Causal Structure and Spectral properties of the Landau Gauge Quark Propagator

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(JLU Giessen)

In collaboration with Jan M. Pawlowski

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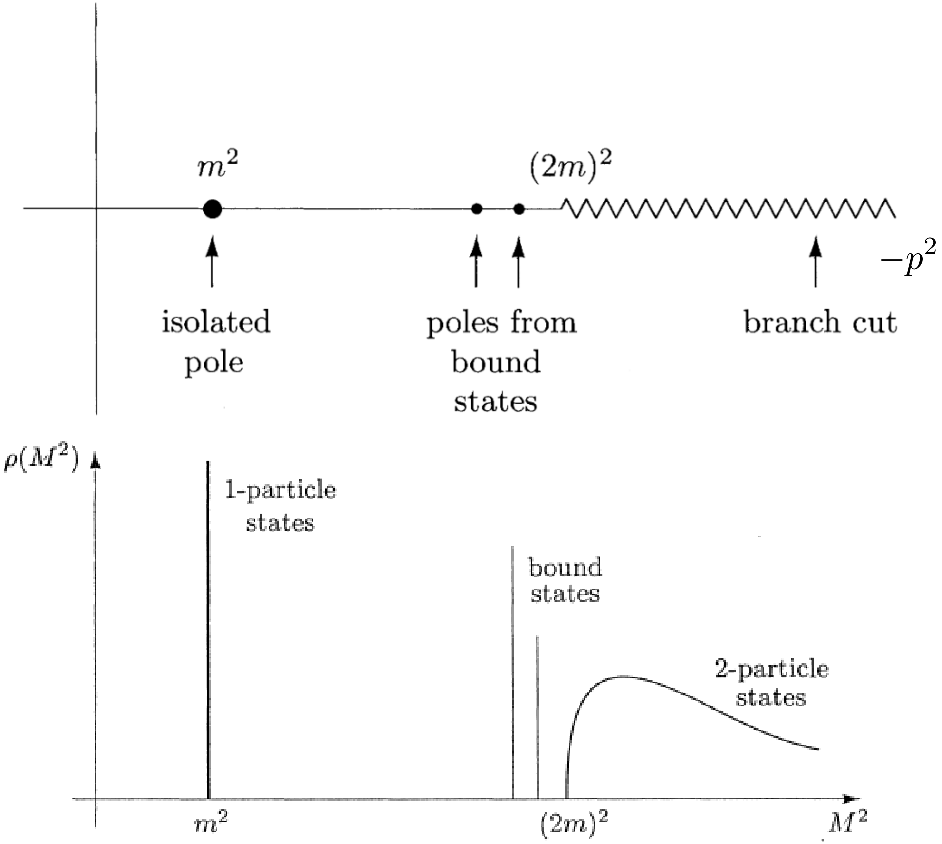
Sevilla 08/09/2026

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

DFG Deutsche Forschungsgemeinschaft

Spectral Functions



Peskin & Schröder

$$\boxed{G(p) = \int_\lambda \frac{\rho(\lambda)}{p^2 + \lambda^2}}$$

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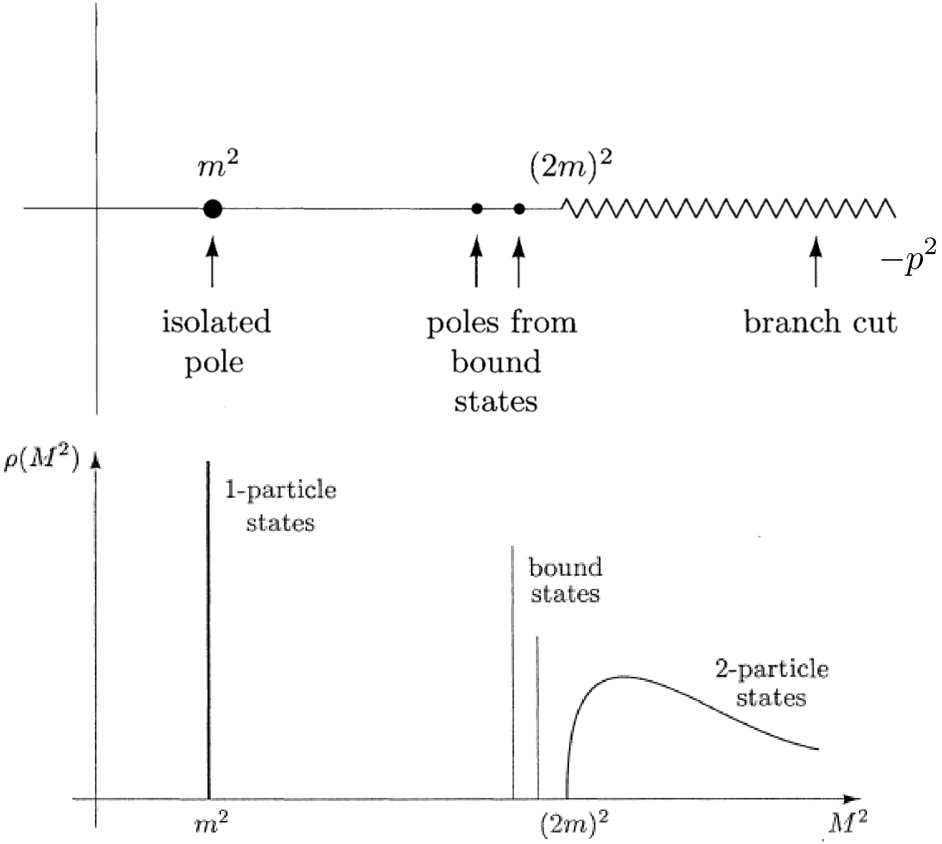

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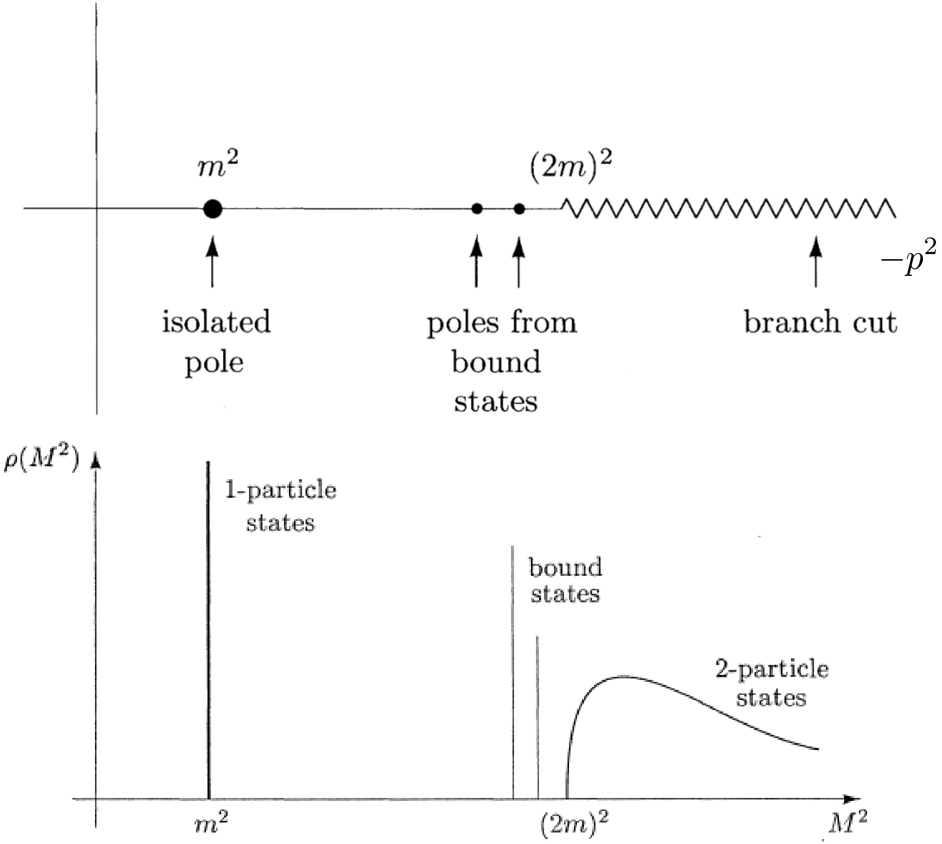

