

Dynamical gluon mass generation, the Schwinger mechanism, spontaneous symmetry breaking, and color confinement in the linear covariant gauges of QCD

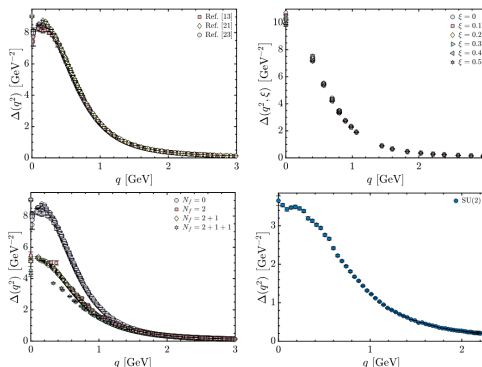
Giorgio Comitini

Liceo Scientifico “E. Majorana”, Scordia (CT) – Italy
Formerly: Department of Physics and Astronomy, University of Catania, Catania (CT) – Italy

NSTAR 2026
Seville, September 7-11, 2026

The evidence

- 30 years of **lattice data** by numerous independent collaborations¹: the **transverse gluon propagator** $\Delta(q^2)$ of **linear covariant gauges** (LCGs) **saturates** to a finite, nonzero value, $\Delta(0) = 1/M^2$, $M^2 \in]0, \infty[$.

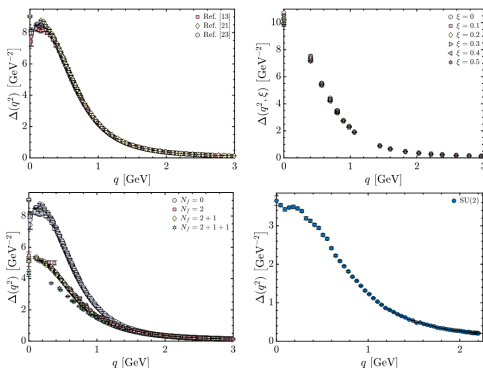


From Ferreira, Papavassiliou, Prog. Part. Nucl. Phys. 144 (2025) 104186

¹Everyone who looked into it.

The evidence

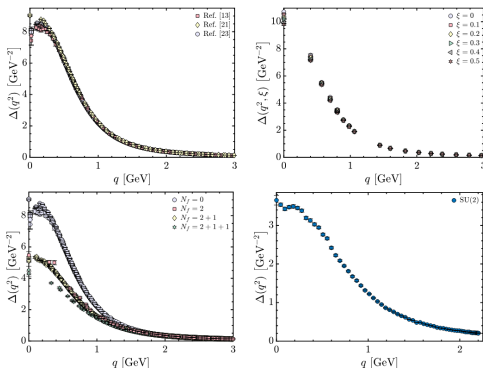
- **Dynamical gluon mass generation:** gluons are **massless** at the level of the **Lagrangian**, but they **acquire a mass** from **interactions**.



From Ferreira, Papavassiliou, Prog. Part. Nucl. Phys. 144 (2025) 104186

The evidence

- More precisely: the **transverse** gluon propagator has **no massless pole** $\implies$ **no massless transverse gluons** in either the physical spectrum (obvious) or the LCG unphysical spectrum (not obvious).
- **Massive poles? Currently we don't know².**



From Ferreira, Papavassiliou, Prog. Part. Nucl. Phys. 144 (2025) 104186

²Probably not.

The problem

- In 1978 T. Kugo and I. Ojima (**KO**) formulated the **covariant operator formalism** of Yang-Mills theory, based on **BRST symmetry**.
- In 1979³ they showed that, if **gluons dynamically acquire a mass**, the global gauge (color) charge Q^a is **BRST-exact**:

$$Q^a = g^{-1} \oint_{S_\infty} d^2 \mathbf{x}^i F_{0i}^a + \left\{ Q_B, g^{-1} \int d^3 \mathbf{x} D_0 \bar{c}^a \right\} \stackrel{(M \neq 0)}{=} \left\{ Q_B, g^{-1} \int d^3 \mathbf{x} D_0 \bar{c}^a \right\}$$

- Physical states $|\text{phys}\rangle$ are **annihilated** by the BRST charge, $Q_B |\text{phys}\rangle = 0$: if gluons acquire a **mass**, physical states are **colorless**
 $\Rightarrow$ **color confinement**

$$\langle \text{phys} | Q^a | \text{phys}' \rangle = \langle \text{phys} | \{ Q_B, \cdot \} | \text{phys}' \rangle = 0$$

- But there's a catch: this is **not automatically true** if Q^a is **spontaneously broken**.

³Kugo, Ojima, Prog. Theor. Phys. Suppl. 66 (1979) 1-130

The problem

- But there's a catch: this is **not automatically true** if Q^a is **spontaneously broken**.
- KO **proposed** that Q^a would **not** be **spontaneously broken** if a certain parameter $1 + u = 0$. In Landau gauge, functional studies indicated that $1 + u = 0$ is equivalent to $\Delta(0) = 0$ (the propagator **vanishes** at zero momentum instead of saturating).
- The condition $1 + u = 0$ would then be **sufficient** for color confinement $\Rightarrow$ **"KO criterion for confinement"**
- Otoh, if $1 + u \neq 0$, e.g., by functional studies, if $\Delta(0) > 0$, KO found that **either** BRST symmetry is broken, or the global gauge (color) charge Q^a is spontaneously broken.
- The lattice **clearly** shows that $\Delta(0) > 0$.
- This is an **open problem**.

The problem

- Many attempts at formulating a **theory of the gluon mass**.
- Only **one** has the following properties:
 - does not change the LCG QCD Lagrangian (same theory as KO)
 - preserves BRST symmetry, color symmetry and Lorentz symmetry
 - predicts $\Delta(0) > 0$ as observed on the lattice
 - explains lattice data such as the Ward-Identity displacements
- That's the **Schwinger mechanism** of Aguilar, Binosi, Ferreira and Papavassiliou (**ABFP**)⁴:
 - based on Schwinger (1962), Cornwall (1982), *et al.*
 - *ansatz*: a **massless, colored** (adjoint), **scalar, longitudinally coupled, unphysical bound state** is formed in the channel of the gluon field A_μ^a
 - the bound state produces **massless poles** in the **elementary dressed vertices of QCD**
 - the vertex poles get **transferred** to the gluon **polarization** $\implies$
 $\implies$ **gluon mass** by the **Schwinger mechanism**

⁴Ferreira, Papavassiliou, Prog. Part. Nucl. Phys. 144 (2025) 104186.

