spin–orbit and tensor interactions do not contribute.

The Hyperradial Schrodinger Equation

- The hyperradial wave functions $\psi_{\gamma\vartheta}(x)$ are the solution of the hyperrdial Schrodinger equation, [6-11]

$$\left(\frac{d^2}{dX^2}+\frac{5}{X}\frac{d}{dX}-\frac{L^2(\Omega)}{X^2}\right)\psi_{\gamma\vartheta}(x)=-2m[E-V(x)]\psi_{\gamma\vartheta}(x)$$

$$L^2(\Omega)=\gamma(\gamma+4),$$

$$m=\frac{2m_\rho m_\lambda}{(m_\rho+m_\lambda)},$$

- The hyperradial function are normalized according to,

$$\int_0^\infty dx\ x^2|\psi_{\gamma\vartheta}(x)|^2=1$$

- It is convenient to introduce the reduced hyperradial wavefunction $u_{\gamma\vartheta}(x)$,

$$u_{\gamma\vartheta}(x)=x^{5/2}\psi_{\gamma\vartheta}(x).$$

- Which obeys the second order differential equation as,

$$\frac{d^2u_{\gamma\vartheta}(x)}{dx^2}-\frac{(\frac{15}{4}+\gamma(\gamma+4))}{x^2}u_{\gamma\vartheta}(x)=-2m(E-V(x))u_{\gamma\vartheta}(x)$$

- The first derivative has disappeared, but something new appeared:

$$V_{\text{centrifugal}} = \frac{(\frac{15}{4} + \gamma(\gamma + 4))}{x^2}$$

- So the transformation does not remove physics. It simply moves part of the geometry into an effective $\frac{1}{x^2}$ potential.

- Thus, the effective potential becomes,

$$V_{\text{eff}} = V(x) + \frac{(\frac{15}{4} + \gamma(\gamma + 4))}{x^2}$$

plus whatever spin-dependent contribution is being included.

- For each choice of quantum numbers, we solve,

$$[\frac{1}{2m}\frac{d^2}{dx^2} + V_{\text{eff}}]\, u_{\gamma\vartheta}(x) = E\, u_{\gamma\vartheta}(x)$$

- Diquark masses taken from Ref. [12]

Quark content	Diquark Type	Mass (GeV)
cc	A	3.226
cb	S	6.519
cb	A	6.526
bb	A	9.778

- Parameters used to produce the spectrum of $Q_1Q_2Q_3Q_4\bar{c}$ pentaquarks are taken from Ref. [13]

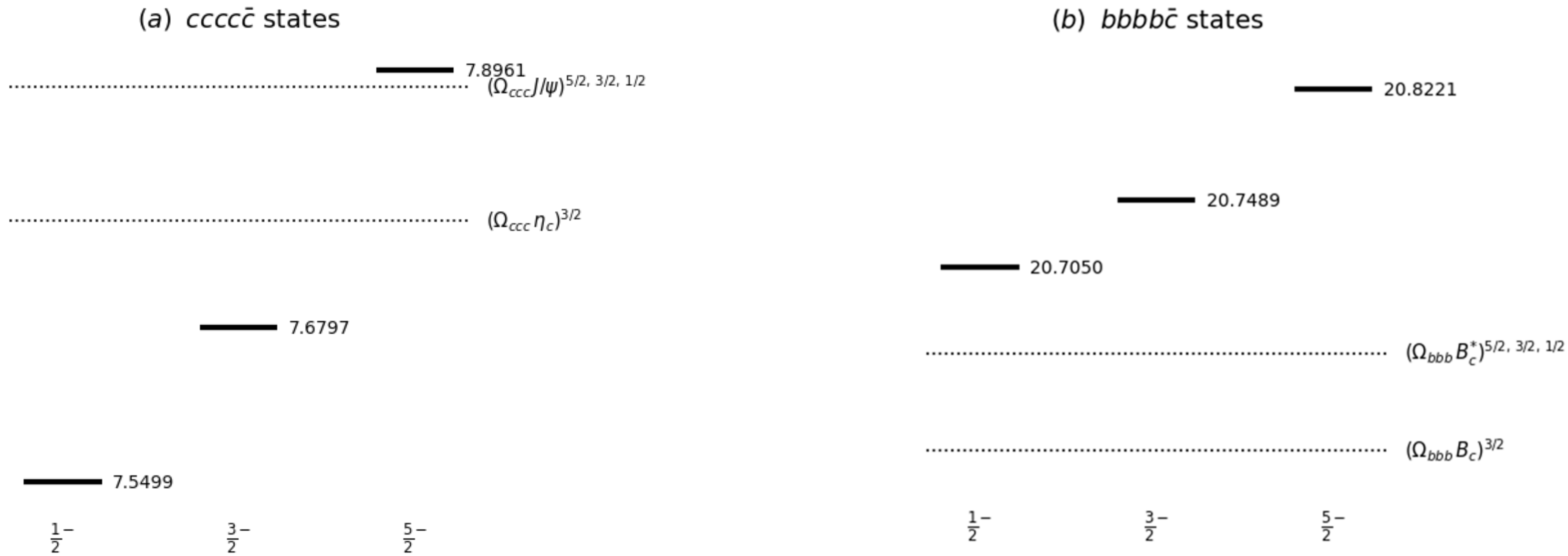
m_c	β	A	x_0
1.57 GeV	0.15 GeV^2	28	1

- The masses of Hadrons used to calculate the thresholds are taken from Ref. [14,15]

