Quark mass dependence of two-pole structures and the SU(3) limit

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Content

Two-pole structures

Examples:
The $\Lambda(1405)$ and the $D_0(2300)$

An strange Baryon: The case of the $\Lambda(1405)$

Two-body thresholds close in energy

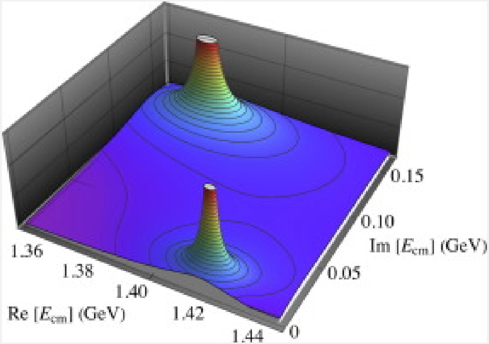
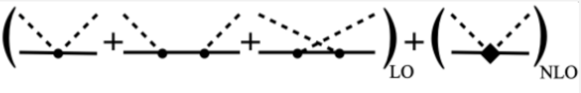


Figure taken from Zhuang, Molina, Geng, Lu, Sci. Bull(2025)

$\pi\Sigma$, $\bar{K}N$



Meson-baryon interaction

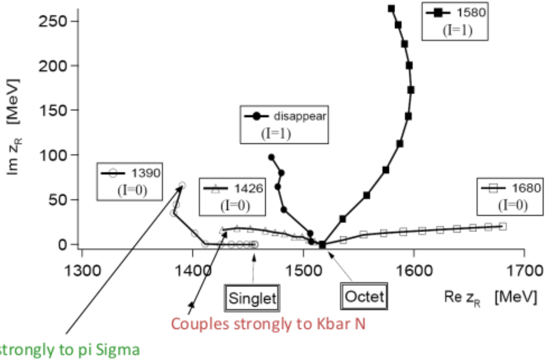
Poles of S=-1 J^P=1/2$^-$ Resonances

Jido, Oller, Oset, Ramos, Meissner NPA03

$$8 \otimes 8 = 1 \oplus 8_s \oplus 8_a \oplus 10 \oplus \overline{10} \oplus 27$$

$$M_i(x) = M_0 + x(M_i - M_0),$$
$$m_i^2(x) = m_0^2 + x(m_i^2 - m_0^2),$$
$$a_i(x) = a_0 + x(a_i - a_0),$$

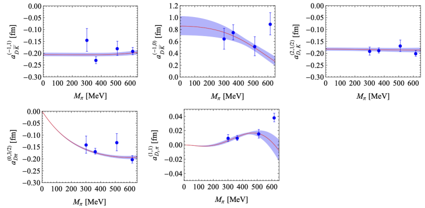
X ∈ [0,1]



Update with LQCD data, Zhuang, Molina, Lu, Geng, Sci. Bull. (2025)

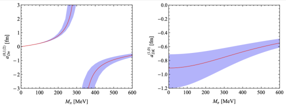
The $D_0(2300)$

Sc. lengths., L.Liu et al. PRD87'13 ($a \simeq 0.12$ fm, $m_\pi \simeq 300 - 620$ MeV).

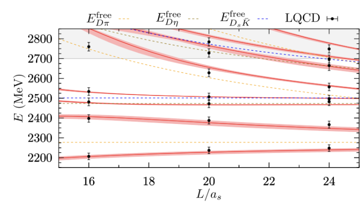


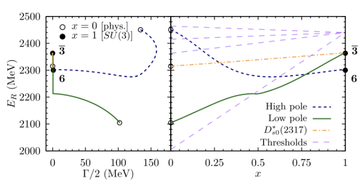
Prediction (S, I)

$(0, 1/2)$ $(1, 0)$



Comparison to LQCD, Moir et al., JHEP10'16, $m_\pi \simeq 400$ MeV.



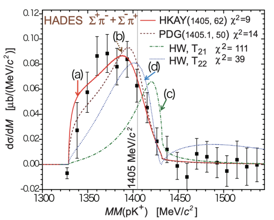


Levels and quark mass in Albaladejo et al. PLB767'17,

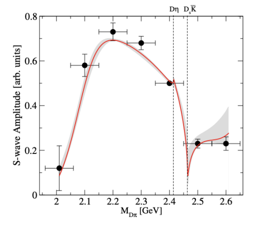
$$m_i = m_i^{\rm phy} + x(m - m_i^{\rm phy}); \quad x = [0, 1].$$

To be or not to be

$\Lambda(1380)$, $\Lambda(1405)$



$D_0^*(2300)$



Not two-pole

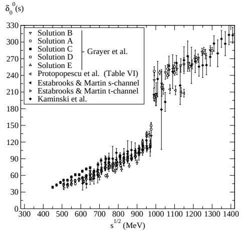


Figure 1: Left: Figure taken from M. Hassanvand et al., PRC87(2013) (theo analyses RIKEN). See also HADES, PRC87(2013) . Center: Fig. taken from Meng-Lin Du et al., PRD98 (2018). See also LHCb article, PRD94(2016). Right: Experimental data for σ, $f_0(980)$.