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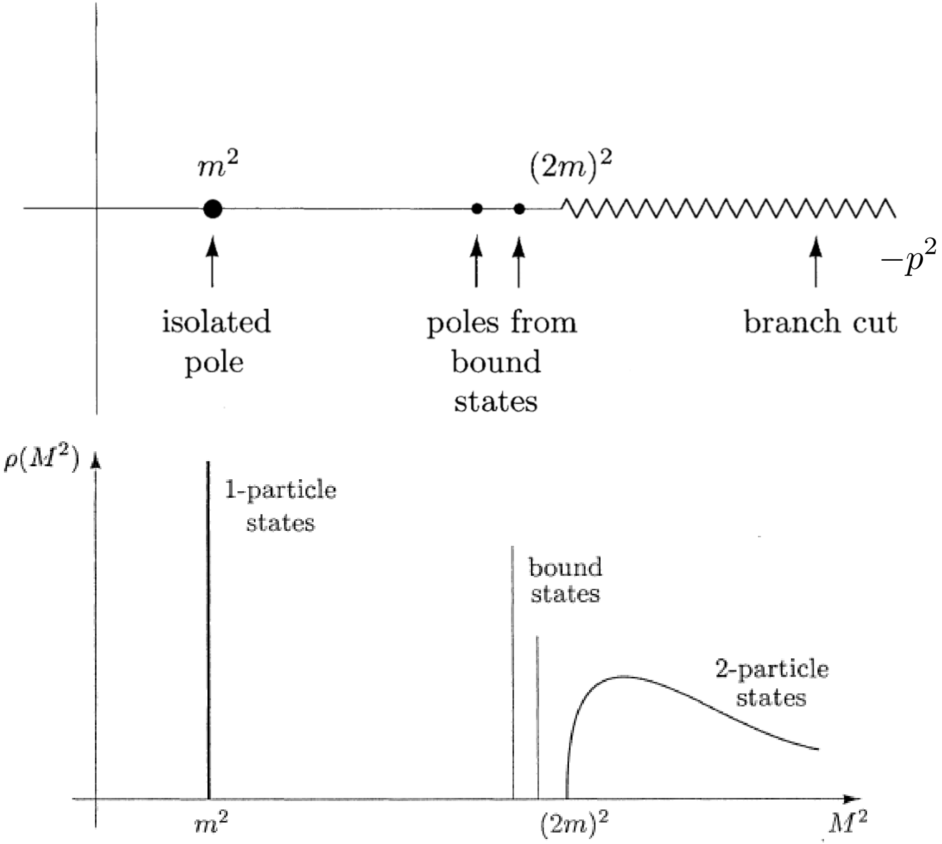

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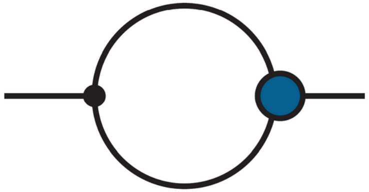
- Spectral functions plays role of (propability) weight for intermediate states of energy λ.
- Causality requires analyticity in upper half plane.
- Propagator determined by its discontinuities.

$$\rho(\omega) = 2\mathrm{Im}\, G\left(p_0 \to -(\omega + 0^+)\right)$$

Spectral Functional equations



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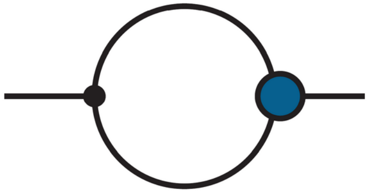

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$$= \int_{\lambda_1,\lambda_2} \rho(\lambda_1)\rho(\lambda_2) \int_q \frac{1}{(q^2+\lambda_1^2)((p+q)^2+\lambda_2^2)}$$

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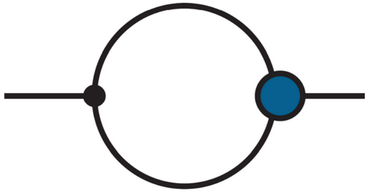

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- Loop integrals can be calculated in dimReg
- Access to the full complex plane

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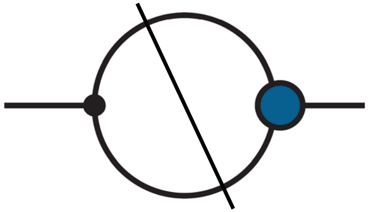

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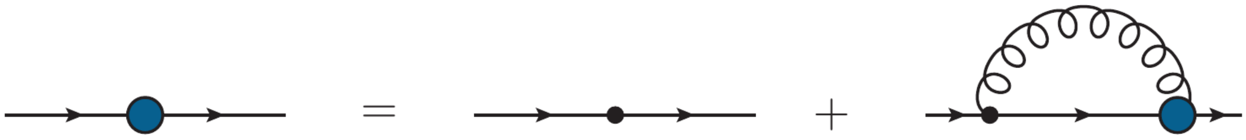
- Loop integrals can be calculated in dimReg
- Access to the full complex plane

- Thresholds/Branch-points depend on support of spectral functions
- Functional form of the discontinuity determined by spectral weights

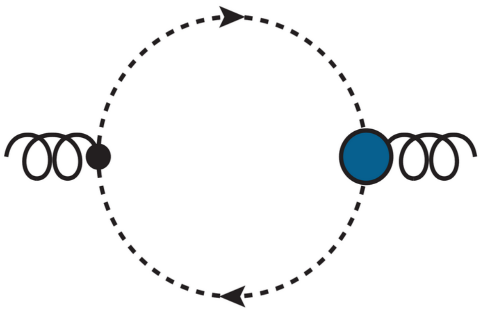
The spectral quark gap equation



J. M. Pawlowski, JW EPJC 85 (2025)

Cutting Rules in QCD

Gluon



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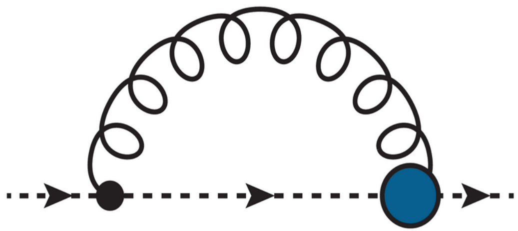

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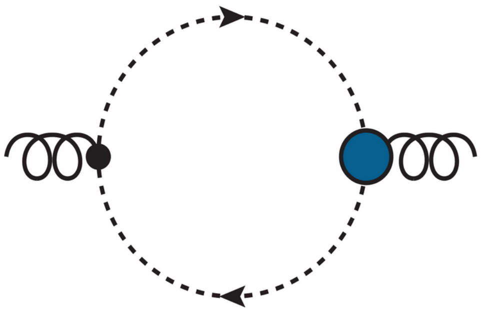

