Exclusive electroproduction of dijets as a probe of generalised parton distributions

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Overview

Introduction to generalised parton distributions (GPDs)

Introduction to exclusive electroproduction of dijet

Analytical results (all polarization cases covered)

Quark dijet production (QED+QCD),

Gluon dijet production

Numerical results at EIC, EicC and HERA kinematics

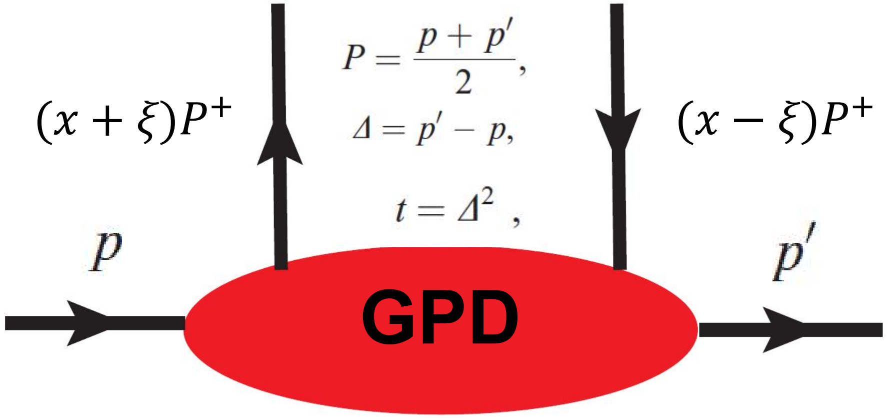
Introduction to generalised parton distributions (GPDs)

Unpolarized GPDs:

Müller et al.,1994; Ji, 1996; Radyushkin, 1997

$$F_{qu}(x,\xi,t) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, \mathrm{e}^{\mathrm{i}xP^+z^-} \langle p'|\bar{q}(-\tfrac{1}{2}z)\gamma^+ W(-\tfrac{1}{2}z,\tfrac{1}{2}z)q(\tfrac{1}{2}z)|p\rangle|_{z^+=0,z_\perp=0}$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}(p')\gamma^+u(p) + E^q(x,\xi,t)\bar{u}(p')\frac{\mathrm{i}\sigma^{+\alpha}\Delta_\alpha}{2m_p}u(p)\right],$$

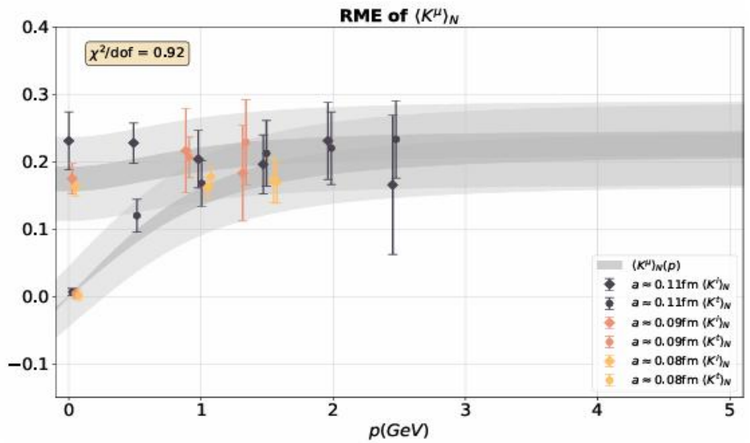
$$F_{gu}(x,\xi,t) = \frac{1}{P^+}\int \frac{\mathrm{d}z^-}{2\pi}\, \mathrm{e}^{\mathrm{i}xP^+z^-} \langle p'|G^{+\mu}(-\tfrac{1}{2}z) W(-\tfrac{1}{2}z,\tfrac{1}{2}z)G^+_\mu(\tfrac{1}{2}z)|p\rangle|_{z^+=0,z_\perp=0}$$
$$= \frac{1}{2P^+}\left[H^g(x,\xi,t)\bar{u}(p')\gamma^+u(p) + E^g(x,\xi,t)\bar{u}(p')\frac{\mathrm{i}\sigma^{+\alpha}\Delta_\alpha}{2m_p}u(p)\right],$$



The moments of GPDs in the forward limit can give different terms in the spin sum-rule:

$$J_{q,g} = \frac{1}{2}\int dx\, x(H_{q,g}(x,0,0) + E_{q,g}(x,0,0)),$$

Ji,1996



The state-of-the-art lattice result of total gluon helicity (ΔG) is 46(9)% of the proton spin. (The first work implementing renormalization, the conversion factor from RI/MOM scheme to $\overline{MS}$ scheme was calculated up to N^3LO)

Pang, Yao, Zhang, JHEP 07 (2024) 222
Zhao, et al., 2512.24315

Introduction to generalised parton distributions (GPDs)

Unpolarized GPDs:

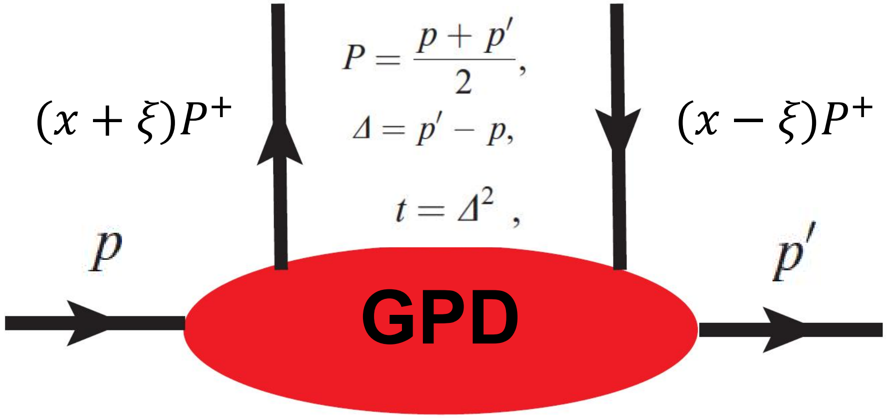
Müller et al.,1994; Ji, 1996; Radyushkin, 1997

$$F_{qu}(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|\bar{q}(-\frac{1}{2}z)\gamma^+ W(-\frac{1}{2}z,\frac{1}{2}z)q(\frac{1}{2}z)|p\rangle\big|_{z^+=0,z_\perp=0}$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}(p')\gamma^+u(p) + E^q(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m_p}u(p)\right],$$

$$F_{gu}(x,\xi,t) = \frac{1}{P^+}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|G^{+\mu}(-\frac{1}{2}z)W(-\frac{1}{2}z,\frac{1}{2}z)G^+_\mu(\frac{1}{2}z)|p\rangle\big|_{z^+=0,z_\perp=0}$$

$$= \frac{1}{2P^+}\left[H^g(x,\xi,t)\bar{u}(p')\gamma^+u(p) + E^g(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m_p}u(p)\right],$$



The moments of GPDs in the forward limit can give different terms in the spin sum-rule:

$$J_{q,g} = \frac{1}{2}\int dx\,x(H_{q,g}(x,0,0) + E_{q,g}(x,0,0)),$$

Ji,1996

“GPDs contain information about how the usual PDFs (the forward matrix elements) are distributed in position space”.

M. Burkardt, 2002

“GPD carry information about the spatial distribution of forces experienced by quarks and gluons inside hadrons”.

M. Polyakov, 2002

Studies of exclusive production of dijet in QCD context

In collinear factorisation framework:

Braun and Ivanov, 2005 (First paper discussing this process using collinear factorisation (unpol GPDs))

Chall et al., 2026 (Recent paper with phenomenology at HERA kinematics)

In small-x framework:

Altinoluk et al., 2015

Hatta,Xiao,Yuan, 2016

Mäntysaari,Mueller,Schenke,2019

Boussarie et al., 2019

...

In k_t factorisation/GTMD framework:

Bartels et al., 1996

Boer, Setyadi, 2021,2023

Linek et al., 2024

Bellotti, Boer, 2026

...

As a probe of gluon orbital angular momentum (OAM):

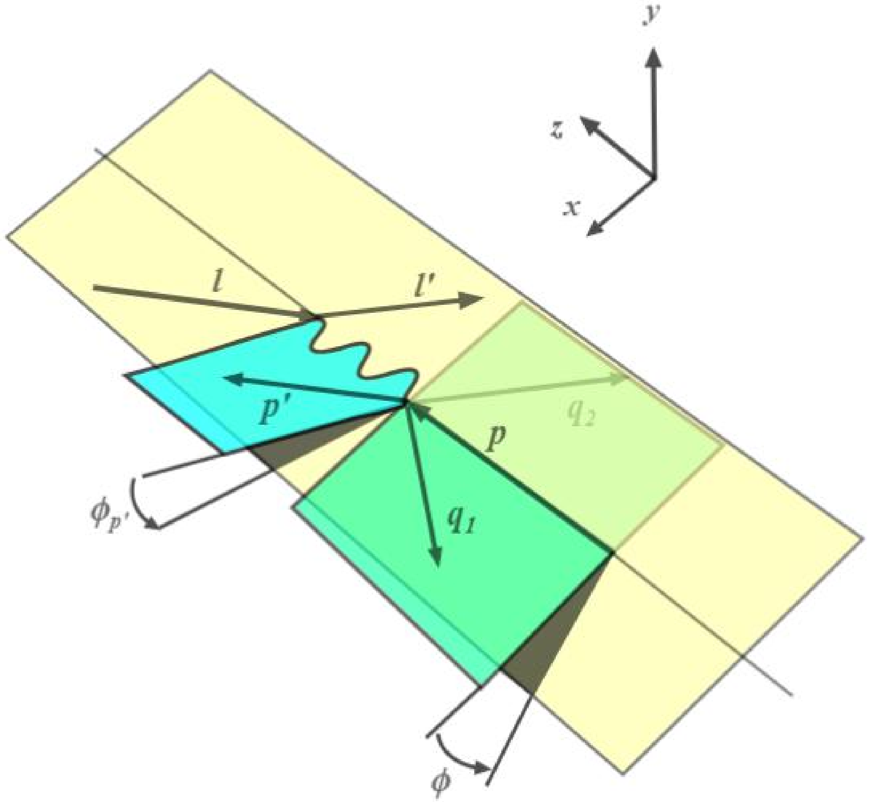
Ji,Yuan,Zhao, 2017

Bhattacharya,Boussarie,Hatta, 2022,2025

...

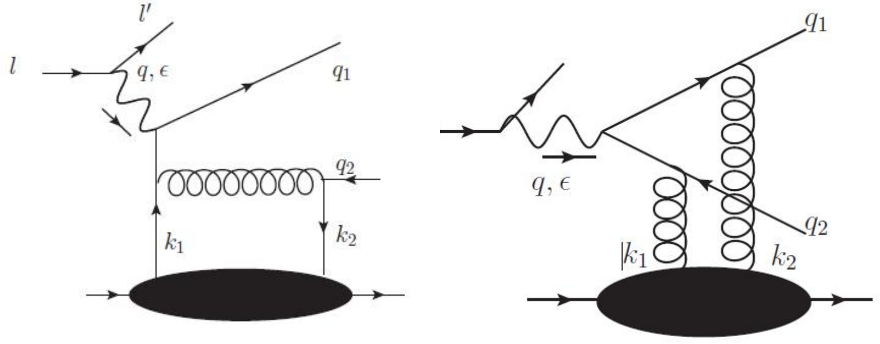
Exclusive electroproduction of dijet: kinematics

$\gamma^* - p$ back-to back frame:



$$y = \frac{q \cdot p}{l \cdot p}, \quad Q^2 = -q^2, \quad W^2 = (p+q)^2, \quad x_{\text{Bj}} = \frac{Q^2}{2p \cdot q} \approx \frac{Q^2}{Q^2 + W^2}.$$

Quark dijet production (LO):

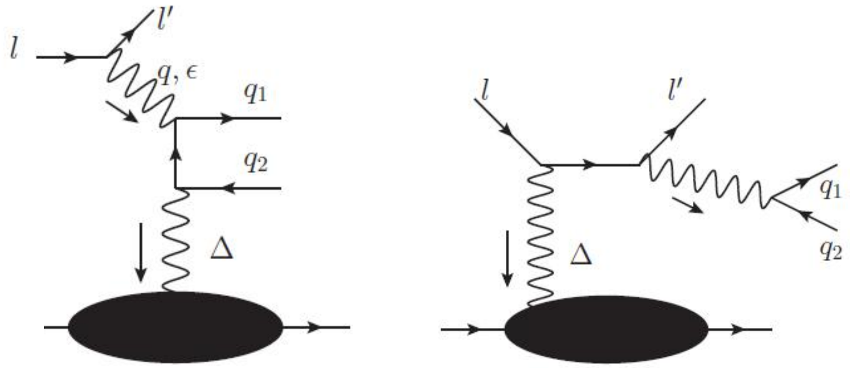


QCD channel

(sensitive to GPDs)