The problem

- Many attempts at formulating a **theory of the gluon mass**.
- Only **one** has the following properties:
 - does not change the LCG QCD Lagrangian (**same theory as KO**)
 - **preserves BRST symmetry, color symmetry** and Lorentz symmetry
 - **predicts $\Delta(0) > 0$** as observed on the lattice
 - explains lattice data such as the Ward-Identity displacements
- That's the **Schwinger mechanism** of Aguilar, Binosi, Ferreira and Papavassiliou (**ABFP**)⁵:

Wait... Didn't you say KO showed $1 + u \neq 0$ ($\Delta(0) > 0$) $\implies$ either one of BRST symmetry or global gauge (color) symmetry is broken?

This is one of the topics of today's talk

⁵Ferreira, Papavassiliou, Prog. Part. Nucl. Phys. 144 (2025) 104186.

New results

- The **ABFP theory** can be *derived in its entirety* (no ansatz!) within the **KO framework**: there, under customary assumptions, it is the **unique** mechanism of gluon mass generation $\Rightarrow$ **KO/ABFP correspondence**
- In the KO framework the **global gauge charge** Q^a is *always spontaneously broken* $\Rightarrow$ **$\Delta(0) = 0$ breaks the global gauge charge just like $\Delta(0) > 0$**
- The **ABFP bound state** is the unphysical Goldstone boson associated with the **spontaneous breaking** of the **global gauge charge** Q^a in LCGs $\Rightarrow$ **a Goldstone boson in the unphysical sector of QCD**
- Yet, *both* **BRST symmetry and color symmetry** (spectrum) are **unbroken** $\Rightarrow$ **using amended KO arguments: no colored asymptotic states in the physical spectrum of QCD (color confinement)**
- **Therefore**: within the KO framework of the LCGs of QCD, the **confinement mechanism** is the **ABFP/Schwinger mechanism**.

The KO/ABFP correspondence

The Kugo-Ojima framework: operatorial BRST quantization of QCD + asymptotic completeness

1) **Define** QCD in linear covariant gauges (FP Lagrangian):

$$\mathcal{L} = -\frac{1}{4} F^a F^a + \frac{\xi}{2} B^a B^a + B^a \partial A^a - i \partial \bar{c}^a D[A] c^a$$

2) Quantize it **canonically**:

$$[A_\mu^a(\mathbf{x}, t), \Pi^{b\nu}(\mathbf{y}, t)] = i \delta_\mu^\nu \delta^{ab} \delta^{(3)}(\mathbf{x} - \mathbf{y}) , \quad \Pi^{a\mu} = F^{a\mu 0} + \eta^{\mu 0} B^a$$

3) Compute its **Noether currents** and **charges**:

$$J_\mu^a = f^{abc} (A^{b\nu} F_{\nu\mu}^c + A_\mu^b B^c + i \partial_\mu \bar{c}^b c^c - i \bar{c}^b D_\mu c^c) , \quad Q^a = \int d^3\mathbf{x} J_0^a$$

$$J_{B\mu} = B^a D_\mu c^a - \partial_\mu B^a c^a + \frac{ig}{2} f^{abc} \partial_\mu \bar{c}^a c^b c^c , \quad Q_B = \int d^3\mathbf{x} J_{B0}$$

The Kugo-Ojima framework: operatorial BRST quantization of QCD + asymptotic completeness

4) **Physical states** are elements of the **BRST cohomology** $\mathcal{H}_{\text{phys}}$,

$$\mathcal{H}_{\text{phys}} = \ker\{Q_B\}/\text{im}\{Q_B\}$$

In the Hilbert space, they are represented by states $|\text{phys}\rangle$ such that

$$Q_B |\text{phys}\rangle = 0, \quad \text{e.g.} \quad Q_B |\text{proton}\rangle = 0, \quad Q_B |\text{antighost}\rangle \neq 0$$

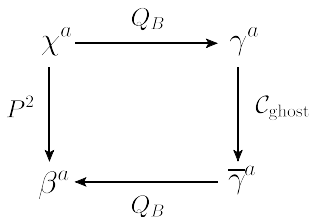
5) **Asymptotic completeness:** the Hilbert space is spanned by scattering (in/out) states:

$$\mathcal{H} = \mathcal{H}_{\text{in}} = \mathcal{H}_{\text{out}}$$

The KO/ABFP correspondence

The Kugo-Ojima framework: operatorial BRST quantization of QCD + asymptotic completeness

- 6) Postulate the non-existence of *certain* massless colored bound states
⇒ get the so-called **elementary BRST quartet**: a quartet of **stable massless colored scalar unphysical particles** created/annihilated by the elementary fields, $A_\mu^a \propto \partial_\mu \chi^a$, B^a , c^a , $\bar{c}^a$



$\gamma^a, \bar{\gamma}^a$: ghost and antighost ($c^a, \bar{c}^a$)

β^a : scalar polarization of the gauge boson ($B^a \propto \partial^\mu A_\mu^a$)

χ^a : “**first parent**”, theory-dependent

- QED: photon’s longitudinal polarization
- Stueckelberg model: Stueckelberg field
- Higgs models: unphysical components of the Higgs field (the unphysical Goldstone bosons)

$$\partial^2 \chi^a + \xi_R M_R^2 \beta^a = 0, \quad \partial^2 \beta^a = 0$$
$$\partial^2 \gamma^a = 0, \quad \partial^2 \bar{\gamma}^a = 0$$

Q: In the KO framework, when a gluon mass is dynamically generated, who is the first parent of the elementary BRST quartet of QCD?

A: It's the ABFP bound state⁶.

⁶New result.