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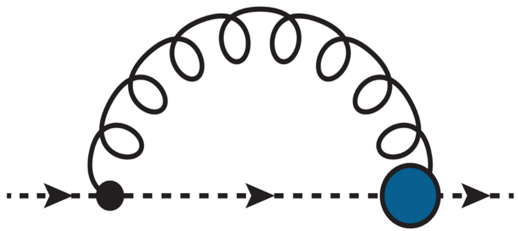

Ghost propagator in Landau gauge

$$G_c(p) \propto \frac{1}{p^2} + \text{subleading}$$

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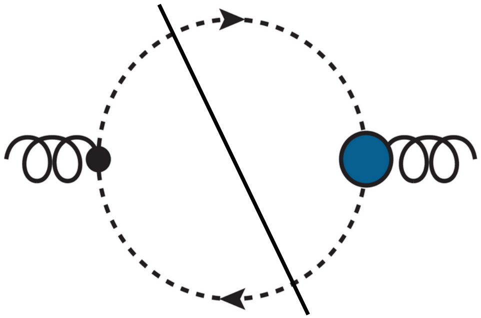

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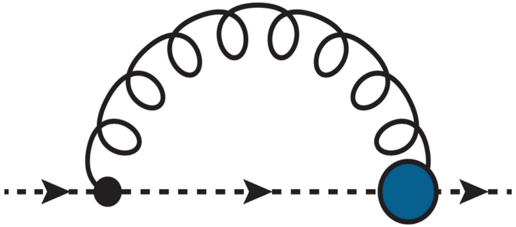

Cut opens at origin!

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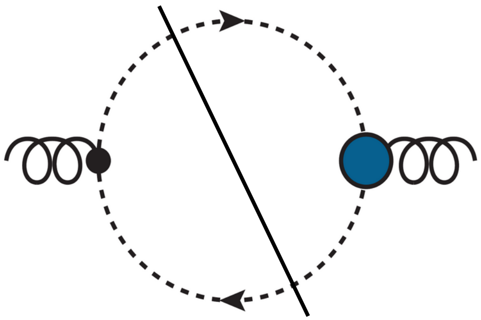

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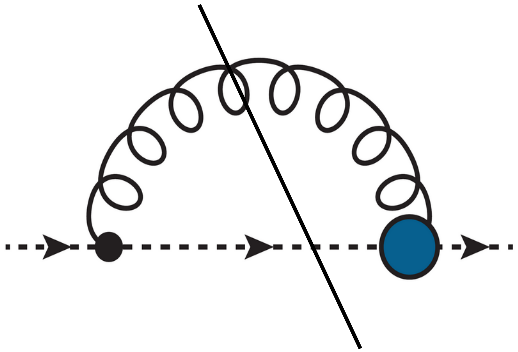

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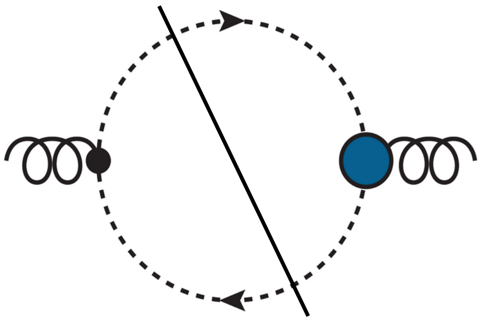

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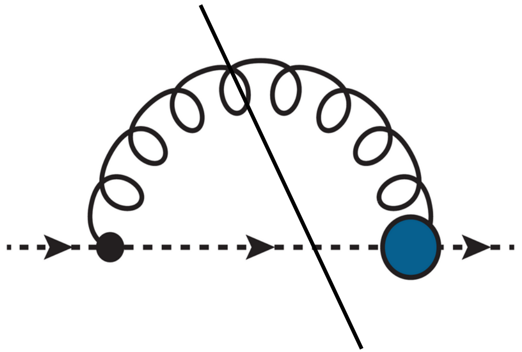

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Ghost propagator in Landau gauge

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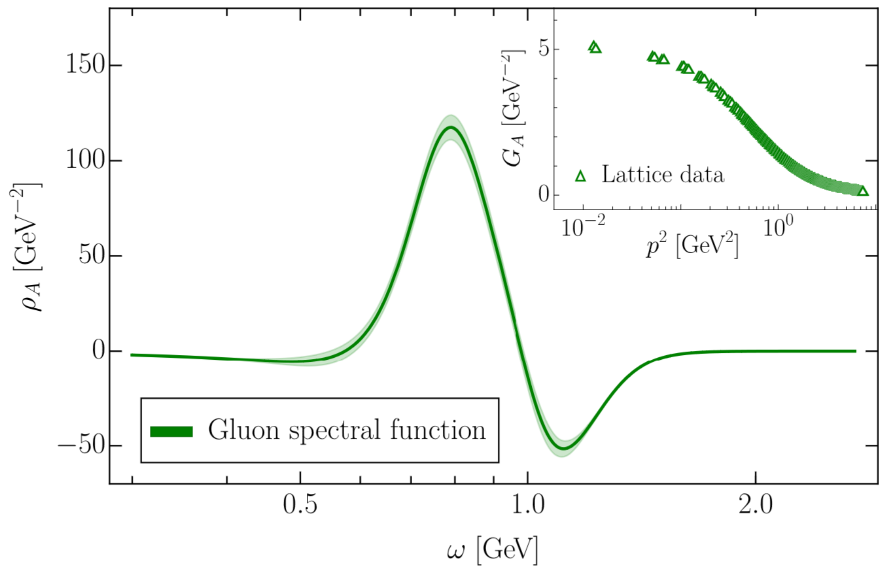
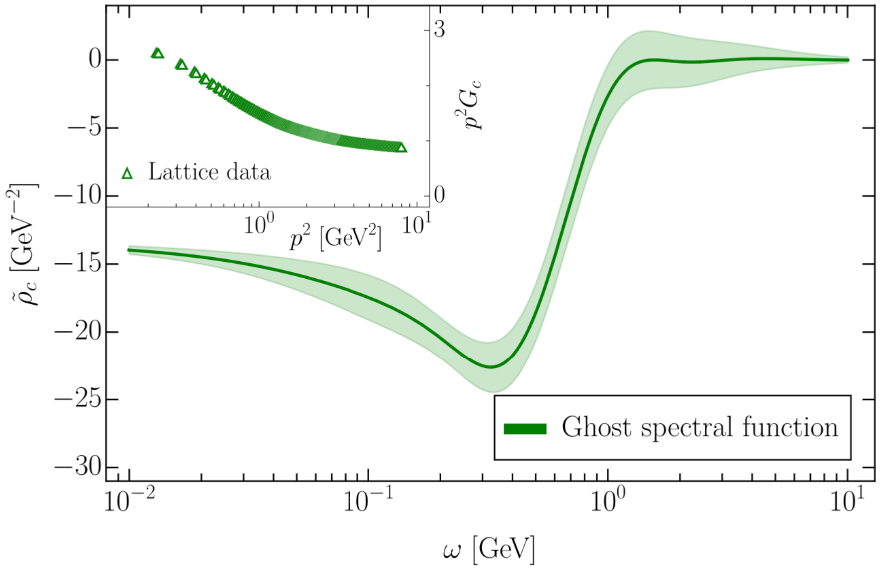
Independent of additional structures in the Ghost/Gluon propagator, both feature a branch-cut on the whole real frequency axis!

