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Hidden-Charm Pentaquark Baryon Resonances in Regge Phenomenology: Spectra and Threshold Patterns

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Introduction and Motivation

- Hadron spectroscopy began with the pion's discovery in 1947 and was firmly established with the development of the quark model in the 1960s.
- In the 1980s, the success of the quark model prompted scientists to consider the possibility of more complex particles called exotic hadrons.
- Over the years, various exotic hadrons—including hybrid mesons, tetraquarks, and pentaquarks—have been reported experimentally [17–24]. The earliest of these, a tetraquark candidate, was observed in 2003. [17]
- Significant progress in discovering exotic hadrons has been made since the Belle Collaboration observed the first candidate $\boldsymbol{X(3872)}$ in 2003 [17] followed by various tetraquark and pentaquark states such as $\boldsymbol{T_{c\bar{c}1}}$ [18], $\boldsymbol{T_{b\bar{b}1}}(10610)$ [19], $\boldsymbol{X(4140)}$ [20], and the pentaquark candidates $\boldsymbol{P_c(4380)^+}$, $\boldsymbol{P_c(4440)^+}$ and $\boldsymbol{P_c(4457)^+}$ reported by LHCb [21].
- Several notable tetraquark candidates have been identified in experiments, including $\boldsymbol{Z(4430)}$ and $\boldsymbol{Y(4660)}$ by Belle [31], $\boldsymbol{Y(4140)}$ by Fermilab [32], $\boldsymbol{X(5568)}$ by the DØ experiment [33], $\boldsymbol{Z(3900)}$ by BESIII [34], and $\boldsymbol{X(6900)}$ by LHCb [35].

- All of the experimentally reported pentaquark candidates belong to the hidden-charm sector and contain a $c\bar{c}$ pair, notably the P_c and P_{cs} structures.
- The LHCb Collaboration has reported several hidden-charm pentaquark candidates, containing a $c\bar{c}$ pair, in the $J/\psi p$ and $J/\psi\Lambda$ channels; however, their internal structure remains unresolved. [36]
- Various theoretical studies using different approaches have been done to study pentaquarks, including lattice QCD, QCD sum rules, and phenomenological models such as hadronic-molecule and compact diquark–diquark–antiquark models. [36-38]
- In the present work, we employ the **Regge phenomenology** under the presumption of linear Regge trajectories. Relationships between the intercept, slope ratios, and pentaquark masses are extracted in the (J, M^2) planes.
- Using these relations, we derived the **mass inequalities** to **calculate the ground and excited states mass bounds** of $nnnc\bar{c}$, $ncnc\bar{c}$ and $nnnn\bar{c}$ pentaquarks in the $(\boldsymbol{J}, \boldsymbol{M^2})$ planes.

Regge Phenomenology

- **Background**

- Regge theory is one of the simplest and most practical phenomenological approaches to hadron spectroscopy.
- Physicist Yoichiro Nambu gave the simplest idea for the Regge trajectories. [43, 44]
- He made the assumption that a strong flux tube is formed by the uniform interaction of a quark-antiquark pair, and that light quarks rotate at radius R at the speed of light at the tube's end.
- It is calculated that the mass that originated in this flux tube is [45]

$$M = 2\int_0^R \frac{\sigma}{\sqrt{1 - v^2(r)}} dr = \pi\sigma R \quad \ldots (1)$$

- Where, σ is the mass density per unit length, or the string tension.

- The flux tube's angular momentum is also calculated as

$$J = 2\int_0^R \frac{\sigma r v(r)}{\sqrt{1 - v^2(r)}} dr = \frac{\pi\sigma R^2}{2} + c' \;\; \ldots (2)$$

- Therefore, using (1) and (2) we can write,

$$J = \frac{M^2}{2\pi\sigma} + c'' \;\; \ldots (3)$$

- As a result, J and M^2 have a linear relationship with one another.
- Plots of hadron Regge trajectories in the (J, M^2) plane are referred as Chew-Frautschi plots. [46]
- Hadrons with the same internal quantum numbers are situated on the same Regge line.
- The most common form of linear Regge trajectories can be stated as follows [41]

$$J = \beta(M) = \beta(0) + \beta' M^2 \;\; \ldots (4)$$

Where, β (0) and β' represent the intercept and slope of the trajectory, respectively.

