Dispersive and model-independent extraction of πN poles from revised total cross-section data

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arXiv:2608.20458 [hep-ph]

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1. Motivation

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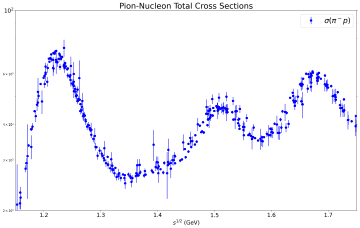

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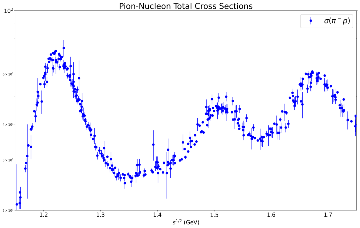

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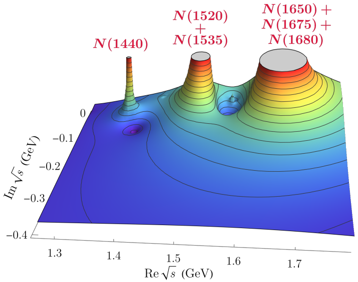

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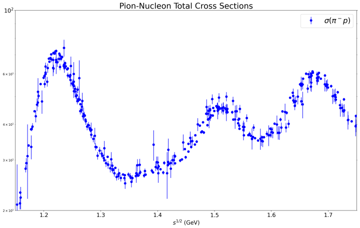

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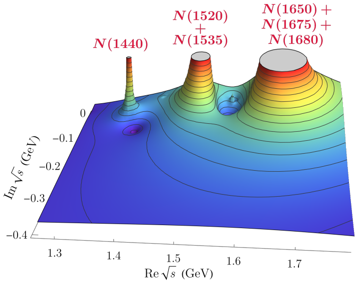

2. Can we estimate the isospin-breaking effects in the $\Delta(1232)$ sector?

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Why dispersive?

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Why dispersive?

∗ Consequence of fundamental principles, generally not imposed when extracting resonances

∗ Integrals decrease the parametrization dependence

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$*$ They hold up to ∞ (Roy-like DR do not)

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Why do we need a robust analytic continuation method?

* We only know exactly how to access the second Riemann sheet

* Also reduces the parametrization dependence

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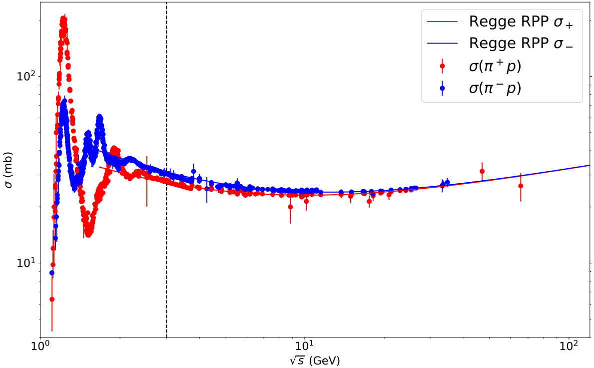

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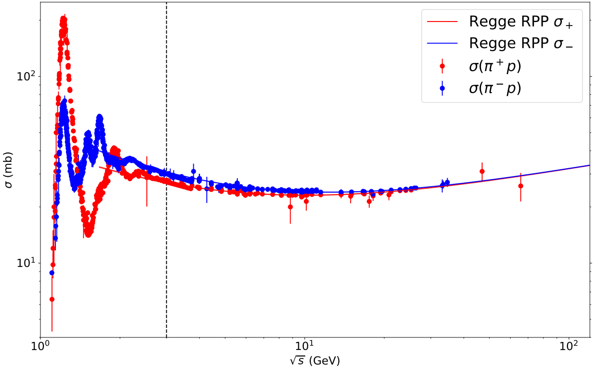
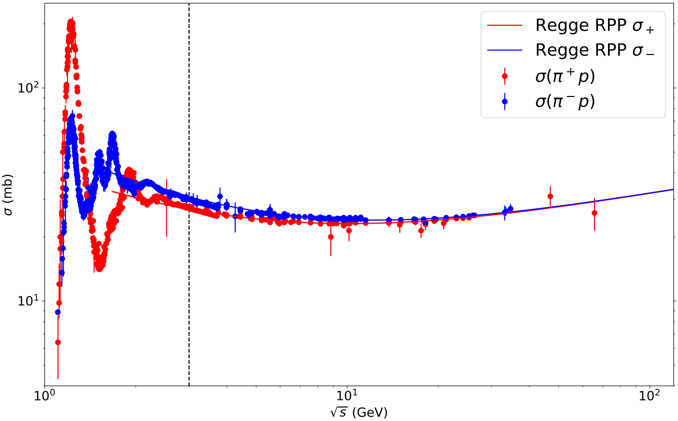


$*$ Determine resonance parameters in a different way (just total cross-section data, no partial waves)

2. πN total cross-section data

For this interaction there are many total cross-section data at low energies!