Status of Spectral functions in QCD





Rec. via GPR from:
Horak et. al. PRD 105 (2022)
Horak et. al. PRD 107 (2023)

Rec. in YM:
Cyrol, Pawlowski, Rothkopf, Wink
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Direct Computation (only Ghost):
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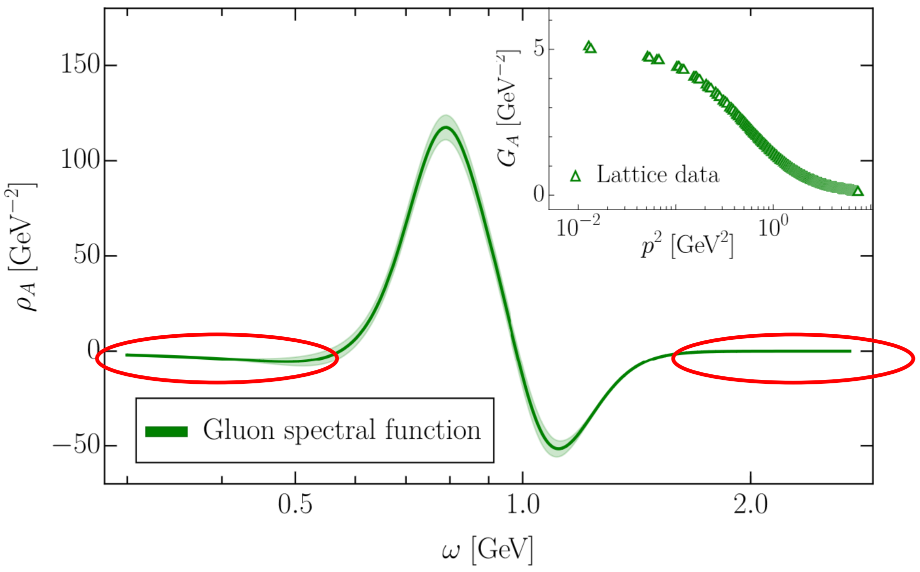

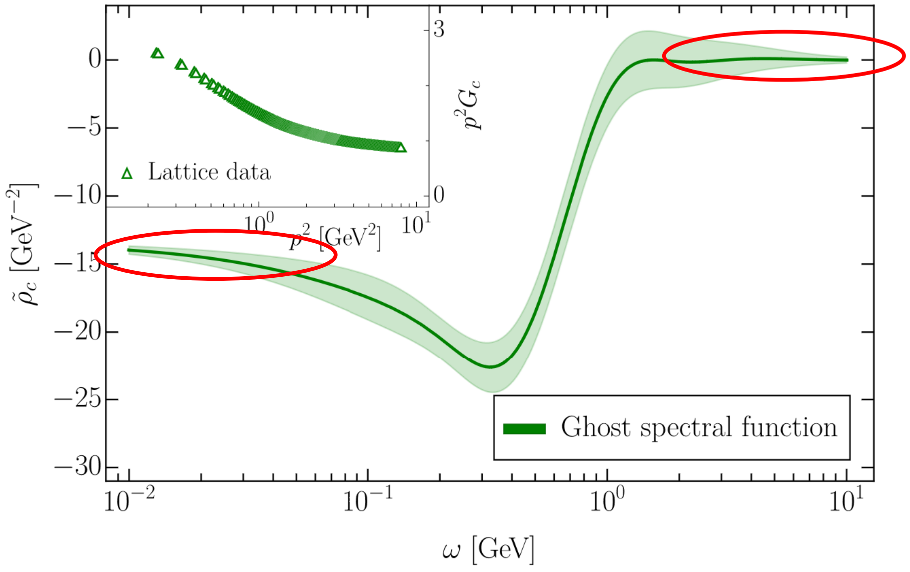

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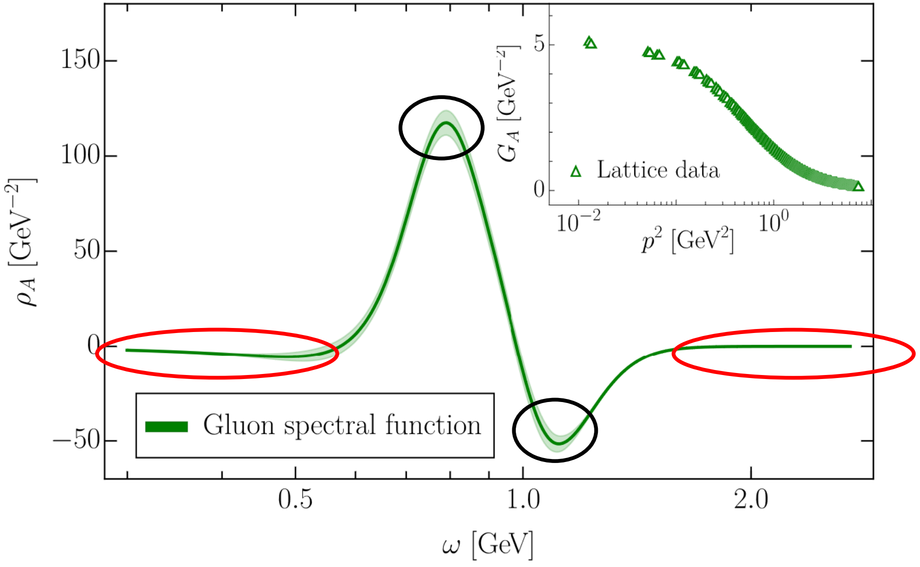

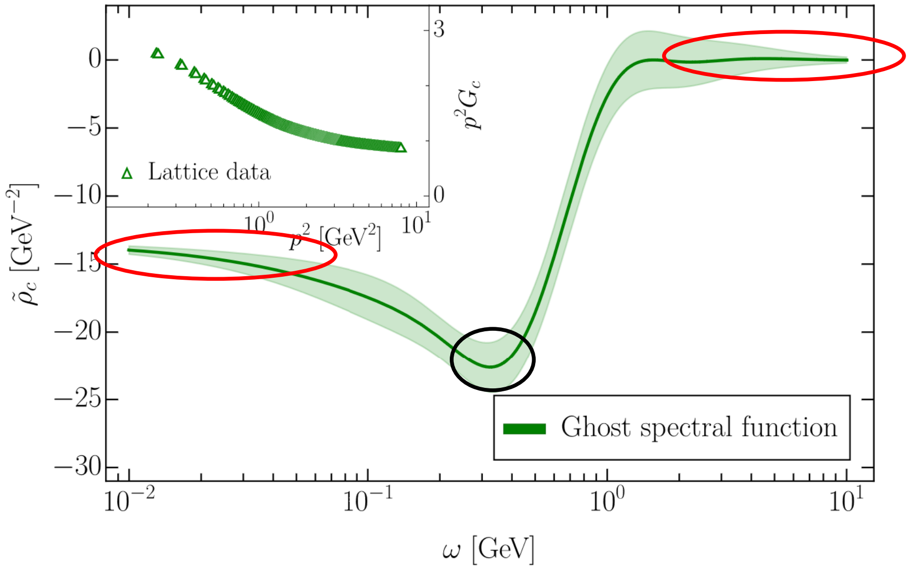

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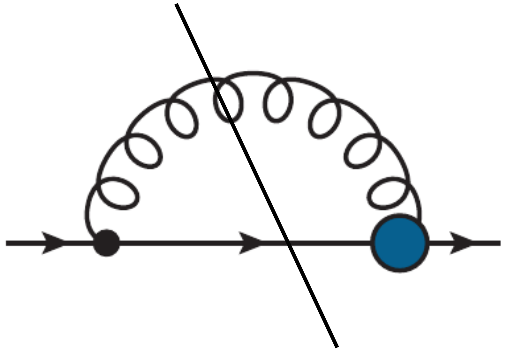
Two scenarios for the quark

