Dynamical gluon mass generation, the Schwinger mechanism, spontaneous symmetry breaking, and color confinement in the linear covariant gauges of QCD

Giorgio Comitini

Liceo Scientifico "E. Majorana", Scordia (CT) – Italy
Formerly: Department of Physics and Astronomy, University of Catania, Catania (CT) – Italy

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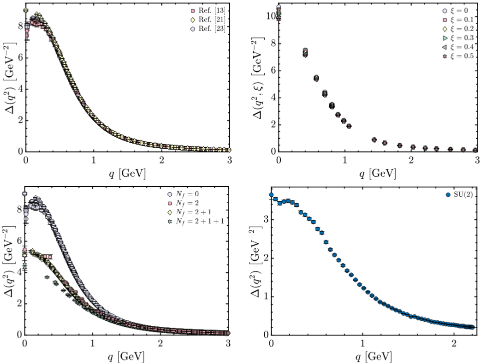
The evidence

- 30 years of **lattice data** by numerous independent collaborations[1]: the **transverse gluon propagator** $\Delta(q^2)$ of **linear covariant gauges** (LCGs) **saturates** to a finite, nonzero value, $\Delta(0) = 1/M^2$, $M^2 \in]0, \infty[$.



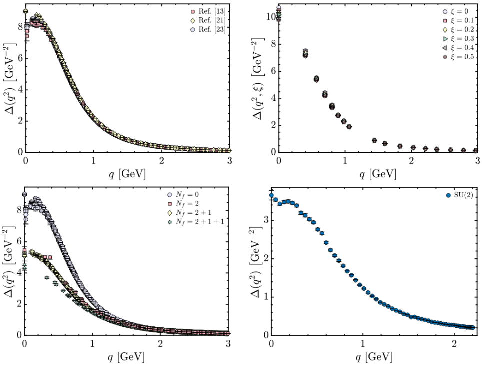
From Ferreira, Papavassiliou, Prog. Part. Nucl. Phys. 144 (2025) 104186

[1] Everyone who looked into it.

Introduction

The evidence

- **Dynamical gluon mass generation**: gluons are **massless** at the level of the **Lagrangian**, but they **acquire a mass** from **interactions**.



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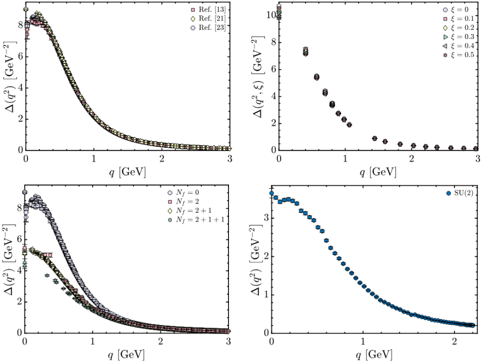
The evidence

- More precisely: the **transverse** gluon propagator has **no massless pole** $\Longrightarrow$ **no massless transverse gluons** in either the physical spectrum (obvious) or the LCG unphysical spectrum (not obvious).
- **Massive poles**? **Currently we don't know**[2].



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[2]Probably not.

The problem

- In 1978 T. Kugo and I. Ojima (**KO**) formulated the **covariant operator formalism** of Yang-Mills theory, based on **BRST symmetry**.

- In 1979[3] they showed that, if **gluons dynamically acquire a mass**, the global gauge (color) charge Q^a is **BRST-exact**:

$$Q^a = g^{-1} \oint_{S_\infty} d^2\mathbf{x}^i \, F^a_{0i} + \left\{ Q_B, g^{-1} \int d^3\mathbf{x} \, D_0 \overline{c}^a \right\} \overset{(M \neq 0)}{=} \left\{ Q_B, g^{-1} \int d^3\mathbf{x} \, D_0 \overline{c}^a \right\}$$

- Physical states $|\text{phys}\rangle$ are **annihilated** by the BRST charge, $Q_B \, |\text{phys}\rangle = 0$: if gluons acquire a **mass**, physical states are **colorless** $\Longrightarrow$ **color confinement**

$$\langle \text{phys} | \, Q^a \, | \text{phys}' \rangle = \langle \text{phys} | \, \{ Q_B, \cdot \} \, | \text{phys}' \rangle = 0$$

- But there's a catch: this is ***not* automatically true** if Q^a is **spontaneously broken**.

