A UNITARY KAON-DEUTERON INTERACTION FOR FEMTOSCOPY

Juan M. Torres-Rincon

Universitat de Barcelona & Institut de Ciències del Cosmos

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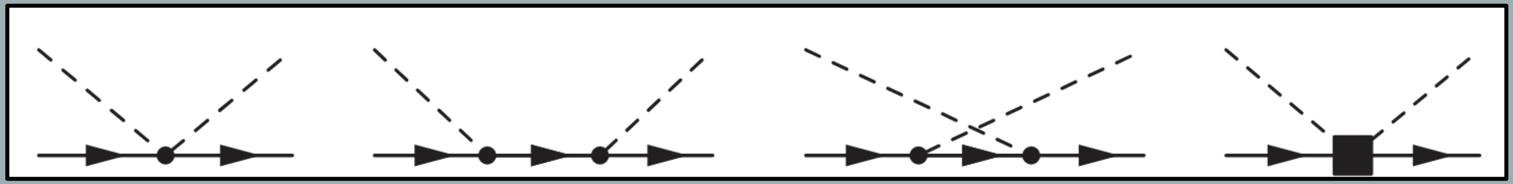
NSTAR 2026

Kaon-Deuteron interaction

Meson-baryon chiral interactions

$$\mathcal{L}_{\Phi B} = \mathrm{Tr}\left[\bar{B}\left(i\not{D} - M_0\right)B + \frac{D}{2}\left(\bar{B}\gamma^{\mu}\gamma_5\{u_{\mu}, B\}\right) + \frac{F}{2}\left(\bar{B}\gamma^{\mu}\gamma_5\left[u_{\mu}, B\right]\right)\right]$$

E. Oset, A. Ramos, *Nucl.Phys.A 635* (1998) 99-120
J.A. Oller, U.G. Meißner, *Phys.Lett.B* 500 (2001) 263-272
T. Hyodo, D. Jido, *Prog. Part. Nucl. Phys. 67*(1):55-98 (2012)



2-body unitarized amplitudes

$$t = \left[1 - VG_2\right]^{-1} V$$

OR model (E. Oset, A. Ramos, *Nucl.Phys.A 635* (1998) 99; D. Jido, E. Oset, A. Ramos, *Phys.Rev.C* 66, 055203 (2002))
Lowest order + $\bar{K}N$ scattering data

BCN model (A. Feijoo, V. Magas, A. Ramos, *Phys.Rev.C* 99, 035211 (2019))
NLO + $\bar{K}N$ scattering data + kaonic hydrogen *1s* state

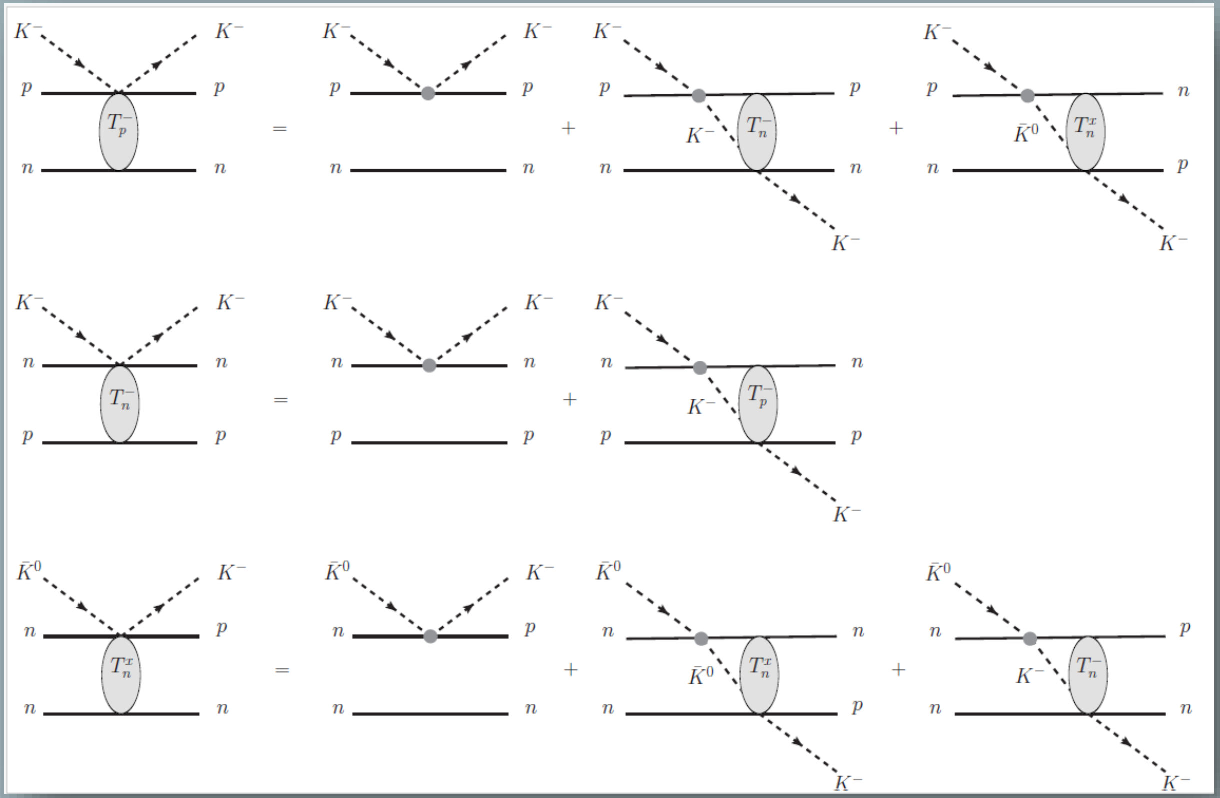
Kaon-Deuteron interaction

Kaon-deuteron interaction based on
2-body meson-baryon chiral interactions

Faddeev equations in the
Fixed Center Approximation (FCA)

$$\boxed{T^{\rm FCA}_{K^-d} = T_p + T_n}$$

$$\begin{aligned} T_p &= t_p + t_p G T_n - t_p^x G T_n^{\rm x} \\ T_n &= t_n + t_n G T_p \\ T_n^{\rm x} &= t_n^x - t_n^0 G T_n^{\rm x} + t_n^x G T_n \end{aligned}$$



S.S. Kamalov, E. Oset, A. Ramos, *Nucl.Phys.A 690* (2001) 494
U. Meißner, U. Raha, A. Rusetsky, *Eur.Phys.J.C* (2006) 473
E. Oset et al., *Nucl.Phys.A 881* (2012) 127 (beyond threshold amplitudes)
A. Martinez-Torres et al. *Phys.Rev.D* 78 (2008) 074031& A. Martinez-Torres, Ph.D. thesis, IFIC (2009)

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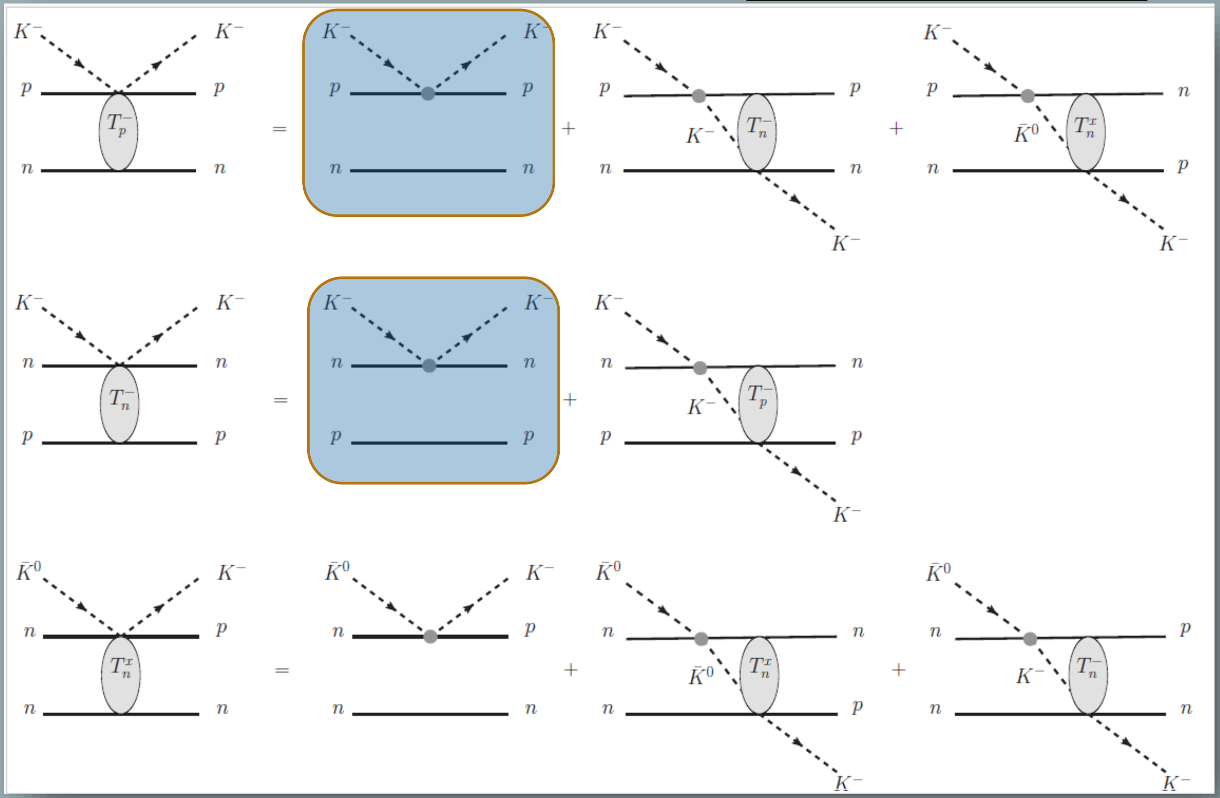
$$\begin{aligned} T_p &= t_p + t_p G T_n - t^x_p G T^{\mathrm{x}}_n \\ T_n &= t_n + t_n G T_p \\ T^{\mathrm{x}}_n &= t^x_n - t^0_n G T^{\mathrm{x}}_n + t^x_n G T_n \end{aligned}$$

Impulse Approximation $\boxed{T^{\mathrm{IA}}_{K^-d} = t_p + t_n}$



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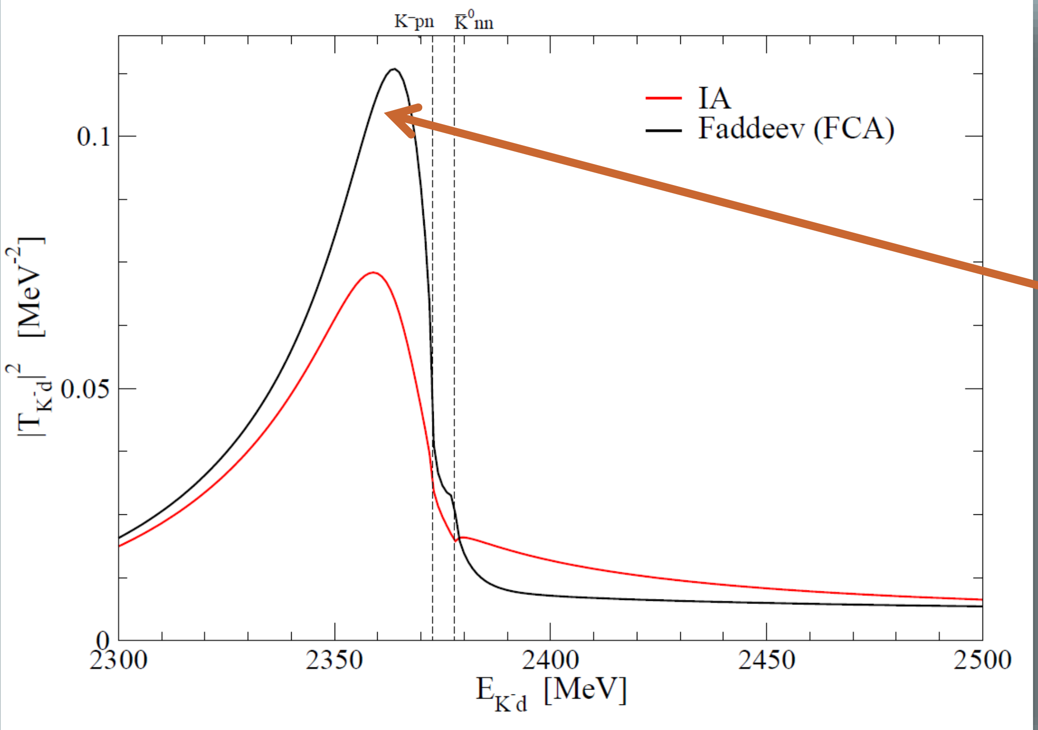