The KO/ABFP correspondence

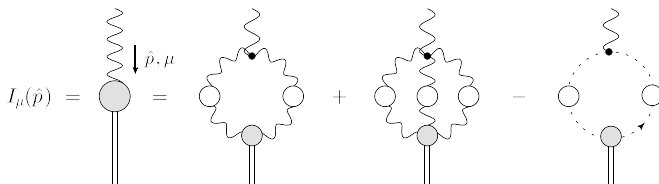
To see this: 1) assume gluons dynamically acquire a mass, and compute the matrix element of the Yang-Mills equations between the vacuum $|\Omega\rangle$ and the one-particle state $|\chi^a(p)\rangle$ of the first parent,

$$0 = \langle \Omega | \partial^\nu F_{\nu\mu}^a + g f^{abc} A^{b\nu} F_{\nu\mu}^c + i g f^{abc} \partial_\mu \bar{c}^b c^c - \partial_\mu B^a | \chi^d(p) \rangle \iff$$

$$\iff g I_\mu(-p) \delta^{ad} e^{-ipx} = i g f^{abc} \langle \Omega | \partial^\nu (A_\nu^b A_\mu^c) + A^{b\nu} F_{\nu\mu}^c + i \partial_\mu \bar{c}^b c^c | \chi^d(p) \rangle$$

$$g I_\mu(p) := Z_3^{-1/2} M_R p_\mu$$

2) write the above equation in diagrammatic form:



The KO/ABFP correspondence

2) write the above equation in diagrammatic form (not 1PI):

$$I_\mu(\hat{p}) = \text{diagram} = \text{diagram} + \text{diagram} - \text{diagram}$$

3) open Ibañez, Papavassiliou, Phys.Rev.D 87 (2013), 034008, p.7 (1PI):

$$\alpha = \text{diagram} = \text{diagram} + \text{diagram} + \text{diagram} + \text{diagram}$$

Gluon-scalar transition amplitude: fundamental ingredient of dynamical gluon mass generation via the ABFP Schwinger mechanism, $I_\mu \propto M_R p_\mu$

4) study the one-particle spectrum of the dressed elementary QCD vertices (a couple of pages of ugly but straightforward calculations):

the three vector fields $\{X_1, X_2, X_3\}$ are linearly independent on $\mathcal{M}$ as

$$\Gamma(X_1, X_2, X_3) = \frac{1}{8} \sqrt{\frac{2}{3}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \quad (B2)$$

near starting point, we observe that the three vector fields $\{X_1, X_2, X_3\}$ do not commute. In fact,

$$[X_1, X_2] = -3E \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \quad (B3)$$

in which the phase field $E(\rho)$ acts on the torus $\mathbb{R}^2/\mathbb{Z}^2$ from the left to the right. The corresponding generator E of the left action is not in the Lie algebra of the three vector fields $\{X_1, X_2, X_3\}$ as

$$[E, X_1] = -\frac{1}{2} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \quad (B4)$$

the discrete translation associated with the action:

$$\begin{aligned} Q_{\frac{1}{2}}^2 \psi_1 &= -3E \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \psi_1 \\ &= -3E \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \psi_1 \\ &= -3E \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \frac{1}{\sqrt{1 - \frac{1}{3} \frac{1}{\rho^2}}} \psi_1 \\ &= \dots \end{aligned}$$

where the dots also include multiple vector fields between the vector fields in $\{X_1, X_2, X_3\}$. The discrete translation $Q_{\frac{1}{2}}^2$ is not in the Lie algebra of the three vector fields $\{X_1, X_2, X_3\}$, and thus all vector fields in the Lie algebra have nontrivial commutation relations with $Q_{\frac{1}{2}}^2$. The Lie algebra of the three vector fields associated with the $\mathbb{H}_2$ is going on the torus $\mathbb{R}^2/\mathbb{Z}^2$ from the left to the right. In fact, the Lie algebra is non-abelian.

We see that the matrix element $\langle 0 | A A | \chi \rangle$ that defines the FSA of χ^* in the two-photon channel – see Eq. (56) –

(ii) $\langle \hat{C}_n^{\dagger}(\vec{x})\hat{C}_m(\vec{x}')|\Omega\rangle^2$ contains the amplitude $\langle \Pi_0|\hat{C}_n^{\dagger}(\vec{x})\hat{C}_m(\vec{x}')|\Omega\rangle^2$ which, by the same analysis of Sec. IV A, has a vanishing core with the three-photon state as first-antighost amplitudes. It can be shown [16] that $\langle \Pi_0|\hat{C}_n^{\dagger}(\vec{x})\hat{C}_m(\vec{x}')|\Omega\rangle = 0$, or the RHS of Eq. (9), where $|\gamma^n\rangle$ is replaced by $|\Pi_0\rangle$. In particular, even if $\langle \Pi_0|\hat{C}_n^{\dagger}(\vec{x})\hat{C}_m(\vec{x}')|\Omega\rangle$ may not vanish on its own, the second line in Eq. (8) does not contribute to gluon mass generation when summed to the three-gluon and ghost-antighost amplitudes. For this reason, we will ignore it.

$$\begin{aligned} \langle \mathcal{F}^k g | (\partial_y)^k \mathcal{F}^k(x) \rangle &= \langle \mathcal{F}^k g | (\partial_y)^k \mathcal{F}^k(x) \rangle = \quad (33) \\ &= -\partial_y^k e^{i y} g^k \end{aligned}$$

Now we Fourier-transform $G_{\text{mean}}^{\text{de}}(x, y, z)$ with respect to the variable x :

$$G_{\text{eff}}^{\text{eff}}(p, k, i) = \int d^4x e^{i p x} G_{\text{eff}}^{\text{eff}}(x, y, i) = -Z_1^{(1)} M_E^{-1} \frac{\not{p}}{p^2 + m} \frac{\not{k}}{2p} x$$

$$\times \left\{ \langle \text{Tr} A_1^i(x) A_2^i(x) \rangle \langle \text{Tr} A_3^i(x) A_4^i(x) \rangle - 2 \langle \text{Tr} A_1^i(x) A_2^i(x) A_3^i(x) A_4^i(x) \rangle + \langle \text{Tr} A_1^i(x) A_2^i(x) \rangle \langle \text{Tr} A_3^i(x) A_4^i(x) \rangle - \langle \text{Tr} A_1^i(x) A_3^i(x) \rangle \langle \text{Tr} A_2^i(x) A_4^i(x) \rangle \right\} + \dots$$