[3]Kugo, Ojima, Prog. Theor. Phys. Suppl. 66 (1979) 1-130

The problem

- But there's a catch: this is ***not* automatically true** if Q^a is **spontaneously broken**.
- KO **proposed** that Q^a would **not** be **spontaneously broken** if a certain parameter $\mathbf{1 + \boldsymbol{u} = 0}$. In Landau gauge, functional studies indicated that $1 + u = 0$ is equivalent to $\Delta(0) = 0$ (the propagator **vanishes** at zero momentum instead of saturating).
- The condition $1 + u = 0$ would then be **sufficient** for color confinement $\Longrightarrow$ "**KO criterion for confinement**"
- Otoh, if $1 + u \neq 0$, e.g., by functional studies, if $\Delta(0) > 0$, KO found that **either** BRST symmetry is broken, or the global gauge (color) charge Q^a is spontaneously broken.
- The lattice **clearly** shows that $\Delta(0) > 0$.
- This is an **open problem**.

The problem

- Many attempts at formulating a **theory of the gluon mass**.
- Only **one** has the following properties:
  - does not change the LCG QCD Lagrangian (same theory as KO)
  - preserves BRST symmetry, color symmetry and Lorentz symmetry
  - predicts $\Delta(0) > 0$ as observed on the lattice
  - explains lattice data such as the Ward-Identity displacements
- That's the **Schwinger mechanism** of Aguilar, Binosi, Ferreira and Papavassiliou (**ABFP**)[4]:
  - based on Schwinger (1962), Cornwall (1982), *et al.*
  - *ansatz*: a **massless**, **colored** (adjoint), **scalar**, **longitudinally coupled**, **unphysical bound state** is formed in the channel of the gluon field A^a_μ
  - the bound state produces **massless poles** in the **elementary dressed vertices of QCD**
  - the vertex poles get **transferred** to the gluon **polarization** $\Longrightarrow$ $\Longrightarrow$ **gluon mass** by the **Schwinger mechanism**

[4] Ferreira, Papavassiliou, Prog. Part. Nucl. Phys. 144 (2025) 104186.

The problem

- Many attempts at formulating a **theory of the gluon mass**.
- Only **one** has the following properties:
  - does not change the LCG QCD Lagrangian (**same theory as KO**)
  - **preserves BRST symmetry, color symmetry** and Lorentz symmetry
  - **predicts $\boldsymbol{\Delta(0) > 0}$** as observed on the lattice
  - explains lattice data such as the Ward-Identity displacements
- That's the **Schwinger mechanism** of Aguilar, Binosi, Ferreira and Papavassiliou (**ABFP**)[5]:

Wait... Didn't you say KO showed $1 + u \neq 0$ $(\Delta(0) > 0) \implies$ either one of BRST symmetry or global gauge (color) symmetry is broken?

This is one of the topics of today's talk

[5] Ferreira, Papavassiliou, Prog. Part. Nucl. Phys. 144 (2025) 104186.

Outline of today's talk

New results

- The **ABFP theory** can be ***derived*** **in its entirety** (no ansatz!) within the **KO framework**: there, under customary assumptions, it is the **unique** mechanism of gluon mass generation $\Longrightarrow$ **KO/ABFP correspondence**
- In the KO framework the **global gauge charge** Q^a is ***always*** **spontaneously broken** $\Longrightarrow$ $\boldsymbol{\Delta(0) = 0}$ **breaks the global gauge charge just like** $\boldsymbol{\Delta(0) > 0}$
- The **ABFP bound state** is the **unphysical Goldstone boson** associated with the **spontaneous breaking** of the **global gauge charge** Q^a in LCGs
  $\Longrightarrow$ **a Goldstone boson in the unphysical sector of QCD**
- Yet, ***both*** **BRST symmetry** ***and*** **color symmetry** (spectrum) are **unbroken**
  $\Longrightarrow$ **using *amended* KO arguments: no colored asymptotic states in the physical spectrum of QCD (color confinement)**
- **Therefore**: within the KO framework of the LCGs of QCD, the **confinement mechanism** is the **ABFP/Schwinger mechanism**.

The KO/ABFP correspondence

The Kugo-Ojima framework: operatorial BRST quantization of QCD + asymptotic completeness

1) **Define** QCD in linear covariant gauges (FP Lagrangian):

$$\mathcal{L} = -\frac{1}{4} F^a F^a + \frac{\xi}{2} B^a B^a + B^a \partial A^a - i\partial\overline{c}^a D[A] c^a$$

2) Quantize it **canonically**:

$$[A^a_\mu(\mathbf{x},t), \Pi^{b\nu}(\mathbf{y},t)] = i\delta^\nu_\mu\, \delta^{ab}\, \delta^{(3)}(\mathbf{x}-\mathbf{y}) \ , \qquad \Pi^{a\mu} = F^{a\mu 0} + \eta^{\mu 0} B^a$$