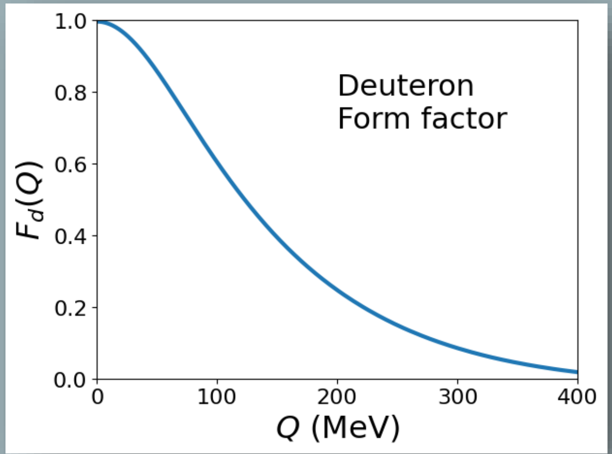
T-matrix of K$^-$ d scattering

À. Ramos, JMT-R, A. de Fagoaga, E. Cabré, *Phys.Rev.D 113*, 036020 (2026)

3-body bound state originated from the $\Lambda(1405)$

See also

E. Oset et al., *Nucl.Phys.A 881* (2012) 127



$$T^{\mathrm{FCA}}_{K^-d} = \frac{t_p + t_n + \left(2t_p t_n - \tilde{t}_x^2\right) G - 2t_n \tilde{t}_x^2 G^2}{1 - t_p t_n G^2 + t_n \tilde{t}_x^2 G^3}$$

$$\tilde{t}_x^2 = t_x^2/(1 + t_0 G)$$

$$G(q^0) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{F_d(\boldsymbol{q})}{(q^0)^2 - \boldsymbol{q}^2 - m_K^2 + \mathrm{i}\eta}$$

Kaon-Deuteron interaction

Scattering Length

$$A_{Kd} = -\frac{1}{4\pi}\frac{M_d}{m_K + M_d}T_{Kd}$$

À. Ramos, JMT-R, A. de Fagoaga, E. Cabré, *Phys.Rev.D 113*, 036020 (2026)

(OR model)

	A_{K^-d} [fm]
Impulse Approx.	$-0.59 + i2.15$
Fixed-Center Approx.	$-2.06 + i1.77$

From K^-d femtoscopy in heavy-ion collisions
ALICE Coll (2602.22833 [nucl-ex])

$$\mathrm{Re}\ A_{K^-d} = -1.44 \pm 0.15\ (\mathrm{stat})\ {}^{+0.10}_{-0.10}\ (\mathrm{syst})\ \mathrm{fm}$$

$$\mathrm{Im}\ A_{K^-d} = +1.34 \pm 0.33\ (\mathrm{stat})\ {}^{+0.21}_{-0.15}\ (\mathrm{syst})\ \mathrm{fm}$$

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Energy shift and width of the *1s* atomic level in kaonic deuterium

$$\varepsilon^d_{1s} - \mathrm{i}\frac{\Gamma^d_{1s}}{2} = -\frac{2\alpha^3\mu_r^2 A_{K^-d}}{1 + 2\alpha\mu_r A_{K^-d}(\ln\alpha - 1)}$$

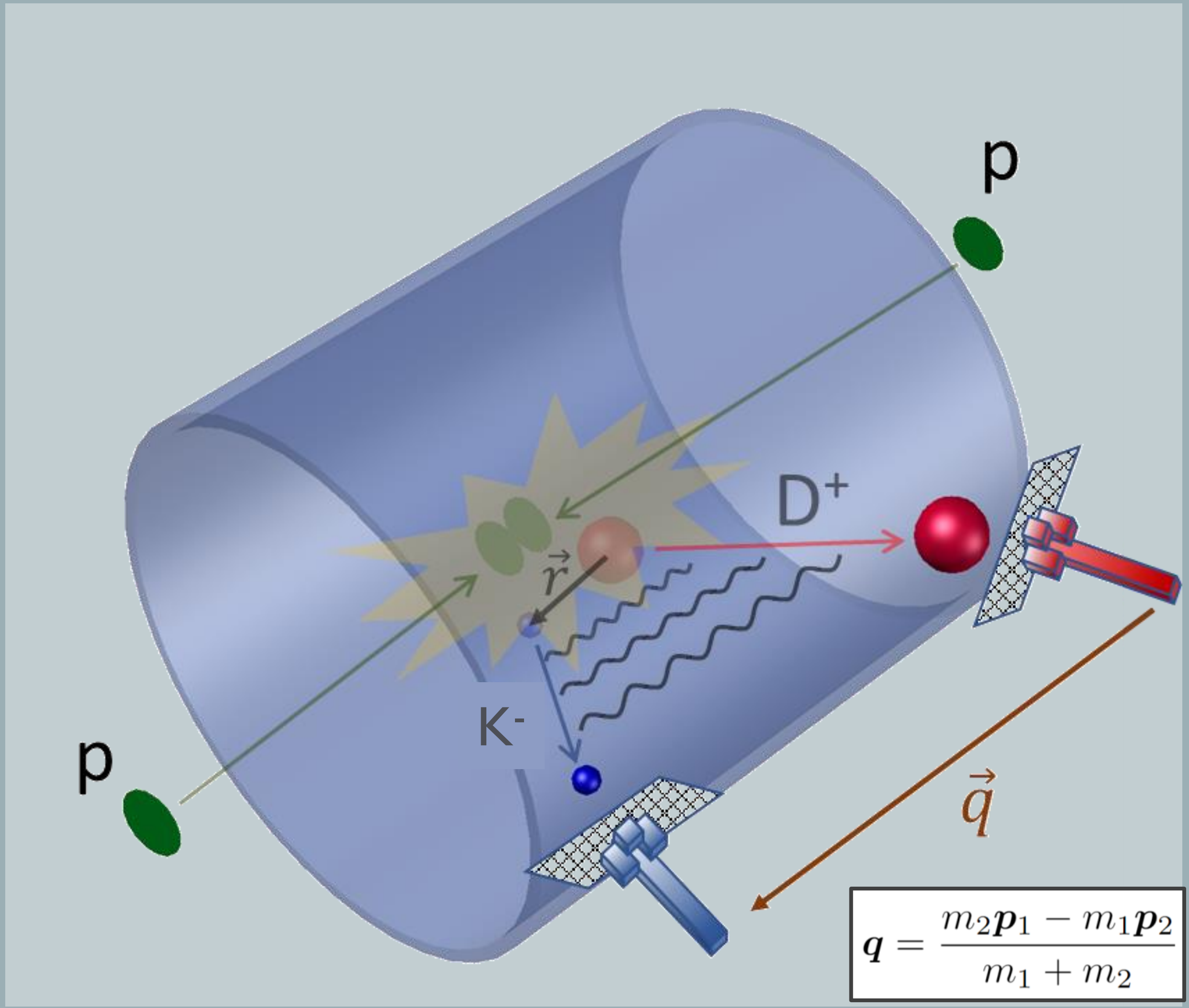
	ε^d_{1s} [eV]	Γ^d_{1s} [eV]
Impulse Approx.	791	2063
Fixed-Center Approx.	1040	1106

Experimental value from SIDDHARTA-2 experiment
(2608.10749 [nucl-ex])

$$\varepsilon^d_{1s} = 810.9 \pm 24.5\ (\mathrm{stat}) \pm 2.1\ (\mathrm{syst})\ \mathrm{eV}$$

$$\Gamma^d_{1s} = 812 \pm 97\ (\mathrm{stat}) \pm 33\ (\mathrm{syst})\ \mathrm{eV}$$

Femtoscopy in RHICs



U. Heinz, B. Jacak, *Ann.Rev.Nucl.Part.Sci.* 49 (1999) 529
M. Lisa, S. Pratt, U. Wiedemann, *Ann.Rev.Nucl.Part.Sci.* 55 (2005) 357

Pair Correlation Function

$$C(\boldsymbol{q}) = \mathcal{N}\,\frac{N_{\mathrm{same}}(\boldsymbol{q})}{N_{\mathrm{mixed}}(\boldsymbol{q})}$$

L. Fabbietti, V. Mantovani Sarti, O. Vazquez Doce, *Ann. Rev. Nucl. Part. Sci,* 71, 377 (2021)

$$C(\boldsymbol{q}) > 1 : \text{attraction}$$
$$C(\boldsymbol{q}) < 1 : \text{repulsion}$$

Koonin-Pratt formula

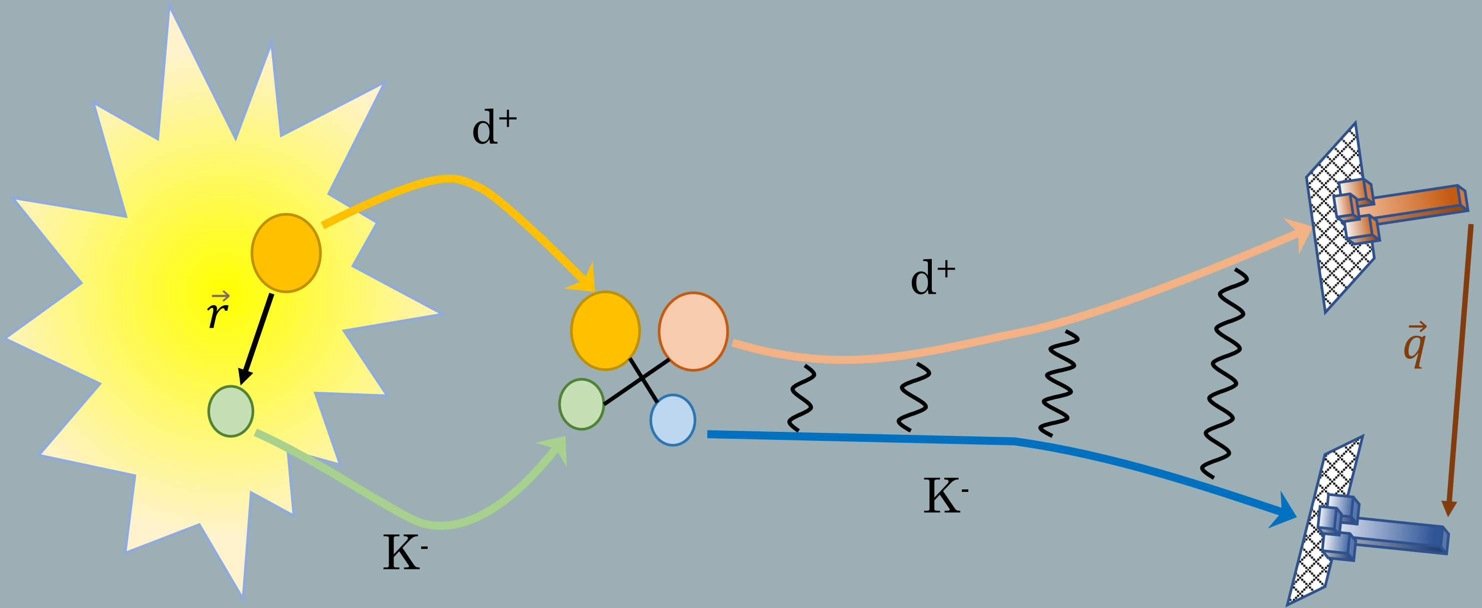
Koonin, *Phys.Lett.B*, 70, 43 (1977)
Pratt, Csorgo, Zimanyi, *Phys.Rev.C*, 42, 2646(1990)