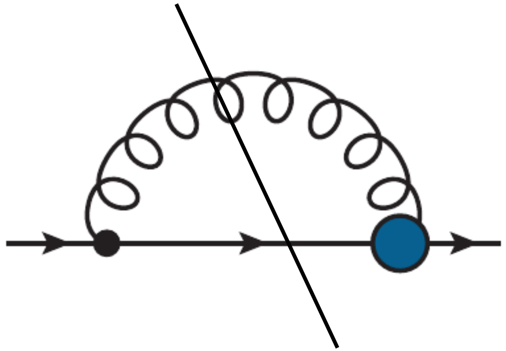


Two scenarios for the quark

A. Real first singularity

… located at

$$p^2 = -m_{\text{pole}}^2 \in \mathbb{R}$$



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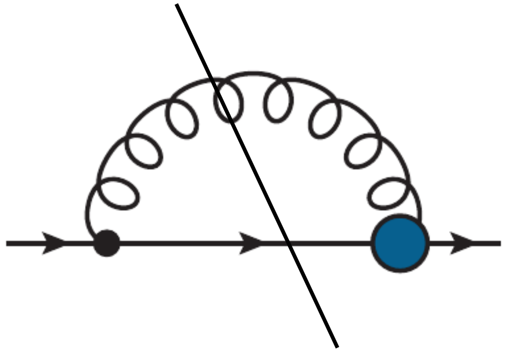

Quark Dressings acquire a cut on the real axis, starting right after the pole.

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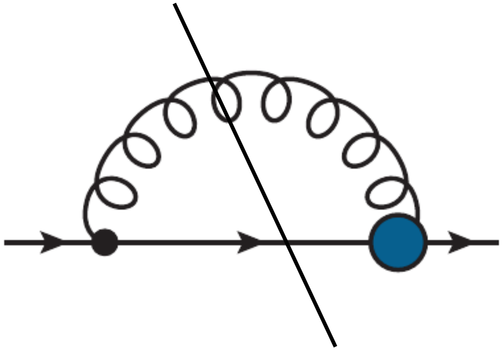

B. Pair of cc poles as first singularity

… located at

$$m_\chi \text{ and } \bar{m}_\chi \quad \text{with } \mathrm{Im}\, m_\chi^2 \neq 0$$

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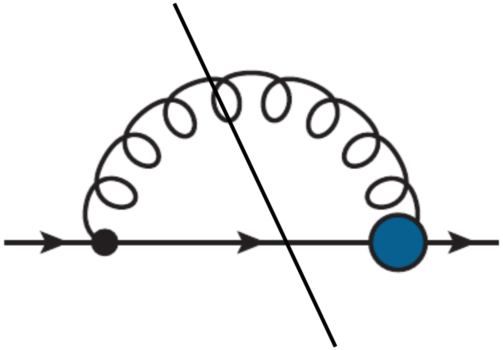

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No Cuts on the real axis ?

CC – cuts starting at m_χ and $\bar{m}_\chi$.

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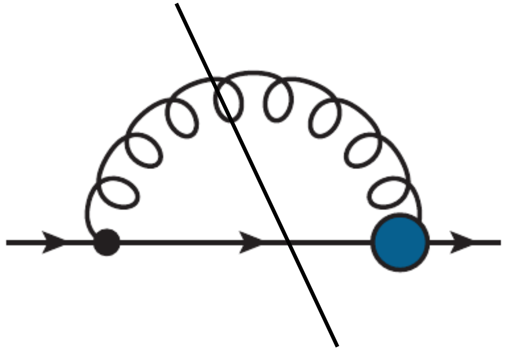

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Recent Results:

Pawlowski, JW EPJC 85 (2025)

Wieland, Alkofer 2606.20235

Alkofer, Ferreira, Miramontes, Morgado, Papavassiliou Phys.Lett B 880 (2026)

strongly indicate a real first pole.

Spectral self-energy

J. M. Pawlowski, JW EPJC 85 (2025)

Rec. via GPR from:
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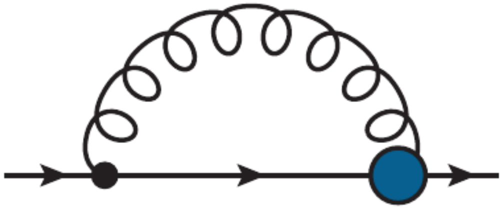

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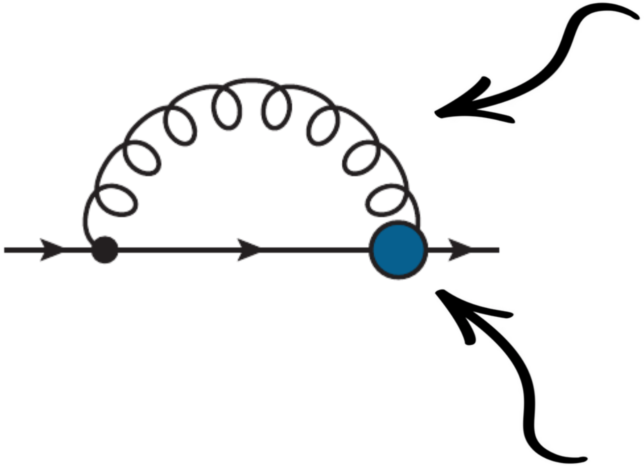
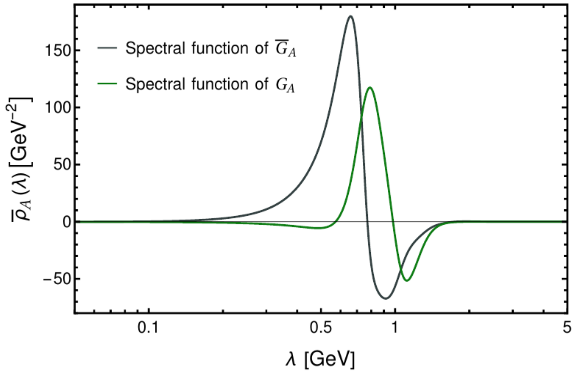
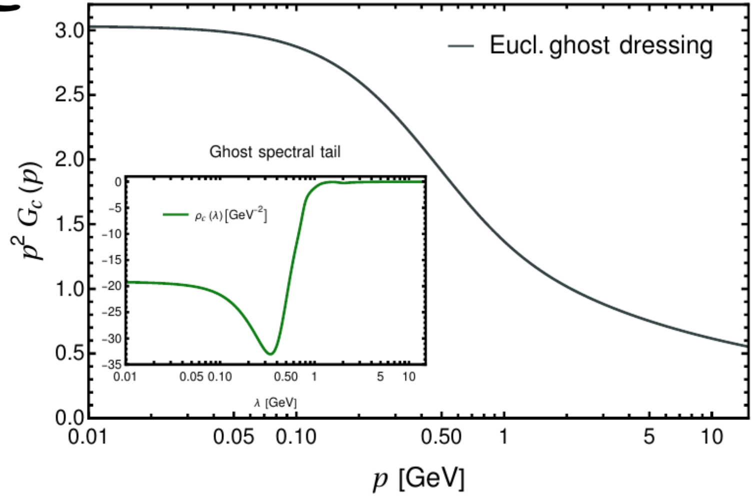




$$\Gamma^\mu \propto \gamma^\mu p^2 G_c(p)$$



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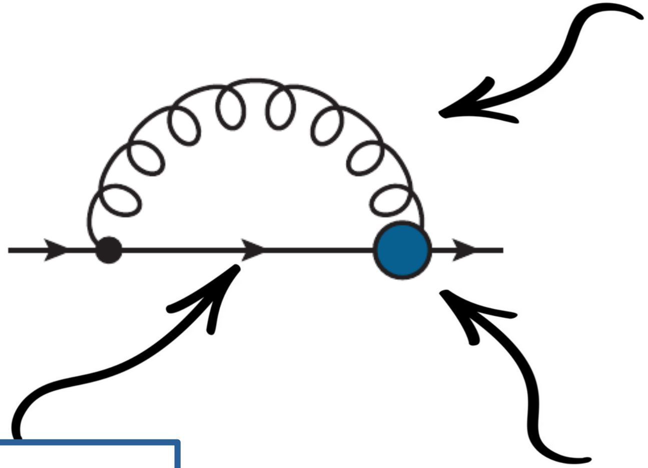