3) Compute its **Noether currents** and **charges**:

$$J^a_\mu = f^{abc}\left(A^{b\nu} F^c_{\nu\mu} + A^b_\mu B^c + i\partial_\mu \overline{c}^b c^c - i\overline{c}^b D_\mu c^c\right) \ , \qquad Q^a = \int d^3\mathbf{x}\ J^a_0$$

$$J_{B\mu} = B^a D_\mu c^a - \partial_\mu B^a c^a + \frac{ig}{2} f^{abc}\, \partial_\mu \overline{c}^a c^b c^c \ , \qquad Q_B = \int d^3\mathbf{x}\ J_{B0}$$

The KO/ABFP correspondence

<u>**The Kugo-Ojima framework**</u>: operatorial BRST quantization of QCD + asymptotic completeness

4) **Physical states** are elements of the **BRST cohomology** $\mathcal{H}_{\text{phys}}$,

$$\mathcal{H}_{\text{phys}} = \text{ker}\{Q_B\}/\text{im}\{Q_B\}$$

In the Hilbert space, they are represented by states $|\text{phys}\rangle$ such that

$$Q_B\,|\text{phys}\rangle = 0\ , \qquad \text{e.g.} \quad Q_B\,|\text{proton}\rangle = 0\ , \quad Q_B\,|\text{antighost}\rangle \neq 0$$

5) **Asymptotic completeness**: the Hilbert space is spanned by scattering (in/out) states:

$$\mathcal{H} = \mathcal{H}_{\text{in}} = \mathcal{H}_{\text{out}}$$

The Kugo-Ojima framework: operatorial BRST quantization of QCD + asymptotic completeness

6) Postulate the non-existence of *certain* massless colored bound states
$\Longrightarrow$ get the so-called **elementary BRST quartet**: a quartet of **stable massless colored scalar unphysical particles** created/annihilated by the elementary fields, $\boldsymbol{A^a_\mu \propto \partial_\mu \chi^a}$, B^a, c^a, $\overline{c}^a$

$$\begin{tikzcd}
\chi^a \arrow[r, "Q_B"] \arrow[d, "P^2"'] & \gamma^a \arrow[d, "\mathcal{C}_{\text{ghost}}"] \\
\beta^a & \overline{\gamma}^a \arrow[l, "Q_B"]
\end{tikzcd}$$

$$\partial^2 \chi^a + \xi_R M_R^2\, \beta^a = 0\,, \quad \partial^2 \beta^a = 0$$
$$\partial^2 \gamma^a = 0\,, \quad \partial^2 \overline{\gamma}^a = 0$$

γ^a, $\overline{\gamma}^a$: ghost and antighost ($c^a, \overline{c}^a$)

β^a: scalar polarization of the gauge boson ($B^a \propto \partial^\mu A^a_\mu$)

χ^a: "**first parent**", theory-dependent

- QED: photon's longitudinal polarization
- Stueckelberg model: Stueckelberg field
- Higgs models: unphysical components of the Higgs field (the unphysical Goldstone bosons)

The KO/ABFP correspondence

Q: In the KO framework, when a gluon mass is dynamically generated, who is the first parent of the elementary BRST quartet of QCD?

A: It's the ABFP bound state[6].

[6] New result.

The KO/ABFP correspondence

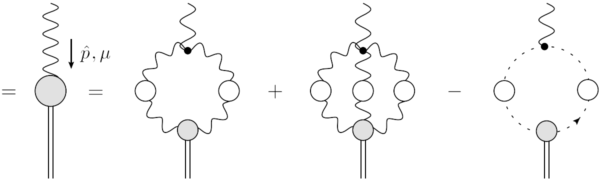
To see this: 1) assume gluons dynamically acquire a mass, and compute the matrix element of the Yang-Mills equations between the vacuum $|\Omega\rangle$ and the one-particle state $|\chi^a(p)\rangle$ of the first parent,

$$0 = \langle\Omega|\, \partial^\nu F^a_{\nu\mu} + gf^{abc}A^{b\nu}F^c_{\nu\mu} + igf^{abc}\partial_\mu\bar{c}^b c^c - \partial_\mu B^a\, |\chi^d(p)\rangle \iff$$

$$\iff gI_\mu(-p)\delta^{ad}\, e^{-ipx} = igf^{abc}\, \langle\Omega|\, \partial^\nu(A^b_\nu A^c_\mu) + A^{b\nu}F^c_{\nu\mu} + i\partial_\mu\bar{c}^b c^c\, |\chi^d(p)\rangle$$

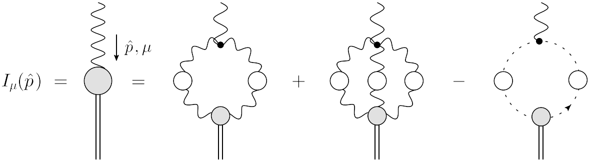
$$gI_\mu(p) := Z_3^{-1/2} M_R\, p_\mu$$

2) write the above equation in diagrammatic form:

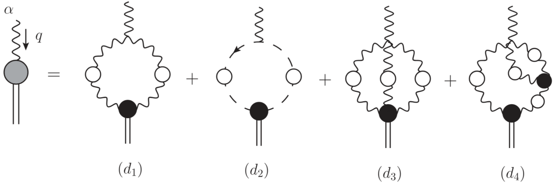
$I_\mu(\hat{p})$ = 

The KO/ABFP correspondence

2) write the above equation in diagrammatic form (not 1PI):

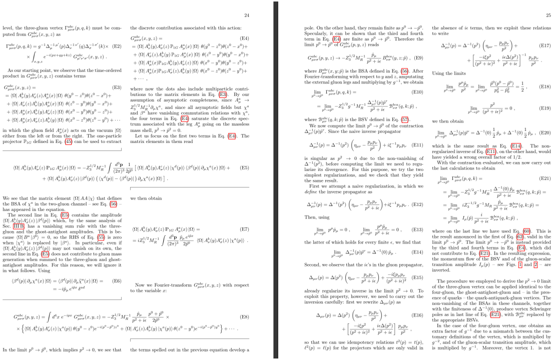


3) open Ibañez, Papavassiliou, Phys.Rev.D 87 (2013), 034008, p.7 (1PI):



Gluon-scalar transition amplitude: fundamental ingredient of dynamical gluon mass generation via the ABFP Schwinger mechanism, $I_\mu \propto M_R \, p_\mu$

4) study the one-particle spectrum of the dressed elementary QCD vertices (a couple of pages of ugly but straightforward calculations):



χ^a **produces massless longitudinal poles in the QCD vertices**: unique condition for the ABFP Schwinger mechanism to be active

The KO/ABFP correspondence

5) with DMG, the 1st parent of the elementary BRST quartet of QCD χ^a is:

- a **massless**, **scalar**, **colored** (adjoint), **longitudinally coupled unphysical** particle in the same **composite channels** and with the same **Bethe-Salpeter amplitudes** as the ABFP bound state
- obeying the **same dynamics** as the ABFP bound state
- producing the **same Schwinger poles** as the ABFP bound state
- **triggering the Schwinger mechanism**[7] like the ABFP bound state

In other words, **it *is* the ABFP bound state.**

KO/ABFP correspondence:

$$\textbf{Gluon mass} \quad \underset{\textbf{ABFP}}{\overset{\textbf{KO}}{\rightleftharpoons}} \quad \textbf{Massless scalar bound state}$$

[7] This can be shown entirely within the KO framework, using DSEs as usual.

Spontaneous breaking of the global gauge charge

- Once the KO/ABFP correspondence is established, we ***really*** want to understand what's going on with spontaneous symmetry breaking.
- **KO** say the global gauge charge Q^a is **spontaneously broken**, **ABFP** say color is **not broken**.
- Keep in mind: no symmetry breaking $\Longrightarrow$ **color confinement**.
- So we compute the vacuum **commutator** between Q^a and the asymptotic (in/out) field χ^a that creates/annihilates the ABFP bound state:

$$\langle\Omega|\,[Q^a, \chi^b(x)]\,|\Omega\rangle = ig^{-1}M(1+u)\,\delta^{ab}$$

- RHS $\neq 0$ implies $Q^a\,|\Omega\rangle \neq 0 \implies$ **SSB**
- This is where the parameter $\mathbf{1} + \boldsymbol{u}$ of the **KO criterion** originates from: if $1 + u = 0$ – KO say – there's no SSB and color is confined.

Spontaneous breaking of the global gauge charge

$$\langle\Omega|\,[Q^a,\chi^b(x)]\,|\Omega\rangle = ig^{-1}M(1+u)\,\delta^{ab}$$

- But there's a problem: in the equation, $1+u$ appears in the **combination** $\boldsymbol{M(1+u)}$, where $M=\Delta^{-1/2}(0)$.
- In their papers, when discussing SSB and confinement, KO **never** multiply the asymptotic field $\chi^a(x)$ by its LSZ coefficient:

$$A^a_\mu \to \boldsymbol{Z_\chi^{1/2}}\,\partial_\mu\chi^a \qquad\qquad (t\to\pm\infty)$$

- As a result, **the original $\boldsymbol{1+u=0}$ KO equation is missing a factor of $\boldsymbol{Z_\chi^{-1/2}}$** on the RHS.
- It turns out that...

$$\boldsymbol{Z_\chi^{-1/2} = M}$$

Spontaneous breaking of the global gauge charge

$$\langle\Omega|\,[Q^a,\chi^b(x)]\,|\Omega\rangle = ig^{-1}M(1+u)\,\delta^{ab}$$

- If $\Delta(0)=0$, $M=\infty$ and $M(1+u)$ is finite even if $1+u=0$!
- $M(1+u)=\widehat{M}$ is the $p^2\to 0$ limit of the **gluon propagator** evaluated in the Pinch-Technique–Background-Field-Method (**PT-BFM**) scheme[8], and **it is *not* equal to zero** for massive gluons.