+

$$\beta = \frac{Q^2 z\bar{z} + m_q^2}{q_\perp^2 + m_q^2 + Q^2 z\bar{z}}$$

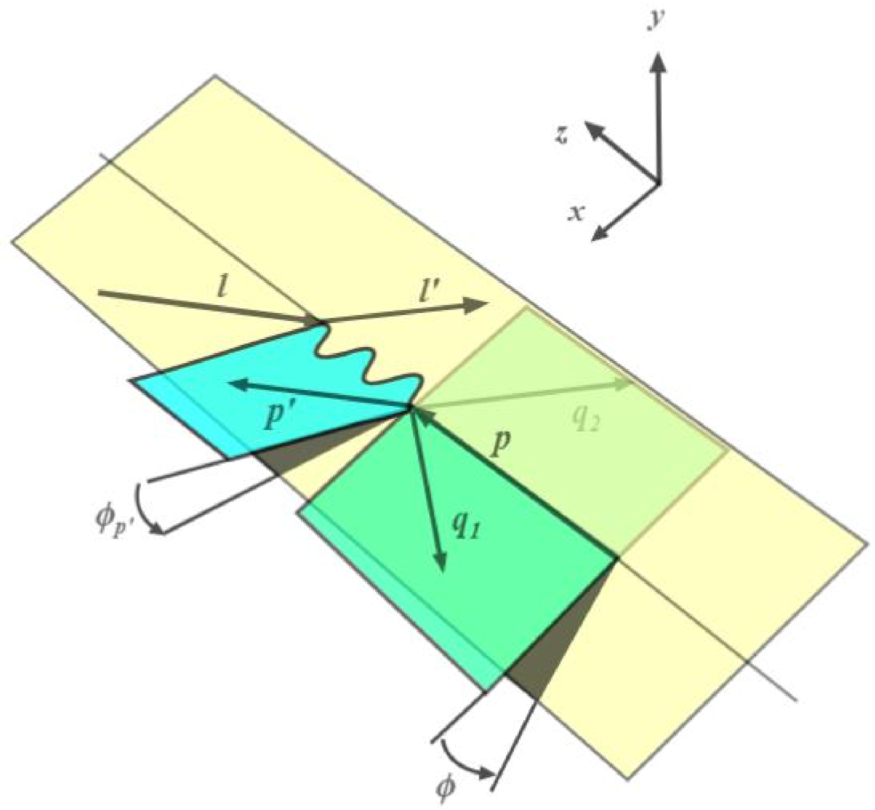


QED/Primakoff channel

(sensitive to EFFs)

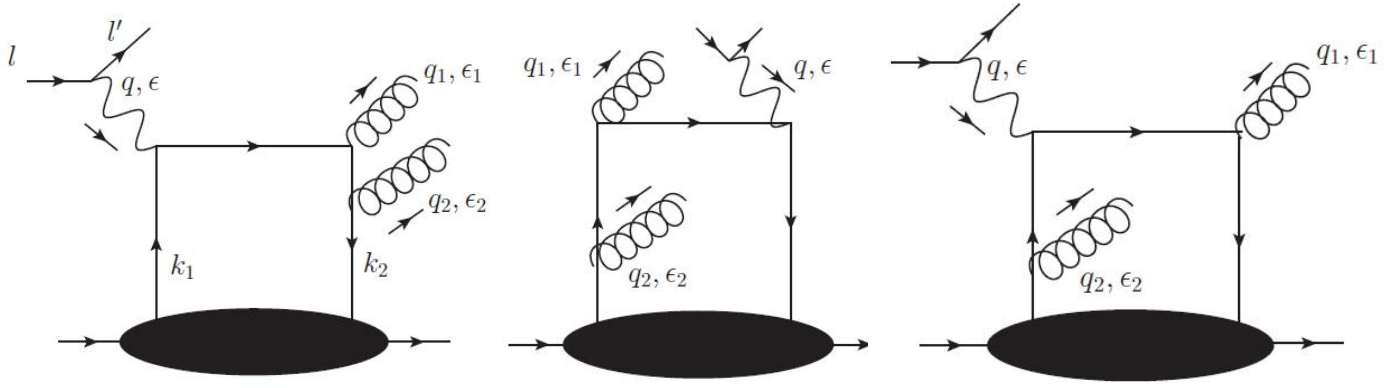
Exclusive electroproduction of dijet: kinematics

$\gamma^* - p$ back-to back frame:



$$y = \frac{q \cdot p}{l \cdot p}, \quad Q^2 = -q^2, \quad W^2 = (p+q)^2, \quad x_{\rm Bj} = \frac{Q^2}{2p \cdot q} \approx \frac{Q^2}{Q^2 + W^2}.$$

Gluon dijet production (LO):

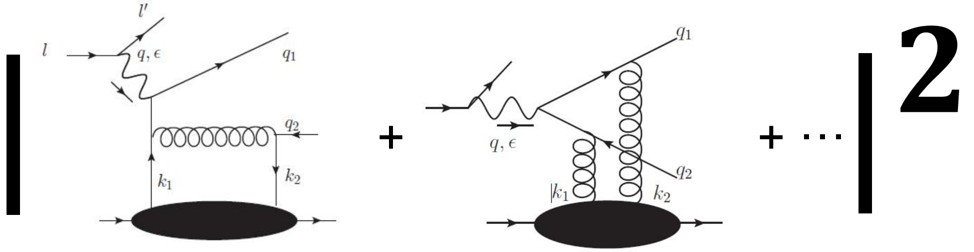


Due to charge conjugation symmetry, only valence quark GPDs contribute to gluon dijet production.

For gluon dijet production, the QED channel starts from NLO.

Analytical results: quark dijet production

$$|M^{q\bar{q}}|^2 = \left|M^{q\bar{q}}_{QCD}\right|^2 + \left|M^{q\bar{q}}_{QED}\right|^2 + 2Re(M^{q\bar{q}}_{QCD} M^{q\bar{q}*}_{QED})$$



Unpolarized beam:

$$\mathrm{Re}\big(M^{q\bar{q},a}_{\mathrm{QCD}} M^{q\bar{q},b*}_{\mathrm{QCD}}\big) = \frac{2\pi\alpha_{\mathrm{em}}}{Q^4}\Big[\frac{8Q^2(1-y)}{y^2}\big|\mathcal{A}^{q\bar{q}}_L\big|^2 + \frac{8Q^2\sqrt{1-y}(2-y)}{y^2}\mathrm{Re}\big(\mathcal{A}^{q\bar{q},x}_T \mathcal{A}^{q\bar{q}*}_L\big) + \frac{2Q^2(2-y)^2}{y^2}\big|\mathcal{A}^{q\bar{q},x}_T\big|^2 + 2Q^2\big|\mathcal{A}^{q\bar{q},y}_T\big|^2\Big],$$

$$\begin{aligned}
|\mathcal{A}^{q\bar{q},x}_T|^2 =& \frac{|C|^2 N_c}{(\mu^2+\mathbf{q}_\perp^2)^2}\Big[\frac{2m_q^2}{\bar{z}z}|I_{gu1}|^2 + \frac{\bar{z}\mathbf{q}_\perp^2}{2z}|4C_F I^{q\bar{q}}_{qu2} + I_{gu2}|^2 + \frac{z\mathbf{q}_\perp^2}{2\bar{z}}|4C_F I^{q\bar{q}}_{qu3} - I_{gu2}|^2 \\
&+ \cos(2\phi)\mathbf{q}_\perp^2 \mathrm{Re}\big((4C_F I^{q\bar{q}}_{qu2} + I_{gu2})(4C_F I^{q\bar{q}}_{qu3} - I_{gu2})^*\big)\Big], \\
|\mathcal{A}^{q\bar{q},y}_T|^2 =& |\mathcal{A}^{q\bar{q},x}_T|^2\big(\cos(2\phi) \to -\cos(2\phi)\big), \\
\mathrm{Re}(\mathcal{A}^{q\bar{q},x}_T \mathcal{A}^{q\bar{q}*}_L) =& -\frac{2|C|^2 N_c \cos(\phi) Q|q_\perp|}{(\mu^2+\mathbf{q}_\perp^2)^2}\Big[\bar{z}\mathrm{Re}\big((4C_F I^{q\bar{q}}_{qu2} + I_{gu2})(2C_F I^{q\bar{q}}_{qu1} + I_{gu1})^*\big) \\
&+ z\mathrm{Re}\big((4C_F I^{q\bar{q}}_{qu3} - I_{gu2})(2C_F I^{q\bar{q}}_{qu1} + I_{gu1})^*\big)\Big], \\
|\mathcal{A}^{q\bar{q}}_L|^2 =& \frac{8|C|^2 N_c Q^2 \bar{z}z}{(\mu^2+\mathbf{q}_\perp^2)^2}|2C_F I^{q\bar{q}}_{qu1} + I_{gu1}|^2.
\end{aligned}$$

$$\left|M^{q\bar{q},u}_{QCD}\right|^2$$

$$\begin{aligned}
|\mathcal{A}^{q\bar{q},x}_T|^2 =& \frac{8|C|^2 N_c \mathbf{q}_\perp^2}{(\mu^2+\mathbf{q}_\perp^2)^2}\Big[\frac{C_F^2\bar{z}}{z}|I^{q\bar{q}}_{qh2}|^2 + \frac{C_F^2 z}{\bar{z}}|I^{q\bar{q}}_{qh3}|^2 + \frac{(2\bar{z}z(\cos(2\phi)-1)+1)}{4\bar{z}z}|I_{gh}|^2 + 2C_F^2\cos(2\phi)\mathrm{Re}(I^{q\bar{q}}_{qh2} I^{q\bar{q}*}_{qh3}) \\
&- \frac{C_F\big(z(1-\cos(2\phi))-1\big)}{z}\mathrm{Re}(I^{q\bar{q}}_{qh2} I^*_{gh}) - \frac{C_F\big(\cos(2\phi)(z-1)-z\big)}{\bar{z}}\mathrm{Re}(I^{q\bar{q}}_{qh3} I^*_{gh})\Big], \\
|\mathcal{A}^{q\bar{q},y}_T|^2 =& |\mathcal{A}^{q\bar{q},x}_T|^2\big(\cos(2\phi) \to -\cos(2\phi)\big), \\
\mathrm{Re}(\mathcal{A}^{q\bar{q},x}_T \mathcal{A}^{q\bar{q}*}_L) =& \frac{16|C|^2 N_c}{(\mu^2+\mathbf{q}_\perp^2)^2}\Big[-C_F^2 Q|q_\perp|\bar{z}\cos(\phi)\mathrm{Re}(I^{q\bar{q}}_{qh1} I^{q\bar{q}*}_{qh2}) - C_F^2 Q|q_\perp|z\cos(\phi)\mathrm{Re}(I^{q\bar{q}}_{qh1} I^{q\bar{q}*}_{qh3}) \\
&- \frac{1}{2}C_F Q|q_\perp|\cos(\phi)\mathrm{Re}(I^{q\bar{q}}_{qh1} I^*_{gh})\Big], \\
|\mathcal{A}^{q\bar{q}}_L|^2 =& \frac{32|C|^2 N_c \bar{z}z}{(\mu^2+\mathbf{q}_\perp^2)^2}C_F^2 Q^2 |I^{q\bar{q}}_{qh1}|^2.
\end{aligned}$$

$$\left|M^{q\bar{q},h}_{QCD}\right|^2$$

$$\begin{aligned}
|\mathcal{A}^x_T|^2 =& -\frac{8|C|^2 N_c C_F^2}{q_\perp^2 \bar{z}z} I^{q\bar{q}}_{qt,\rho} I^{q\bar{q},\rho*}_{qt} - \frac{|C|^2 N_c}{(\mu^2+\mathbf{q}_\perp^2)^4}\mathrm{Re}\Big[I_{gt,\mu\nu} I^*_{gt,\rho\sigma}\Big(q_\perp^\nu q_\perp^\sigma g_\perp^{\mu\rho}\frac{2(\mu^2+\mathbf{q}_\perp^2)^2}{\bar{z}z} + 8e_x^\mu q_\perp^\nu e_x^\rho q_\perp^\sigma(\mu^2+\mathbf{q}_\perp^2)^2 \\
&+ 8q_\perp^\mu q_\perp^\nu q_\perp^\rho q_\perp^\sigma(4\mathbf{q}_\perp^2\cos^2(\phi)+Q^2) - 32e_x^\mu q_\perp^\nu q_\perp^\rho q_\perp^\sigma|q_\perp|\cos(\phi)(\mu^2+\mathbf{q}_\perp^2)\Big)\Big], \\
|\mathcal{A}^y_T|^2 =& |\mathcal{A}^x_T|^2\big(e_x \to e_y, \cos(\phi) \to \sin(\phi)\big), \\
\mathrm{Re}(\mathcal{A}^x_T \mathcal{A}^*_L) =& \frac{4|C|^2 N_c}{(\mu^2+\mathbf{q}_\perp^2)^4}\mathrm{Re}\Big[I_{gt,\mu\nu} I^*_{gt,\rho\sigma}\Big(2Q(1-2z)e_x^\mu q_\perp^\nu q_\perp^\rho q_\perp^\sigma(\mu^2+\mathbf{q}_\perp^2) - 4Q(1-2z)|q_\perp|\cos(\phi)q_\perp^\mu q_\perp^\nu q_\perp^\rho q_\perp^\sigma\Big)\Big], \\
|\mathcal{A}_L|^2 =& \frac{32|C|^2 N_c}{(\mu^2+\mathbf{q}_\perp^2)^4} I_{gt,\mu\nu} I^*_{gt,\rho\sigma} Q^2 \bar{z}z q_\perp^\mu q_\perp^\nu q_\perp^\rho q_\perp^\sigma.
\end{aligned}$$