Hadrons	J^p	Mass (GeV)	Hadrons	J^p	Mass (GeV)
η_c	0^-	2.9839	Ω_{ccc}	$3/2^-$	4.7856
J/ψ	1^-	3.0969	Ω_{ccb}	$1/2^-$	7.9903
η_b	0^-	9.3990	Ω^*_{ccb}	$3/2^-$	8.0218
γ	1^-	9.4603	Ω_{bbc}	$1/2^-$	11.1650
B_c	0^-	6.2749	Ω^*_{bbc}	$3/2^-$	11.1964
B^*_c	1^-	6.3380	Ω_{bbb}	$3/2^-$	14.3097

3.
Results and Discussion

Mass specta of $ccc c\bar{c}$ and $bbbb\bar{c}$ pentaquark

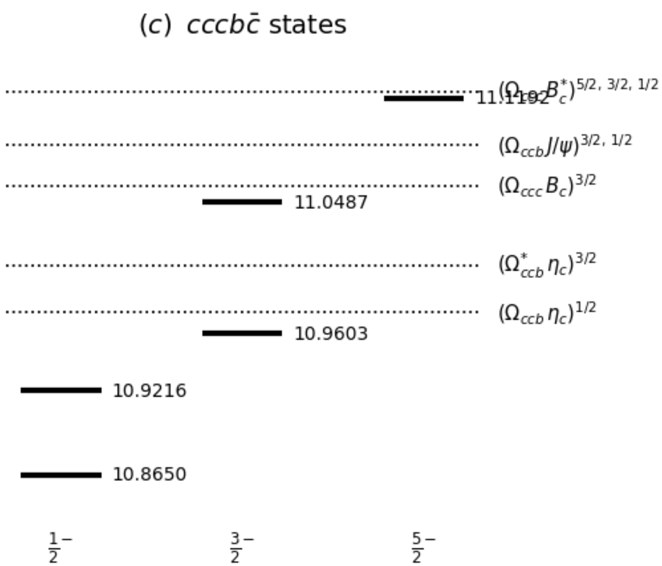


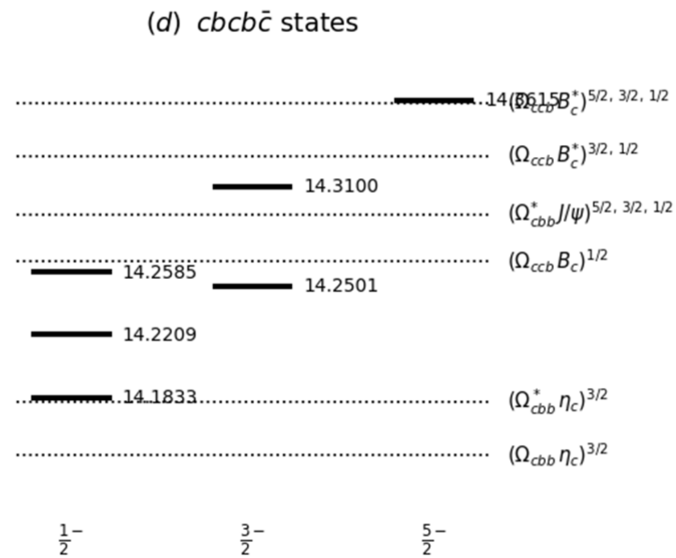
State $\lvert S_{d_1}, S_{d_2}, S_Q; J >$	J^p	Present [GeV]	Ref.[16] [GeV]
$\lvert 1,1,1/2;\ 1/2 >$	$1/2^-$	7.5499	
$\lvert 1,1,1/2;\ 3/2 >$	$3/2^-$	7.6797	7.864
$\lvert 1,1,1/2;\ 5/2 >$	$5/2^-$	7.8961	

State $\lvert S_{d_1}, S_{d_2}, S_Q; J >$	J^p	Present [GeV]	Ref.[19] [GeV]
$\lvert 1,1,1/2;\ 1/2 >$	$1/2^-$	20.7050	20.699
$\lvert 1,1,1/2;\ 3/2 >$	$3/2^-$	20.7489	20.652
$\lvert 1,1,1/2;\ 5/2 >$	$5/2^-$	20.8221	

- No stable pentaquark state exist in $bbbb\bar{c}$ pentaquark subsystem.
- The state (7.5961, $5/2^-$) of $cccc\bar{c}$ pentaquark couple almostly to the $\Omega_{ccc}J/\psi$ syatem, which can be written as direct product of Ω_{ccc} baryon and J/ψ meson. This kind of pentaquarks behaves similarly to the ordinary scattering state made of baryon and meson if the inner interaction is not strong but could also be a resonance.