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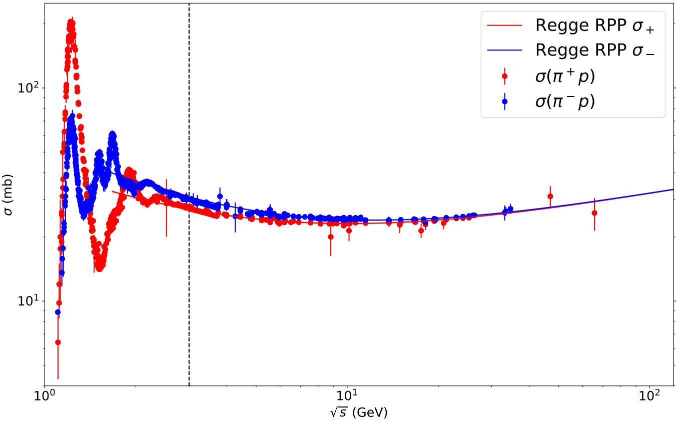

Strategy:

$$\boxed{\sigma_{\text{tot}}(\pi^\pm p) \Rightarrow \text{Im}D^\pm \Rightarrow \text{Re}D^\pm \Rightarrow \text{Resonances}}$$

For $\pi^a(q) + N(p) \to \pi^b(q') + N(p')$

$$\boxed{T_{fi}^{ba}(s,t) = \delta^{ba} T_{fi}^{+}(s,t) + i\epsilon^{bac}\tau^c T_{fi}^{-}(s,t)}$$

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$$\boxed{T^{\pm}_{fi}(s,t) = \bar{u}_f(p')\left\{D^{\pm}(s,t) - \frac{[\not{q}', \not{q}]}{4m_N} B^{\pm}(s,t)\right\} u_i(s,t)}$$

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The relation between these amplitudes and the total cross-section data is the following (**optical theorem**):

$$\boxed{\text{Im} D_{\pm}(s,0) = k_L \sigma_{\text{tot}}(\pi^\pm p) \equiv k_L \sigma_\pm}$$

with

$$D^{+} = \frac{1}{2}(D_{+} + D_{-}), \quad D^{-} = \frac{1}{2}(D_{-} - D_{+}),$$

and k_L the pion momentum in the laboratory frame

With these relations it is straightforward to obtain **once-subtracted FDRs**!
($\bar{D}^{\pm}$ are the pseudovector-Born-term-subtracted amplitudes)

$$\boxed{\mathrm{Re}\bar{D}^{+}(k_L) = 4\pi\left(1+\frac{M_\pi}{m_N}\right)\bar{a}^{+}_{0+} + \frac{k_L^2}{\pi}\mathcal{P}\int_0^\infty \frac{\sigma_+(k'_L)+\sigma_-(k'_L)}{k_L'^2-k_L^2}dk'_L}$$

$$\boxed{\mathrm{Re}\bar{D}^{-}(k_L) = \frac{g^2\omega_L}{2m_N^2} + \frac{\omega_L}{\pi}\mathcal{P}\int_0^\infty \frac{k_L'^2}{\omega'_L}\frac{\sigma_-(k'_L)-\sigma_+(k'_L)}{k_L'^2-k_L^2}dk'_L}$$

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with

$$M_\pi \equiv \text{pion mass},\qquad m_N \equiv \text{proton mass},\qquad \omega_L = \sqrt{k_L^2+M_\pi^2},$$
$$g^2 \equiv \text{pseudoscalar coupling constant},$$

and $\bar{a}_{0+}^{+}$ is related to even isospin S-wave scattering length, precisely extracted from **pionic-atoms data** Baru et al. PLB 694, 473 (2011), Baru et al. Nucl.Phys.A 872, 69 (2011)

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* Fit to Roy-Steiner analysis near threshold (tiny region) to stabilize the fit
  Hoferichter et al., Phys. Rept 625 (2016)