$$\left|M^{q\bar{q},t}_{QCD}\right|^2$$

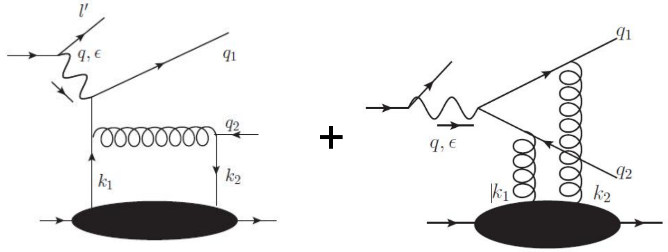
$$\begin{aligned}
|\mathcal{A}^x_T|^2 =& \frac{2|C|^2 N_c}{(\mu^2+\mathbf{q}_\perp^2)^3}\Big[\Big(\frac{\mu^2+\mathbf{q}_\perp^2(2z(\cos(2\phi)+1)-1)}{z}q_\perp^\mu q_\perp^\nu - 2(\mu^2+\mathbf{q}_\perp^2)\cos(\phi)|q_\perp|e_x^\mu q_\perp^\nu\Big)\mathrm{Re}\big((4C_F I^{q\bar{q}}_{qu2} + I_{gu2})I^*_{gt,\mu\nu}\big) \\
&- \Big(4C_F I^{q\bar{q}}_{qu2} + I_{gu2} \to 4C_F I^{q\bar{q}}_{qu3} - I_{gu2}, z \to 1-z\Big)\Big], \\
|\mathcal{A}^y_T|^2 =& |\mathcal{A}^x_T|^2\big(e_x \to e_y, \cos(2\phi) \to -\cos(2\phi), \cos(\phi) \to \sin(\phi)\big), \\
\mathrm{Re}(\mathcal{A}^x_T \mathcal{A}^*_L) =& \frac{4|C|^2 N_c Q}{(\mu^2+\mathbf{q}_\perp^2)^3}\Big[|q_\perp|\bar{z}\cos(\phi)q_\perp^\mu q_\perp^\nu \mathrm{Re}\big((4C_F I^{q\bar{q}}_{qu2} + I_{gu2})I^*_{gt,\mu\nu}\big) + |q_\perp|z\cos(\phi)q_\perp^\mu q_\perp^\nu \mathrm{Re}\big((4C_F I^{q\bar{q}}_{qu3} - I_{gu2})I^*_{gt,\mu\nu}\big) \\
&+ \Big(2|q_\perp|(1-2z)\cos(\phi)q_\perp^\mu q_\perp^\nu - (1-2z)(\mu^2+\mathbf{q}_\perp^2)e_x^\mu q_\perp^\nu\Big)\mathrm{Re}\big((2C_F I^{q\bar{q}}_{qu1} + I_{gu1})I^*_{gt,\mu\nu}\big)\Big], \\
|\mathcal{A}_L|^2 =& -\frac{32|C|^2 N_c Q^2 \bar{z}z q_\perp^\mu q_\perp^\nu}{(\mu^2+\mathbf{q}_\perp^2)^3}\mathrm{Re}\big((2C_F I^{q\bar{q}}_{qu1} + I_{gu1})I^*_{gt,\mu\nu}\big).
\end{aligned}$$

2Re$\left(M^{q\bar{q},u}_{QCD} M^{q\bar{q},t*}_{QCD}\right)$

Analytical results: quark dijet production

$$|M^{q\bar{q}}|^2 = \left|M^{q\bar{q}}_{QCD}\right|^2 + \left|M^{q\bar{q}}_{QED}\right|^2 + 2Re(M^{q\bar{q}}_{QCD}M^{q\bar{q}*}_{QED})$$

$\Bigg|$



$+\cdots\Bigg|^2$

Unpolarized beam:

$$\mathrm{Re}\left(M^{q\bar{q},a}_{\mathrm{QCD}}M^{q\bar{q},b*}_{\mathrm{QCD}}\right) = \frac{2\pi\alpha_{\mathrm{em}}}{Q^4}\left[\frac{8Q^2(1-y)}{y^2}\left|\mathcal{A}^{q\bar{q}}_L\right|^2+\frac{8Q^2\sqrt{1-y}(2-y)}{y^2}\mathrm{Re}\left(\mathcal{A}^{q\bar{q},x}_T\mathcal{A}^{q\bar{q}*}_L\right)+\frac{2Q^2(2-y)^2}{y^2}\left|\mathcal{A}^{q\bar{q},x}_T\right|^2+2Q^2\left|\mathcal{A}^{q\bar{q},y}_T\right|^2\right],$$

$$\begin{aligned}
\left|\mathcal{A}^{q\bar{q},x}_T\right|^2 =& \frac{2|C|^2N_c\mathbf{q}_\perp^2\sin(2\phi)}{(\mu^2+\mathbf{q}_\perp^2)^2}\Big[-\mathrm{Im}\big((4C_FI^{q\bar{q}}_{qu3}-I_{gu2})(2C_FI^{q\bar{q}*}_{qh2}+I^*_{gh})\big)+\mathrm{Im}\big((4C_FI^{q\bar{q}}_{qu2}+I_{gu2})(2C_FI^{q\bar{q}*}_{qh3}+I^*_{gh})\big)\Big],\\
\left|\mathcal{A}^{q\bar{q},y}_T\right|^2 =& -\left|\mathcal{A}^{q\bar{q},x}_T\right|^2,\\
\mathrm{Re}\left(\mathcal{A}^{q\bar{q},x}_T\mathcal{A}^{q\bar{q}*}_L\right) =& \frac{4|C|^2N_cQ|q_\perp|\sin(\phi)}{(\mu^2+\mathbf{q}_\perp^2)^2}\Big[\mathrm{Im}\Big(\big(2C_F(-\bar{z}I^{q\bar{q}}_{qh2}+zI^{q\bar{q}}_{qh3})+(2z-1)I_{gh}\big)(2C_FI^{q\bar{q}}_{qu1}+I_{gu1})^*\Big)\\
&-\bar{z}C_F\mathrm{Im}\Big((4C_FI^{q\bar{q}}_{qu2}+I_{gu2})I^{q\bar{q}*}_{qh1}\Big)+zC_F\mathrm{Im}\Big((4C_FI^{q\bar{q}}_{qu3}-I_{gu2})I^{q\bar{q}*}_{qh1}\Big)\Big],\\
\left|\mathcal{A}^{q\bar{q}}_L\right|^2 =& 0.
\end{aligned}$$

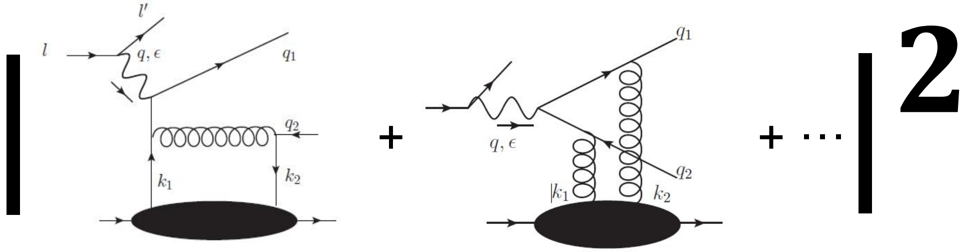
2Re$(M^{q\bar{q},u}_{QCD}M^{q\bar{q},h*}_{QCD})$

$$\begin{aligned}
\left|\mathcal{A}^{q\bar{q},x}_T\right|^2 =& \frac{4|C|^2N_c|q_\perp|}{(\mu^2+\mathbf{q}_\perp^2)^3}\Bigg\{\Big[\Big(-\frac{2\cos(\phi)}{z}(\mu^2+\mathbf{q}_\perp^2)q^\nu_\perp e^\mu_y+4|q_\perp|\sin(2\phi)q^\nu_\perp q^\mu_\perp-\frac{2(2z-1)\sin(\phi)}{z}(\mu^2+\mathbf{q}_\perp^2)q^\nu_\perp e^\mu_x\Big)\\
&\cdot\mathrm{Im}\Big(C_FI^{q\bar{q}}_{qh2}I^*_{gt,\mu\nu}\Big)+\Big(I^{q\bar{q}}_{qh2}\to I^{q\bar{q}}_{qh3}, z\to 1-z\Big)\Big]+\Big(\frac{(1-2z)^2\sin(\phi)}{z\bar{z}}(\mu^2+\mathbf{q}_\perp^2)q^\nu_\perp e^\mu_x\\
&+4|q_\perp|\sin(2\phi)q^\nu_\perp q^\mu_\perp-\frac{\cos(\phi)}{z\bar{z}}(\mu^2+\mathbf{q}_\perp^2)q^\nu_\perp e^\mu_y\Big)\mathrm{Im}\Big(I_{gh}I^*_{gt,\mu\nu}\Big)\Bigg\},\\
\left|\mathcal{A}^{q\bar{q},y}_T\right|^2 =& -\left|\mathcal{A}^{q\bar{q},x}_T\right|^2\left(\sin(\phi)\leftrightarrow\cos(\phi), e_x\leftrightarrow e_y\right),\\
\mathrm{Re}\Big(\mathcal{A}^{q\bar{q},x}_T\mathcal{A}^{q\bar{q}*}_L\Big) =& \frac{8|C|^2N_cQ}{(\mu^2+\mathbf{q}_\perp^2)^3}\Bigg\{2C_F|q_\perp|\sin(\phi)q^\nu_\perp q^\mu_\perp\mathrm{Im}\Big((\bar{z}I^{q\bar{q}}_{qh2}-zI^{q\bar{q}}_{qh3})I^*_{gt,\mu\nu}\Big)+C_F\Big[(\mu^2+\mathbf{q}_\perp^2)q^\nu_\perp e^\mu_y-2|q_\perp|\sin(\phi)q^\nu_\perp q^\mu_\perp\Big]\\
&\cdot\mathrm{Im}\Big(I^{q\bar{q}}_{qh1}I^*_{gt,\mu\nu}\Big)-|q_\perp|(2z-1)\sin(\phi)q^\nu_\perp q^\mu_\perp\mathrm{Im}\Big(I_{gh}I^*_{gt,\mu\nu}\Big)\Bigg\},\\
\left|\mathcal{A}^{q\bar{q}}_L\right|^2 =& 0.
\end{aligned}$$

2Re$(M^{q\bar{q},h}_{QCD}M^{q\bar{q},t*}_{QCD})$

Analytical results: quark dijet production

$$|M^{q\bar{q}}|^2 = \left|M^{q\bar{q}}_{QCD}\right|^2 + \left|M^{q\bar{q}}_{QED}\right|^2 + 2Re(M^{q\bar{q}}_{QCD} M^{q\bar{q}*}_{QED})$$



Beam asymmetry:

$$\delta\mathrm{Re}\big(M^{q\bar{q},a}_{\mathrm{QCD}} M^{q\bar{q},b*}_{\mathrm{QCD}}\big) = 16 \cdot \frac{2\pi\alpha_{\mathrm{em}}}{Q^4}\left[\frac{Q^2(2-y)}{2y}\mathrm{Im}\big(\mathcal{A}^{q\bar{q},x}_T \mathcal{A}^{q\bar{q},y*}_T\big) + \frac{Q^2\sqrt{1-y}}{y}\mathrm{Im}\big(\mathcal{A}^{q\bar{q}}_L \mathcal{A}^{q\bar{q},y*}_T\big)\right].$$

$$\mathrm{Im}\big(\mathcal{A}^{q\bar{q},x}_T \mathcal{A}^{q\bar{q},y*}_T\big) = 0,$$

$$\mathrm{Im}\big(\mathcal{A}^{q\bar{q}}_L \mathcal{A}^{q\bar{q},y*}_T\big) = -\frac{2|C|^2 N_c Q|q_\perp|\sin(\phi)}{(\mu^2+\mathbf{q}_\perp^2)^2}\Big[\bar{z}\mathrm{Im}\big((2C_F I^{q\bar{q}}_{qu1} + I_{gu1})(4C_F I^{q\bar{q}}_{qu2} + I_{gu2})^*\big) + z\mathrm{Im}\big((2C_F I^{q\bar{q}}_{qu1} + I_{gu1})(4C_F I^{q\bar{q}}_{qu3} - I_{gu2})^*\big)\Big].$$

$$\delta\left|M^{q\bar{q},u}_{QCD}\right|^2$$

$$\mathrm{Im}\big(\mathcal{A}^{q\bar{q},x}_T \mathcal{A}^{q\bar{q},y*}_T\big) = 0,$$

$$\mathrm{Im}\big(\mathcal{A}^{q\bar{q}}_L \mathcal{A}^{q\bar{q},y*}_T\big) = -\frac{8|C|^2 N_c C_F Q|q_\perp|\sin(\phi)}{(\mu^2+\mathbf{q}_\perp^2)^2}\mathrm{Im}\Big(I^{q\bar{q}}_{qh1}\big(2C_F\bar{z}I^{q\bar{q}}_{qh2} + 2C_F z I^{q\bar{q}}_{qh3} + I_{gh}\big)^*\Big).$$

$$\delta\left|M^{q\bar{q},h}_{QCD}\right|^2$$

$$\mathrm{Im}\big(\mathcal{A}^{q\bar{q},x}_T \mathcal{A}^{q\bar{q},y*}_T\big) = \frac{16|C|^2 N_c C_F^2}{\mathbf{q}_\perp^2 z\bar{z}}\mathrm{Im}\Big(I^{q\bar{q}}_{qt,1} I^{q\bar{q}*}_{qt,2}\Big) + \frac{4|C|^2 N_c \mathbf{q}_\perp^2(1-2z\bar{z})(\mu^4 - \mathbf{q}_\perp^4)}{z\bar{z}(\mu^2+\mathbf{q}_\perp^2)^4}\mathrm{Im}\Big(I_{gt,11} I^*_{gt,12}\Big),$$

