Quark propagator without complex conjugate poles

[Based on Phys.Lett.B 880 (2026) 140856]

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With: R. Alkofer, M. N. Ferreira, A. S. Miramontes and J. Papavassiliou

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CSIC
Consejo Superior de Investigaciones Científicas

Vniversitat de València

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Excelencia Severo Ochoa

Introduction

Goal

To study the analytic structure of the quark propagator

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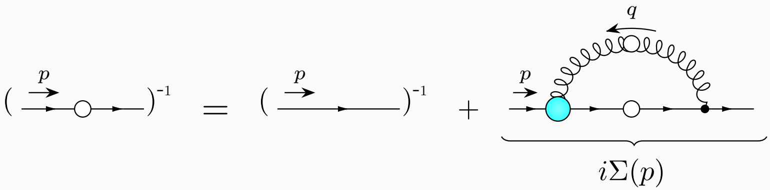

$$S(p) = \sigma_V(p^2)\not{p} + \sigma_S(p^2)\,, \qquad S^{-1}(p) = A(p^2)\not{p} - B(p^2)$$

$$S^{-1}(p) = S_0^{-1}(p) - i\Sigma(p)\,, \quad \Sigma(p) = -\frac{4g^2}{3}\int_q \gamma_\mu S(p+q)\overline{\Gamma}^\mu(q,p,-p-q)\Delta(q^2)\,.$$

The quark-gluon vertex (II)

$$\overline{\Gamma}^{\mu}(q,p,-k) = \sum_{i=1}^{8} \lambda_i(q,p,-k)\bar{\tau}_i^{\mu}(p,-k)$$

$$\bar{\tau}_i^{\mu}(p,-k) = P^{\mu\nu}(q)\tau_i^{\nu}(p,-k)$$

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\tau_1^{\nu} &= \gamma^{\nu}\,, & \tau_2^{\nu} &= (k+p)^{\nu}\,,\\
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[Aguilar,Ferreira,Oliveira,Papavassiliou,Linhares,EPJC84(2024)11,1231]

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[Aguilar,Ferreira,Oliveira,Papavassiliou,Linhares,EPJC84(2024)11,1231]

Name		Form factor	Tensor
Tree-Level Component	(TLC)	$\lambda_1(q,p,-k)$	γ^{ν}
Anomalous Chromomagnetic Moment	(ACM)	$\lambda_4(q,p,-k)$	$(\not{k}-\not{p})\gamma^{\nu}$
Spin-Momentum Curvature	(SMC)	$\lambda_7(q,p,-k)$	$\frac{1}{2}[\not{k},\not{p}]\gamma^{\nu}$

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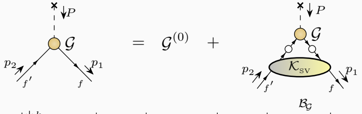
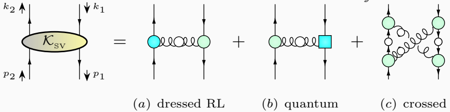
[Aguilar,Ferreira,Oliveira,Papavassiliou,Linhares,EPJC84(2024)11,1231]

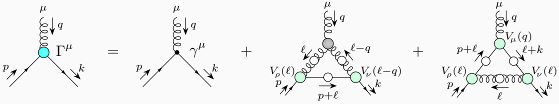
Symmetric vertex approximation

[Ferreira,Miramontes,JM,Papavassiliou,Pawlowski,EPJC86(2026)4,325
Ferreira,Miramontes,JM,Papavassiliou,EPJC86(2026)7,858]





See A.Miramontes' talk
Wednesday, Session A, 10:00



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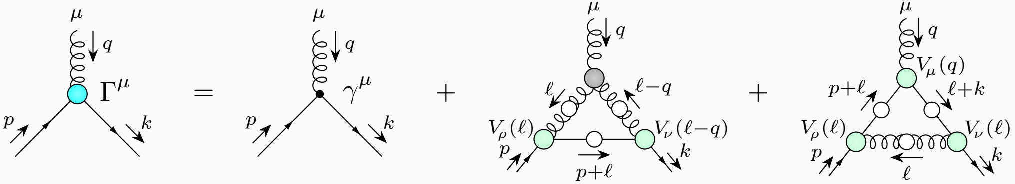
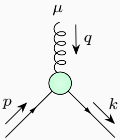


$$V^{\mu}(q) = V(q^2)\gamma^{\mu} =$$



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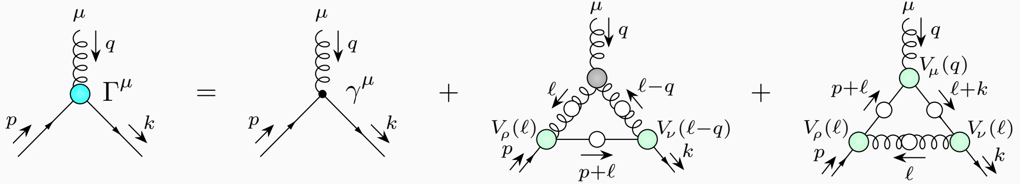