Regge Phenomenology

• Methodology

- We can have following two relationships between Regge slopes and intercepts for three body particles with different flavors as follows [41, 47-54]

$$\beta_{iiq}(0) + \beta_{jjq}(0) = 2\beta_{ijq}(0) \ldots (5)$$

$$\frac{1}{\beta'_{iiq}} + \frac{1}{\beta'_{jjq}} = \frac{2}{\beta'_{ijq}} \ldots (6)$$

where i, j and q represent quark flavors.

- If, we solve above two relations for the same J quantum number in (J, M^2) then we can get the following quadratic type equation.

$$M_{j\bar{j}}^{2}\left(\frac{\beta'_{jjq}}{\beta'_{iiq}}\right)^{2}+\left(M_{iiq}^{2}+M_{jjq}^{2}-4M_{ijq}^{2}\right)\left(\frac{\beta'_{jjq}}{\beta'_{iiq}}\right)+M_{iiq}^{2}=0\ \ldots(7)$$

- By, solving this quadratic equation we get,

$$\frac{\beta'_{jjq}}{\beta'_{iiq}}=\frac{1}{2M^{2}{}_{jjq}}\times\left[\left(4M_{ijq}^{2}-M_{iiq}^{2}-M_{jjq}^{2}\right)+\sqrt{\left(4M_{ijq}^{2}-M_{iiq}^{2}-M_{jjq}^{2}\right)^{2}-4M_{iiq}^{2}M_{jjq}^{2}}\right]\ldots(8)$$

$$\frac{\beta'_{ijq}}{\beta'_{jjq}}=\frac{1}{4M^{2}{}_{ijq}}\times\left[\left(4M_{ijq}^{2}+M_{jjq}^{2}-M_{iiq}^{2}\right)+\sqrt{\left(4M_{ijq}^{2}-M_{iiq}^{2}-M_{jjq}^{2}\right)^{2}-4M_{iiq}^{2}M_{jjq}^{2}}\right]\ldots(9)$$

- These equations give us the important relations between the slope ratios and masses of three body hadronic systems.

- ## Linear mass inequalities and quadratic mass inequalities

- Since the Regge slopes β'_{jjq} and β'_{iiq} are positive real numbers, their ratio $\beta'_{jjq}/\beta'_{iiq}$ is also real. Thus, from Eq.(8) we obtain,

$$\left|4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2\right| \geq 2M_{iiq}M_{jjq} \quad \ldots (10)$$

- By performing a series of mathematical manipulations on the above equation, we obtain the following inequality.

$$M_{ijq} > \frac{M_{iiq} + M_{jjq}}{2} \quad \ldots (11)$$

- Studies have shown that Regge trajectory slopes decrease with increasing quark mass [47-49, 56-61]. Thus, for $m_j > m_i$, we have $\beta'_{jjq}/\beta'_{iiq} < 1$. From Eq.(8), it then follows that

$$\frac{1}{2M^2{}_{jjq}} \times \left[\left(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2\right) + \sqrt{\left(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2\right)^2 - 4M_{iiq}^2M_{jjq}^2}\right] < 1 \quad \ldots (12)$$

- As the square root term in the Eq.(12) is positive, we can conclude that

$$2M_{jjq}^2 - \left(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2\right) > 0 \quad \ldots (13)$$

- By, equations (12) and (13) we get,

$$\left(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2\right)^2 - 4M_{iiq}^2 M_{jjq}^2 < \left[2M_{jjq}^2 - \left(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2\right)\right]^2 \quad \ldots (14)$$

- The last two equations can be used to get the following inequality:

$$M_{ijq} < \sqrt{\frac{M_{iiq}^2 + M_{jjq}^2}{2}} \quad \ldots (15)$$

- Combining equations (11) and (15) we get following mass bound for $M_{j\bar{j}}$

$$\frac{M_{iiq} + M_{jjq}}{2} < M_{ijq} < \sqrt{\frac{M_{iiq}^2 + M_{jjq}^2}{2}} \quad \ldots (16)$$

- The above mass inequality defines the upper and lower bounds for M_{ijq}. In the next section, we use this to estimate the mass ranges of pentaquarks with charmed quarks.