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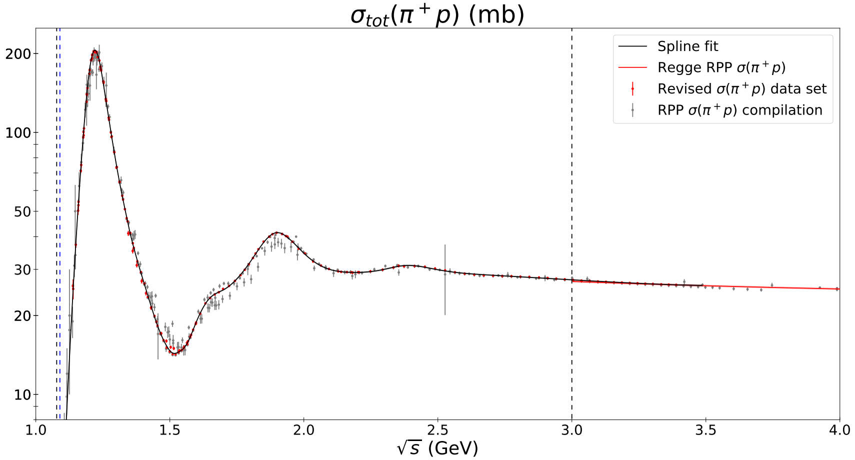

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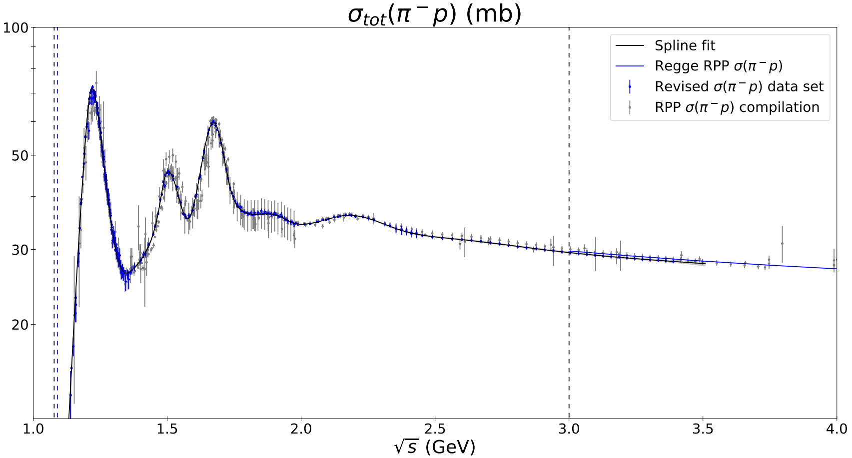

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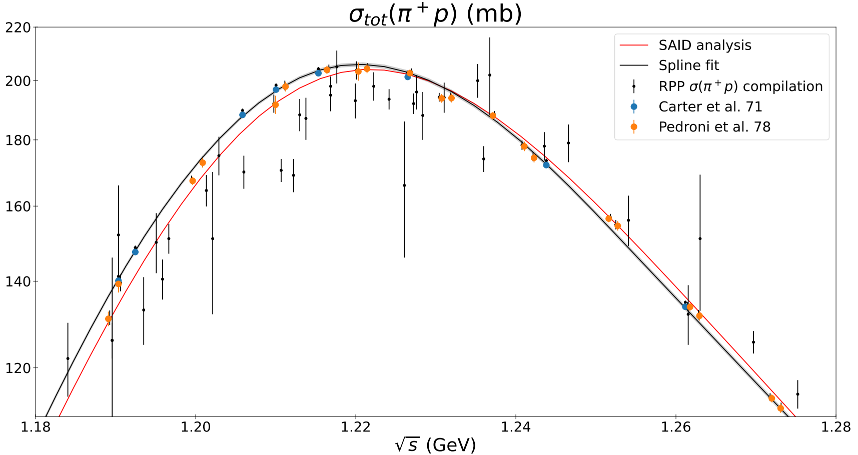

