Parton tomography of helium-4 nucleus

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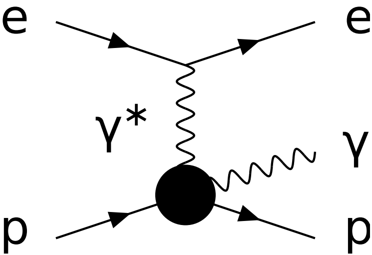
based on:
V. Martínez-Fernández, B. Pire, P. Sznajder, J. Wagner
arXiv: 2604.25677 [hep-ph], Phys.Rev.C.XXX
arXiv: 2605.18519 [hep-ph], Phys.Rev.Lett.XXX

DVCS and GPDs



Factorization into **GPDs** and perturbative coefficient function - on the level of **amplitude**.

$$\textbf{DIS}: \qquad \sigma = \textbf{PDF} \otimes \textbf{partonic cross section}$$
$$\textbf{DVCS}: \qquad \mathcal{M} = \textbf{GPD} \otimes \textbf{partonic amplitude}$$

Exciting because of:

- Ji sum rule
- Hadron tomography
- Connection to energy-momentum tensor

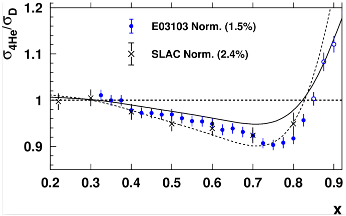
DVCS on Helium-4 with high precision

Why helium-4?

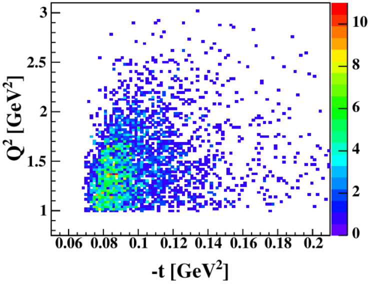
- provides access to the quark–gluon structure of light nuclei. Important nuclear modification effects;
- existing JLab6 data, with more data expected from JLab12;
- only one GPD at LO/LT, with one additional contribution entering at NLO;
- isospin symmetry helps reduce the number of model parameters;

Why higher precision?

- most existing studies are limited to LO/LT accuracy;
- higher-twist effects are important at JLab kinematics;
- as well as NLO corrections in α_s;



JLAB, PRL103



CLAS, PRL119

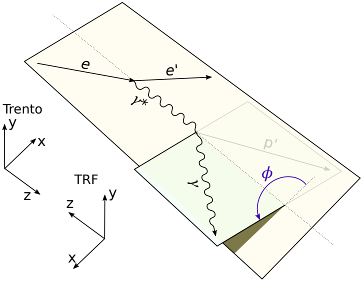
Cross-section

For a beam charge sign χ and helicity s, and outgoing photon polarization λ:

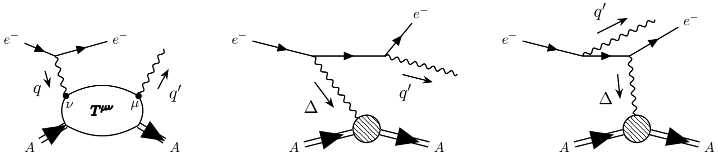
$$\frac{d^4\sigma_\chi^{s\lambda}}{dx_A\ dQ^2\ dt\ d\phi} = \frac{\alpha_{\rm EM}^3}{8\pi}\frac{x_A y^2}{Q^4\sqrt{1+\omega^2}}\left|\frac{\mathcal{M}_\chi^{s\lambda}}{e^3}\right|^2 ,$$

where $y = pq/(pk)$, $\omega = 2x_A M/Q$, and:

$$x_A = \frac{Q^2}{2pq} \approx \frac{x_B}{A} ,$$



Amplitudes



DVCS (on the left) and the two cases of Bethe-Heitler (center and right).