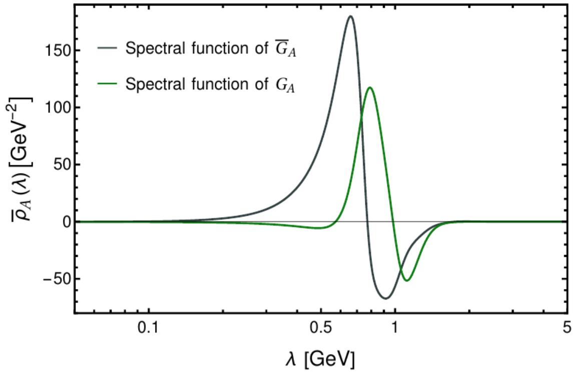

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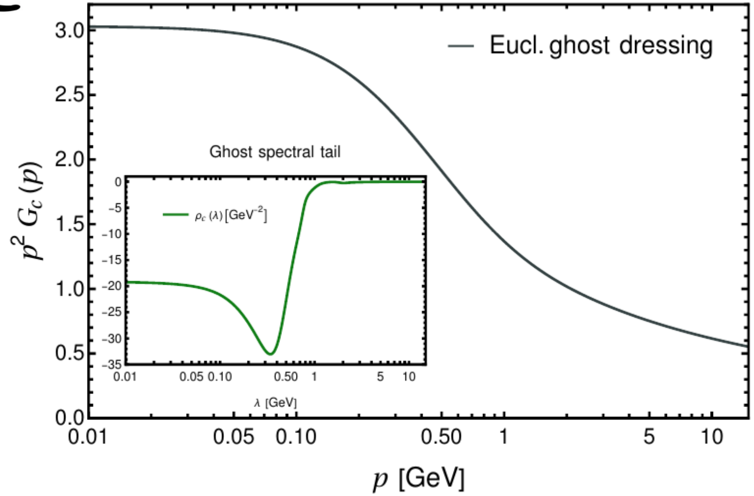
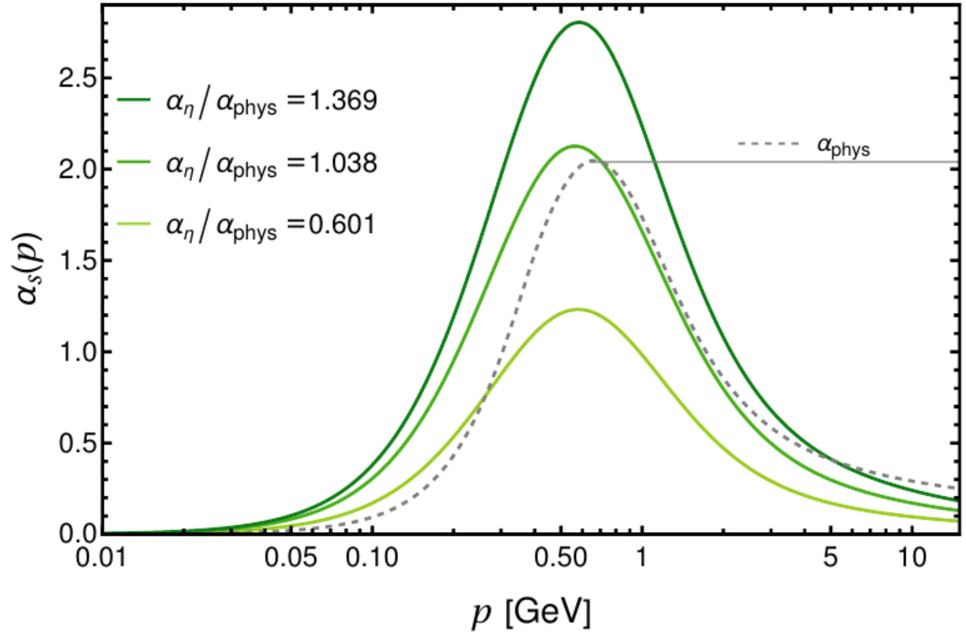
Compute selfconsistently:

$$G_q(p) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \frac{\rho_q(\lambda)}{\mathrm{i}\not{p} + \lambda}$$

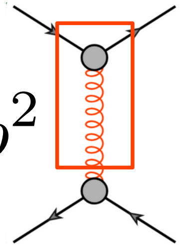
$$= \frac{-\mathrm{i}\not{p} + M_q(p)}{Z_q(p)\left[p^2 + M_q^2(p)\right]}$$

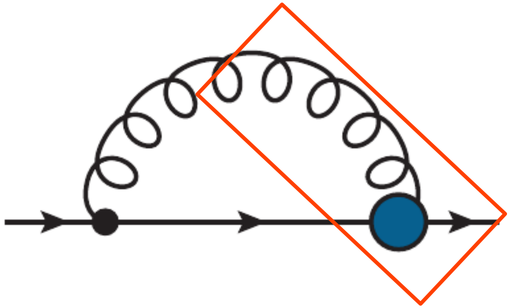


The causal quark gluon coupling



$$\alpha_s(p) = p^2$$





α_s^{phys} by **Ihssen, Pawlowski, Sattler, Wink arXiv: 2408.08413 (2024)**

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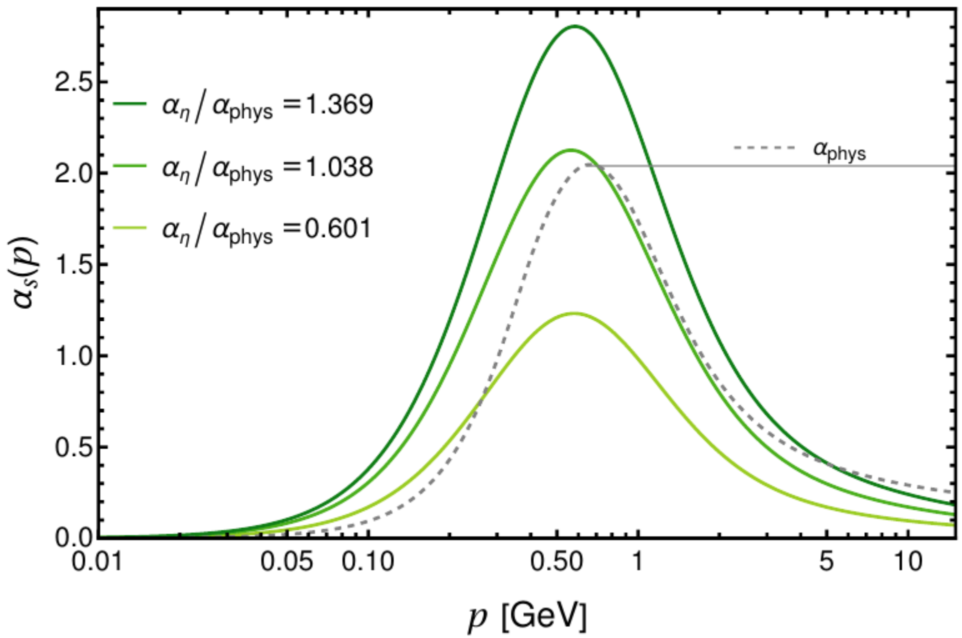
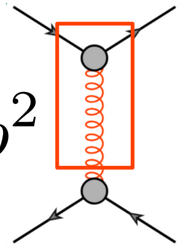


→ Peak position is well reproduced with given input.