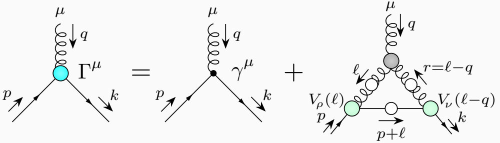
1. Neglect Abelian contribution: $\sim 2‰$ of full result
2. Take the soft-gluon limit

The quark-gluon vertex (III)

[Ferreira,Miramontes,JM,Papavassiliou,EPJC86(2026)8,954]

$$\overline{\Gamma}^{\mu}(q,p,-k) = \sum_{i=1}^{8} \lambda_i^{sg}(p^2)\bar{\tau}_i^{\mu}(p,-k)$$

$$\lambda_i^{sg}(p^2) := \lim_{q\to 0} \lambda_i(q,p,-k)$$



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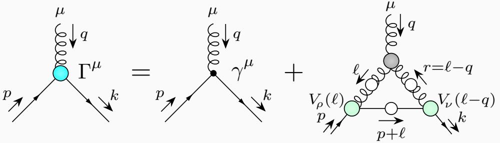

1. Write down vertex SDE (general kinematics): $\overline{\Gamma}^{\mu}(q,p,-k) = \overline{\Gamma}_0^{\mu}(q) + \overline{\Gamma}_{\rm nab}(q,p,-k)\,,$

$$\overline{\Gamma}^{\mu}_{\rm nab}(q,p,-k) = \frac{3ig^2}{2}\int_\ell \overline{\Gamma}_0^{\nu}(r)S(p+\ell)\overline{\Gamma}_0^{\rho}(\ell)\overline{\Gamma}^{\mu\nu\rho}(q,r,-\ell)\Delta(\ell^2)\Delta(r^2)V(\ell^2)V(r^2)\,,$$

with $\overline{\Gamma}_0^{\mu}(q) := P^{\mu\nu}(q)\gamma^{\nu}$ and $\overline{\Gamma}^{\mu\nu\rho}(q,r,-\ell) = P^{\mu\alpha}(q)P^{\nu\beta}(r)P^{\rho\gamma}(\ell)\Gamma^{\alpha\beta\gamma}(q,r,-\ell).$

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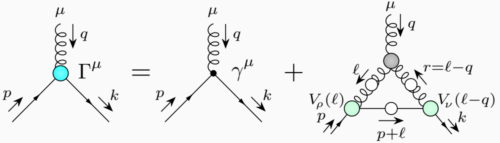

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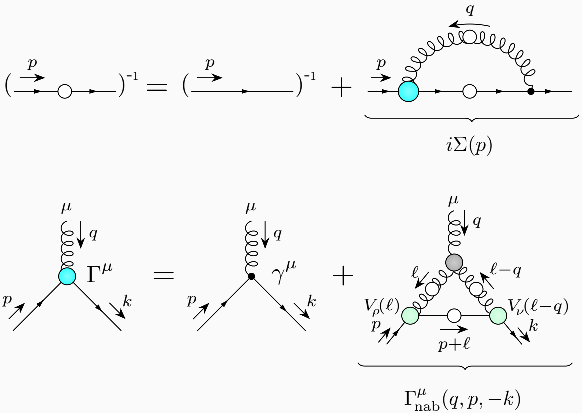
2. Project out form factors: $\lambda_i(q,p,-k) = {\rm Tr}[\mathcal{P}_i^{\mu}(p,-k)\overline{\Gamma}^{\mu}(q,p,-k)]$

$$\lambda_i^{sg}(p^2) = \delta_{i1} + \underbrace{\lim_{q\to 0}{\rm Tr}\left[\mathcal{P}_i^{\mu}(q,p,-k)\overline{\Gamma}_{\rm nab}(q,p,-k)\right]}_{\lambda_{i,Q}^{sg}(p^2)}$$