$$\mathrm{Im}\big(\mathcal{A}^{q\bar{q}}_L \mathcal{A}^{q\bar{q},y*}_T\big) = \frac{8|C|^2 N_c Q\mathbf{q}_\perp^2|q_\perp|(1-2z)}{(\mu^2+\mathbf{q}_\perp^2)^4}\Big[\cos(\phi)\big(\mu^2 + 2\mathbf{q}_\perp^2\cos(2\phi)\big) - \cos(3\phi)\mathbf{q}_\perp^2\Big]\mathrm{Im}\Big(I_{gt,11} I^*_{gt,12}\Big).$$

$$\delta\left|M^{q\bar{q},t}_{QCD}\right|^2$$

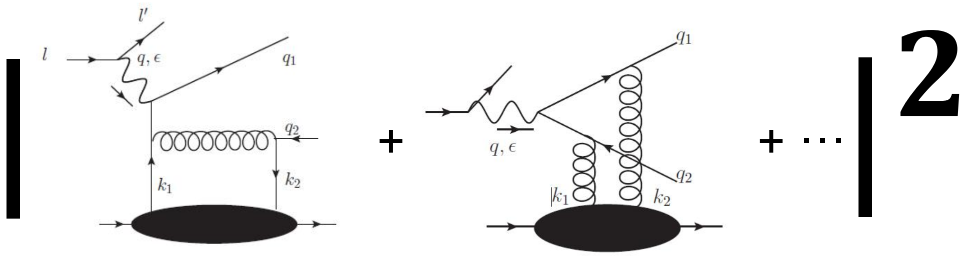
$$\begin{aligned}\mathrm{Im}\big(\mathcal{A}^{q\bar{q},x}_T \mathcal{A}^{q\bar{q},y*}_T\big) = &-\frac{|C|^2 N_c \mathbf{q}_\perp^2}{z\bar{z}(\mu^2+\mathbf{q}_\perp^2)^2}\Big[4C_F\mathrm{Re}\Big(\bar{z}^2(4C_F I^{q\bar{q}}_{qu2} + I_{gu2})I^{q\bar{q}*}_{qh2} - z^2(4C_F I^{q\bar{q}}_{qu3} - I_{gu2})I^{q\bar{q}*}_{qh3}\Big)\\ &- 2z\bar{z}\cos(2\phi)\mathrm{Re}\Big(I_{gh}\big(4C_F I^{q\bar{q}}_{qu3} - 4C_F I^{q\bar{q}}_{qu2} - 2I_{gu2}\big)^*\Big) + 2\mathrm{Re}\Big(I_{gh}\big(\bar{z}^2(4C_F I^{q\bar{q}}_{qu2} + I_{gu2})\\ &- z^2(4C_F I^{q\bar{q}}_{qu3} - I_{gu2})\big)^*\Big)\Big],\end{aligned}$$

$$\begin{aligned}\mathrm{Im}\big(\mathcal{A}^{q\bar{q}}_L \mathcal{A}^{q\bar{q},y*}_T\big) = &\frac{-4|C|^2 N_c Q|q_\perp|\cos(\phi)}{(\mu^2+\mathbf{q}_\perp^2)^2}\Big[\mathrm{Re}\Big((2C_F I^{q\bar{q}}_{qu1} + I_{gu1})\big(2C_F z I^{q\bar{q}}_{qh3} - 2C_F\bar{z}I^{q\bar{q}}_{qh2} - (1-2z)I_{gh}\big)^*\Big)\\ &+ \mathrm{Re}\Big(C_F I^{q\bar{q}}_{qh1}\big(z(4C_F I^{q\bar{q}}_{qu3} - I_{gu2}) - \bar{z}(4C_F I^{q\bar{q}}_{qu2} + I_{gu2})\big)^*\Big)\Big]\end{aligned}$$

$$2\delta\mathrm{Re}(M^{q\bar{q},u}_{QCD} M^{q\bar{q},h*}_{QCD})$$

Analytical results: quark dijet production

$$|M^{q\bar{q}}|^2 = \left|M^{q\bar{q}}_{QCD}\right|^2 + \left|M^{q\bar{q}}_{QED}\right|^2 + 2Re(M^{q\bar{q}}_{QCD} M^{q\bar{q}*}_{QED})$$



Beam asymmetry:

$$\delta\mathrm{Re}\left(M^{q\bar{q},a}_{\mathrm{QCD}} M^{q\bar{q},b*}_{\mathrm{QCD}}\right) = 16 \cdot \frac{2\pi\alpha_{\mathrm{em}}}{Q^4}\left[\frac{Q^2(2-y)}{2y}\mathrm{Im}\left(\mathcal{A}_T^{q\bar{q},x}\mathcal{A}_T^{q\bar{q},y*}\right) + \frac{Q^2\sqrt{1-y}}{y}\mathrm{Im}\left(\mathcal{A}_L^{q\bar{q}}\mathcal{A}_T^{q\bar{q},y*}\right)\right].$$

$$\mathrm{Im}\left(\mathcal{A}_T^{q\bar{q},x}\mathcal{A}_T^{q\bar{q},y*}\right) = -\frac{2|C|^2 N_c \mathbf{q}_\perp^2}{z\bar{z}(\mu^2+\mathbf{q}_\perp^2)^2}\mathrm{Im}\Big(\big(\bar{z}^2(4C_F I^{q\bar{q}}_{qu2} + I_{gu2}) - z^2(4C_F I^{q\bar{q}}_{qu3} - I_{gu2})\big)\big(\sin(2\phi) I^*_{gt,11} - \cos(2\phi) I^*_{gt,12}\big)\Big),$$

$$\begin{aligned}\mathrm{Im}\left(\mathcal{A}_L^{q\bar{q}}\mathcal{A}_T^{q\bar{q},y*}\right) = \frac{2|C|^2 N_c Q|q_\perp|}{(\mu^2+\mathbf{q}_\perp^2)^3}\Big[&2(1-2z)\sin(\phi)\big(\mu^2 + 2\mathbf{q}_\perp^2\cos(2\phi) + \mathbf{q}_\perp^2\big)\mathrm{Im}\Big(\big(2C_F I^{q\bar{q}}_{qu1} + I_{gu1}\big) I^*_{gt,11}\Big) \\ &+ \mathbf{q}_\perp^2(\sin(\phi) - \sin(3\phi))\mathrm{Im}\Big(\big(\bar{z}(4C_F I^{q\bar{q}}_{qu2} + I_{gu2}) + z(4C_F I^{q\bar{q}}_{qu3} - I_{gu2})\big) I^*_{gt,11}\Big) \\ &+ 2(2z-1)\big(\cos(\phi)\mu^2 + \cos(3\phi)\mathbf{q}_\perp^2\big)\mathrm{Im}\Big(\big(2C_F I^{q\bar{q}}_{qu1} + I_{gu1}\big) I^*_{gt,12}\Big) \\ &- \mathbf{q}_\perp^2(\cos(\phi) - \cos(3\phi))\mathrm{Im}\Big(\big(\bar{z}(4C_F I^{q\bar{q}}_{qu2} + I_{gu2}) + z(4C_F I^{q\bar{q}}_{qu3} - I_{gu2})\big) I^*_{gt,12}\Big)\Big].\end{aligned}$$

$$2\delta\mathrm{Re}(M^{q\bar{q},u}_{QCD} M^{q\bar{q},t*}_{QCD})$$

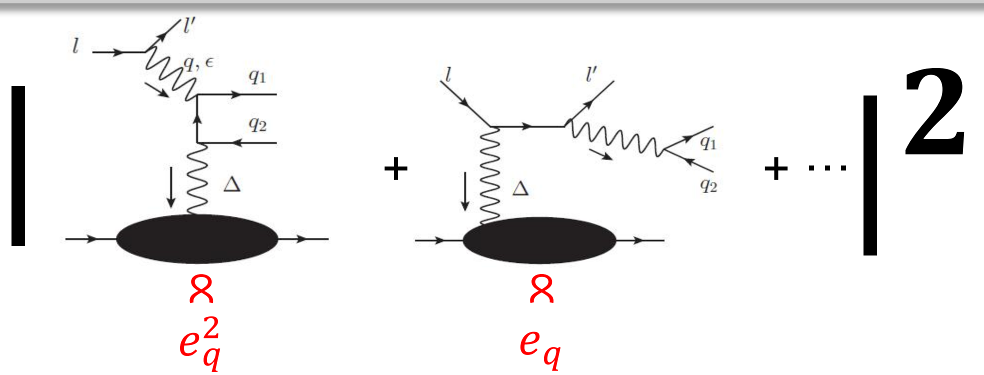
$$\begin{aligned}\mathrm{Im}\left(\mathcal{A}_T^{q\bar{q},x}\mathcal{A}_T^{q\bar{q},y*}\right) = \frac{4|C|^2 N_c \mathbf{q}_\perp^2(\mathbf{q}_\perp^2 - \mu^2)}{z\bar{z}(\mu^2+\mathbf{q}_\perp^2)^3}&\mathrm{Re}\Big(\big((1-2z\bar{z})I_{gh} + 2C_F\bar{z}^2 I^{q\bar{q}}_{qh2} + 2C_F z^2 I^{q\bar{q}}_{qh3}\big) \\ &\cdot \big(\sin(2\phi) I_{gt,12} + \cos(2\phi) I_{gt,11}\big)^*\Big),\end{aligned}$$

$$\begin{aligned}\mathrm{Im}\left(\mathcal{A}_L^{q\bar{q}}\mathcal{A}_T^{q\bar{q},y*}\right) = \frac{4|C|^2 N_c Q|q_\perp|}{(\mu^2+\mathbf{q}_\perp^2)^3}\Big[&\mathbf{q}_\perp^2(\cos(\phi) + \cos(3\phi))\mathrm{Re}\Big(\big(-2C_F\bar{z} I^{q\bar{q}}_{qh2} + 2C_F z I^{q\bar{q}}_{qh3} + (2z-1)I_{gh}\big) I^*_{gt,11}\Big) \\ &+ \mathbf{q}_\perp^2(\sin(\phi) + \sin(3\phi))\mathrm{Re}\Big(\big(-2C_F\bar{z} I^{q\bar{q}}_{qh2} + 2C_F z I^{q\bar{q}}_{qh3} + (2z-1)I_{gh}\big) I^*_{gt,12}\Big) \\ &- 2C_F\mathrm{Re}\Big(I^{q\bar{q}}_{qh1}\big(I_{gt,11}(\cos(3\phi)\mathbf{q}_\perp^2 - \cos(\phi)\mu^2) + I_{gt,12}(\sin(3\phi)\mathbf{q}_\perp^2 - \sin(\phi)\mu^2)\big)^*\Big)\Big].\end{aligned}$$

$$2\delta\mathrm{Re}(M^{q\bar{q},h}_{QCD} M^{q\bar{q},t*}_{QCD})$$

Analytical results: quark dijet production

$$|M^{q\bar{q}}|^2 = \left|M^{q\bar{q}}_{QCD}\right|^2 + \left|\boldsymbol{M^{q\bar{q}}_{QED}}\right|^2 + 2Re(M^{q\bar{q}}_{QCD} M^{q\bar{q}*}_{QED})$$



$$|M^{q\bar{q}}_{\rm QED}|^2 = \frac{1}{2}\cdot\frac{256\pi^4\alpha^4_{\rm em}e_q^2 N_c}{t^2(\mu^2+\mathbf{q}_\perp^2)^4}\Big[(\Sigma_f e_f \tilde{J}^x_f)(\Sigma_{f'} e_{f'} \tilde{J}^{x*}_{f'})\cdot\Sigma^4_{i=0}a^i_{xx}\cos(i\phi) + (\Sigma_f e_f \tilde{J}^y_f)(\Sigma_{f'} e_{f'} \tilde{J}^{y*}_{f'})\cdot\Sigma^4_{i=0}a^i_{yy}\cos(i\phi)$$
$$+\mathrm{Re}\Big[(\Sigma_f e_f \tilde{J}^x_f)(\Sigma_{f'} e_{f'} \tilde{J}^{y*}_{f'})\Big]\cdot\Sigma^4_{i=1}a^i_{xy}\sin(i\phi)\Big],$$

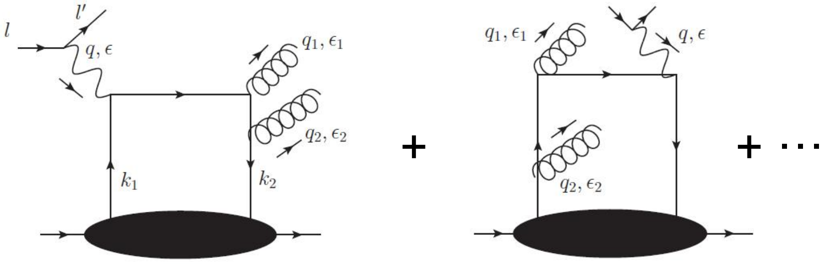
$$\tilde{J}^\alpha_f = (g^{\alpha\beta} - \frac{\Delta^\alpha n^\beta_-}{\Delta\cdot n_-})\langle p'|\overline{\psi}_f(0)\gamma_\beta\psi_f(0)|p\rangle$$

$$a^2_{xx} = 16\mathbf{q}_\perp^2\Bigg\{\frac{e_q^2(\mu^2+\mathbf{q}_\perp^2)^2\left[2m_q^2(y-2)^2+Q^2\left(2((y-8)y+8)z\bar{z}-(y-1)^2-1\right)\right]}{Q^2y^2} - \frac{2z^2\bar{z}^2\left((m_q^2+\mathbf{q}_\perp^2)^2-Q^4z^2\bar{z}^2\right)^2}{(m_q^2+\mathbf{q}_\perp^2)^2}$$
$$+\frac{2e_q(y-2)z(2z-1)\bar{z}\left(\mu^2+\mathbf{q}_\perp^2\right)^2\left(3m_q^2-Q^2z\bar{z}+3\mathbf{q}_\perp^2\right)}{y\left(m_q^2+\mathbf{q}_\perp^2\right)}\Bigg\},$$

Compared to the QED channel in DDVCS, the QED channel in exclusive dijet production has more complicated charge structures, which results in nontrivial cancellation or enhancement between diagrams depending on the outgoing jet type.