Pentaquark mass spectra

• The five-quark state in the diquark-diquark-antiquark picture

- Here, we calculate the mass spectra of $[nc][nc]\bar{c}$, $[nn][nn]\bar{c}$ and $[nn][nc]\bar{c}$ pentaquarks while considering them as the compact bound states of two diquarks and an anti charm quark.
- Each diquark is treated as a correlated quark pair without internal spatial excitation, so that all orbital degrees of freedom reside in the relative motion between the two diquarks and between the diquark–diquark cluster and the antiquark.
- A diquark does not form a color singlet by itself and thus acts as an effective constituent that can only appear inside hadrons.
- We mainly adopt diquarks in the antisymmetric color anti-triplet $\bar{3}$ channel, which is attractive under one-gluon exchange; two such diquarks combine as $\bar{3} \otimes \bar{3} = 3 \oplus \bar{6}$ to form a color-triplet subsystem that couples with the anti-charm quark ($\bar{3}$) into an overall singlet, $3 \otimes \bar{3} = 1 \oplus 8$.
- Although sextet diquarks and more complicated color couplings are in principle allowed, they are energetically disfavored and therefore neglected in the present effective description

Mass spectra of $nnnc\bar{c}$, $nnnn\bar{c}$ and $ncnc\bar{c}$ pentaquark in (J, M^2) plane

- Here, we use equation (16), which is an inequality to evaluate the ground-state mass bounds of the $nnnc\bar{c}$ pentaquark.
- $nnnc\bar{c}$ pentaquark is considered to be composed of nn and nc diquarks and $\bar{c}$ anti-quark.
- In eq (16), if we take $\boldsymbol{i} = [\boldsymbol{nn}]$, $\boldsymbol{j} = [\boldsymbol{nc}]$, and $\boldsymbol{q} = \bar{\boldsymbol{c}}$ we get the following relation:

$$\frac{M_{nnnn\bar{c}} + M_{ncnc\bar{c}}}{2} < M_{nnnc\bar{c}} < \sqrt{\frac{M^2_{nnnn\bar{c}} + M^2_{ncnc\bar{c}}}{2}} \ldots (17)$$

- Here, we take ground state masses of $nnnn\bar{c}$ and $ncnc\bar{c}$ pentaaquark as an input from **Ref [68] and Ref [69]**, respectively, since no experimental results are available for this.
- Substituting the masses of $nnnn\bar{c}$ and $ncnc\bar{c}$ (for $J^P = (1/2)^-, (3/2)^-, (5/2)^-$), we get the ground-state mass bounds of $nnnc\bar{c}$ pentaquark, such as 4.140–4.484 GeV for $J^P = (1/2)^-$, 4.199–4.497 GeV for $J^P = (3/2)^-$and 4.407–4.659 GeV for $J^P = (5/2)^-$.

- **Calculations of Excited state mass bounds:**
- Now, using equation (4) we have,

$$M_{J+k(nnnc\bar{c})} = \sqrt{M^2_{J(nnnc\bar{c})} + \frac{k}{\beta'_{nnnc\bar{c}}}} \quad \dots (18)$$

- Now, by getting the value of $\beta'_{nnnc\bar{c}}$ from Ref. [70], we can get mass ranges of excited states of $nnnc\bar{c}$ pentaquark.
- By inserting the suitable values in equation (9), we can get

$$\frac{1}{\beta'_{ncnc\bar{c}}}$$

$$= \frac{1}{\beta'_{nnnc\bar{c}}\,(4M^2_{nnnc\bar{c}})}$$

$$\times \left[\left(4M^2_{nnnc\bar{c}} + M^2_{nnnn\bar{c}} - M^2_{ncnc\bar{c}}\right) + \sqrt{(4M^2_{nnnc\bar{c}} - M^2_{nnnn\bar{c}} - M^2_{ncnc\bar{c}})^2 - 4M^2_{nnnn\bar{c}}M^2_{ncnc\bar{c}}}\right] \quad \dots (19)$$

- By inserting the values of $M_{nnnn\bar{c}}$, $M_{ncnc\bar{c}}$, and $\beta'_{nnnc\bar{c}}$ into the above equation, $\beta'_{ncnc\bar{c}}$ can be represented as a function of $M_{nnnc\bar{c}}$.
- And using equation (6) we can get bound for $\beta'_{nnnn\bar{c}}$. Therefore, we can get excited states mass bounds for $M_{nnnn\bar{c}}$ and $M_{ncnc\bar{c}}$ pentaquarks.