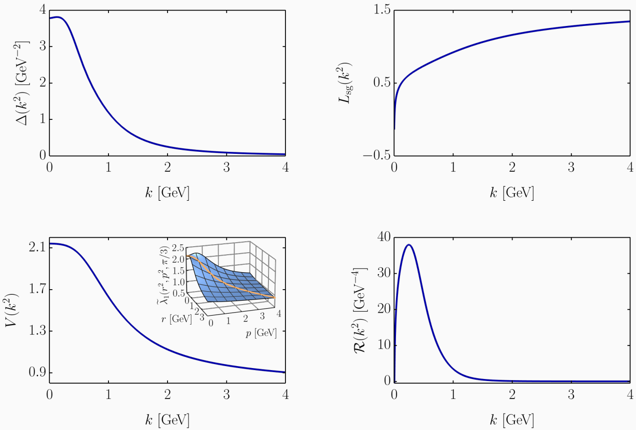
The system of Schwinger-Dyson equations

Goal

Solve the system of quark propagator and quark-gluon vertex SDEs on the complex p^2-plane



- **Gluon propagator $\boldsymbol{\Delta(k^2)}$:**
  Fit to lattice data[1]
- **Three-gluon vertex $\boldsymbol{L_{sg}(k^2)}$:**
  Fit to lattice data[1]
- **Function $\boldsymbol{V(k^2)}$**
  SDE determination[2]: $V(k^2) = \lambda_1(k,k,k)$

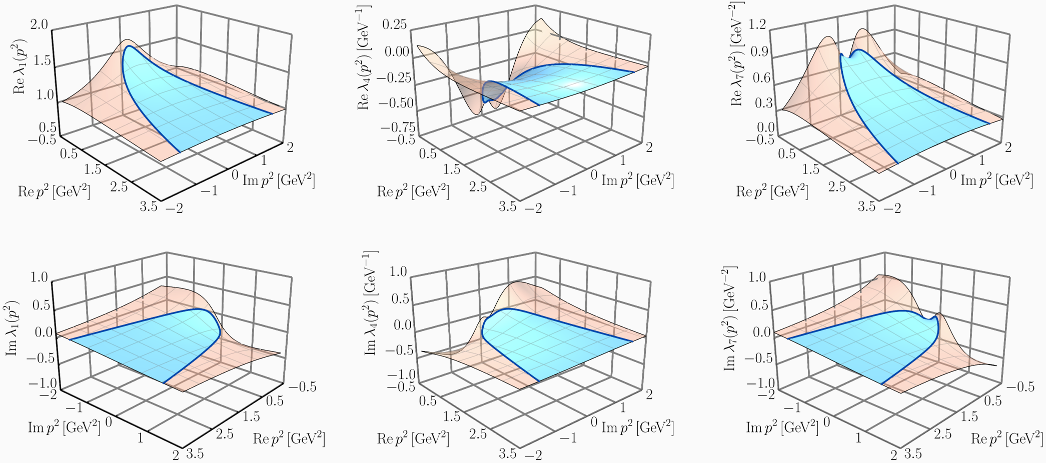


[1] [Aguilar,Ferreira,Ibañez,Papavassiliou,EPJC83(2023)10,967]
[2] [Ferreira,Miramontes,JM,Papavassiliou,EPJC86(2026)4,325]

Results

Results: Quark-gluon vertex

[Ferreira,Miramontes,JM,Papavassiliou,EPJC86(2026)8,954]



Tree-Level Component

TLC: $\tau_1^\mu \to \gamma^\mu$

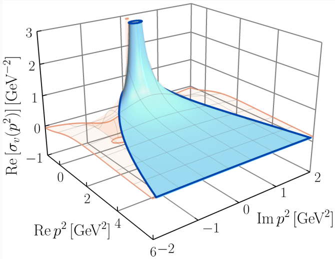
Anomalous Chrom. Moment

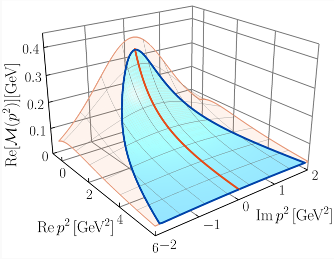
ACM: $\tau_4^\mu \to (\not{k} - \not{p})\gamma^\mu$

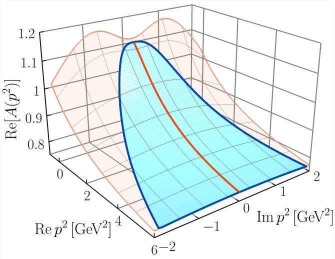
Spin-Momentum Curvature

SMC: $\tau_7^\mu \to -\frac{1}{2}[\not{k}, \not{p}]\gamma^\mu$

Results: Quark propagator (I)







$$S(p) = \sigma_V(p^2)\not{p} + \sigma_S(p^2)\,, \qquad S^{-1}(p) = A(p^2)\not{p} - B(p^2)$$