Mass specta of cccb$\bar{c}$ and cbcb$\bar{c}$ pentaquark

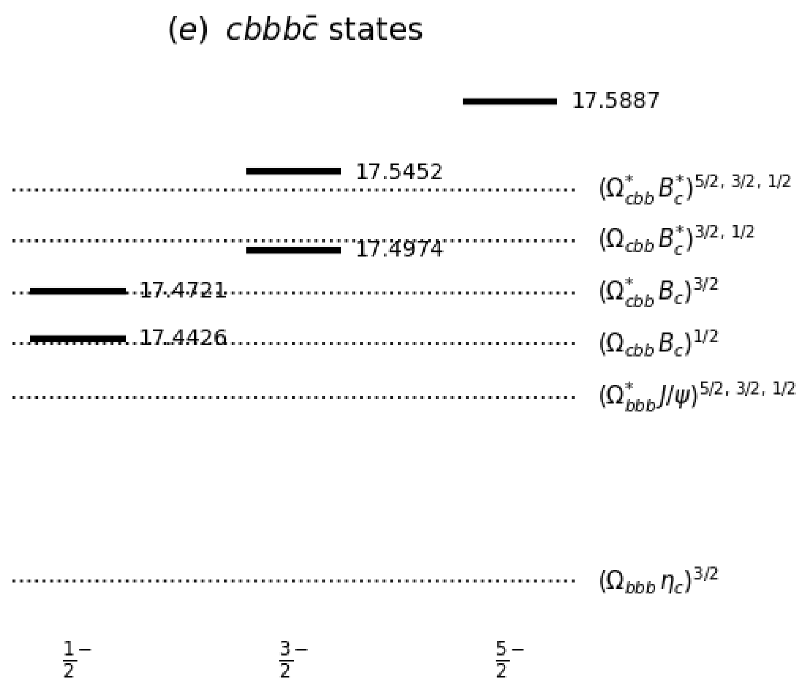




State $\vert S_{d_1}, S_{d_2}, S_Q; J >$	J^p	Present [GeV]	Ref.[19] [GeV]
$\vert 1,0,1/2; 1/2 >$	$1/2^-$	10.9216	
$\vert 1,0,1/2; 3/2 >$	$3/2^-$	11.0487	
$\vert 1,1,1/2; 1/2 >$	$1/2^-$	10.8650	11.048
$\vert 1,1,1/2; 3/2 >$	$3/2^-$	10.9603	11.038
$\vert 1,1,1/2; 5/2 >$	$5/2^-$	11.1192	11.124

State $\vert S_{d_1}, S_{d_2}, S_Q; J >$	J^p	Present [GeV]	Ref.[19] [GeV]
$\vert 0,0,1/2; 1/2 >$	$1/2^-$	14.2585	14.253
$\vert 1,0,1/2; 1/2 >$	$1/2^-$	14.2209	
$\vert 1,0,1/2; 3/2 >$	$3/2^-$	14.3100	14.298
$\vert 1,1,1/2; 1/2 >$	$1/2^-$	14.1833	14.183
$\vert 1,1,1/2; 3/2 >$	$3/2^-$	14.2501	14.274
$\vert 1,1,1/2; 5/2 >$	$5/2^-$	14.3615	14.295

Mass specta of cbbbc̄ pentaquark



State $\lvert S_{d_1}, S_{d_2}, S_Q; J >$	J^p	Present [GeV]	Ref.[19] [GeV]
$\lvert 1,0,1/2; 1/2 >$	$1/2^-$	17.4721	17.523
$\lvert 1,0,1/2; 3/2 >$	$3/2^-$	17.5452	17.535
$\lvert 1,1,1/2; 1/2 >$	$1/2^-$	17.4426	17.399
$\lvert 1,1,1/2; 3/2 >$	$3/2^-$	17.4974	
$\lvert 1,1,1/2; 5/2 >$	$5/2^-$	17.5887	

Conclusion:

- All-heavy pentaquark spectra are studied within the hCQM using a diquark–diquark–antiquark configuration.
- Hyperradial Schrödinger equation provides the energy spectrum; spin-spin interaction generates the J^P-dependent hyperfine splittings.
- Predicted masses increase systematically with heavy-quark content: 7.55–7.90, 10.92–11.12, 14.18–14.36, 17.44–17.59, and 20.71–20.82 GeV for the investigated configurations.
- Several states occur near baryon–meson thresholds, highlighting the importance of threshold and decay-channel analysis.
- The obtained spectra provide useful theoretical predictions for the identification and future experimental study of all-heavy pentaquarks.

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Thank You….