Analytical results: gluon dijet production

Unpolarized beam:

$$\Bigg|$$  $$+ \cdots \Bigg|^2$$

$$\mathrm{Re}\big(M^{gg,a}M^{gg,b*}\big) = \frac{2\pi\alpha_{\rm em}}{Q^4}\left[\frac{8Q^2(1-y)}{y^2}\big|\mathcal{A}_L^{gg}\big|^2 + \frac{8Q^2\sqrt{1-y}(2-y)}{y^2}\mathrm{Re}\big(\mathcal{A}_T^{gg,x}\mathcal{A}_L^{gg*}\big) + \frac{2Q^2(2-y)^2}{y^2}\big|\mathcal{A}_T^{gg,x}\big|^2 + 2Q^2\big|\mathcal{A}_T^{gg,y}\big|^2\right].$$

$$\begin{aligned}
\big|\mathcal{A}_T^{gg,x}\big|^2 =& \frac{8C_F^2}{\mathbf{q}_\perp^2}\Big[-2\cos(2\phi)z\bar{z} - 2\bar{z}z + 1\Big](\Sigma_f C_f I^{gg}_{qu2,f})(\Sigma_{f'}C^*_{f'}I^{gg*}_{qu2,f'}),\\
\big|\mathcal{A}_T^{gg,y}\big|^2 =& \big|\mathcal{A}_T^{gg,x}\big|^2\big(\cos(2\phi)\to -\cos(2\phi)\big),\\
\mathrm{Re}\big(\mathcal{A}_T^{gg,x}\mathcal{A}_L^{gg*}\big) =& \frac{32C_F^2 Q z\bar{z}(1-2z)\cos(\phi)}{|q_\perp|(\mu^2+\mathbf{q}_\perp^2)}\mathrm{Re}\Big[(\Sigma_f C_f I^{gg}_{qu1,f})(\Sigma_{f'}C^*_{f'}I^{gg*}_{qu2,f'})\Big],\\
\big|\mathcal{A}_L^{gg}\big|^2 =& \frac{128C_F^2Q^2\bar{z}^2z^2}{(\mu^2+\mathbf{q}_\perp^2)^2}(\Sigma_f C_f I^{gg}_{qu1,f})(\Sigma_{f'}C^*_{f'}I^{gg*}_{qu1,f'}).
\end{aligned}$$

$$\boldsymbol{|M^{gg,u}|^2}$$

$$\begin{aligned}
\big|\mathcal{A}_T^{gg,x}\big|^2 =& \frac{8C_F^2}{\mathbf{q}_\perp^2}\Big[2\cos(2\phi)z\bar{z} - 2z\bar{z} + 1\Big](\Sigma_f C_f I'^{gg}_{qh1,f})(\Sigma_{f'}C^*_{f'}I'^{gg*}_{qh1,f'}),\\
\big|\mathcal{A}_T^{gg,y}\big|^2 =& \big|\mathcal{A}_T^{gg,x}\big|^2\big(\cos(2\phi)\to -\cos(2\phi)\big),\\
\mathrm{Re}\big(\mathcal{A}_T^{gg,x}\mathcal{A}_L^{gg*}\big) =& 0,\\
\big|\mathcal{A}_L^{gg}\big|^2 =& 0.
\end{aligned}$$

$$C_f = 2i\pi\alpha_s\sqrt{4\pi\alpha_{em}}e_f$$

$$\boldsymbol{|M^{gg,h}|^2}$$

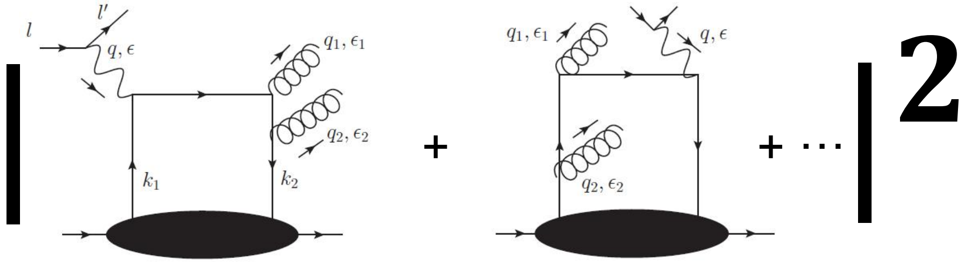
$$\begin{aligned}
\big|\mathcal{A}_T^{gg,x}\big|^2 =& \frac{32C_F^2 z\bar{z}\sin(2\phi)}{\mathbf{q}_\perp^2}\mathrm{Im}\Big((\Sigma_f C_f I^{gg}_{qu2,f})(\Sigma_{f'}C_{f'}I^{gg*}_{qh1,f'})\Big)\\
\big|\mathcal{A}_T^{gg,y}\big|^2 =& -\big|\mathcal{A}_T^{gg,x}\big|^2,
\end{aligned}$$

$$\begin{aligned}
\mathrm{Re}\big(\mathcal{A}_T^{gg,x}\mathcal{A}_L^{gg*}\big) =& \frac{32C_F^2Qz\bar{z}(2z-1)\sin(\phi)}{|q_\perp|(\mu^2+\mathbf{q}_\perp^2)}\mathrm{Im}\Big((\Sigma_f C_f I^{gg}_{qu1,f})(\Sigma_{f'}C_{f'}I^{gg*}_{qh1,f'})\Big),\\
\big|\mathcal{A}_L^{gg}\big|^2 =& 0.
\end{aligned}$$

Re$(M^{gg,u}M^{gg,h*})$

Analytical results: gluon dijet production

Beam asymmetry:



$$\delta\mathrm{Re}\big(M^{gg,a} M^{gg,b*}\big) = 16 \cdot \frac{2\pi\alpha_{\mathrm{em}}}{Q^4}\Big[\frac{Q^2(2-y)}{2y}\mathrm{Im}\big(\mathcal{A}_T^{q\bar{q},x}\mathcal{A}_T^{q\bar{q},y*}\big) + \frac{Q^2\sqrt{1-y}}{y}\mathrm{Im}\big(\mathcal{A}_L^{q\bar{q}}\mathcal{A}_T^{q\bar{q},y*}\big)\Big].$$

$$\mathrm{Im}\big(\mathcal{A}_T^{gg,x}\mathcal{A}_T^{gg,y*}\big) = 0,$$
$$\mathrm{Im}\big(\mathcal{A}_L^{gg}\mathcal{A}_T^{gg,y*}\big) = \frac{32C_F^2 Qz\bar{z}(1-2z)\sin(\phi)}{|q_\perp|(\mu^2+\mathbf{q}_\perp^2)}\mathrm{Im}\Big((\Sigma_f C_f I^{gg}_{qu1,f})(\Sigma_f C'_f I^{gg}_{qu2,f'})\Big).$$

$\delta|M^{gg,u}|^2$

$$\mathrm{Im}\big(\mathcal{A}_T^{gg,x}\mathcal{A}_T^{gg,y*}\big) = 0,$$
$$\mathrm{Im}\big(\mathcal{A}_L^{gg}\mathcal{A}_T^{gg,y*}\big) = 0.$$

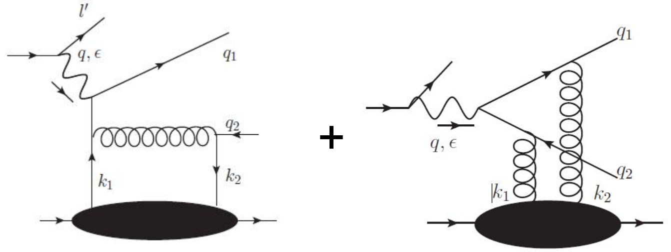
$\delta|M^{gg,h}|^2$

$$\mathrm{Im}\big(\mathcal{A}_T^{gg,x}\mathcal{A}_T^{gg,y*}\big) = -\frac{16C_F^2(1-2z\bar{z})}{\mathbf{q}_\perp^2}\mathrm{Re}\Big((\Sigma_f C_f I^{gg}_{qu2,f})(\Sigma_{f'} C_{f'} I^{gg}_{qh1,f'})\Big),$$
$$\mathrm{Im}\big(\mathcal{A}_L^{gg}\mathcal{A}_T^{gg,y*}\big) = \frac{32C_F^2 Qz\bar{z}(2z-1)\cos(\phi)}{|q_\perp|(\mu^2+\mathbf{q}_\perp^2)}\mathrm{Re}\Big((\Sigma_f C_f I^{gg}_{qu1,f})(\Sigma_{f'} C_{f'} I^{gg}_{qh1,f'})\Big).$$

$2\delta\mathrm{Re}(M^{gg,u}M^{gg,h*})$

Analytical results: quark dijet production

$$|M^{q\bar{q}}|^2 = \left|\boldsymbol{M^{q\bar{q}}_{QCD}}\right|^2 + \left|M^{q\bar{q}}_{QED}\right|^2 + 2Re(M^{q\bar{q}}_{QCD} M^{q\bar{q}*}_{QED})$$



Structure of CFFs:

$$I^{q\bar{q}}_{qu2/qh2} = \int_{-1}^{1} dx F_{qu/h}(x,\xi,t) \left(\frac{z}{x-\xi+i\varepsilon} - \frac{\beta\bar{z}}{\bar{\beta}(x+\xi-i\varepsilon)} + \frac{\bar{z}}{\bar{\beta}(x-\xi(1-2\beta)-i\varepsilon)} \right),$$

$$\beta = \frac{Q^2 z\bar{z} + m_q^2}{q_\perp^2 + m_q^2 + Q^2 z\bar{z}}$$

Some CFFs are sensitive to GPDs at $x \neq \xi$.

$$I_{gu1} = \int_{-1}^{1} dx F_{gu}(x,\xi,t) \left(\frac{\bar{\beta}}{(x+\xi-i\varepsilon)^2} + \frac{\bar{\beta}}{(x-\xi+i\varepsilon)^2} - \frac{1-2\beta}{(x+\xi-i\varepsilon)(x-\xi+i\varepsilon)} \right), \quad \frac{F_{gu/h}(x,\xi,t)}{(x+\xi-i\varepsilon)^2} \to \frac{1}{(x+\xi-i\varepsilon)} \frac{\partial}{\partial x} F_{gu/h}(x,\xi,t),$$

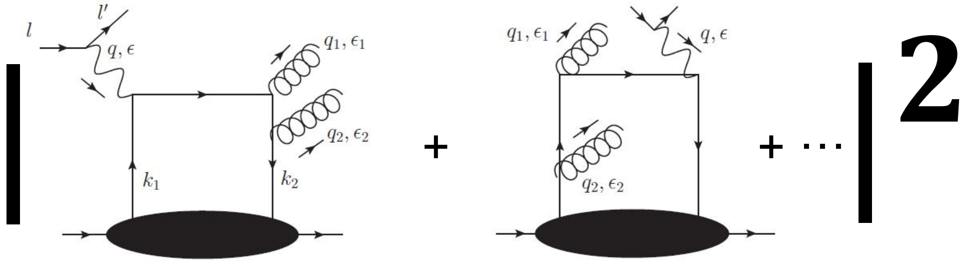
We have analysed the LO evolution of $\frac{\partial}{\partial x} F_g(x,\xi,t)$, and found it won't generate discontinuities at $x = \pm\,\xi$ (backup).

$$F_{qu}(x,\xi,t) = \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m_p} u(p) \right].$$

Radyushkin, 1999
Müller, Schäfer, 2005

Analytical results: gluon dijet production

Structure of CFFs:



CFFs that appear in the cross section:

Contribute

$$I^{gg}_{qu1,f} = \int_{-1}^{1} dx F_{qu,f}(x,\xi,t)\left(\frac{1}{x+\xi-i\varepsilon} - \frac{1}{x-\xi+i\varepsilon}\right)$$