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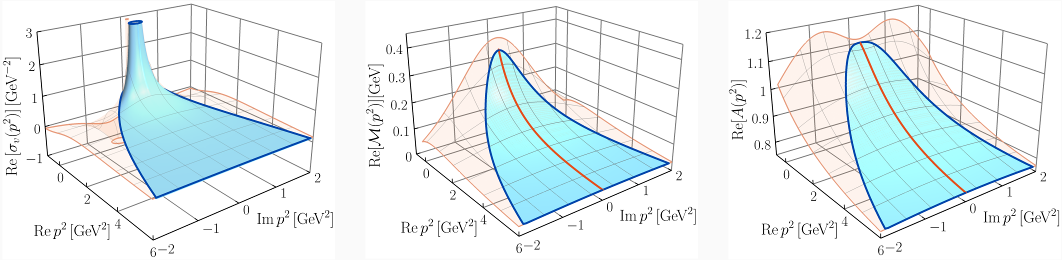
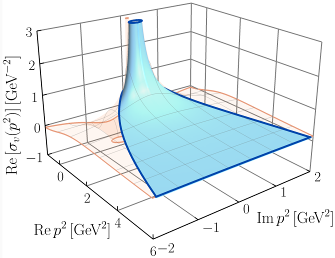
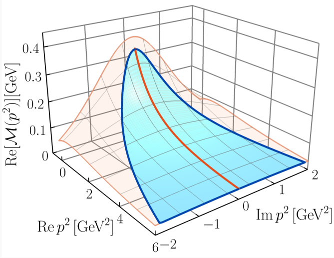


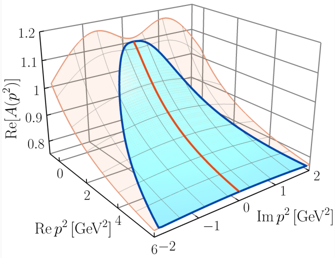
$$z = p^2 = x + iy\,, \qquad \lim_{z\to p_i^2}(z-p_i^2)^m\sigma_V(z)$$

- Order-one poles: $m = 1$
- Stability of pole position: less than 5% variation in second pole
- No evidence for clustered poles/zeroes nearby

Results: Quark propagator (I)







Quark propagator shows **real poles with opposite-sign residues**

#	Location in p^2 [GeV^2]	Residue
Pole 1	-0.16	1.12
Pole 2	-0.58	-0.51

See also: M. Ferreira (Tuesday, A, 15:30); M.Huber (Wednesday, A, 09:30); Wieland, Alkofer, arXiv:2604.20235.

Results: Quark propagator (II)

Pole 1 signals a propagating mode of a massive Dirac fermion

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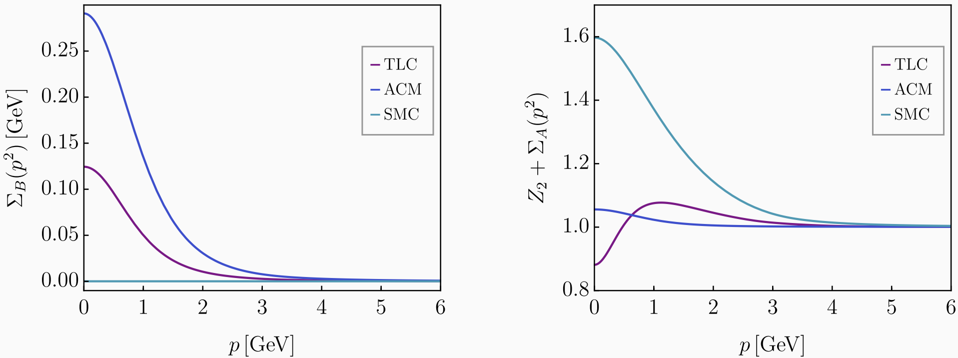
Spectral functions assume negative values $\rightarrow$ Violation of reflection-positivity $\rightarrow$ No positive-definite Hilbert space $\rightarrow$ No quark asymptotic states[3]

[3] [Alkofer,Smekal,Phys.Rept.353(2001)281; Oehme,Int.J.Mod.Phys.A10(1995)1995-2014] [Kugo,Ojima,Prog.Theor.Phys.Suppl.66(1979)1-130; Alkofer,Greensit,J.Phys.G34(2007)S3]

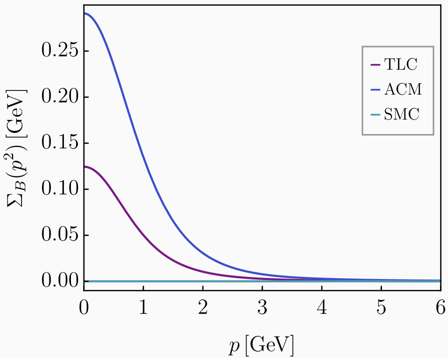
Results: The interplay between ACM and SMC (I)