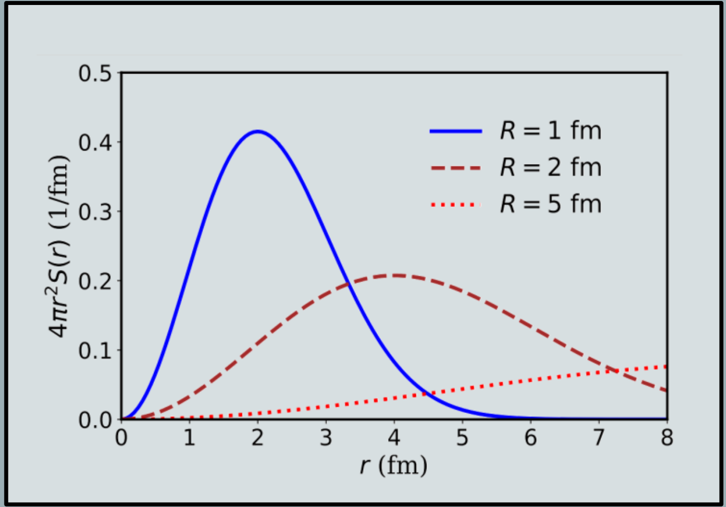
Single-channel KP formula

$$C(\boldsymbol{q}) = \int d^3r\ S(\boldsymbol{r})|\Psi(\boldsymbol{q};\boldsymbol{r})|^2$$

Pair wave function with CM momentum $\boldsymbol{q}$

Source function





Gaussian source function

$$S(r) = \frac{1}{(2\sqrt{\pi}R)^3}\exp\left(-\frac{r^2}{4R^2}\right)$$

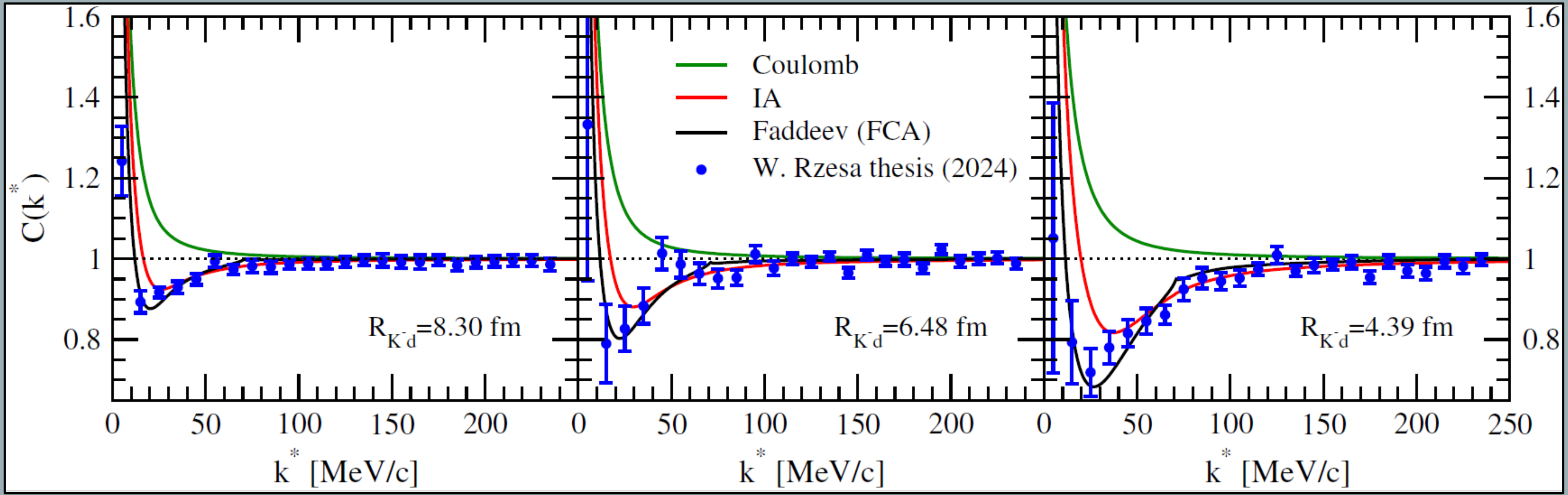
K^- d correlation function

À. Ramos, JMT-R, A. de Fagoaga, E. Cabré, *Phys.Rev.D 113*, 036020 (2026)

Pb-Pb collisions @ $s^{1/2}_{NN}$=5.02 TeV

W. Rzesa, Ph.D. thesis,
Warsaw U. of Tech. (2024) (ALICE Coll.)

Centrality: 0-10% 10-30% 30-50%



Source radii only **indicative** from experimental fits

FCA works very well! No new parameters, only model assumptions

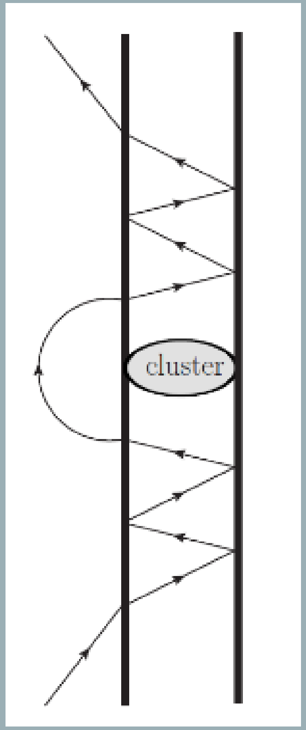
Kaon-Deuteron interaction

Lack of unitarity in the particle-cluster scattering concerns in FCA raised by

- P. Encarnación, A. Feijoo, and E. Oset, *Phys. Rev. D* 111, 114023 (2025),
- N. Ikeno and E. Oset, *Phys. Rev D 111* (2025) 114023

Solved by coherent propagation in

- B. Agatão, P. Brandão, A. Martínez Torres, K. P. Khemchandani, L. M. Abreu, and E. Oset, *Eur. Phys. J. C* 85, 1136 (2025)



Unitarity not realized at the particle-cluster threshold

Small effect for the deuteron case (binding energy of 2.2 MeV).
Tiny shift in nucleon masses adjust unitary branch point
*) À. Ramos, JMT-R, A. de Fagoaga, E. Cabré, *Phys.Rev.D 113*, 036020 (2026)

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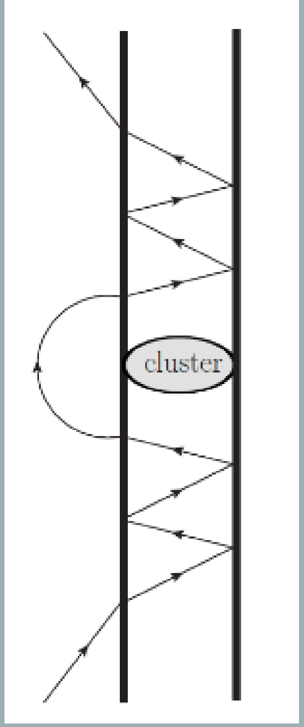

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Effective range expansion

$$\boxed{-\frac{4\pi\sqrt{s}}{M_d}(T_{Kd})^{-1} = \frac{1}{A_{Kd}} + \frac{1}{2}r_0 q_{\rm CM}^2 - \mathrm{i}q_{\rm CM}}$$

Unitary Factor should be 1!

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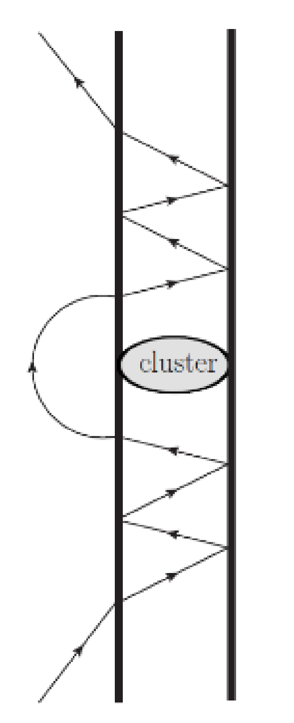

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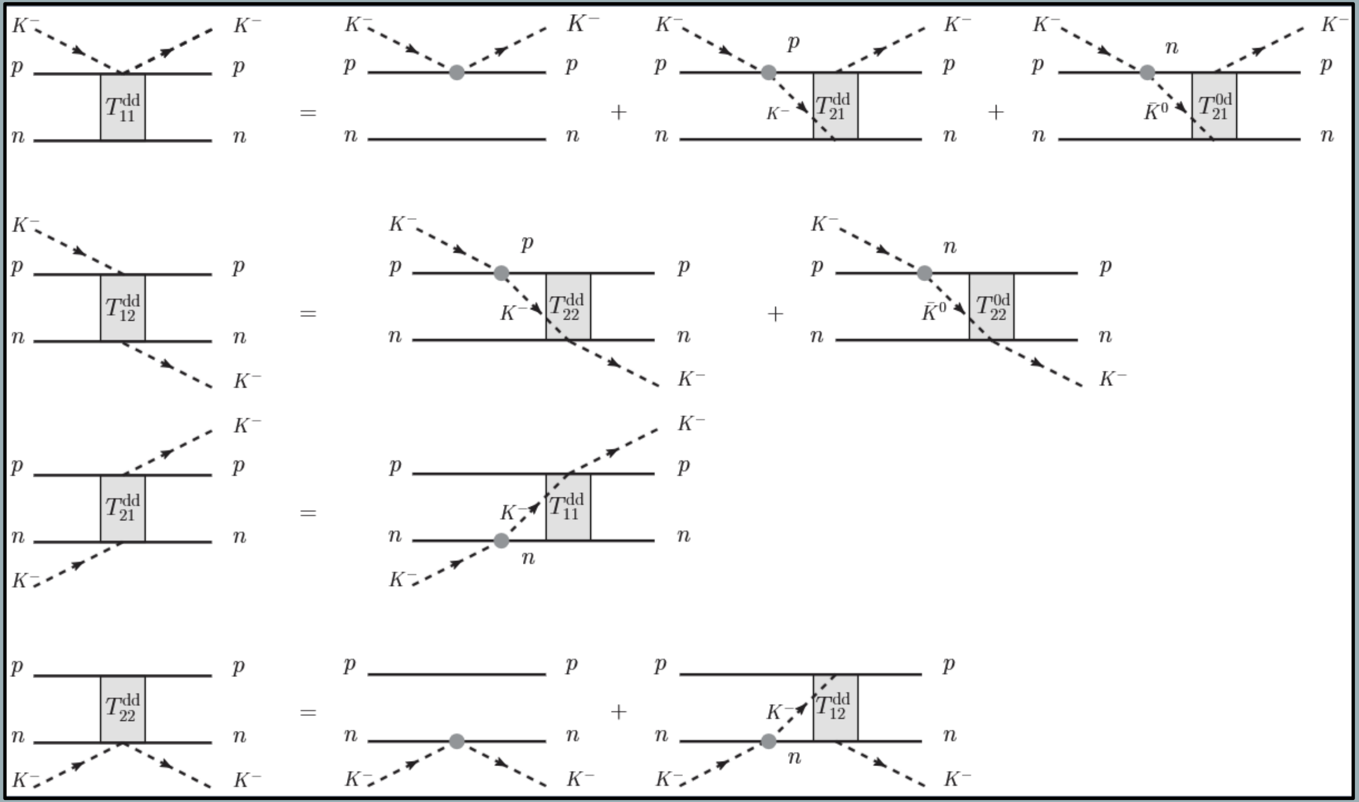
Unitary Factor should be 1!

