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Theoretical and Mathematical Sciences

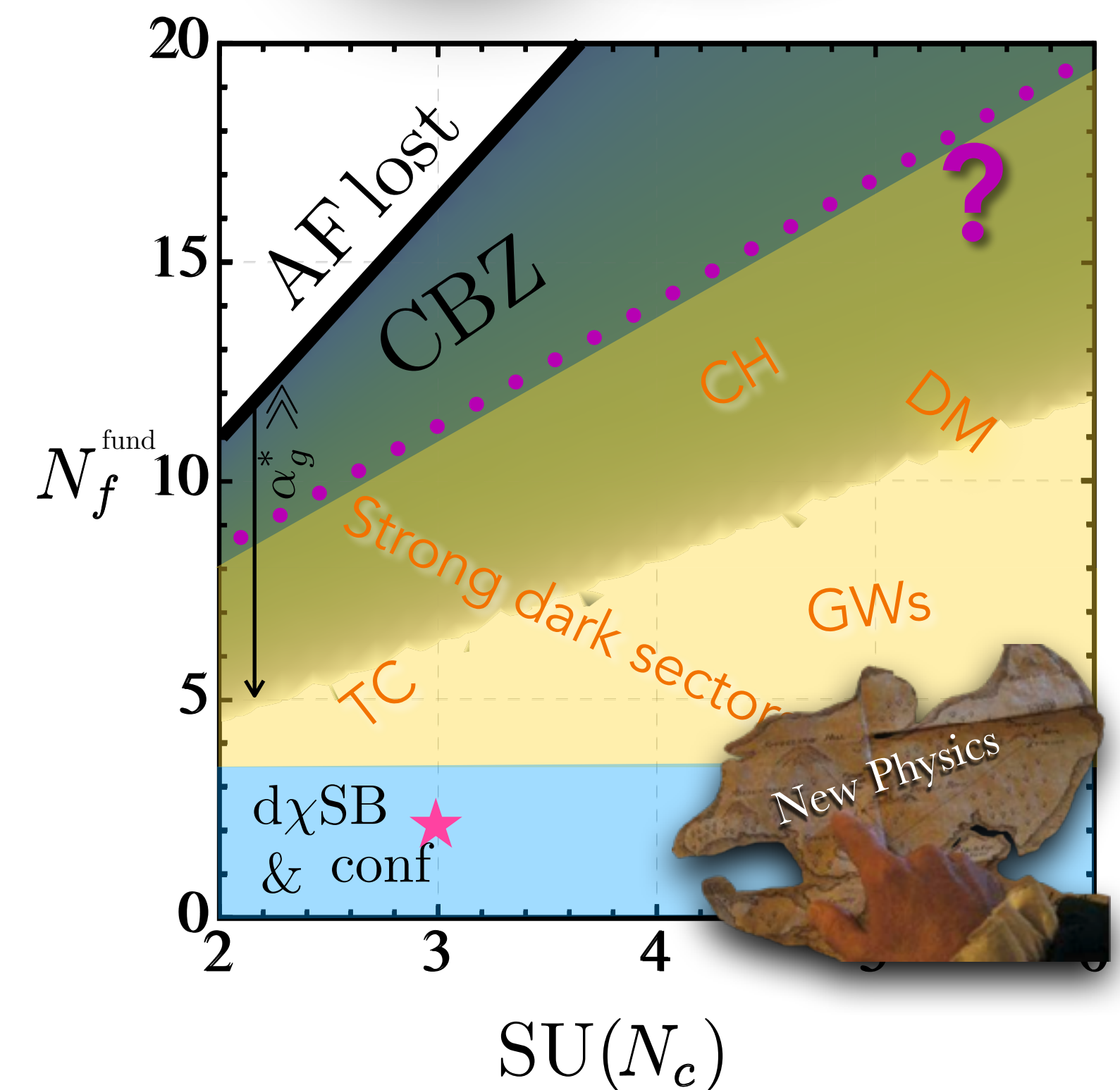
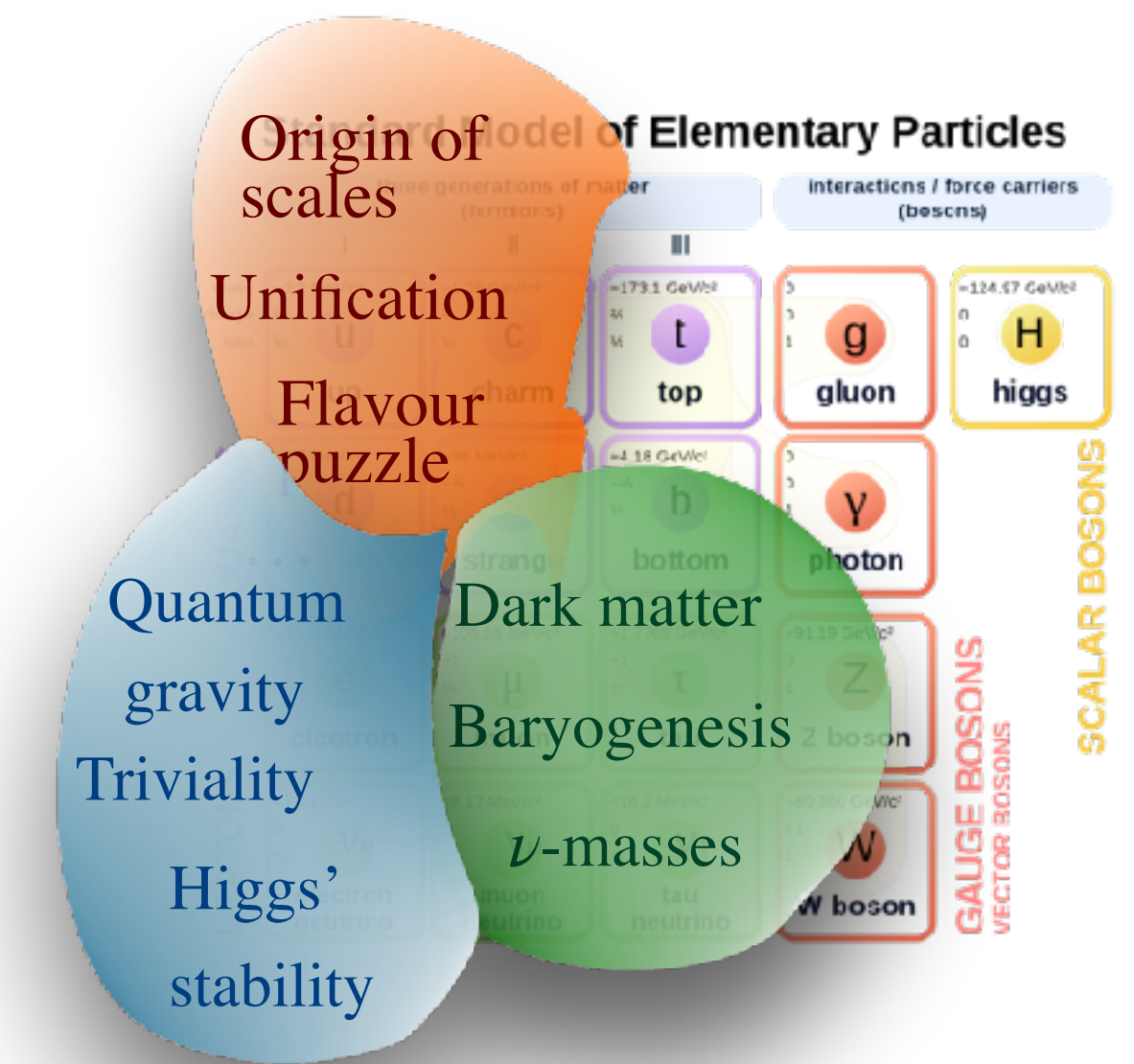
# Phases of **QCD-like** and **chiral-gauge** theories

**Álvaro Pastor Gutiérrez**

# Why beyond QCD?

- **Theoretical** relevance:
  - Understand physical theory from different limits (large  $N_c$ ,  $N_f$ , chiral limit ...)
  - Delimit phase structure and where new dynamics reside
- **Phenomenological** interest in physics beyond the SM:
  - Strong dynamics = rich dynamics
  - Most challenging
  - Unexplored landscape

A task for non-perturbative approaches!

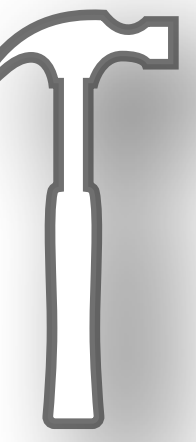


# Content

1. Conceptual basis of the functional RG approach to gauge-fermion theories
  - i. Colour confinement
  - ii. Dynamical symmetry breaking
2. Quantum phase transition to the CBZ conformal window
3. IR phases of chiral-gauge theories



# Functional Renormalisation Group



$$\mathcal{Z}[J] = \int \mathcal{D}\phi \exp \left[ -S[\phi] + \int_x \phi \cdot J \right]$$

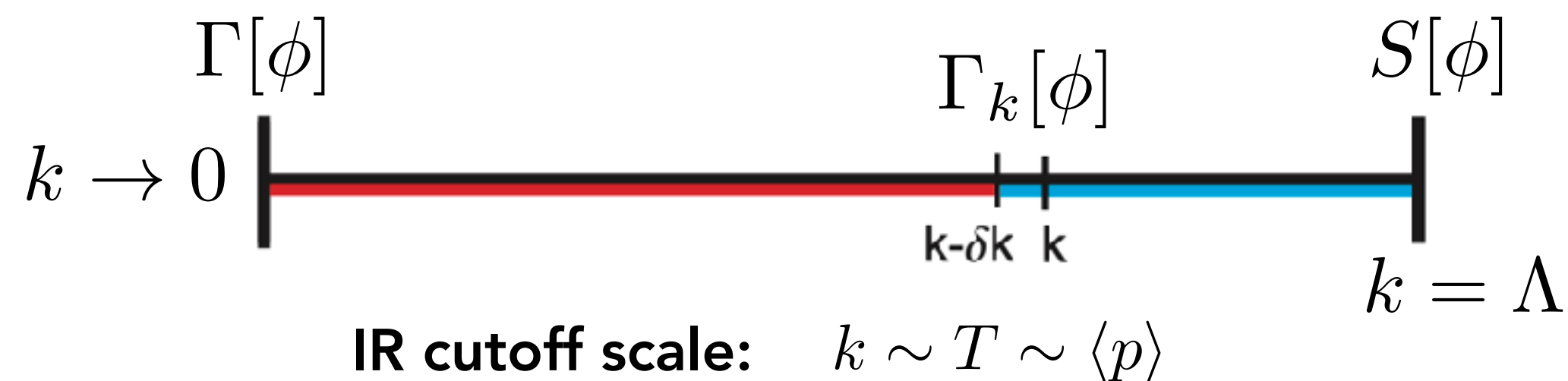
- Progressive integration of momentum shells in the path integral:

$$\int [\mathcal{D}\phi]_{p>k} = \int \mathcal{D}\phi \exp(-\Delta S_k[\phi]) \quad \Delta S_k[\phi] = \int_p \phi(p) R_k(p) \phi(-p)$$

- **Effective average action:**

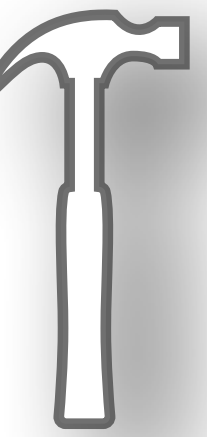
$$\Gamma_k[\phi] = \int_x J(x)\phi(x) - \mathcal{W}_k[J] - \Delta S_k[\phi] \quad \text{Wetterich '89}$$

- Average action of fields over a  $k^{-d}$  space-time volume





# Functional Renormalisation Group



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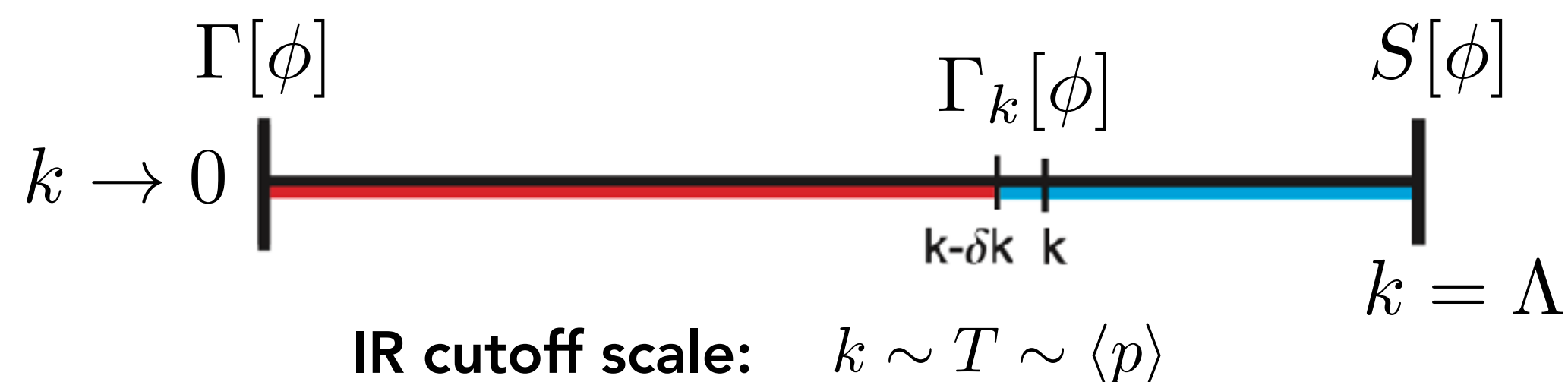
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$$\Gamma_k[\phi] = \int_x J(x)\phi(x) - \mathcal{W}_k[J] - \Delta S_k[\phi] \quad \text{Wetterich '89}$$

- Average action of fields over a  $k^{-d}$  space-time volume



Flow equation:

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \left[ \frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k \right] = \frac{1}{2} \text{ (circle with a cross) }$$

$$\partial_t \equiv k \partial_k$$

Wetterich '93

- Exact derivation
- Non-perturbative
- One loop
- Analytic regulators
- Versatile
- UV-IR finite
- Approximate resolution
- Systematic truncation schemes
- Chiral regulators

$$R_{\psi/\chi, k}(p^2) = i Z_{\psi/\chi} \bar{\sigma}_\mu p_\mu r_{\psi/\chi}(x)$$

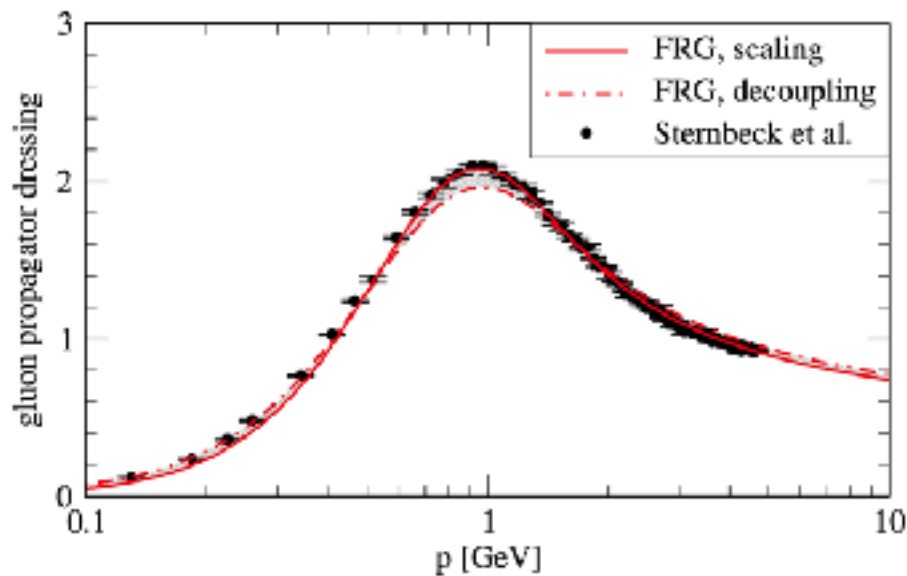
$$r_{\psi/\chi}(x) = (1/\sqrt{x} - 1) \theta(1 - x)$$

# Some results in gauge-fermion theories

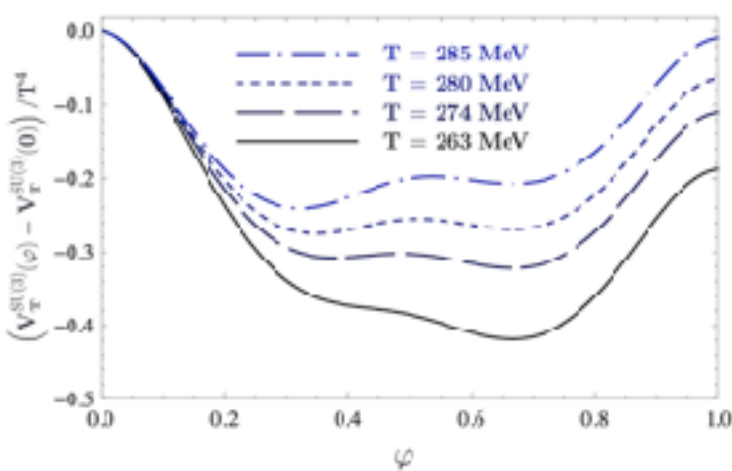
a.k.a. QCD

## Colour confinement

Cyrol,Fister,Mitter,Pawlowski [1605.01856]

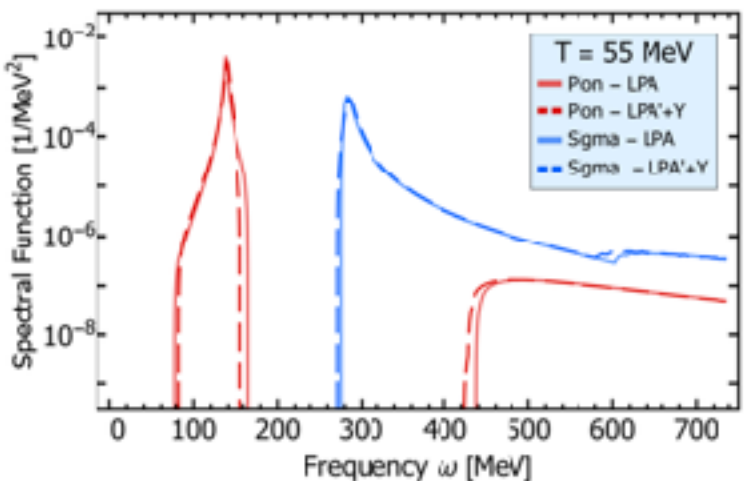
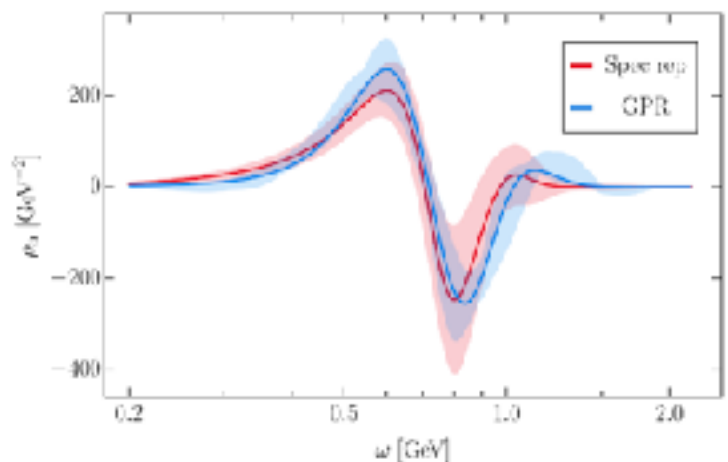


Fister,Pawlowski [1301.4163]



## Real time properties

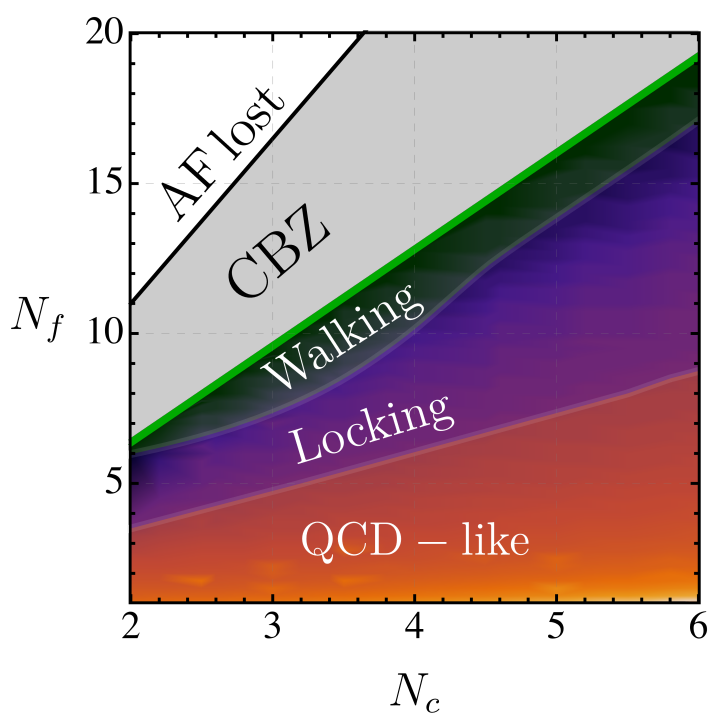
Horak,Pawlowski,Turnwald,Urbán,Wink,  
Zafeiropoulos[1711.07444]



Pawlowski,Wink,Strodthoff[1711.07444]

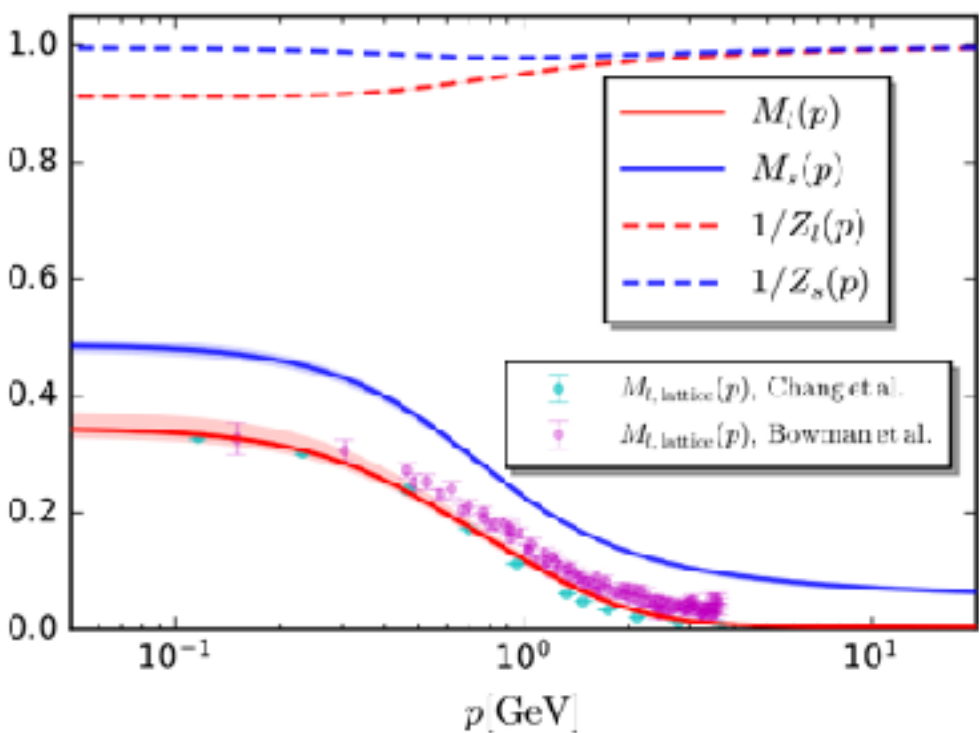
## Many flavour and colours

Gies,Jaeckel[hep-ph/0507171]  
Braun,Gies[0912.4168]  
Braun,Fischer,Gies[1012.4279]  
Goertz,APG,Pawlowski[2412.12254]

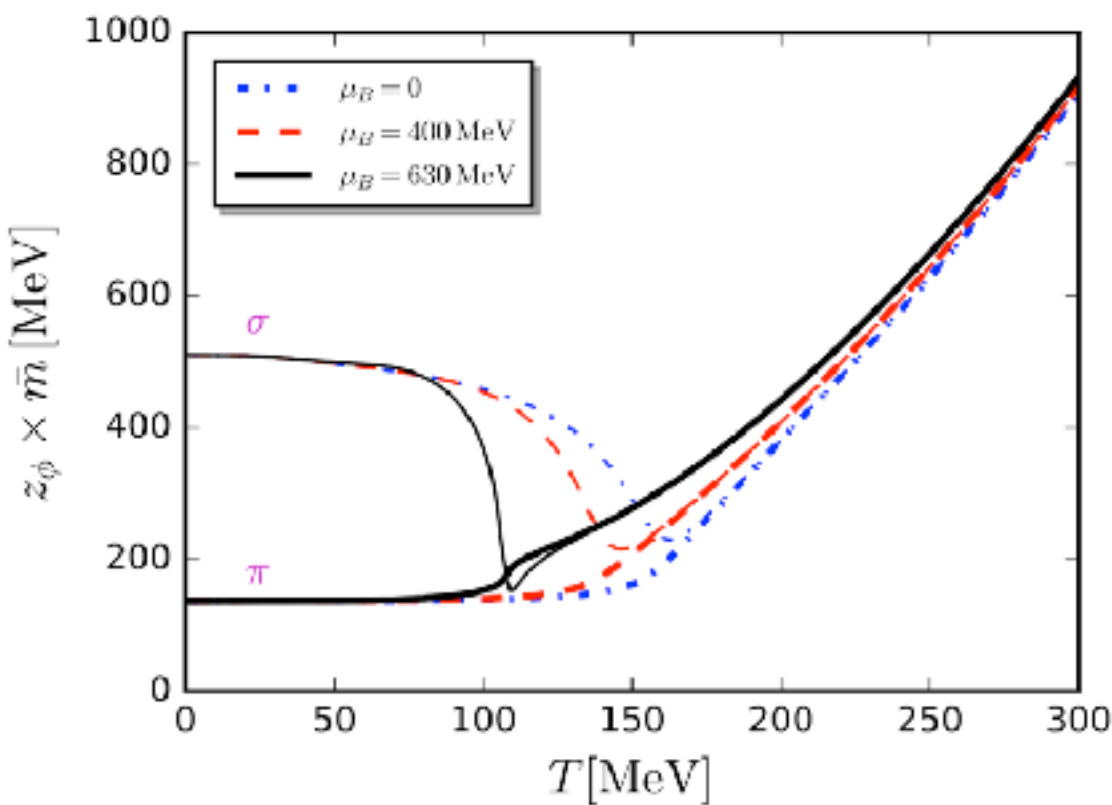


## Dynamical chiral symmetry and bound states

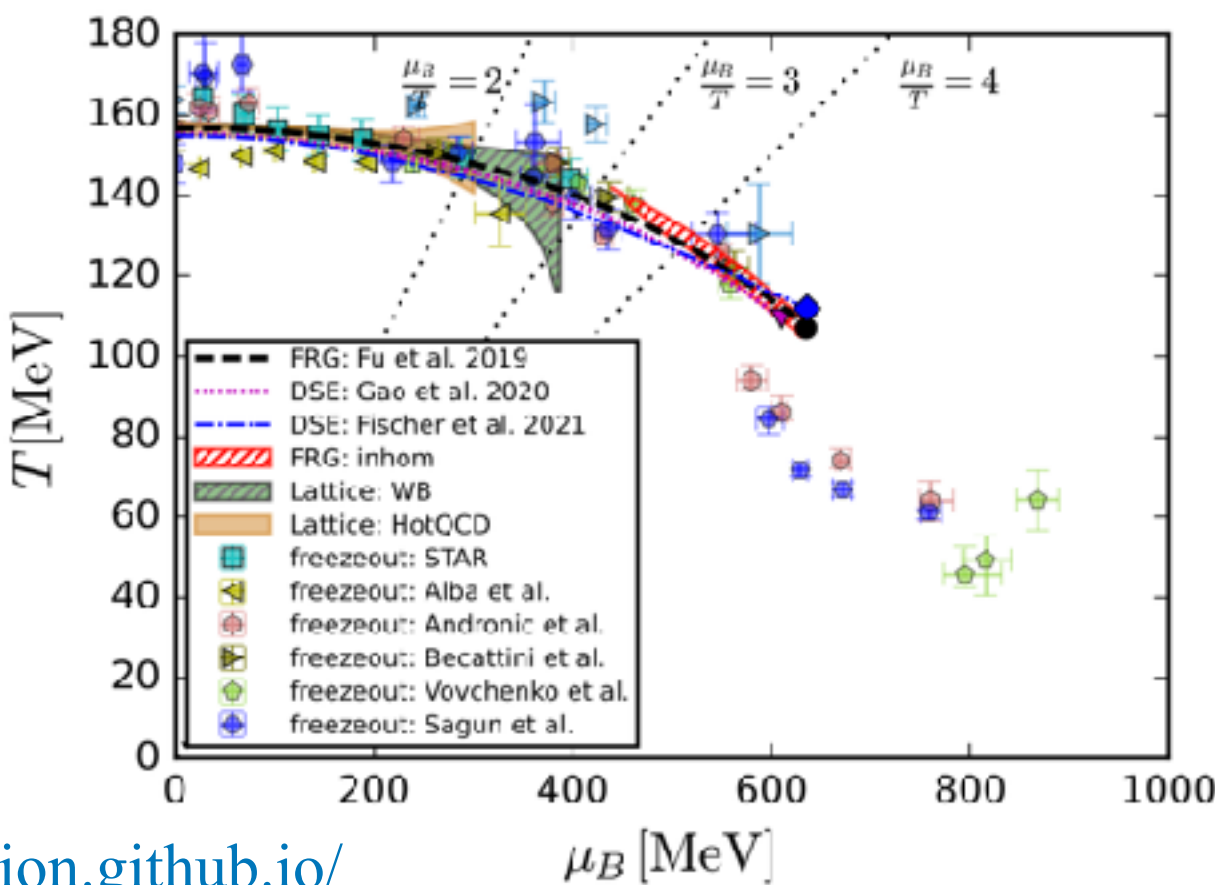
Fu,Huang,Pawlowski,Tan,Zhou[2502.14388]