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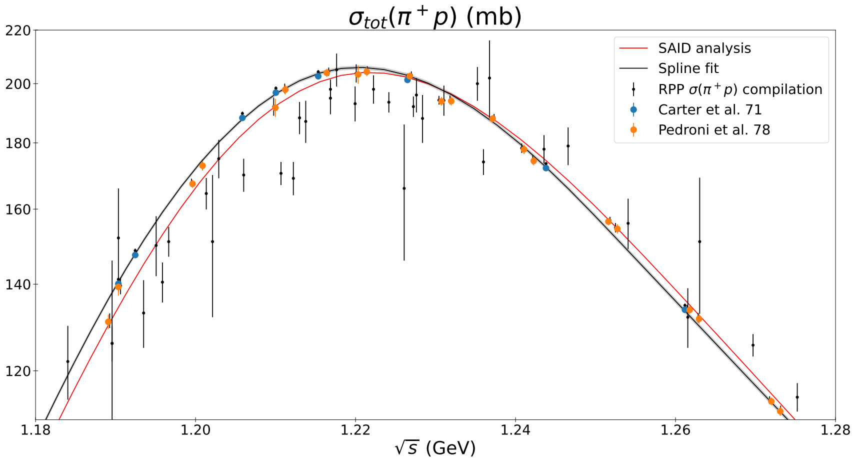

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$*$ In the resonance region, fair agreement with SAID partial-wave analysis (different data)

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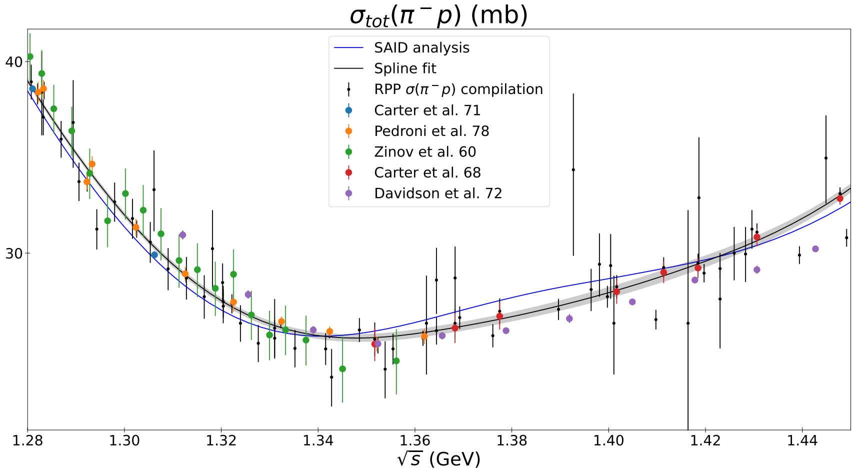

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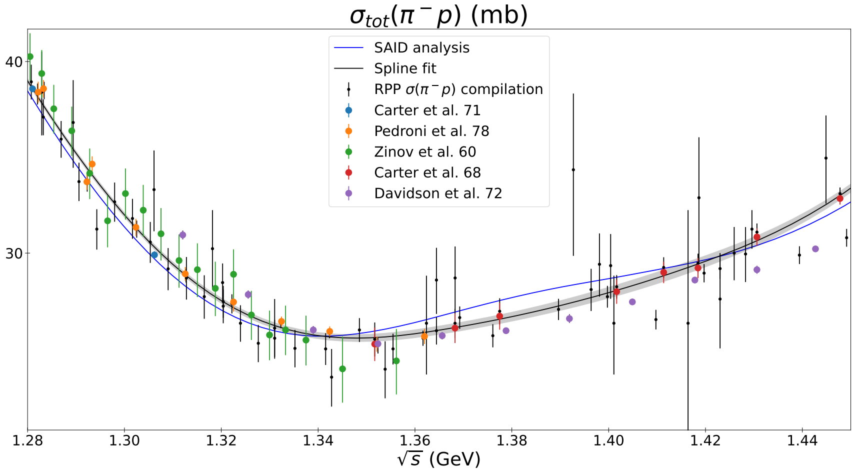

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$\ast$ Special case: $N(1440)$ Roper resonance $\Rightarrow$ Clear differences

$\Rightarrow$ The Roper effect is imperceptible in the $\sigma_{\rm tot}(\pi^\pm p)$ data