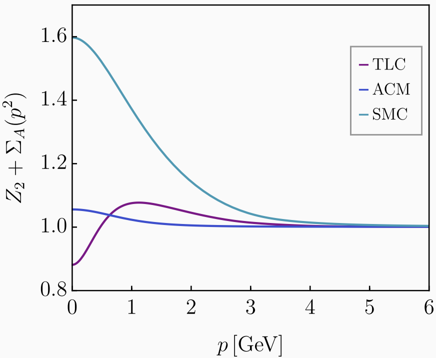


- TLC, ACM, SMC: essence of $S(p)$

Note: $p^2\Sigma_{\rm A}(p^2) = \mathrm{Tr}[\not{p}\Sigma(p)]\,, \quad \Sigma_{\rm B}(p^2) = \mathrm{Tr}[\Sigma(p)]$

Results: The interplay between ACM and SMC (I)





- TLC, ACM, SMC: essence of $S(p)$
- ACM: primary mechanism for dynamical quark mass generation
- SMC: momentum-dependence quark propagator

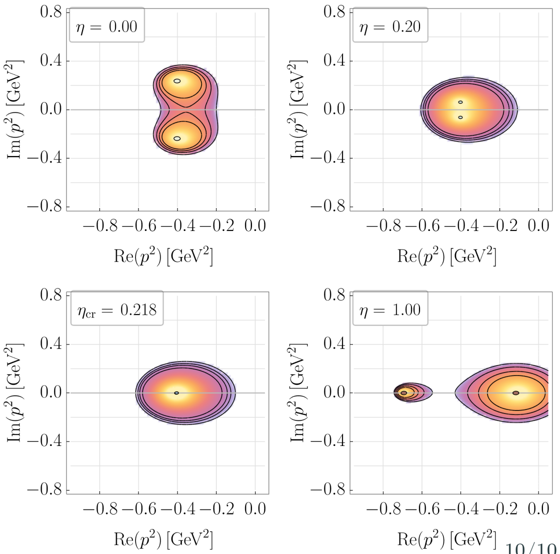
Note: $p^2\Sigma_{\rm A}(p^2) = \mathrm{Tr}[\not{p}\Sigma(p)]\,, \quad \Sigma_{\rm B}(p^2) = \mathrm{Tr}[\Sigma(p)]$

Results: The interplay between ACM and SMC (II)

$$\overline{\Gamma}^{\mu}_{\eta}(q,p,-k) = \lambda_1^{sg}(p^2)\bar{\tau}_1^{\mu} + \lambda_4^{sg}(p^2)\bar{\tau}_4^{\mu} + \eta\lambda_7^{sg}(p^2)\bar{\tau}_7^{\mu}\,,$$

with $\eta \in [0,1]$.

quark-gluon vertex	poles	$\mathcal{M}(0)$ [MeV]
λ_1	real	180
λ_1, λ_4	cc	546
$\lambda_1, \lambda_4, \lambda_7$	real	310
all form factors	real	350



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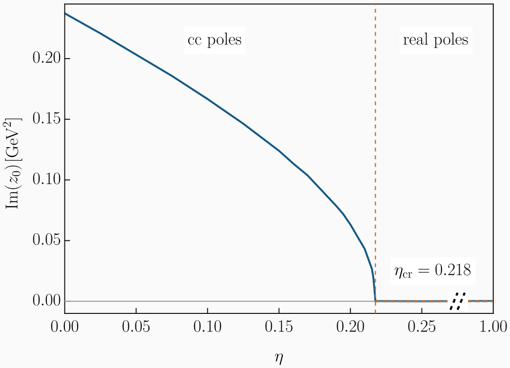

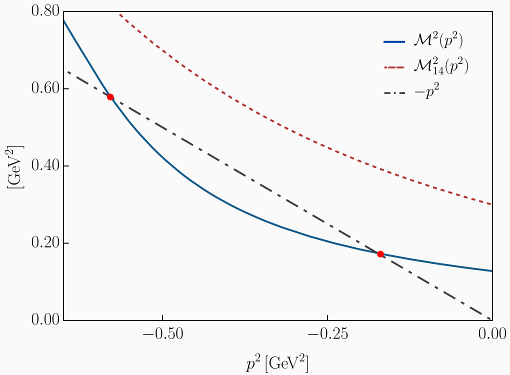

$$\sigma_V(p^2) \sim \frac{1}{p^2 + \mathcal{M}^2(p^2)}\,,$$

Poles: solutions to

$$p^2 + \mathcal{M}^2(p^2) = 0\,.$$

Conclusions and future steps

Summary

- Quark propagator and quark-gluon vertex on the complex p^2-plane
- Quark propagator shows **real poles with opposite-sign residues**
- Vertex controls the propagator's analytic structure:
  - Tree-level component (TLC)
  - Anomalous chromomagnetic moment (ACM)
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Future steps

- Extension beyond the soft-gluon limit
- Exploration of implications in the study of hadrons

Thank you!