Fu,Pawlowski,Rennecke [1909.02991]



## Finite temperature and chemical potential



<https://fqcd-collaboration.github.io/>

# Some results in gauge-fermion theories

a.k.a. QCD

Colour confinement

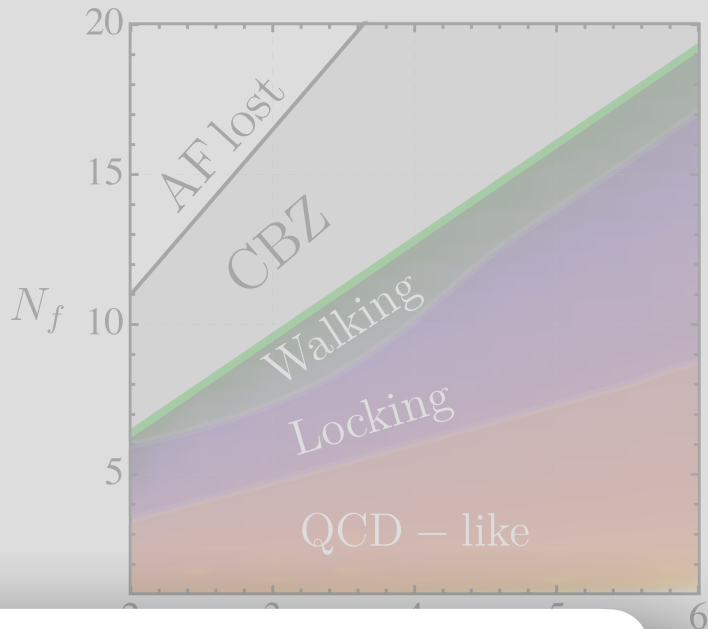
Real time properties

Many flavour and colours

Physics in correlation functions:

Colour confinement, dynamical symmetry breaking and/or anything else

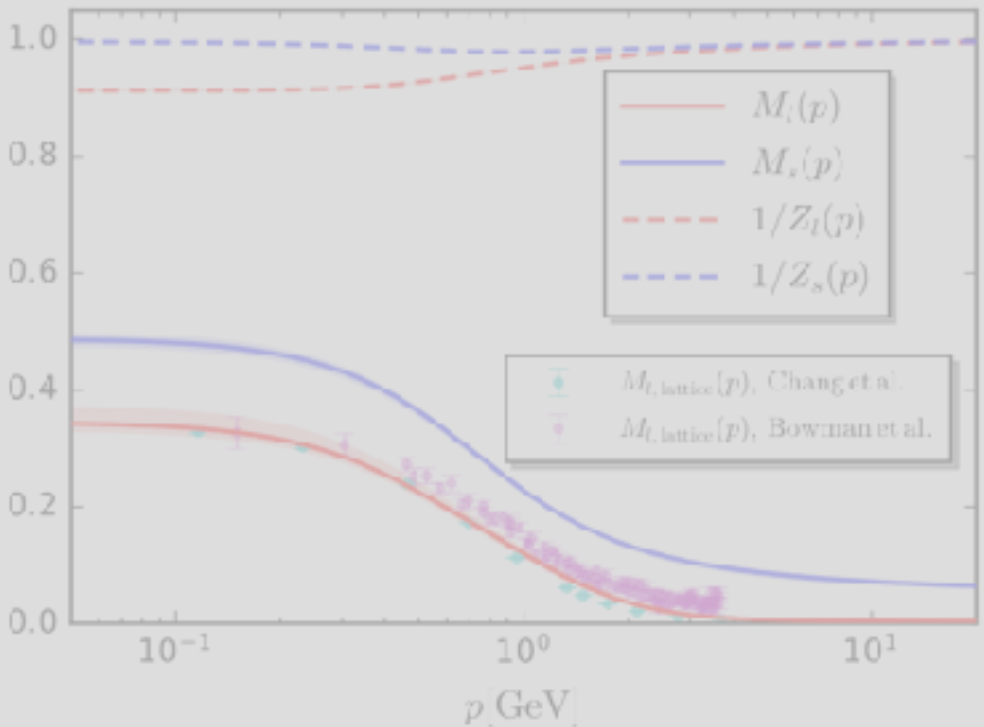
Gies,Jaeckel[hep-ph/0507171]  
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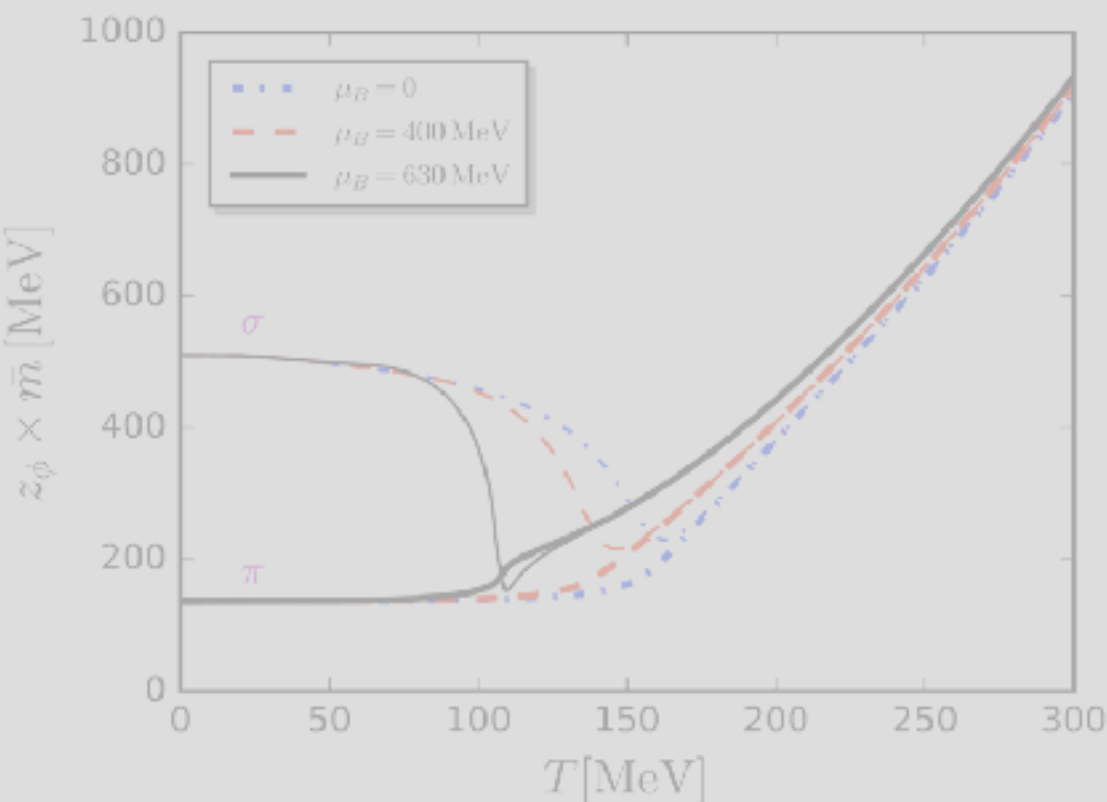
Pawlowski,Wink,Strodthoff[1711.07444]

Dynamical chiral symmetry and bound states

Fu,Huang,Pawlowski,Tan,Zhou[2502.14388]



Fu,Pawlowski,Rennecke [1909.02991]



Fundamental, first principles:

- symmetries
- relevant deformation
- predictions for LEFTs

<https://fqcd-collaboration.github.io/>



# Colour confinement and the gluon mass gap

Alkofer,von Smekal'96'00 Pawłowski,Litim,Nedelko,von Smekal[hep-th/0312324] Fischer, Maas, Pawłowski[0810.1987] Cyrol,Fister,Mitter,Pawłowski [1605.01856]...

- **Gluon mass gap** generated by **quantum fluctuations**

$$\Gamma_k^{(AA)}(p^2) = Z_{A,k}(p)(p^2 + m_{\text{gap},k}^2) = \hat{Z}_{A,k}(p) p^2$$

- How we know its confinement?

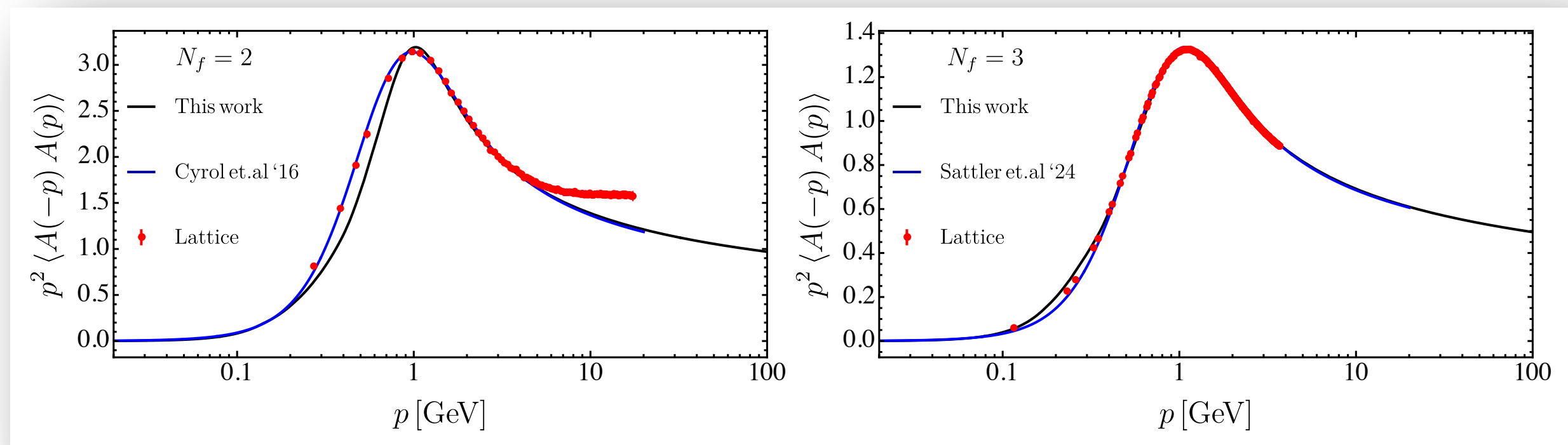
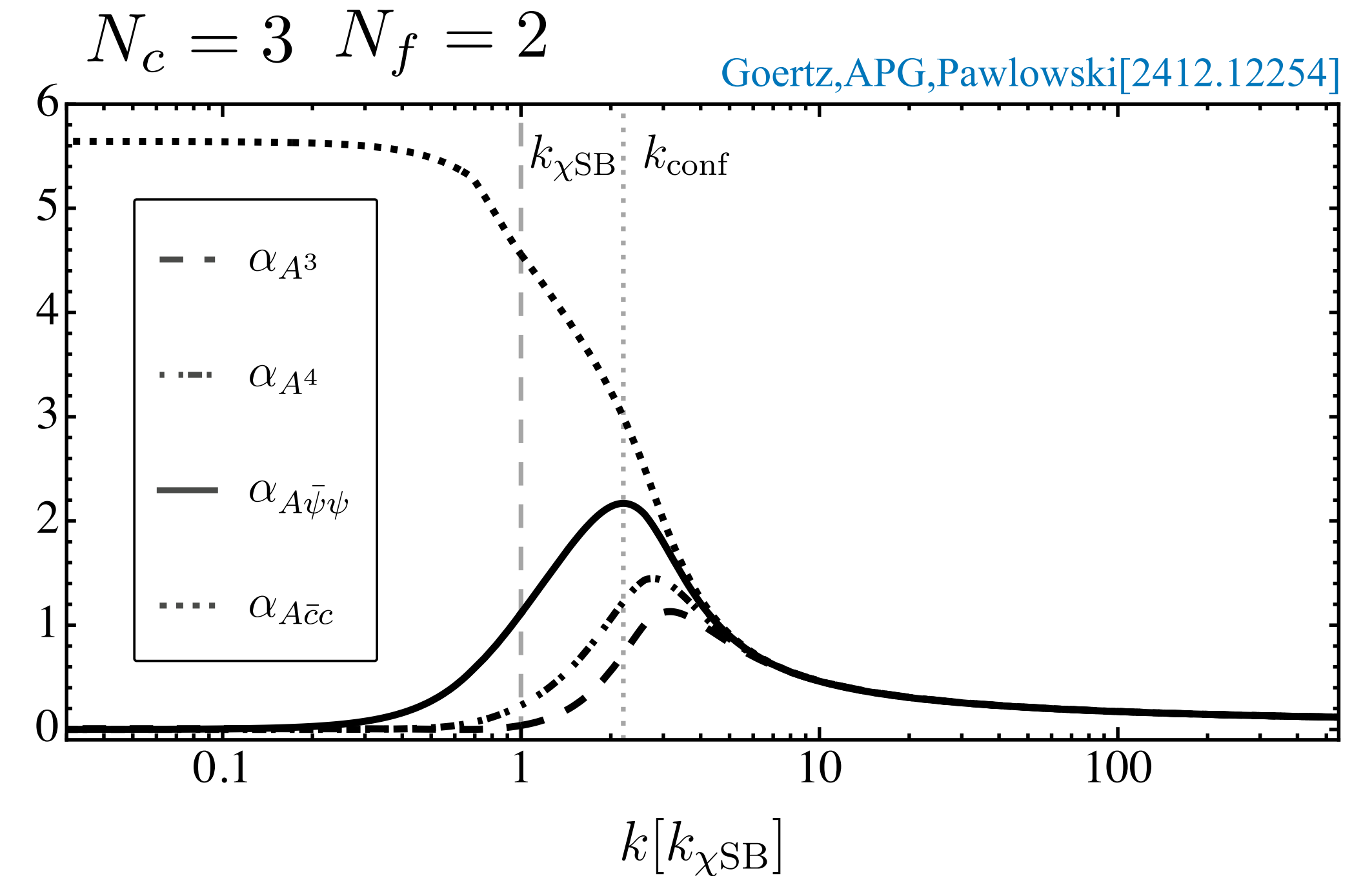
- **Kugo-Ojima criterion satisfied** [Kugo, Ojima'79](#) [Nakanishi,Ojima'90](#)

- Sufficient conditions: **Massive spectrum, colour symmetry and global BRST symmetry**
- **Unique** IR scaling of correlation functions

$$\lim_{p \rightarrow 0} Z_A(p^2) \propto (p^2)^{-2\kappa} \quad \lim_{p \rightarrow 0} Z_c(p^2) \propto (p^2)^\kappa$$

[Kugo\[hep-th/9511033\]](#)

[Zwanziger\[hep-th/0109224\]](#)



# Dynamical symmetry breaking in the effective action

a chiral QCD example

$$S = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} + \bar{\psi} \gamma_\mu D_\mu \psi$$



# Dynamical symmetry breaking in the effective action

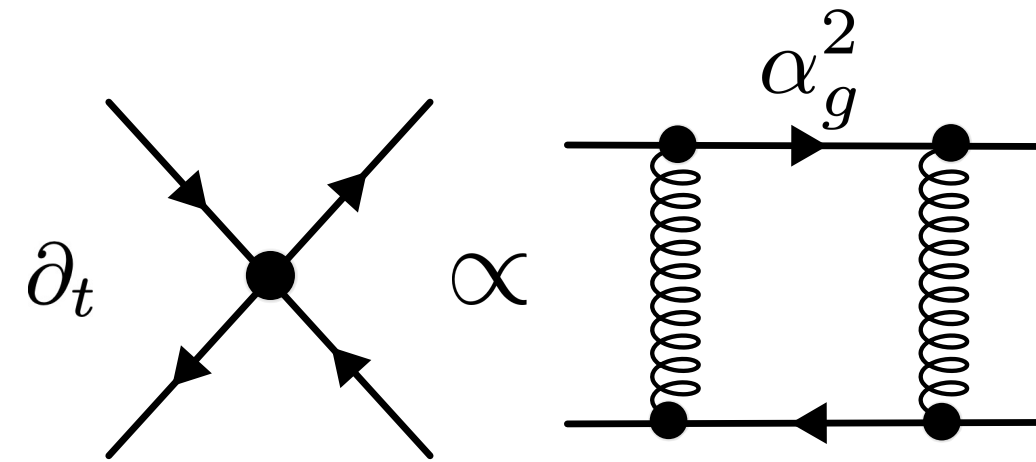
a chiral QCD example

$$\Gamma = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} + \lambda_i (\bar{\psi} \mathcal{T}_i \psi)^2 + \kappa_i (\bar{\psi} \mathcal{T}_i \psi)^3 + \dots$$

# Dynamical symmetry breaking in the effective action

a chiral QCD example

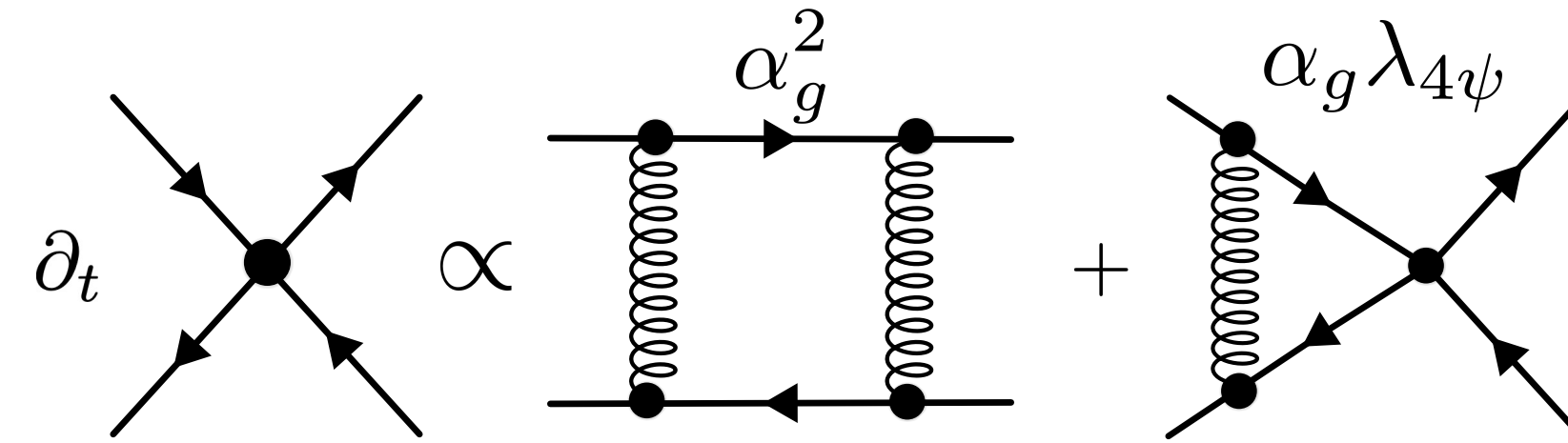
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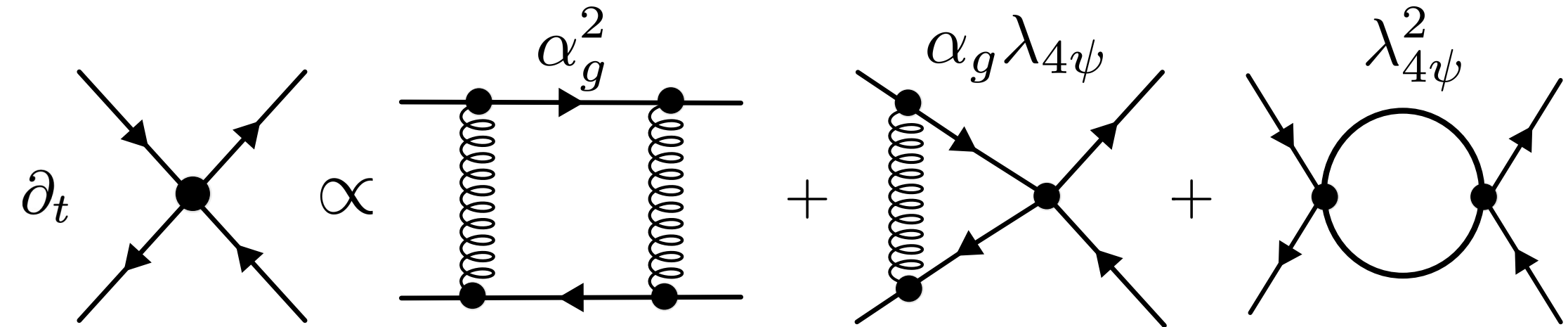




# Dynamical symmetry breaking in the effective action

a chiral QCD example

$$\Gamma = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} + \lambda_i (\bar{\psi} \mathcal{T}_i \psi)^2 + \kappa_i (\bar{\psi} \mathcal{T}_i \psi)^3 + \dots$$

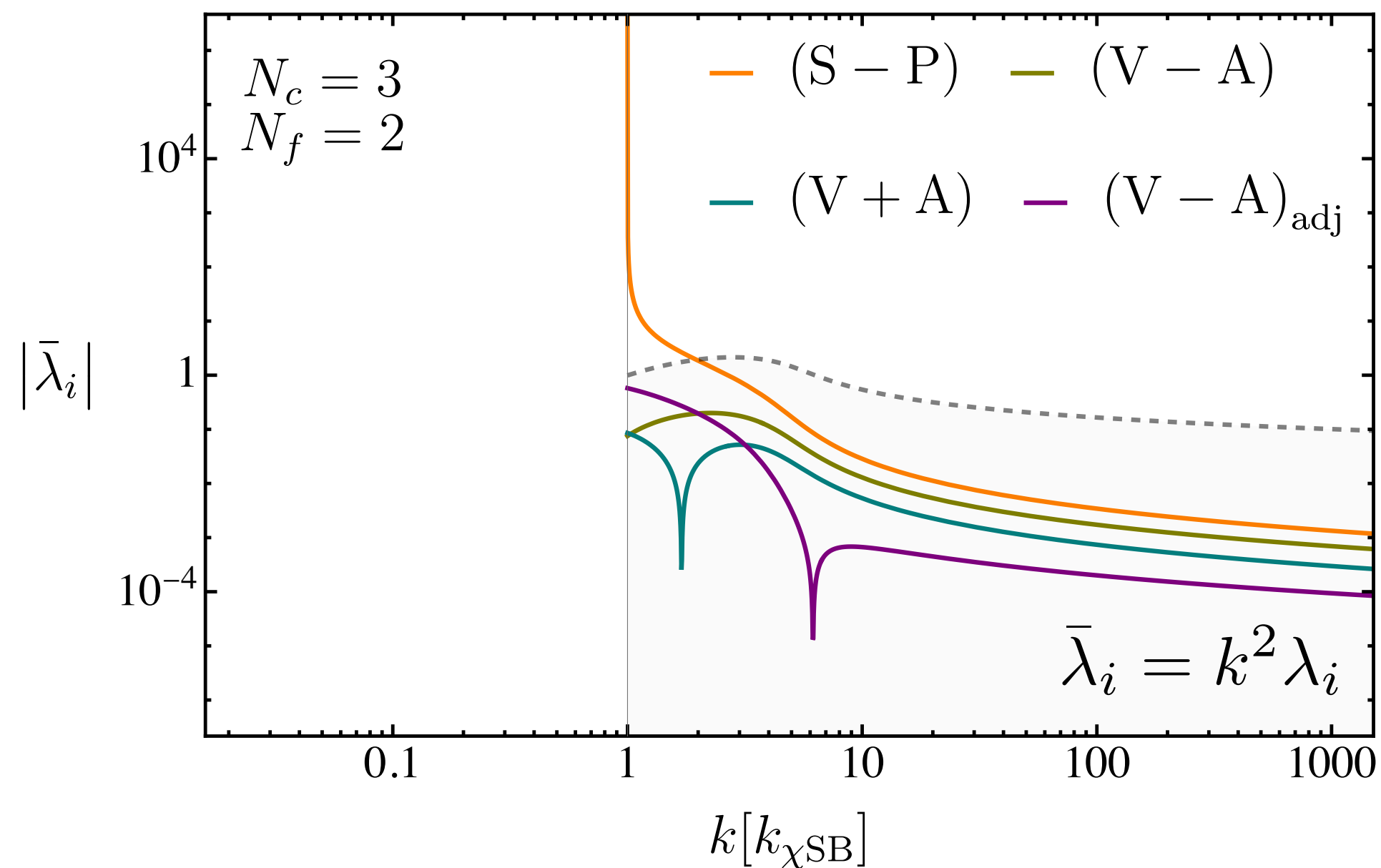
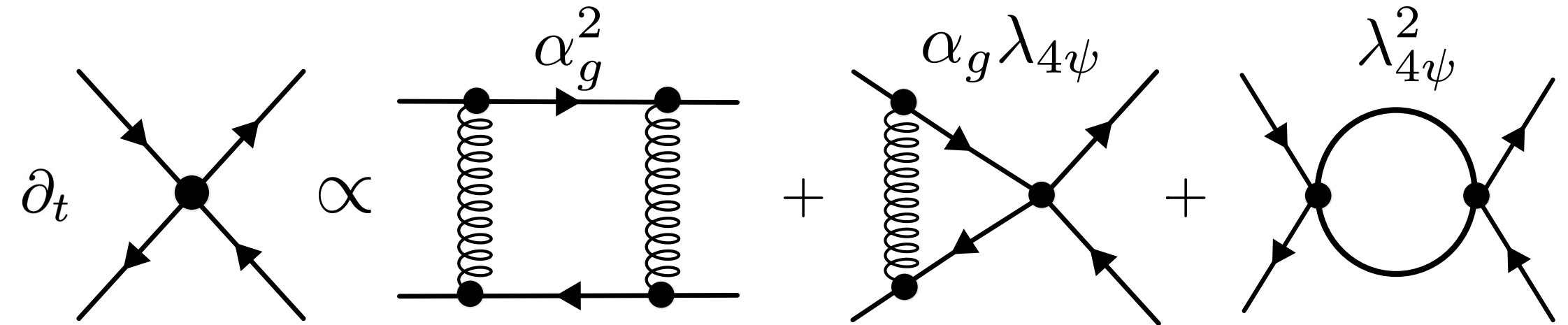




# Dynamical symmetry breaking in the effective action

a chiral QCD example

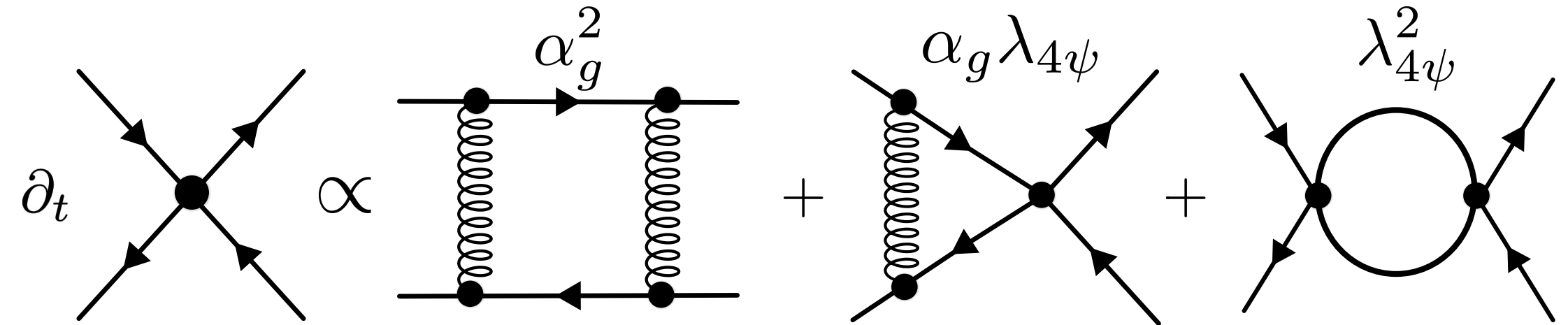
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# Dynamical symmetry breaking in the effective action

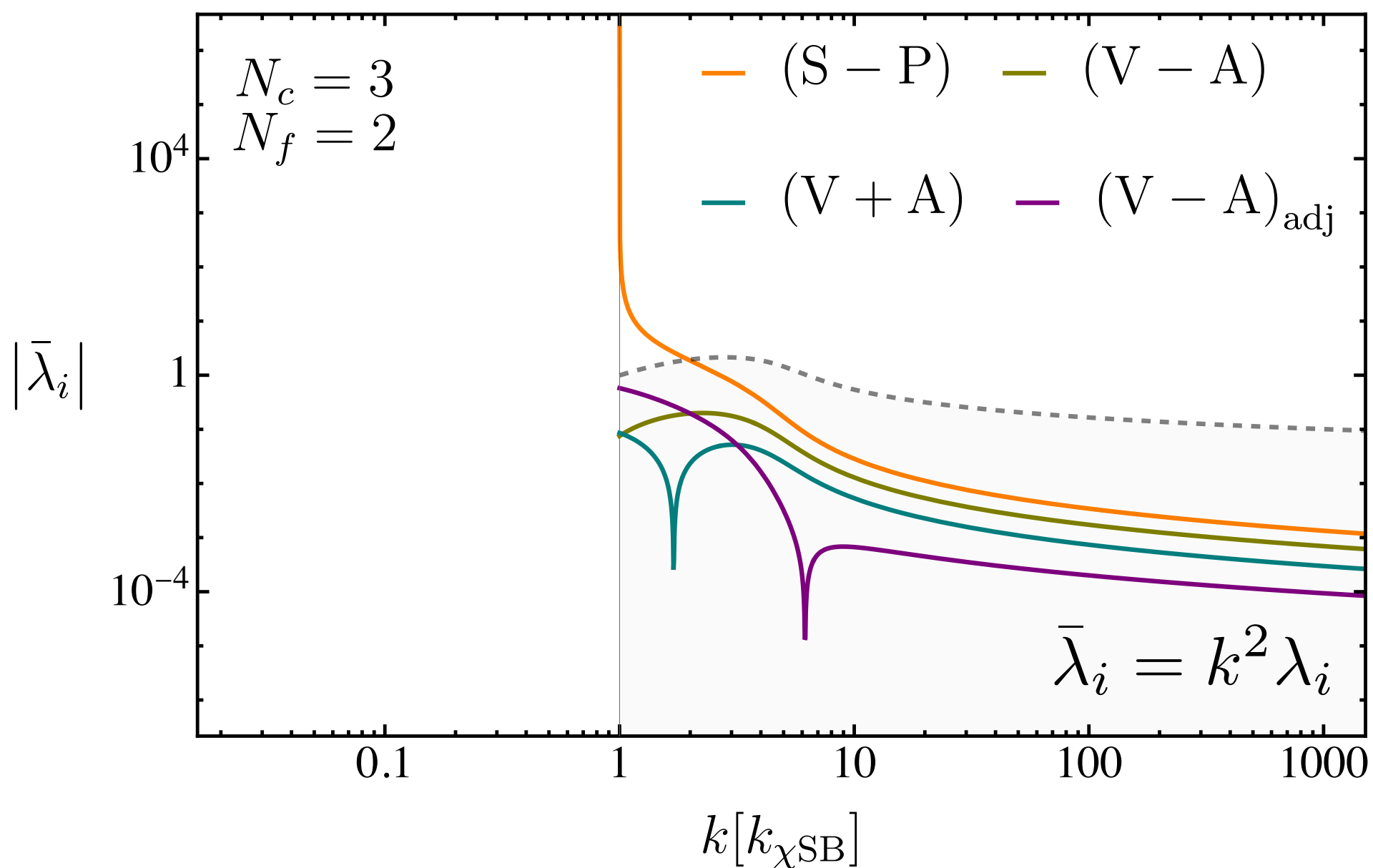
a chiral QCD example

$$\Gamma = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} + \lambda_i (\bar{\psi} \mathcal{T}_i \psi)^2 + \kappa_i (\bar{\psi} \mathcal{T}_i \psi)^3 + \dots$$