$$i\mathcal{M}_{\chi}^{s\lambda} = i\mathcal{M}_{\chi,\,\mathrm{DVCS}}^{s\lambda} + i\mathcal{M}_{\mathrm{BH}}^{s\lambda} + i\mathcal{M}_{\mathrm{BHX}}^{s\lambda}\,.$$

For DVCS:

$$i\mathcal{M}_{\chi,\,\mathrm{DVCS}}^{s\lambda} = \frac{ie^3\chi}{q^2}\bar{u}(k',s)\gamma_\nu u(k,s) T^{\mu\nu}\varepsilon'_\mu(-\lambda)\,,$$

where the Compton tensor:

$$T^{\mu\nu} = i\int d^4z\; e^{iq'z}\langle p'|\mathcal{T}\left\{j^\mu(z)j^\nu(0)\right\}|p\rangle = \sum_{\Lambda,\Lambda'} T^{\mu\nu}_{(\Lambda\Lambda')}\mathcal{A}^{\Lambda\Lambda'}$$

Helicity-dependent amplitudes:

$$\mathcal{A}^{\Lambda\Lambda'} = (-1)^{\Lambda-1}\left(\varepsilon'_\mu(\Lambda')\right)^* T^{\mu\nu}\varepsilon_\nu(\Lambda)$$

Amplitudes

Spinor techinques (Kleiss-Stierling) - direct calculation of amplitudes.

$$i\mathcal{M}^{s\lambda}_{\chi,\,\mathrm{DVCS}} = \frac{-ie^3\chi}{Q^2}\Bigg[\mathcal{A}^{++}\left(\frac{\sqrt{2}}{2|\bar{p}_\perp|}\right)\left(\sum_{j=1}^{3} B_j g(s,k',L_j,k) - i\lambda p_\mu \epsilon_\perp^{\mu\nu} j_\nu(s,k',k)\right)$$
$$+\mathcal{A}^{+-}\left(\frac{\sqrt{2}}{2|\bar{p}_\perp|}\right)\left(\sum_{j=1}^{3} B_j g(s,k',L_j,k) + i\lambda p_\mu \epsilon_\perp^{\mu\nu} j_\nu(s,k',k)\right)$$
$$-\mathcal{A}^{\mathbf{0}+}\ \frac{2}{Q} g(s,k',q',k)\Bigg]$$

where blue parts given it terms of scalar functions of four-momenta for example:

$$g(s,\ell,a,k) = \bar{u}(\ell,s)\not{a}u(k,s) = \delta_{s+}s(\ell,a)t(a,k) + \delta_{s-}t(\ell,a)s(a,k)\,,$$

where:

$$s(k,k') = \bar{u}(k,+)u(k',-) = -\left[t(k',k)\right]^*$$

can be expressed as:

$$s(k,k') = (k^2 + ik^3)\sqrt{\frac{k'^0 - k'^1}{k^0 - k^1}} - (k \leftrightarrow k')\,.$$

Similarly for purely electromagnetic Bethe-Heitler amplitudes, expressed in terms of elastic form factors.

Accuracy: NLO and kinematic HT

$$\mathcal{A}^{++}(\xi,t) = \int_{-1}^{1} dx\ \frac{1}{\xi}\Bigg\{ \left[\mathbf{C_0(x)} + \frac{\alpha_S C_F}{4\pi}\ C^q_{NLO}(x) + \frac{t}{\mathbb{Q}^2}\ C_{HT}(x)\right] H^q(x,\xi,t) + \frac{\alpha_S T_F}{4\pi}\frac{1}{\xi}\ C^g_{NLO}(x)\ H^g(x,\xi,t)\Bigg\}$$

$$\mathcal{A}^{+-}(\xi,t) = \int_{-1}^{1} dx\ \frac{1}{\xi}\left\{ \frac{t}{\mathbb{Q}^2}\ C^{+-}_{HT}(x)\ H^q(x,\xi,t) + \frac{\alpha_S T_F}{4\pi}\frac{1}{\xi}\ C^{+-}_{NLO}(x)\ H^g_T(x,\xi,t)\right\}$$

$$\mathcal{A}^{0+}(\xi,t) = \int_{-1}^{1} dx\ \frac{1}{\xi}\left\{ \frac{\sqrt{-t}}{\mathbb{Q}}\ C^{0+}_{HT}(x)\ H^q(x,\xi,t)\right\}$$

