Quarkonium-nucleon femtoscopy as a probe of nucleon gravitational form factors

R. Ejima, D. Fujii, M. Kawaguchi, arXiv:2607.00650 (2026)

ASRC (JAEA)　　Daisuke Fujii
先端基礎研究センター（原子力研究開発機構）　藤井　大輔

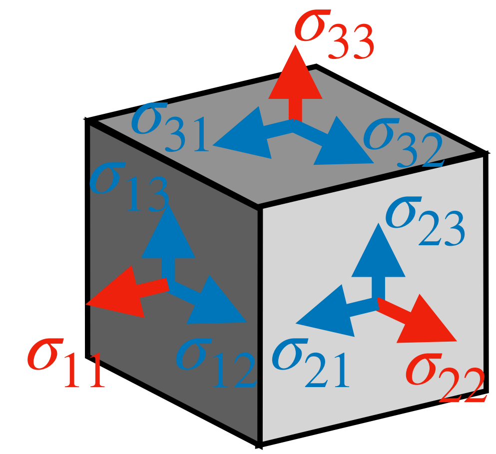
Collaborators:
Ren Ejima (Tokyo Univ.)
Mamiya Kawaguchi (Anhui Univ. of Science and Technology)

Outline

- Gravitational Form Factors (GFFs) and the quarkonium-nudcleon interaction
- Correlation functions from a GFF-based charmonium-nucleon potential
- Sensitivity of the correlation function to the nucleon D-term
- Summary

EMT and GFFs

Spatial component of the matrix element of GFFs : D(t)



$$T_{ij} = -\sigma_{ij}$$

- **Matrix elements of EMT** for **Proton** (spin-1/2)

→ **GFFs: A(t), B(t), D(t)**

Spin indices

$$\langle p', s' \,|\, \hat{T}^{\mu\nu}(x) \,|\, p, s \rangle = u(p', s')\Big[i\gamma^{(\mu}P^{\nu)}A(t) + i\frac{P^{(\mu}\sigma^{\nu)}_{\rho}\Delta^{\rho}}{2M}B(t) + \boxed{\frac{\Delta^{\mu}\Delta^{\nu} - \eta^{\mu\nu}\Delta^{2}}{4M}D(t)} \Big] u(p, s)e^{i\Delta_{\mu}x^{\mu}}$$

Proton states

Spatial component

$P^{\mu} = (p_1^{\mu} + p_2^{\mu})/2, \;\; \Delta^{\mu} = p_2^{\mu} - p_1^{\mu}, \;\; t = -\Delta^2$

The last unknown global property of the nucleon

$D_0 = D(0)$ is called as D-term

- **Spatial components of EMT → Stress tensor**

Energy density

Momentum density = energy flux

$$T_{\mu\nu} = \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$$

Shear force

$s(r)$

Pressure

$p(r)$

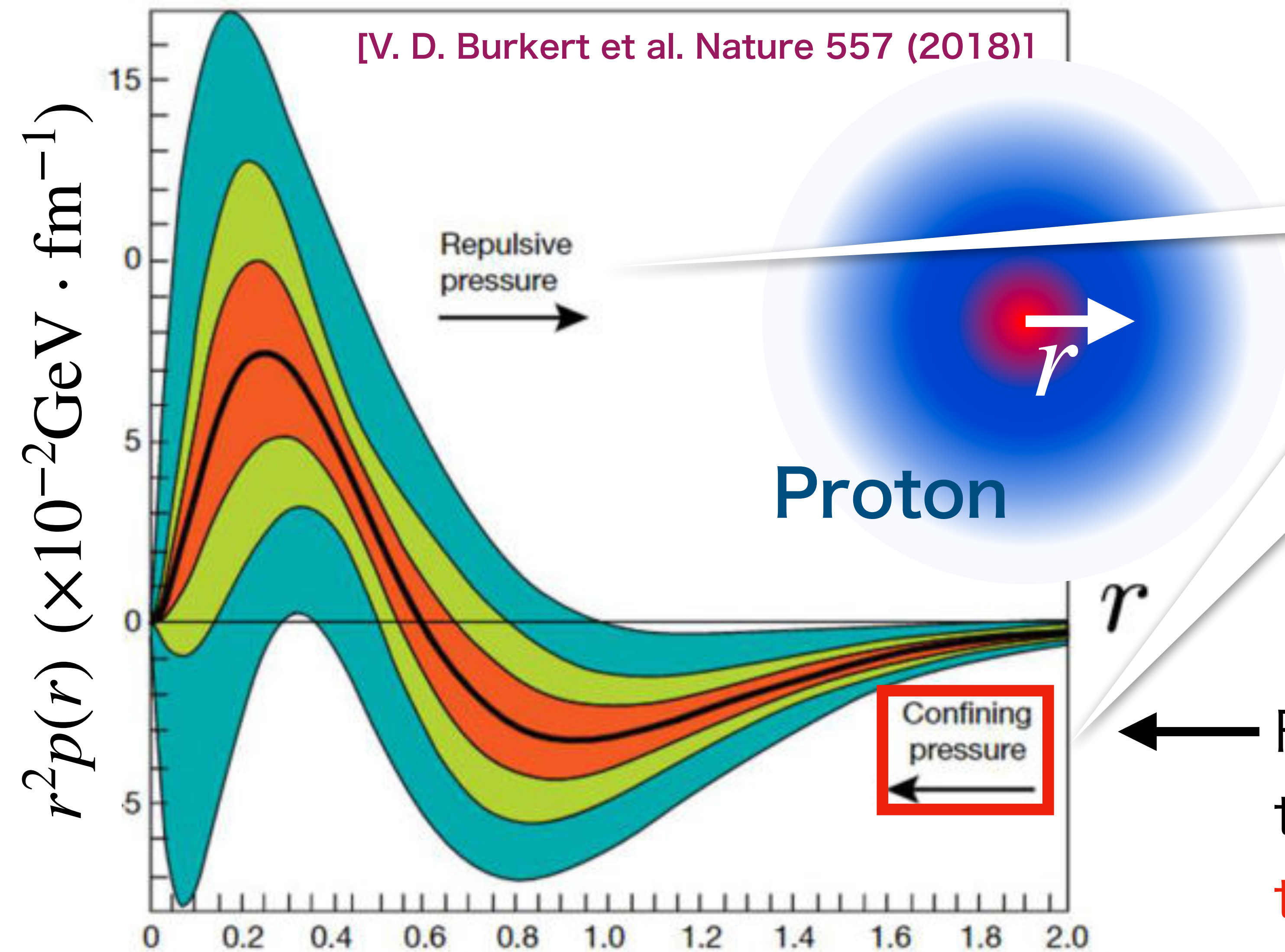
Stress tensor = Momentum flux

Spatial component

The FT of D(t) encodes the Stress tensor in hadrons

GFFs reveal the spatial distribution of strong forces inside and around the nucleon

- **Pressure inside proton**～Stabilizing force distribution of hadrons



Pushes quarks and gluons outwards from the proton

Confine quarks and gluons inside the proton

Force distribution generated by the **strong interaction** around the nucleon core

Central question of this study

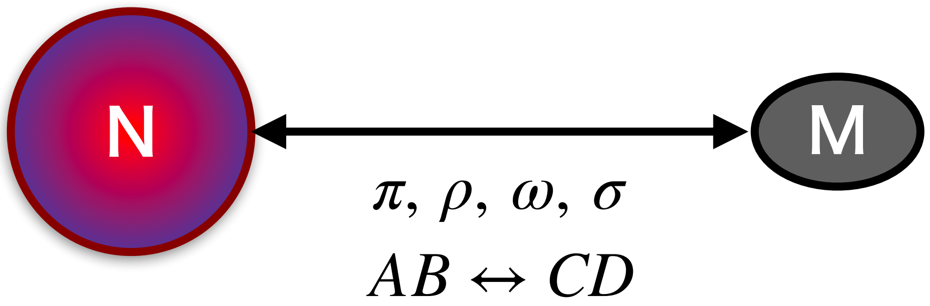
Is there a connection between the force distribution around the nucleon encoded in GFFs and the two-particle interaction probed by femtoscopy?