Stratonovich'57 Hubbard'59

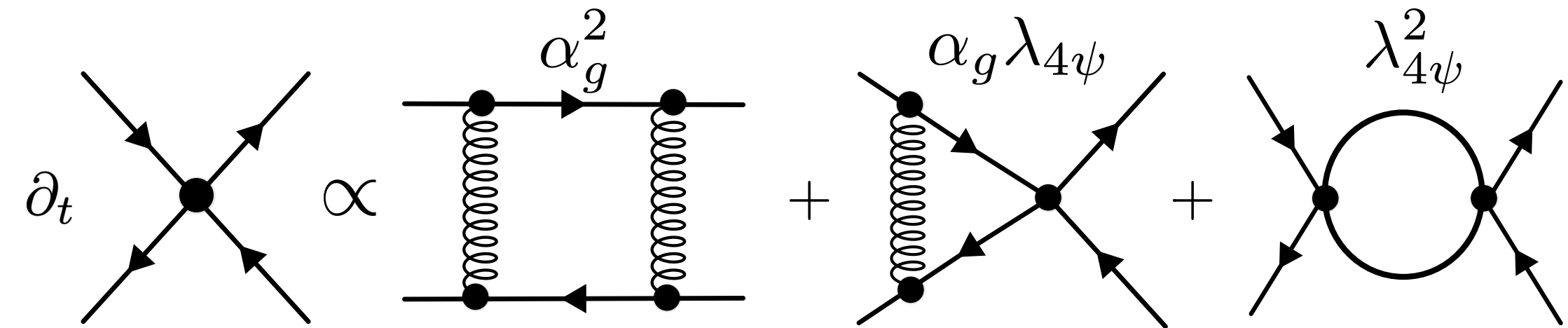
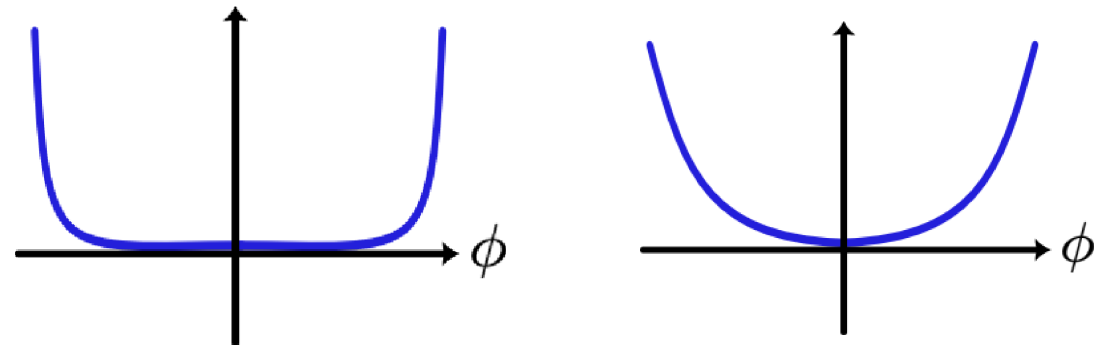
$$\left. \begin{array}{c} \vec{p}_1 \\ \vec{p}_2 \end{array} \right\} \left. \begin{array}{c} \vec{p}_4 \\ \vec{p}_3 \end{array} \right\} \Big| (\phi) = \sum_{\phi \in \{\sigma, \pi^a\}} \left. \begin{array}{c} \vec{p}_1 \\ \vec{p}_2 \end{array} \right\} \left. \begin{array}{c} \vec{p}_4 \\ \vec{p}_3 \end{array} \right\} \Big| \begin{array}{l} (p_1 + p_3)^2 = 0 \\ (p_2 + p_4)^2 = 0 \end{array}$$



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$$\Gamma = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} + \lambda_i (\bar{\psi} \mathcal{T}_i \psi)^2 + \kappa_i (\bar{\psi} \mathcal{T}_i \psi)^3 + \dots$$



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$$\lambda_i \propto 1/m_{\phi_i}^2$$

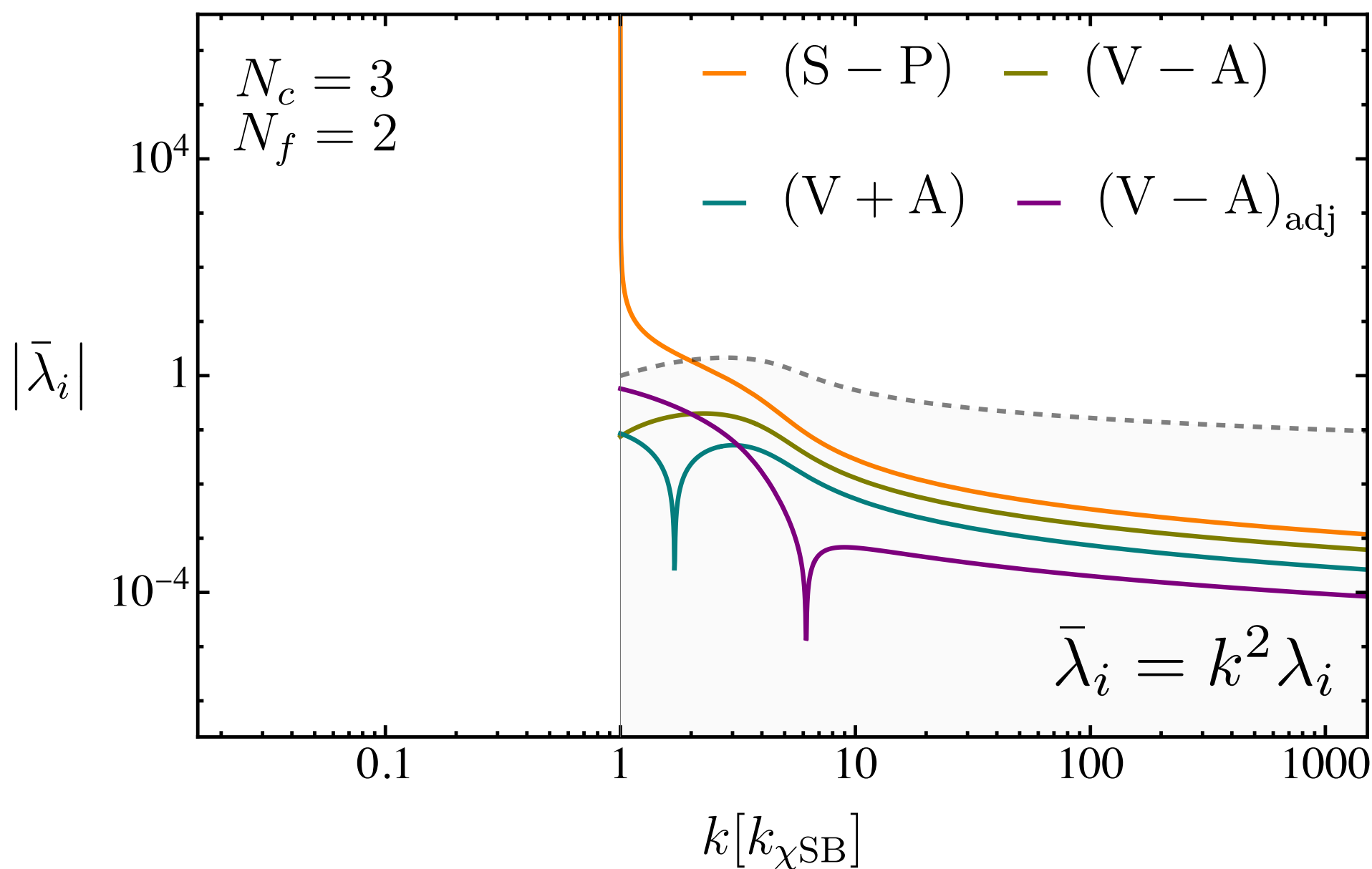
$$m_{\phi_i}^2 = \partial_{\phi_i^2} V(\phi_i)$$

$$\lambda_i \rightarrow \infty \quad m_{\phi_i}^2 = 0$$

Infinite correlation length:  
 $\xi \rightarrow \infty$

Phase transition

Nambu, Jona-Lasinio'60'61



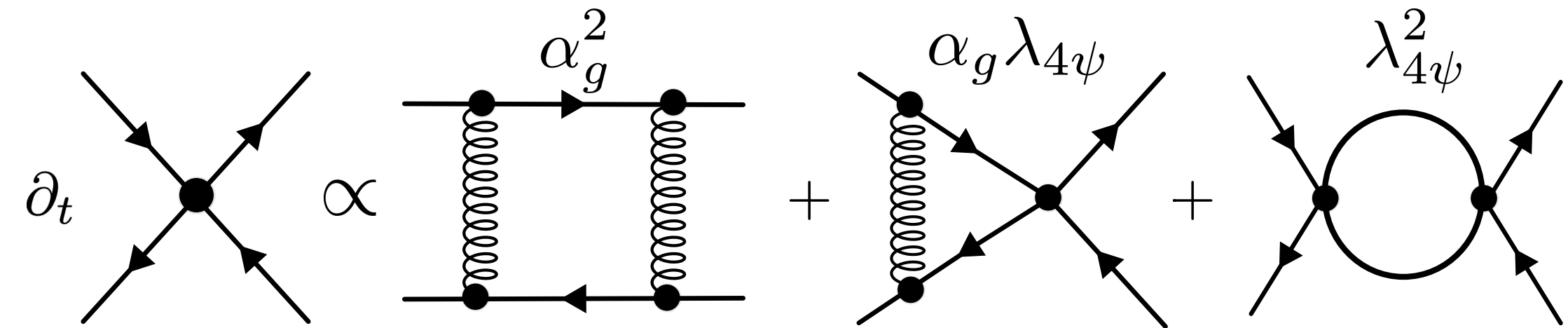
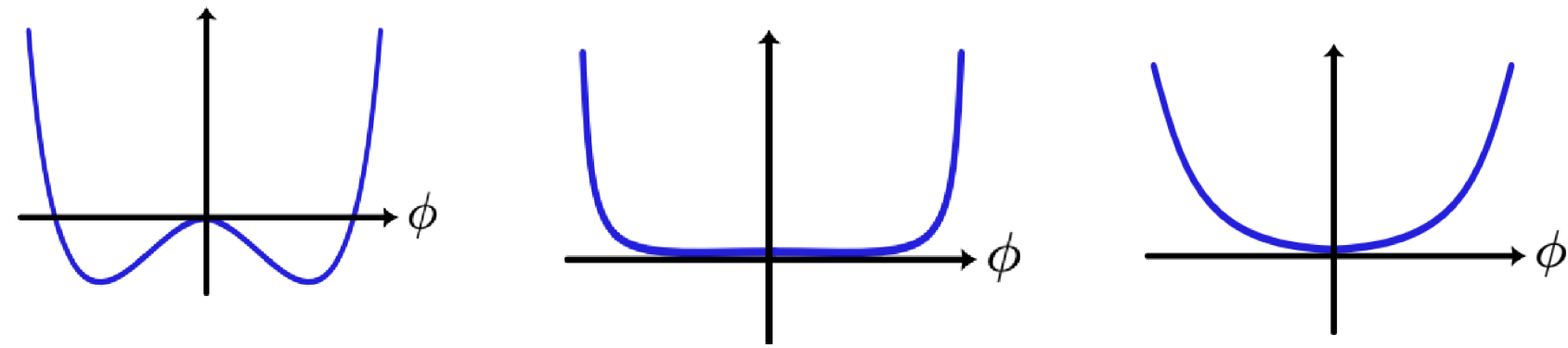


# Dynamical symmetry breaking in the effective action

a chiral QCD example

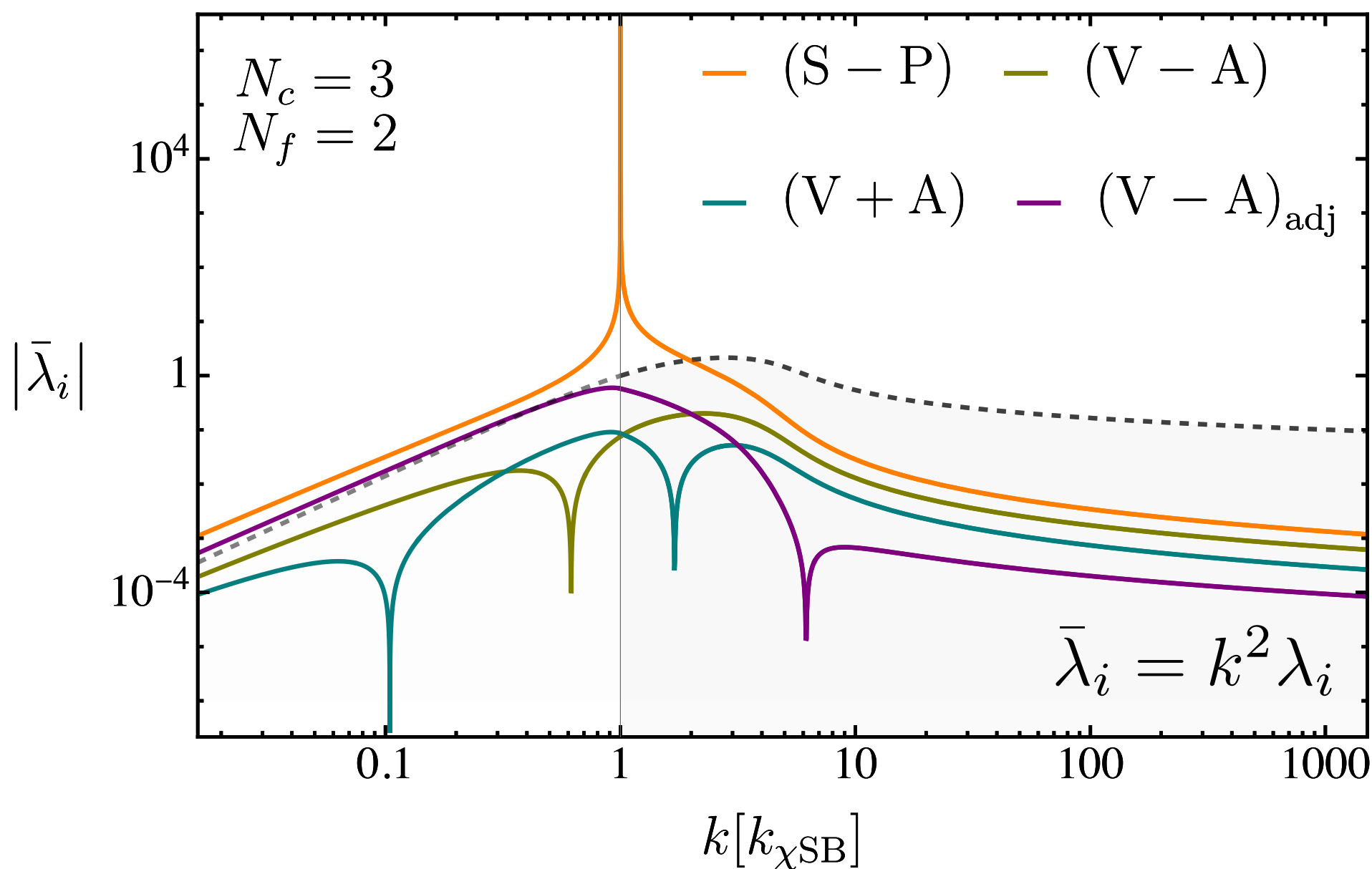
$$\Gamma = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} + \lambda_i (\bar{\psi} \mathcal{T}_i \psi)^2 + \kappa_i (\bar{\psi} \mathcal{T}_i \psi)^3 + \dots$$

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \rightarrow SU(N_f)_V$$



Stratonovich'57 Hubbard'59

$$\left. \begin{array}{c} \vec{p}_1 \\ \vec{p}_2 \end{array} \right| \left. \begin{array}{c} \vec{p}_4 \\ \vec{p}_3 \end{array} \right| (\phi) = \sum_{\phi \in \{\sigma, \pi^a\}} \left. \begin{array}{c} \vec{p}_1 \\ \vec{p}_2 \end{array} \right| \left. \begin{array}{c} \vec{p}_4 \\ \vec{p}_3 \end{array} \right| \left. \begin{array}{c} (p_1 + p_3)^2 = 0 \\ (p_2 + p_4)^2 = 0 \end{array} \right|$$



$$\lambda_i \propto 1/m_{\phi_i}^2$$

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Nambu, Jona-Lasinio'60'61

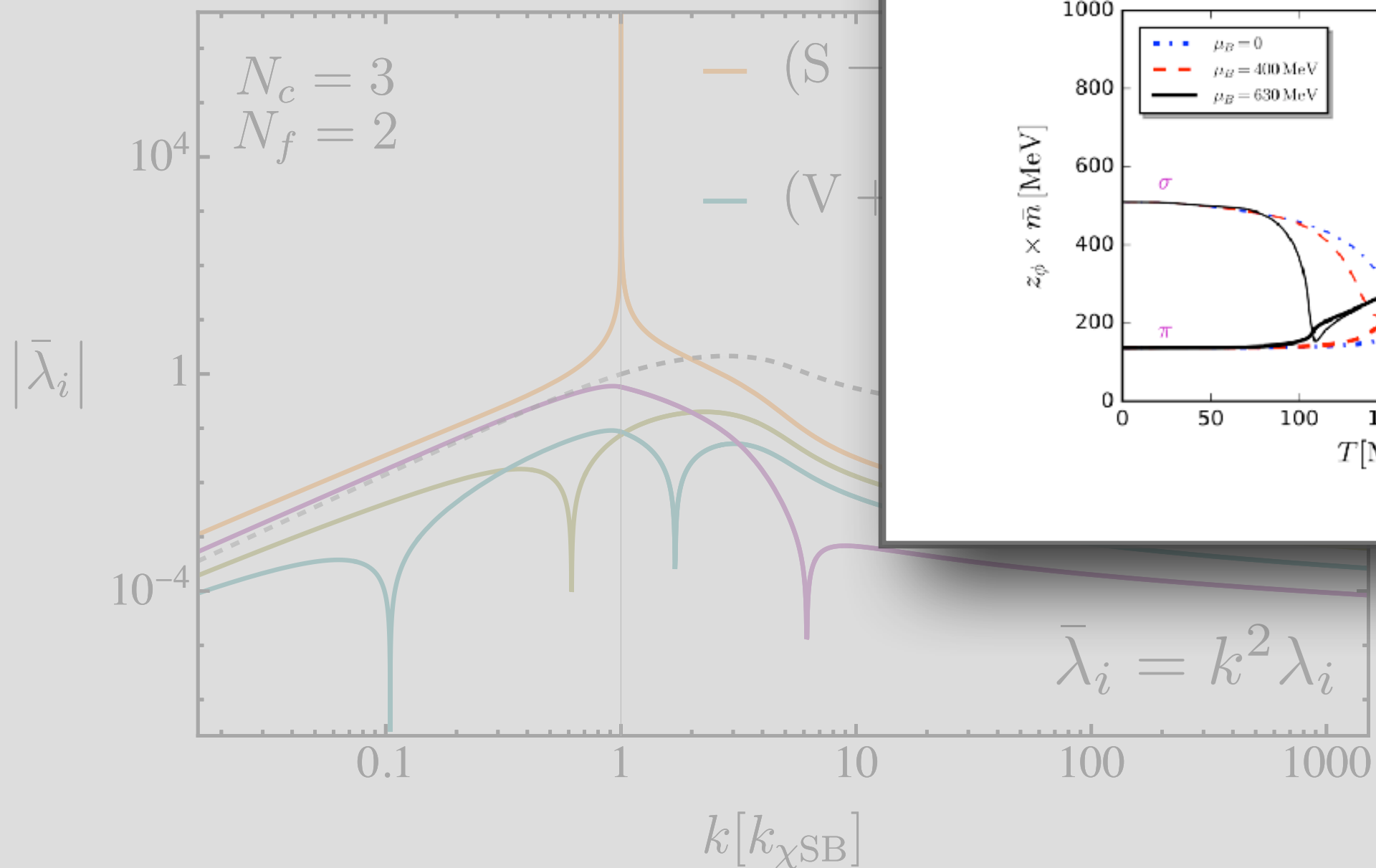
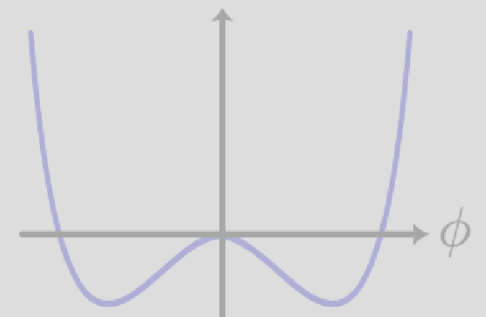


# Dynamical scale effective action

a chiral QCD example

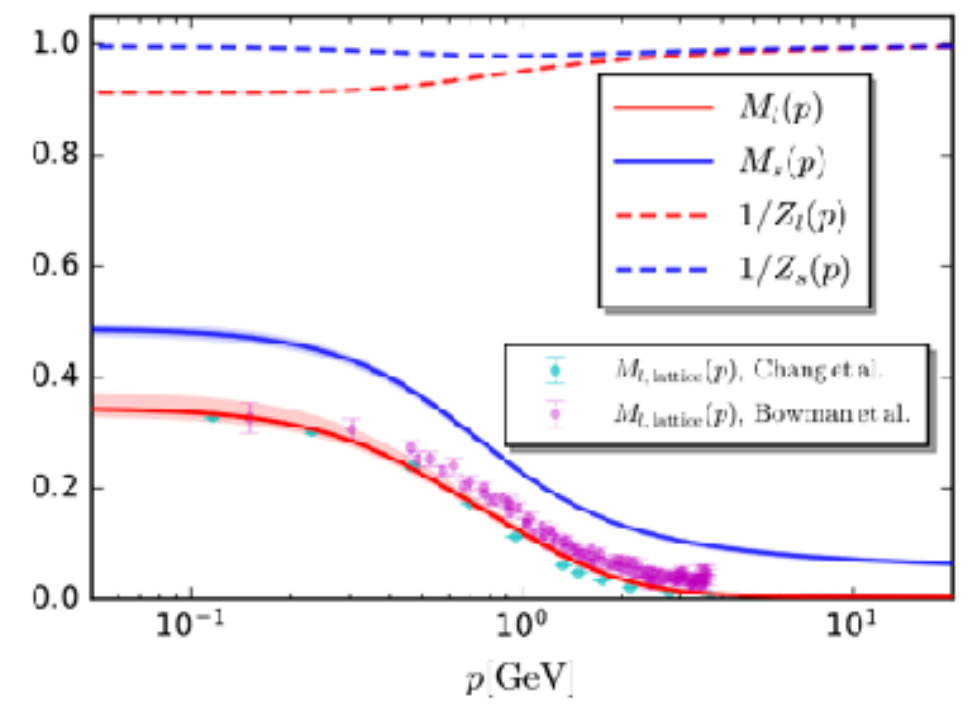
$$\Gamma = \int x \frac{1}{4}$$

$$SU(N_f)_L \times SU(N_f)_R \times U$$

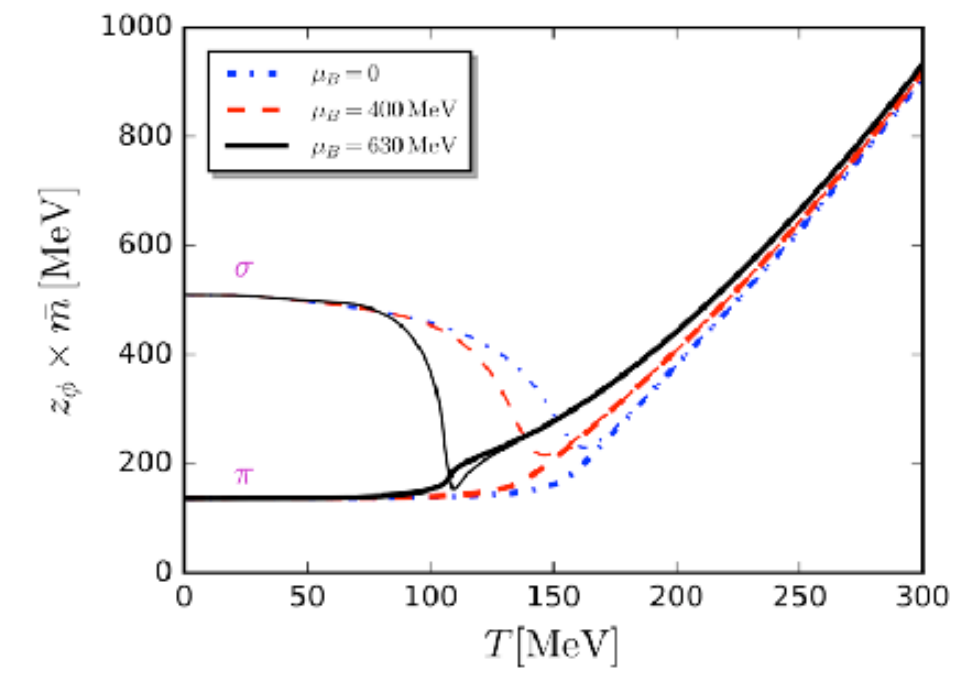


## Scale-dependent field transformations

Fu,Huang,Pawlowski,Tan,Zhou[2502.14388]

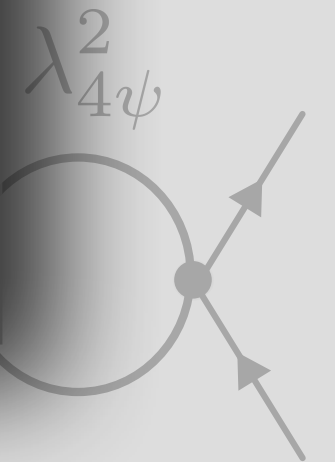


Fu,Pawlowski,Rennecke [1909.02991]

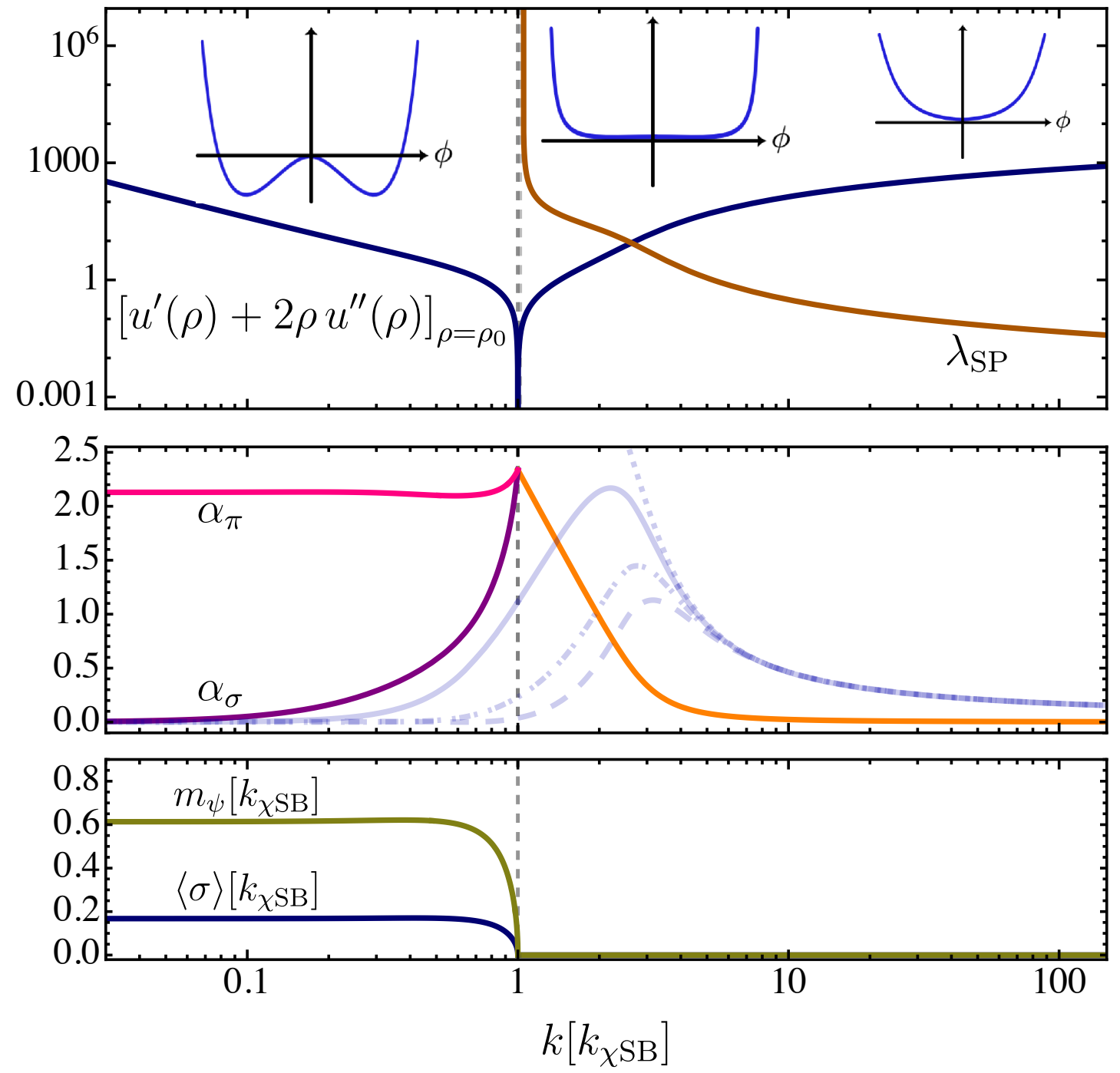


Generalised flow

$$\left( \partial_t + \int \dot{\phi}_c \frac{\delta}{\delta \phi_c} \right) \Gamma_k[\phi]$$
$$= \frac{1}{2} \text{Tr} \left[ \left( \frac{1}{\Gamma_k^{(2)} + R_k} \right)_{ij} \left( \partial_t \delta^{jn} + 2 \frac{\delta \dot{\phi}_n}{\delta \phi_j} \right) R_k^{mi} \right]$$