	FCA (OR model*)	FCA (BCN model)
Unitary Factor	0.69	0.62

Kaon-Deuteron interaction

A. Feijoo, À. Ramos, JMT-R, in preparation



Rewriting of Faddeev partitions

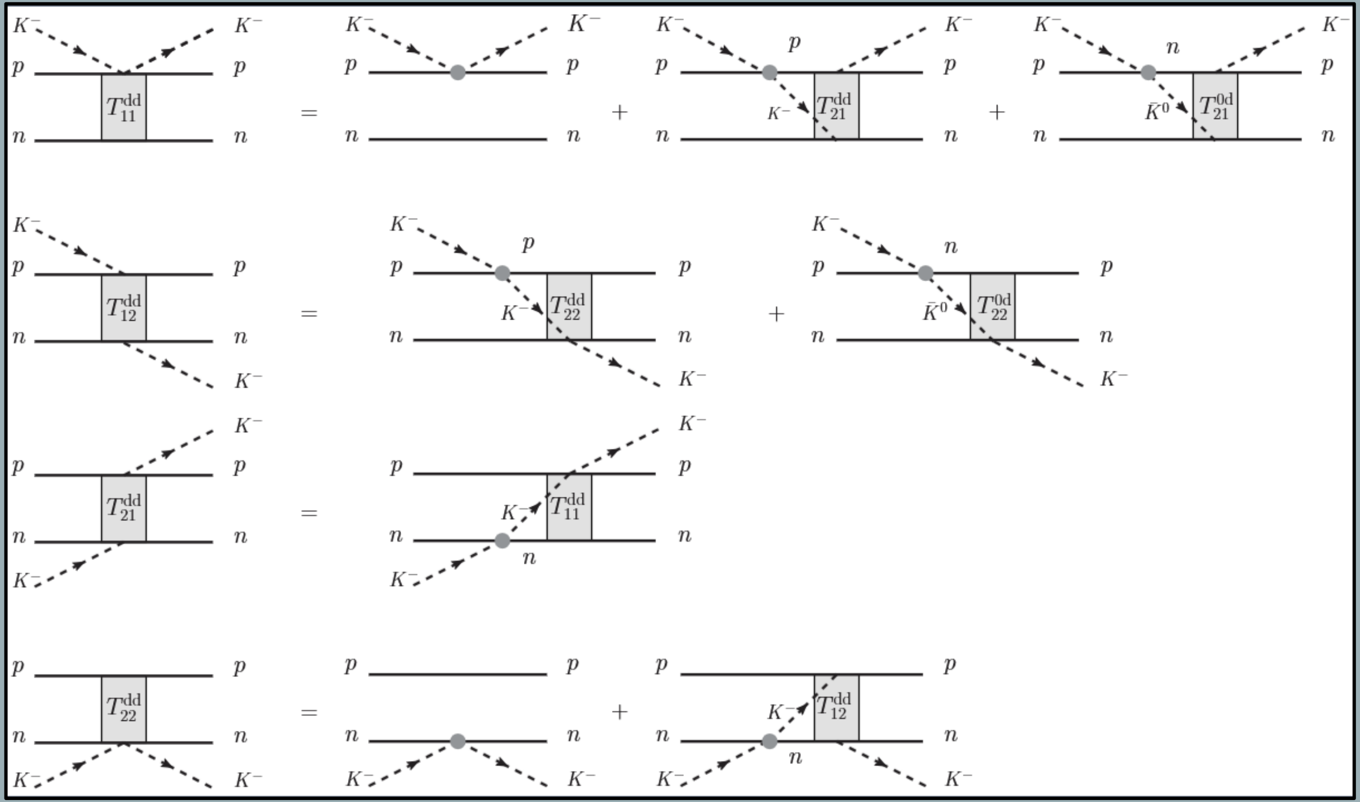
Keep track of

1) first and last kaon interaction with particle 1 or 2

2) Channel type
   - 2.1) direct (K^- p n)
   - 2.2) exchange (K^- n p)
   - 2.3) neutral ($\bar{K}^0$ n n)

Kaon-Deuteron interaction

A. Feijoo, À. Ramos, JMT-R, in preparation



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$$\mathbf{T} = \begin{pmatrix} T^{\rm dd}_{11} & T^{\rm dd}_{12} & T^{\rm de}_{11} & T^{\rm de}_{12} & T^{\rm d0}_{11} & T^{\rm d0}_{12} \\ T^{\rm dd}_{21} & T^{\rm dd}_{22} & T^{\rm de}_{21} & T^{\rm de}_{22} & T^{\rm d0}_{21} & T^{\rm d0}_{22} \\ T^{\rm ed}_{11} & T^{\rm ed}_{12} & T^{\rm ee}_{11} & T^{\rm ee}_{12} & T^{\rm e0}_{11} & T^{\rm e0}_{12} \\ T^{\rm ed}_{21} & T^{\rm ed}_{22} & T^{\rm ee}_{21} & T^{\rm ee}_{22} & T^{\rm e0}_{21} & T^{\rm e0}_{22} \\ T^{\rm 0d}_{11} & T^{\rm 0d}_{12} & T^{\rm 0e}_{11} & T^{\rm 0e}_{12} & T^{\rm 00}_{11} & T^{\rm 00}_{12} \\ T^{\rm 0d}_{21} & T^{\rm 0d}_{22} & T^{\rm 0e}_{21} & T^{\rm 0e}_{22} & T^{\rm 00}_{21} & T^{\rm 00}_{22} \end{pmatrix}$$

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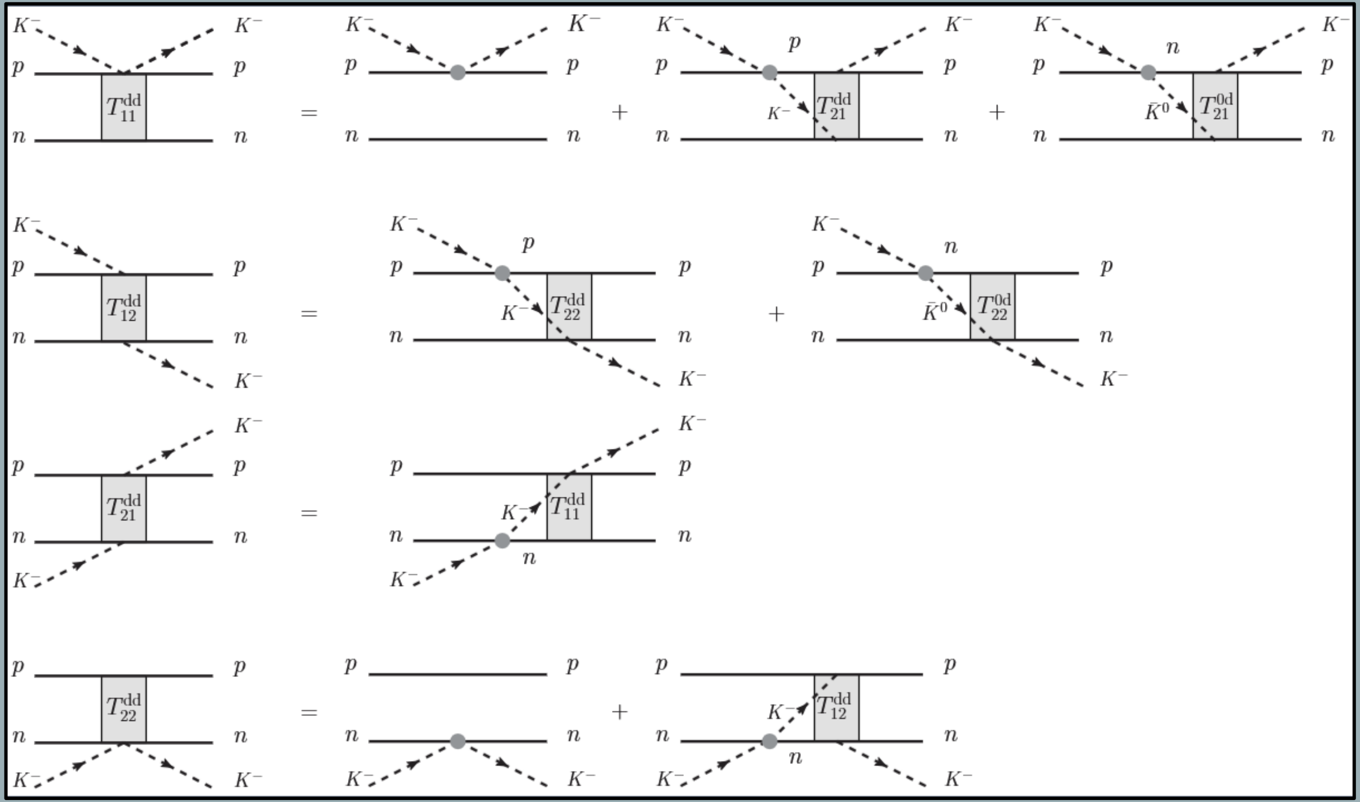
A. Feijoo, À. Ramos, JMT-R, in preparation



$$T^{\mathrm{FCA}}_{K^-d} = T^p + T^n = \sum_{i,j=1}^{2} (T^{\mathrm{dd}}_{ij} - T^{\mathrm{de}}_{ij})$$

This coupled system gives the previous FCA

Rewriting of Faddeev partitions

Keep track of

1) first and last kaon interaction with particle 1 or 2

2) Channel type

- 2.1) direct (K^- p n)
- 2.2) exchange (K^- n p)
- 2.3) neutral ($\bar{K}^0$ n n)

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Kaon-Deuteron interaction

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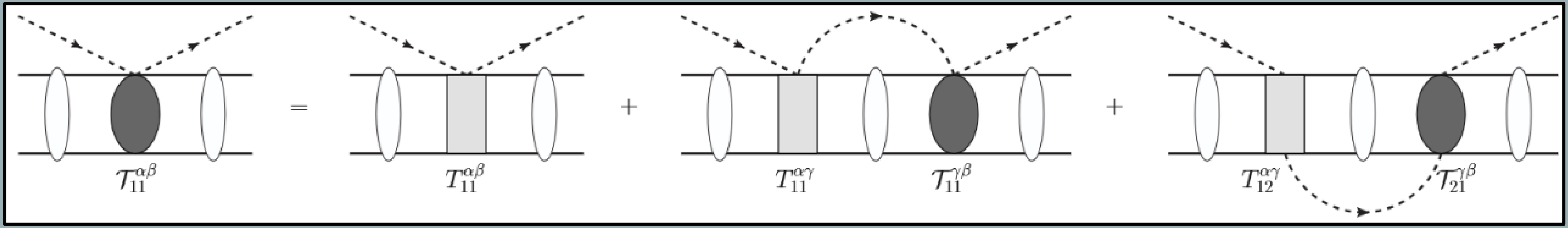
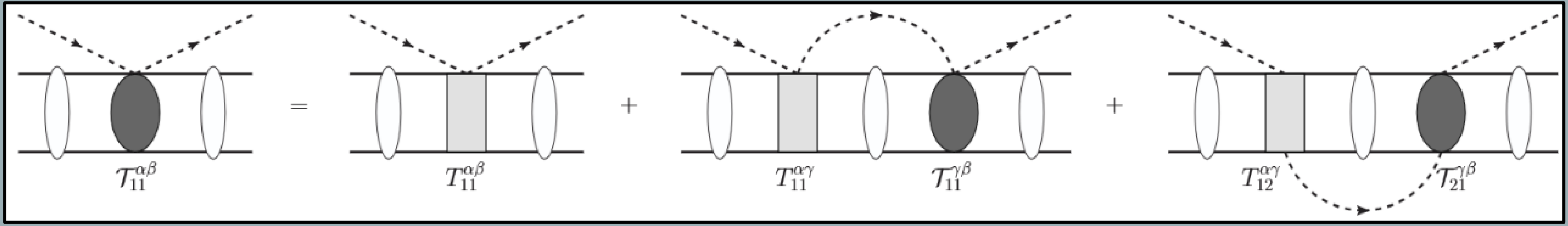


$$\mathbf{T}^{-1} = \begin{pmatrix} \frac{t_0}{t_0 t_1 - t_x^2} & -G_- & 0 & 0 & -\frac{t_x}{t_0 t_1 - t_x^2} & 0 \\ -G_- & t_2^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & t_2^{-1} & -G_- & 0 & 0 \\ 0 & 0 & -G_- & \frac{t_0}{t_0 t_1 - t_x^2} & 0 & -\frac{t_x}{t_0 t_1 - t_x^2} \\ -\frac{t_x}{t_0 t_1 - t_x^2} & 0 & 0 & 0 & \frac{t_1}{t_0 t_1 - t_x^2} & -G_0 \\ 0 & 0 & 0 & -\frac{t_x}{t_0 t_1 - t_x^2} & -G_0 & \frac{t_1}{t_0 t_1 - t_x^2} \end{pmatrix}$$

Kaon-Deuteron interaction

A. Feijoo, À. Ramos, JMT-R, in preparation



Unitary Amplitude

$$\mathcal{T}^{-1} = \mathbf{T}^{-1} - \mathbf{G_c}$$

$$\mathbf{G_c} = \mathrm{diag}\,\left(G_-^{(1)}, G_-^{(2)}, G_-^{(1)}, G_-^{(2)}, G_0^{(1)}, G_0^{(2)}\right)$$

$$\mathbf{T}^{-1} = \begin{pmatrix} \frac{t_0}{t_0t_1-t_x^2} & -G_- & 0 & 0 & -\frac{t_x}{t_0t_1-t_x^2} & 0 \\ -G_- & t_2^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & t_2^{-1} & -G_- & 0 & 0 \\ 0 & 0 & -G_- & \frac{t_0}{t_0t_1-t_x^2} & 0 & -\frac{t_x}{t_0t_1-t_x^2} \\ -\frac{t_x}{t_0t_1-t_x^2} & 0 & 0 & 0 & \frac{t_1}{t_0t_1-t_x^2} & -G_0 \\ 0 & 0 & 0 & -\frac{t_x}{t_0t_1-t_x^2} & -G_0 & \frac{t_1}{t_0t_1-t_x^2} \end{pmatrix}$$

$$G_{\mathrm{m}}^{(i)} = \int \frac{\mathrm{d}^3q}{(2\pi)^3} \frac{\left(F_d^{(i)}(\boldsymbol{q})\right)^2}{2\omega_{\mathrm{m}}(\boldsymbol{q})\left(\sqrt{s_{Kd}} - \omega_{\mathrm{m}}(\boldsymbol{q}) - E_{\mathrm{m}}(\boldsymbol{q}) + \mathrm{i}\eta\right)}$$

$$T_{K^-d}^{\mathrm{uni}} = \sum_{i,j=1}^{2} \left(\mathcal{T}_{ij}^{\mathrm{dd}} - \mathcal{T}_{ij}^{\mathrm{de}}\right)$$