Other possible examples: $K_1(1270)$, $\Xi(1820)$, $Y(4260)$ ...

The $\Lambda(1405)$

Chiral Unitary Approach for PB

Channels for $I = 0, S = -1$: $\pi\Sigma, \bar{K}N, \eta\Lambda, K\Xi$

- Weinberg-Tomozawa+Born+NLO: $\boxed{V_{ij} = V_{ij}^{\rm LO} + V_{ij}^{\rm NLO}}$

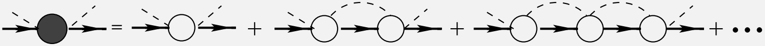
where $V^{\rm LO} = V^{\rm WT} + V^{\rm Born}$. WT and NLO terms (Feijoo2015),

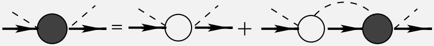
$$V_{ij}^{\rm WT} = -\frac{N_i N_j}{4f^2}\left[C_{ij}(2\sqrt{s} - M_i - M_j)\right], \qquad (1)$$

$$V_{ij}^{\rm NLO} = -\frac{N_i N_j}{f^2}(D_{ij} - 2k_\mu k'^\mu L_{ij}), \qquad (2)$$

$N_i = \sqrt{(M_i + E_i)/2M_i}$. M_i, E_i baryon mass and energy of the channel i

Born, direct s and crossed u (Khemchandani, Martinez-Torres,Oller, 2019)





Interaction $I = 0; S = -1$

SU(3) limit LO+WT+Born s, u channels+NLO

Channel basis - SU(3) basis

$$\begin{bmatrix} |\pi\Sigma\rangle \\ |\bar{K}N\rangle \\ |\eta\Lambda\rangle \\ |K\Xi\rangle \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{\frac{3}{2}}}{2} & -\sqrt{\frac{3}{5}} & 0 & -\frac{1}{2\sqrt{10}} \\ -\frac{1}{2} & -\frac{1}{\sqrt{10}} & \frac{1}{\sqrt{2}} & -\frac{\sqrt{\frac{3}{5}}}{2} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{\sqrt{5}} & 0 & \frac{3\sqrt{\frac{3}{10}}}{2} \\ \frac{1}{2} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{2}} & \frac{\sqrt{\frac{3}{5}}}{2} \end{bmatrix} \begin{bmatrix} |1\rangle \\ |8\rangle \\ |8'\rangle \\ |27\rangle \end{bmatrix}$$

$$C^{\mathrm{WT}}_{\mathrm{SU(3)}} = \begin{bmatrix} 6 & & & \\ & 3 & & \\ & & 3 & \\ & & & -2 \end{bmatrix}$$

$$C^{\mathrm{Born}-s}_{\mathrm{SU(3)}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{10D^2}{3} & -2\sqrt{5}DF & 0 \\ 0 & -2\sqrt{5}DF & 3F^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C^{\mathrm{Born}-u}_{\mathrm{SU(3)}} = \begin{bmatrix} \frac{10D^2}{3} - 6F^2 & 0 & 0 & 0 \\ 0 & -D^2 - 3F^2 & 0 & 0 \\ 0 & 0 & 3F^2 - \frac{5D^2}{3} & 0 \\ 0 & 0 & 0 & \frac{2}{3}(D^2 + 3F^2) \end{bmatrix},$$

$$C^{\mathrm{NLO1}}_{\mathrm{SU(3)}} = \begin{bmatrix} \frac{4}{3}m^2(3b_0 + 7b_D) & 0 & 0 & 0 \\ 0 & \frac{2}{3}m^2(6b_0 + b_D) & -2\sqrt{5}m^2 b_F & 0 \\ 0 & -2\sqrt{5}m^2 b_F & 2m^2(2b_0 + 3b_D) & 0 \\ 0 & 0 & 0 & 4m^2(b_0 + b_D) \end{bmatrix},$$