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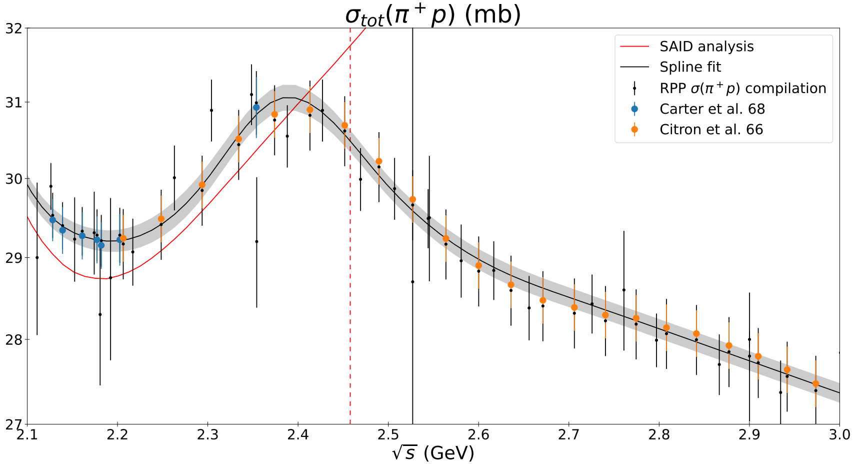

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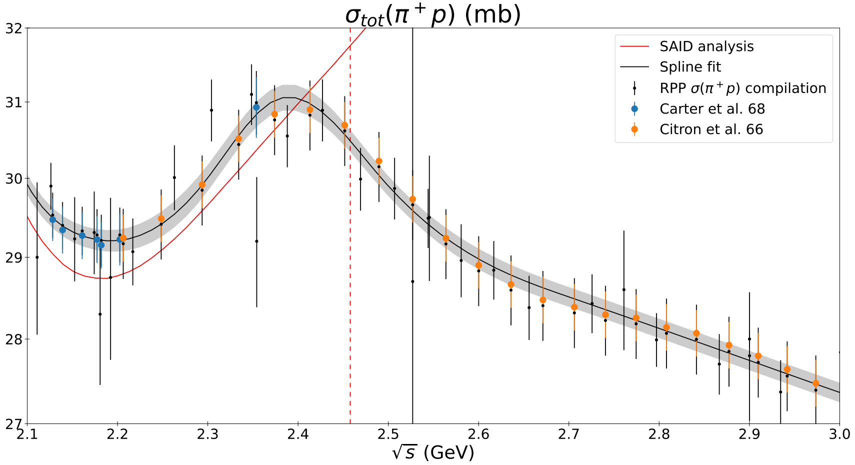

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* Huge differences at higher energies! (Above SAID upper validity limit)

3. πN FDRs' output

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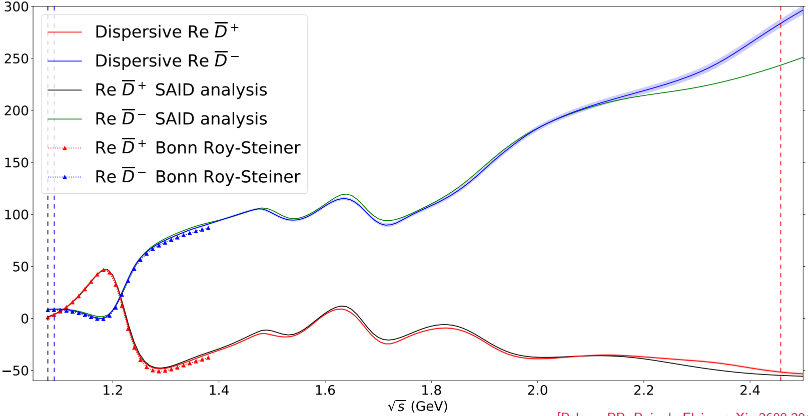

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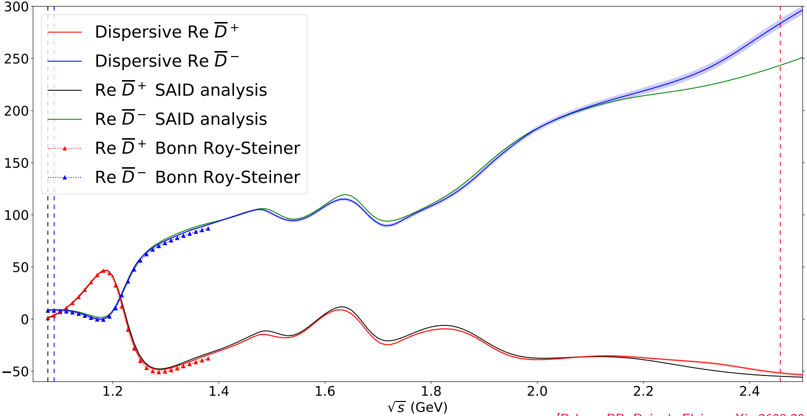

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* Good agreement with both analyses up to 2.2 GeV