Back-up slides

The quark-gluon vertex equations (I)

Dynamical equations for the form factors are of the form:

$$\lambda_i^{sg}(p^2) = \int_k \mathcal{R}(k^2) K_i(k,p)\sigma_{\rm V}(k+p)\,, \qquad i = 1,6,7$$

$$\lambda_i^{sg}(p^2) = \int_k \mathcal{R}(k^2) K_i(k,p)\sigma_{\rm S}(k+p)\,, \qquad i = 2,4,8$$

$$\lambda_5^{sg}(p^2) = \int_k \mathcal{R}(k^2) K_5(k,p)\sigma_{\rm V}(k+p) + \int_k \widetilde{K}_5(k,p)\left[\frac{1}{2}\mathcal{R}'(k^2)\sigma_{\rm V}(k+p) + \mathcal{R}(k^2)\sigma_{\rm V}'(k+p)\right]$$

with,

$$\mathcal{R}(k^2) = \Delta^2(k^2)V^2(k^2)L_{sg}(k^2)\,, \quad S(p) = \sigma_{\rm V}(p)\not{p} + \sigma_{\rm S}(p)\,, \quad \Gamma^{\mu\alpha\beta}(q,r,p) = L_{sg}(s^2)\Gamma_0^{\mu\alpha\beta}(q,r,p)$$

$$\boxed{\lambda_i^{sg}(p^2) \sim \int_k \mathcal{R}(k^2) K_i(k,p)\sigma_{\rm V/S}(k+p)}$$

The quark-gluon vertex equations (II)

$$K_1 = -\frac{4}{3}(3k^2 + 2\sqrt{k^2p^2}\cos\omega)\sin^2\omega\,, \quad K_2 = -6\sqrt{\frac{k^2}{p^2}}\cos\omega\,, \quad K_3 = 0\,,$$

$$K_4 = 4(1+\cos^2\omega)\,, \qquad K_5^a = -\frac{4}{3}\frac{k^2}{p^2}\cos^2\omega(3\sqrt{k^2p^2} + 2\sqrt{p^2}\cos\omega)\sin^2\omega\,,$$

$$K_5^b = -\frac{1}{3p^2}(3k^2 + 2p^2(3 + 2\cos^4\omega) + 10\sqrt{k^2p^2}(5-\cos^2\omega)\cos\omega - 4(3k^2 - 2p^2)\cos^2\omega)\,,$$

$$K_6 = \frac{1}{3p^2}(-3k^2 + \sqrt{k^2p^2}\cos\omega + 12k^2\cos^2\omega + 8\sqrt{k^2p^2}\cos^3\omega)\,,$$

$$K_7 = -8\sqrt{\frac{k^2}{p^2}}\cos\omega - 4(1+\cos^2\omega)\,, \qquad K_8 = -\frac{2}{3p^2}(4\cos^2\omega - 1)\,.$$

Systematics of Wick rotation

$$I(p^2) = \int \frac{d^4k}{(2\pi)^4} f(k_0, \mathbf{k}, p)\,, \qquad I_{\mathrm{E}}(p^2) = i \int \frac{d^4k^{\mathrm{E}}}{(2\pi)^4} f(k_0^{\mathrm{E}}, \mathbf{k}, p)$$

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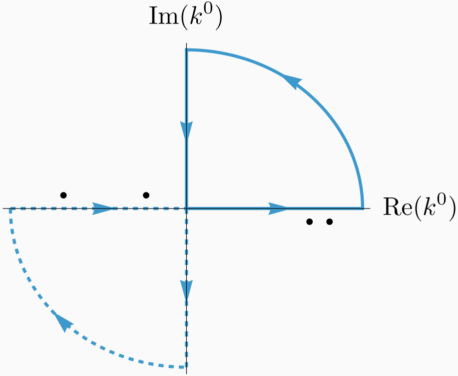

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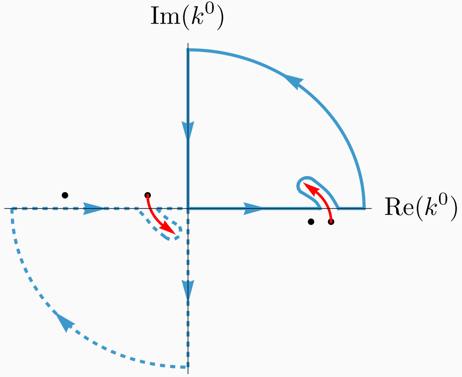