In the limit $\kappa^2 \rightarrow \kappa_0^2$, which involves $\kappa^2 \rightarrow 0$, we see that the terms omitted in the previous equation disappear,

the discrete contribution associated with this action:

$$\begin{aligned}
G_{\text{even}}^{\text{odd}}(x, y, z) = & \quad (E4) \\
= & \|\mathcal{A}_1^{\text{odd}}(x)\mathcal{A}_1^{\text{odd}}(y)\mathcal{P}_{\text{odd}}\mathcal{A}_1^{\text{odd}}(z)\|\mathcal{R}\|\mathcal{A}_1^{\text{odd}}\mathcal{A}_1^{\text{odd}} - \mathcal{A}_1^{\text{odd}}\mathcal{A}_1^{\text{odd}} - \mathcal{A}_1^{\text{odd}}\mathcal{A}_1^{\text{odd}} + \\
& \|\mathcal{A}_1^{\text{odd}}(x)\mathcal{A}_1^{\text{odd}}(y)\mathcal{P}_{\text{odd}}\mathcal{A}_1^{\text{odd}}\|\mathcal{R}\|\mathcal{A}_1^{\text{odd}}\mathcal{A}_1^{\text{odd}} - \mathcal{A}_1^{\text{odd}}\mathcal{A}_1^{\text{odd}} - \mathcal{A}_1^{\text{odd}}\mathcal{A}_1^{\text{odd}} + \\
& \|\mathcal{A}_1^{\text{odd}}(x)\mathcal{P}_{\text{odd}}\mathcal{A}_1^{\text{odd}}(z)\mathcal{A}_1^{\text{odd}}(y)\mathcal{R}\|\mathcal{A}_1^{\text{odd}}\mathcal{A}_1^{\text{odd}} - \mathcal{A}_1^{\text{odd}}\mathcal{A}_1^{\text{odd}} - \mathcal{A}_1^{\text{odd}}\mathcal{A}_1^{\text{odd}} + \\
& \|\mathcal{A}_1^{\text{odd}}(x)\mathcal{P}_{\text{odd}}\mathcal{A}_1^{\text{odd}}(z)\mathcal{A}_1^{\text{odd}}(y)\mathcal{R}\|\mathcal{A}_1^{\text{odd}}\mathcal{A}_1^{\text{odd}} - \mathcal{A}_1^{\text{odd}}\mathcal{A}_1^{\text{odd}} - \mathcal{A}_1^{\text{odd}}\mathcal{A}_1^{\text{odd}} +
\end{aligned}$$

where now the dots also include multiparticle contributions to the matrix elements in Eq. (14). By our assumption of asymptotic completeness, since $J_0^+ = Z_0^{1/2} M_0^+ Z_0^{-1/2}$, and since all asymptotic fields have χ^+ and β^+ have vanishing commutation relations with χ^+ , the four terms in Eq. (14) saturate the discrete spectrum associated with the log χ^+ going on the massless

Let us focus on the first two terms in Eq. (E4). The matrix elements in them read

$$\begin{aligned}
& \lim_{y \rightarrow 0} \frac{1}{y} \Gamma_{\text{eff}}^{\text{gluon}}(p, q, y) = \\
& = \lim_{y \rightarrow 0} -\frac{\alpha_s^2}{y} M_2^{\text{gluon}} \left[\frac{\Delta^2(p)^2}{p^2 + \Delta^2(p)} - \frac{1}{p^2} \Gamma_{\text{eff}}^{\text{gluon}}(p, q, y) \right],
\end{aligned}
\quad (23)$$

where $\mathcal{B}_{\mu\nu}^{(R)}(q, k; \beta)$ is the BSV defined in Eq. (51). We now compute the limit $n^2 \rightarrow n^0$ of the construct

$$\Delta_{\mu\nu}^{(0)}(p)/V. \text{ Since the naive inverse propagator}$$

is singular as $p^2 \rightarrow 0$ due to the non-vanishing $\Delta^{-1}(p^2)$, before computing the limit we need to regularize its divergence. For this purpose, we try the simplest regularizations, and we check that they yield

First we attempt a naive regularization, in which we define the inverse propagator as

$$\Delta_{\text{ren}}^{-1}(p) = \Delta^{-1}(p^2) \left(\epsilon_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 + i\epsilon} \right) + \mathcal{O}^{-1} p_\mu p_\nu. \quad (\text{B22})$$

Then, using

$$\lim_{\lambda \rightarrow 0} \lambda^2 \hat{A}_\lambda = 0, \quad \lim_{\lambda \rightarrow 0} \frac{\partial^2 \hat{A}_\lambda}{\partial \lambda^2} = 0, \quad (E_2)$$

$$\frac{\partial^2 \mathcal{L}}{\partial \theta^2} = \frac{\partial^2 \mathcal{L}}{\partial \theta^2} - \frac{\partial^2 \mathcal{L}}{\partial \theta^2} = 0 \quad (10)$$

the latter of which holds for every finite s , we find that

$$\lim_{s \rightarrow \infty} \Delta_s^{-1}(g)g^s = \Delta^{-1}(g)g. \quad (E)$$

Second, we observe that the δ 's in the gluon propagator

$$\Delta_{\text{eff}}(p) = \Delta(p^2) \left(v_m - \frac{p_0 p_0}{p^2 + \alpha} \right) + \frac{-2\Delta_0 p_0}{(p^2 + \alpha)^2} \quad (\text{E2})$$

exploit this property, however, we need to carry out the inversion carefully: first we rewrite $\Delta_{\mu\nu}(y)$ as

$$\Delta_{\text{ext}}(y) = \Delta(y^2) \left(\alpha_{\text{ext}} - \frac{\partial \Delta(y)}{\partial y} \right) + \quad (E2)$$

$$+ \left| \frac{-\partial^2 \bar{f}}{(p^2 + m^2)^2} + \frac{\partial^2 \Delta(\bar{f})}{p^2 + m^2} \right| \frac{\partial_0 \partial_0}{p^2},$$

the absence of δ is trivial, then we exploit these relations to write

$$\Delta_{\mu}^{-1}y = \Delta^{-1}(y^2) \left(e - \frac{\partial y^2}{y^2} \right) + \quad (8)$$

$$+ \left[\frac{-4\epsilon p^2}{(p^2 + u)^2} + \frac{\alpha_2 M(p^2)}{p^2 + u} \right]^{-1} \frac{p_+ p_-}{p^2}.$$