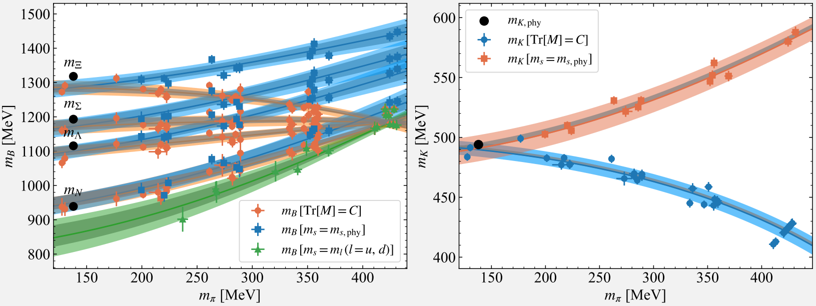
$$C^{\mathrm{NLO2}}_{\mathrm{SU(3)}} = \begin{bmatrix} -6d_2 + 9d_3 + 2d_4 & 0 & 0 & 0 \\ 0 & -3d_2 + d_3 + 2d_4 & -\sqrt{5}d_1 & 0 \\ 0 & -\sqrt{5}d_1 & 9d_2 - d_3 + 2d_4 & 0 \\ 0 & 0 & 0 & 2d_2 + d_3 + 2d_4 \end{bmatrix},$$

CoBChPT

- Input: RQCD, JHEP23. Chiral trajectories $\mathrm{Tr}[M] = \mathrm{C}$, $m_s = m_{s,\mathrm{phy}}$, and $m_s = m_l$, $l = u, d$. m_P, f_P, L_i^r's, from a global fit over the CLS ensembles, Molina and Ruiz de Elvira, JHEP20
- CoBChPT. J. Martín-Camalich, L. S. Geng. M. J. Vicente-Vacas, JHEP05'23

$$m_B = m_0 + m_B^{(2)} + m_B^{(3)} , \tag{3}$$

Figure 3: Baryon masses over the different chiral trajectories.



Loop function in the FV

Scattering amplitude in the FV:

$$\tilde{T}^{-1} = V_0^{-1} - \tilde{G}\,,$$

Loop function in the FV (Martinez-Torres, Dai, Koren, Jido, Oset, PRD85'12)

$$\widetilde{G}(P^0, \vec{P}) = G^{DR}(P^0, \vec{P}) + \lim_{q_{\max}\to\infty} \Delta G(P_0, \vec{P}, q_{\max}),$$

$$\Delta G = \tilde{G}^{co} - G^{co}, \quad \tilde{G}^{co} = \frac{2M_B}{L^3} \sum_{n}^{q_{\max}} \frac{E}{P_0} I(q^*)\,,$$

Energy levels $\boxed{\det(I - V_0 \tilde{G}) = 0}$ (4)

DR and co, Oller, PPNP20:

$$a(\mu) = -\frac{2}{m_i + M_i}\left[m_i \log(1 + \sqrt{1 + \frac{m_i^2}{q_{\max}^2}}) + M_i \log(1 + \sqrt{1 + \frac{M_i^2}{q_{\max}^2}})\right] + 2\log(\frac{\mu}{q_{\max}}). \quad (5)$$

$q_{\max} = 623(20)(20)$ at NLO. Oset, Ramos, NPA98 (630 MeV)

Energy Levels for $\bar{K}N - \pi\Sigma$ scattering ($I = 0$)

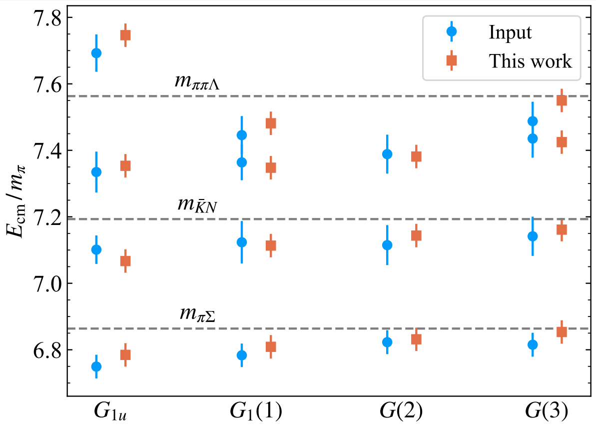


Figure 4: Finite-volume spectrum (circle points) in the center-of-mass frame used as input to constrain the free parameters of the chiral Lagrangian up to NLO. Input: J. Bulava, BaSc, PRD24, PRL24.

LECs, cutoff, SC, poles, couplings ..