→ Tune height of the peak with global factor η.



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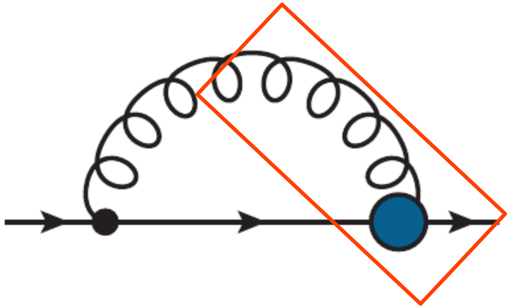

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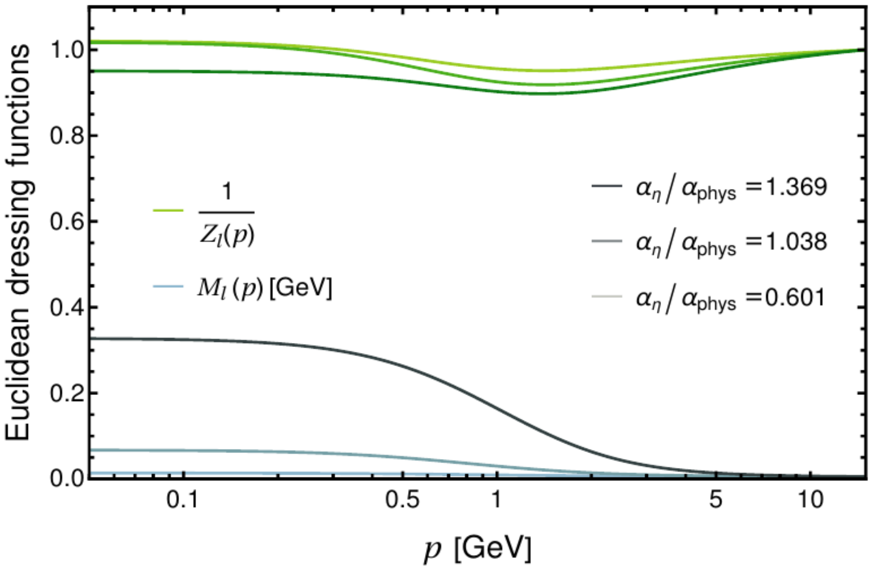
The quark spectral function



$\rightsquigarrow$ Causal vertex model reproduces phenomenology of $\mathrm{D}\chi\mathrm{SB}$.

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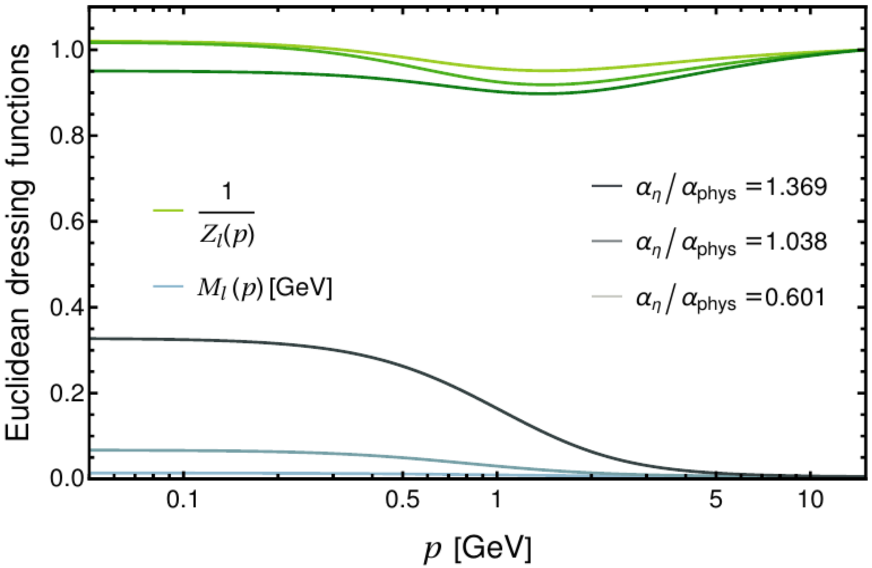
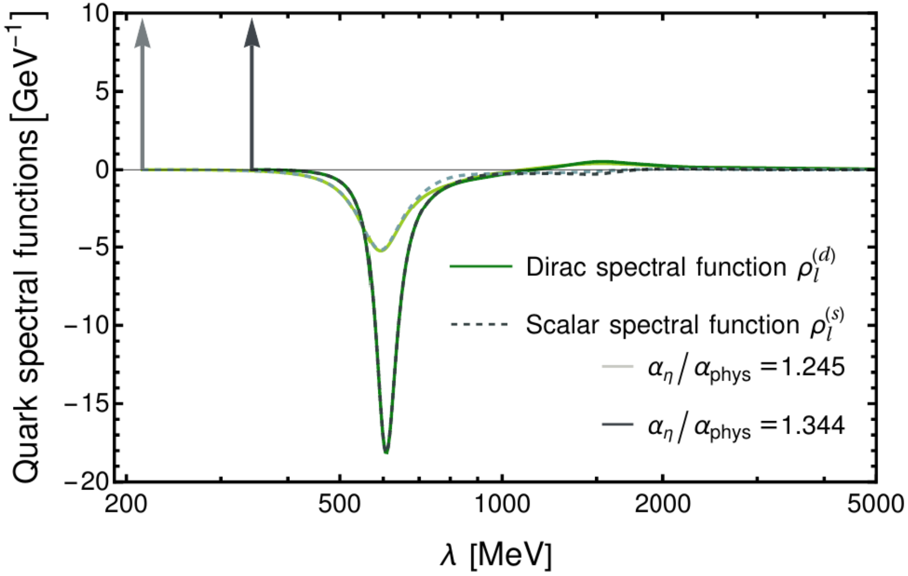
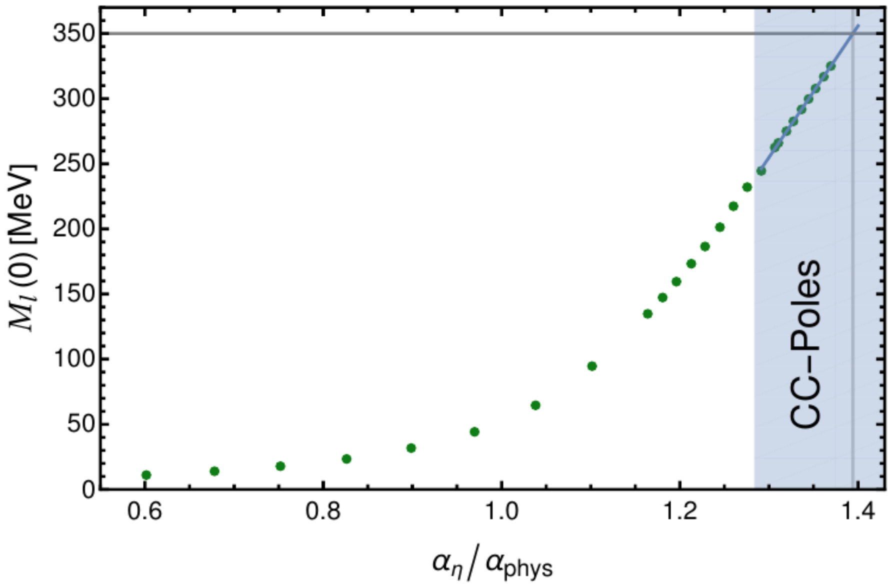




- Causal vertex model reproduces phenomenology of $\mathrm{D}\chi\mathrm{SB}$.