Using the limits

$$\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)} = \frac{1}{2}, \quad (1)$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{\sqrt{2}} I$$

$$\lim_{p^2 \rightarrow p_0^2} \frac{p^2}{[p^2 + i\epsilon]^2} = 0, \quad (3)$$

$$\lim_{\rho \rightarrow \rho^*} \Delta_{\rho\rho}^{-1}(y) \rho^2 = \Delta^{-1}(0) \left(\frac{1}{2} \dot{\rho}_\rho + \Delta^{-1}(0) \frac{1}{2} \dot{\rho}_\rho \right), \quad (3)$$

which is the same result as Eq. (E14). The regularized inverse of Eq. (E11), on the other hand, we

With the construction evaluated, we can now carry the last calculations to obtain

$$\lim_{p' \rightarrow p} \Gamma_{\text{pole}}^{\text{str}}(p, q, k) = \Delta^{-1}(p) \Delta^{-1}(q) \Delta^{-1}(k) \quad (8)$$

$$= \lim_{\rho \rightarrow \rho^*} -Z_0^{(1/2)} \rho^{-1} M_0^{-1} \frac{\partial}{\partial \rho} \frac{(1/\rho) \tilde{\rho}_0}{\rho^2 + \tilde{\rho}_0} \mathfrak{B}_{\text{ex}}^{\text{ex}}(\rho, \tilde{\rho}; f)$$

$$= \lim_{\epsilon \rightarrow 0^+} L_\epsilon(f) \frac{i}{k^2 + i\epsilon} \mathfrak{H}_{\text{res}}^{\text{Im}}(q, k; f),$$

where on the last line we have used Eq. (68). The result announced in the first of Eq. (63), valid in

limit $p^0 \rightarrow p^z$. The limit $y^0 \rightarrow -y^z$ is instead probed by the third and fourth terms in Eq. (E4), which not contribute to Eq. (E21). In the resulting expres-

the momentum flow of the IBV and of the gluon- γ transition amplitude $J_\mu(y)$ – see Figs. 1 and 2 – inverted.

The procedure we employed to derive the $p^2 \rightarrow 0$ limit of the three-gluon vertex can be applied identically to

mesons, the ghost-antighost-gluon and $\bar{\psi}\psi$ in the presence of quarks – the quark-antiquark-gluon vertices, two-mesons of the BBs in these channels, loop

with the inclusion of $\Delta^{(n)}(0)$, produce vertex Schwinger poles as in last line of Eq. (E2), with $\mathcal{B}_{\mu\nu}^{(n)}$ replacing the appropriate ISVs.

in the case of the low-gain vertex, one obtains extra factor of g^{-1} due to a mismatch between the two definitions of the vertex, which is multiplied

g^{-1} , and of the gluon-sea transition amplitude, ω is multiplied by g^{-1} . Moreover, the vertex 1. is

χ^a produces massless longitudinal poles in the QCD vertices:
unique condition for the ABFP Schwinger mechanism to be active

5) with DMG, the 1st parent of the elementary BRST quartet of QCD χ^a is:

- a **massless, scalar, colored** (adjoint), **longitudinally coupled unphysical** particle in the same **composite channels** and with the same **Bethe-Salpeter amplitudes** as the ABFP bound state
- obeying the **same dynamics** as the ABFP bound state
- producing the **same Schwinger poles** as the ABFP bound state
- triggering the Schwinger mechanism⁷ like the ABFP bound state

In other words, **it is the ABFP bound state.**

KO/ABFP correspondence:

Gluon mass $\xrightleftharpoons[\text{ABFP}]{\text{KO}}$ **Massless scalar bound state**

⁷This can be shown entirely within the KO framework, using DSEs as usual. 

Spontaneous breaking of the global gauge charge

- Once the KO/ABFP correspondence is established, we **really** want to understand what's going on with spontaneous symmetry breaking.
- **KO** say the global gauge charge Q^a is **spontaneously broken**, **ABFP** say color is **not broken**.
- Keep in mind: no symmetry breaking $\implies$ **color confinement**.
- So we compute the vacuum **commutator** between Q^a and the asymptotic (in/out) field χ^a that creates/annihilates the ABFP bound state:

$$\langle \Omega | [Q^a, \chi^b(x)] | \Omega \rangle = ig^{-1} M(1+u) \delta^{ab}$$

- $\text{RHS} \neq 0$ implies $Q^a | \Omega \rangle \neq 0 \implies$ **SSB**
- This is where the parameter $1+u$ of the **KO criterion** originates from: if $1+u=0$ – KO say – there's no SSB and color is confined.

Spontaneous breaking of the global gauge charge

$$\langle \Omega | [Q^a, \chi^b(x)] | \Omega \rangle = ig^{-1} M(1+u) \delta^{ab}$$

- But there's a problem: in the equation, $1+u$ appears in the **combination** $M(1+u)$, where $M = \Delta^{-1/2}(0)$.
- In their papers, when discussing SSB and confinement, KO **never** multiply the asymptotic field $\chi^a(x)$ by its LSZ coefficient:

$$A_\mu^a \rightarrow Z_\chi^{1/2} \partial_\mu \chi^a \quad (t \rightarrow \pm\infty)$$

- As a result, **the original $1+u=0$ KO equation is missing a factor of $Z_\chi^{-1/2}$ on the RHS.**
- It turns out that...

$$Z_\chi^{-1/2} = M$$

Spontaneous breaking of the global gauge charge

$$\langle \Omega | [Q^a, \chi^b(x)] | \Omega \rangle = ig^{-1} M(1+u) \delta^{ab}$$

- If $\Delta(0) = 0$, $M = \infty$ and $M(1+u)$ is finite even if $1+u = 0$!
- $M(1+u) = \hat{M}$ is the $p^2 \rightarrow 0$ limit of the **gluon propagator** evaluated in the Pinch-Technique–Background-Field-Method (**PT-BFM**) scheme⁸, and it is **not equal to zero** for massive gluons.