Pole positions and couplings:

	m_π (MeV)	Pole (MeV)	$\lvert g_{\pi\Sigma}\rvert$	$\lvert g_{\bar{K}N}\rvert$	$\left\lvert \frac{g_{\pi\Sigma}}{g_{\bar{K}N}} \right\rvert$	$\lvert g_{\eta\Lambda}\rvert$	$\lvert g_{K\Xi}\rvert$
4 channels	138	$1376(10)(10) - i142(19)(5)$	2.8(1)(2)	2.3(6)(3)	1.2(5)(1)	1.5(3)(3)	1.0(1)(2)
		$1418(11)(2) - i11(6)(3)$	1.1(5)(2)	3.0(2)(2)	0.4(1)(1)	2.0(2)(5)	0.4(1)(3)
	200	$1366^{(43)(19)}_{(6)(3)} - i57^{(42)(23)}_{(57)(57)}$	3.7(7)(4)	1.8(8)(6)	2.1(5)(3)	1.2(3)(3)	0.8(2)(2)
		$1450(15)(11) - i15(9)(5)$	1.5(7)(4)	3.1(5)(4)	0.5(2)(1)	2.1(3)(6)	0.5(1)(3)
2 channels	200	$1387(7)(6)$	3.4(10)(10)	1.8(5)(8)	1.9(10)(5)	...	...
		$1455(14)(6) - i29(11)(7)$	2.2(7)(3)	3.8(6)(3)	0.6(1)(1)	...	...

TABLE III. Pole positions and couplings of the $\Lambda(1405)$ for m_π = 138 and 200 MeV.

Pole positions and couplings in LQCD ($m_\pi \simeq 200$ MeV):

$z_1 = 1392(9)(2)(16)$ MeV, $z_2 = 1455(13)(2)(17) - i11.5(4.4)(4)(0.1)$

Ratios of the couplings:

$$\left|\frac{g^{(1)}_{\pi\Sigma}}{g^{(1)}_{\bar{K}N}}\right| = 1.9(4)_{\mathrm{st}}(6)_{\mathrm{md}}, \quad \left|\frac{g^{(2)}_{\pi\Sigma}}{g^{(2)}_{\bar{K}N}}\right| = 0.53(9)_{\mathrm{st}}(10)_{\mathrm{md}},$$

Comparison with experiment

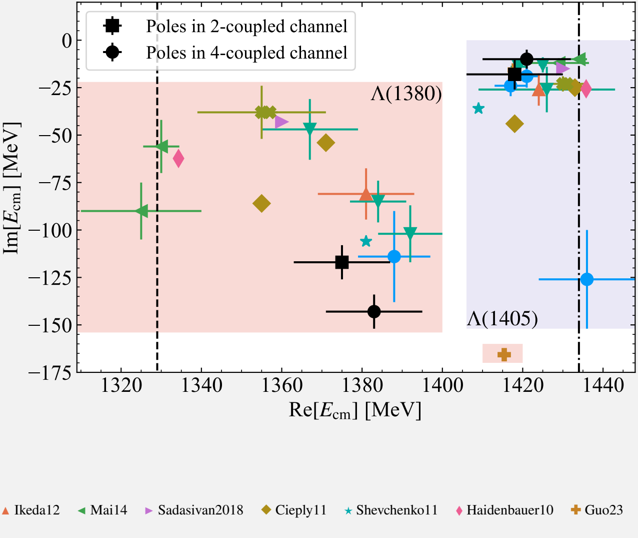


Figure 5: Pole positions of the two $\Lambda(1405)$ in comparison with other studies.

SU(3) limit

Coupling to the representations in the SU(3) limit. We define

$$|8_a> = \cos\theta|8> -\sin\theta|8'>,$$
$$|8_b> = \cos\theta|8> +\sin\theta|8'>, \tag{6}$$

$$I = 0; \quad a_{\bar{K}N} = a_{\pi\Sigma} = -2.08; \quad m_\pi = 423 \text{ MeV}; \quad \theta \simeq -60^o$$

	WT			LO [WT+Born]			LO+NLO		
	z_1	z_2	z_3	z_1	z_2	z_3	z_1	z_2	z_3
Pole (MeV)	1556(2)	1606(1)	1606(1)	1548(3)	1601(1)	1607(1)	1573(6)(6)	1589(7)(5)	1603(9)(10)
$\lvert g_{(1)}\rvert$	3.0(1)	0	0	3.0(1)	0	0	2.8(1)(1)	0	0
$\lvert g_{(8_a)}\rvert$	0	0.8(1)	0	0	1.8(1)	0	0	2.4(8)(2)	0
$\lvert g_{(8_b)}\rvert$	0	0	0.8(1)	0	0	0.4(1)	0	0	1.7(6)(5)

TABLE IV. Pole positions and couplings to the relevant multiplets in the SU(3) limit.