Hidden-charm $nnnc\bar{c}$: predicted S-F spectra

Spin family	S	P	D	F
Doublet s = 1/2	$1/2^-$	$3/2^+$	$5/2^-$	$7/2^+$
	4.140–4.484	**4.352–4.680**	**4.554–4.868**	**4.747–5.049**
Quartet s = 3/2	$3/2^-$	$5/2^+$	$7/2^-$	$9/2^+$
	4.199–4.497	**4.418–4.702**	**4.627–4.898**	**4.826–5.087**
Sextet s = 5/2	$5/2^-$	$7/2^+$	$9/2^-$	$11/2^+$
	4.407–4.659	**4.644–4.885**	**4.870–5.100**	**5.086–5.306**
Masses in GeV; each cell gives J^P and its mass interval. Leading stretched sequences only.				

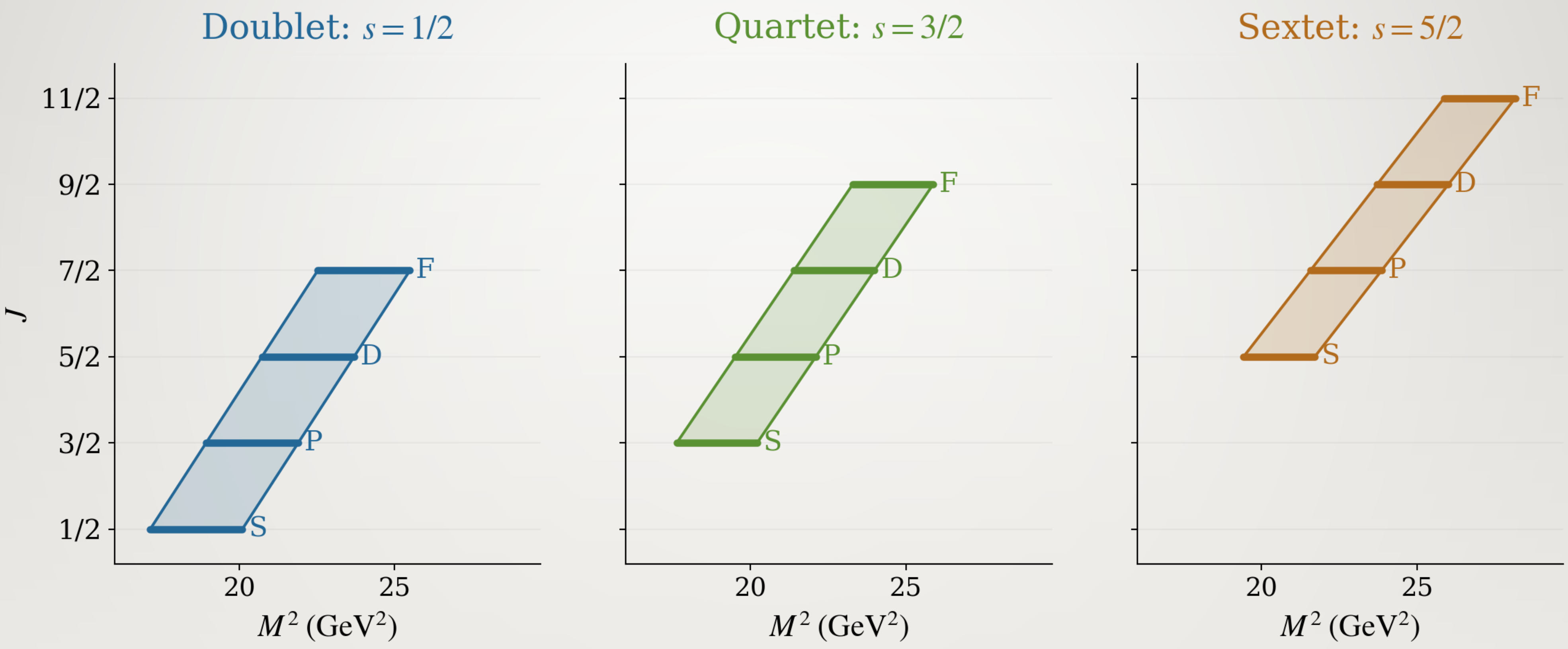
Anti-charmed $nnnn\bar{c}$: predicted P-F spectra