- Real pole at the onset of scattering for converged solution for all coupling strengths.
- Spectral representation for small Enhancements, including the physical one.

Emergence of non-causal structures



For $\alpha_\eta/\alpha_{\mathrm{phys}} \geq 1.283$ additional cc poles emerge in the complex plane.

Spectral representation is lost for η that reproduce $M_l(0) \approx 350\mathrm{MeV}$.

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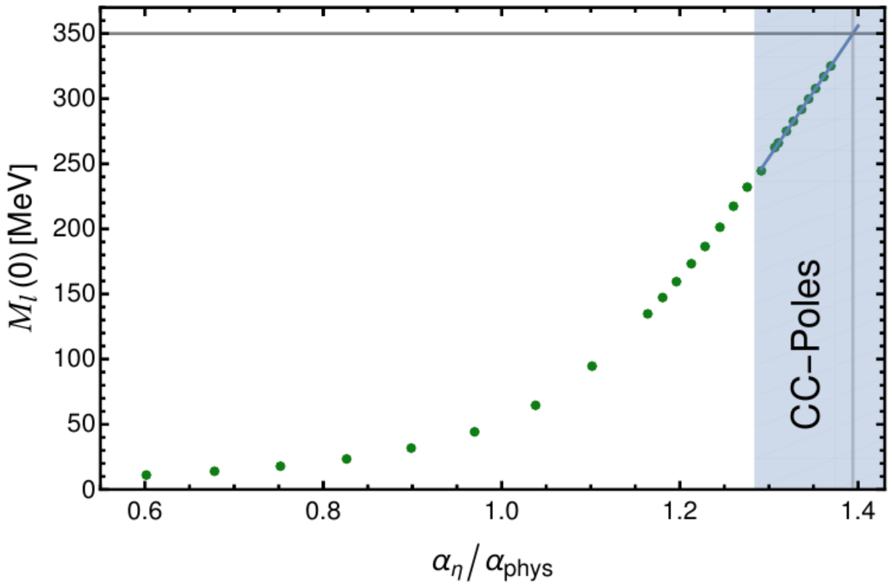

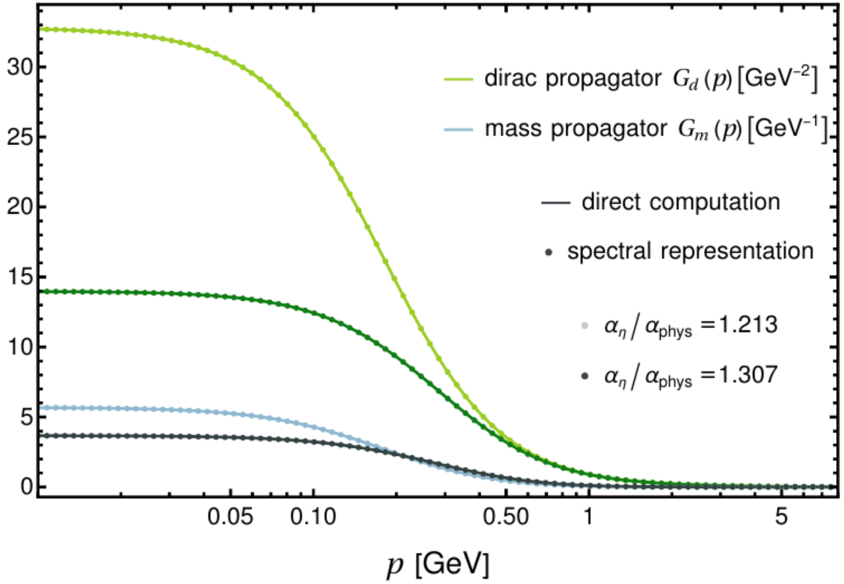

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- Spectral representation is lost for η that reproduce $M_l(0) \approx 350\text{MeV}$.

- Spectral representation effectively fulfilled up to permille level.
- CC poles likely not effect computation of light hadron properties.

Conclusion

- Quark spectral functions with **causal** Rainbow-Ladder model

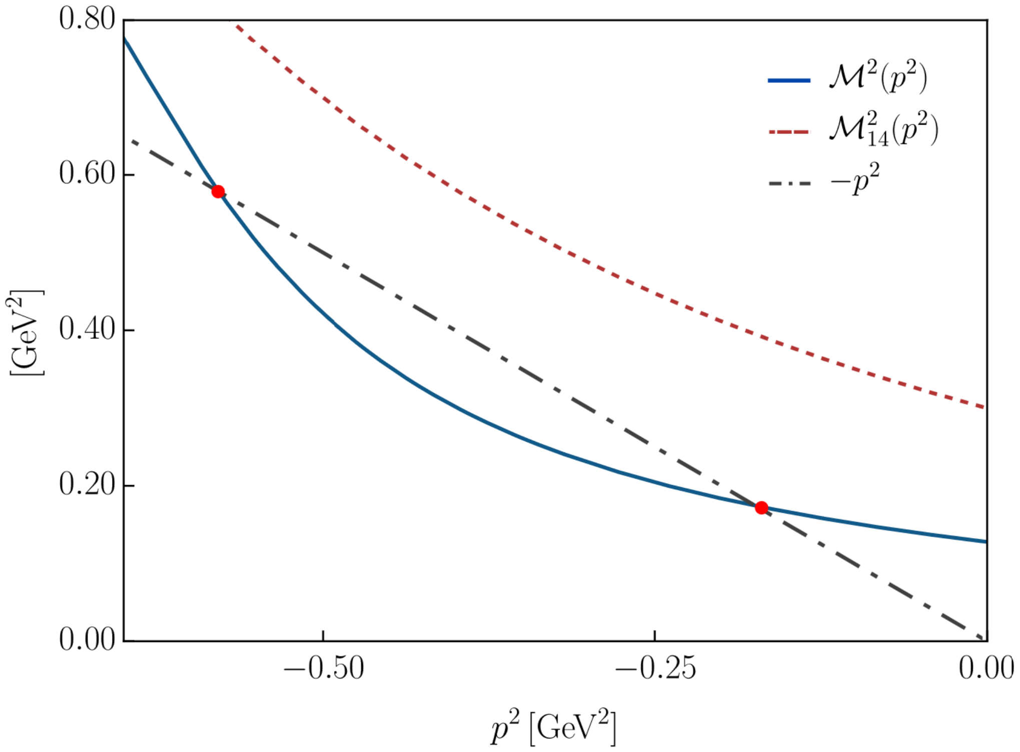
- Real **first** singularity and a continuous spectral function.

- Enhancement of vertex strength above a critical value leads to **additional** cc-poles.

- Phenomenological applications possible with effective KL-representation!

Backup

Scenario B) CC-poles as first singularity – What can we know about the real axis?



- CC poles alone do not give rise to an imaginary part on the real axis.
- Without essential singularity at infinity, Mass-function has to come down again.
- Crossing the $-p^2$ – line leads to real pole.
- By the same logic as before, cut induced right after.

Back to Scenario A) real first pole:

- Cut after the first pole translate to Cut/Imaginary part of the Massfunction on real Axis.
- Second crossing in presence of imaginary part for M^2

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