Goertz,APG,Pawlowski[2412.12254]



7 Hubbard'59

$$(p_1 + p_3)^2 = 0$$
$$(p_2 + p_4)^2 = 0$$

$$V(\phi_i)$$

relation length:

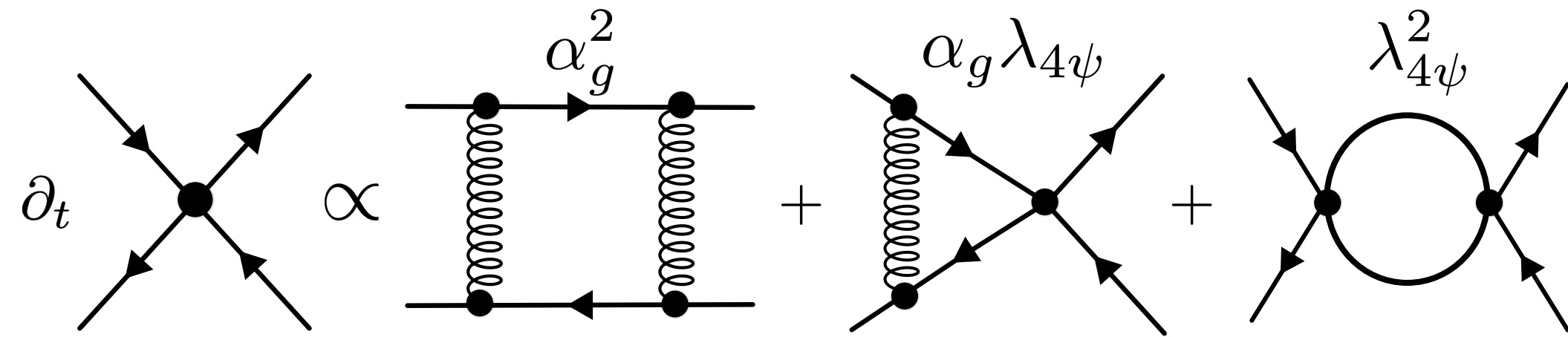
$$\rightarrow \infty$$

ona-Lasinio'60'61



# Mechanism underlying dSB

Nambu, Jona-Lasinio '60'61 Kosterlitz '74 Miransky '85 Jungnickel, Wetterich [hep-ph/9505267] Gies, Jaeckel [hep-ph/0507171] B. Kaplan, Lee, T. Son, Stephanov [0905.4752] Braun [1108.4449]



$$\partial_t \bar{\lambda}_i \propto 2 \bar{\lambda}_i + c_{A,i} \cdot \alpha_g^2 + c_{B,ij} \cdot \alpha_g \bar{\lambda}_j + c_{C,ijk} \cdot \bar{\lambda}_j \bar{\lambda}_k + \dots$$

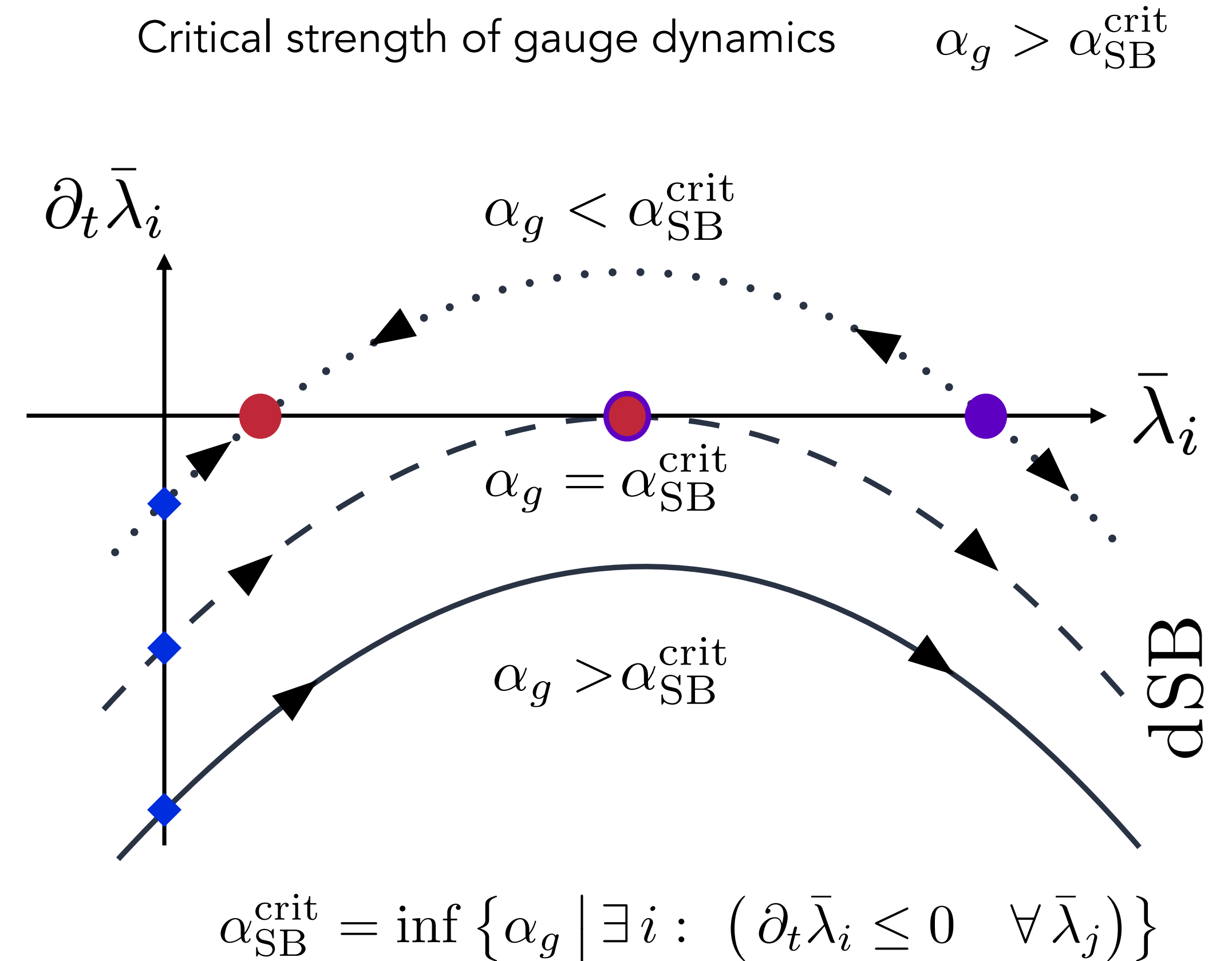
- Necessary conditions for dSB (FP merger):

- Resonant structure:

$$\frac{c_{A,i}}{c_{C,iii}} > 0$$

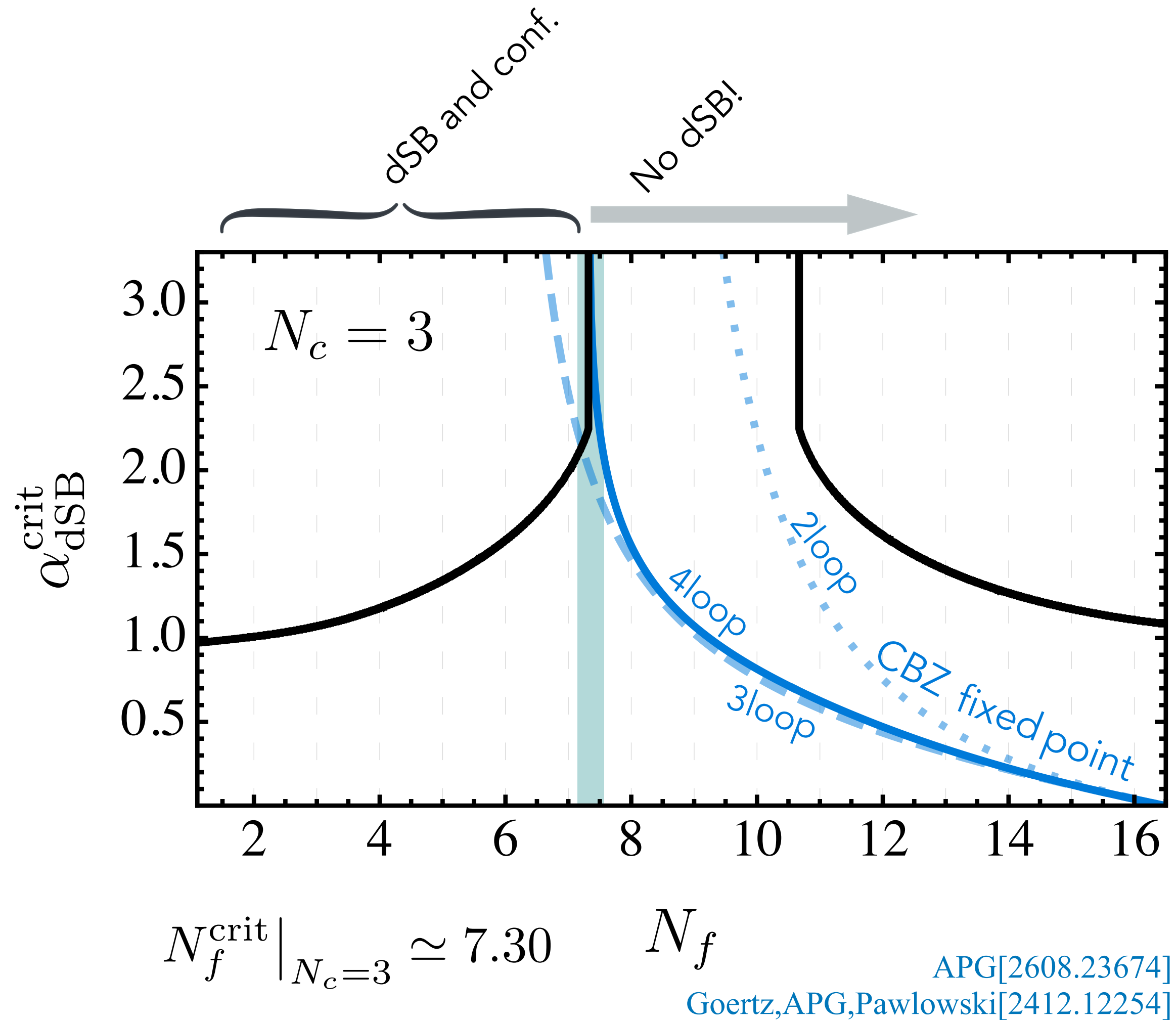
- Critical strength of gauge dynamics is reached:

$$\alpha_g > \alpha_{\text{SB}}^{\text{crit}}$$



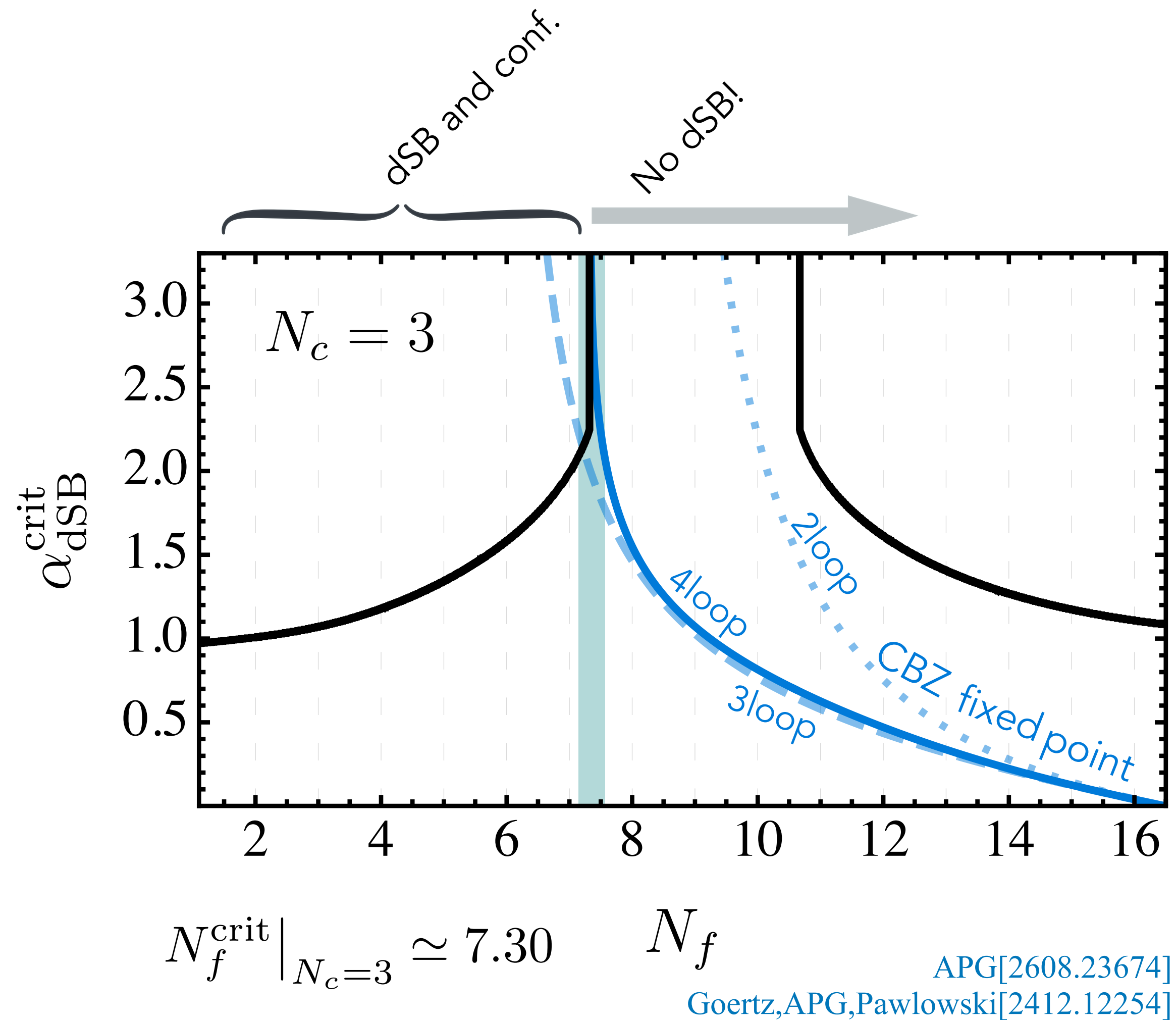
Goertz, APG, Pawłowski [2412.12254]  
Li, APG, Vatani, Xu [2507.21208]

# Transition to the conformal window



- Self-consistent fRG **prediction** of the conformal window
  - Non-perturbative effects at the critical theory
    - ★ Absence of walking regimes and Miransky/BKT scaling
    - ★ 1st order quantum phase transition
  - Symmetric mass gap and conf. wo/ dSB @ the critical theory?!

# Transition to the conformal window



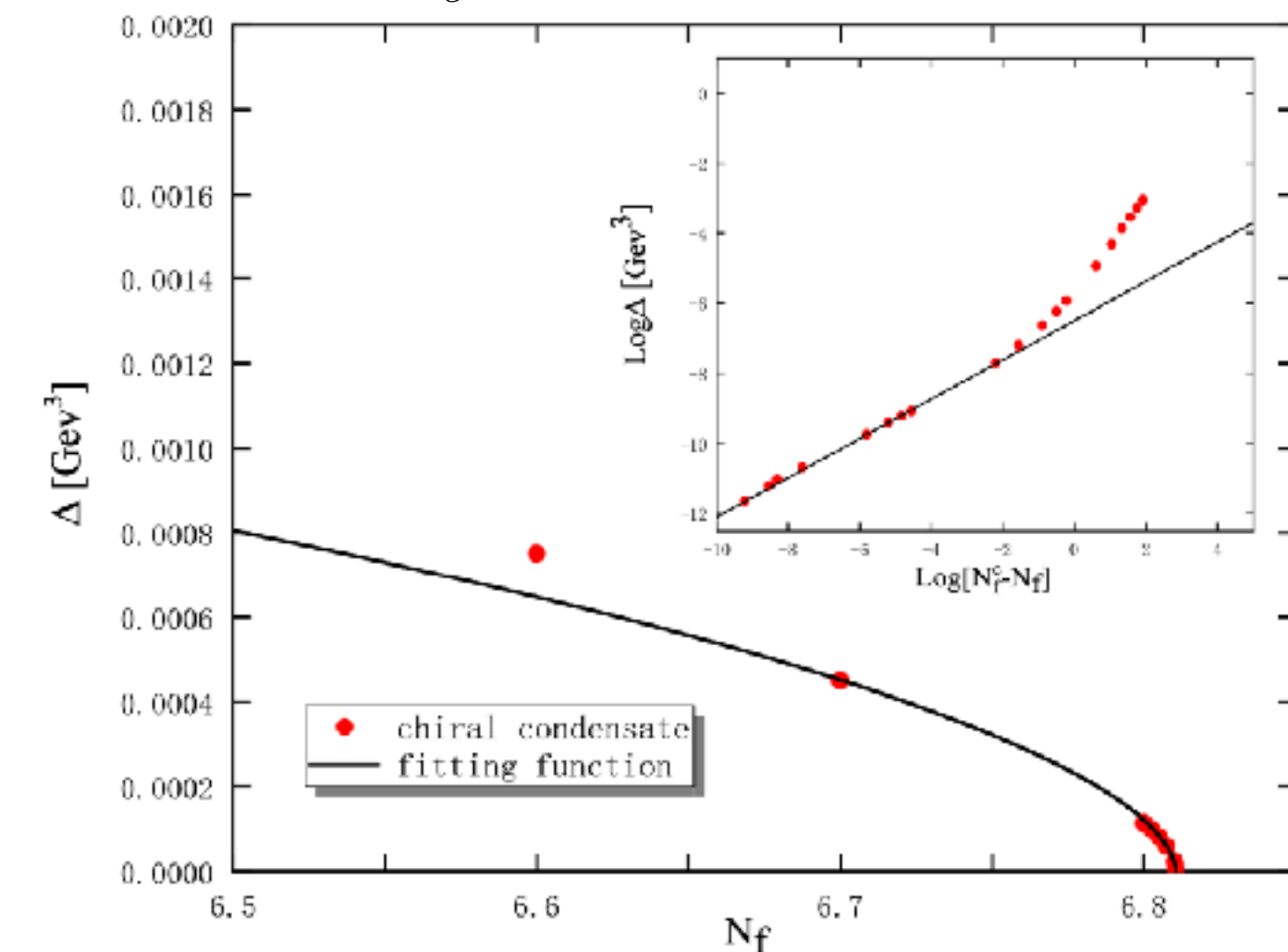
Recent news from other methods:

Lattice computations:

$$N_f^{\text{crit}}|_{N_c=3} \simeq (7 - 8) \quad \text{Hasenfratz,Rebbi,Witzel[2210.16760]} \\ \text{D'Elia,Naccia,Zambello[2604.0728],...}$$

SDE:

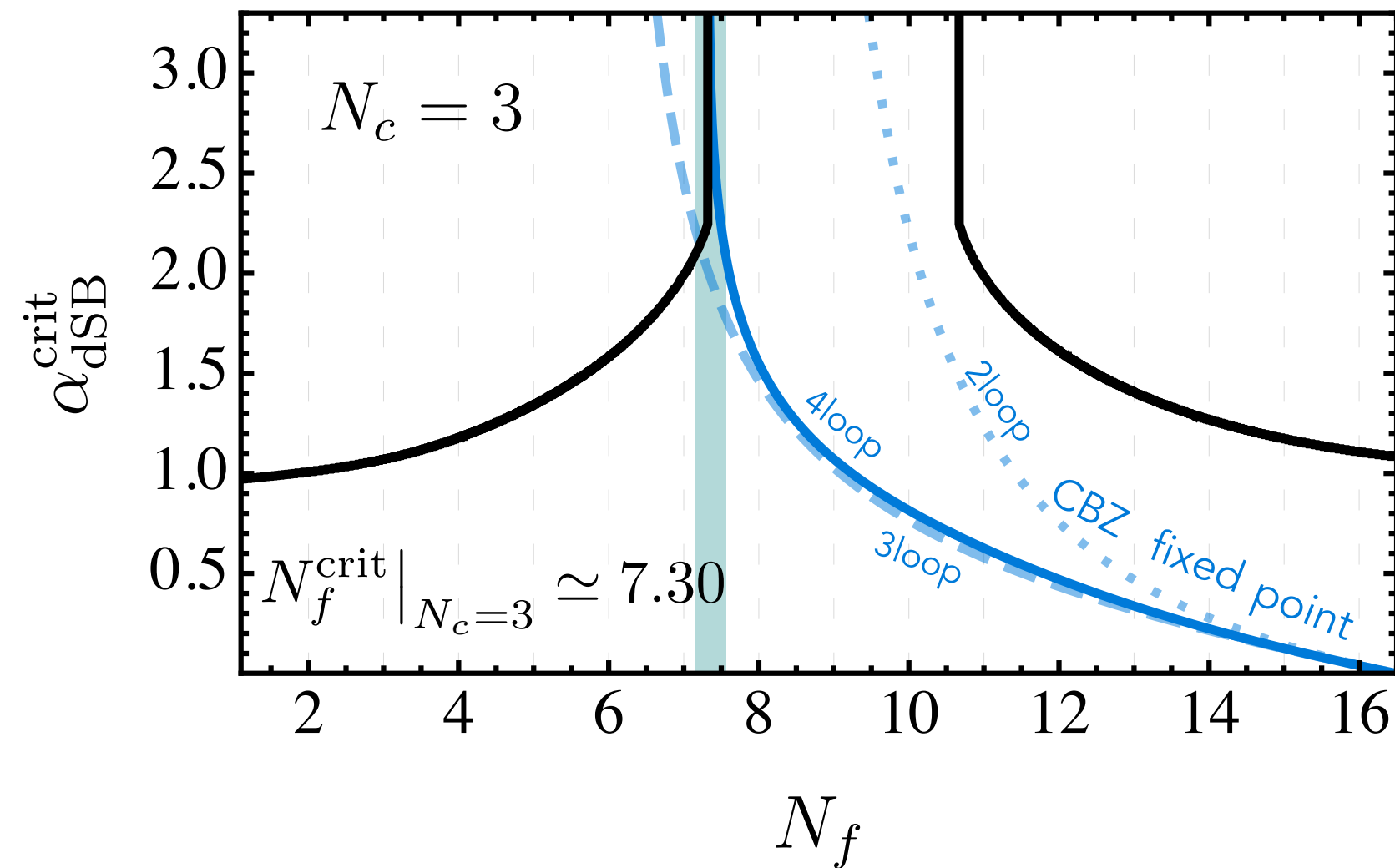
$$N_f^{\text{crit}}|_{N_c=3} \simeq 6.81$$



Chen,Lu,Wang,Liu,Gao[2602.14489]



# Quantum phase transition and near-critical region



- Disfavoured by divergent  $\alpha_g^{\text{crit}}$ :
  - Miransky/BKT scaling
  - walking theories
- Symmetric mass gaps and confinement without dSB
  - Recap: no dSB at
  - Colour confinement
  - Symmetric propagator and gap

$$\Gamma_k^{(\bar{\psi}\psi)}(p) = Z_{\psi,k}(p) i\gamma_\mu p_\mu \quad \text{with} \quad \lim_{p \rightarrow 0} |\Gamma_{k \rightarrow 0}^{(\bar{\psi}\psi)}(p)|^{-1} < \infty \quad \longrightarrow \quad Z_{\psi,k \rightarrow 0}(p) \propto p^{-(1+\varepsilon)} \quad \text{with} \quad \varepsilon \geq 0$$

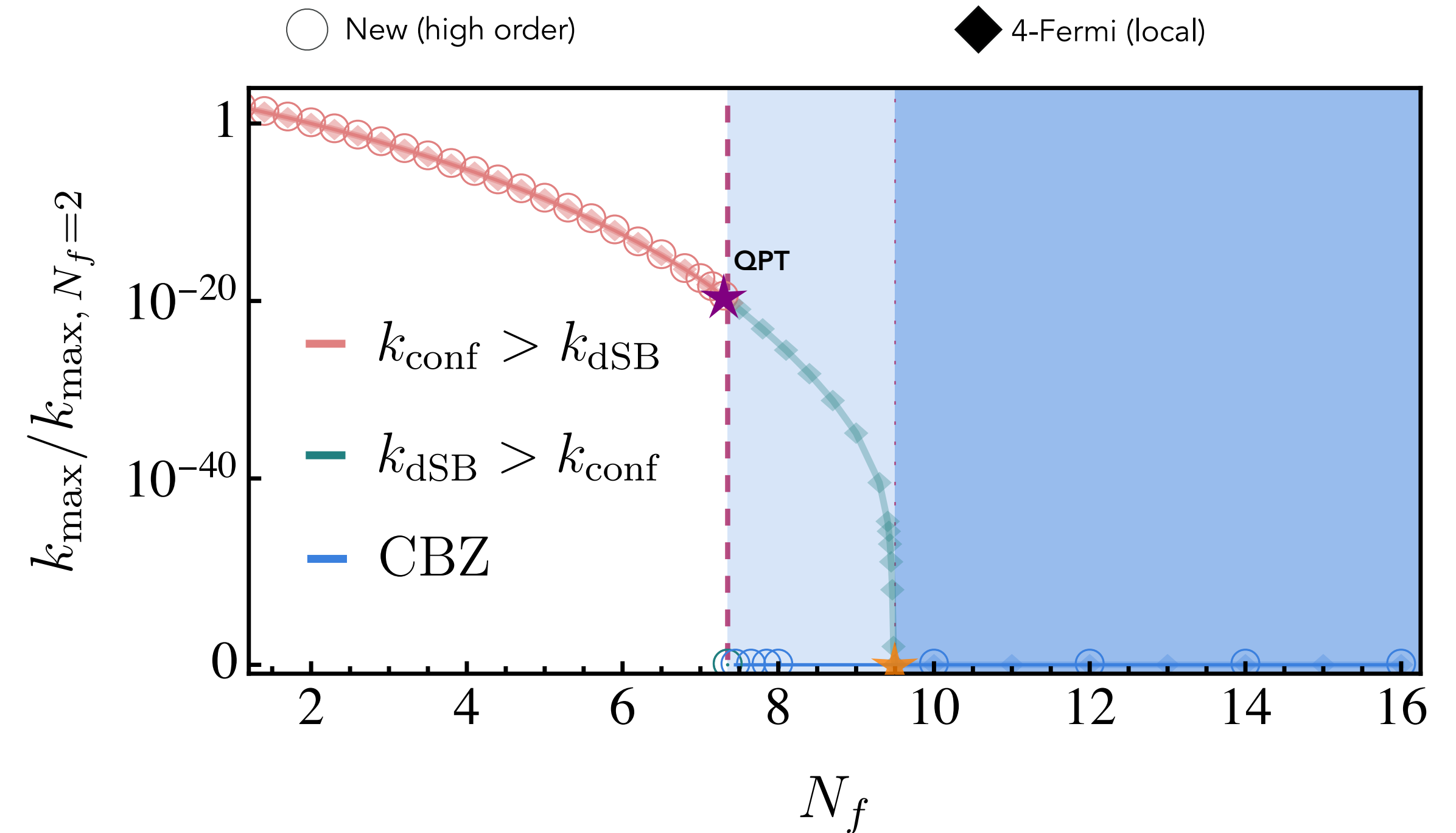
$$\eta_{\psi,k \rightarrow 0}(p=0) \geq 1$$

Is satisfied for:

$$\eta_\psi|_{\alpha_g^{\text{crit}}} \geq 1 \quad \text{for} \quad N_f > 7.2$$

and

$$\eta_\psi|_{\alpha_g^*} \geq 1 \quad \text{for} \quad N_f < \left( 7.3|_{3\text{loop}}, 7.6|_{4\text{loop}} \right)$$



# Chiral gauge theories

$$S = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \psi^\dagger \bar{\sigma}^\mu D_\mu \psi + \chi^\dagger \bar{\sigma}^\mu D_\mu \chi$$

Chiral: Weyl fermions, only left (right)-handed

- Phenomenology:
  - SM and BSM physics (GUTs, preon models, ...)
    - Georgi,Glashow'74 Raby,Dimopoulos,Susskind '79 Bars,Yankielowitz'81
    - Bolognesi,Konishi,Orso[2605.10416] Cacciapaglia,Sannino,Wagner [2605.08294]
- Theory:
  - unaccessible** to lattice formulations (doubler & sign problems)
  - conjectured **rich** and **unexplored** dynamics
    - ★ Exotic condensates
    - ★ *tumbling*: recursive patterns of SB
      - Wang,You[2204.14271]
      - Karasik,Önder,Tong[2208.07842]
      - Tong[2104.03997]
    - ★ Symmetric Mass Generation

|        | $SU(N_c)$       | $SU(N_c - 4)$ | $U(1)$       |
|--------|-----------------|---------------|--------------|
| $\psi$ | $\bar{\square}$ | $\square$     | $-(N_c - 2)$ |
| $\chi$ | $\square$       | 1             | $N_c - 4$    |