Spin family	S input	P	D	F
Doublet s = 1/2	$1/2^-$	$3/2^+$	$5/2^-$	$7/2^+$
	3.032	**3.234–3.315**	**3.424–3.576**	**3.604–3.818**
Quartet s = 3/2	$3/2^-$	$5/2^+$	$7/2^-$	$9/2^+$
	3.043	**3.254–3.339**	**3.453–3.610**	**3.641–3.863**
Sextet s = 5/2	$5/2^-$	$7/2^+$	$9/2^-$	$11/2^+$
	3.249	**3.480–3.564**	**3.697–3.854**	**3.901–4.124**
Masses in GeV; each cell gives J^P and its mass interval. Leading stretched sequences only.				

Multicharm $ncnc\bar{c}$: predicted P-F spectra

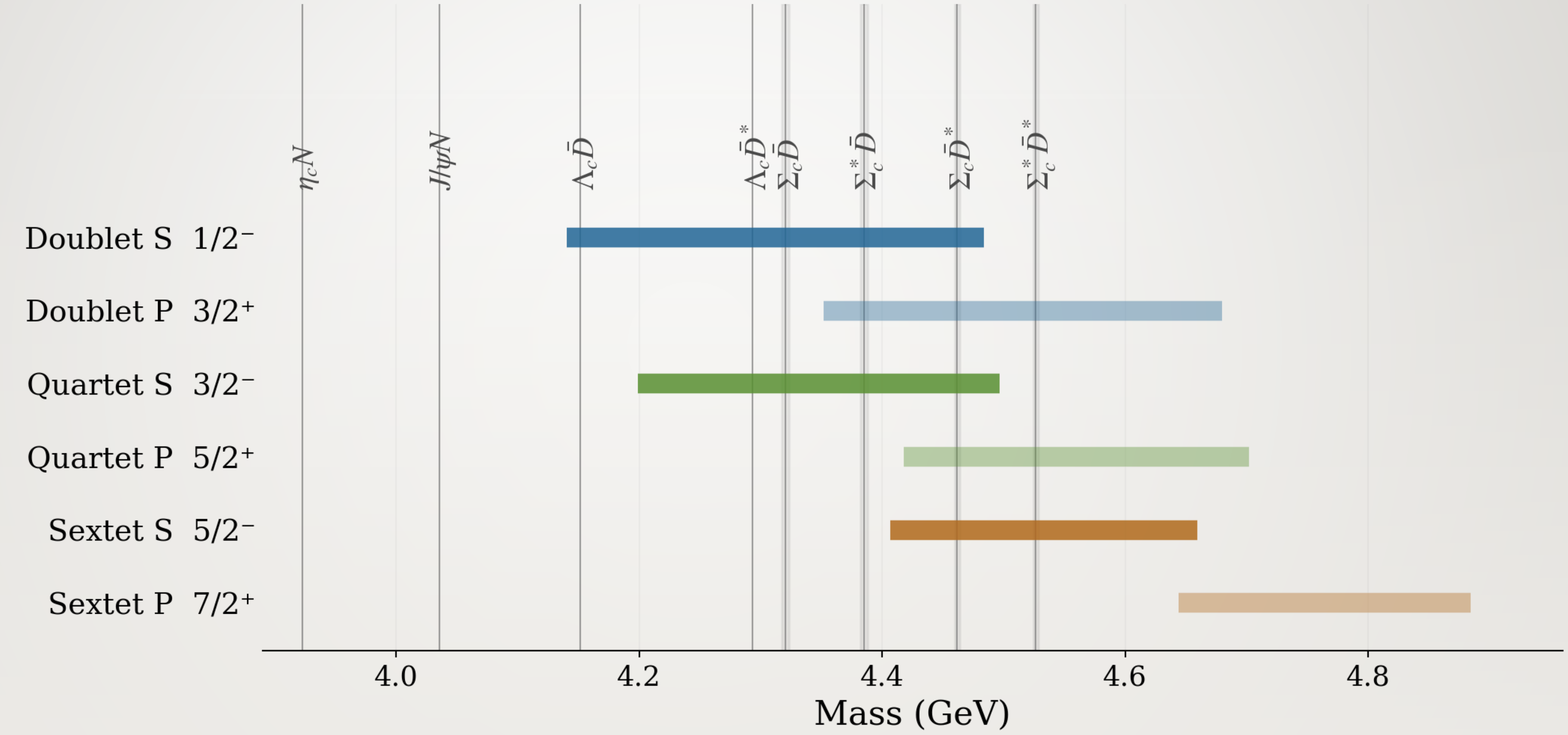
Spin family	S input	P	D	F
Doublet **s = 1/2**	$1/2^-$	$3/2^+$	$5/2^-$	$7/2^+$
	5.2490–5.5695	**5.417–5.775**	**5.581–5.972**	**5.739–6.164**
Quartet **s = 3/2**	$3/2^-$	$5/2^+$	$7/2^-$	$9/2^+$
	5.3552–5.5838	**5.529–5.798**	**5.697–6.005**	**5.860–6.205**
Sextet **s = 5/2**	$5/2^-$	$7/2^+$	$9/2^-$	$11/2^+$
	5.5647–5.7327	**5.755–5.967**	**5.938–6.193**	**6.117–6.411**
Masses in GeV; each cell gives J^P and its mass interval. Leading stretched sequences only.				