How can

Femtoscopy be related to the stress distribution

encoded in GFFs?〜General hadron-hadron interactions are complicated

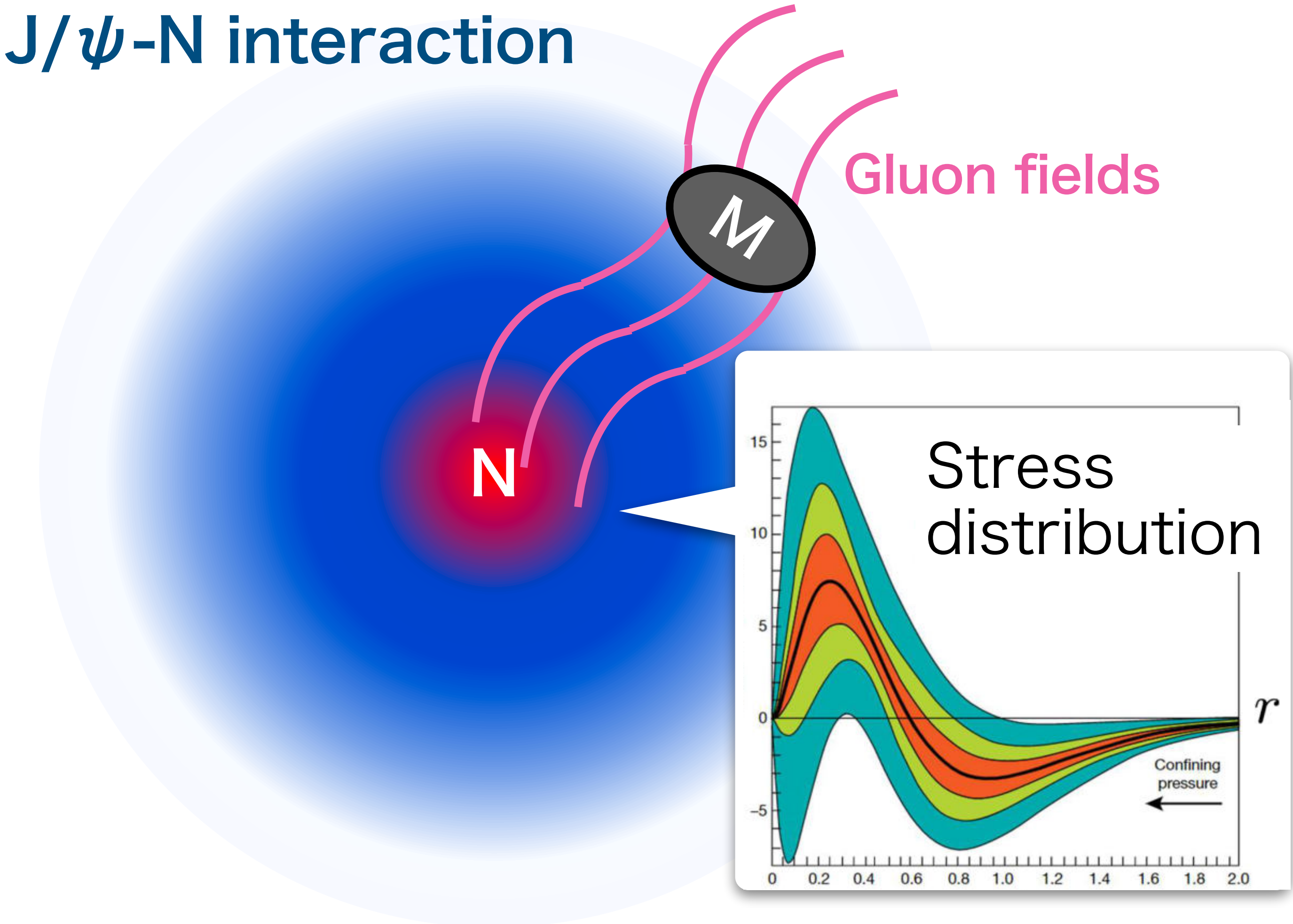
General hadron-hadron interaction



- Quark exchange
- Meson exchange
- Coupled channel
- ⋯

Difficult to clarify the relation between the femtoscopy and the nucleon stress distribution in general systems

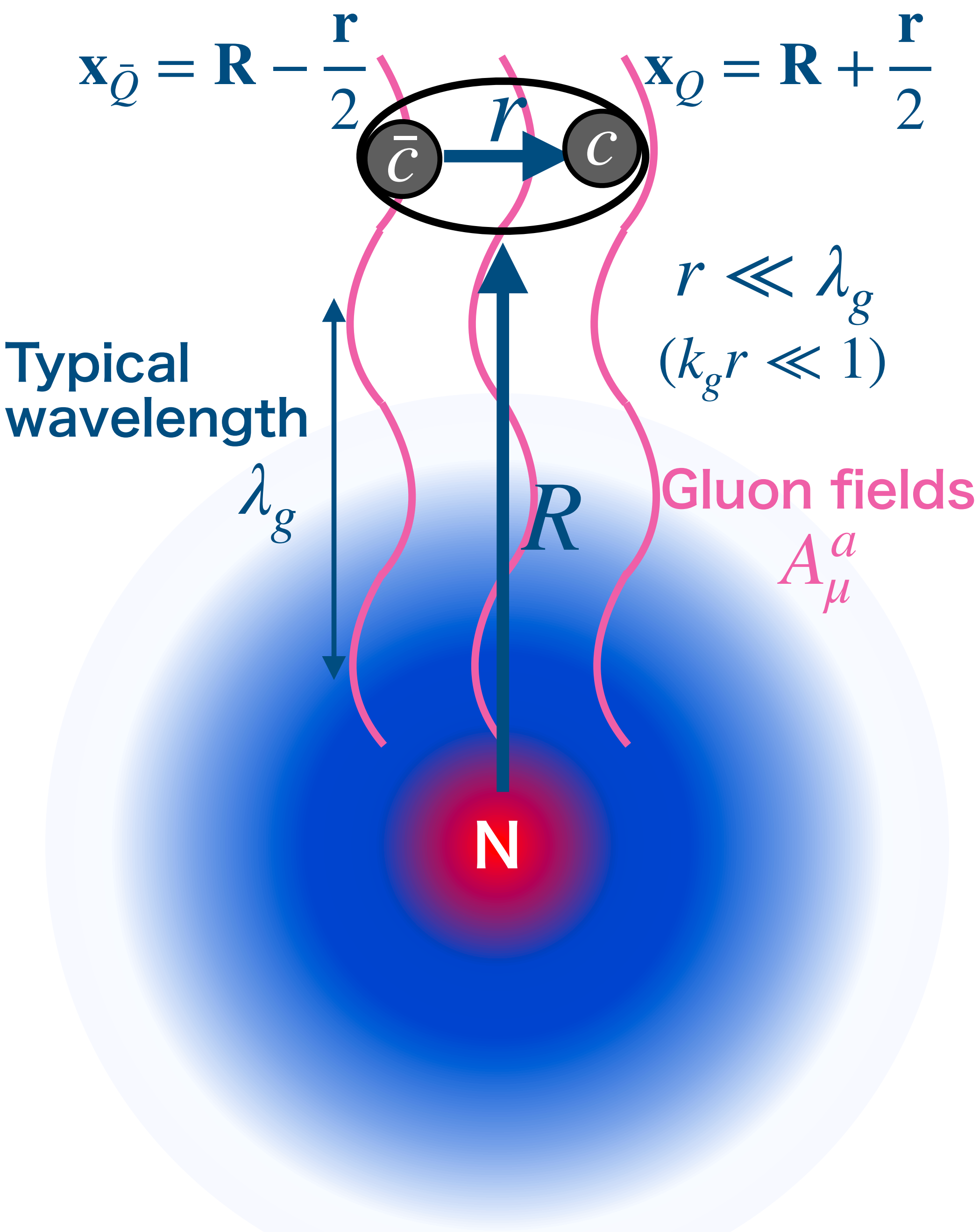
J/ψ-N interaction



They do not share any valence quarks
→Quark, Meson exchange are suppressed

The J/ψ-N interaction is dominated by gluon-field dynamics

Multipole expansion for heavy quarkonium:
Relation between **J/ψ-N interaction** and **gluons** in nucleons



For J/ψ, the $c\bar{c}$ separation can be regarded as small compare with λ_g

- **Multipole exp.** applies when the system size is much smaller than λ_g ($r \ll \lambda_g$)

$$A^a_\mu\left(\mathbf{R} \pm \frac{\mathbf{r}}{2}\right) = A^a_\mu(\mathbf{R}) \pm \underbrace{\frac{r^i}{2}\overset{k_g}{\partial_i} A^a_\mu(\mathbf{R})}_{\mathcal{O}(k_g r)} + \mathcal{O}((k_g r)^2)\,.$$

- **Leading interaction** of multipole exp. is

$$H_{E1} = -\overset{\text{Color-electric dipole moment}}{\mathbf{d}^a} \cdot \underset{\text{Chromoelectric fields}}{\mathbf{E}^a(\mathbf{R})}$$

$$(c\bar{c})_1 \xrightarrow{E1} (c\bar{c})_8 \xrightarrow{G_8} (c\bar{c})_8 \xrightarrow{E1} (c\bar{c})_1$$

The E1 int. changes $(c\bar{c})_1$ into $(c\bar{c})_8$ state.
→Elastic J/ψ-N scattering starts from two E1 int. via $(c\bar{c})_8$

- **Effective interaction** M. B. Voloshin (1982)