Bars-Yankielowitz (BY)

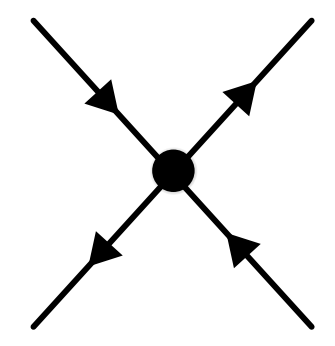
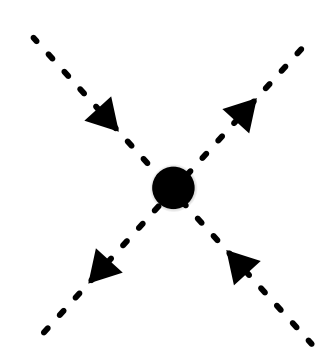
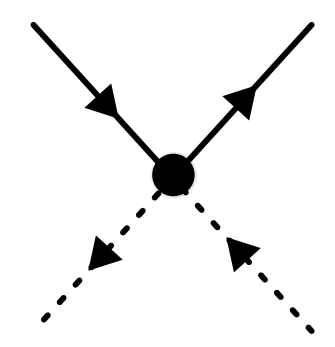
|        | $SU(N_c)$       | $SU(N_c + 4)$ | $U(1)$       |
|--------|-----------------|---------------|--------------|
| $\psi$ | $\bar{\square}$ | $\square$     | $-(N_c - 2)$ |
| $\chi$ | $\square$       | 1             | $N_c + 4$    |

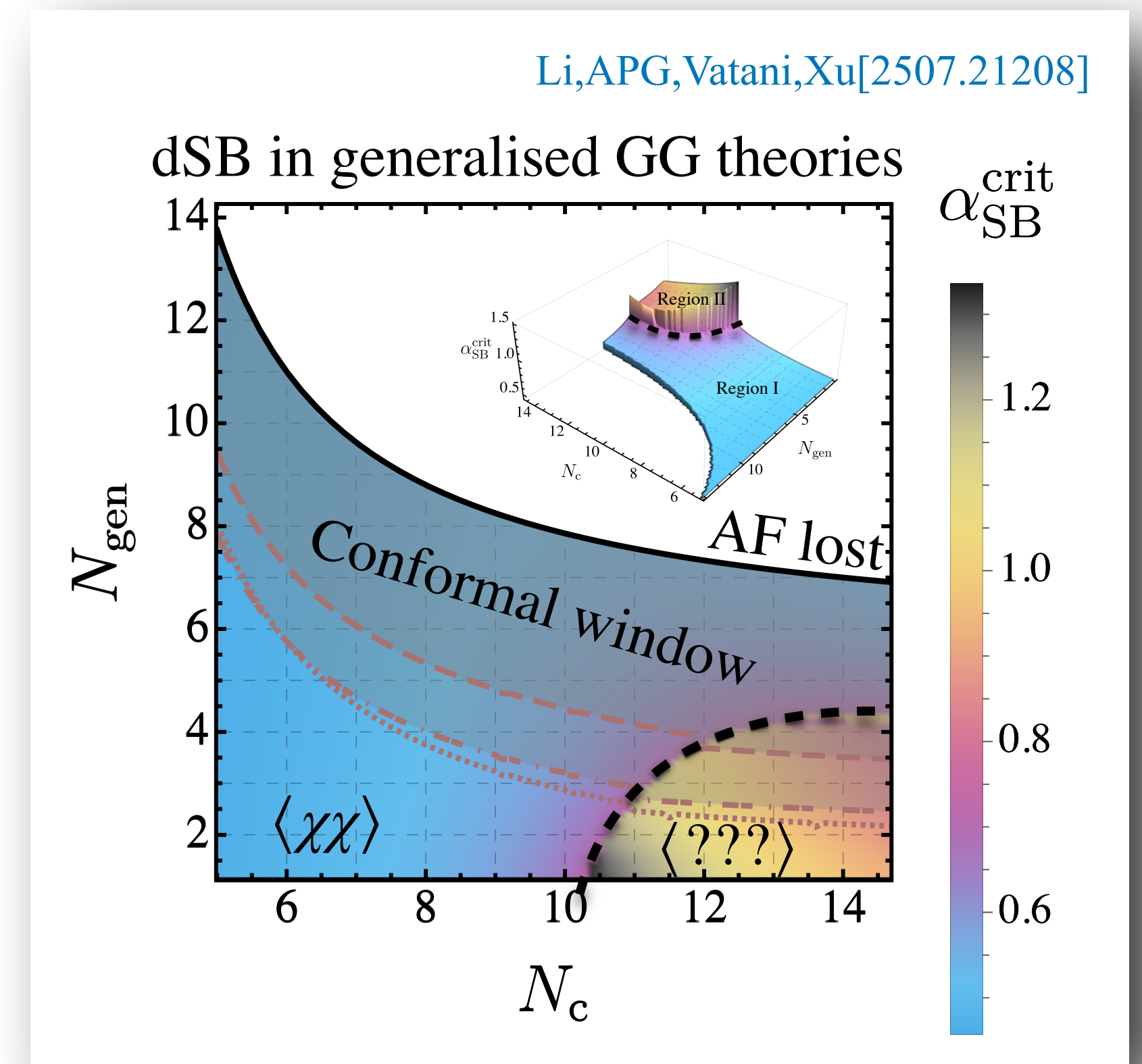
A task for functional approaches!



# Fierz-complete four-fermion basis in chiral gauge theories

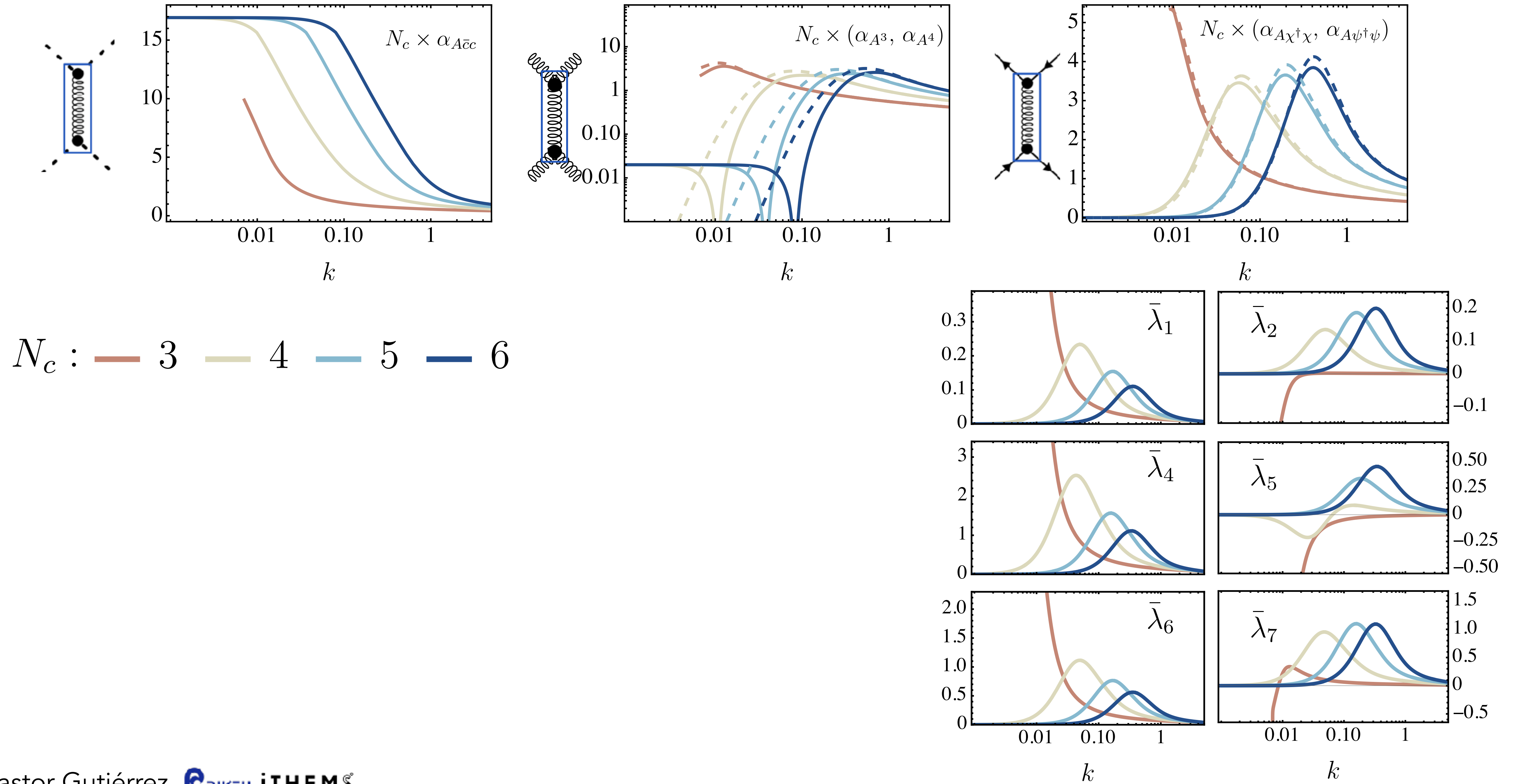
$$\Gamma_{\text{fermion},k}[\chi^\dagger, \chi, \psi^\dagger, \psi] = - \int_x \underbrace{Z_\psi^2 \sum_{i=1}^2 \lambda_i \mathcal{O}_i^\psi}_{\text{blue}} + \underbrace{Z_\chi^2 \sum_{i=4}^5 \lambda_i \mathcal{O}_i^\chi}_{\text{teal}} + \underbrace{Z_\psi Z_\chi \sum_{i=6}^7 \lambda_i \mathcal{O}_i^{\chi\psi}}_{\text{red}}$$

|                                                                                                                                      |   |                                                                                                       |
|--------------------------------------------------------------------------------------------------------------------------------------|---|-------------------------------------------------------------------------------------------------------|
|                                                     | { | $\mathcal{O}_1^\psi = (\psi^\dagger \bar{\sigma}^\mu \psi)(\psi^\dagger \bar{\sigma}^\mu \psi)$       |
| $\mathcal{O}_2^\psi = (\psi^\dagger f_1 \bar{\sigma}^\mu \psi_{f_2})(\psi^\dagger f_2 \bar{\sigma}^\mu \psi_{f_1})$                  |   |                                                                                                       |
|                                                   | { | $\mathcal{O}_4^\chi = (\chi^\dagger \bar{\sigma}^\mu \chi)(\chi^\dagger \bar{\sigma}^\mu \chi)$       |
| $\mathcal{O}_5^\chi = (\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi)(\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi)$        |   |                                                                                                       |
|                                                   | { | $\mathcal{O}_6^{\chi\psi} = (\psi^\dagger \bar{\sigma}^\mu \psi)(\chi^\dagger \bar{\sigma}^\mu \chi)$ |
| $\mathcal{O}_7^{\chi\psi} = (\psi^\dagger \bar{\sigma}^\mu T_{\text{anti}} \psi)(\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi)$ |   |                                                                                                       |



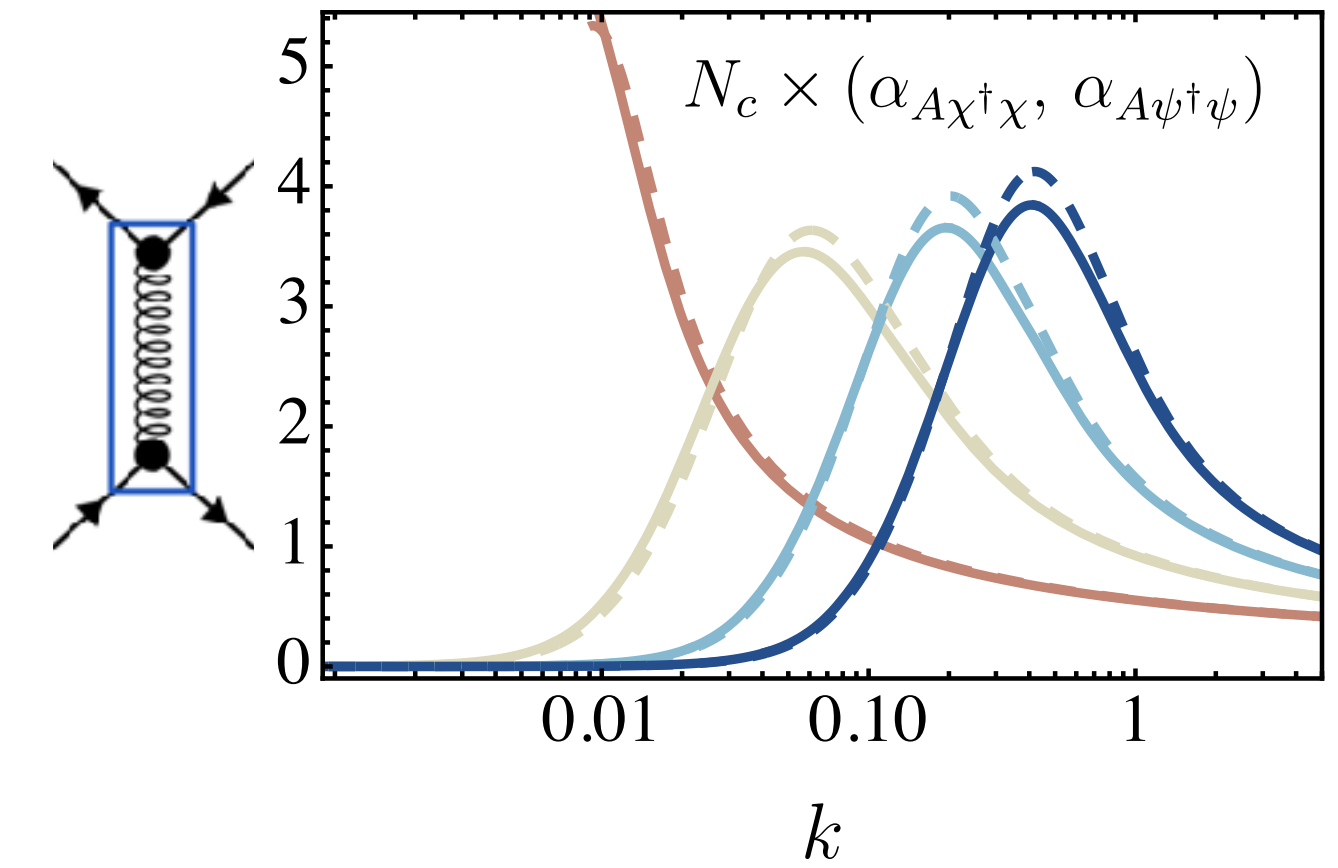
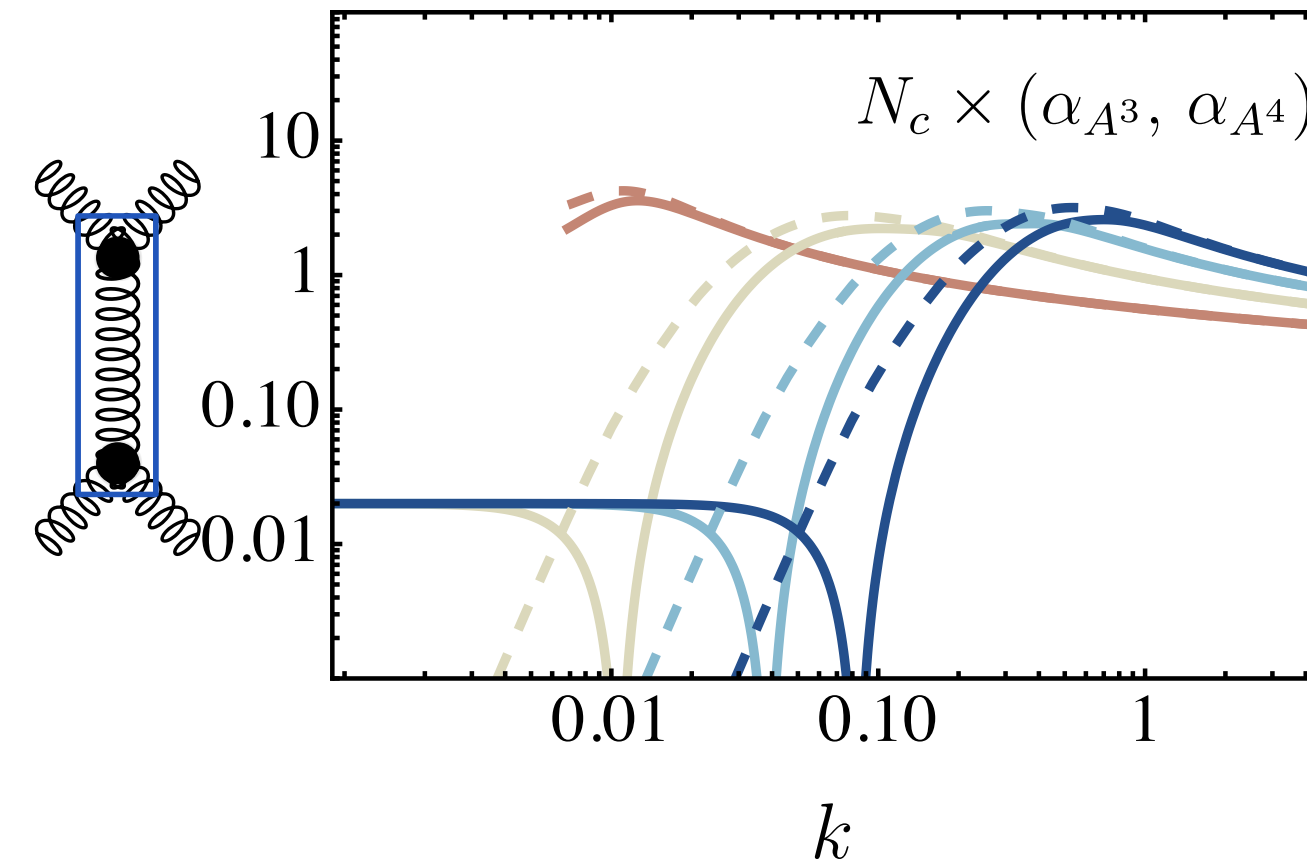
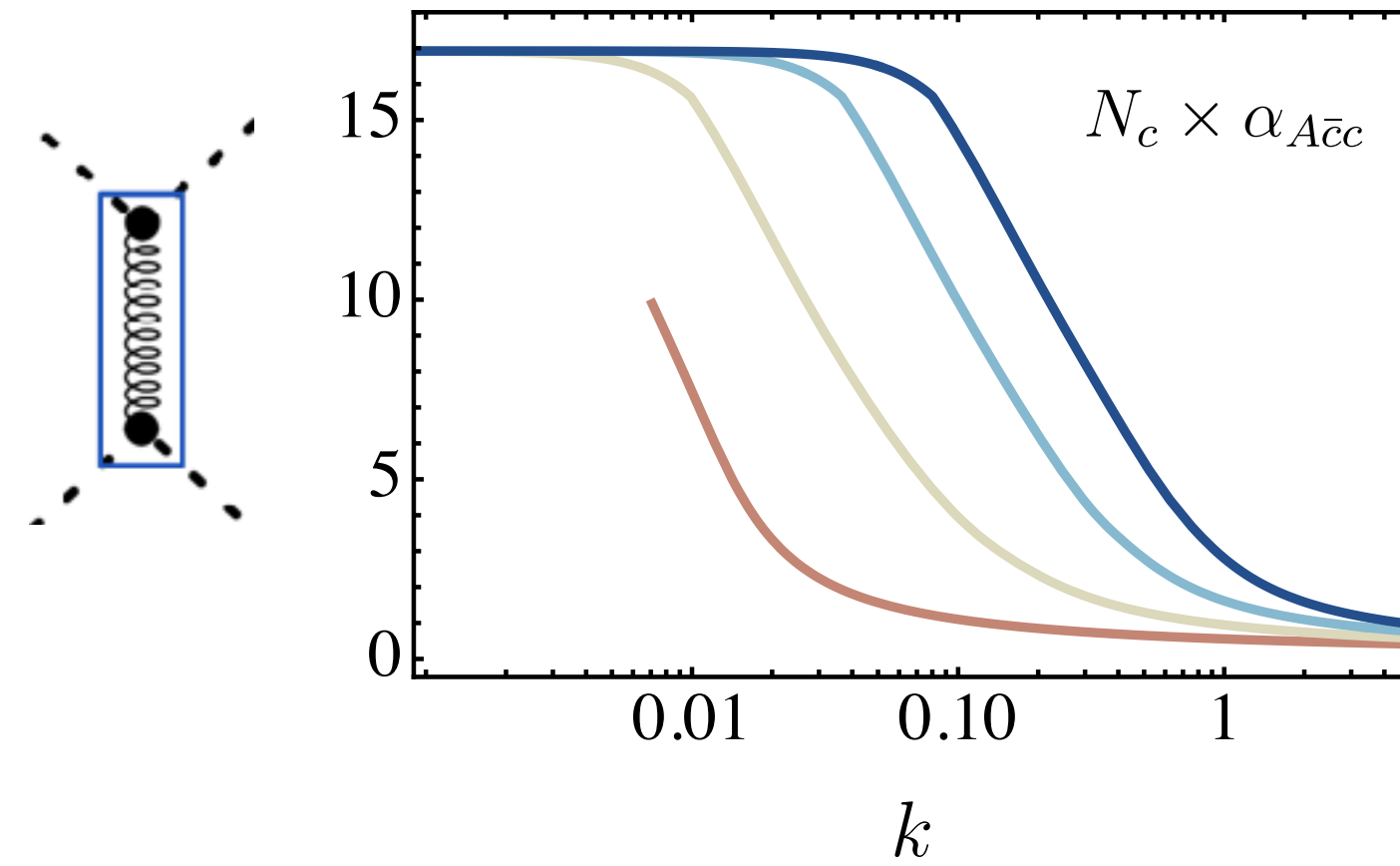
# Colour-confinement and dSB in BY

Li,APG,Vatani[2603.19355]



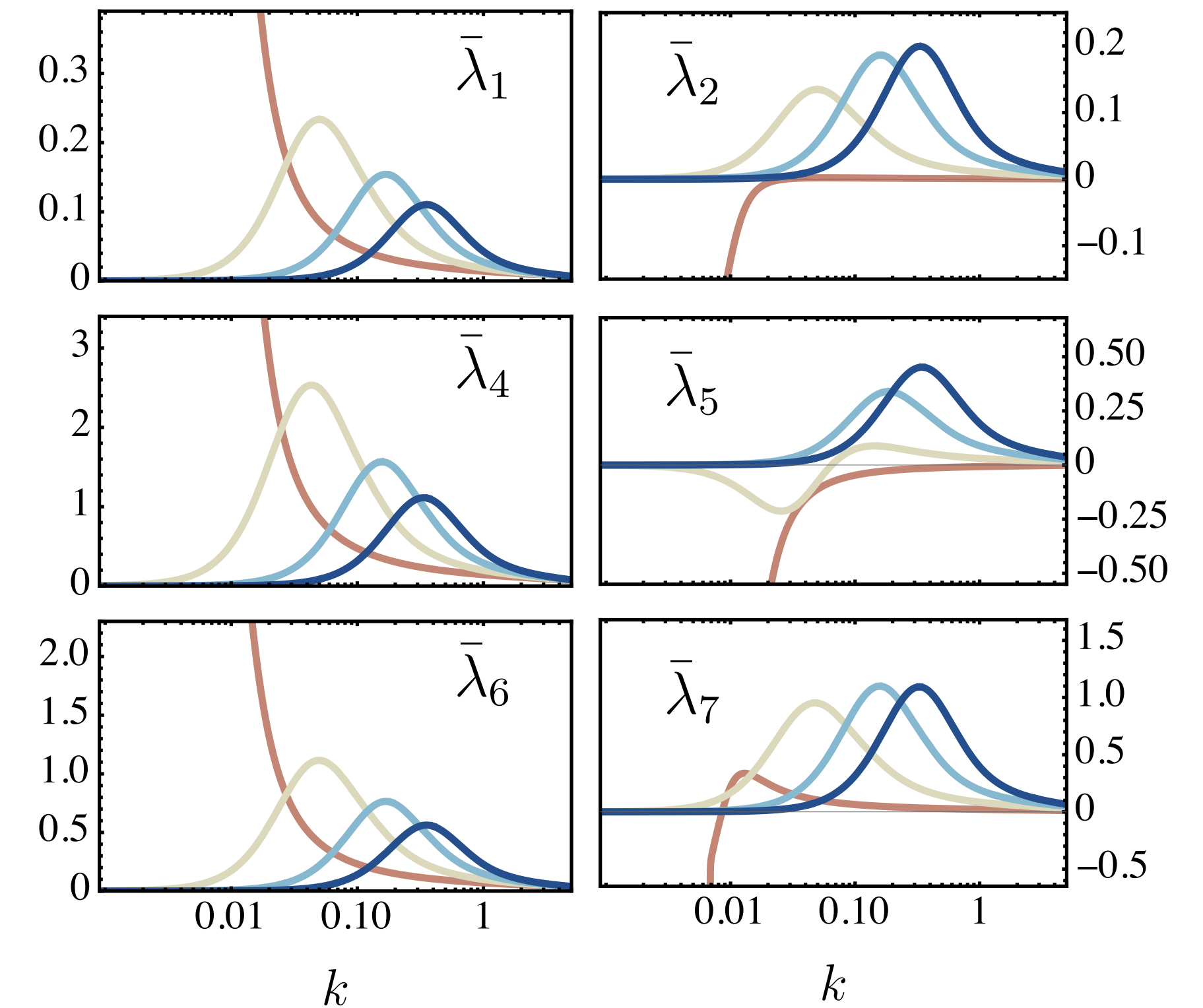
# Colour-confinement and dSB in BY

Li,APG,Vatani[2603.19355]



$N_c$  : — 3 — 4 — 5 — 6

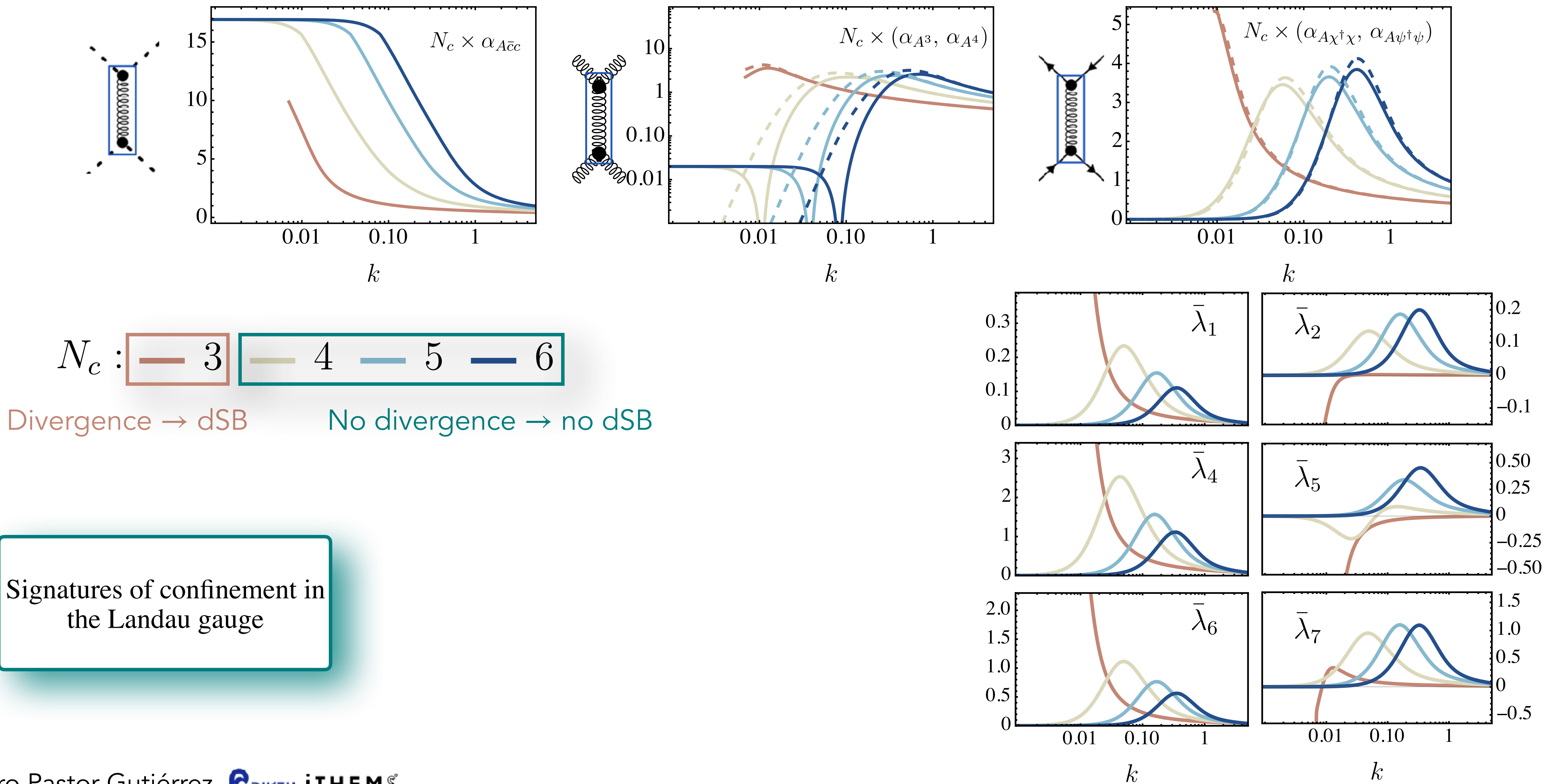
Signatures of confinement in the Landau gauge