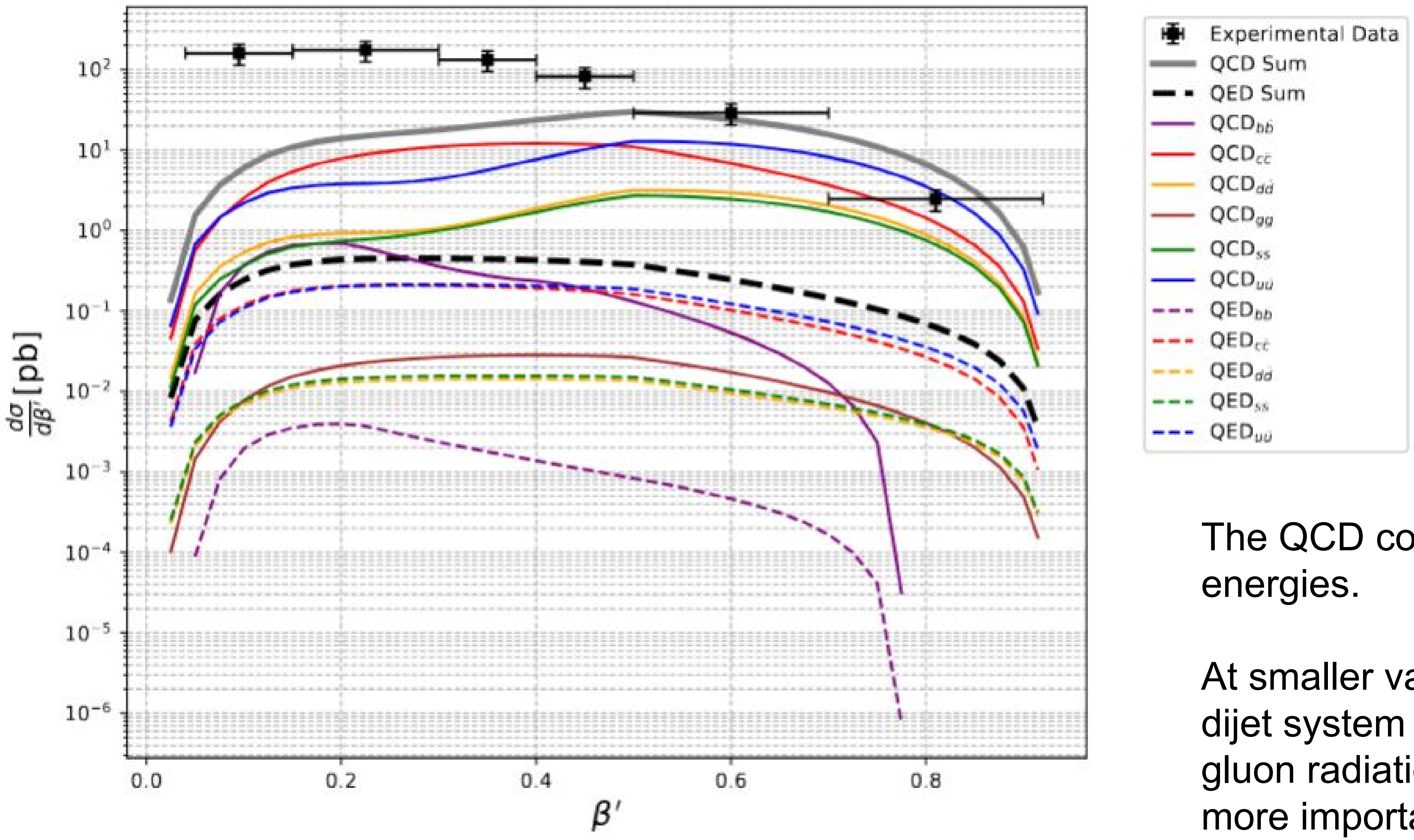
$$I^{gg}_{qu2,f} = \int_{-1}^{1} dx F_{qu,f}(x,\xi,t)\left((1-2\beta)\left(\frac{1}{x+\xi-i\varepsilon} - \frac{1}{x-\xi+i\varepsilon}\right) + {\color{red}\frac{1}{x+\xi(1-2\beta)+i\varepsilon} - \frac{1}{x-\xi(1-2\beta)-i\varepsilon}}\right)$$

$$I^{gg}_{qh1,f} = \int_{-1}^{1} dx F_{qh,f}(x,\xi,t)\left(\frac{1}{x+\xi-i\varepsilon} + \frac{1}{x-\xi+i\varepsilon} - {\color{red}\frac{1}{x+\xi(1-2\beta)+i\varepsilon} - \frac{1}{x-\xi(1-2\beta)-i\varepsilon}}\right)$$

Only C-odd combination of quark GPDs survives under convolution.

The CFFs are sensitive to quark GPDs at $x \neq \xi$, due to two independent hard scales.

Numerical results with HERA kinematics



$$\beta' = \frac{Q^2}{2q\cdot(p-p')} = \frac{Q^2}{Q^2+M^2-t}$$

Integration range:

$y \in [0.1, 0.64]$,
$t \in [-1.5, -0.01] GeV^2$,
$z \in [0,1], q_\perp^2 \in [4,50] GeV^2$,
$\phi \in [0, 2\pi], \phi_{p'} \in [0, 2\pi]$,
$Q^2 \in [25, 300] GeV^2$,
$\eta < 2$,
$W^2 \in [90, 250] GeV^2$.

The QCD contribution dominates at HERA energies.

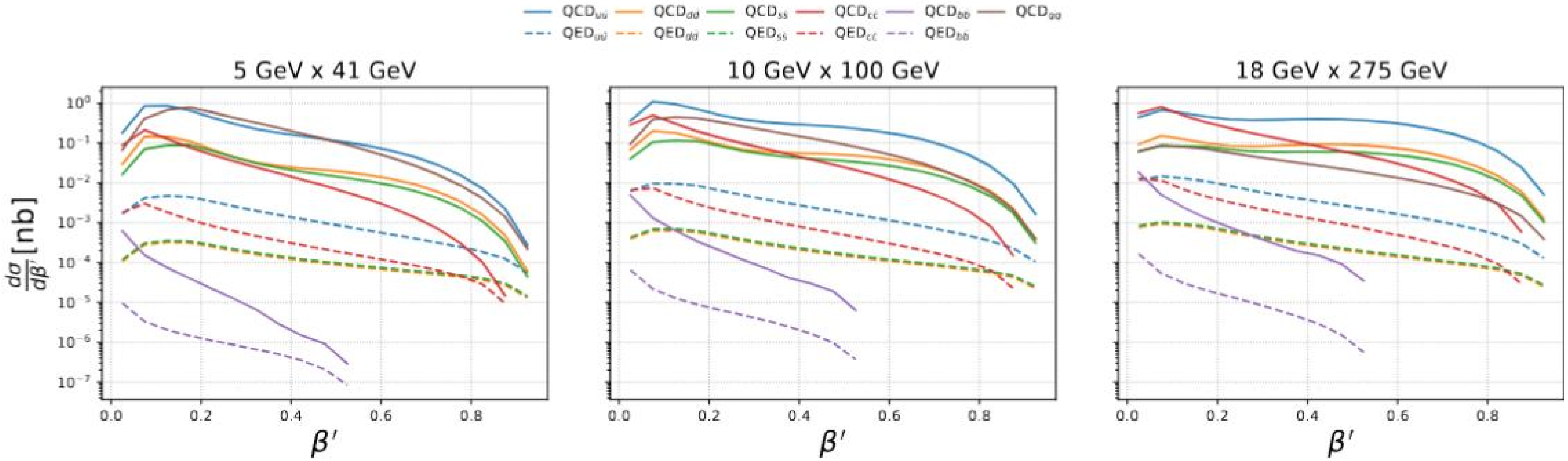
At smaller value of β', the invariant mass of dijet system gets larger, the contribution from gluon radiation ($q\bar{q}g$ configuration) becomes more important.

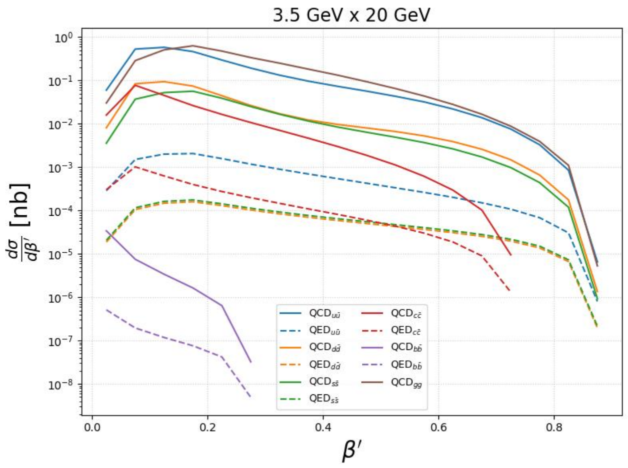
Bartels, Jung, Wusthoff, 1999
Boussarie et al., 2019

Chall et al., 2026; Goloskokov, Kroll, 2007; Abramowicz et al., 2016 (ZEUS)

Numerical results with EIC and EicC kinematics

$$\beta' = \frac{Q^2}{2q\cdot(p-p')} = \frac{Q^2}{Q^2+M^2-t}$$





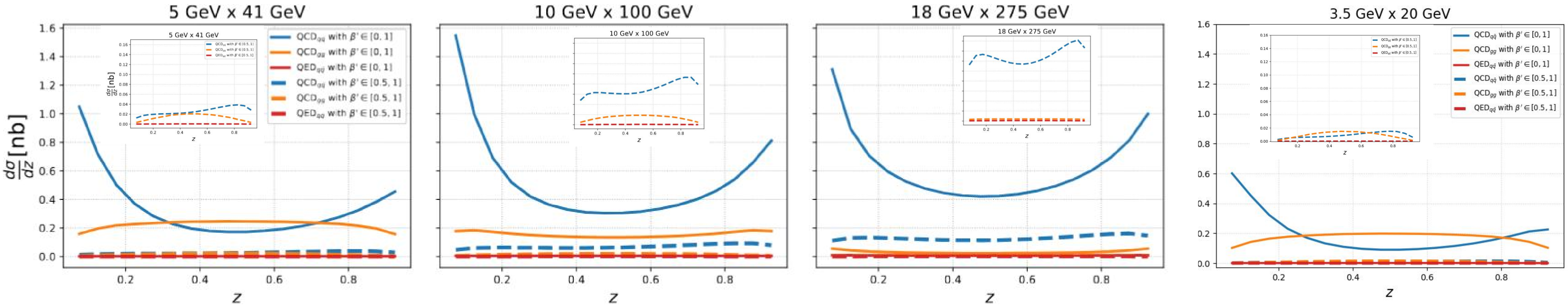
Integration range (for EIC):

$$Q^2 \in [1,100]GeV^2, t \in [-1.5,-0.01]GeV^2, z \in [0.05,0.95], q_\perp^2 \in [1,50]GeV^2, \phi \in [0,2\pi], \phi_{p'} \in [0,2\pi], y \in [0.01,0.95].$$

Integration range (for EicC):

$$Q^2 \in [1,30]GeV^2, t \in [-1.0,-0.01]GeV^2, z \in [0.05,0.95], q_\perp^2 \in [1,50]GeV^2, \phi \in [0,2\pi], \phi_{p'} \in [0,2\pi], y \in [0.01,0.95].$$

Numerical results with EIC and EicC kinematics

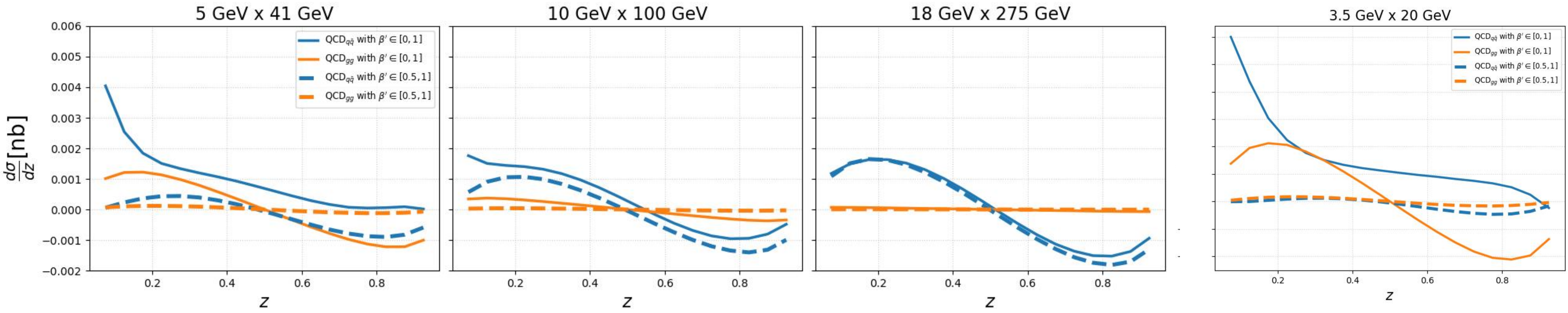


Integration range (for EIC):
$Q^2 \in [1,100]GeV^2, t \in [-1.5,-0.01]GeV^2, q_\perp^2 \in [1,50]GeV^2, \phi \in [0,2\pi], \phi_{p'} \in [0,2\pi], y \in [0.01,0.95].$

Integration range (for EicC):
$Q^2 \in [1,30]GeV^2, t \in [-1.0,-0.01]GeV^2, q_\perp^2 \in [1,50]GeV^2, \phi \in [0,2\pi], \phi_{p'} \in [0,2\pi], y \in [0.01,0.95].$

The deviation from symmetry with respect to $z = 0.5$ indicates the **contributions from valence quark GPDs become significant** at the endpoint region of z.