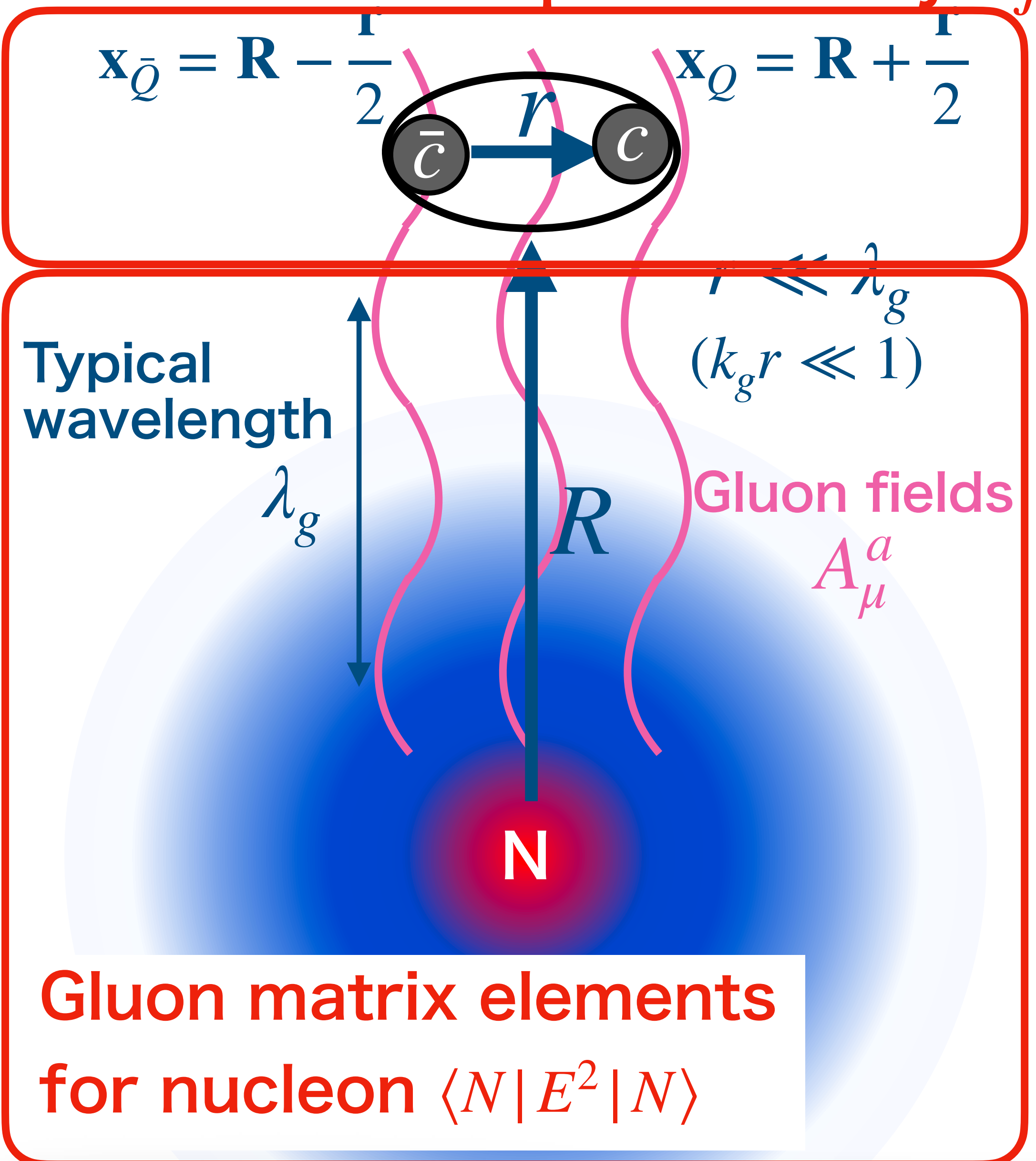
$$\left\langle J/\psi, N \left| H_{\text{eff}} \right| J/\psi, N \right\rangle = -\frac{1}{2}\alpha_{J/\psi}\left\langle N \left| \mathbf{E}^2(\mathbf{R}) \right| N \right\rangle$$

$$\alpha_{J/\psi} = \frac{1}{12}\left\langle J/\psi \left| d^a_i \, G_8 \, d^a_i \right| J/\psi \right\rangle$$

Chromoelectric polarizability

Multipole expansion for heavy quarkonium:
Relation between J/ψ-N interaction and gluons in nucleons

Chromoelectric polarizability $\alpha_{J/\psi}$



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$$A^a_\mu\left(\mathbf{R} \pm \frac{\mathbf{r}}{2}\right) = A^a_\mu(\mathbf{R}) \pm \underbrace{\frac{r^i}{2}\partial_i}_{\mathcal{O}(k_g r)} A^a_\mu(\mathbf{R}) + \mathcal{O}((k_g r)^2)\,.$$

k_g

- **Leading interaction** of multipole exp. is

Color-electric dipole moment

$$H_{E1} = -\,\mathbf{d}^a \cdot \mathbf{E}^a(\mathbf{R})$$

Chromoelectric fields

$$(c\bar{c})_1 \xrightarrow{E1} (c\bar{c})_8 \xrightarrow{G_8} (c\bar{c})_8 \xrightarrow{E1} (c\bar{c})_1$$

The E1 int. changes $(c\bar{c})_1$ into $(c\bar{c})_8$ state.
→Elastic J/ψ-N scattering starts from two E1 int. via $(c\bar{c})_8$

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$$\left\langle J/\psi\, N \left| H_{\rm eff} \right| J/\psi\, N \right\rangle = -\frac{1}{2}\alpha_{\psi}\left\langle N \left| \mathbf{E}^2(\mathbf{R}) \right| N \right\rangle$$

Chromoelectric polarizability

Thanks to scale separation, the J/ψ structure is encoded in α, while the nucleon structure is encoded in the gluon matrix element

The Charmonium structure is encoded in

Chromoelectric polarizability α:

which is still not precisely known

- **Perturbative QCD calculation for α** (which describes the response to E)
(the Coulombic, heavy quark and large Nc limit) G. Bhanot and V. Y. Peskin (1979)

$$\alpha(nS)_{\rm pert} = \frac{16\pi n^2 c_n}{3\underline{g^2} N_c^2}\underline{a_0^3}$$

Bohr radius

$$a_0 = \frac{16\pi}{g^2 N_c m_Q} \qquad c_n = 5n^2(7n^2-3)/16$$

Coupling const. evaluated by charmonium size scale

$\alpha(1S)_{\rm pert} \approx 0.2\ {\rm GeV}^{-3}$,	$\alpha(2S)_{\rm pert} \approx 14\ {\rm GeV}^{-3}$	M. I. Eides, et al, PRD (2016)
$\alpha(1S)_{\rm pert} \approx 4.1\ {\rm GeV}^{-3}$,	$\alpha(2S)_{\rm pert} \approx 296\ {\rm GeV}^{-3}$	J. Ferretti, PLB (2018)

←The perturbative estimates strongly depend on the input parameters (Bohr radius)

- **Phenomenological estimates for α**

  - J/ψ-N EFT + lattice QCD data $\alpha(1S)_{\rm pheno} \approx 0.24\ {\rm GeV}^{-3}$
  J. Tarrus Castella and G. A. Krein, PRD (2018)
  - Analysis of the lattice QCD J/ψ-N potential $\alpha(1S)_{\rm pheno} \approx 1.6 \pm 0.8\ {\rm GeV}^{-3}$
  M. V. Polyakov and P. Schweitzer, PRD (2018)

←The phenomenological estimates also have large uncertainties

  - No reliable determination of $\alpha(2S)_{\rm pheno}$ is currently available

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$$\alpha(nS)_{\text{pert}} = \frac{16\pi n^2 c_n}{3\underline{g^2} N_c^2}\underline{a_0^3}$$

Bohr radius

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←**The perturbative estimates strongly depend on the input parameters (Bohr radius)**

- **Phenomenological estimates for α**

We use this value because it is directly related to the J/ψ-N potential

・Analysis of the lattice QCD J/ψ-N potential $\alpha(1S)_{\text{pheno}} \approx \boxed{1.6} \pm 0.8\ \text{GeV}^{-3}$
M. V. Polyakov and P. Schweitzer, PRD (2018)

For ψ(2S), we vary its value as a parameter

・No reliable determination of $\alpha(2S)_{\text{pheno}}$ is currently available

The nucleon structure is encoded in

Gluon matrix element $\langle N|E^2|N\rangle$

which is related to GFFs for nucleons

- **The chromoelectric field operator is expressed in terms of the EMT for nucleons**

Coupling const. at charmonium size scale

$$\mathbf{E}^2 = -\frac{1}{4}g^2 G^{\alpha}_{\mu\nu}G^{\alpha\mu\nu} + g^2 T^{G}_{00} \sim \frac{8\pi^2}{bg_s^2} g^2 T^{\mu}{}_{\mu} + g^2 \xi T_{00}$$

Written by total EMT

Trace anomaly

$$T^{\mu}{}_{\mu} = \frac{\beta(g_s)}{2g_s} G^{a}_{\mu\nu}G^{a\mu\nu} + \sum_q m_q \bar{q}q \sim -\frac{bg_s^2}{32\pi^2} G^{a}_{\mu\nu}G^{a\mu\nu}$$

1-loop, chiral limit

Leading coeff. of β function

$b = (11N_c - 2N_f)/3$

Coupling const. at low (nucleon) scale

$T^{G}_{00} = \xi T_{00}$ $\quad \xi$: fraction of gluon contribution to M_N

- **The gluon matrix element is expressed in terms of the nucleon energy density and pressure**

Fourier transform $\quad \nu = 1 + \xi b g_s^2/(8\pi^2)$

$$\left\langle J/\psi, N \left| H_{\rm eff} \right| J/\psi, N \right\rangle = -\frac{1}{2}\alpha(nS) \left\langle N \left| \mathbf{E}^2 \right| N \right\rangle \longrightarrow \boxed{-\alpha(nS)\frac{4\pi^2}{b}\left(\frac{g}{g_s}\right)^2 \left[\nu\epsilon(r) - 3p(r)\right]}$$

J/ψ-N potential $V_{\rm eff}(r)$

→Connection between the nucleon GFFs and the J/ψ-N interaction

Purpose and strategy of this study

Strategy

- Determine **GFFs**, A(t), J(t), and D(t), from the **Lattice QCD data**
- Obtain energy density **ε (r)** and pressure **p(r)** by doing Fourier Transform of **Lattice GFFs**
- Construct the **Charmonium-N potential V(r)** from ε (r) and p(r)
- Extract the **Relative wave function ψ** by solving the **Schrödinger equation** with V(r)
- Calculate the **Correlation function C(r)**

Purpose

- Check the consistency of **Lattice GFF-based J/ψ-N potential** with **HAL QCD results** and Connect the **Nucleon mechanical structure** to a **Femtoscopy observable**
- Examine the **D-term sensitivity** of the J/ψ-N and ψ(2S)-N **Correlation functions**

Can charmonium-N femtoscopy provide a complementary probe of the nucleon GFFs?