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Case 2:

$$I(p^2) = I_{\mathrm{E}}(p^2)\,.$$

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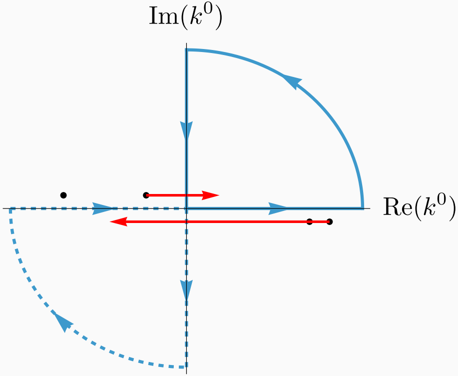

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Case 2:

$$I(p^2) = I_{\mathrm{E}}(p^2)\,.$$

Case 3:

$$I(p^2) = I_{\mathrm{E}}(p^2) + i \int \frac{d^3\mathbf{k}}{(2\pi)^3} \sum_{\substack{\text{poles} \\ \text{inside}}} \mathrm{Res}\, f(k_0, \mathbf{k}, p)$$

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Problem:	Incorporating residue requires knowledge of the entire integrand for complex momentum
Consequence:	Whole complex plane not directly accessible

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- $\sigma_{\mathrm{V/S}}(k+p)$: expected to develop poles, *e.g.* (complex conjugate)

$$\sigma_{\mathrm{V/S}}(k+p) = \frac{Z}{(k+p)^2+M^2} + \frac{Z^*}{(k+p)^2+(M^*)^2} + \text{Regular terms}$$

Analyticity region (II)

$$J(p^2) = \int \frac{d^4k}{(2\pi)^4} \frac{\tilde{f}(k,p)}{(k^2-m^2)^2[(k+p)^2-M^2]}\,, \qquad M = |M|e^{i\varphi_{\rm M}}\,.$$

$$k_{0,1}^{\pm} = \pm\sqrt{\mathbf{k}^2+m^2} \mp i\epsilon\,, \quad k_{0,2}^{\pm} = p_0 \pm \sqrt{\mathbf{k}^2+M^2} \mp i\epsilon$$

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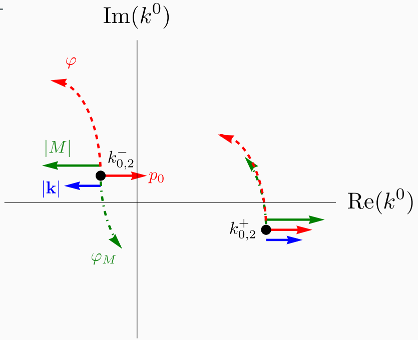
Poles move into a different quadrant iff

$$\mathrm{Re}(k_{0,2}^{+}) < 0\,, \quad \mathrm{Re}\left(k_{0,2}^{-}\right) > 0$$

which implies

$$\mathrm{Re}^2(p_0) > \mathrm{Re}^2(\sqrt{\mathbf{k}^2 + M^2})$$



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$\mathbf{k}^2 > 0$ pushes poles away from the crossover,

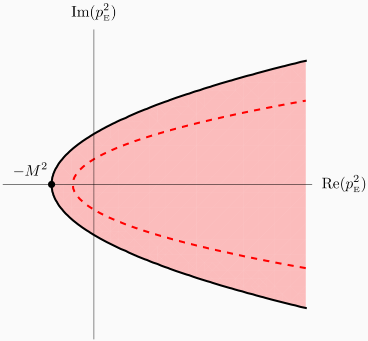
$$\mathrm{Re}^2(p_0) > \mathrm{Re}^2(M^2) = |M|^2\cos^2\varphi_{\rm M}.$$

Introducing Euclidean momentum $p_{\rm E}$

$$p_0 = \sqrt{-p_{\rm E}^2}\,, \quad p_{\rm E}^2 = x + iy$$

Analytic region

$$y^2 < 4|M|^2\cos^2\varphi_{\rm M}(x + |M|^2\cos^2\varphi_{\rm M})\,.$$



Renormalization

[Ferreira,Miramontes,JM,Papavassiliou,Pawlowski,EPJC:86(2026)4,325]