In the LCGs, the global gauge charge is *always* spontaneously broken, even when $1+u=0$ ($\Delta(0)=0$)

[8]Double-checked by evaluating explicitly the LHS, see next slide.

Spontaneous breaking of the global gauge charge

- Moreover, the ABFP bound state is a **Goldstone boson**!

$$\langle\Omega|\,[Q^a,\chi^b(x)]\,|\Omega\rangle = ig^{-1}\,\widehat{M}\,\delta^{ab} \neq 0$$

- In terms of the **color current**,

$$\langle\Omega|\,J^a_\mu(x)\,|\chi^b(p)\rangle = -i\,\widetilde{I}_\mu(-p)\,\delta^{ab}\,e^{-ipx} = i\,\widehat{M}/g\;p_\mu\,\delta^{ab}\,e^{-ipx}$$

$\widetilde{I}_\mu(p)$ is gluon-scalar transition amplitude evaluated in the **PT-BFM**.

The ABFP bound state is the Goldstone boson associated with the SB of the global gauge (color) charge

Spontaneous breaking of the global gauge charge

- But this makes **no sense**.
  - The lattice LCG Green functions are color-symmetric.
  - The ABFP Schwinger mechanism (and Green functions) are color-symmetric by design, and they reproduce the lattice data.
- Green functions are **vacuum correlators**. If *all* vacuum correlators respect color symmetry, color symmetry **cannot** be spontaneously broken. Yet Q^a ***is*** broken.
- Clearly we are **missing something**.
- What is Q^a **doing** to break symmetry?

Spontaneous breaking of the global gauge charge

- What is Q^a **doing** to break symmetry? Let's find out.
- Standard assumptions $\Longrightarrow$ the (broken) transformations of the elementary BRST quartet are

$$[Q^a, \chi^b(x)] = \underline{ig^{-1}\widehat{M}\,\delta^{ab}} + if^{abc}\chi^c(x)\ , \quad [Q^a, \beta^b(x)] = if^{abc}\beta^c(x)\ ,$$
$$[Q^a, \gamma^b(x)] = if^{abc}\gamma^c(x)\ , \quad [Q^a, \overline{\gamma}^b(x)] = if^{abc}\overline{\gamma}^c(x)$$

The <u>constant</u> $\propto \delta^{ab}$ in $[Q^a, \chi^b(x)]$ is the symmetry-breaking term.

- In terms of the asymptotic fields, the generator Q^a reads

$$Q^a = \boldsymbol{g^{-1}\widehat{M}\int d^3\mathbf{x}\ \dot{\beta}^a} + f^{abc}\int d^3\mathbf{x}\ \left(\dot{\chi}^b\beta^c - \chi^b\dot{\beta}^c - \dot{\beta}^b\beta^c + i\dot{\overline{\gamma}}^b\gamma^c - i\overline{\gamma}^b\dot{\gamma}^c\right) + \cdots$$

The ellipses contain contributions due to other asymptotic fields. Those **do not** yield other symmetry-breaking terms. The **first term** $\propto \dot{\beta}^a$ is the one breaking the symmetry.

Spontaneous breaking of the global gauge charge

$$Q^a = \boldsymbol{g^{-1}\widehat{M}\int d^3\mathbf{x}\ \dot{\beta}^a} + f^{abc}\int d^3\mathbf{x}\ \left(\dot{\chi}^b\beta^c - \chi^b\dot{\beta}^c - \dot{\beta}^b\beta^c + i\,\dot{\overline{\gamma}}^b\gamma^c - i\,\overline{\gamma}^b\dot{\gamma}^c\right) + \cdots$$

- Q^a is the ill-defined, constant-parameter limit ($\Lambda^a \to \lambda^a$) of a well-defined operator $\mathcal{Q}[\Lambda, \lambda]$

$$\mathcal{Q}[\Lambda,\lambda] = \boldsymbol{g^{-1}\widehat{M}\int d^3\mathbf{x}\ \left(\Lambda^a\dot{\beta}^a - \dot{\Lambda}^a\beta^a\right)} +$$

$$+\ \lambda^a f^{abc}\int d^3\mathbf{x}\ \left(\dot{\chi}^b\beta^c - \chi^b\dot{\beta}^c - \dot{\beta}^b\beta^c + i\,\dot{\overline{\gamma}}^b\gamma^c - i\,\overline{\gamma}^b\dot{\gamma}^c\right) + \lambda^a(\cdots)$$

where the $\Lambda^a(x)$'s are functions such that $\partial^2\Lambda^a = 0$ and the λ^a's are constants