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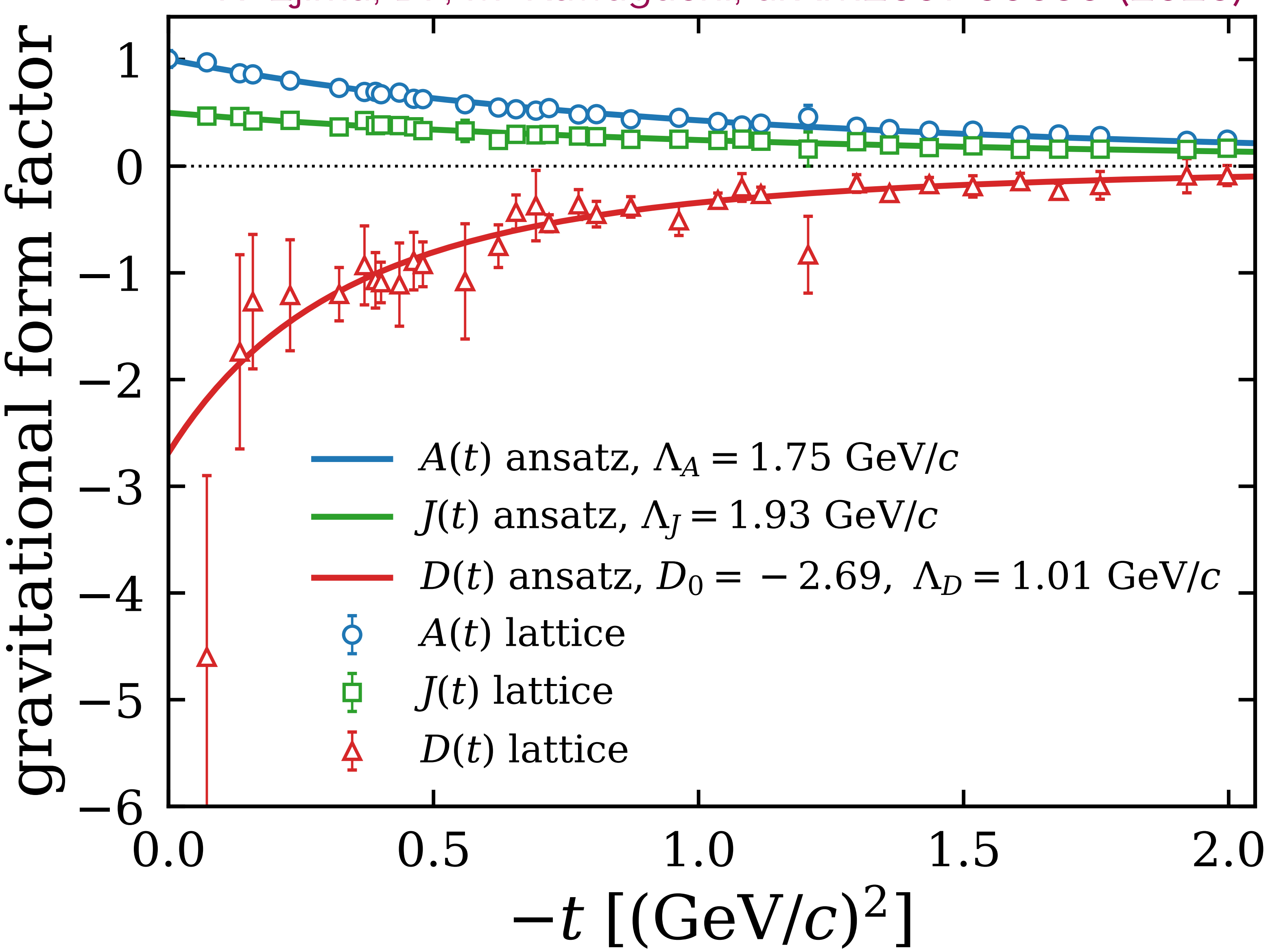
Nucleon energy and pressure distribution from lattice GFFs

- We determine the nucleon GFFs by fitting the following ansatz to lattice QCD data D. C. Hackett, et al, PRL (2024)

$$A(t) = \frac{1}{(1 - t/\Lambda_A)^3}, \qquad J(t) = \frac{1/2}{(1 - t/\Lambda_J)^3}, \qquad D(t) = \frac{D_0}{(1 - t/\Lambda_D)^3}.$$

R. Ejima, DF, M. Kawaguchi, arXiv:2607.00650 (2026)



The fitted parameters

$$\Lambda_A = 1.75\ \mathrm{GeV}, \quad \Lambda_J = 1.93\ \mathrm{GeV}$$

$$\Lambda_D = 1.01\ \mathrm{GeV}, \quad D_0 = -2.69.$$

$$A(t), J(t), D(t) \xrightarrow{\text{F.T.}} \epsilon(r), p(r)$$

→The input for the J/ψ-N potential

J/ψ-N potential from the nucleon energy density and pressure

J/ψ-N potential

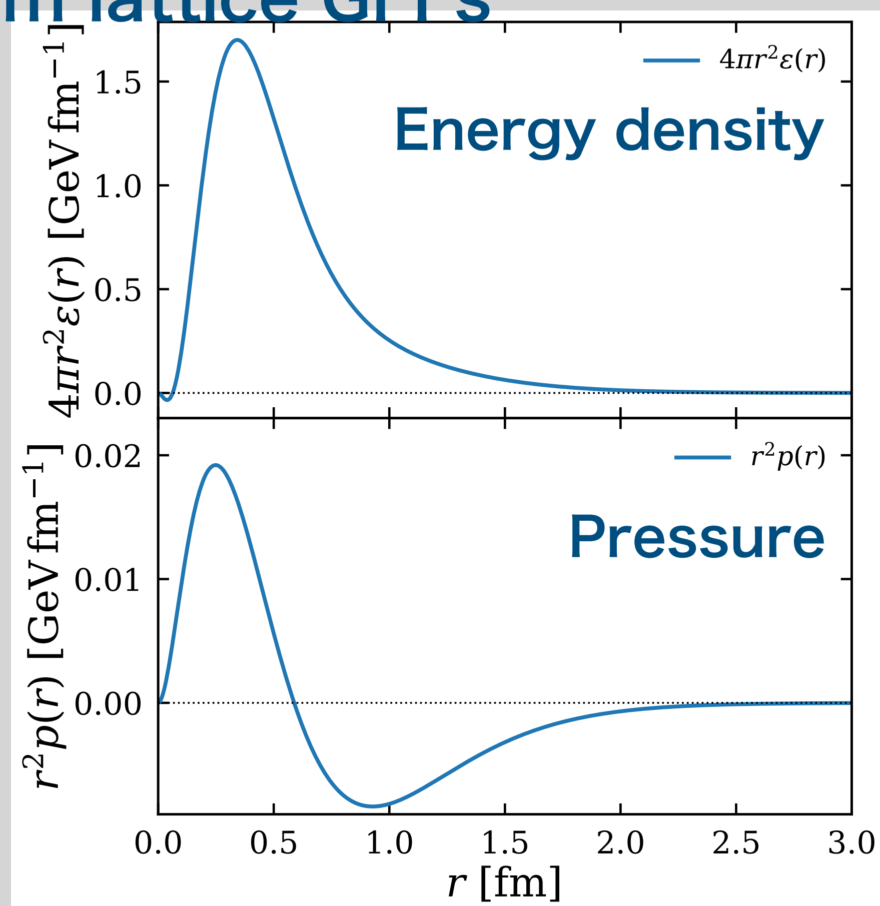
$$V_{\mathrm{eff}}(r) = -\alpha(nS)\frac{4\pi^2}{b}\left(\frac{g^2}{g_s^2}\right)\left[\nu\epsilon(r) - 3p(r)\right]$$

$$\left(\alpha_{1S},\, g^2/g_s^2,\, \nu\right) = \left(1.6\ \mathrm{GeV}^{-3},\, 1,\, 1.5\right)$$