Numerical results with EIC and EicC kinematics

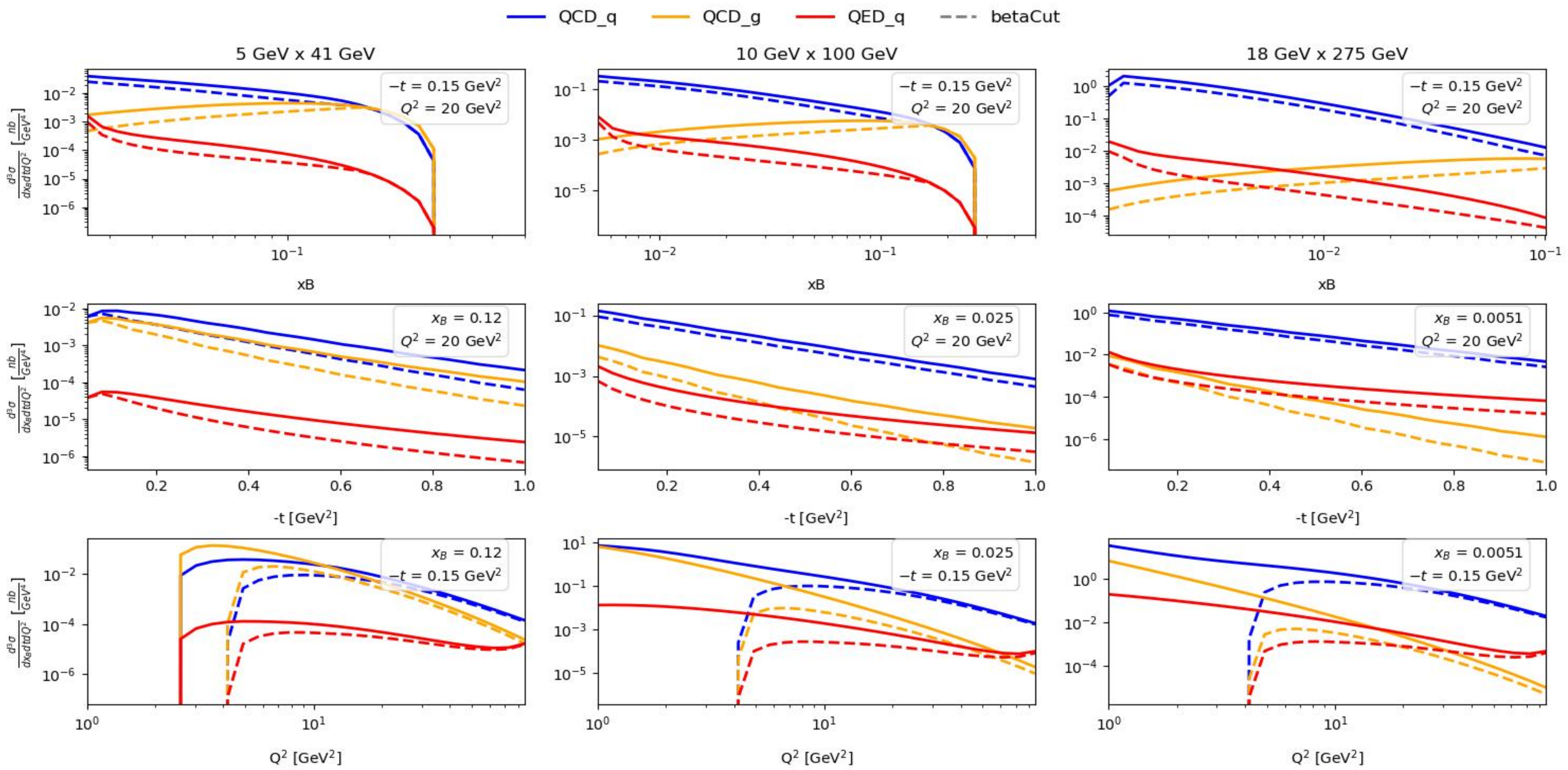


Integration range (for EIC):
$Q^2 \in [1,100]GeV^2, t \in [-1.5, -0.01]GeV^2, q_\perp^2 \in [1,50]GeV^2, \phi = \pi/2, \phi_{p'} \in [0,2\pi], y \in [0.01,0.95]$.

Integration range (for EicC):
$Q^2 \in [1,30]GeV^2, t \in [-1.0, -0.01]GeV^2, q_\perp^2 \in [1,50]GeV^2, \phi = \pi/2, \phi_{p'} \in [0,2\pi], y \in [0.01,0.95]$.

The deviation from asymmetry with respect to $z = 0.5$ indicates the **contributions from valence quark GPDs become significant**.

Numerical results with EIC kinematics



Integration range:

$$z \in [0.05, 0.95],$$
$$q_\perp^2 \in [1, 50] GeV^2,$$
$$\phi \in [0, 2\pi],$$
$$\phi_{p'} \in [0, 2\pi],$$

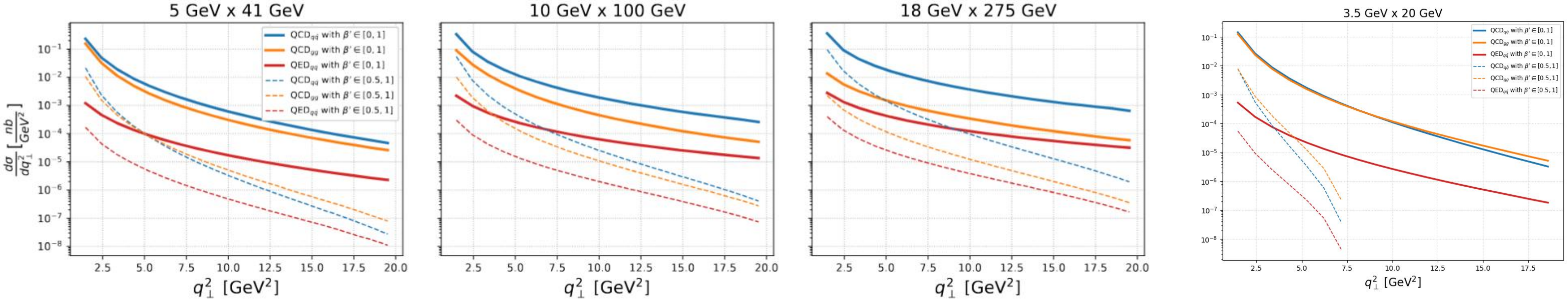
$$\boldsymbol{Q^2 z\bar{z} + q_\perp^2 + m_q^2 > 1 GeV^2}.$$

The effect of β' cutoff ($\beta' > 0.5$) is less important for larger Q^2 ($\sim 20 GeV^2$), as expected from $\beta' = \frac{Q^2}{Q^2 + M^2 - t}$.

The gluon dijet production becomes more significant at larger x_B, due to it is only sensitive to valence quark GPDs.

Numerical results with EIC and EicC kinematics

$$\beta = \frac{Q^2 z\bar{z} + m_q^2}{Q^2 z\bar{z} + q_\perp^2 + m_q^2}$$



$Q^2 \in [1,100]GeV^2, t \in [-1.5, -0.01]GeV^2, \phi \in [0,2\pi], \phi_{p'} \in [0,2\pi], y \in [0.01,0.95], z \in [0.05,0.95]$. **For EIC**

$Q^2 \in [1,30]GeV^2, t \in [-1.0, -0.01]GeV^2, \phi \in [0,2\pi], \phi_{p'} \in [0,2\pi], y \in [0.01,0.95], z \in [0.05,0.95]$. **For EicC**

For small values of $q_\perp^2$, jet reconstruction becomes difficult (pileup of particles from different jets).

As $q_\perp^2$ gets larger, the contribution from QED channel becomes more significant, one of the reasons is $\frac{|QED|^2}{|QCD|^2} \propto \frac{\alpha_{em}^2}{\alpha_s^2}$ with μ^2 in α_s chosen to be $Q^2 z\bar{z} + q_\perp^2 + m_q^2$.

Summary and outlook

In this talk, we consider LO factorization of exclusive electroproduction of dijet. The phenomenology analyses are realized within PARTONS framework.

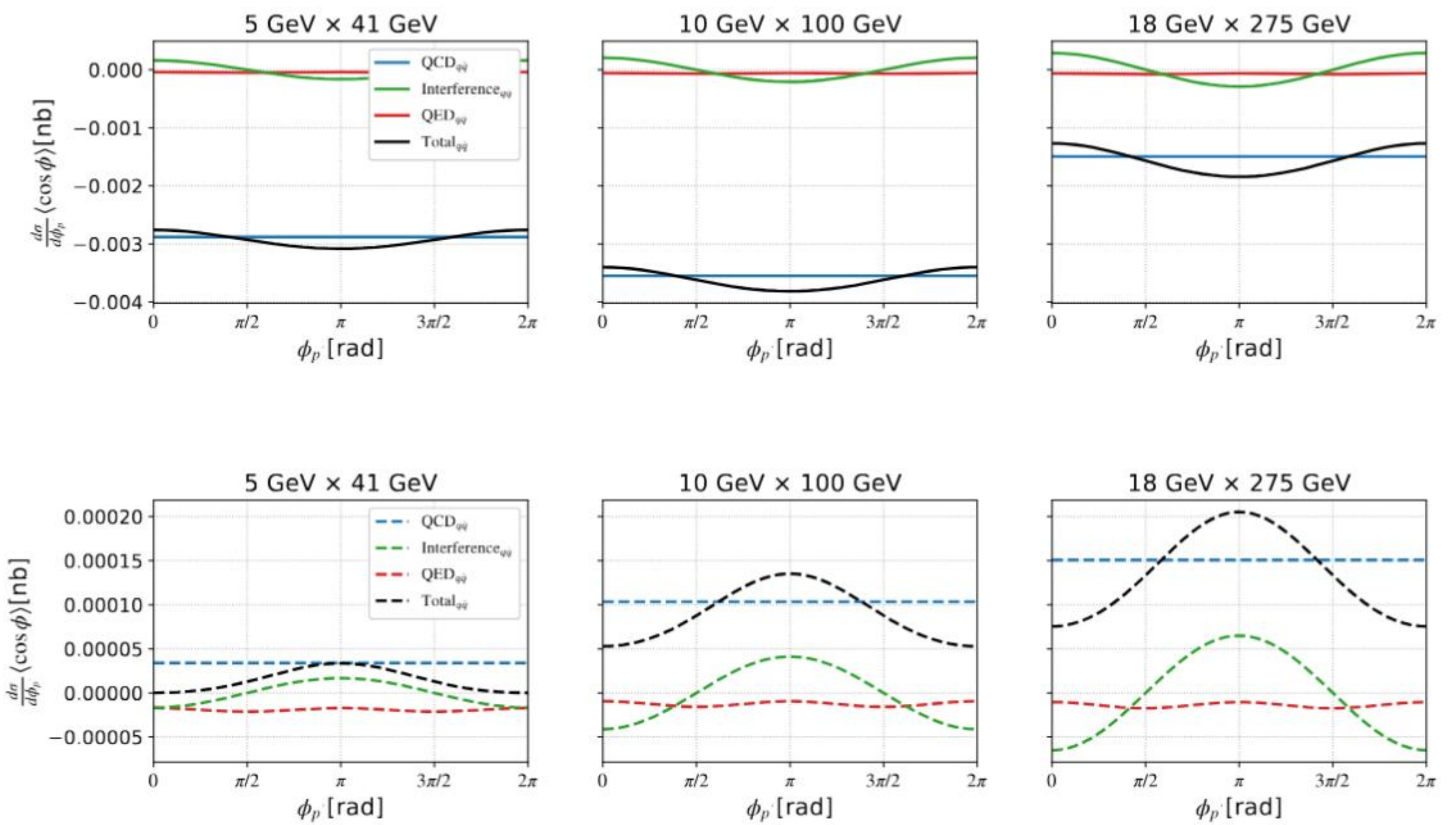
The QED contributions are found to be subdominant in most observables and kinematics regions considered above. Compared with HERA kinematics, the cross sections receive enhanced contributions from valence quark GPDs in EIC and EicC kinematics.

The factorization at NLO within a specific jet algorithm (anti-kt···) is worth to investigate in the future.

Thanks for your attention!