NLO coefficient functions given by: [Ji, Osborne, PRD 58]
[Pire, Szymanowski, Wagner, PRD 83]
[Belitsky, Mueller, PLB 486]

HT coefficient functions given by: [Braun, Ji, Manashov, JHEP 01, 078]
[Martinez-Fernandez, Pire, Sznajder, Wagner, PRD111]

Model

Our model is based on double distributions:

$$H^i(x,\xi,t;\mu^2) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta+\xi\alpha-x) F^i(\beta,\alpha,t;\mu^2)\,.$$

$$F^i(\beta,\alpha,t) = f_i(\beta,t)\, h_i(\beta,\alpha)\,.$$

For the profile function we use the classic Radyushkin's Ansatz. The generalized PDF is

$$f_i(\beta,t) = f_i^A(\beta)\,\frac{k_i(|\beta|,t)}{k_i(|\beta|,0)}\,,$$

where for $f_i^A(\beta)$ we use nNNPDF30 parameterizations.

For valence quarks, $i=\{u_{\rm val}, d_{\rm val}\}$, we use:

$$k_i(|\beta|,t) = \left(\frac{1}{1-p_0(1-|\beta|)^2 t}\right)^{p_1} \prod_{j=1}^{n} \left(\frac{|p_{2,j}+t|}{|p_{2,j}+t|-p_{3,j}(1-|\beta|)^2 t}\right)^{p_{4,j}}\,.$$

For sea quarks and gluons:

$$k_i(|\beta|,t) = \exp\Big(p(1-|\beta|^2)t\Big)\,.$$

Full evolution equations for GPDs (we utilize the `APFEL++`)

Model for H_T^g

We also need a model for H_T^g, which contributes at NLO to the $\mathcal{A}^{+-}$ amplitude. Since little is known about this GPD, we bound it by $H^g(x,\xi,0)$ as $t \to 0$, while for $t \ll 0$ we use the positivity inequality :

$$|H_T^g(x,\xi,t)| < \frac{1}{\xi}\sqrt{\frac{x^2-\xi^2}{1-\xi^2}}\sqrt{\frac{t}{t-t_0}}\sqrt{\frac{t_0}{t-t_0}}\sqrt{g\left(\frac{x+\xi}{1+\xi}\right)g\left(\frac{x-\xi}{1-\xi}\right)},$$

which for $\xi \to 0$

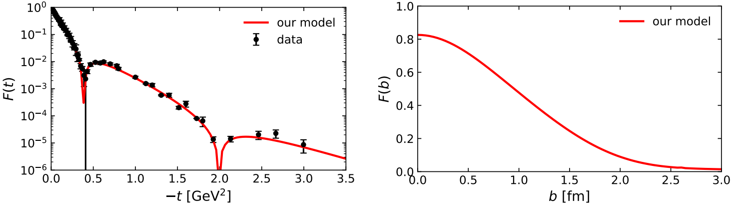
$$|H_T^g(x,\xi\to 0,t)| < x\,g(x)\sqrt{\frac{1}{-t/(4M^2)}}\,.$$

This suggests the following Ansatz, which we adopt in this analysis:

$$H_T^g(x,\xi,t) = H^g(x,\xi,0)\sqrt{\frac{1}{1-t/(4M^2)}}\,.$$

Fitting

- Elastic Form Factors:

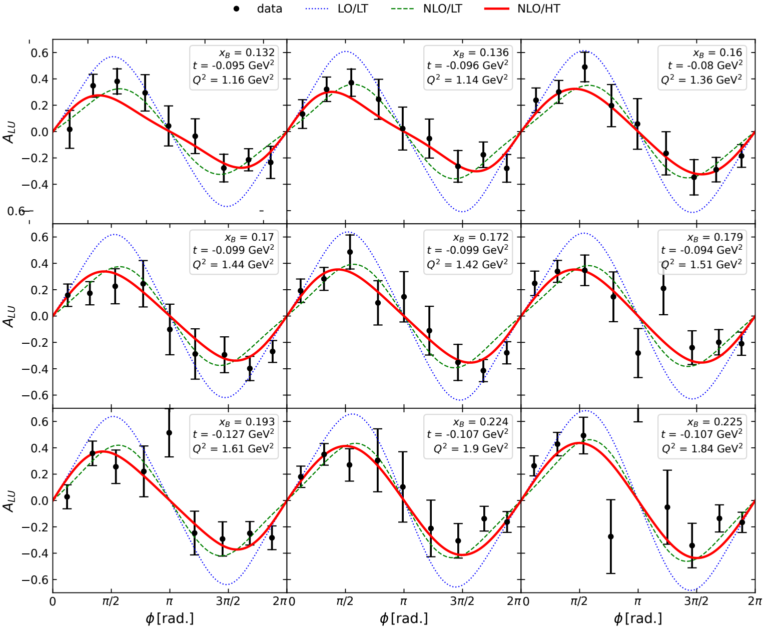


- DVCS data from JLAB:

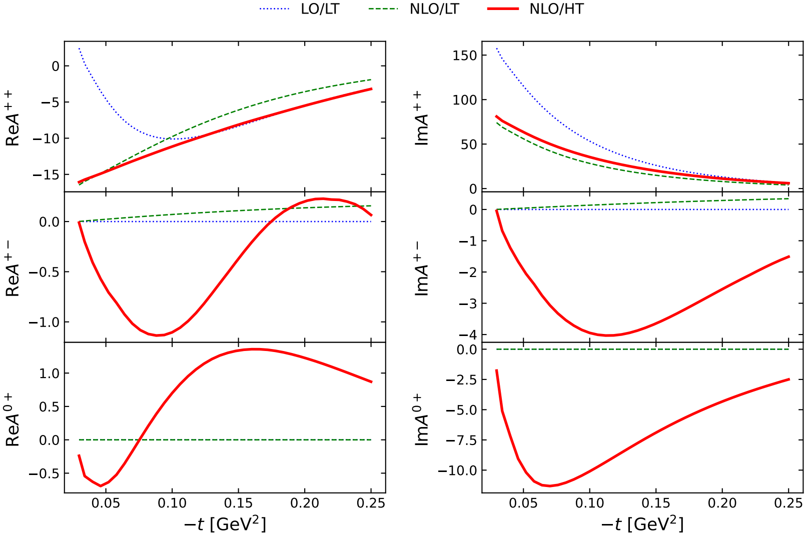
$$A_{LU}(\phi) = \frac{\sigma_-^{++} + \sigma_-^{+-} - \sigma_-^{-+} - \sigma_-^{--}}{\sigma_-^{++} + \sigma_-^{+-} + \sigma_-^{-+} + \sigma_-^{--}}$$

Precision in α_s expansion	Precision in kinematic twist expansion	GPDs involved	Non-vanishing amplitudes	Fitted value of p [GeV^{-2}]	Global χ^2/n
LO	LT	H^q	$\mathcal{A}^{++}$	> 70	1.31
NLO	LT	H^q, H^g, H^g_T	$\mathcal{A}^{++}$, $\mathcal{A}^{+-}$	$23.4^{+7.4}_{-3.9}$	1.28
NLO	HT	H^q, H^g, H^g_T	$\mathcal{A}^{++}$, $\mathcal{A}^{+-}$, $\mathcal{A}^{0+}$	$22.0^{+5.5}_{-2.1}$	1.00