# Colour-confinement and dSB in BY

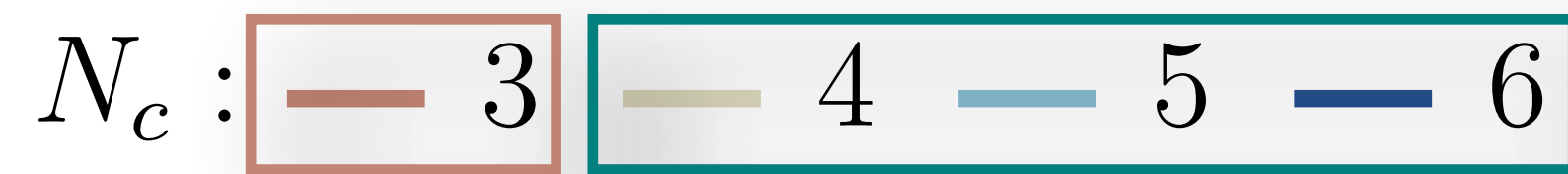
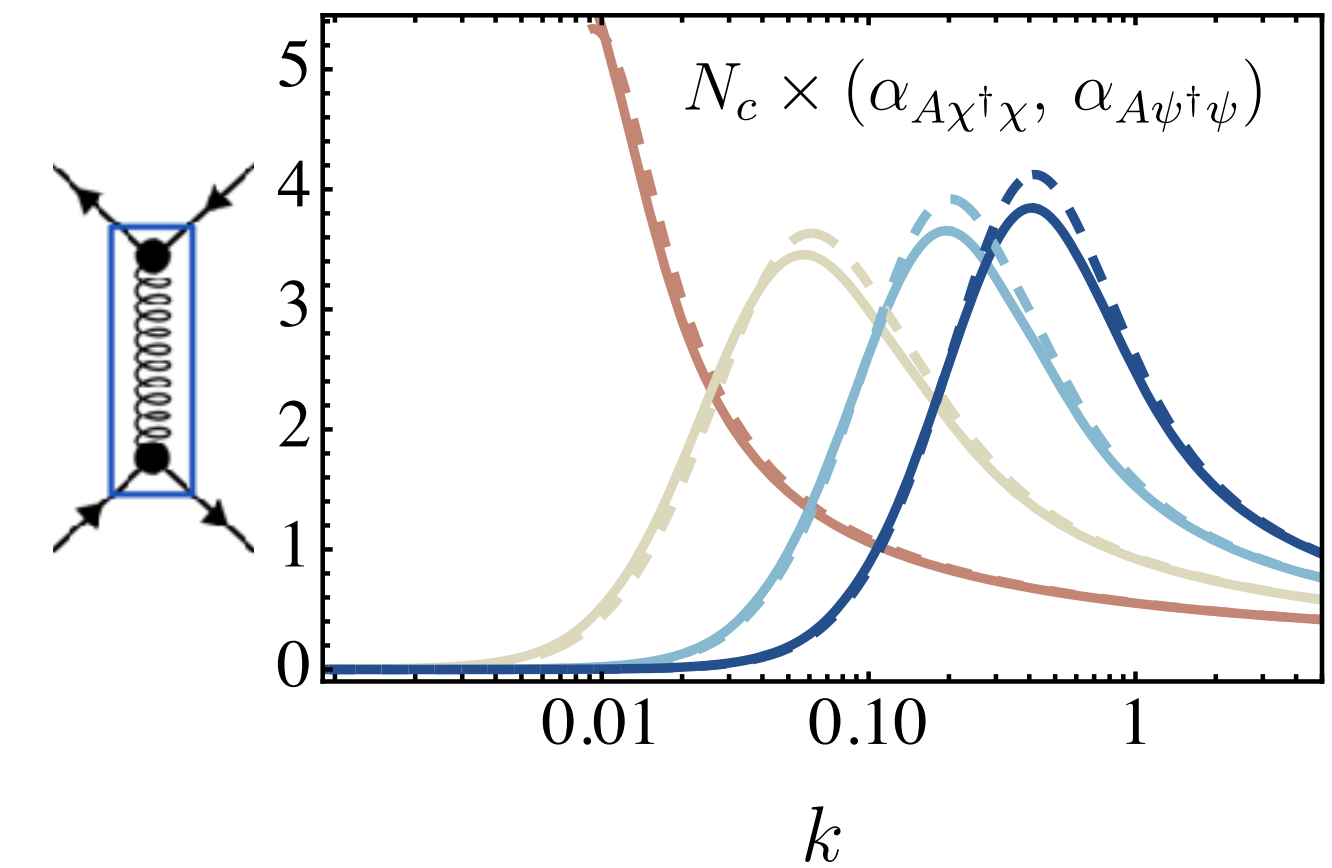
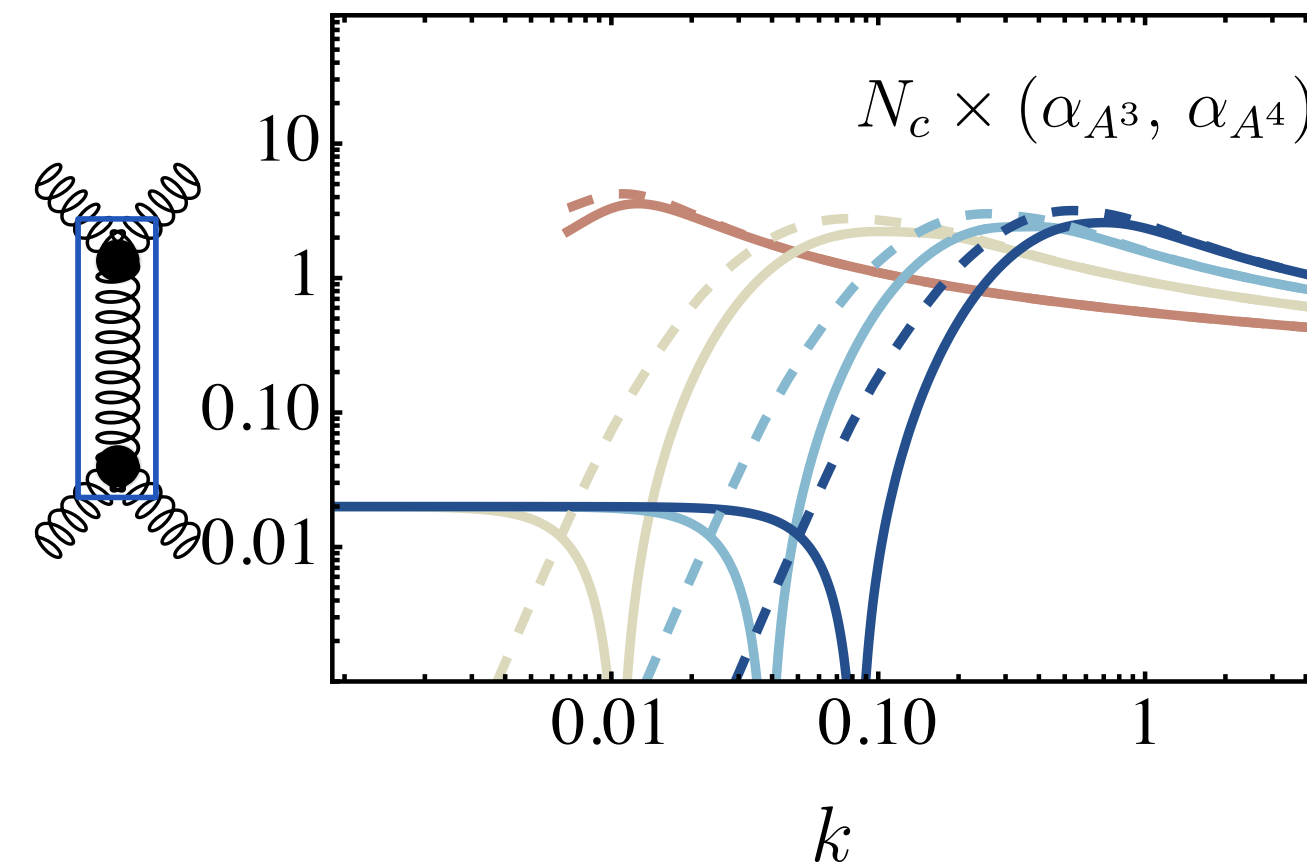
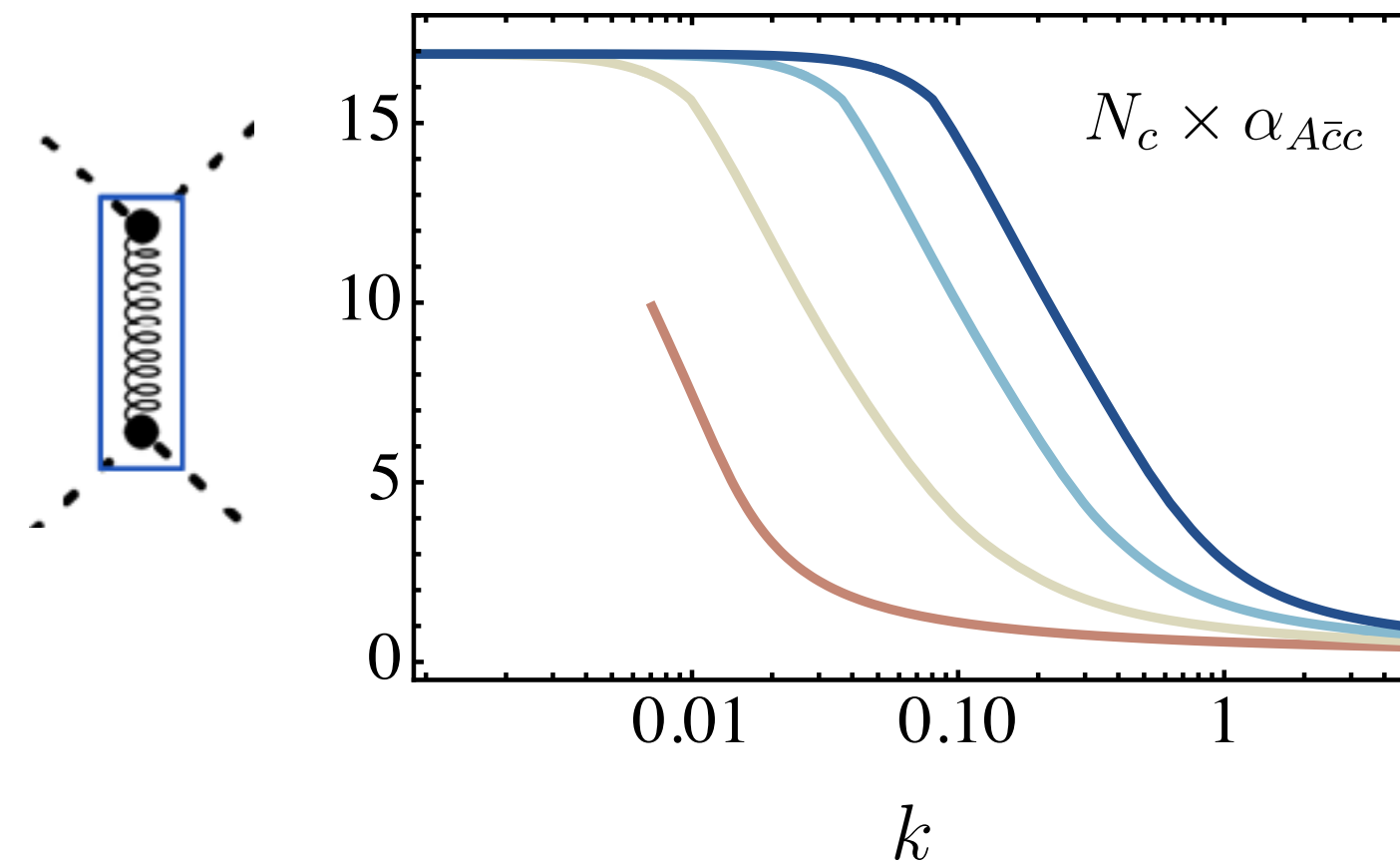
Li,APG,Vatani[2603.19355]





# Colour-confinement and dSB in BY

Li,APG,Vatani[2603.19355]



Divergence  $\rightarrow$  dSB

No divergence  $\rightarrow$  no dSB

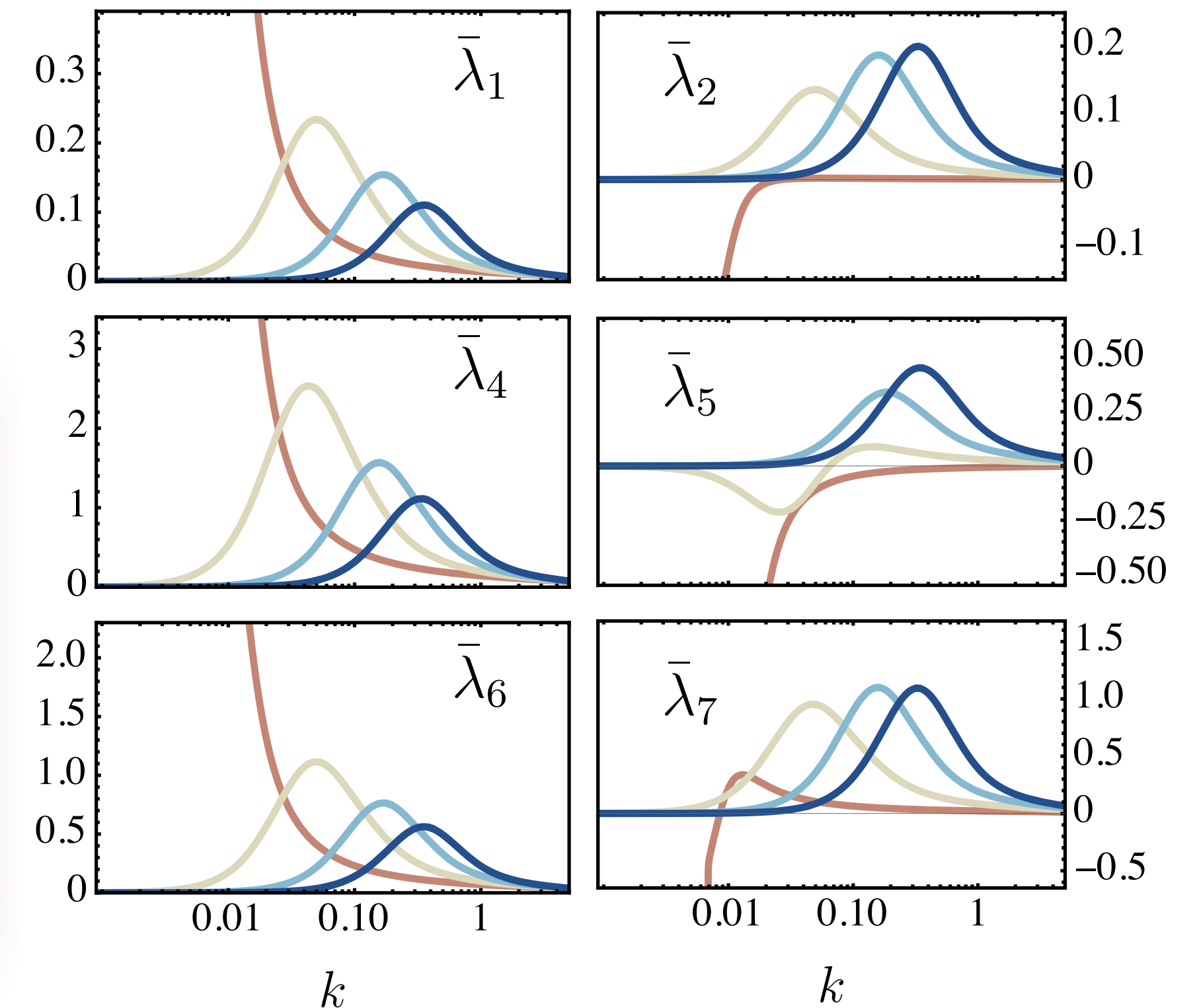
Signatures of confinement in the Landau gauge

Dominant four-fermion operators:

$$\mathcal{O}_4^\chi = (\chi^\dagger \bar{\sigma}^\mu \chi)(\chi^\dagger \bar{\sigma}^\mu \chi)$$

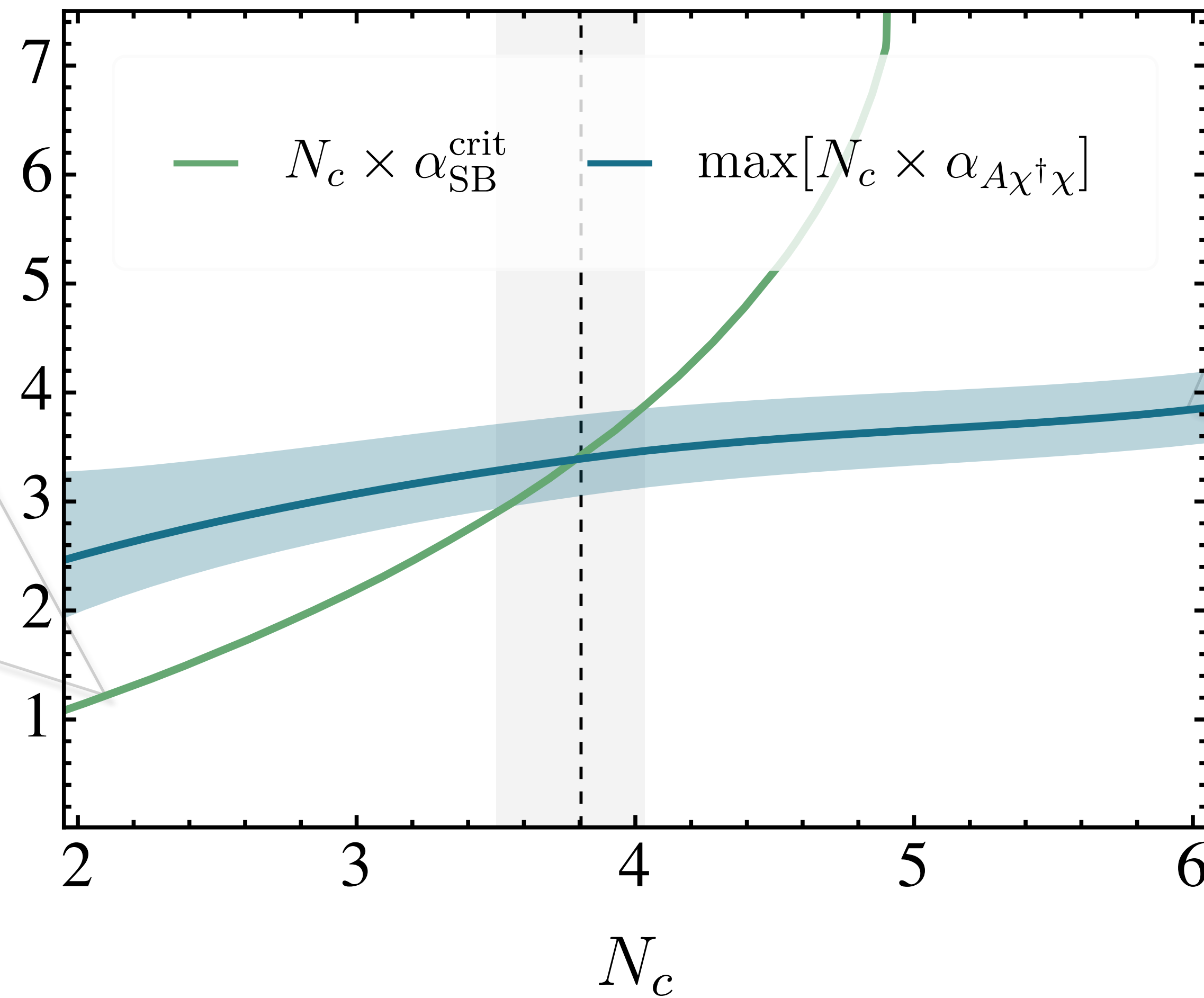
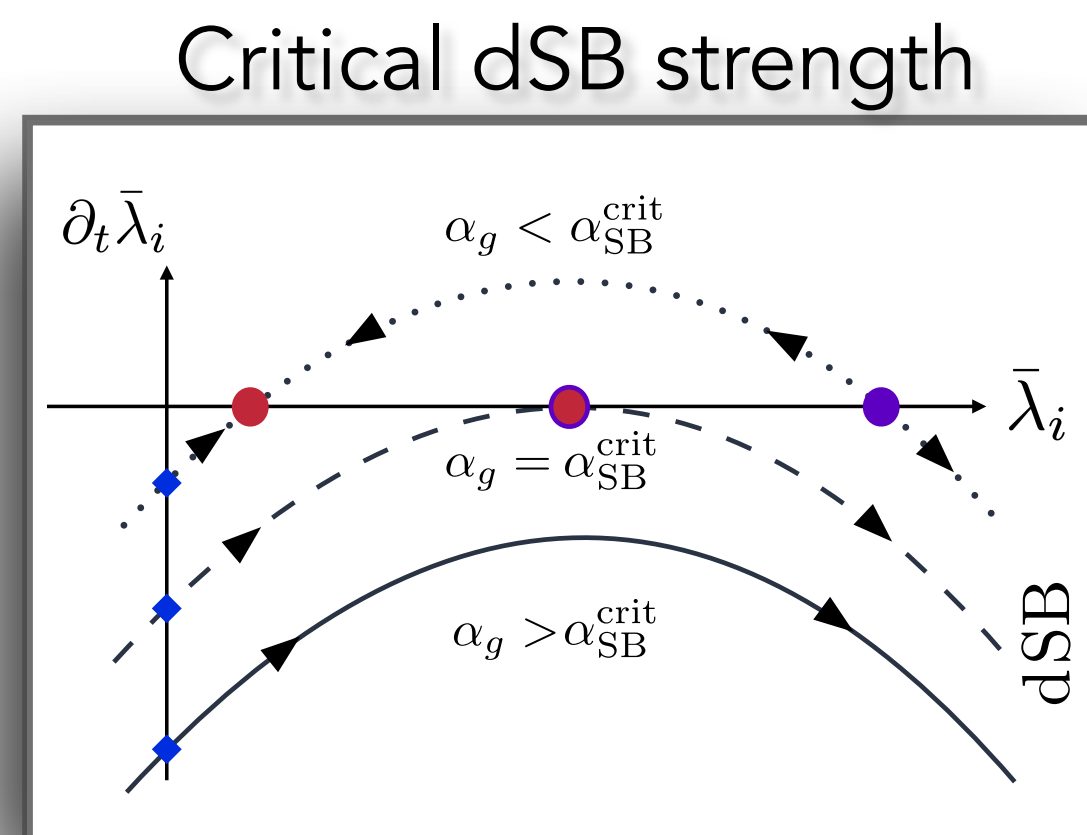
$$\mathcal{O}_5^\chi = (\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi)(\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi)$$

Diquark-like condensate:  $\sim \langle \chi \chi \rangle$

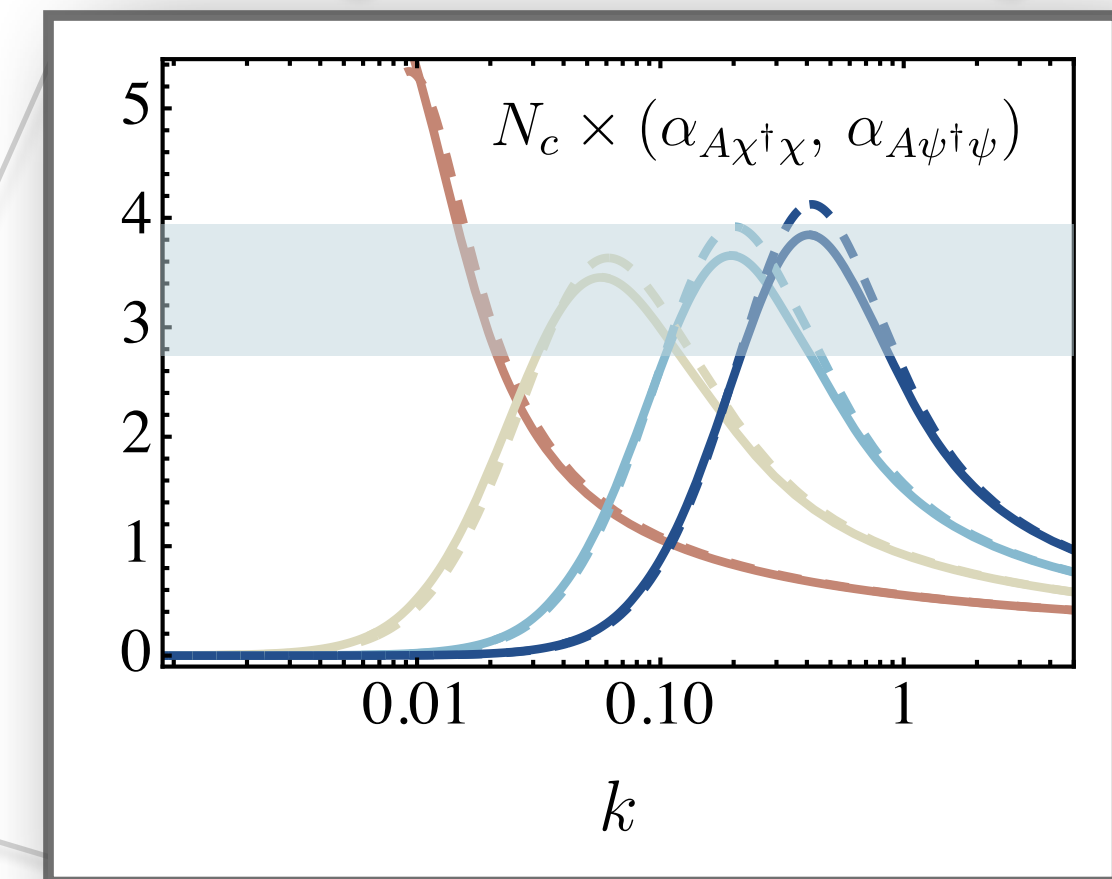




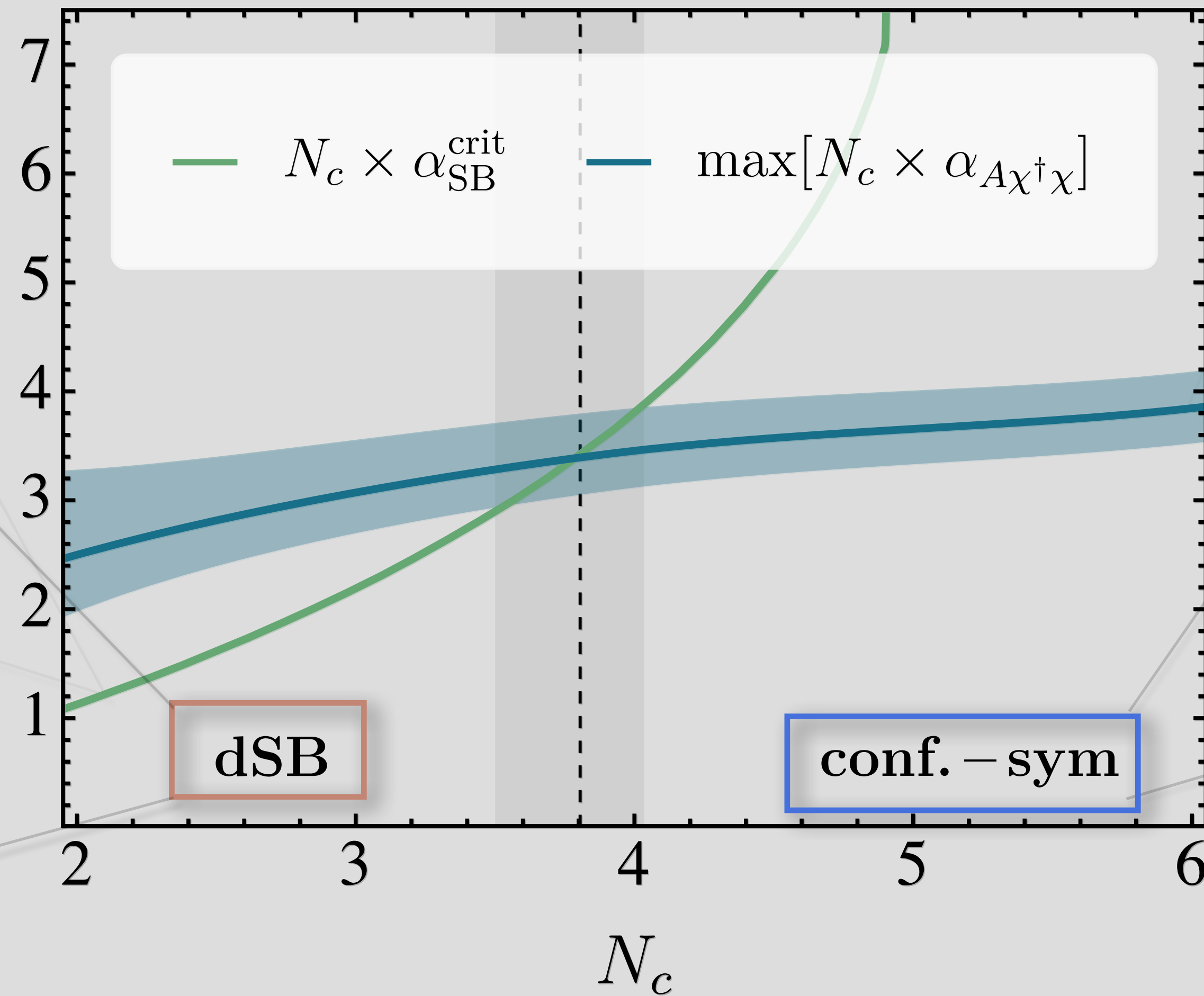
# Phase diagram



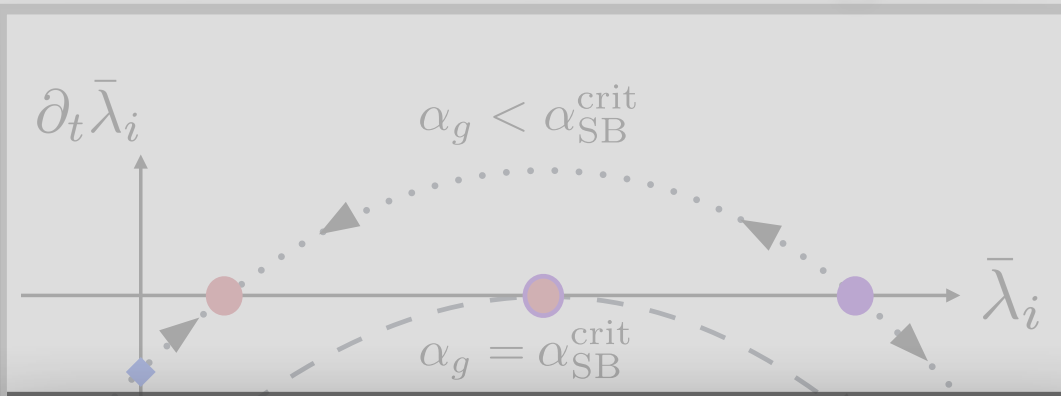
Gauge-fermion strength



# Phase diagram



Critical dSB strength

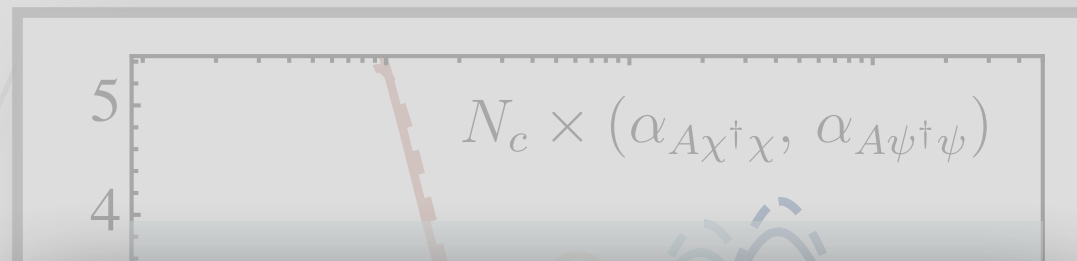


## Highest condensate

Di-quark like  $\sim \langle \chi\chi \rangle$

(Partial) higgsing of gluons,  
symmetry tumbling, ... (tbd)

Gauge-fermion strength



## Confinement without dSB

(agreement with large  $N_c$  analysis!)