M. V. Polyakov and P. Schweitzer (2018)

+

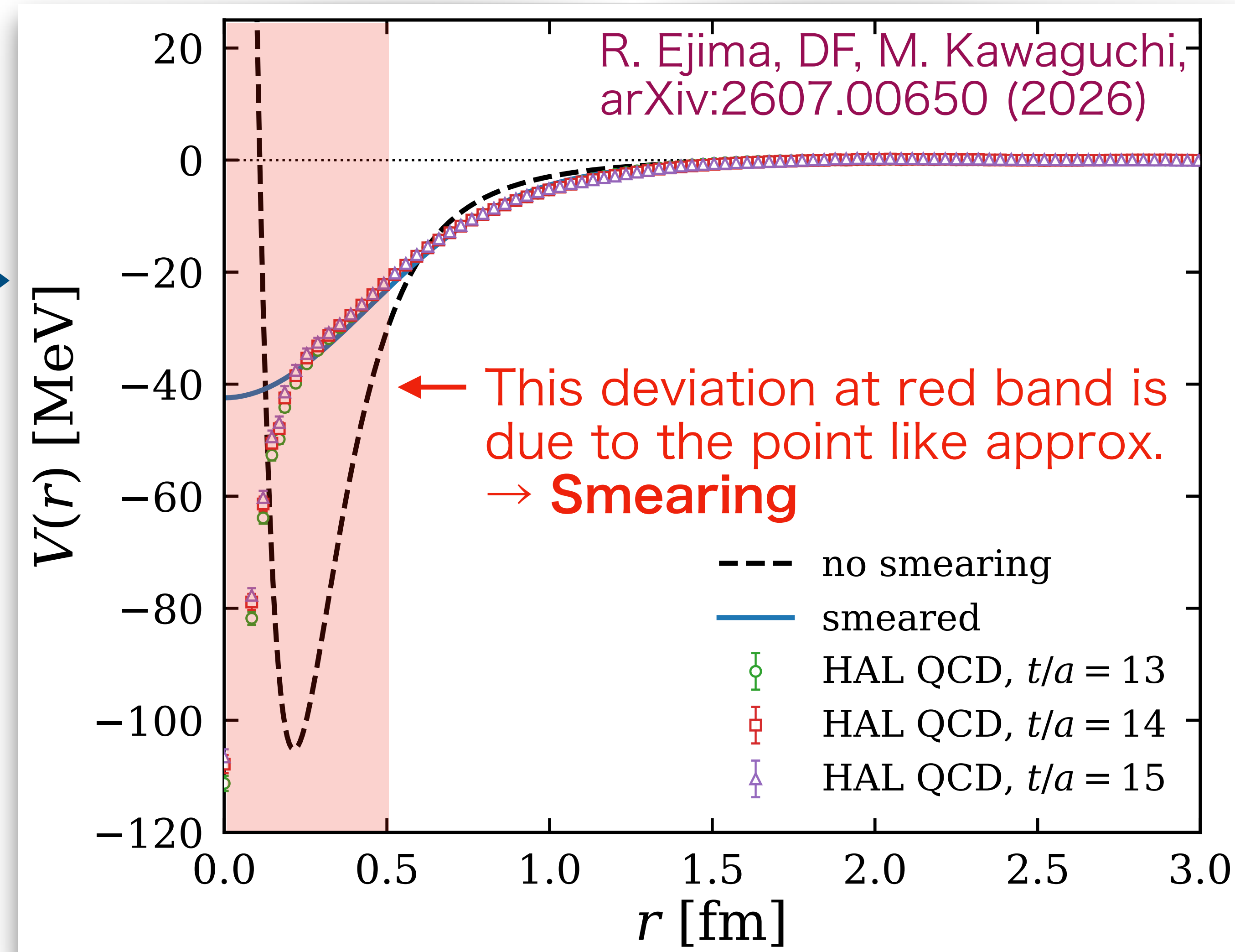
From lattice GFFs



Finite-size effect of J/ψ

$$V_{\mathrm{smear}}(r) = \int d^3r' \rho_{Q\bar{Q}}(|r - r'|)V(r')$$

$$\rho_{Q\bar{Q}} = \frac{1}{(2\pi\sigma_{Q\bar{Q}}^2)^{3/2}}\exp\left(-\frac{r^2}{2\sigma_{Q\bar{Q}}^2}\right) \qquad \sigma_{Q\bar{Q}} = 0.3\ \mathrm{fm}$$



- The potential from the **lattice GFFs** is consistent with **HAL QCD** at long distances
- Finite-size smearing **improves** the behavior of the potential at short distances

The scattering phase shift from the J/ψ-N potential

- **We solve the Schrödinger equation using V(r)**

$$\left[-\frac{\nabla^2}{2\mu}+V(r)\right]\psi(r,k^*)=E_{\rm cm}\psi(r,k^*)$$

$$E_{\rm cm}=k^{*2}/2\mu, \qquad \mu=m_{J/\psi}M_N/(m_{J/\psi}+M_N)$$

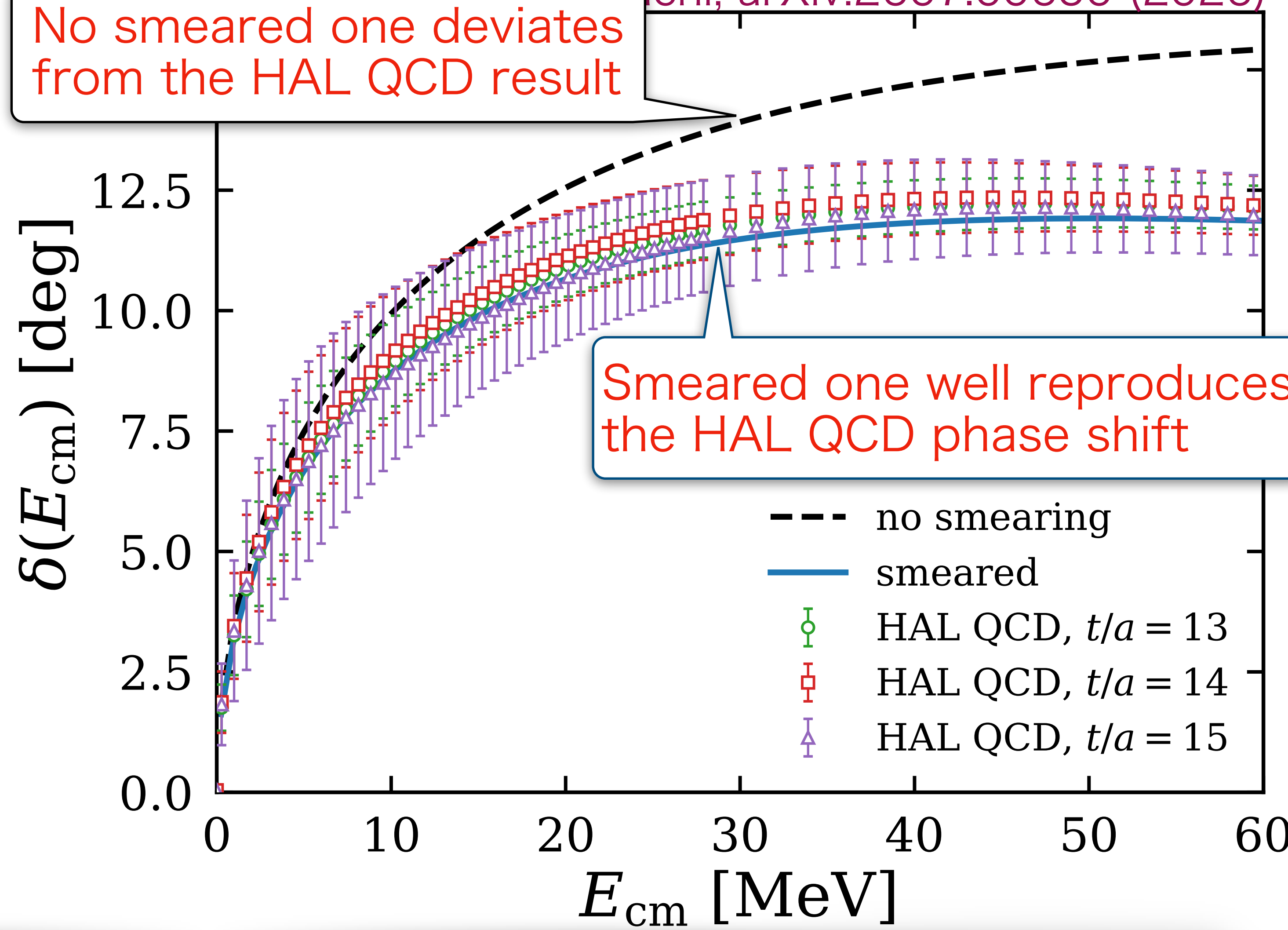
$$\psi(r,k^*)=\frac{1}{\sqrt{4\pi}}\frac{u(r,k^*)}{rk^*}$$

S-wave dominance at low k^*

...chi, arXiv:2607.00650 (2026)

No smeared one deviates from the HAL QCD result

Smeared one well reproduces the HAL QCD phase shift



- **The phase shift is extracted from the asymptotic WF**

$$u(r,k^*)\xrightarrow[r\to\infty]{}\sin\left(k^*r+\boxed{\delta_0(k^*)}\right)$$

The agreement of the phase shift justifies the use of the relative WF for femtoscopy