Up to NLO, i.e., $\Delta E^{(8)} = E^{(8_b)} - E^{(8_a)} = 14$ MeV. Splitting in agreement with Guo, Kamiya, Mai, Meissner, but masses quite different: 1704 and 1788 MeV, PLB23

Trajectories of the poles towards SU(3)

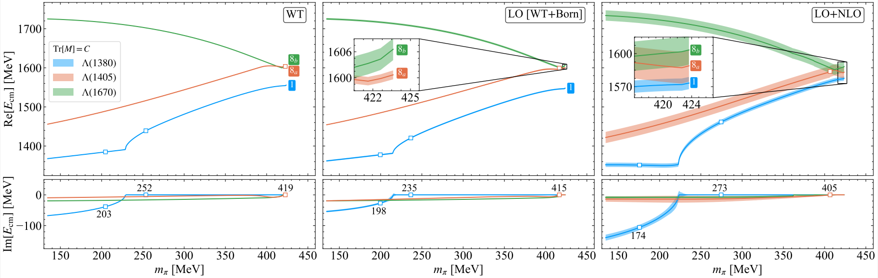


Figure 6: Trajectories of the three poles up to NLO for the Λ(1405) and Λ(1670) along the curve Tr[M] = C.

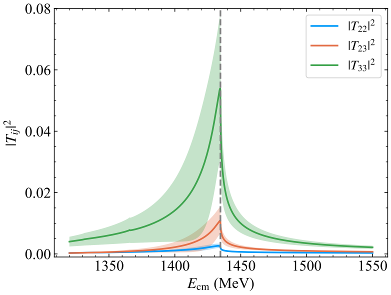
We do not observe that the trajectories exchange when the NLO is included (Guo, Kamiya, Mai, Meissner PLB23). Result in qualitative agreement with Jido, Oset, Ramos, Meissner, NPA03, and Bruns, Cieply, NPA22.

See Z. Zhuangs's talk on Thursday (B, 16.30)

Prediction for $I = 1$

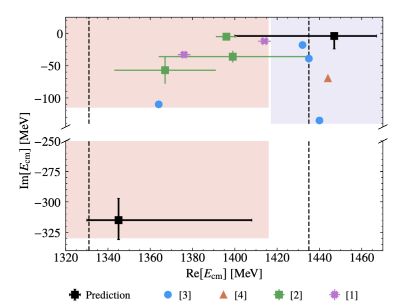
TABLE V. Pole positions and couplings of the two $I = 1$ states at the physical point.

Pole (MeV)	$\lvert g_{\pi\Lambda}\rvert$	$\lvert g_{\pi\Sigma}\rvert$	$\lvert g_{\bar{K}N}\rvert$	$\lvert g_{\eta\Sigma}\rvert$	$\lvert g_{K\Xi}\rvert$
$1345^{(63)}_{(15)} - i315^{(16)}_{(18)}$	0.3(1)	2.2(2)	2.3(2)	0.8(4)	1.6(1)
$1447^{(25)}_{(47)} - i4^{(20)}_{(4)}$	0.9(1)	1.2(2)	2.5(2)	1.6(4)	0.3(1)



Consistent with recent Belle data

$m_\pi = 138$ MeV



[1] Z.-H. Guo et al, PRC, 87, 035202 (2013) [2] K. P. et al, PRC, 100, 015208 (2019)

[3] J.-X. Lu et al, PRL, 130, 071902 (2023) [4] J. A. Oller et al, PLB, 500:263-272 (2001)

(PRL130’23 exp.

$m = 1434.3 \pm 0.6 \pm 0.9$,

$\Gamma = 11.5 \pm 2.8 \pm 5.3$ MeV)

The $D_0^*(2300)$

πD scattering: HMChPT at NLO

LO lagrangian for light-heavy meson interactions (Wise, PRD45'92),

$$\mathcal{L}^{(1)}_{\mathcal{P}\phi} = \mathcal{D}_\mu \mathcal{P} \mathcal{D}_\mu \mathcal{P}^\dagger - M_H^2 \mathcal{P}\mathcal{P}^\dagger \tag{7}$$

with $\mathcal{P} = (D^0, D^+, D_s^+)$, $\mathcal{D}_\mu \mathcal{P} := \mathcal{P}(\overleftarrow{\partial}_\mu + \Gamma_\mu^\dagger)$, $\Gamma_\mu = \frac{1}{2}(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger)$, $u^2 = \exp(i\frac{\sqrt{2}\phi}{f})$. Leads to,

$$\text{Weinberg – Tomozawa} \qquad V_{\mathrm{WT}} = -\frac{C_{\mathrm{LO}}}{4 f_i f_j}(s-u),$$

For NLO, see Guo, Hanhart, Krewald, Meissner, PLB666'08, EPJA40'09,

$$V_{\mathrm{NLO}} = \frac{-1}{f_i f_j}\left(C_{h_0} h_0 + C_{h_1} h_1 + H_{24}(s,t,u) + H_{35}(s,t,u)\right),$$

Free parameters, $h_{0,\cdots 5}$, and $a(\mu)$.