[Bars, Yankielowicz '81](#)

[Raby, Dimopoulos, Susskind '79](#)

[Eichten, Peccei, Preskill, Zeppenfeld '86](#)

Massless baryons saturate  
the anomaly

[Raby, Dimopoulos, Susskind '79](#)

[Eichten, Peccei, Preskill, Zeppenfeld '86](#)

What is this dynamics?

SMG?, ...

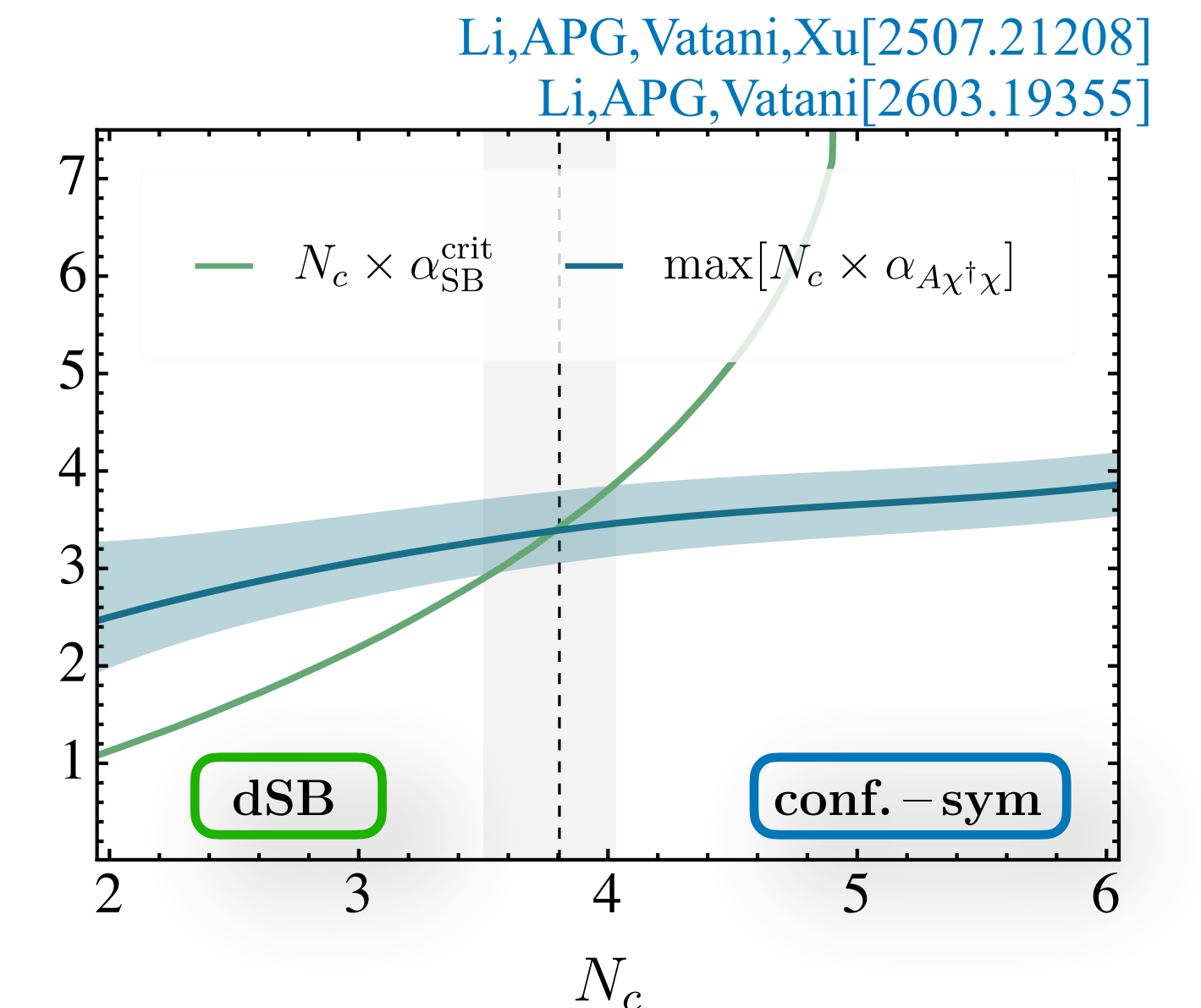
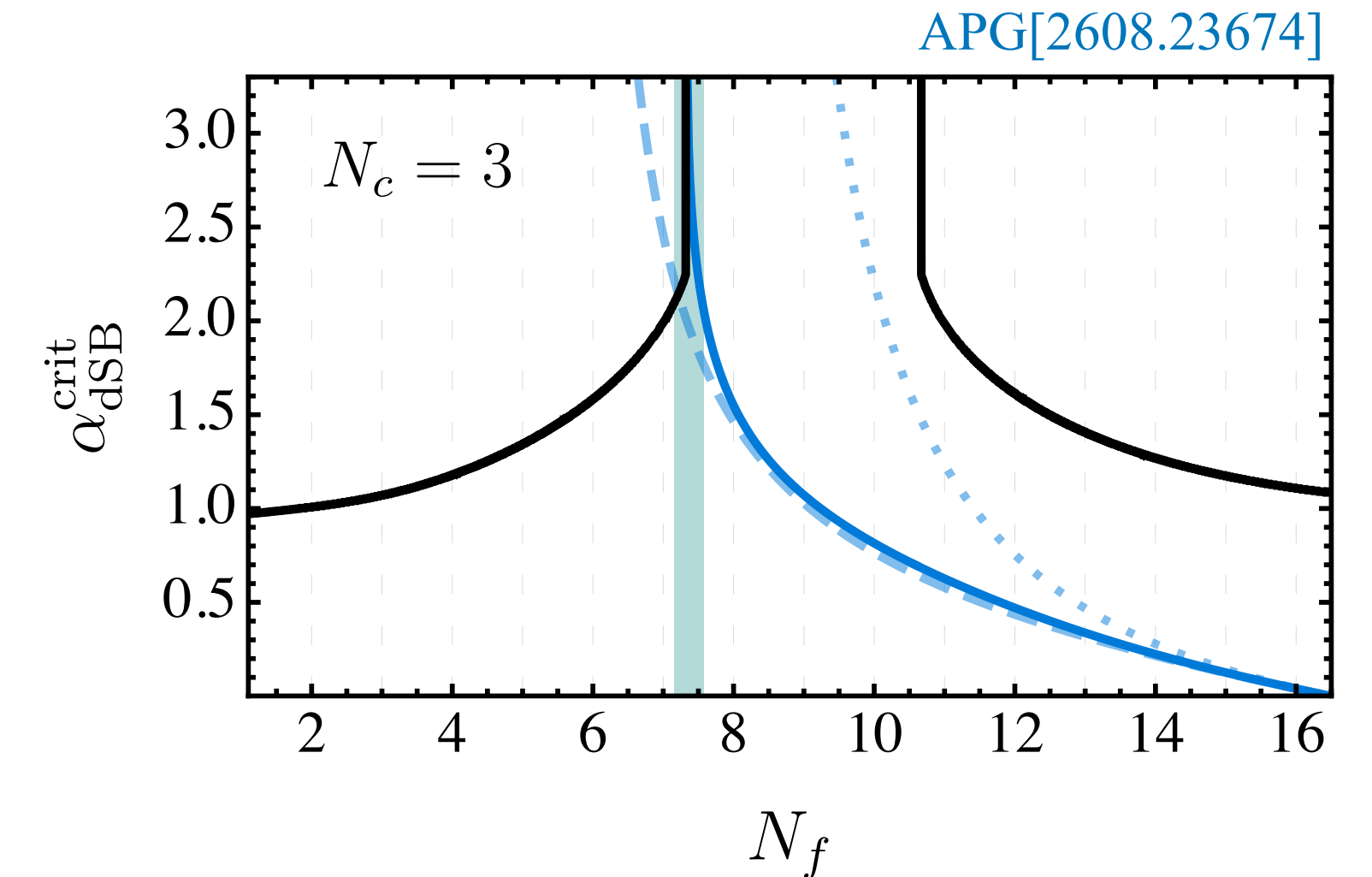
[Wang, You \[2204.14271\]](#)

[Karasik, Önder, Tong \[2208.07842\]](#)

# Conclusions

fRG approach: approximate, first-principles, versatile and nonperturbative

- QCD conformal phase transition:
  - Non-perturbative dynamics at the critical theory
  - $N_f^{\text{crit}}|_{N_c=3} \simeq 7.30$
  - New picture of the QPT: no walking, conf wo/ dSB, sym. mass gaps?
- First steps towards chiral-gauge theories:
  1. **dSB,  $\langle\chi\chi\rangle$  condensate, tumbling**
  2. **Confinement without dSB**
    - ★ Massless hadrons, SMG, ?, ...



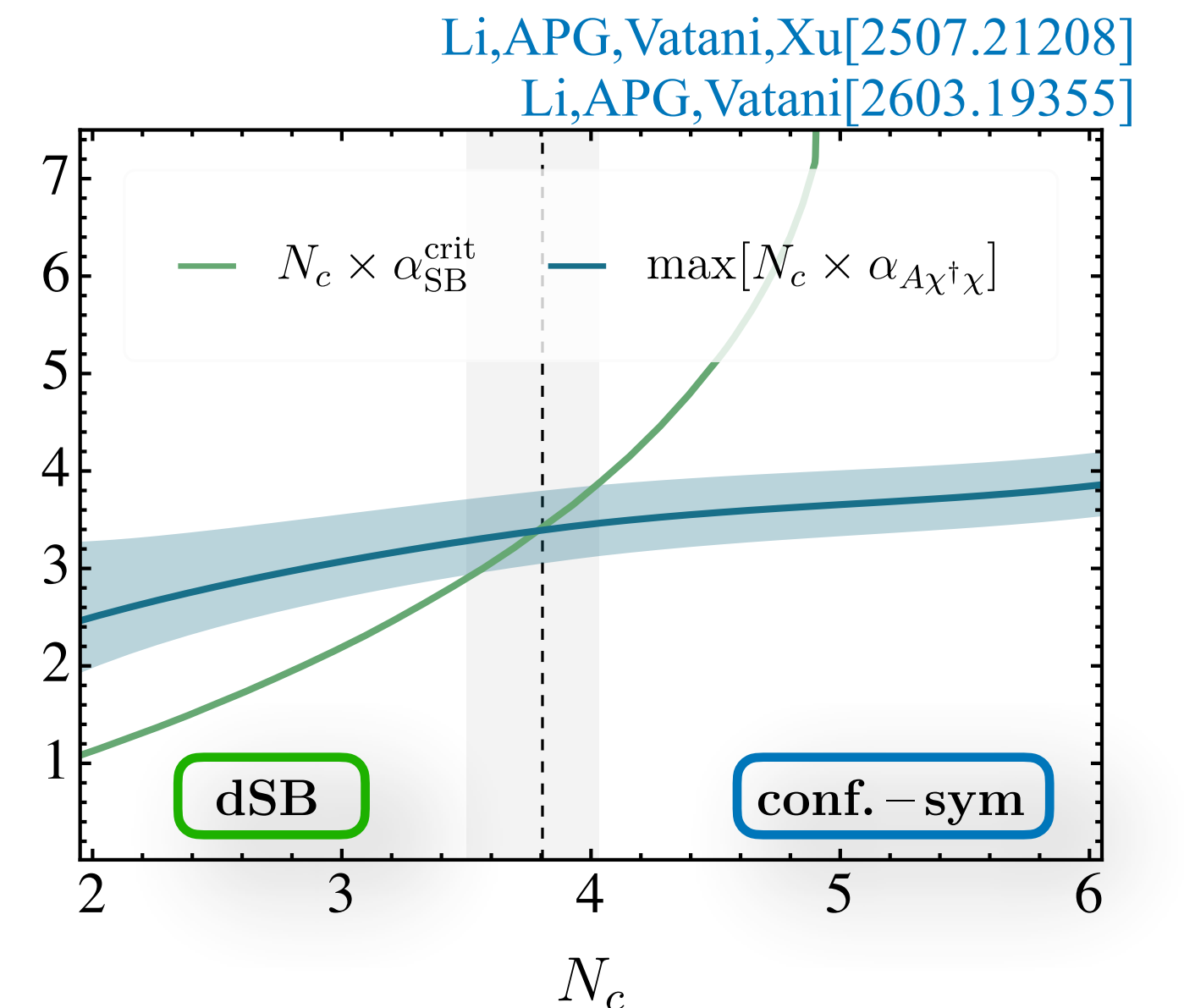
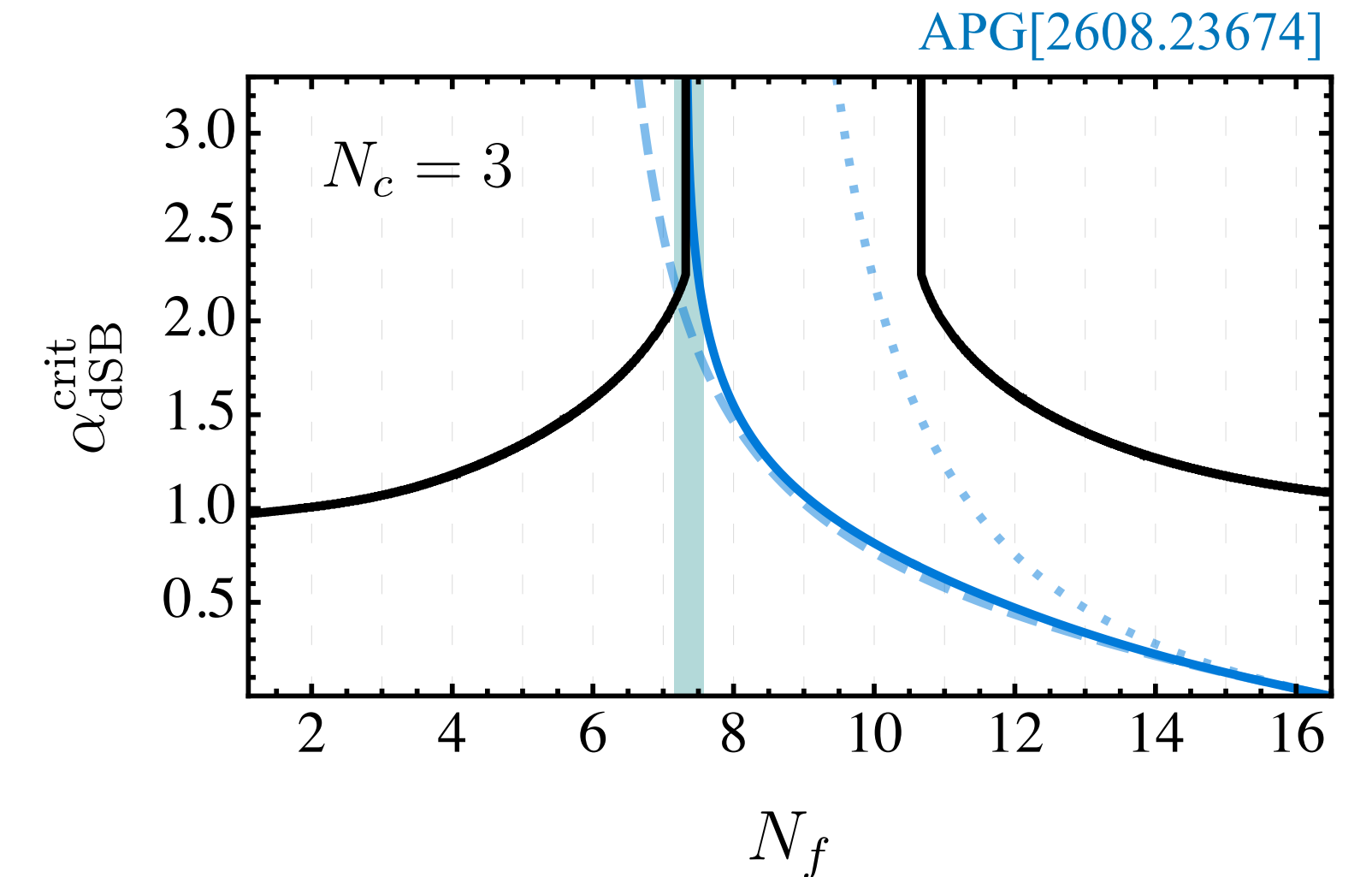


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    - ★ Massless hadrons, SMG, ?, ...

Thank you for your attention!



Additional slides



# Effective action 1

$$\Gamma_k[A_\mu, \bar{c}, c, \chi^\dagger, \chi, \psi^\dagger, \psi] = \Gamma_{\text{gauge}, k}[A_\mu, \bar{c}, c] + \Gamma_{\text{gauge-fermion}, k}[A_\mu, \chi^\dagger, \chi, \psi^\dagger, \psi] + \Gamma_{\text{fermion}, k}[\chi^\dagger, \chi, \psi^\dagger, \psi]$$

$$\begin{aligned} \Gamma_{\text{gauge}, k}[A, c, \bar{c}] &= \frac{1}{2} \int_p A_\mu^a(p) \left[ \tilde{Z}_{A, k} (p^2 + m_{\text{gap}, k}^2) \Pi_{\mu\nu}^\perp(p) + \frac{1}{\xi} Z_{A, k}^\parallel (p^2 + m_{\text{STI}, k}^2) \frac{p_\mu p_\nu}{p^2} \right] A_\nu^a(-p) \\ &+ \frac{1}{3!} \int_{p_1, p_2} \tilde{Z}_{A, k}^{3/2} g_{A^3, k} \left[ \mathcal{T}_{A^3}^{(1)}(p_1, p_2) \right]_{\mu_1 \mu_2 \mu_3}^{a_1 a_2 a_3} \prod_{i=1}^3 A_{\mu_i}^{a_i}(p_i) + \frac{1}{4!} \int_{p_1, p_2, p_3} \tilde{Z}_{A, k}^2 g_{A^4, k} \left[ \mathcal{T}_{A^4}^{(1)}(p_1, p_2, p_3) \right]_{\mu_1 \mu_2 \mu_3 \mu_4}^{a_1 a_2 a_3 a_4} \prod_{i=1}^4 A_{\mu_i}^{a_i}(p_i) \\ &+ \int_p Z_{c, k} \bar{c}^a(p) p^2 \delta^{ab} c^b(-p) + \int_{p_1, p_2} Z_{c, k} \tilde{Z}_{A, k}^{1/2} g_{c\bar{c}A, k} \left[ \mathcal{T}_{A\bar{c}c}^{(1)}(p_1, p_2) \right]_\mu^{a_1 a_2 a_3} \bar{c}^{a_2}(p_2) c^{a_1}(p_1) A_\mu^{a_3}(-p_1 - p_2) \end{aligned}$$

$$\Gamma^{(\Phi_{i_1} \cdots \Phi_{i_n})}(p_1, \dots, p_n) = \frac{\delta}{\delta \Phi_{i_1}(p_1)} \cdots \frac{\delta}{\delta \Phi_{i_n}(p_n)} \Gamma[\Phi] = \tilde{\Gamma}^{(\Phi_{i_1} \cdots \Phi_{i_n})}(p_1, \dots, p_n) \mathcal{T}_{\Phi_{i_1} \cdots \Phi_{i_n}}^{(1)}(p_1, \dots, p_n)$$

# Effective action 2

$$\Gamma_{\text{gauge-fermion},k} [A_\mu, \psi^\dagger, \psi, \chi^\dagger, \chi] =$$

$$\int_p Z_{\psi,k} \psi^{\dagger,i,f}(p) \bar{\sigma}_\mu \partial_\mu \psi^{i,f}(-p) - i \int_{p_1,p_2} \tilde{Z}_{\psi,k} \tilde{Z}_{A,k}^{1/2} g_{A\psi^\dagger\psi,k} \psi^{\dagger,i_1 f_1}(p_2) \left[ \mathcal{T}_{A\psi^\dagger\psi}^{(1)}(p_1, p_2) \right]_\mu^{a i_1 i_2 f_1 f_2} A_\mu^a(-p_1 - p_2) \psi$$

$$+ \int_p Z_{\chi,k} \chi^{\dagger,\alpha}(p) \bar{\sigma}_\mu \partial_\mu \chi^\alpha(-p) - i \int_{p_1,p_2} Z_{\chi,k} \tilde{Z}_{A,k}^{1/2} g_{A\chi^\dagger\chi,k} \chi^{\dagger,\alpha_1}(p_2) \left[ \mathcal{T}_{A\chi^\dagger\chi}^{(1)}(p_1, p_2) \right]_\mu^{a \alpha_1 \alpha_2} A_\mu^a(-p_1 - p_2) \chi^{\alpha_2}(p_1)$$

$$\left[ \mathcal{T}_{A\psi^\dagger\psi}^{(1)}(p_1, p_2) \right]_\mu^{a i_1 i_2 f_1 f_2} = \bar{\sigma}_\mu (T_{\bar{F}}^a)^{i_1 i_2} \delta^{f_1 f_2} \quad \text{and} \quad \left[ \mathcal{T}_{A\chi^\dagger\chi}^{(1)}(p_1, p_2) \right]_\mu^{a \alpha_1 \alpha_2} = \bar{\sigma}_\mu (T_S^a)^{\alpha_1 \alpha_2}$$

$$\Gamma_{\text{fermion},k} [\chi^\dagger, \chi, \psi^\dagger, \psi] = - \int_x Z_\psi^2 \sum_{i=1}^2 \lambda_i \mathcal{O}_i^\psi + Z_\chi^2 \sum_{i=4}^5 \lambda_i \mathcal{O}_i^\chi + Z_\psi Z_\chi \sum_{i=6}^7 \lambda_i \mathcal{O}_i^{\chi\psi},$$

# Deriving full correlation functions

$$\mathcal{M} \propto \Gamma_k^{(\bar{\psi}\psi A)}_{k \rightarrow 0} \left[ \Gamma_k^{(AA)} \right]^{-1} \Gamma_k^{(\bar{\psi}\psi A)}_{k \rightarrow 0}$$

Vertex expansion of the effective average action

$$\Gamma_k[\Phi] = \sum_n \int_{\mathbf{p}} \Gamma_k^{(\Phi_{i_1} \dots \Phi_{i_n})}(\mathbf{p}) \Phi_{i_n}(p_n) \dots \Phi_{i_1}(p_1) \quad \text{with} \quad \Gamma_k^{(\Phi_{i_1} \dots \Phi_{i_n})}(p_1, \dots, p_n) = \frac{\delta}{\delta \Phi_{i_1}(p_1)} \dots \frac{\delta}{\delta \Phi_{i_n}(p_n)} \Gamma_k[\Phi]$$

Example: fermion-gauge vertex and flow of the coupling

$$\partial_t \Gamma_k^{(\bar{\psi}\psi A)} = \partial_t \left( Z_A^{1/2} Z_\psi g_{\bar{\psi}\psi A} \cdot \mathcal{T}_\mu \right) \quad \text{with} \quad \partial_t g_{\bar{\psi}\psi A} = \frac{\text{Tr} \left[ \mathcal{T}_\mu \partial_t \Gamma_k^{(\bar{\psi}\psi A)} \right]}{\text{Tr} \left[ \mathcal{T}_\mu^2 \right] Z_A^{1/2} Z_\psi} + \left( \frac{1}{2} \eta_A + \eta_\psi \right) g_{\bar{\psi}\psi A}$$

with anomalous dimensions  $\eta_{\phi_i} = -\frac{\partial_t Z_{\phi_i}}{Z_{\phi_i}} = -\frac{\partial_{p^2} \partial_t \Gamma_k^{(\phi_i \phi_i)}}{Z_{\phi_i}}$

$$\partial_t \Gamma_k^{(\bar{\psi}\psi)} = \partial_t \longrightarrow^{-1} = \tilde{\partial}_t \left( \text{diagram 1} + \text{diagram 2} \right)$$

$$\partial_t \Gamma_k^{(AA)} = \partial_t \text{diagram 3}^{-1} = \tilde{\partial}_t \left( \text{diagram 4} - \frac{1}{2} \text{diagram 5} - \text{diagram 6} - \text{diagram 7} \right)$$

$$\partial_t \Gamma_k^{(\bar{\psi}\psi A)} = \partial_t \text{diagram 8} = \tilde{\partial}_t \left( - \text{diagram 9} - \text{diagram 10} - \text{diagram 11} \right)$$

**Diagrammatic flows to be computed**



# STIs and mSTIs and confinement

$$\Delta S_k[\phi] = \int_p \phi(p) R_k \phi(-p)$$

- Insertion of mass-like regulator: Slavnov-Taylor identities (STIs) for gauge invariance  $\rightarrow$  modified STIs (mSTIs)

[Becchi\[9607188\]](#), [Bonini, D'Attanasio, Marchesini'95](#) [Ellwanger\[9402077\]](#)

$$\int_x \frac{\delta \Gamma}{\delta \mathcal{Q}_i(x)} \frac{\delta \Gamma}{\delta \Phi_i(x)} = 0 \quad \rightarrow \quad \int_x \frac{\delta \Gamma_k}{\delta \mathcal{Q}_i(x)} \frac{\delta \Gamma_k}{\delta \Phi_i(x)} = \text{Tr} R^{ij} G_{jl} \frac{\delta^2 \Gamma_k[\Phi, \mathcal{Q}]}{\delta \Phi_l \delta \mathcal{Q}^i} \quad \begin{array}{l} \Phi_i = \{A_\mu, c, \bar{c}, \dots\} \\ \mathcal{Q}^i : \text{BRST sources of the } \Phi_i \text{ fields} \end{array}$$

[Pawlowski\[0512261\]](#) [Gies\[0611146\]](#)

- In the physical limit  $k \rightarrow 0$  and  $R_k \rightarrow 0$  and mSTI  $\rightarrow$  STI and no gauge symmetry breaking is present (\*)

[Pawlowski, Schneider, Wink\[2202.11123\]](#)

- "easy" approach: physics at  $k$ -dependent correlation functions

[Ihssen, Pawlowski\[2503.22638\]](#)

- Minimise mSTI and STI deviations for  $k \neq 0$

[Morris, Rosten\[0606189\]](#)  
[Wetterich\[1607.02989\]](#)

- Hands-on approach: include problematic power-law corrections to the flow of the gauge mass gap only in the confinement regime

$$\partial_t \bar{m}_{\text{gap}}^2 = (-2 + \eta_A) \bar{m}_{\text{gap}}^2 + \underbrace{\overline{\text{Flow}}_{AA}(p^2 = 0)}$$