- $\mathcal{Q}[\Lambda, \lambda]$ generates the following transformations of the ***asymptotic* gluon field** $A_\mu^{a\,\text{as.}}$ ($g_R^{-1} = Z_3^{-1/2}g^{-1}(1+u)$):

$$[\mathcal{Q}[\Lambda,\lambda], A_\mu^{a\,\text{as.}}] = i\left(\boldsymbol{g_R^{-1}\partial_\mu\Lambda^a} + f^{abc}A_\mu^{b\,\text{as.}}\lambda^c\right) \qquad (\partial^2\Lambda^a = 0)$$

Spontaneous breaking of the global gauge charge

$$[\mathcal{Q}[\Lambda,\lambda], A_\mu^{a\,\text{as.}}] = i\left(\boldsymbol{g_R^{-1}\partial_\mu\Lambda^a} + f^{abc}A_\mu^{b\,\text{as.}}\lambda^c\right) \qquad (\partial^2\Lambda^a = 0)$$

$$0 = \langle\Omega|\, A_\mu^{a\,\text{as.}}(x)\,|\Omega\rangle \quad \to \quad \langle\Omega|\, A_\mu^{a\,\text{as.}}(x) + \delta A_\mu^{a\,\text{as.}}(x)\,|\Omega\rangle = g_R^{-1}\partial_\mu\Lambda^a(x) \neq 0$$

- Those are **not** mere color transformations: they are **asymptotic residual gauge transformations**! (Symmetries of the asymptotic field equations)
- χ^a enters $A_\mu^{a\,\text{as.}} = \boldsymbol{M_R^{-1}\partial_\mu\chi^a} + \cdots$ as a derivative, and it carries a representation of the **derivative** part of the gauge transformation.
- In the constant limit in which Q^a is recovered from $\mathcal{Q}[\Lambda,\lambda]$, $\Lambda^a \to \lambda^a$, the derivative in the transformation of $A_\mu^{a\,\text{as.}}$ **survives** as a **symmetry-breaking constant** in the transformation of χ^a
- The constant is **not** realizing **broken color transformations**, but the **derivative term** of a ***gauge*** **transformation**.
- In linear covariant gauges, that transformation is **well-known** to be **always** spontaneously broken, **even in QED**![9]

[9] See e.g. Nakanishi, Ojima, *Covariant operator formalism of gauge theories and quantum gravity*, World Scientific (1990), p.68

Spontaneous breaking of the global gauge charge

$$[\mathcal{Q}[\Lambda,\lambda], A_\mu^{a\,\text{as.}}] = i\left(\boldsymbol{g_R^{-1}\partial_\mu\Lambda^a} + f^{abc}A_\mu^{b\,\text{as.}}\lambda^c\right) \qquad (\partial^2\Lambda^a = 0)$$

- Therefore Q^a is **not** the generator of color transformations. To obtain the generator of **color transformations** $\widetilde{Q}^a$, we need to **subtract** the generator of the derivative from Q^a:

$$\widetilde{Q}^a = Q^a - \boldsymbol{g^{-1}\widehat{M}\int d^3\mathbf{x}\ \dot{\beta}^a}$$

- $\widetilde{Q}^a$ is an **unbroken** ($\widetilde{Q}^a\,|\Omega\rangle = 0$), **well-defined** color charge operator, generating the asymptotic transformations

$$[\widetilde{Q}^a,\chi^b] = if^{abc}\chi^c\,, \quad [\widetilde{Q}^a,\beta^b] = if^{abc}\beta^c\,, \quad [\widetilde{Q}^a,\gamma^b] = if^{abc}\gamma^c\,, \quad [\widetilde{Q}^a,\overline{\gamma}^b] = if^{abc}\overline{\gamma}^c\,,$$
$$[\widetilde{Q}^a,X] = [Q^a,X]\,, \qquad [\widetilde{Q}^a,\widetilde{Q}^b] = if^{abc}\,\widetilde{Q}^c$$

where X is any other asymptotic field.