4. Resonance determination: FDR_{C_N}

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* Need of a robust and general analytic continuation: **continued fractions**

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$\text{FDR}_{\text{C}_\text{N}}$: "Interpolate" the FDR output with this functional form in a real s segment, which contains **poles** ($\sim N/2$) (already tested in $\pi\pi$)

Pelaez, Rodas, Ruiz de Elvira, PRL 130, 051902 (2023), Pelaez, PR, Ruiz de Elvira, PRD 113, 034018 (2026)

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* **We only keep stable poles under N, segments and fit parameters!**

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$\ast$ But we can distinguish the isospin of resonances!

$$\boxed{D^{\frac{1}{2}} = D^{+} + 2D^{-}} \qquad \boxed{D^{\frac{3}{2}} = D^{+} - D^{-}}$$

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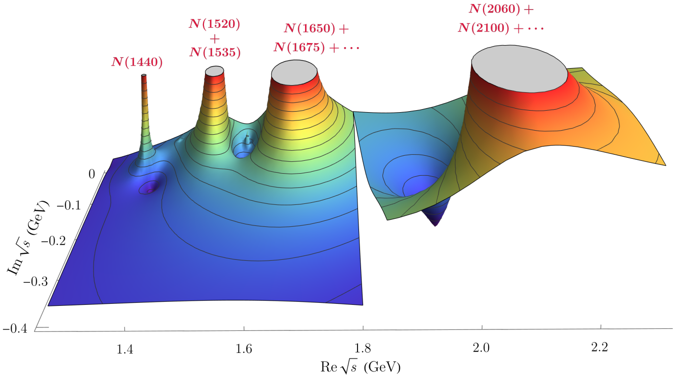
* We can then extract isospin-definite M, Γ, and residues!

4. πN pole-resonance extraction $I = 1/2$



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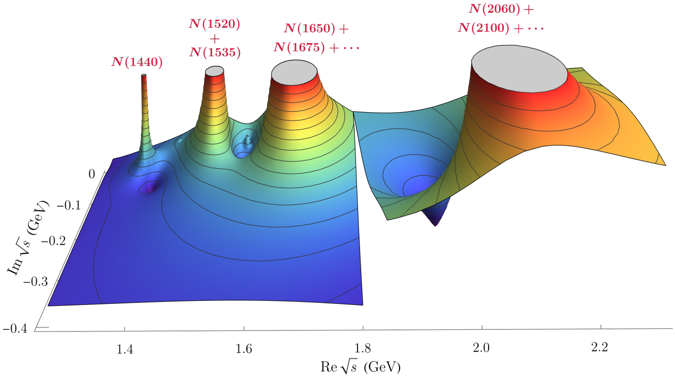

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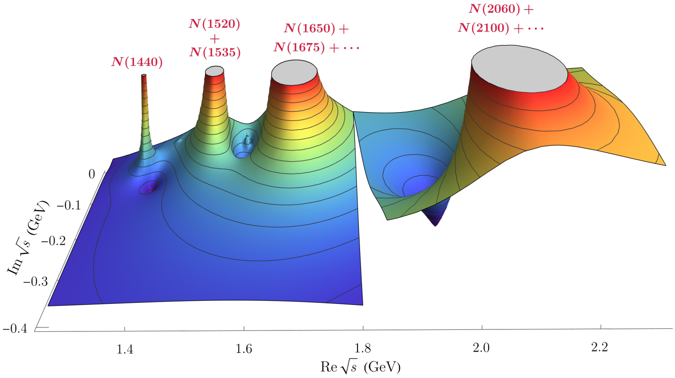
$*$ We obtain 4 stable $I = \frac{1}{2}$ poles that we associate with these resonances



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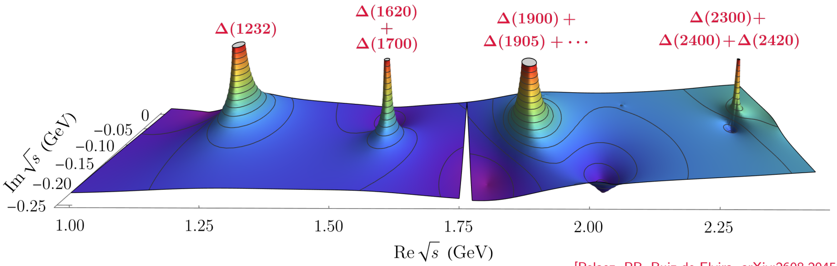
4. πN pole-resonance extraction $I = 3/2$