- Upgrade with full momentum dependencies

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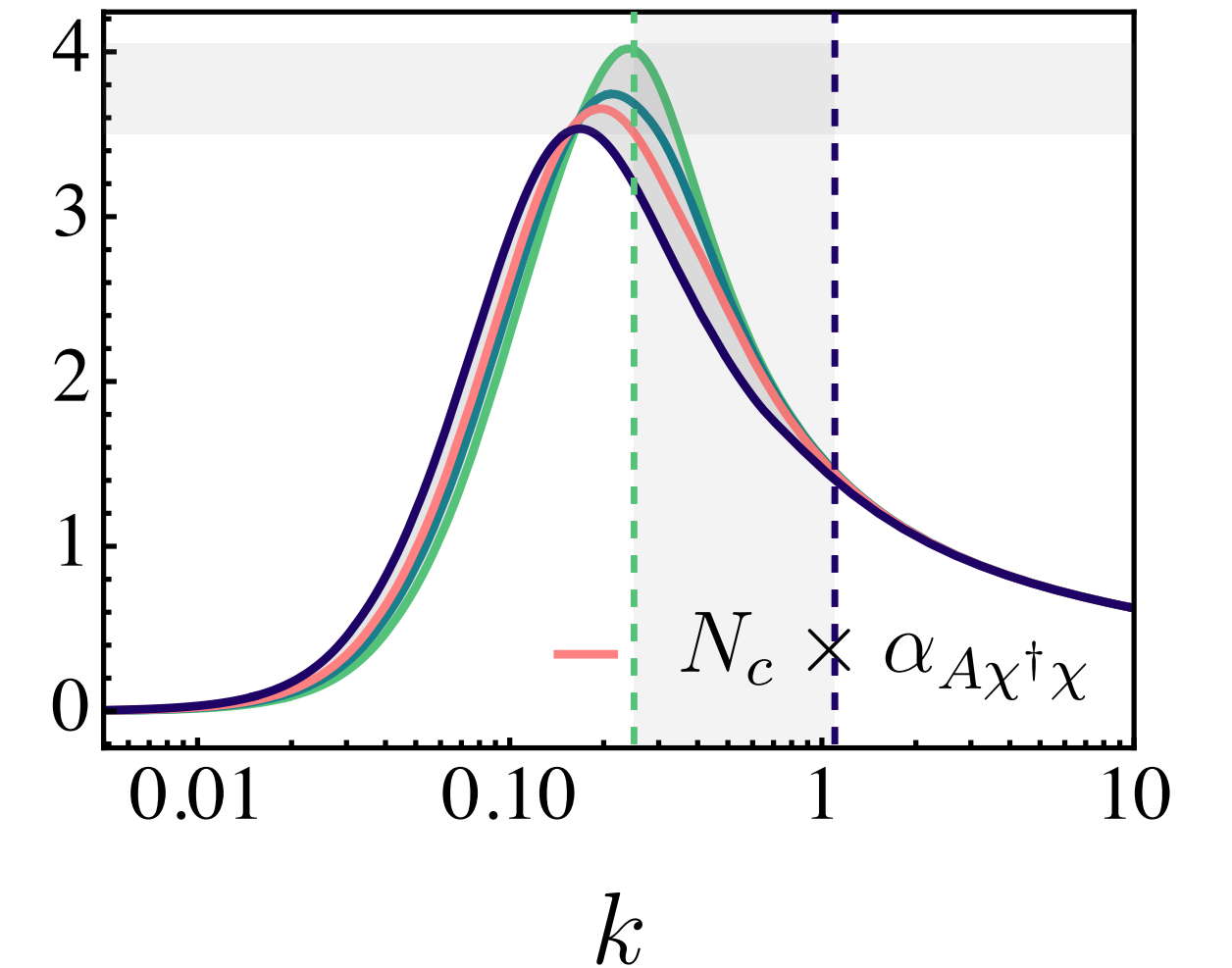
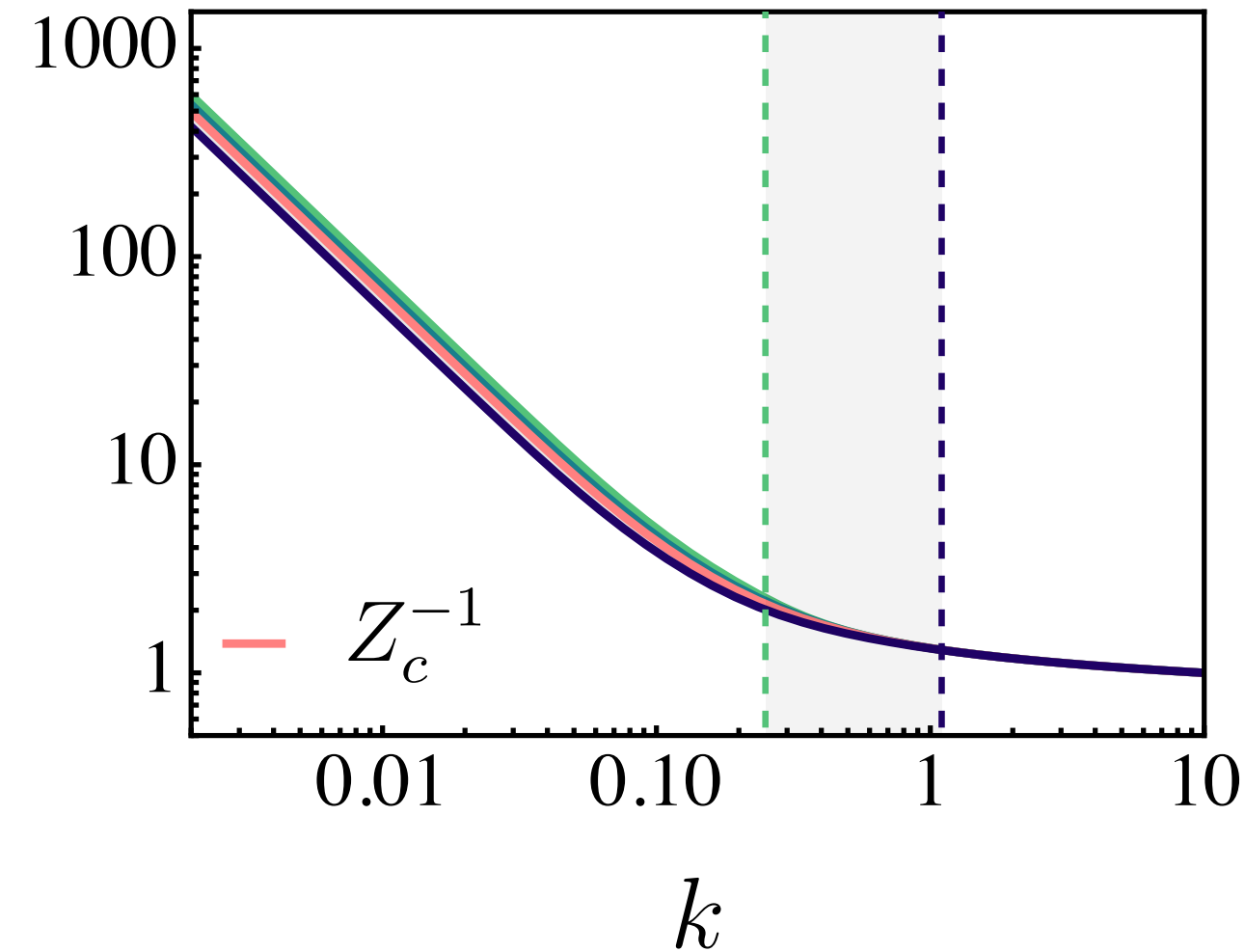
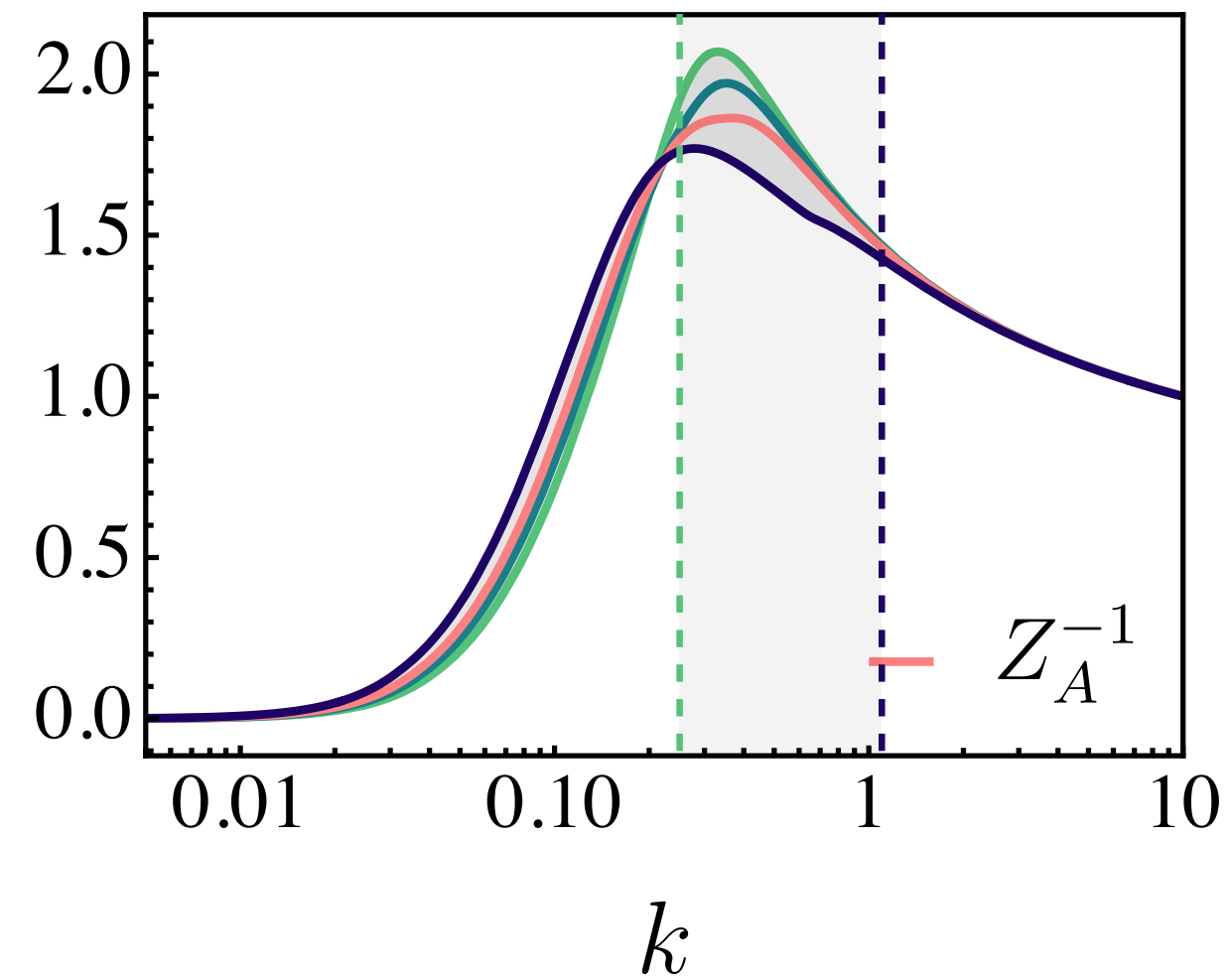
Chiral symmetry intact:

$$R_{\psi/\chi, k}(p^2) = i Z_{\psi/\chi} \bar{\sigma}_\mu p_\mu r_{\psi/\chi}(x)$$

$$r_{\psi/\chi}(x) = (1/\sqrt{x} - 1) \theta(1 - x)$$



# Systematics: minimasing the mSTI/STI deviation



- control onset of power-law divergences in the flow of the gauge mass gap.

$$\partial_t \bar{m}_{\text{gap}}^2 = (-2 + \eta_A) \bar{m}_{\text{gap}}^2 + \overline{\text{Flow}}_{AA}(p^2 = 0)$$

The diagram shows the decomposition of the four-point function  $\partial_t$  into three one-loop diagrams. The first diagram is a tree-level contact interaction represented by a black dot. The second diagram is a box diagram with two internal gluon lines (coiled) and two external fermion lines (straight), labeled with  $\alpha_g^2$ . The third diagram is a bubble diagram with two internal fermion lines (straight) and two external fermion lines (straight), labeled with  $\lambda_{4\psi}^2$ . The diagrams are separated by plus signs, indicating they are summed together.

Gies, Wetterich '01



Gies, Wetterich '01Pawlowski[hep-th/0512261]

$$\Gamma_k[\Phi] \supset \int_x \bar{h} \bar{\psi} (T_f^0 \sigma + i \gamma_5 T_f^a \pi^a) \psi + Z_\phi (\partial_\mu \phi)^2 + V(\phi^2) + \dots$$



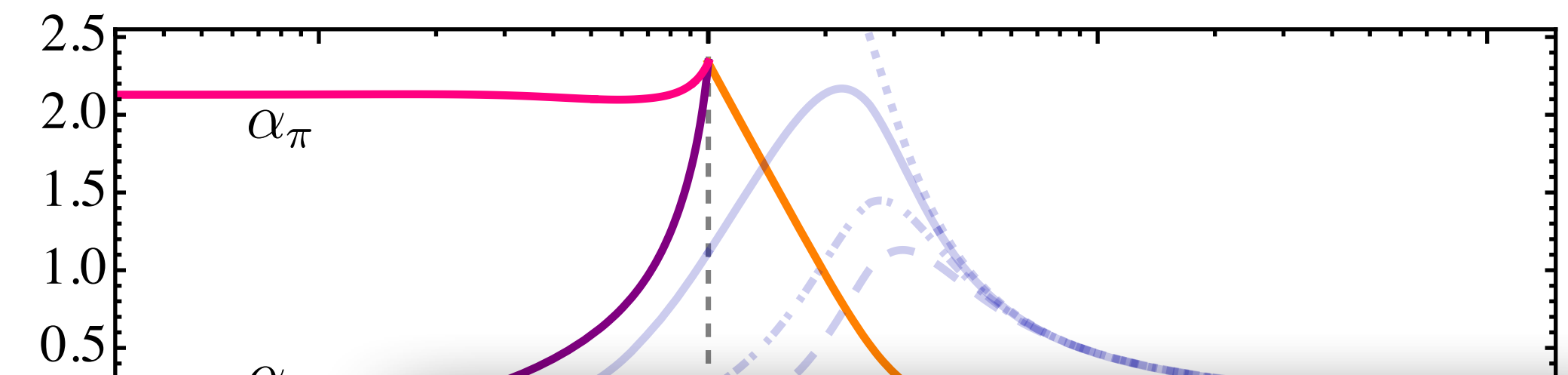
The diagram shows the time derivative of the four-point function,  $\partial_t$ , which is proportional to the sum of three diagrams. The first diagram is a box diagram with two horizontal fermion lines and two vertical gluon lines, labeled  $\alpha_g^2$ . The second diagram is a box diagram with two horizontal fermion lines and two vertical gluon lines, labeled  $\alpha_g \lambda_{4\psi}$ . The third diagram is a box diagram with two horizontal fermion lines and two vertical gluon lines, labeled  $\lambda_{4\psi}^2$ .

Gies, Wetterich '01

Fukushima, Pawłowski, Strodthoff [2103.01129]

Gies, Wetterich '01Pawlowski[hep-th/0512261]

$$\begin{aligned} & \left( \partial_t + \int \dot{\phi}_c \frac{\delta}{\delta \phi_c} \right) \Gamma_k[\phi] \\ &= \frac{1}{2} \text{Tr} \left[ \left( \frac{1}{\Gamma_k^{(2)} + R_k} \right)_{ij} \left( \partial_t \delta^{jn} + 2 \frac{\delta \dot{\phi}_n}{\delta \phi_j} \right) R_k^{mi} \right] \\ & \quad k[k_{\chi\text{SB}}] \end{aligned}$$





# Dynamical bosonisation and chiral symmetry

$$\Gamma_k[\Phi] \supset - \int_x \bar{\lambda}_\sigma (\bar{\psi} \mathcal{T}_{(S-P)} \psi)^2 + \dots$$

The diagram shows an equality between two expressions. On the left, a four-point vertex with incoming momenta  $\vec{p}_1, \vec{p}_2$  and outgoing momenta  $\vec{p}_3, \vec{p}_4$  is shown. To its right is a vertical bar containing the label  $(\phi)$  and two conditions:  $(p_1 + p_3)^2 = 0$  and  $(p_2 + p_4)^2 = 0$ . This is followed by an equals sign. On the right side of the equals sign, there is a summation over  $\phi$  of a two-point vertex (represented by two lines meeting at a point) connected to another two-point vertex by a dashed line. To the right of this is another vertical bar containing the same two conditions:  $(p_1 + p_3)^2 = 0$  and  $(p_2 + p_4)^2 = 0$ .

$$\Gamma_{\text{fermion}, k}[\bar{\psi}, \psi] \longrightarrow \Gamma_{\text{fermion}, k}[\bar{\psi}, \psi]_{\mathcal{T}_{(S-P)}=0} + \Gamma_{\text{boson}, k}[\bar{\psi}, \psi, \phi]$$

Generalised flow

$$\left( \partial_t + \int \dot{\phi}_c \frac{\delta}{\delta \phi_c} \right) \Gamma_k[\phi]$$

$$= \frac{1}{2} \text{Tr} \left[ \left( \frac{1}{\Gamma_k^{(2)} + R_k} \right)_{ij} \left( \partial_t \delta^{jn} + 2 \frac{\delta \dot{\phi}_n}{\delta \phi_j} \right) R_k^{mi} \right]$$

$$\phi^i = (\sigma, \pi^a, a^a, \eta) \quad \& \quad \langle \partial_t \hat{\phi}_k^i \rangle = \dot{\mathcal{A}}_{i,k} \bar{\psi} T_i \psi$$

- Inclusion of non-perturbative effects
  - Higher-order operators
  - Inclusion of momentum dependencies beyond point-like limit

$$\Gamma_{\text{boson}, k}[\bar{\psi}, \psi, \phi] = \int_x \left\{ Z_{\phi_i} \text{Tr}[(\partial_\mu \Sigma^\dagger)(\partial^\mu \Sigma)] + V(\Sigma^\dagger \Sigma) + Z_{\phi_i}^{1/2} Z_\psi h_i \bar{\psi} T_i \phi_i \psi \right\}$$

$$\Sigma = T_f^0(\sigma + i\eta) + T_f^a(a^a + i\pi^a)$$

$$\rho = \text{Tr}[\Sigma^\dagger \Sigma] = \frac{1}{2} (\sigma^2 + \eta^2 + \mathbf{a}^2 + \boldsymbol{\pi}^2) \quad \tau = \text{Tr} \left[ \left( \Sigma^\dagger \Sigma - \frac{\rho}{N_f} 1_{N_f} \right)^2 \right]$$

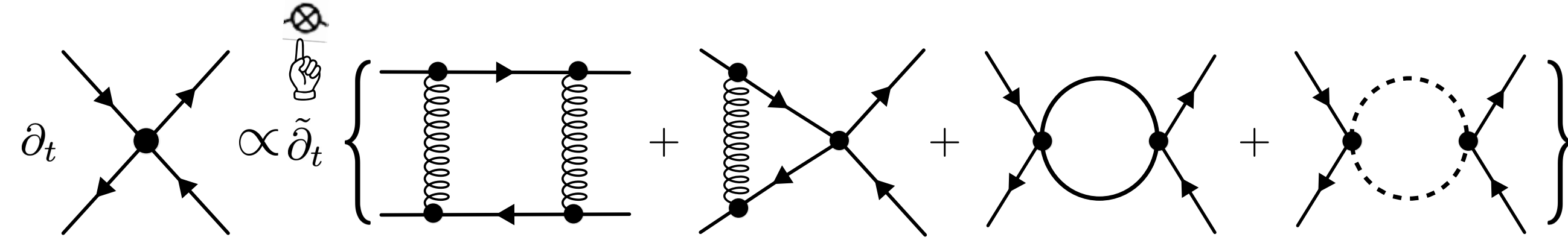
$$V(\rho, \tau) = \sum_{n=0}^{N_{\max}} \frac{\lambda_n}{n!} \rho^n + \sum_{m=0}^{M_{\max}} \frac{\lambda_m^{(\tau)}}{m!} \tau^m + \sum_{l=0}^{L_{\max}} \frac{\lambda_l^{(\rho\tau)}}{l!} \rho^l \tau$$

$$\phi_{\text{EoM}}^i(p) = - \frac{Z_\psi h_i}{Z_{\phi_i}^{1/2} (p^2 + m_{\phi_i}^2)} \int_q \bar{\psi}(q) T_i \psi(p - q)$$

# Computing the four-Fermi flows (gGG example)

- Flow of four-point functions

$$\partial_t \Gamma_k^{(\bar{\psi}_a \psi_b \bar{\psi}_c \psi_d)} = -\partial_t (Z_\psi^2 \lambda_1) \mathcal{T}_{1,L}^{abcd} - \partial_t (Z_\psi^2 \lambda_2) \mathcal{T}_{2,L}^{abcd}$$



$$\begin{aligned} \partial_t \bar{\lambda}_1 &= (2 + 2\eta_\psi) \bar{\lambda}_1 + \frac{k^2}{Z_\psi^2 \mathcal{N}} \left\{ \partial_t \left[ \mathcal{P}_{1,R}^{abcd} \Gamma_k^{(\bar{\psi}_a \psi_b \bar{\psi}_c \psi_d)} \right] (N_c N_\psi + 1) - \partial_t \left[ \mathcal{P}_{2,R}^{abcd} \Gamma_k^{(\bar{\psi}_a \psi_b \bar{\psi}_c \psi_d)} \right] (N_c + N_\psi) \right\} \\ \partial_t \bar{\lambda}_2 &= (2 + 2\eta_\psi) \bar{\lambda}_2 + \frac{k^2}{Z_\psi^2 \mathcal{N}} \left\{ \partial_t \left[ \mathcal{P}_{2,R}^{abcd} \Gamma_k^{(\bar{\psi}_a \psi_b \bar{\psi}_c \psi_d)} \right] (N_c N_\psi + 1) - \partial_t \left[ \mathcal{P}_{1,R}^{abcd} \Gamma_k^{(\bar{\psi}_a \psi_b \bar{\psi}_c \psi_d)} \right] (N_c + N_\psi) \right\} \end{aligned}$$

Anomalous dimensions:

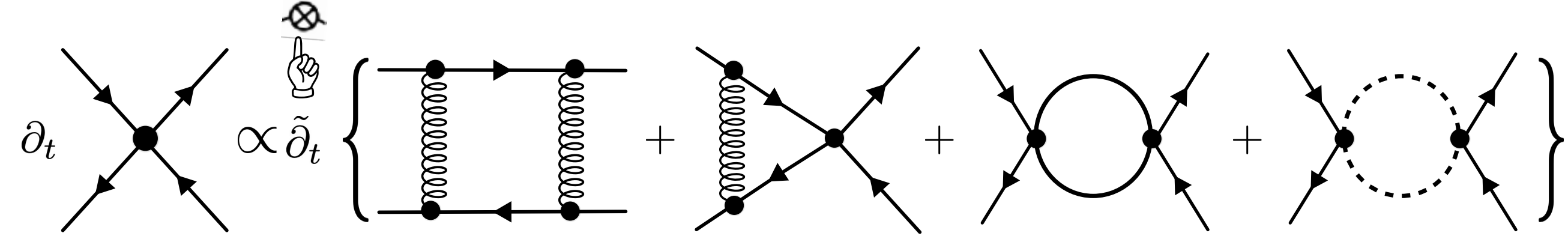
$$\eta_i = -\frac{\partial_t Z_i}{Z_i}$$



# Computing the four-Fermi flows (gGG example)

- Flow of four-point functions

$$\partial_t \Gamma_k^{(\bar{\psi}_a \psi_b \bar{\psi}_c \psi_d)} = -\partial_t (Z_\psi^2 \lambda_1) \mathcal{T}_{1,L}^{abcd} - \partial_t (Z_\psi^2 \lambda_2) \mathcal{T}_{2,L}^{abcd}$$



$$\partial_t \bar{\lambda}_1 = (2 + 2\eta_\psi) \bar{\lambda}_1 + \frac{k^2}{Z_\psi^2 \mathcal{N}} \left\{ \partial_t \left[ \mathcal{P}_{1,R}^{abcd} \Gamma_k^{(\bar{\psi}_a \psi_b \bar{\psi}_c \psi_d)} \right] (N_c N_\psi + 1) - \partial_t \left[ \mathcal{P}_{2,R}^{abcd} \Gamma_k^{(\bar{\psi}_a \psi_b \bar{\psi}_c \psi_d)} \right] (N_c + N_\psi) \right\}$$

$$\partial_t \bar{\lambda}_2 = (2 + 2\eta_\psi) \bar{\lambda}_2 + \frac{k^2}{Z_\psi^2 \mathcal{N}} \left\{ \partial_t \left[ \mathcal{P}_{2,R}^{abcd} \Gamma_k^{(\bar{\psi}_a \psi_b \bar{\psi}_c \psi_d)} \right] (N_c N_\psi + 1) - \partial_t \left[ \mathcal{P}_{1,R}^{abcd} \Gamma_k^{(\bar{\psi}_a \psi_b \bar{\psi}_c \psi_d)} \right] (N_c + N_\psi) \right\}$$

Anomalous dimensions:

$$\eta_i = -\frac{\partial_t Z_i}{Z_i}$$

- System of flows:

$$\partial_t \bar{\lambda}_1 = (2 + 2\eta_\psi) \bar{\lambda}_1 + \frac{1}{16\pi^2} \left[ \frac{g^4 (3N_c^2 + 4) (5\eta_A + 3\eta_\psi - 45)}{160N_c^2} + \frac{g^2 (5\eta_A + 6\eta_\psi - 60) (\bar{\lambda}_1 - \bar{\lambda}_2 N_c)}{10N_c} \right. \\ \left. - \frac{1}{20} (\eta_\chi - 5) N_\chi \left( \bar{\lambda}_6^2 (N_c - 1) N_c + \frac{(N_c - 2)}{2N_c} \bar{\lambda}_7^2 \right) + \frac{4}{5} (\eta_\psi - 5) \left( \bar{\lambda}_2^2 - \bar{\lambda}_1 \bar{\lambda}_2 (N_c + N_\psi) - \bar{\lambda}_1^2 \frac{(N_c N_\psi - 1)}{2} \right) \right]$$

$$\partial_t \bar{\lambda}_2 = (2 + 2\eta_\psi) \bar{\lambda}_2 + \frac{1}{16\pi^2} \left[ \frac{g^4 (N_c^2 - 8) (5\eta_A + 3\eta_\psi - 45)}{160N_c} + \frac{g^2 (5\eta_A + 6\eta_\psi - 60) (\bar{\lambda}_2 - \bar{\lambda}_1 N_c)}{10N_c} \right. \\ \left. - \frac{1}{40} (\eta_\chi - 5) \bar{\lambda}_7^2 (N_c - 2) N_\chi + \frac{2}{5} (\eta_\psi - 5) (4\bar{\lambda}_1 \bar{\lambda}_2 - \bar{\lambda}_2^2 (N_c + N_\psi)) \right]$$

⋮

# What do we know?

## Method

Monte Carlo Lattice simulations

Nielsen, Ninomiya '80 '81 Eichten, Preskill '86

Most Attractive Channel

Raby, Dimopoulos, Susskind '79

Dimopoulos, Raby, Susskind '80 Eichten, Feinberg '82

Large N

Eichten, Peccei, Preskill, Zeppenfeld '86

Karasik, Önder, Tong [2208.07842]

Anomaly mediated SUSY breaking

Csáki, Murayama, Telem [2105.03444]

Anomaly matching, discrete symmetries and higher forms

't Hooft '80 Bolognesi, Konishi, Luzio [2101.02601]

Bars, Yankielowicz '81

## Major challenges

Doubler and sign problems

Too naive

Finite N

Non-SUSY limit

Not sufficiently constraining

## Results

*Color-flavor-locked* IR with  $\langle \chi \psi \rangle$

If confinement, no dSB

Conformal theories for few  $N < 13$  and for larger,  $\langle \chi \psi \rangle$  and  $\langle \chi \chi \rangle$

Massless  $\mathcal{B} \sim \langle \chi \psi \psi \rangle$  baryons with no dSB, or *color-flavor-locked* phase

# What do we know?

| Method                                                                                                                                             | Major challenges                                                                            | Results                                                                                                       |
|----------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------|
| Monte Carlo Lattice simulations<br><small>Nielsen,Ninomiya'80'81 Eichten,Preskill'81</small>                                                       | <div>In conclusion:<br/>Highly challenging and <u>no</u> good understanding of the IR</div> | Conformal theories for few $N < 13$ and for larger, $\langle \chi\psi \rangle$ and $\langle \chi\chi \rangle$ |
| Most Attractive Channel<br><small>Raby,Dimopoulos,Susskind '79<br/>Dimopoulos,Raby,Susskind '80 Eichten,Feinberg '82</small>                       |                                                                                             | Color-flavor-locked IR with $\langle \chi\psi \rangle$                                                        |
| Large N<br><small>Eichten,Peccei,Preskill,Zeppenfeld'86<br/>Karasik,Önder,Tong[2208.07842]</small>                                                 |                                                                                             | If confinement, no dSB                                                                                        |
| Anomaly mediated SUSY breaking<br><small>Csáki,Murayama,Telem[2105.03444]</small>                                                                  |                                                                                             | Conformal theories for few $N < 13$ and for larger, $\langle \chi\psi \rangle$ and $\langle \chi\chi \rangle$ |
| Anomaly matching, discrete symmetries and higher forms<br><small>'t Hooft '80 Bolognesi,Konishi,Luzio[2101.02601]<br/>Bars,Yankielowicz'81</small> |                                                                                             | Massless $\mathcal{B} \sim \langle \chi\psi\psi \rangle$ baryons with no dSB, or color-flavor-locked phase    |
|                                                                                                                                                    | Finite N                                                                                    |                                                                                                               |
|                                                                                                                                                    | Non-SUSY limit                                                                              |                                                                                                               |
|                                                                                                                                                    | Not sufficiently constraining                                                               |                                                                                                               |



# What do we know?

| Method                                                                                                                                                                                  | Major challenges              | Results                                                                                                             |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------|---------------------------------------------------------------------------------------------------------------------|
| Monte Carlo Lattice simulations<br><a href="#">Nielsen, Ninomiya '80'81</a> <a href="#">Eichten, Preskill '81</a>                                                                       |                               |                                                                                                                     |
| Most Attractive Channel<br><a href="#">Raby, Dimopoulos, Susskind '79</a><br><a href="#">Dimopoulos, Raby, Susskind '80</a> <a href="#">Eichten, Feinberg '82</a>                       |                               | <i>color-flavor-locked</i> IR with $\langle \chi \psi \rangle$                                                      |
| Large N<br><a href="#">Eichten, Peccei, Preskill, Zeppenfeld '86</a><br><a href="#">Karasik, Önder, Tong [2208.07842]</a>                                                               | Finite N                      | If confinement, no dSB                                                                                              |
| Anomaly mediated SUSY breaking<br><a href="#">Csáki, Murayama, Telem [2105.03444]</a>                                                                                                   |                               | Conformal theories for few $N < 13$ and larger, $\langle \chi \psi \rangle$ and $\langle \chi \chi \rangle$         |
| Anomaly matching, discrete symmetries and higher forms<br><a href="#">'t Hooft '80</a> <a href="#">Bolognesi, Konishi, Luzio [2101.02601]</a><br><a href="#">Bars, Yankielowicz '81</a> | Not sufficiently constraining | Massless $\mathcal{B} \sim \langle \chi \psi \psi \rangle$ baryons with no dSB, or <i>color-flavor-locked</i> phase |

**In conclusion:**  
Highly challenging and no good understanding of the IR

A promising path: functional methods