- So **both** KO and ABFP are right. The global gauge charge Q^a *is* spontaneously broken, **and** the color symmetry of the spectrum (in/out asymptotic states) *is not* violated.

$$\widetilde{Q}^a = Q^a - g^{-1}\widehat{M}\int d^3\mathbf{x}\ \dot{\beta}^a$$

- Having identified the correct color charge operator, we can turn to **color confinement**. As we know, when a gluon mass is dynamically generated, we have

$$Q^a = \left\{Q_B, g^{-1}\int d^3\mathbf{x}\ D_0\overline{c}^a\right\}$$

- Moreover, **the symmetry-breaking term is BRST exact**[10],

$$g^{-1}\widehat{M}\int d^3\mathbf{x}\ \dot{\beta}^a = \left\{Q_B, g^{-1}\widetilde{Z}_3^{1/2}(1+u)\int d^3\mathbf{x}\ \partial_0\overline{\gamma}^a\right\}$$

- Therefore

$$\widetilde{Q}^a = \left\{Q_B, g^{-1}\int d^3\mathbf{x}\ \left[D_0\overline{c}^a - \widetilde{Z}_3^{1/2}(1+u)\partial_0\overline{\gamma}^a\right]\right\}$$

[10] $1+u$ still cancels out from final expressions.

- The color charge operator $\widetilde{Q}^a$ is an **unbroken** ($\widetilde{Q}^a \left|\Omega\right\rangle = 0$), **well-defined**, **BRST-exact** operator whose action is known on in/out asymptotic states[11]

$$\widetilde{Q}^a = \left\{ Q_B, g^{-1} \int d^3\mathbf{x} \left[D_0 \overline{c}^a - \widetilde{Z}_3^{1/2} (1+u) \partial_0 \overline{\gamma}^a \right] \right\}$$

- BRST-exact $\Longrightarrow$ it **vanishes** on physical states

$$Q_B \left|\text{phys}\right\rangle = Q_B \left|\text{phys}'\right\rangle = 0 \qquad \Longrightarrow \qquad \left\langle \text{phys}'\right| \widetilde{Q}^a \left|\text{phys}\right\rangle = 0$$

$$\boldsymbol{\widetilde{Q}^a \, \mathcal{H}_{\text{phys}} = 0}$$

- The above equation expresses the **confinement of color** in QCD: **absence of colored asymptotic states from the physical spectrum**.

[11] Because it is constructed from asymptotic fields.

Full color symmetry: soft theorems (preliminary)

- The color charge operator $\widetilde{Q}^a$ is built from **asymptotic fields**: it represents a **pair** of **asymptotic** (in/out) **charges**

$$(\widetilde{Q}^a)^{\text{in/out}} = Q^a - g^{-1}\widehat{M}\int d^3\mathbf{x}\,(\dot{\beta}^a)^{\text{in/out}}\,, \qquad S(\widetilde{Q}^a)^{\text{out}} = (\widetilde{Q}^a)^{\text{in}} S$$

- This is **sufficient** for the symmetry of the **spectrum** and for **color confinement**, not for **full color symmetry**. That requires color to be conserved in unphysical **scattering processes** (full S-matrix):

$$(\widetilde{Q}^a)^{\text{in}} = (\widetilde{Q}^a)^{\text{out}} = \widetilde{Q}^a\,, \qquad [\widetilde{Q}^a, S] = 0$$

- More generally, the spontaneously broken symmetry induces **broken Ward identities** – equivalently, **soft theorems**

$$\sum_k T^a_{I_k J_k}\,\langle\Omega|\,T\left\{\mathcal{O}_{J_k}(x_k)\prod_{j\neq k}\mathcal{O}_{I_j}(x_j)\right\}|\Omega\rangle = -i\widehat{M}/g\,\lim_{p\to 0}\,\langle\beta^a(p);\text{out}|\,T\left\{\prod_k \mathcal{O}_{I_k}(x_k)\right\}|\Omega\rangle$$

Full color symmetry: soft theorems (preliminary)

$$\sum_k T^a_{I_k J_k} \, \langle\Omega| \, T\left\{\mathcal{O}_{J_k}(x_k)\prod_{j\neq k}\mathcal{O}_{I_j}(x_j)\right\}|\Omega\rangle = -i\widehat{M}/g \, \lim_{p\to 0} \, \langle\beta^a(p);\text{out}| \, T\left\{\prod_k \mathcal{O}_{I_k}(x_k)\right\}|\Omega\rangle$$

- To preserve color symmetry in full, LHS=0, hence

$$\lim_{p\to 0} \, \langle\beta^a(p);\text{out}| \, T\left\{\prod_k \mathcal{O}_{I_k}(x_k)\right\}|\Omega\rangle = 0$$

- This is **trivially** verified in the BRST cohomology: if $[Q_B, \mathcal{O}_{I_k}(x)]_\mp = 0$,

$$\langle\beta^a(p);\text{out}| \, T\left\{\prod_k \mathcal{O}_{I_k}(x_k)\right\}|\Omega\rangle \propto \langle\overline{\gamma}^a(p);\text{out}|\left[Q_B, \; T\left\{\prod_k \mathcal{O}_{I_k}(x_k)\right\}\right]_\mp |\Omega\rangle = 0$$