Final result too large to be shown

Kaon-Deuteron interaction

Particular cases

Set all kaon-deuteron propagators equal

$$G_-^{(1)} = G_-^{(2)} = G_- \quad ; \quad G_0^{(1)} = G_0^{(2)} = G_0$$

Unitarized Impulse Approximation!

$$\left(T_{\text{uni}}^{K^-d}\right)^{-1} = \frac{1}{t_p + t_n} - G_-$$

Neglect charge-exchange process

$$t_x = 0$$

We recover previous result in

B. Agatão, P. Brandão, A. Martínez Torres, K. P. Khemchandani, L. M. Abreu, and E. Oset, *Eur. Phys. J. C* 85, 1136 (2025)

$$\left(T_{\text{uni}}^{K^-d}\right)^{-1} = \frac{t_p + t_n + (2G - G_-^{(1)} - G_-^{(2)})t_p t_n}{1 - G_-^{(1)} t_p - G_-^{(2)} t_n - \left(G^2 - G_-^{(1)} G_-^{(2)}\right) t_p t_n}$$

Kaon-Deuteron interaction

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Unitary Factor	0.69	1.00	1.00
Scattering Length [fm]	$-2.06 + i1.77$	$-1.04 + i0.90$	$-0.85 + i1.07$

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ALICE Coll (arXiv:2602.22833 [nucl-ex])

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$$\mathrm{Im}\ A_{K^-d} = +1.34 \pm 0.33\ (\mathrm{stat})\ ^{+0.21}_{-0.15}\ (\mathrm{syst})\ \mathrm{fm}$$

	FCA (OR model*)	FCA-UNIT (OR model)	FCA-UNIT (BCN model)
ε^d_{1s} [eV]	1040	583	525
Γ^d_{1s} [eV]	1106	767	958

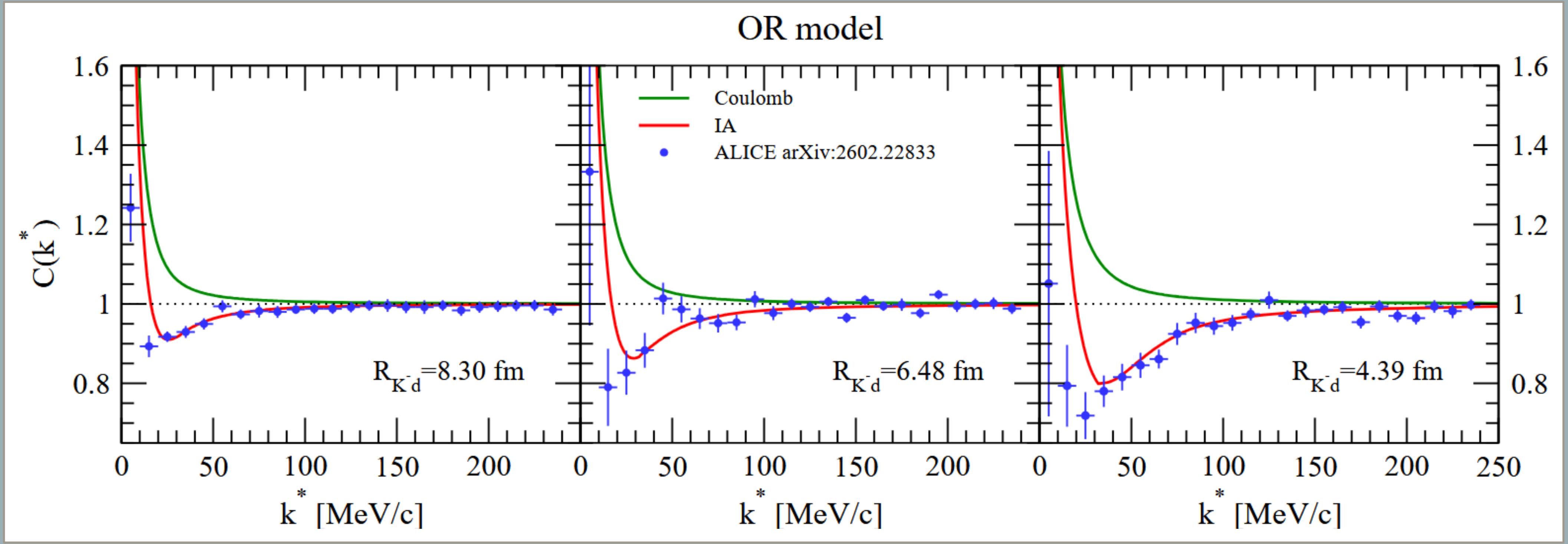
Experimental value from SIDDHARTA-2 experiment
(arXiv:2608.10749 [nucl-ex])

$$\varepsilon^d_{1s} = 810.9 \pm 24.5\ (\mathrm{stat}) \pm 2.1\ (\mathrm{syst})\ \mathrm{eV}$$

$$\Gamma^d_{1s} = 812 \pm 97\ (\mathrm{stat}) \pm 33\ (\mathrm{syst})\ \mathrm{eV}$$

Kaon-Deuteron interaction

Experimental from K^- d femtoscopy in heavy ions ALICE Coll (arXiv:2602.22833 [nucl-ex])

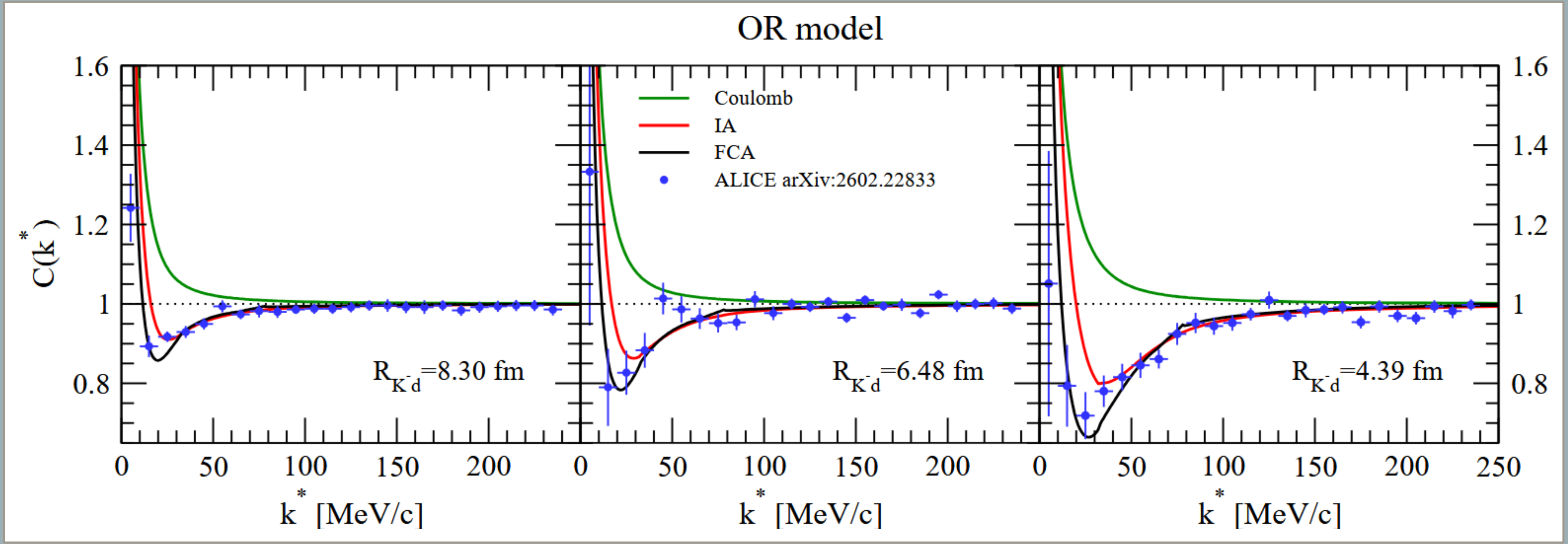


OR model

Source radii only **indicative** from experimental fits (they are attached to a different interaction)

Kaon-Deuteron interaction

Experimental from K⁻ d femtoscopy in heavy ions ALICE Coll (arXiv:2602.22833 [nucl-ex])



OR model

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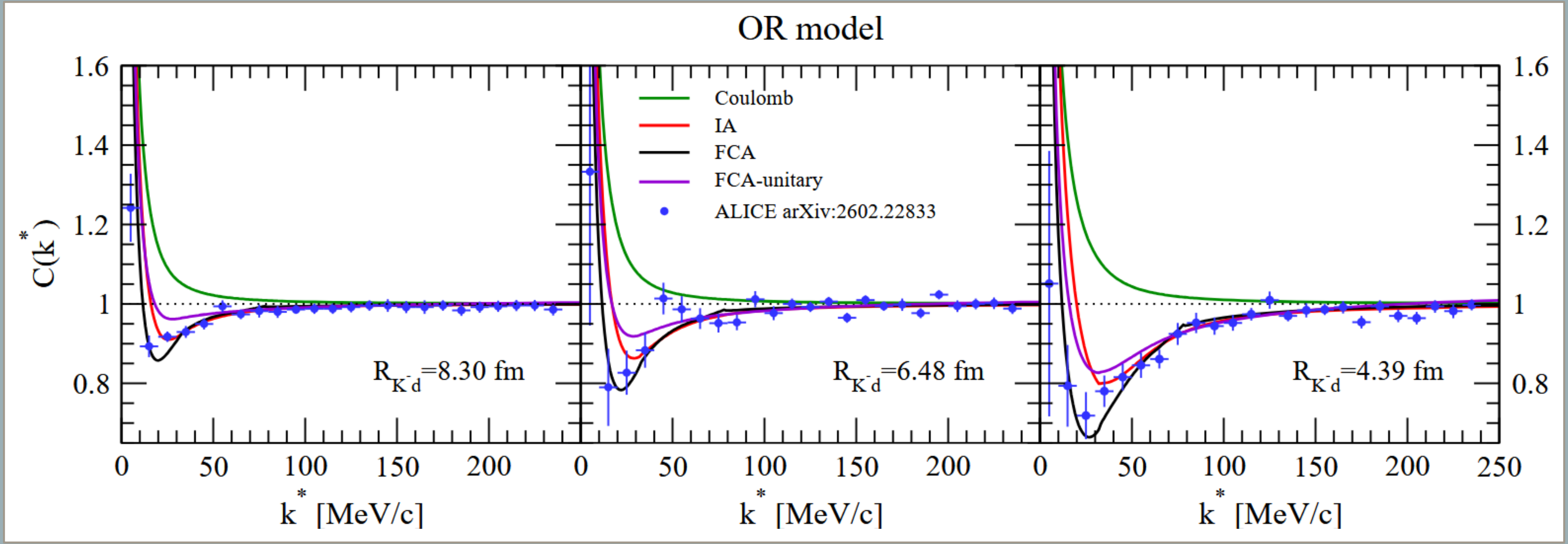
Kaon-Deuteron interaction

Experimental from K- d femtoscopy in heavy ions ALICE Coll (arXiv:2602.22833 [nucl-ex])