In the LCGs, the global gauge charge is *always* spontaneously broken, even when $1+u = 0$ ($\Delta(0) = 0$)

⁸Double-checked by evaluating explicitly the LHS, see next slide.

Spontaneous breaking of the global gauge charge

- Moreover, the ABFP bound state is a **Goldstone boson**!

$$\langle \Omega | [Q^a, \chi^b(x)] | \Omega \rangle = i g^{-1} \hat{M} \delta^{ab} \neq 0$$

- In terms of the **color current**,

$$\langle \Omega | J_\mu^a(x) | \chi^b(p) \rangle = -i \tilde{I}_\mu(-p) \delta^{ab} e^{-ipx} = i \hat{M} / g p_\mu \delta^{ab} e^{-ipx}$$

$\tilde{I}_\mu(p)$ is gluon-scalar transition amplitude evaluated in the **PT-BFM**.

The ABFP bound state is the Goldstone boson associated with the SB of the global gauge (color) charge

Spontaneous breaking of the global gauge charge

- But this makes **no sense**.
 - The lattice LCG Green functions are color-symmetric.
 - The ABFP Schwinger mechanism (and Green functions) are color-symmetric by design, and they reproduce the lattice data.
- Green functions are **vacuum correlators**. If *all* vacuum correlators respect color symmetry, color symmetry **cannot** be spontaneously broken. Yet Q^a **is** broken.
- Clearly we are **missing something**.
- What is Q^a **doing** to break symmetry?

Spontaneous breaking of the global gauge charge

- What is Q^a **doing** to break symmetry? Let's find out.
- Standard assumptions $\implies$ the (broken) transformations of the elementary BRST quartet are

$$[Q^a, \chi^b(x)] = \underline{ig^{-1}\hat{M}\delta^{ab}} + if^{abc}\chi^c(x), \quad [Q^a, \beta^b(x)] = if^{abc}\beta^c(x), \\ [Q^a, \gamma^b(x)] = if^{abc}\gamma^c(x), \quad [Q^a, \bar{\gamma}^b(x)] = if^{abc}\bar{\gamma}^c(x)$$

The constant $\propto \delta^{ab}$ in $[Q^a, \chi^b(x)]$ is the symmetry-breaking term.

- In terms of the asymptotic fields, the generator Q^a reads

$$Q^a = g^{-1}\hat{M} \int d^3\mathbf{x} \dot{\beta}^a + f^{abc} \int d^3\mathbf{x} \left(\dot{\chi}^b \beta^c - \chi^b \dot{\beta}^c - \dot{\beta}^b \beta^c + i \dot{\bar{\gamma}}^b \gamma^c - i \bar{\gamma}^b \dot{\gamma}^c \right) + \dots$$

The ellipses contain contributions due to other asymptotic fields. Those **do not** yield other symmetry-breaking terms. The **first term** $\propto \dot{\beta}^a$ is the one breaking the symmetry.

Spontaneous breaking of the global gauge charge

$$Q^a = g^{-1} \hat{M} \int d^3\mathbf{x} \dot{\beta}^a + f^{abc} \int d^3\mathbf{x} \left(\dot{\chi}^b \beta^c - \chi^b \dot{\beta}^c - \dot{\beta}^b \beta^c + i \dot{\bar{\gamma}}^b \gamma^c - i \bar{\gamma}^b \dot{\gamma}^c \right) + \dots$$

- Q^a is the ill-defined, constant-parameter limit ($\Lambda^a \rightarrow \lambda^a$) of a well-defined operator $\mathcal{Q}[\Lambda, \lambda]$

$$\begin{aligned} \mathcal{Q}[\Lambda, \lambda] = & g^{-1} \hat{M} \int d^3\mathbf{x} \left(\Lambda^a \dot{\beta}^a - \dot{\Lambda}^a \beta^a \right) + \\ & + \lambda^a f^{abc} \int d^3\mathbf{x} \left(\dot{\chi}^b \beta^c - \chi^b \dot{\beta}^c - \dot{\beta}^b \beta^c + i \dot{\bar{\gamma}}^b \gamma^c - i \bar{\gamma}^b \dot{\gamma}^c \right) + \lambda^a (\dots) \end{aligned}$$

where the $\Lambda^a(x)$'s are functions such that $\partial^2 \Lambda^a = 0$ and the λ^a 's are constants

- $\mathcal{Q}[\Lambda, \lambda]$ generates the following transformations of the **asymptotic gluon field** $A_\mu^{a \text{ as.}}$ ($g_R^{-1} = Z_3^{-1/2} g^{-1} (1 + u)$):

$$[\mathcal{Q}[\Lambda, \lambda], A_\mu^{a \text{ as.}}] = i \left(g_R^{-1} \partial_\mu \Lambda^a + f^{abc} A_\mu^{b \text{ as.}} \lambda^c \right) \quad (\partial^2 \Lambda^a = 0)$$

Spontaneous breaking of the global gauge charge

$$[\mathcal{Q}[\Lambda, \lambda], A_\mu^{a\text{as.}}] = i \left(\mathbf{g}_R^{-1} \partial_\mu \Lambda^a + f^{abc} A_\mu^{b\text{as.}} \lambda^c \right) \quad (\partial^2 \Lambda^a = 0)$$

$$0 = \langle \Omega | A_\mu^{a\text{as.}}(x) | \Omega \rangle \rightarrow \langle \Omega | A_\mu^{a\text{as.}}(x) + \delta A_\mu^{a\text{as.}}(x) | \Omega \rangle = g_R^{-1} \partial_\mu \Lambda^a(x) \neq 0$$