Correlation function from the GFF-based J/ψ-N potential

$$V(r) \longrightarrow \psi(r,k^*) \longrightarrow C(k^*)$$

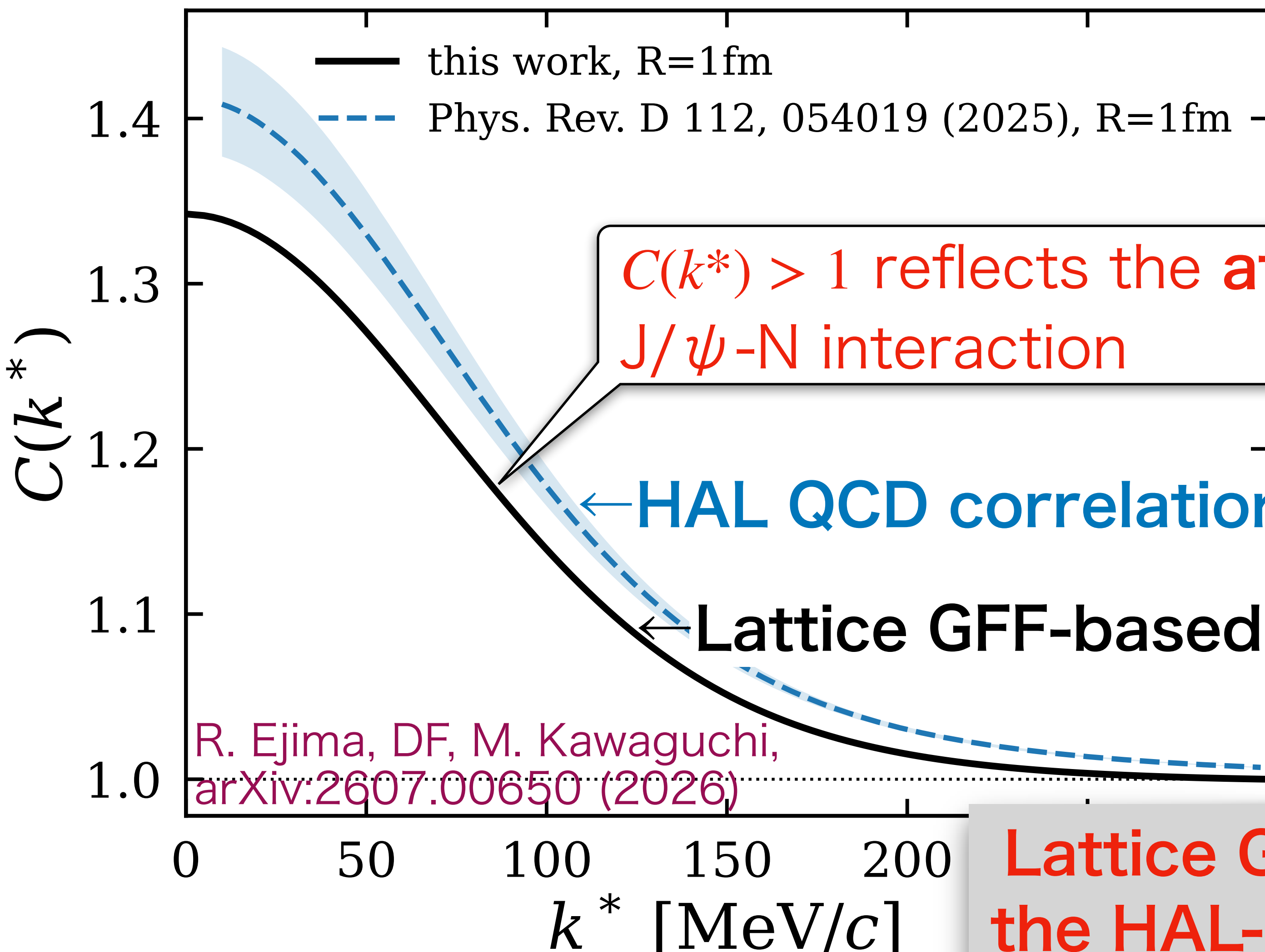
Koonin-Pratt formula

$$C(k^*) = 1 + \int dr^3 S(r) \Big(|\psi(r,k^*)|^2 - |\psi_{\mathrm{free}}(r,k^*)|^2 \Big)$$

The correlation function by averaging the interaction effect of the relative WF over the S(r)

$$S(r) = \exp\left[-r^2/(4R^2)\right]/(4\pi R^2)^{3/2}$$

←Describe the relative separation of the two particles at emission



$C(k^*) > 1$ reflects the **attractive** J/ψ-N interaction

←HAL QCD correlation function

←Lattice GFF-based correlation function

Lattice GFF-based potential reproduces the HAL-QCD-based correlation function

Mechanical structure in nucleon → Two-particle correlation

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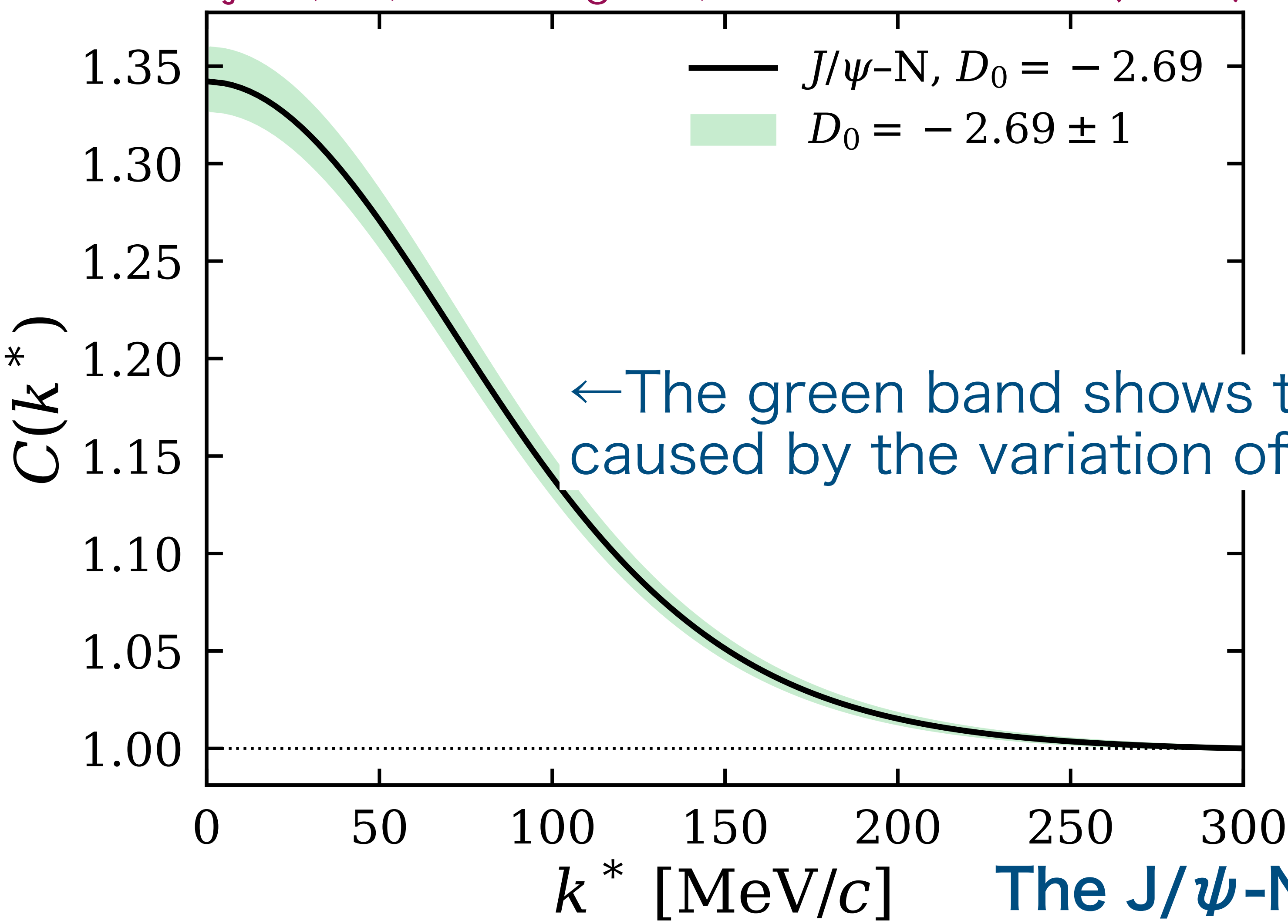
Sensitivity of the J/ψ-N correlation function to the D-term

- **Vary D-term by ±1 around the central value D(0)=-2.69, while keeping all other parameters fixed**

R. Ejima, DF, M. Kawaguchi, arXiv:2607.00650 (2026)



←The green band shows the change caused by the variation of D-term

The J/ψ-N correlation function shows very weak sensitivity to D-term

How does the D-term sensitivity change in the ψ(2S)-N system with a larger α ?

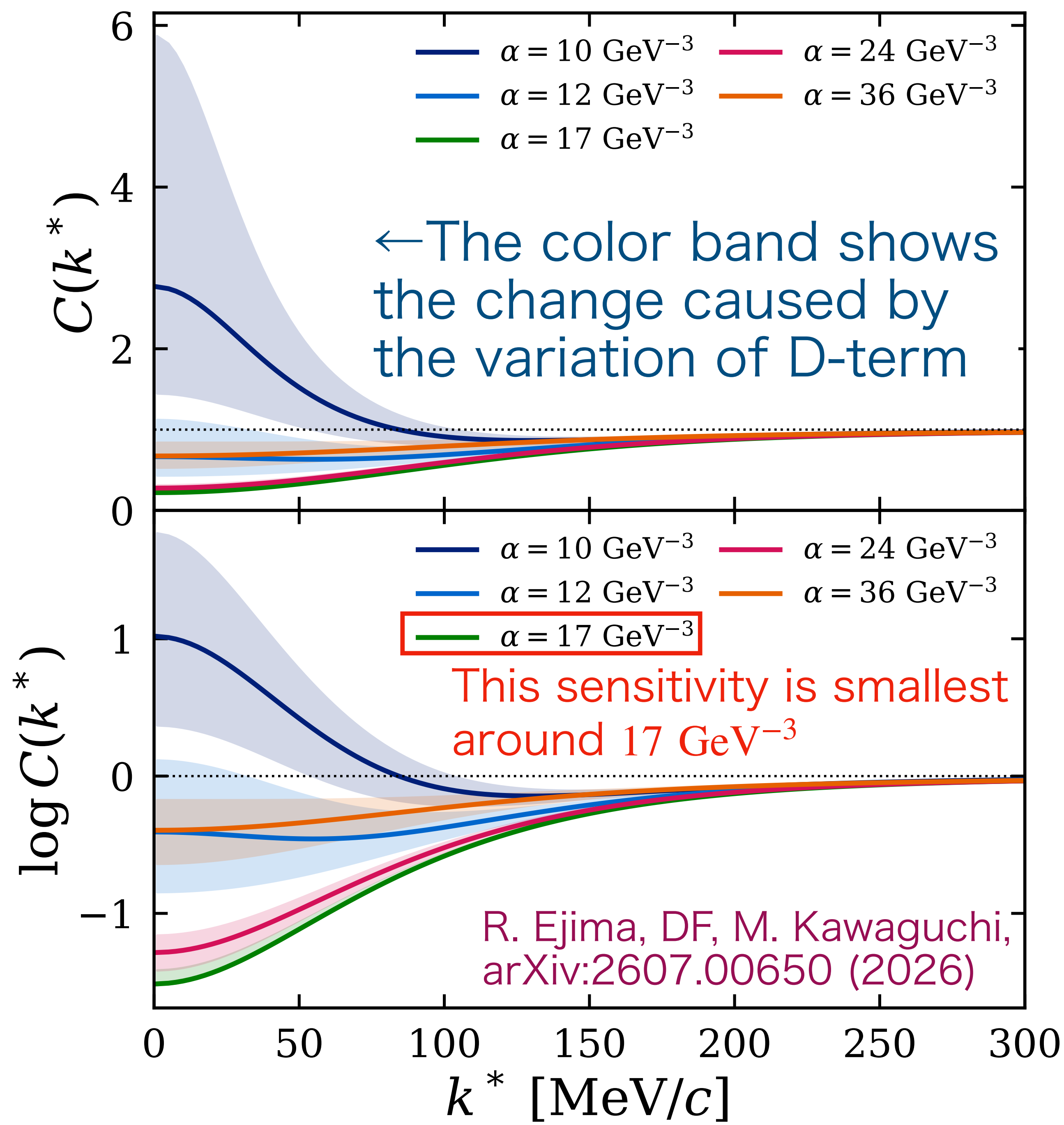
Sensitivity of the $\psi(2S)$-N correlation function to the D-term