Hidden charm linear trajectories of the leading sequences



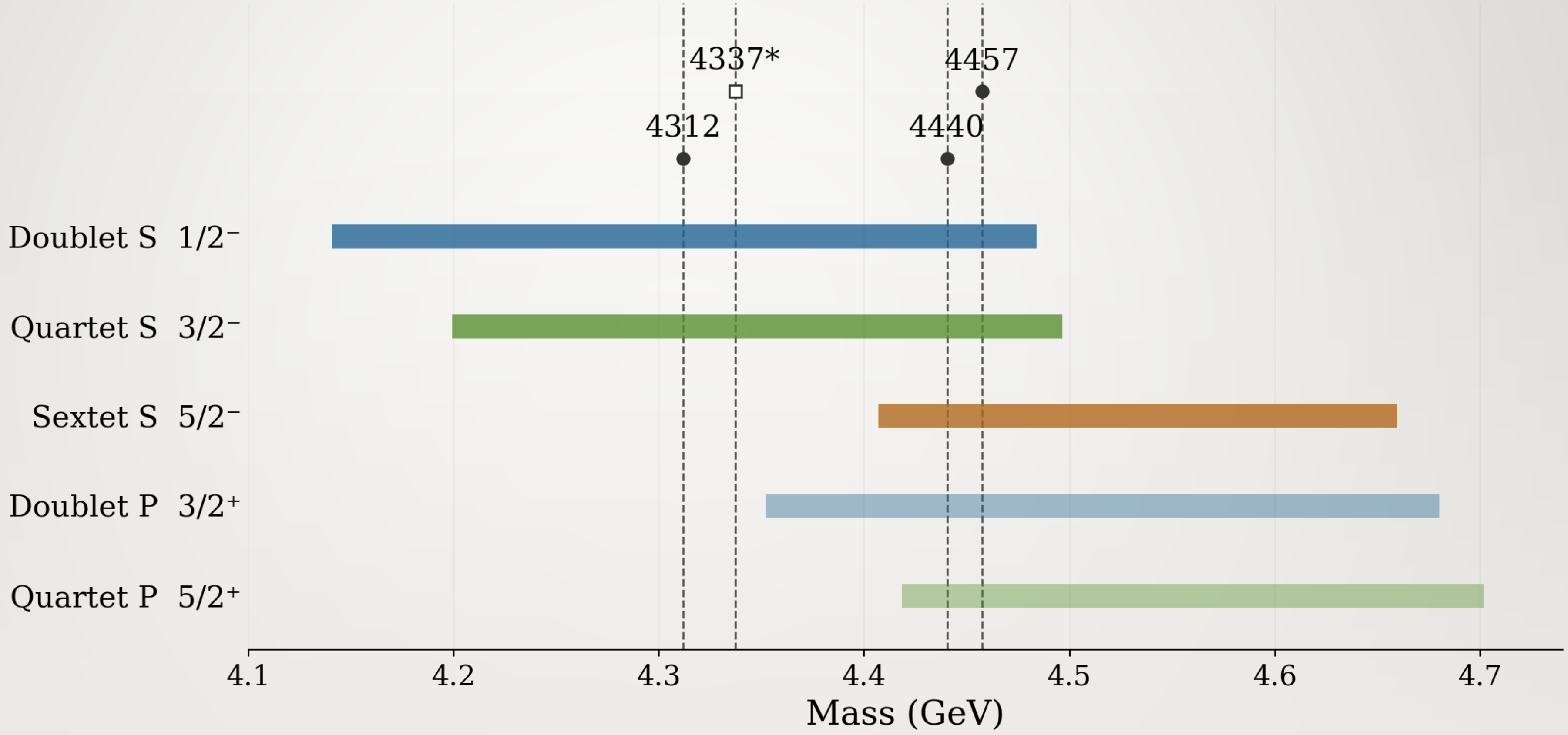
The straight-line ordering follows from the assumed Regge form