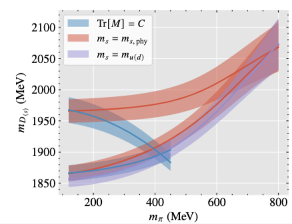
Prediction for the $D_0^*(2300)$

$8 \otimes \bar{3} = 6 \oplus \overline{15} \oplus \bar{3}$; Channels: πD, ηD, $\bar{K} D_s$. Relation to the SU(3) basis; Fit to $m_{D^{(*)}_{(s)}}$ one-loop NLO, F. Gil-Dominguez, R. Molina, PLB'23,

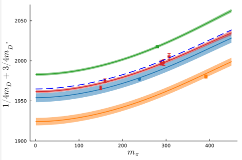
$$\begin{pmatrix} |\bar{3}\rangle \\ |6\rangle \\ |\overline{15}\rangle \end{pmatrix} = \begin{pmatrix} -\frac{3}{4} & -\frac{1}{4} & -\sqrt{\frac{3}{8}} \\ \sqrt{\frac{3}{8}} & -\sqrt{\frac{3}{8}} & -\frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} & -\sqrt{\frac{3}{8}} \end{pmatrix} \begin{pmatrix} |\pi D\rangle \\ |\eta D\rangle \\ |\bar{K} D_s\rangle \end{pmatrix}$$

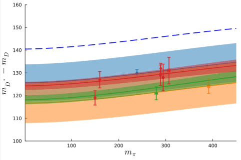
Interaction very attractive in the $\bar{3}$,
attractive in the 6
and repulsive in the $\overline{15}$

$$C_{\mathrm{LO}} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$



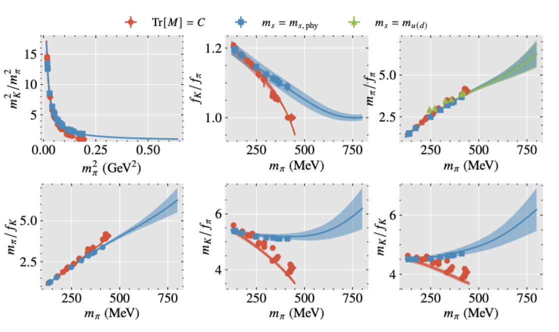
Charm meson masses NLO HHChPT





Decay constant and mass fits

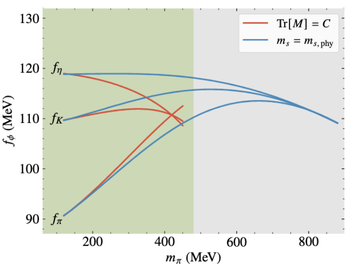
We perform a fit to m_P, f_P, $P = \pi, K, \eta$ LQCD data up to NNLO, $N_f = 3$.
$f_a = f_0(1 + \bar{f}_a^{(4)} + \bar{f}_{ct,a}^{(6)} + \bar{f}_{loop,a}^{(6)})$, $\simeq 21$ pars. Amoros, Bijnens, NPB568'00



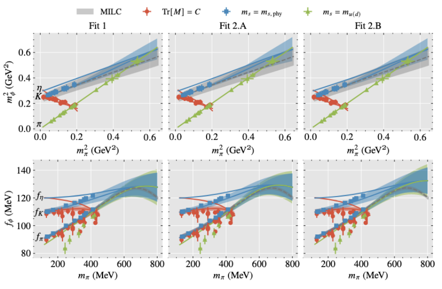
- Fit 1: Global fit with all free pars.
- Fit 2: The LASSO method is applied to the N2LO LECs. Fit 2A, starting with N2LO fits $C_{ir} = 0$ to data to fix L_{ir} first, then refit. Fit 2B, leaving C_{ir} free.

Decay constant and mass fits

Previous result at NLO

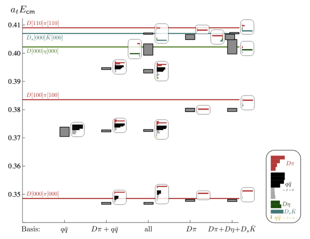


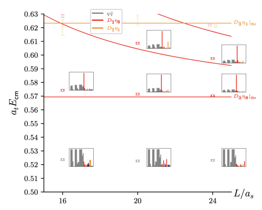
New result at N2LO



The $D_0(2300)$

Fit to energy levels: DK, $I = 0$, $D\pi$, $D\eta$, $D_s\bar{K}$, $I = 1/2$, for $m_\pi = 239, 391, 688$ MeV, HadSpec, JHEP 02 (2021) 100, JHEP 07 (2021) 123, JHEP 07 (2024) 012

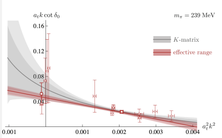


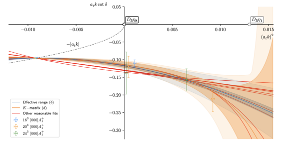


$$a_t\sqrt{s_0} = (0.361 \pm 0.011) - \tfrac{i}{2}(0.070 \pm 0.037)$$
$$a_t c = (0.32 \pm 0.13)\exp i\pi(-0.59 \pm 0.41)$$