Experimental data for the A_{LU} asymmetry compared to our model

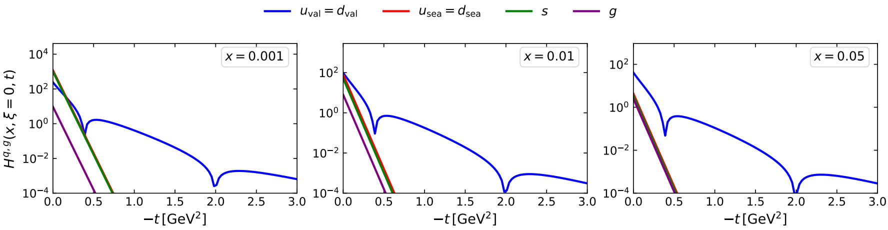


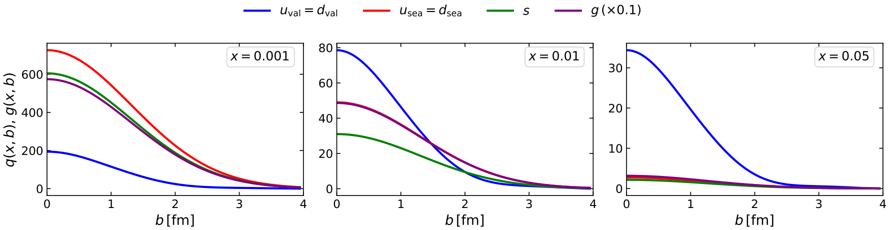
Helicity Amplitudes



$x_B = 0.18$ and $Q^2 = 1.5\,\mathrm{GeV}^2$.

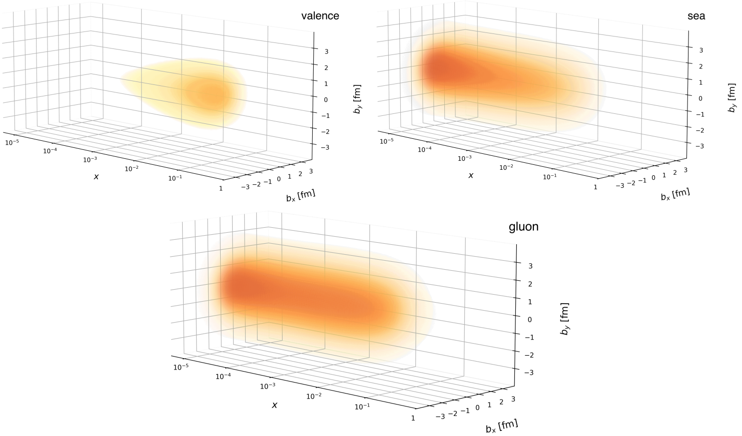
Fitted GPDs





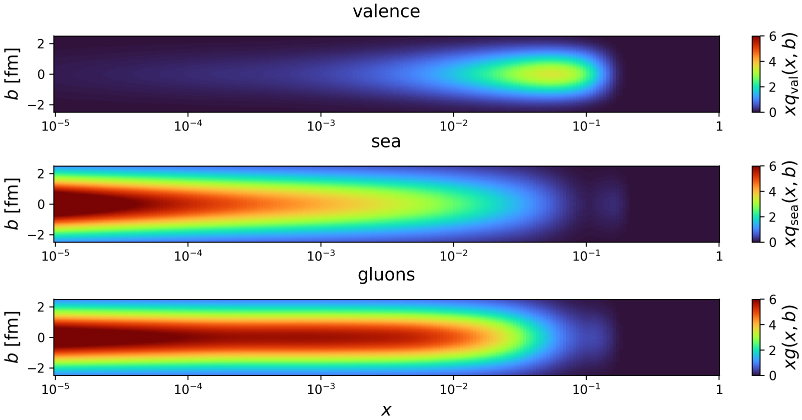
$$\mu^2 = 2\,\mathrm{GeV}^2.$$

Tomography



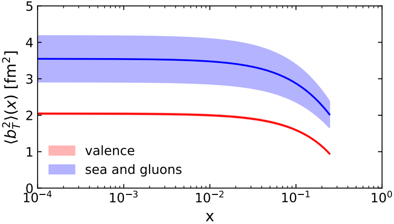
Tomographic pictures of $^4\mathrm{He}$ nuclei, $x\, q(x, b_x, b_y)$ and $x\, g(x, b_x, b_y)$. The valence distribution indicates contributions coming from both up and down quarks, while the sea distribution includes contributions from up, down and strange quarks and antiquarks.