- Those are **not** mere color transformations: they are **asymptotic residual gauge transformations!** (Symmetries of the asymptotic field equations)
- χ^a enters $A_\mu^{a\text{as.}} = \mathbf{M}_R^{-1} \partial_\mu \chi^a + \dots$ as a derivative, and it carries a representation of the **derivative** part of the gauge transformation.
- In the constant limit in which $\mathcal{Q}^a$ is recovered from $\mathcal{Q}[\Lambda, \lambda]$, $\Lambda^a \rightarrow \lambda^a$, the derivative in the transformation of $A_\mu^{a\text{as.}}$ **survives** as a **symmetry-breaking constant** in the transformation of χ^a
- The constant is **not** realizing **broken color transformations**, but the **derivative term** of a ***gauge* transformation**.
- In linear covariant gauges, that transformation is **well-known** to be **always** spontaneously broken, **even in QED!**⁹

⁹See e.g. Nakanishi, Ojima, *Covariant operator formalism of gauge theories and quantum gravity*, World Scientific (1990), p.68

Spontaneous breaking of the global gauge charge

$$[\mathcal{Q}[\Lambda, \lambda], A_\mu^{a \text{ as.}}] = i \left(\mathbf{g}_R^{-1} \partial_\mu \Lambda^a + f^{abc} A_\mu^{b \text{ as.}} \lambda^c \right) \quad (\partial^2 \Lambda^a = 0)$$

- Therefore Q^a is **not** the generator of color transformations. To obtain the generator of **color transformations** $\tilde{Q}^a$, we need to **subtract** the generator of the derivative from Q^a :

$$\tilde{Q}^a = Q^a - \mathbf{g}^{-1} \hat{M} \int d^3\mathbf{x} \dot{\beta}^a$$

- $\tilde{Q}^a$ is an **unbroken** ($\tilde{Q}^a |\Omega\rangle = 0$), **well-defined** color charge operator, generating the asymptotic transformations

$$\begin{aligned} [\tilde{Q}^a, \chi^b] &= i f^{abc} \chi^c, & [\tilde{Q}^a, \beta^b] &= i f^{abc} \beta^c, & [\tilde{Q}^a, \gamma^b] &= i f^{abc} \gamma^c, & [\tilde{Q}^a, \bar{\gamma}^b] &= i f^{abc} \bar{\gamma}^c, \\ [\tilde{Q}^a, X] &= [Q^a, X], & [\tilde{Q}^a, \tilde{Q}^b] &= i f^{abc} \tilde{Q}^c \end{aligned}$$

where X is any other asymptotic field.

- So **both** KO and ABFP are right. The global gauge charge Q^a is spontaneously broken, **and** the color symmetry of the spectrum (in/out asymptotic states) is not violated.

$$\tilde{Q}^a = Q^a - g^{-1} \hat{M} \int d^3 \mathbf{x} \dot{\beta}^a$$

- Having identified the correct color charge operator, we can turn to **color confinement**. As we know, when a gluon mass is dynamically generated, we have

$$Q^a = \left\{ Q_B, g^{-1} \int d^3 \mathbf{x} D_0 \bar{c}^a \right\}$$

- Moreover, **the symmetry-breaking term is BRST exact**¹⁰,

$$g^{-1} \hat{M} \int d^3 \mathbf{x} \dot{\beta}^a = \left\{ Q_B, g^{-1} \tilde{Z}_3^{1/2} (1+u) \int d^3 \mathbf{x} \partial_0 \bar{\gamma}^a \right\}$$

- Therefore

$$\tilde{Q}^a = \left\{ Q_B, g^{-1} \int d^3 \mathbf{x} \left[D_0 \bar{c}^a - \tilde{Z}_3^{1/2} (1+u) \partial_0 \bar{\gamma}^a \right] \right\}$$

¹⁰ $1+u$ still cancels out from final expressions.

- The color charge operator $\tilde{Q}^a$ is an **unbroken** ($\tilde{Q}^a |\Omega\rangle = 0$), **well-defined**, **BRST-exact** operator whose action is known on in/out asymptotic states¹¹

$$\tilde{Q}^a = \left\{ Q_B, g^{-1} \int d^3\mathbf{x} \left[D_0 \bar{c}^a - \tilde{Z}_3^{1/2} (1+u) \partial_0 \bar{\gamma}^a \right] \right\}$$

- BRST-exact $\implies$ it **vanishes** on physical states

$$Q_B |\text{phys}\rangle = Q_B |\text{phys}'\rangle = 0 \quad \implies \quad \langle \text{phys}' | \tilde{Q}^a | \text{phys} \rangle = 0$$

$$\tilde{Q}^a \mathcal{H}_{\text{phys}} = 0$$

- The above equation expresses the **confinement of color** in QCD:
absence of colored asymptotic states from the physical spectrum.

¹¹Because it is constructed from asymptotic fields.

Full color symmetry: soft theorems (**preliminary**)

- The color charge operator $\tilde{Q}^a$ is built from **asymptotic fields**: it represents a **pair of asymptotic (in/out) charges**

$$(\tilde{Q}^a)^{\text{in/out}} = Q^a - g^{-1} \hat{M} \int d^3 \mathbf{x} (\dot{\beta}^a)^{\text{in/out}}, \quad S(\tilde{Q}^a)^{\text{out}} = (\tilde{Q}^a)^{\text{in}} S$$

- This is **sufficient** for the symmetry of the **spectrum** and for **color confinement**, not for **full color symmetry**. That requires color to be conserved in unphysical **scattering processes** (full S -matrix):

$$(\tilde{Q}^a)^{\text{in}} = (\tilde{Q}^a)^{\text{out}} = \tilde{Q}^a, \quad [\tilde{Q}^a, S] = 0$$

- More generally, the spontaneously broken symmetry induces **broken Ward identities** – equivalently, **soft theorems**

$$\sum_k T_{I_k J_k}^a \langle \Omega | T \left\{ \mathcal{O}_{J_k}(x_k) \prod_{j \neq k} \mathcal{O}_{I_j}(x_j) \right\} | \Omega \rangle = -i \hat{M} / g \lim_{p \rightarrow 0} \langle \beta^a(p); \text{out} | T \left\{ \prod_k \mathcal{O}_{I_k}(x_k) \right\} | \Omega \rangle$$