$\implies$ BRST symmetry enforces **full color symmetry** ($\widetilde{Q}^a = 0 \to 0$) in the **physical sector**

Full color symmetry: soft theorems (preliminary)

$$\lim_{p\to 0} \langle \beta^a(p); \text{out}|\, T\left\{\prod_{k=1}^{n} \mathcal{O}_{I_k}(x_k)\right\}|\Omega\rangle = 0$$

- It is **not** trivially verified in the **unphysical sector**
- For instance, the **VEV** of the **elementary gauge fields** is **protected**:

$$f^{abc}\,\langle A^c_\mu(x)\rangle = -\widehat{M}/g \lim_{p\to 0} \langle \beta^a(p); \text{out}|\, A^b_\mu(x)\,|\Omega\rangle = i(1+u)/g \lim_{p\to 0}\left(p_\mu e^{ipx}\delta^{ab}\right) = 0$$

$$f^{abc}\,\langle X^c(x)\rangle = -\widehat{M}/g \lim_{p\to 0} \langle \beta^a(p); \text{out}|\, X^b(x)\,|\Omega\rangle = 0 \qquad \text{for} \qquad X^a = B^a, c^a, \bar{c}^a$$

- But in the SU(2) Higgs-Kibble model (LCGs), the **Higgs field's VEV** can yield LHS=RHS$\neq 0$

$$-\frac{1}{2}\,\sigma^i\,\langle\Phi(x)\rangle = -i\widehat{M}/g \lim_{p\to 0} \langle \beta^i(p); \text{out}|\, \Phi(x)\,|\Omega\rangle \qquad \Longleftrightarrow \qquad \frac{v}{2} = Z_\pi^{1/2}\,\widehat{M}/g$$

(exact nonperturbative generalization of tree level $M = gv/2$)

Full color symmetry: soft theorems (preliminary)

$$\lim_{p\to 0} \langle \beta^a(p); \text{out}|\, T\left\{\prod_{k=1}^{n} \mathcal{O}_{I_k}(x_k)\right\} |\Omega\rangle = 0$$

- Color symmetry is preserved **in full** if the NL field decouples as $p \to 0$: **Adler zeros** in the unphysical (colored) sector.
- ***Similar* conditions** in ABFP, to preserve BRST+color (same reason):

Now, combining Eqs. (5.10) and (5.22), we obtain for $q \to 0$ that

$$\lim_{q\to 0} V_\alpha(r,p,q) = \lim_{q\to 0} 2q_\alpha \frac{(q\cdot r)}{q^2} C(r^2) = 2|r|\hat{q}_\alpha \cos\theta\, C(r^2)\,, \tag{5.23}$$

which does not act as a pole. In fact, the r.h.s. of Eq. (5.23) is strictly finite. Nevertheless, $V_\alpha(r,p,q)$ is irregular; its value at vanishing q is not unique, but instead depends on $\hat{q}$ and θ, which specify the path through which the $q \to 0$ limit is approached. As we will see in Section 5.5, this irregularity is enough to generate a gluon mass scale (see also the discussions in [141,162]). In fact, as we show there, the resulting gluon mass scale does *not* depend on the way that the limit $q \to 0$ is implemented.

Note that the fact that the Vs have no poles in $q = 0$ may be viewed as a consequence of the central assumption that the gluon mass generation leaves the gauge symmetry intact. Indeed, as shown in Section 6.2 and Appendix E, the key conditions described by Eqs. (5.19) and (5.21) originate from the STIs of the theory. A related mechanism, but with the global color symmetry broken, has been discussed in [92], and leads to stronger vertex irregularities, with the corresponding Vs acting as poles in $q = 0$.

- Needs investigation using ABFP+PT-BFM+KO.

Summary

In the **KO framework**, under **standard assumptions**, when a **gluon mass** is **dynamically generated**...

- The **global gauge (color) charge** Q^a is **spontaneously broken**. $\Delta(0)$, $1+u$ play **no role** (missing LSZ coefficient in KO equation).
- The **ABFP Schwinger mechanism** is the **unique mechanism** of **dynamical gluon mass generation**.
- The **ABFP bound state** is Q^a's **unphysical Goldstone boson.**
- Q^a does **not** generate mere **color transformations**, but rather (structurally broken) **asymptotic residual gauge transformations.**
- The **color symmetry** of the **spectrum** (in/out states) is **preserved**.
- The corresponding **asymptotic color charge** $\widetilde{Q}^a$ is well-defined and **BRST-exact**, hence **confining**.
- **Full color symmetry** (S-matrix+Green's functions) protected by **Adler zeros/soft theorems**?

Thank you for your attention!