- **Turn to the $\psi(2S)$-N system, where α is expected to be much lager**

No lattice-QCD $\psi(2S)$-N potential is currently available, so we vary $\alpha(2S)$ as an input parameter

D-term sensitivity

- strongly depends on $\alpha(2S)$
- becomes more pronounced at small k^*
- is not simply enhanced by increasing $\alpha(2S)$



Compared with J/ψ-N, the $\psi(2S)$-N correlation function may provide better sensitivity to the nucleon D-term

To clarify the D-term sensitivity, we examine

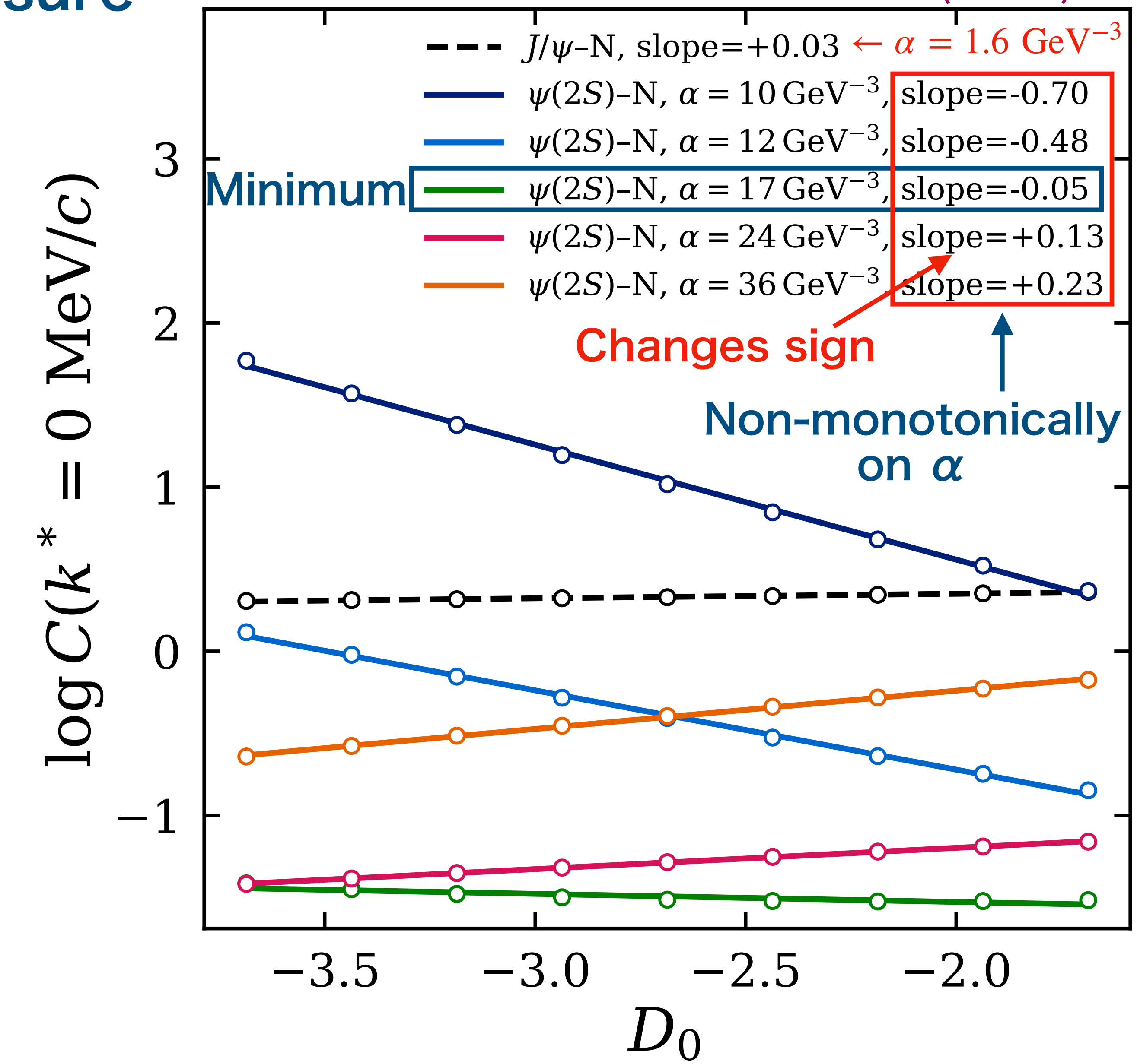
the D-term dependence of correlation function
at the near threshold point $k^* = 0$

R. Ejima, DF, M. Kawaguchi, arXiv:2607.00650 (2026)

The slope provides a quantitative measure of the D-term sensitivity

- The J/ψ-N correlation function has very weak sensitivity
- The sensitivity is minimal around $\alpha(2S) \sim 17\ \mathrm{GeV}^{-3}$
- At larger $\alpha(2S)$, the slope changes sign and becomes positive.

The D-term sensitivity depends non-monotonically on α and is minimal around $\alpha = 17\ \mathrm{GeV}^{-3}$



A precise determination of α is essential for extracting the D-term from charmonium-N femtoscopy

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Summary

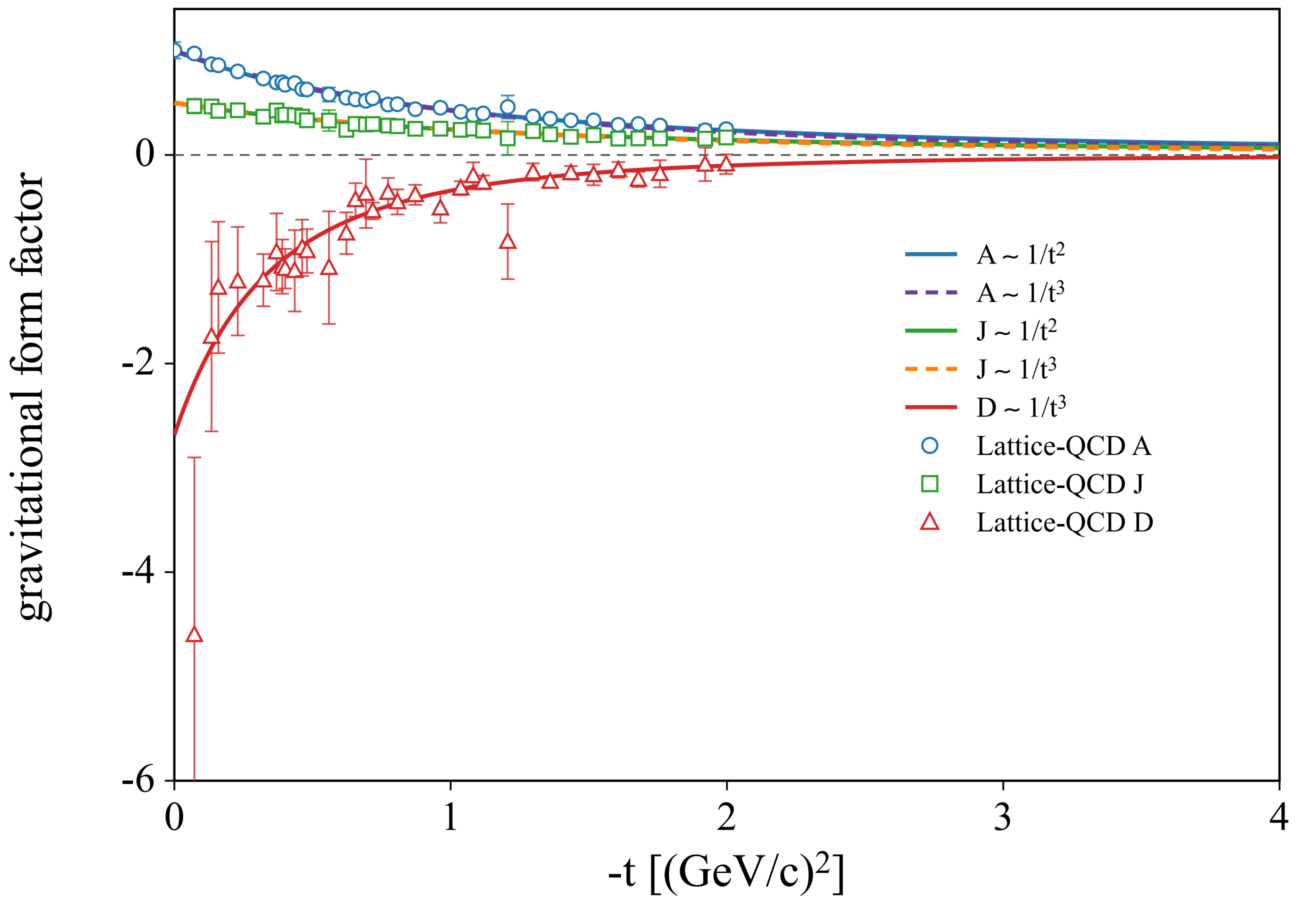
- The QCD multipole expansion connects the **Charmonium-N potential** to the nucleon energy density and pressure based on the **GFFs**
- After finite-size smearing, the **GFF-based J/ψ-N potential** reproduces the **HAL-QCD potential** and **Scattering phase shift**
- The resulting **J/ψ-N correlation function** is consistent with the **HAL-QCD-based results**
- A precies determination of the chromoelectric polarizability α is essential for extracting the D-term