$a_t m = 0.5225 - 0.5239$ and $a_t c = 0.349 - 0.630$

$m = 2432 - 2439$ MeV and $c = 1620 - 2930$ MeV

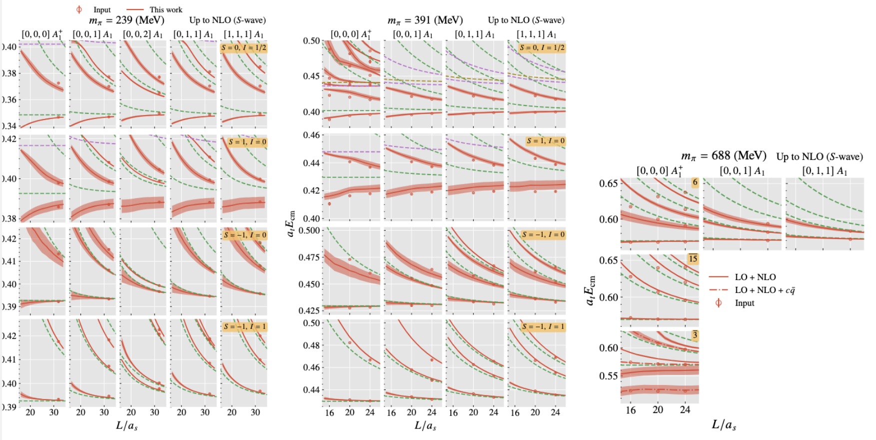
239 

700 

Energy levels

Two fits: I) UChPT NLO (wo $\bar{3}$ data); II) NLO+$Q\bar{q}$; $V = V_{NLO} + V_{c\bar{q}}$;

$V_{c\bar{q}} = \frac{g^2}{s - m_{c\bar{q}}^2}$; $m_{c\bar{q}} = m_D + m_{c\bar{q}}^0 - m_{D^0}$; $g = g_0 + (m_\pi - m_{\pi,\mathrm{phys}})g_1$



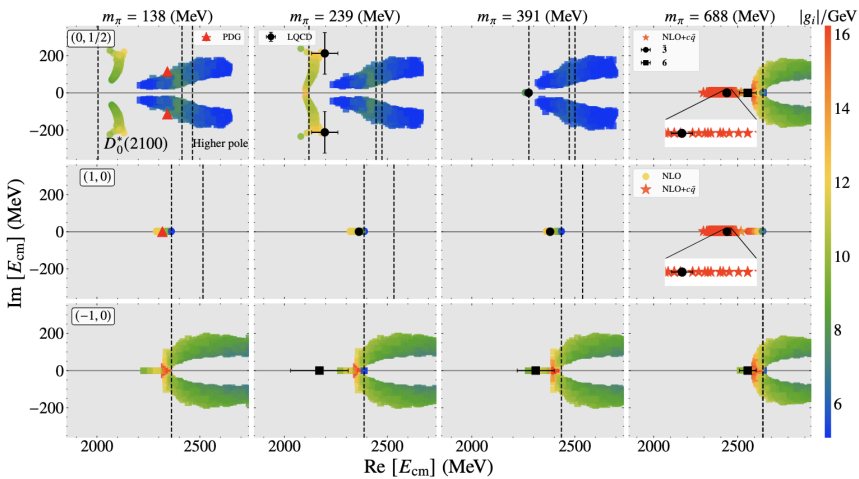
Pole positions

m_π (MeV)	R.S.	Pole (MeV)	$\lvert g_{D\pi}\rvert$ (GeV)	$\lvert g_{D\eta}\rvert$ (GeV)	$\lvert g_{D_s\bar{K}}\rvert$ (GeV)
138	$[-++]$	$2094(7)(1) - i111(7)(13)$	9.6(2)(2)	2.0(2)(3)	6.8(5)(3)
	$[---]$	$2463(58)(27) - i108(14)(12)$	4.4(1)(6)	6.3(1)(5)	6.1(4)(3)
239	$[-++]$	$2125(7)(1) - i84(9)(17)$	10.6(1)(1)	2.5(2)(4)	7.8(5)(3)
	$[---]$	$2492(66)(19) - i113(16)(9)$	4.7(2)(5)	6.1(2)(3)	5.9(2)(4)
391	$[+++]$	$2277(7)(4)$	2.3(1)(16)	0.6(4)(1)	1.8(8)(5)
	$[---]$	$2549(68)(15) - i121(16)(7)$	4.9(1)(4)	5.8(1)(3)	5.3(2)(2)