Renormalization in $\widetilde{\text{MOM}}$ scheme

[Aguilar,Ferreira,Oliveira,Papavassiliou,Linhares,EPJC84(2024)11,1231;Aguilar,Ferreira,Ibañez,Papavassiliou,EPJC83(2023)10,967
Skullerud,Kizilersu,JHEP09(2002)013;Smekal,Maltman,Sternbeck,PLB681(2009)336-342
Kizilersu,Oliveira,Silva,Skullerud,Sternbeck,PRD103(2021)11,114515]

$$\Delta_{\rm R}^{-1}(\mu^2) = \mu^2\,, \qquad A_{\rm R}(\mu^2) = 1\,, \qquad B_{\rm R}(\mu^2) = m_{\rm R}\,, \qquad \lambda_{1,{\rm R}}^{sg}(\mu^2) = 1\,.$$

The renormalized quark propagator and quark-gluon vertex SDE:

$$S_{\rm R}^{-1}(p) \;=\; Z_2 \not{p} + i g_{\rm R}^2 C_f Z_1 \int_q \gamma^\nu S_{\rm R}(p+q) \Gamma_{\rm R}^\mu(q,p,-p-q) \Delta_{\mu\nu}^{\rm R}(q)\,,$$

$$\Gamma_{\rm R}^\mu(q,p,-k) \;=\; Z_1 \gamma^\mu + \Gamma_{\rm nab}^\mu(q,p,-k)\,.$$

Replacement:

[Fischer,Alkofer,PRD67,094020(2003);Aguilar,Papavassiliou,PRD83,014013(2011);
Aguilar,Cardona,Ferreira,Papavassiliou,PRD98,014002(2018)]

$$Z_1 \mathcal{K}_{\rm gap}(p,p+q,q) \to C(q) \mathcal{K}_{\rm gap}(p,p+q,q)\,,$$

where $\mathcal{C}(q)$ evolving as the classical form factor of the quark-gluon vertex

Numerical procedure

Initial "guess" for $A(p^2)$ and $B(p^2)$

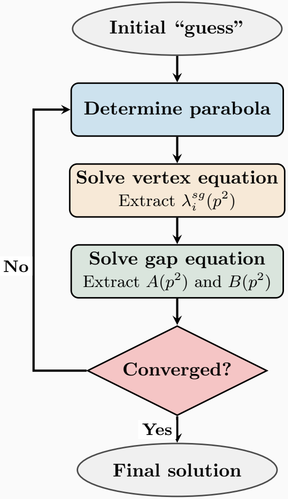
1. Find singularities and determine accessible region:
[Ferreira,Miramontes,JM,Papavassiliou,EPJC86(2026)8,954]

$$y^2 = x\,\alpha + \beta \quad \text{with } p^2 = x + iy$$

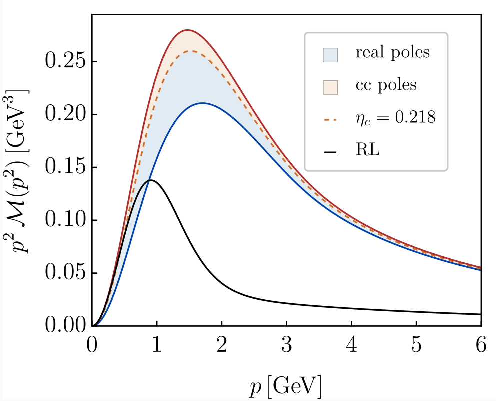
2. Compute, numerically, vertex form factors $\lambda_i^{sg}(p^2)$

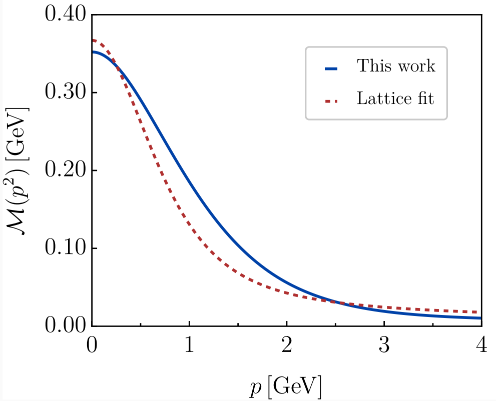
3. Compute, numerically, quark propagator: $A(p^2)$, $B(p^2)$

4. Check for convergence: $\texttt{tol} = 10^{-6}$.



Comparisons





Spin-momentum curvature

$$\tau_7^{\nu} = -\frac{1}{2}[\not{k}, \not{p}]\gamma^{\nu} = i\sigma^{\alpha\beta}k_{\alpha}p_{\beta}\gamma^{\nu} \,,$$

with $\sigma^{\alpha\beta} = \frac{i}{2}[\gamma^{\alpha}, \gamma^{\beta}]$ the generator of Lorentz transformations for spin-1/2 fermions: c.f. **spin**.

- Vanishes for parallel/anti-parallel momenta
- Vanishes for on-shell fermions
- Does not vanish for off-shell fermions