Back-up

Numerical results with EIC kinematics



Integration range:

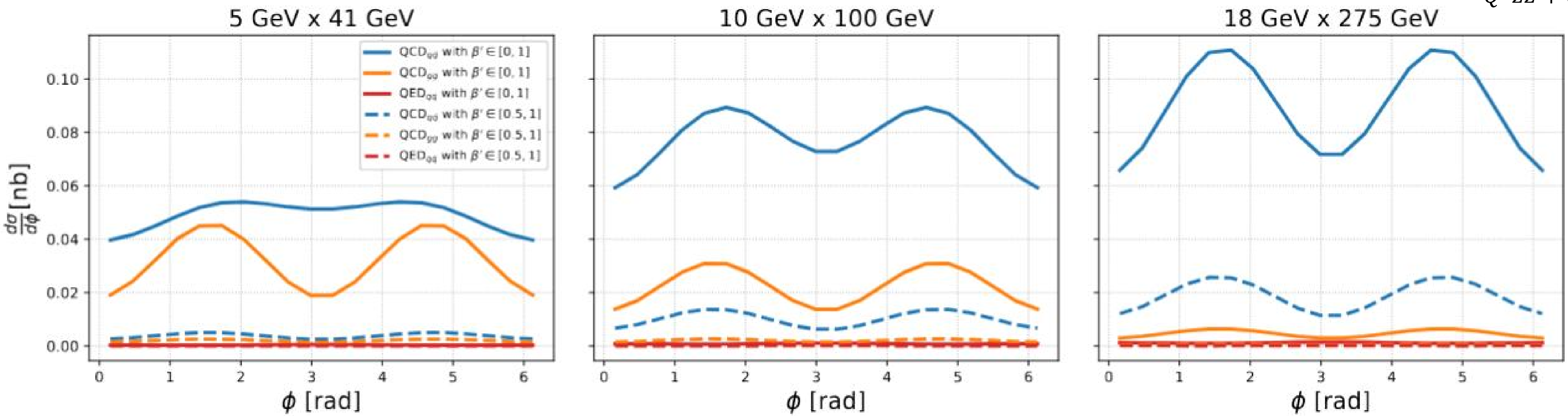
$z \in [0.05,0.95]$,
$q_\perp^2 \in [1,50]GeV^2$,
$\phi \in [0,2\pi]$,
$y \in [0.01,0.95]$,
$Q^2 \in [1,100]GeV^2$,
$t \in [-1.5,-0.01]GeV^2$

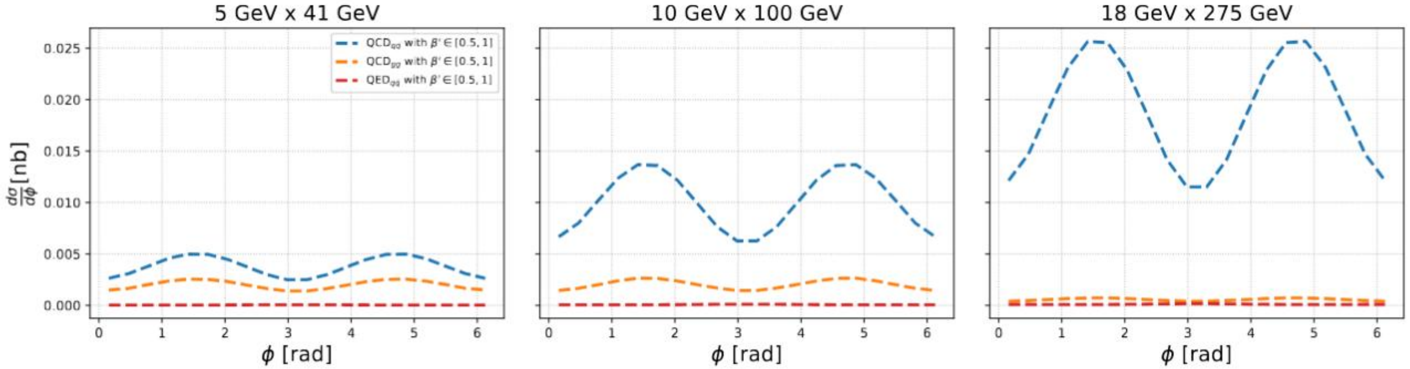
The observable $\frac{d\sigma}{d\phi_{p'}}\langle cos\phi\rangle$ receives sizable contributions from the QED channel, especially for the lower panels.

$Re(M_{QCD}^{q\bar{q}} M_{QED}^{q\bar{q}*}) \propto cos\phi_{p'}, sin\phi_{p'}$ from Lorentz covariance.

Distribution in ϕ at EIC kinematics

$$\beta = \frac{Q^2 z\bar{z} + m_q^2}{Q^2 z\bar{z} + q_\perp^2 + m_q^2}$$





Integration range:
$Q^2 \in [1,100]GeV^2$,
$t \in [-1.5, -0.01]GeV^2$,
$q_\perp^2 \in [1,50]GeV^2, z \in [0.05,0.95]$,
$\phi_{p'} \in [0,2\pi], y \in [0.01,0.95]$.

LO evolution of gluon GPDs

From evolution of gluon GPDs (0504030):

$$\frac{d}{d\ln\mu}\boldsymbol{F}(x,\eta) = -\int_{-1}^{1} dy\,\boldsymbol{K}\left(\frac{\eta+x}{2},\frac{\eta-x}{2}\middle|\frac{\eta+y}{2},\frac{\eta-y}{2}\right)\boldsymbol{F}(y,\eta)\,,$$

$$F_g^{(1)}(x,\xi,t) = \frac{\partial}{\partial x}F_g(x,\xi,t)$$

Unpolarized gluon-in-quark channel (denoted by superscript “gq”):

$$K_{(0)}^{gq;V}(x_1,x_2|y_1,y_2) = C_F\left[(y_1-y_2)\vartheta^0_{111}(x_1,-x_2,x_1-y_1) + x_1x_2\vartheta^1_{111}(x_1,-x_2,x_1-y_1)\right]\,,$$

$\Rightarrow$ $\frac{d}{dln\mu}\left(F_{gu}^{(1)gq}(\xi^+,\xi,t) - F_{gu}^{(1)gq}(\xi^-,\xi,t)\right) = \int_{-1}^{1} dy\frac{1}{\xi}F_{qu}^{s}(y,\xi,t)$ = 0 due to $F_{qu}^{s}(y,\xi,t)$ is **odd** in y.

Polarized gluon-in-quark channel:

$$K_{(0)}^{gq;A}(x_1,x_2|y_1,y_2) = C_F\left[(x_1-x_2)\vartheta^0_{111}(x_1,-x_2,x_1-y_1) + x_1x_2\vartheta^1_{111}(x_1,-x_2,x_1-y_1)\right]\,,$$

$\Rightarrow$ $\frac{d}{dln\mu}\left(F_{gh}^{(1)gq}(\xi^+,\xi,t) - F_{gh}^{(1)gq}(\xi^-,\xi,t)\right) = 0.$

LO evolution of gluon GPDs

Gluon-in-gluon channel (denoted by superscript "gg", $i = \{u, h, t\}$):

$$
\begin{aligned}
\frac{d}{d\ln\mu} F_{gi}^{gg}(x,\xi,t) = \int_{-1}^{1} dy \Bigg\{ & \Big[\frac{x_1}{y_1}\frac{x_1}{x_1 - y_1}\theta_{11}^{0}(x_1, x_1 - y_1) + \frac{x_2}{y_2}\frac{x_2}{x_2 - y_2}\theta_{11}^{0}(x_2, x_2 - y_2)\Big]\big(F_{gi}(y,\xi,t) - F_{gi}(x,\xi,t)\big) \\
& + \Big[\frac{x_1}{y_1}\frac{x_1}{x_1 - y_1}\theta_{11}^{0}(x_1, x_1 - y_1) + \frac{x_2}{y_2}\frac{x_2}{x_2 - y_2}\theta_{11}^{0}(x_2, x_2 - y_2) - \frac{y_1}{y_1 - x_1}\theta_{11}^{0}(y_1, y_1 - x_1) \\
& - \frac{y_2}{y_2 - x_2}\theta_{11}^{0}(y_2, y_2 - x_2)\Big] F_{gi}(x,\xi,t) \\
& + \Delta K_{ggi}(x,y,\xi) F_{gi}(y,\xi,t) + \Big(\frac{1}{2}\frac{\beta_0}{C_A} + 2\Big) F_{gi}(x,\xi,t)\Bigg\},
\end{aligned}
$$

The contribution from fourth line:

$$
\Delta K_{ggi}(x,y,\xi) = \begin{cases} 2\dfrac{x_1 x_2 + y_1 y_2}{y_1 y_2}\vartheta_{111}^{0}(x_1, -x_2, x_1 - y_1) + 2\dfrac{x_1 x_2}{y_1 y_2}\dfrac{x_1 y_1 + x_2 y_2}{(x_1 + x_2)^2}\vartheta_{11}^{0}(x_1, -x_2), i = u \\ 2\dfrac{x_1 y_2 + y_1 x_2}{y_1 y_2}\vartheta_{111}^{0}(x_1, -x_2, x_1 - y_1) + 2\dfrac{x_1 x_2}{y_1 y_2}\vartheta_{11}^{0}(x_1, -x_2), i = h \\ 0, i = t \end{cases}
$$

$$
\Rightarrow \quad \frac{d}{d\ln\mu}\Big(F_{gi}^{(1)gg}(\xi^{+},\xi,t) - F_{gi}^{(1)gg}(\xi^{-},\xi,t)\Big) \supset 0,
$$

for $i = u, h, t$.

LO evolution of gluon GPDs

Gluon-in-gluon channel (denoted by superscript "gg", $i = \{u, h, t\}$):

$$\frac{d}{d\ln\mu} F_{gi}^{gg}(x,\xi,t) = \int_{-1}^{1} dy \Big\{ \Big[\frac{x_1}{y_1}\frac{x_1}{x_1-y_1}\theta_{11}^{0}(x_1, x_1-y_1) + \frac{x_2}{y_2}\frac{x_2}{x_2-y_2}\theta_{11}^{0}(x_2, x_2-y_2)\Big]\big(F_{gi}(y,\xi,t) - F_{gi}(x,\xi,t)\big)$$

$$+ \Big[\frac{x_1}{y_1}\frac{x_1}{x_1-y_1}\theta_{11}^{0}(x_1, x_1-y_1) + \frac{x_2}{y_2}\frac{x_2}{x_2-y_2}\theta_{11}^{0}(x_2, x_2-y_2) - \frac{y_1}{y_1-x_1}\theta_{11}^{0}(y_1, y_1-x_1)$$

$$- \frac{y_2}{y_2-x_2}\theta_{11}^{0}(y_2, y_2-x_2)\Big] F_{gi}(x,\xi,t)$$

$$+ \Delta K_{ggi}(x,y,\xi) F_{gi}(y,\xi,t) + \Big(\frac{1}{2}\frac{\beta_0}{C_A} + 2\Big) F_{gi}(x,\xi,t)\Big\},$$

The contribution from second and third lines:

$$\frac{d}{d\ln\mu} F_{gi}^{gg}(x,\xi,t) \supset F_{gi}(x,\xi,t) \cdot \begin{cases} 4\ln\left(1-\xi^2\right) + \frac{8(x-\xi^2)}{\xi^2-1} - 8\ln(1-x), & x > \xi \\ -\frac{8\xi}{\xi+1} + 8\ln(\xi+1) - 4\ln\left(1-x^2\right), & 0 < x < \xi \end{cases}$$

The r.h.s. and the first derivative of it with respect to x are continuous at $x = \xi$.

LO evolution of gluon GPDs

Gluon-in-gluon channel (denoted by superscript “gg”, $i=\{u,h,t\}$):

$$\frac{d}{d\ln\mu}F^{gg}_{gi}(x,\xi,t)=\int_{-1}^{1}dy\Big\{\Big[\frac{x_1}{y_1}\frac{x_1}{x_1-y_1}\theta^0_{11}(x_1,x_1-y_1)+\frac{x_2}{y_2}\frac{x_2}{x_2-y_2}\theta^0_{11}(x_2,x_2-y_2)\Big]\big(F_{gi}(y,\xi,t)-F_{gi}(x,\xi,t)\big)$$
$$+\Big[\frac{x_1}{y_1}\frac{x_1}{x_1-y_1}\theta^0_{11}(x_1,x_1-y_1)+\frac{x_2}{y_2}\frac{x_2}{x_2-y_2}\theta^0_{11}(x_2,x_2-y_2)-\frac{y_1}{y_1-x_1}\theta^0_{11}(y_1,y_1-x_1)$$
$$-\frac{y_2}{y_2-x_2}\theta^0_{11}(y_2,y_2-x_2)\Big]F_{gi}(x,\xi,t)$$
$$+\Delta K_{ggi}(x,y,\xi)F_{gi}(y,\xi,t)+\Big(\frac{1}{2}\frac{\beta_0}{C_A}+2\Big)F_{gi}(x,\xi,t)\Big\},$$

$$\int_{-1}^{x}\frac{\partial_x G(x,y)}{(y-\xi)^2}dy,\ \int_{x}^{1}\frac{\partial_x G(x,y)}{(y-\xi)^2}dy\propto\frac{1}{x-\xi},$$

At scale μ^2, $F^{(1)}_g(x,\xi,t)$ is continuous at $x=\xi$ & $F^{(2)}_g(x,\xi,t)$ is finite in the neighbourhood of $x=\xi$.

The contribution from first line: $\left(G(x,y)=\dfrac{F_g(y,\xi,t)-F_g(x,\xi,t)}{x-y}\right)$

$$\frac{d}{dln\mu}\left(F^{(1)gg}_{gi}(\xi^+,\xi,t)-F^{(1)gg}_{gi}(\xi^-,\xi,t)\right)$$
$$\supset\ lim_{x\to\xi^+}\left(2(x-\xi)\int_x^1\frac{G(x,y)}{(y-\xi)^2}+F^{(1)}_g(x,\xi,t)+(x-\xi)^2\int_x^1\frac{\partial_x G(x,y)}{(y-\xi)^2}dy\right)$$
$$-\,lim_{x\to\xi^-}\left(-2(x-\xi)\int_{-1}^x\frac{G(x,y)}{(y-\xi)^2}+F^{(1)}_g(x,\xi,t)-(x-\xi)^2\int_{-1}^x\frac{\partial_x G(x,y)}{(y-\xi)^2}dy\right)$$
$$=-F^{(1)}_g(\xi^+,\xi,t)+F^{(1)}_g(\xi^-,\xi,t)=0.$$

$$lim_{x\to\xi^+}2(x-\xi)\int_x^1\frac{G(x,y)}{(y-\xi)^2}dy=lim_{x\to\xi^+}2(x-\xi)\left(\int_x^1\frac{G(x,y)-G(x,\xi)}{(y-\xi)^2}dy+\int_x^1\frac{G(x,\xi)}{(y-\xi)^2}dy\right)$$
$$=lim_{x\to\xi^+}2(x-\xi)\left(\int_x^1\frac{\frac{\partial G(x,y)}{\partial y}}{(y-\xi)}dy-F^{(1)}_g(\xi^+,\xi,t)\left(\frac{1}{x-\xi}-\frac{1}{1-\xi}\right)\right)\quad \textbf{IBP}$$
$$=-2F^{(1)}_g(\xi^+,\xi,t),$$