m_π (MeV)	R.S.	Threshold (MeV)	Pole (MeV)	$\lvert g_{\bar{3}}\rvert$ (GeV)	$\lvert g_6\rvert$ (GeV)
420 ($\mathrm{Tr}[M] = C$)	$[+]$	2317	$2286(2)(13)$	10.7(3)(10)	0
	$[-]$		$2462(70)(10) - i137(18)(3)$	0	9.1(2)(2)
688	$[+]$	2650	$2612(6)(14)$ (NLO)	13.2(5)(20)	0
			$2443(25)(1)$ (NLO + $c\bar{q}$)	16.8(4)(5)	0
	$[-]$		$2735^{(60)(120)}_{(40)(180)} - i107^{(60)(10)}_{(50)(60)}$	0	9.2(3)(4)
780	$[+]$	2841	$2816(7)(11)$	13.0(5)(27)	0
	$[-]$		$2846(50)(5) - i61^{(10)(30)}_{(30)(30)}$	0	10.5(3)(10)

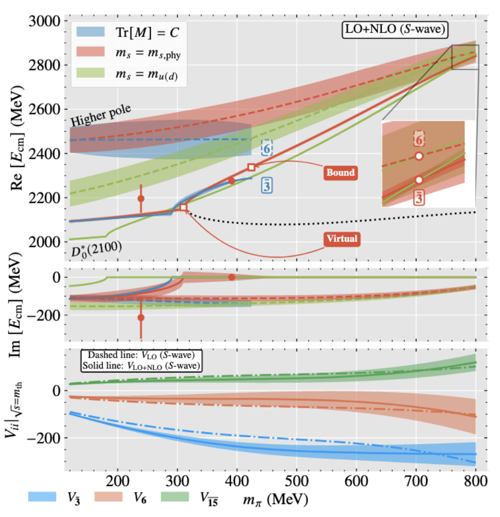
Fit consistent with work on DK, Gil-Dominguez and Molina, PRD109'24

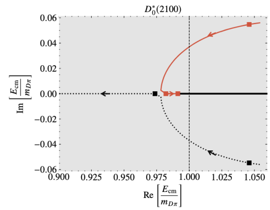
Pole positions

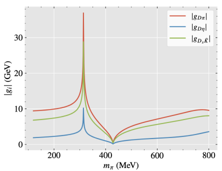


In fit II, the compositeness at the SU(3) limit for the $D_0(2100)$ is $X = 0.63(11)(26)$, compatible with the LQCD result, $0.56(2)(10)$, while in fit I, $X \simeq 1$.

Quark mass dependence

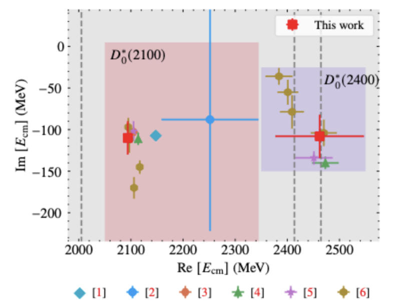




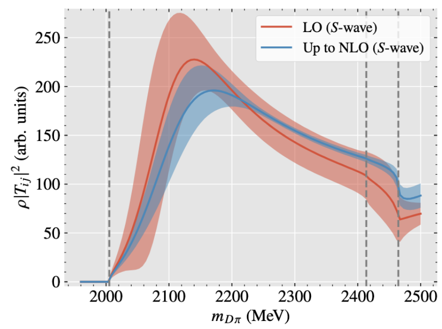


Similar to the σ. See also, J. Luo, B. Wu, P. P. Shi and M. L. Du, 2604.18286.

Physical point



[1] D.G et al, PRD.76.074016 (2007)
[2] H. Yan et al, PRD.111.014503 (2015)
[3] M. A. et al, PRD.89.014026 (2014)
[4] ZHG et al, PRD.92.094008 (2015)
[5] M. A. et al, PLB.767.465 (2017)
[6] ZHG et al, EPJC.79.13 (2019)



Left: Comparison to other works at the physical point. Right: Result at NLO.

Conclusions

Conclusions

- The combination of finite volume-EFT with LQCD is an useful tool to study the quark mass dependence of resonances
- We have conducted a precise study for the $\Lambda(1405)$ quark mass dependence and the two-pole trajectories to the SU(3) limit that supports the two-pole structure
- We find also that this LQCD data analysis is consistent with experimental data Z. Zhuang, Molina, Junxu-Lu, L. S. Geng, SciBull25
- We have conducted an extrapolation of the quark mass dependence of the $D_0^*(2300)$ to the SU(3) limit, where we have a bound state coupling to the $\bar{3}$ and a resonance in the 6. To do that we needed to extend the formulas of the meson masses and decay constants beyond NLO. See Zhuang, Gil-Dominguez, Molina, 2606.03826[hep-ph]

Comparison to other works