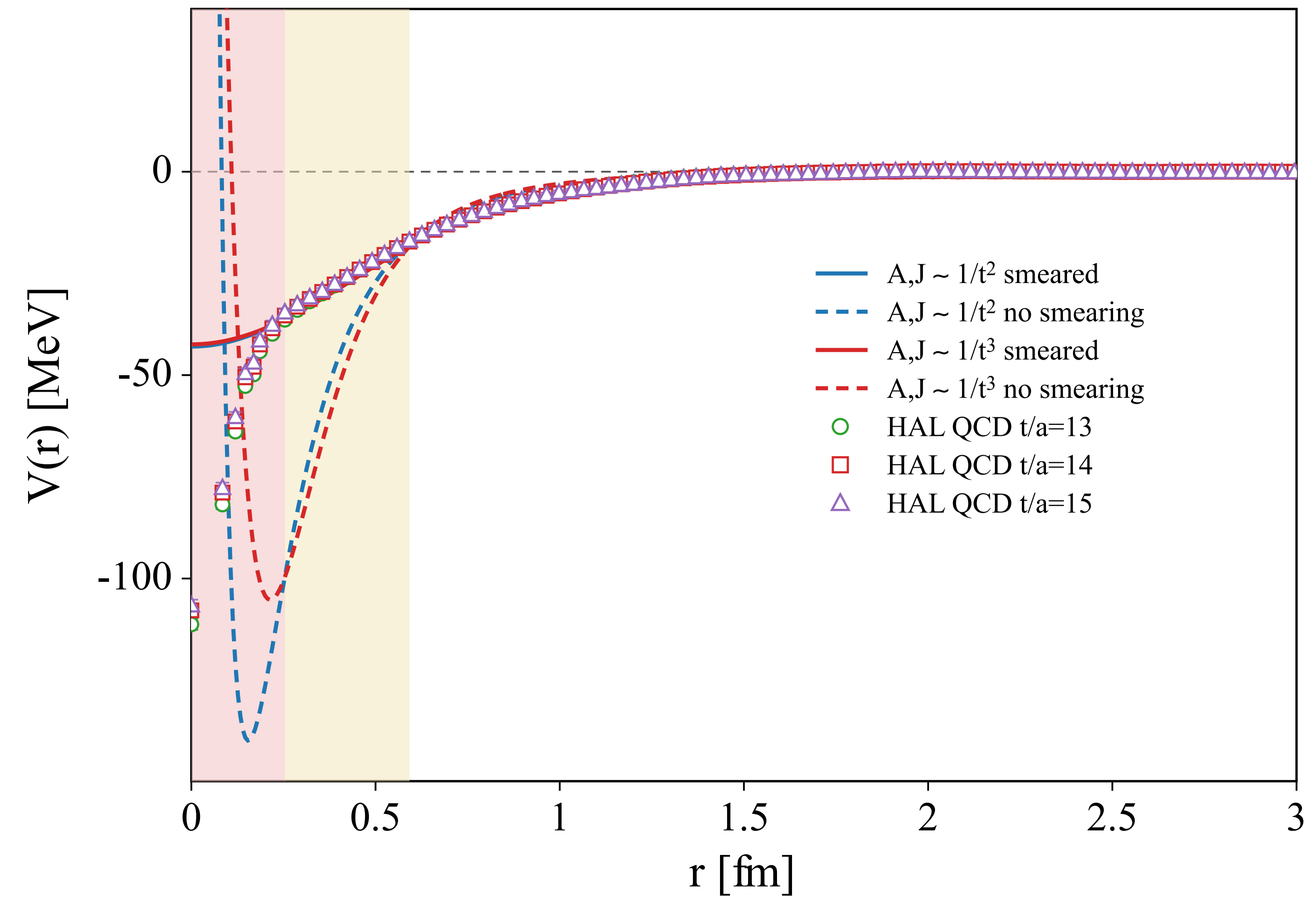
Outlook

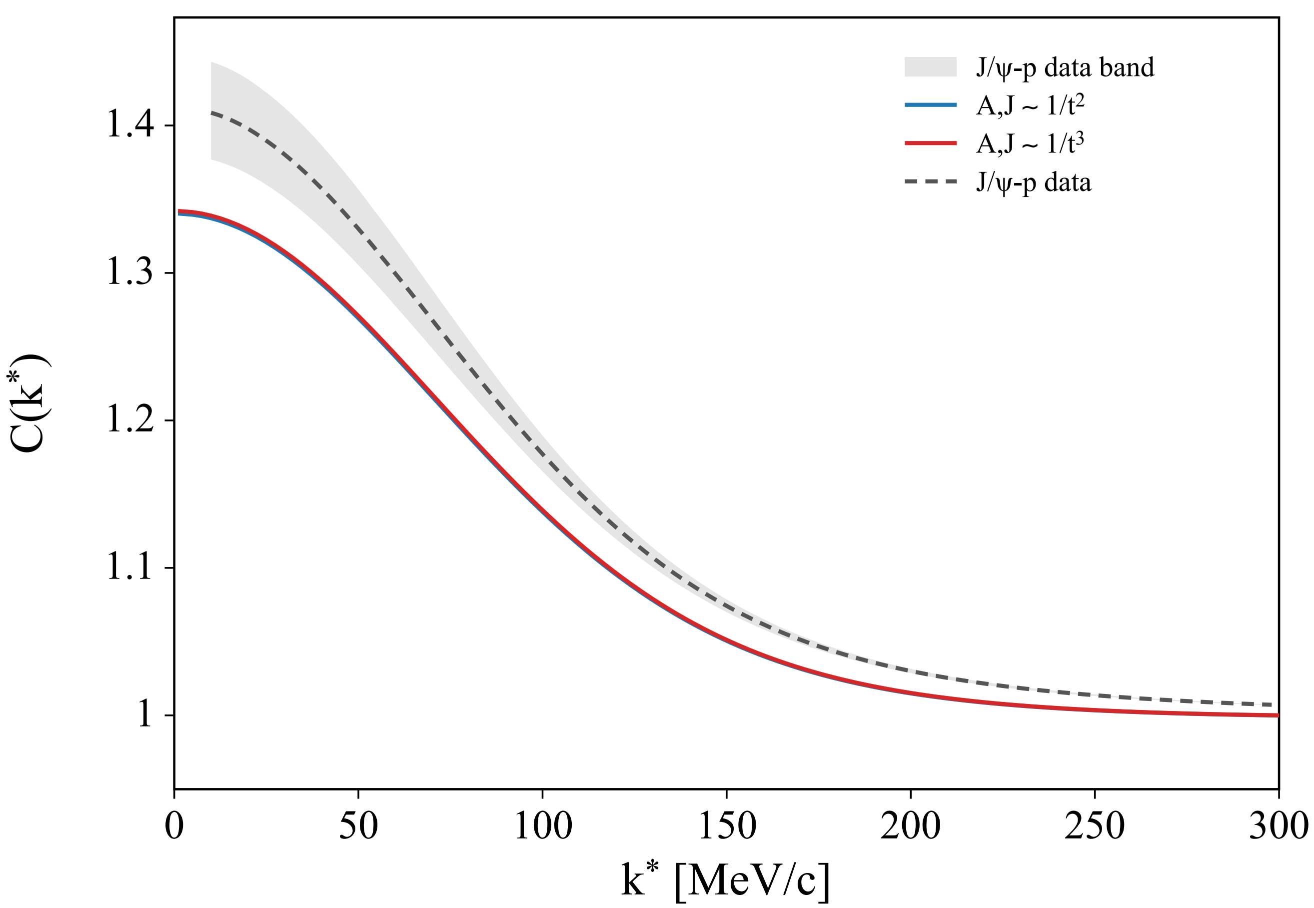
- Lattice-QCD information on the ψ(2S)-N potential and better constraints on α
- Systematic studies of the source size, GFF parameterization, and charmonium finite-size effect

Charmonium-N femtoscopy may provide a complementary probe of the nucleon's mechanical structure

Buck up







The correlation function only weakly sensitive to the large-t behavior of the GFFs

Reducing parameter dependence from our discussion

- **We define the effective radius of the potential as follows:**

$$\langle r_V^2 \rangle = \frac{\displaystyle\int r^2 V(r)\, d^3r}{\displaystyle\int V(r)\, d^3r} = -\frac{1}{M_N^2}\left(\frac{3}{2}+\frac{3}{\nu}\right)D(0) - \frac{3}{4M_N^2} + 6A'(0)$$

The uncertain parameters cancel out

$$V(r) = -\,\alpha_{nS}\frac{4\pi^2}{b}\left(\frac{g^2}{g_s^2}\right)\left[\nu\epsilon(r) - 3p(r)\right]$$

We can calculate the this effective radius from femtoscopy or lattice QCD

Could we determine the D-term by measuring the potential with femtoscopy and calculating its effective radius.

`V(r)=-\alpha_{nS}\frac{4\pi^2}{b}\left(\frac{g^2}{g_s^2}\right)\left[\nu\epsilon(r)-3p(r)\right]`