Tomography


valence
b [fm]
$xq_{\mathrm{val}}(x,b)$
sea
b [fm]
$xq_{\mathrm{sea}}(x,b)$
gluons
b [fm]
$xg(x,b)$
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1
x

Tomography

$$\langle b_T^2\rangle_i(x) = \frac{\int d^2\boldsymbol{b_T}\ \boldsymbol{b_T}^2 f_i^A(x,\boldsymbol{b_T})}{\int d^2\boldsymbol{b_T}\ f_i^A(x,\boldsymbol{b_T})}\,.$$



$$\langle b_T^2\rangle_q = \frac{\int dx \int d^2\boldsymbol{b_T}\ \boldsymbol{b_T}^2 f_q^A(x,\boldsymbol{b_T})}{\int dx \int d^2\boldsymbol{b_T}\ f_q^A(x,\boldsymbol{b_T})}\,, \qquad \langle b_T^2\rangle_g = \frac{\int \frac{dx}{x} \int d^2\boldsymbol{b_T}\ \boldsymbol{b_T}^2 f_g^A(x,\boldsymbol{b_T})}{\int \frac{dx}{x} \int d^2\boldsymbol{b_T}\ f_g^A(x,\boldsymbol{b_T})}\,.$$

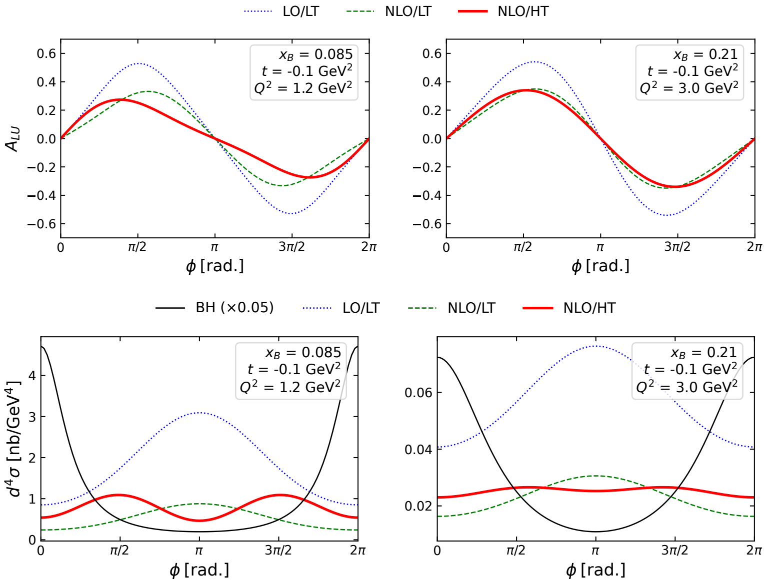
$\langle b_T^2\rangle_{q_{\rm val}} = 1.943(28)\,{\rm fm}^2$, $\langle b_T^2\rangle_{q_{\rm sea}} = 3.51(63)\,{\rm fm}^2$, $\langle b_T^2\rangle_g = 3.50(62)\,{\rm fm}^2$

Comparing to the mean electric charge radius $r^e_{^4{\rm He}}$, extracted from EFF:

$$r^e_{^4{\rm He}} = \sqrt{\frac{3}{2}\langle b_T^2\rangle_{q_{\rm val}}} = 1.71(21)\,{\rm fm}\,,$$

compared to $1.6785(21)\,{\rm fm}$ reported in Rev. Mod. Phys. 97 (2025) (CODATA)

Predictions for JLAB12



$y = 0.75,\ E_{beam} = 10.06\,\mathrm{GeV}.$

Summary

- First GPD fit based on kinematic HT and NLO (also with full GPD evolution)
- First helium tomography
- Outlook: TCS and other exclusive processes on helium
- Outlook: Same precision needed for DVCS on nucleons
- Genuine higher twist corrections - open topic