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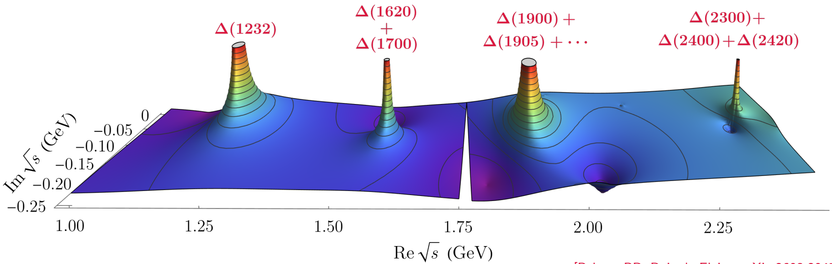

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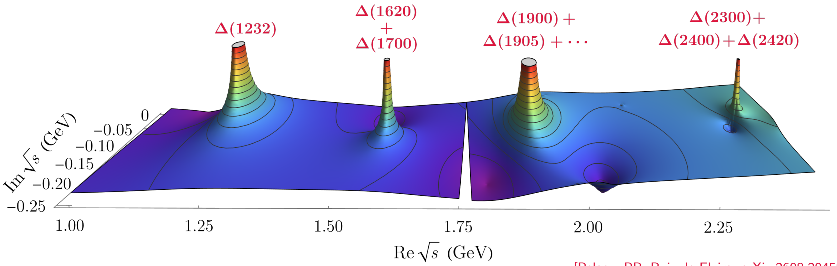

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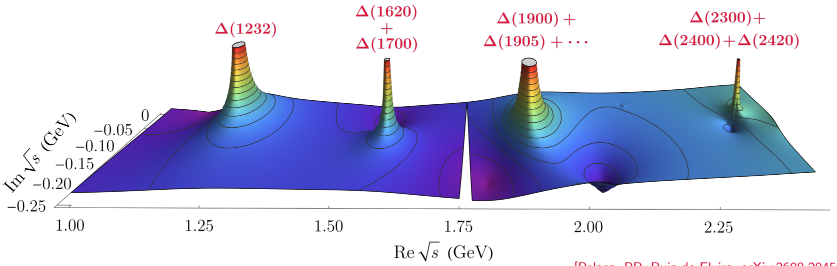

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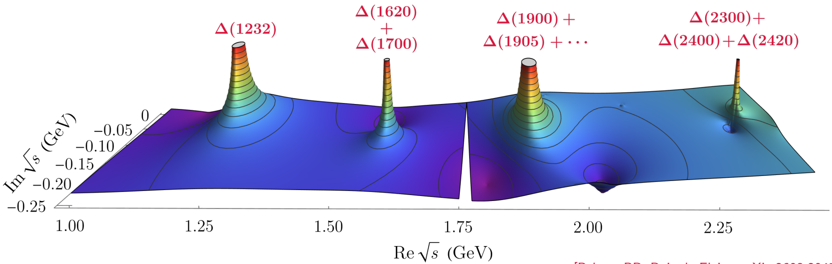
∗ However, we have more information: residue of the poles



[Pelaez, PR, Ruiz de Elvira, arXiv:2608.20458]

∗ We obtain 4 stable $I = \frac{3}{2}$ poles that we associate with these resonances

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⇒ By doing a Cauchy integral enclosing several j resonances

$$\boxed{R_{\rm FDR} = \sum_j R_j}$$

∗ Pole residues help identify the contributing resonances!

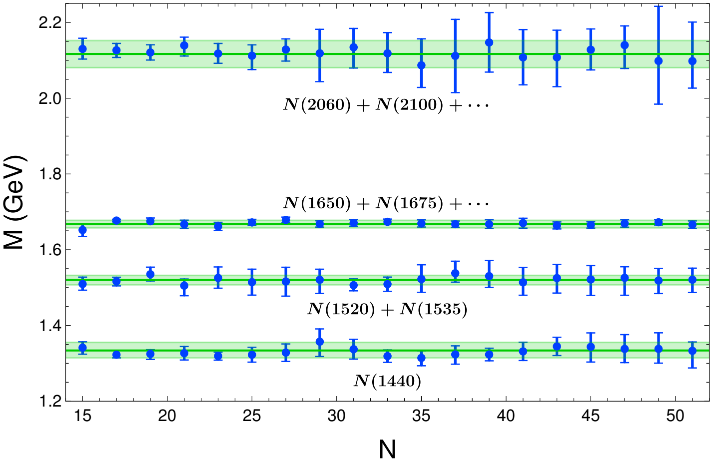
Pole	$\|R_{\rm FDR}\|$ (GeV)	$\|R_{\rm PDG}\|$ (GeV)
$N(1440)$	6^{+4}_{-3}	7.8 ± 0.7
$N(1520) + N(1535)$	18^{+9}_{-8}	12.8 ± 1.6
$N(1650) + \cdots + N(1720)$	32 ± 13	34 ± 6
$N(2060) + \cdots + N(2250)$	133^{+51}_{-47}	77 ± 20
$\Delta(1232)$	25 ± 10	18.3 ± 0.7
$\Delta(1620) + \Delta(1700)$	10 ± 4	7 ± 4
$\Delta(1905) + \Delta(1910) + \Delta(1950)$	32 ± 6	36 ± 7
$\Delta(2420)$	14 ± 3	17 ± 8