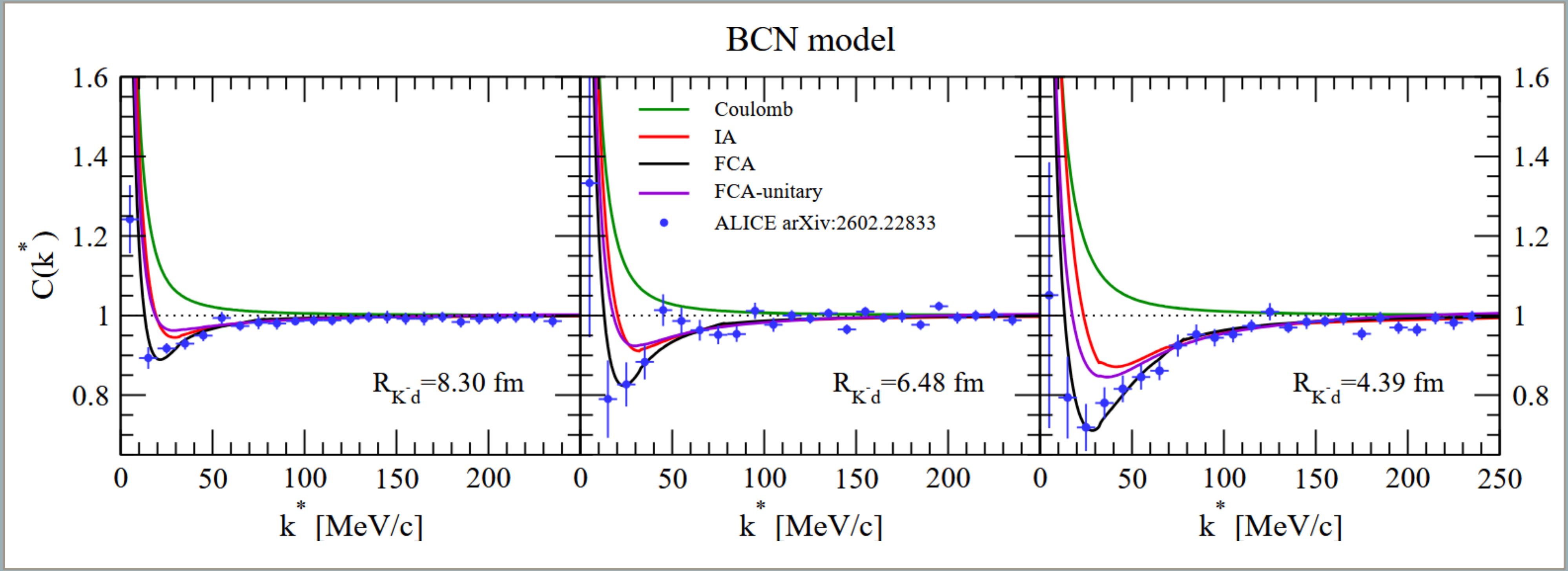


OR model

Source radii only **indicative** from experimental fits (they are attached to a different interaction)

Kaon-Deuteron interaction

Experimental from K^- d femtoscopy in heavy ions ALICE Coll (arXiv:2602.22833 [nucl-ex])



BCN model

Source radii only **indicative** from experimental fits (they are attached to a different interaction)

FCA with exact unitarity deviates from data

K^-d correlation function

Summary

Kaon-deuteron interaction in FCA provide a good description of ALICE femtoscopy data

- Unitarity has been restored! Scattering length and kaonic deuterium energy shift and width have improved from previous results.
- Femtoscopy correlation function with untarized FCA has deviated from data.
  Radii need to be modified/fitted. However, interaction model might need also to be assessed.
- OUTLOOK: Calibrate break-up, recombination, rearrangement processes in Faddeev equations.

A UNITARY KAON-DEUTERON INTERACTION FOR FEMTOSCOPY

Juan M. Torres-Rincon
Universitat de Barcelona & Institut de Ciències del Cosmos

Sevilla, September 9, 2026,

15th International Workshop on the Physics of Excited Nucleons

NSTAR 2026

Meson-baryon chiral effective theory

$$\mathcal{L}_{\Phi B} = \mathrm{Tr}\left[\bar{B}\left(i\not{D} - M_0\right)B + \frac{D}{2}\left(\bar{B}\gamma^{\mu}\gamma_5\{u_{\mu}, B\}\right) + \frac{F}{2}\left(\bar{B}\gamma^{\mu}\gamma_5\left[u_{\mu}, B\right]\right)\right]$$

$$D_{\mu}B = \partial_{\mu}B + [\Gamma_{\mu}, B] \qquad \Gamma_{\mu} = \frac{1}{2}\left[u^{\dagger}, \partial_{\mu}u\right] = \frac{1}{2}\left(u^{\dagger}\partial_{\mu}u + u\partial_{\mu}u^{\dagger}\right) \qquad u_{\mu} = \frac{i}{2}\{u^{\dagger}, \partial_{\mu}u\} = \frac{i}{2}\left(u^{\dagger}\partial_{\mu}u - u\partial_{\mu}u^{\dagger}\right)$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix} \qquad \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

Ecker, *Prog. Part. Nucl. Phys., 35*:1-80 (1995), Bernard, Kaiser, Meißner, *Int. J. Mod. Phys. E, 4*:193-346 (1995), Oset, Ramos, *Nucl.Phys.A 635* (1998) 99-120, Hyodo, Jido, *Prog. Part. Nucl. Phys. 67*(1):55-98 (2012)

Meson-baryon chiral effective theory

$$\mathcal{L}_{\mathrm{WT}} = \frac{1}{4f^2}\mathrm{Tr}\ \left[\bar{B}i\gamma^\mu\left[\Phi\partial_\mu\Phi - (\partial_\mu\Phi)\Phi, B\right]\right]$$

$$V_{ij} = \frac{-C_{ij}}{4f_i f_j}\bar{u}(j)\gamma_\mu u(i)(k_i + k_j)^\mu$$

$Q = -1$ $\quad K^- n, \pi^0\Sigma^-, \pi^-\Sigma^0$
$\pi^-\Lambda, \eta\Sigma^-, K^0\Xi^-$

$Q = 0$ $\quad K^- p, \bar{K}^0 n, \pi^0\Lambda, \pi^0\Sigma^0, \pi^+\Sigma^-$
$\pi^-\Sigma^+, \eta\Lambda, \eta\Sigma^0, K^+\Sigma^-, K^0\Sigma^0$

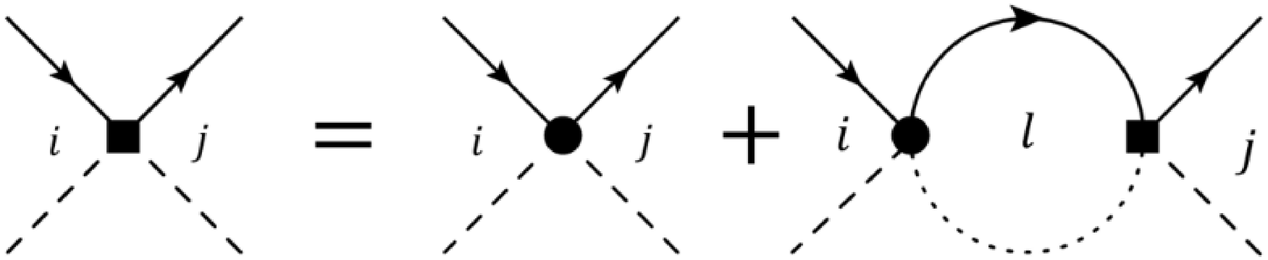
$$V_{ij}^{s-\text{wave, on-shell}} = -\frac{C_{ij}}{4f_i f_j}\sqrt{\frac{E_j + M_j}{2M_j}}\sqrt{\frac{E_i + M_i}{2M_i}}\left(2\sqrt{s} - M_i - M_j\right)$$

Isospin limit: $f_i = f_j = 1.123 f_\pi$

E. Oset, A. Ramos, *Nucl.Phys.A 635* (1998) 99-120, Hyodo, Jido, *Prog. Part. Nucl. Phys. 67*(1):55-98 (2012)

T-matrix equation with on-shell factorization

$$t_{ij} = V_{ij} + \sum_l V_{il} G_l t_{lj}$$



$$G_l(P) = i \int \frac{d^4q}{(2\pi)^4} \frac{2M_l}{(P-q)^2 - M_l^2 + i\epsilon} \frac{1}{q^2 - m_l^2 + i\epsilon}$$

$$\boxed{t = [1 - VG]^{-1} V}$$

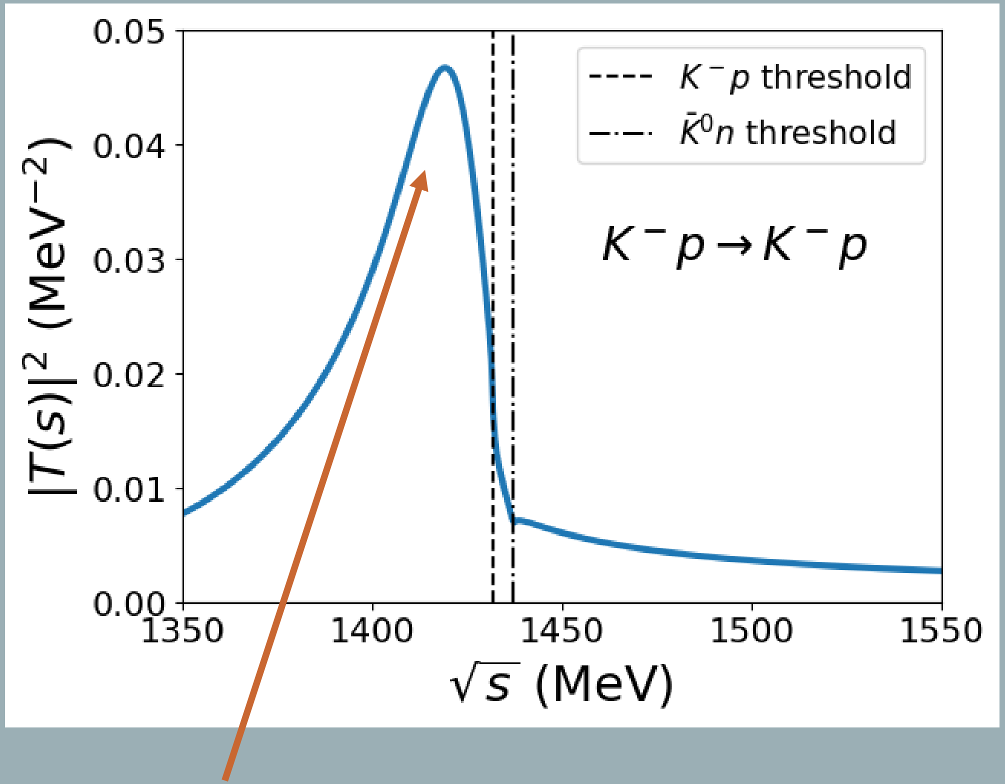
Dimensional regularization for loop functions at μ=630 MeV

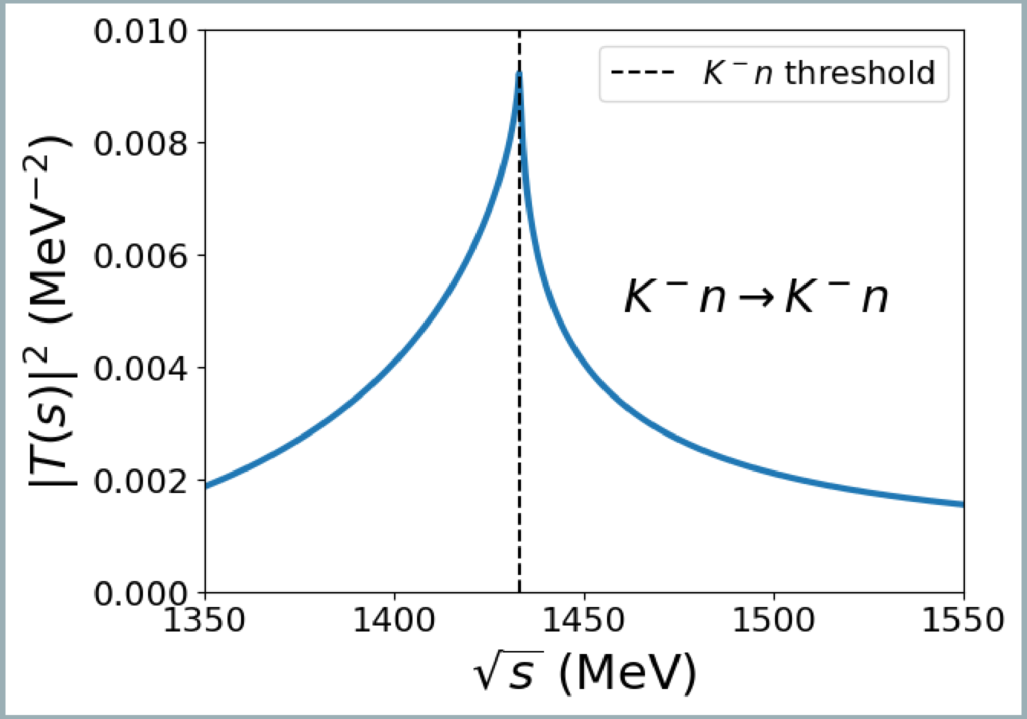
E. Oset, A. Ramos, *Nucl.Phys.A 635* (1998) 99-120