Hidden charm ground bands intersections with several thresholds



Thresholds: sums of PDG 2026 central masses; constituent widths are not folded in

Reported P_c masses overlaps with several bands



2019 observations; * Pc(4337): evidence in the 2022 analysis

Comparison of the lowest hidden-charm masses with other theoretical studies

State	J^P	Present work	Extended CMI [89]	Diquark model [90]	Coupled-channel [91]	QCD sum rules [92]
$1^2S_{1/2}$	$1/2^-$	4.140–4.484	4.327	4.238	4.313	4.20 ± 0.11
$1^4S_{3/2}$	$3/2^-$	4.199–4.497	4.367	4.284	4.369	4.34 ± 0.11
$1^6S_{5/2}$	$5/2^-$	4.407–4.659	4.546	4.451	4.522	4.38 ± 0.11

Results and physical interpretation

- The Regge construction constrains the hidden-charm ground-state masses to $(4.140 - 4.484)$ GeV for $(J^P = 1/2^-)$, $(4.199 - 4.497)$ GeV for $(J^P = 3/2^-)$, and $(4.407 - 4.659)$ GeV for $(J^P = 5/2^-)$.
- The extracted slopes generate systematically ordered S to F wave sequences. The masses increase with orbital angular momentum according to $\Delta M^2 = 1/\alpha'$), while the parity alternates as $(P = (-1)^{L+1})$.
- The predicted hidden-charm bands intersect several $(\eta_c N)$, $(J/\psi N)$, $(\Lambda_c \bar{D})$, *and* $(\Sigma_c^{()} \overline{D^{(*)}})$ thresholds. These threshold crossings identify experimentally relevant channels, although they do not by themselves establish a molecular interpretation or determine decay widths.

- Several reported (P_c) masses are compatible with more than one predicted S or P wave band. Therefore, mass agreement alone does not provide a unique (J^P) assignment or determine the internal structure of a state.
- The same correlated Regge construction also produces ordered P to F wave spectra for the anti-charmed ($nnnn\bar{c}$) and multicharm ($ncnc\bar{c}$) sectors, providing a common description of the three flavour configurations.
- The quoted intervals are model-input envelopes rather than statistical confidence intervals or resonance widths. Their sizes mainly reflect the spread of the theoretical (S)-wave inputs and the assumptions of linear Regge trajectories and additivity.
- The predicted $nnnc\bar{c}$ mass intervals overlap well with results from the extended chromomagnetic, diquark, coupled-channel, and QCD-sum-rule approaches, placing the lowest states broadly in the 4.2–4.55 GeV region. This agreement supports the predicted mass scale, although the present intervals are too broad to establish a unique dynamical interpretation or state assignment.

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