[Pelaez, PR, Ruiz de Elvira, arXiv:2608.20458]

And same for the residue phase!

4. FDR_{C_N}: Stability of the analytic continuation

* **Stable** results for different N (e.g. for the $I = \frac{1}{2}$ poles)



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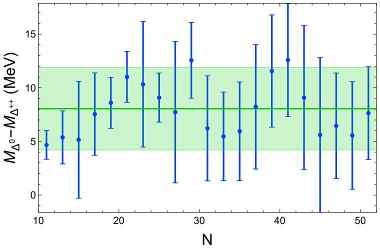
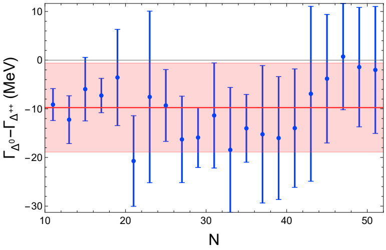
$\Rightarrow$ **Problem:** FDRs involve both $\sigma_\pm \rightarrow$ **non isospin invariant amplitude**

$*$ FDRs for $D_\pm$ in the $\Delta(1232)$ region are dominated by the RHC, i.e. $\sigma_\pm$

$\Rightarrow$ **LHC contribution small** compared to our uncertainties

Results:





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$$\mathbf{M_{\Delta^0} - M_{\Delta^{++}} = 8 \pm 4\,MeV}$$
$$\mathbf{\Gamma_{\Delta^0} - \Gamma_{\Delta^{++}} = -10 \pm 9\,MeV}$$
$$\mathbf{|R_{\Delta^0}| - |R_{\Delta^{++}}|/3 = -0.8 \pm 1.2\,GeV}$$
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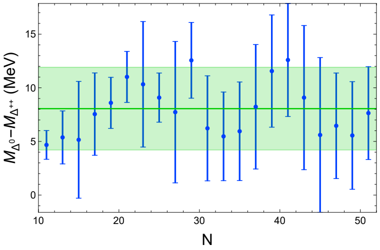

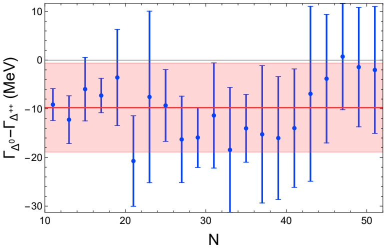

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$*$ Large uncertainties, but clear hints of $\mathbf{M_{\Delta^0} > M_{\Delta^{++}}}$ in agreement with PDG!

Summary

★ Consistent with the PDG estimate and Arndt et al. PRC 74, 045205 (2006) for isolated resonances (Roper is difficult to obtain from $\sigma_{\text{tot}}(\pi^{\pm}p)$ data!)

	$\sqrt{s_{\Delta(1232)}}$ (MeV)
FDR_{C_N}	$(1211 \pm 4) - i\,(52 \pm 4)$
PDG estimate	$(1210 \pm 1) - i\,(50 \pm 1)$
Arndt et al.	$1211 - i\,49.5$

	$\sqrt{s_{N(1440)}}$ (MeV)
FDR_{C_N}	$\left(1334^{+21}_{-26}\right) - i\left(81^{+25}_{-21}\right)$
PDG estimate	$(1370 \pm 10) - i\left(95^{+8}_{-5}\right)$
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★ IB effects in Δ^0/Δ^{++} estimated $\Rightarrow$ Clear indication of $M_{\Delta^0} > M_{\Delta^{++}}$

Thank you for your attention