J.A. Oller, E. Oset, *Nucl.Phys.A 620* (1997) 438-456, J.A. Oller, U.G. Meißner, *Phys.Lett.B* 500 (2001) 263-272

T matrices for (elastic) K^-p and K^-n

t_p





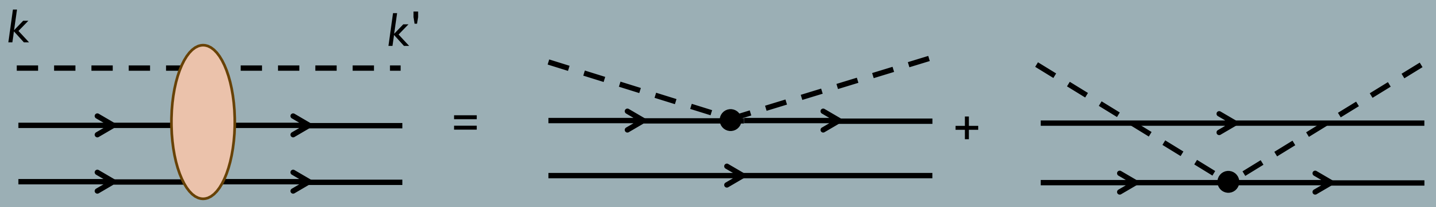
t_n

Λ(1405) resonance below K^-p threshold (but above $\pi\Sigma$)

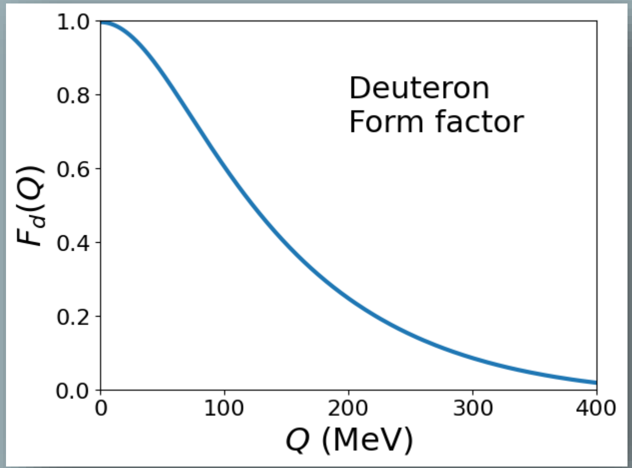
E. Oset, A. Ramos, *Nucl.Phys.A 635* (1998) 99-120; J.A. Oller, U.G. Meißner, *Phys.Lett.B* 500 (2001) 263-272,;
S. Navas et al. (Particle Data Group), *Phys. Rev. D* 110, 030001 (2024)

Kaon-Deuteron interaction

Impulse Approximation



$$F_d(Q) = \int e^{-i\mathbf{Q}\cdot\mathbf{r}} \mid \phi_d(\mathbf{r}) \mid^2 \, d\mathbf{r}$$



Argonne AV18 wave functions

$$T_{Kd}(k', k) \; = \; [t_p(k', k) + t_n(k', k)] \, F_d(Q)$$

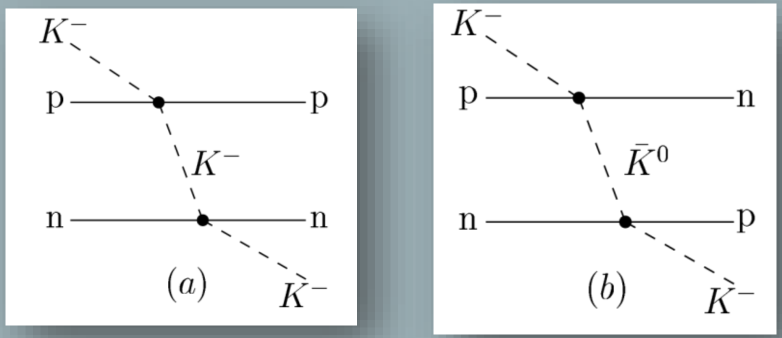
Chiral Kp interaction

Chiral Kn interaction

S.S. Kamalov, E. Oset, A. Ramos, *Nucl.Phys.A 690* (2001) 494-508

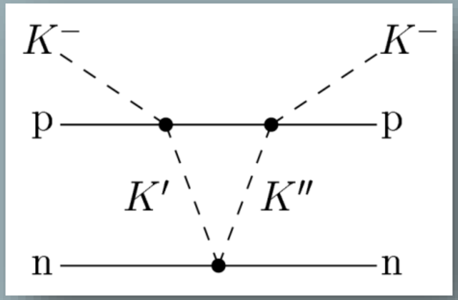
Kaon-Deuteron interaction

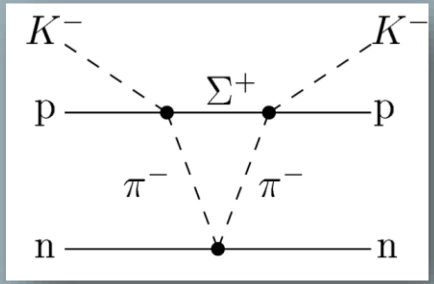
Double scattering approximation



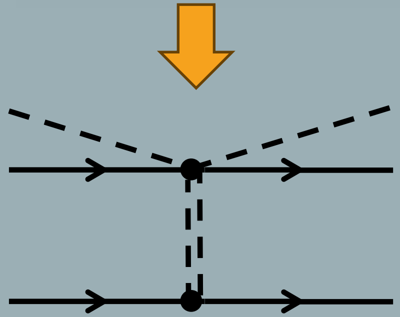
Cancellation of off-shell 2-body T operators with chiral 3-body contact terms

Triple scattering approximation





Only 1% contribution to the K^-d amplitude



Fixed-Center Approximation

S.S. Kamalov, E. Oset, A. Ramos, *Nucl.Phys.A 690* (2001) 494-508;
E. Oset et al., *Nucl.Phys.A 881* (2012) 127-140;
A. Martinez-Torres et al. *Phys.Rev.D* 78 (2008) 074031; A. Martinez-Torres, Ph.D. thesis, IFIC (2009)

Deuterons in RHICs

- Deuteron production in HICs still under debate (coalescence versus thermal models)

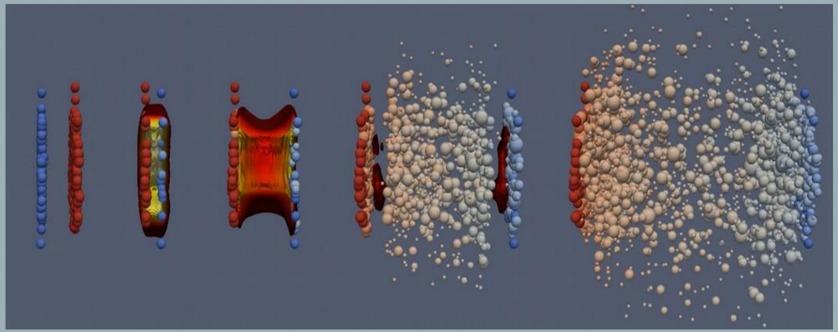
L. P. Csernai and J. I. Kapusta, *Phys. Rept.* 131, 223 (1986); P. Danielewicz and G. F. Bertsch, *Nucl. Phys. A* 533, 712 (1991)., , R. Scheibl and U. W. Heinz, *Phys. Rev. C* 59, 1585 (1999), A. Andronic, P. Braun-Munzinger, and J. Stachel, *Nucl. Phys. A* 772, 167 (2006), A. Andronic, P. Braun-Munzinger, K. Redlich, and J. Stachel, *Nature* 561, 321 (2018), V. Vovchenko et al. Phys. Lett. B 800, 135131 (2020); T. Neidig, et al. *Phys. Lett. B* 827, 136891 (2022); J. Rais, H. van Hees and C. Greiner, Phys. Rev.C 106 (2022), 064004…

- Deuteron **femtoscopy** might help to distinguish between them

S. Mrówczýnski and P. Słoń, *Acta Phys. Polon. B* 51, 1739 (2020)

- Hints of deuteron early formation via deuteron-pion femtoscopy

O. Vázquez Doce, D. Mihaylov, L. Fabbietti, *Eur. Phys. J.A* 61 (2025) 3, 53



MADAI collaboration,
Hannah Petersen and Jonah Bernhard

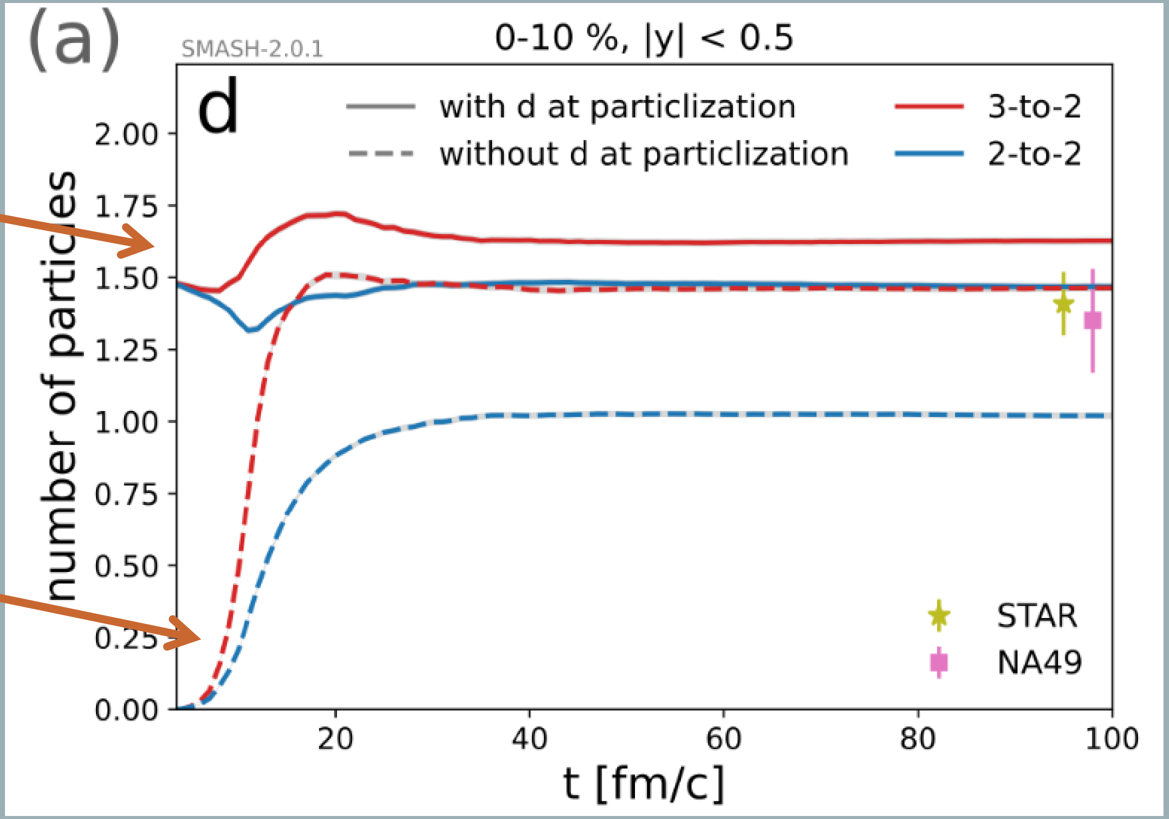
Deuterons in RHICs

Time evolution of deuteron yield in Au Au collisions at $s^{1/2}_{NN}$ =7.7 GeV

smash

Statistical Thermal Model:
Production at freeze out

Coalescence Model:
Dynamical production of composed hadrons



"Stochastic rates" to account for boost-invariant, 3-to-2 reactions:

$N\, p\, n \leftrightarrow N\, d$

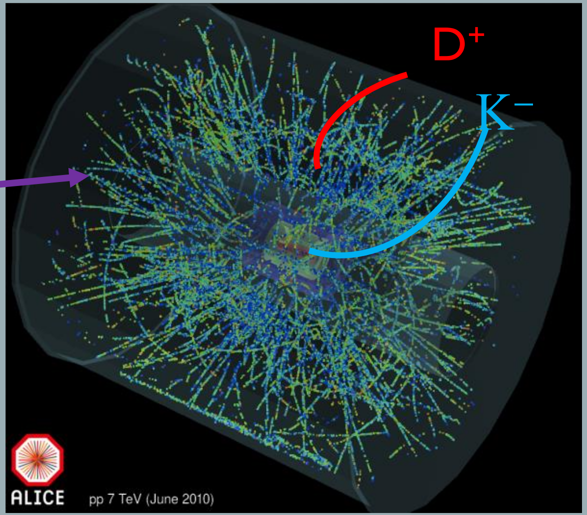
$\pi\, p\, n \leftrightarrow \pi\, d$

J. Staudenmaier, D. Oliinychenko, JMT-R and H. Elfner
Phys.Rev.C 104 (2021) 3, 034908

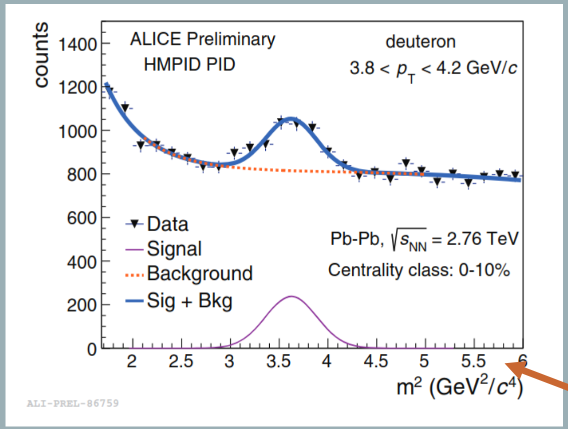
Femtoscopy in RHICs

Pair Correlation Function

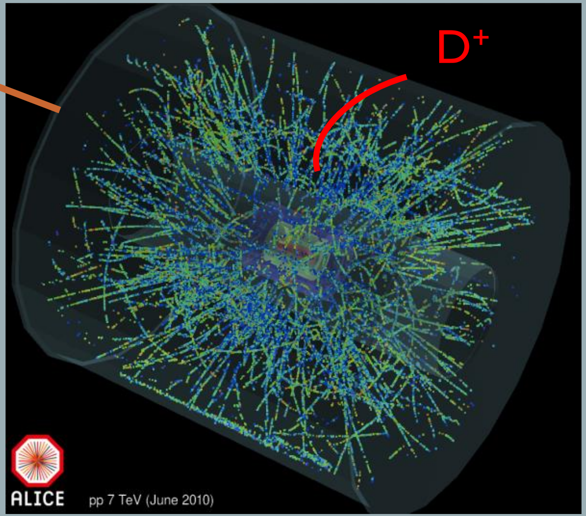
$$C(\boldsymbol{q}) = \mathcal{N}\,\frac{N_{\mathrm{same}}(\boldsymbol{q})}{N_{\mathrm{mixed}}(\boldsymbol{q})}$$

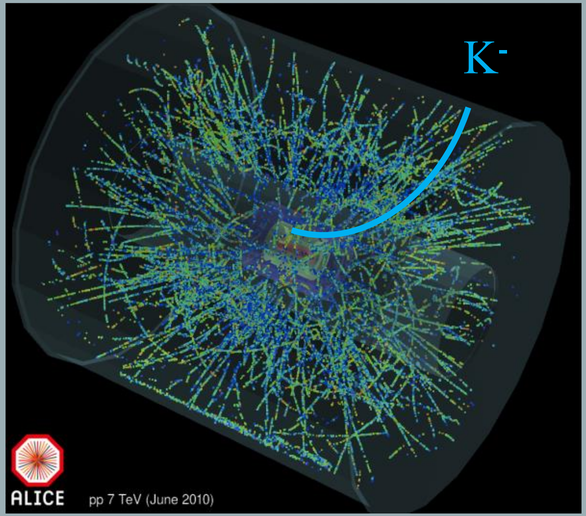


'Event mixing' technique



D$^+$ reconstruction





10^6-10^7 events

Correlation function

KP Correlation function with Coulomb interaction

$$C(q) = \int d^3r\, S(r)|\Phi^{\mathrm{C}}(\boldsymbol{q};\boldsymbol{r})|^2 + 4\pi \int r^2 dr\, S(r) \sum_{\ell} (2\ell+1) \left[|\varphi_\ell(q;r)|^2 - |\Phi^{\mathrm{C}}_\ell(qr)|^2 \right]$$

Complete Coulomb wave function

ℓ-wave strong + Coulomb wf

ℓ-wave Coulomb wf

As in 2-body, we keep only s-wave interactions

Two-body approach

S. Mrowczynski, 2501.08283

The three-body approach to hadron-deuteron correlations changes into the two-body approach if the three-particle hadron-deuteron wave function factorizes into the deuteron wave-function and the wave function of a hadron motion relative to the deuteron. The assumption of factorization is less restrictive for the kaon-deuteron than the proton-deutereon system, as there is no ambiguity which particle should be bound to the neutron.

Kaon-deuteron akin to having a faithful two-body description

We consider an effective two-body approach from Faddeev equations

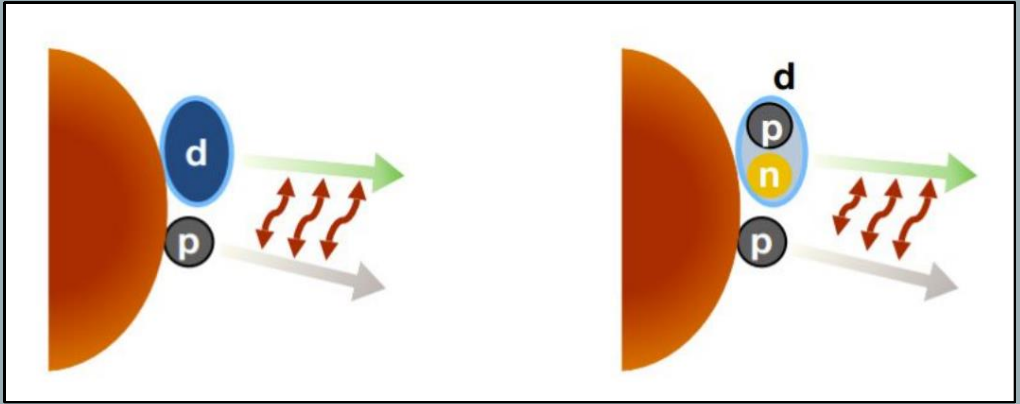


Image Credit: ALICE coll.

Kaon-Deuteron Correlation Function

We use T-matrix to compute the wave function (Lippmann-Schwinger equation)

$$\varphi_i(q;r) = j_0(qr)\delta_{if} + \int_0^\infty \frac{4\pi q'^2\, dq'}{(2\pi)^3} \frac{T_{if}(q',q;\sqrt{s})\, j_0(q'r)}{2\omega_{H,i}\, 2\omega_{\phi,i}(\sqrt{s}-\omega_{H,i}-\omega_{\phi,i}+\mathrm{i}\eta)}$$

$$T(q',q;\sqrt{s}) = T^{\mathrm{S}}(\sqrt{s}) + T^{\mathrm{C}}(q',q;\sqrt{s})$$

Total Coulomb wave function and ℓ =0 partial wave

$$\Phi^{\mathrm{C}}(r,z;k) = e^{-\pi\gamma/2}\Gamma(1+i\gamma)e^{ikz}\ {}_1F_1(-i\gamma;1;ik(r-z))$$

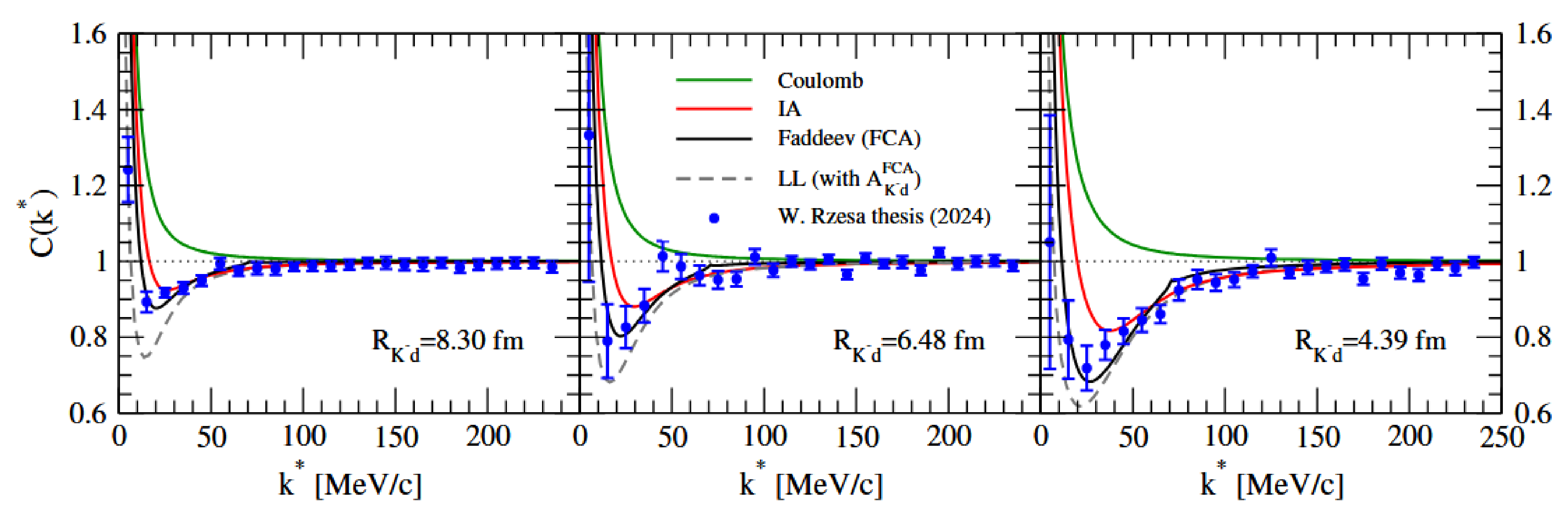
$$\gamma = Z_1 Z_2 \mu\alpha/k$$

$$\Phi^{\mathrm{C}}_{0f}(qr) = (qr)^{-1}e^{i\sigma_0}F_0(\gamma;qr)$$

$$\sigma_l = \arg\Gamma(1+l+i\gamma)$$

C.J. Joachain, *Quantum Collision Theory* (1975)

ALICE results in PbPb collisions



$$|\psi(r^*, k^*)| = \sqrt{A_C(\eta)}\left[\exp(-ik^*r^*)F(-i\eta, 1, i\xi) + f_c(k^*)\frac{G}{r^*}\right]$$

$$S(\vec{r}) \sim \exp\left(-\frac{r^2}{2R^2_{Kd}}\right)$$

Asymptotic part of the wave function

W. Rzesa (ALICE Coll.) *PhD Thesis*

Effect of charge-exchange channel

A. Feijoo, À. Ramos, JMT-R, in preparation

	FCA-UNIT (OR model)	FCA-UNIT (BCN model)
Unitary Factor	1.00/1.00	1.00/1.00
Scattering Length [fm]	$-1.04/-1.12 + i0.90/0.84$	$-0.85/-0.93 + i1.07/1.00$

Inelastic channel removal $t_x = 0$

ALICE

2602.22833 [nucl-ex]

$$\mathrm{Re}\ A_{K^-d} = -1.44 \pm 0.15\ (\mathrm{stat})\ ^{+0.10}_{-0.10}\ (\mathrm{syst})\ \mathrm{fm}$$

$$\mathrm{Im}\ A_{K^-d} = +1.34 \pm 0.33\ (\mathrm{stat})\ ^{+0.21}_{-0.15}\ (\mathrm{syst})\ \mathrm{fm}$$

	FCA-UNIT (OR model)	FCA-UNIT (BCN model)
ε^d_{1s} [eV]	583/607	525/547
Γ^d_{1s} [eV]	767/701	958/876

Inelastic channel removal $t_x = 0$

INFN LNF

2608.10749 [nucl-ex]

$$\varepsilon^d_{1s} = 810.9 \pm 24.5\ (\mathrm{stat}) \pm 2.1\ (\mathrm{syst})\ \mathrm{eV}$$

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