Full color symmetry: soft theorems (**preliminary**)

$$\sum_k T_{I_k J_k}^a \langle \Omega | T \left\{ \mathcal{O}_{J_k}(x_k) \prod_{j \neq k} \mathcal{O}_{I_j}(x_j) \right\} | \Omega \rangle = -i\hat{M}/g \lim_{p \rightarrow 0} \langle \beta^a(p); \text{out} | T \left\{ \prod_k \mathcal{O}_{I_k}(x_k) \right\} | \Omega \rangle$$

- To preserve color symmetry in full, LHS=0, hence

$$\lim_{p \rightarrow 0} \langle \beta^a(p); \text{out} | T \left\{ \prod_k \mathcal{O}_{I_k}(x_k) \right\} | \Omega \rangle = 0$$

- This is **trivially** verified in the BRST cohomology: if $[Q_B, \mathcal{O}_{I_k}(x)]_{\mp} = 0$,

$$\langle \beta^a(p); \text{out} | T \left\{ \prod_k \mathcal{O}_{I_k}(x_k) \right\} | \Omega \rangle \propto \langle \bar{\gamma}^a(p); \text{out} | \left[Q_B, T \left\{ \prod_k \mathcal{O}_{I_k}(x_k) \right\} \right]_{\mp} | \Omega \rangle = 0$$

$\implies$ BRST symmetry enforces **full color symmetry** ($\tilde{Q}^a = 0 \rightarrow 0$)
in the **physical sector**

Full color symmetry: soft theorems (**preliminary**)

$$\lim_{p \rightarrow 0} \langle \beta^a(p); \text{out} | T \left\{ \prod_{k=1}^n \mathcal{O}_{I_k}(x_k) \right\} | \Omega \rangle = 0$$

- It is **not** trivially verified in the **unphysical sector**
- For instance, the **VEV** of the **elementary gauge fields** is **protected**:

$$f^{abc} \langle A_\mu^c(x) \rangle = -\widehat{M}/g \lim_{p \rightarrow 0} \langle \beta^a(p); \text{out} | A_\mu^b(x) | \Omega \rangle = i(1+u)/g \lim_{p \rightarrow 0} (p_\mu e^{ipx} \delta^{ab}) = 0$$

$$f^{abc} \langle X^c(x) \rangle = -\widehat{M}/g \lim_{p \rightarrow 0} \langle \beta^a(p); \text{out} | X^b(x) | \Omega \rangle = 0 \quad \text{for} \quad X^a = B^a, c^a, \bar{c}^a$$

- But in the SU(2) Higgs-Kibble model (LCGs), the **Higgs field's VEV** can yield $\text{LHS}=\text{RHS} \neq 0$

$$-\frac{1}{2} \sigma^i \langle \Phi(x) \rangle = -i\widehat{M}/g \lim_{p \rightarrow 0} \langle \beta^i(p); \text{out} | \Phi(x) | \Omega \rangle \quad \Longleftrightarrow \quad \frac{v}{2} = Z_\pi^{1/2} \widehat{M}/g$$

(exact nonperturbative generalization of tree level $M = gv/2$)

Full color symmetry: soft theorems (**preliminary**)

$$\lim_{p \rightarrow 0} \langle \beta^a(p); \text{out} | T \left\{ \prod_{k=1}^n \mathcal{O}_{I_k}(x_k) \right\} | \Omega \rangle = 0$$

- Color symmetry is preserved **in full** if the NL field decouples as $p \rightarrow 0$: **Adler zeros** in the unphysical (colored) sector.
- **Similar conditions** in ABFP, to preserve BRST+color (same reason):

Now, combining Eqs. (5.10) and (5.22), we obtain for $q \rightarrow 0$ that

$$\lim_{q \rightarrow 0} V_a(r, p, q) = \lim_{q \rightarrow 0} 2a \frac{(q \cdot r)}{q^2} C(r^2) = 2|r|\hat{q}_a \cos \theta C(r^2), \quad (5.23)$$

which does not act as a pole. In fact, the r.h.s. of Eq. (5.23) is strictly finite. Nevertheless, $V_a(r, p, q)$ is irregular; its value at vanishing q is not unique, but instead depends on $\hat{q}$ and θ , which specify the path through which the $q \rightarrow 0$ limit is approached. As we will see in Section 5.5, this irregularity is enough to generate a gluon mass scale (see also the discussions in [141,162]). In fact, as we show there, the resulting gluon mass scale does *not* depend on the way that the limit $q \rightarrow 0$ is implemented.

Note that the fact that the V s have no poles in $q = 0$ may be viewed as a consequence of the central assumption that the gluon mass generation leaves the gauge symmetry intact. Indeed, as shown in Section 6.2 and Appendix E, the key conditions described by Eqs. (5.19) and (5.21) originate from the STIs of the theory. A related mechanism, but with the global color symmetry broken, has been discussed in [92], and leads to stronger vertex irregularities, with the corresponding V s acting as poles in $q = 0$.

- Needs investigation using ABFP+PT-BFM+KO.

Summary

In the **KO framework**, under **standard assumptions**, when a **gluon mass** is **dynamically generated**...

- The **global gauge (color) charge** Q^a is **spontaneously broken**. $\Delta(0)$, $1 + u$ play **no role** (missing LSZ coefficient in KO equation).
- The **ABFP Schwinger mechanism** is the **unique mechanism** of **dynamical gluon mass generation**.
- The **ABFP bound state** is Q^a 's **unphysical Goldstone boson**.
- Q^a does **not** generate mere **color transformations**, but rather (structurally broken) **asymptotic residual gauge transformations**.
- The **color symmetry** of the **spectrum** (in/out states) is **preserved**.
- The corresponding **asymptotic color charge** $\tilde{Q}^a$ is well-defined and **BRST-exact**, hence **confining**.
- **Full color symmetry** (S -matrix+Green's functions) protected by **Adler zeros/soft theorems**?

Thank you for your attention!