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Effect of the lightest scalar meson as a dilaton on hadron-hadron scatterings

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Talk @ The 15th International Workshop on the Physics of Excited Nucleons (NSTAR 2026)
September 7th to 11th, 2026, Seville, Spain

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Outline

I. Introduction

II. Chiral-scale effective field theory

III. Hadron-hadron scattering at leading order

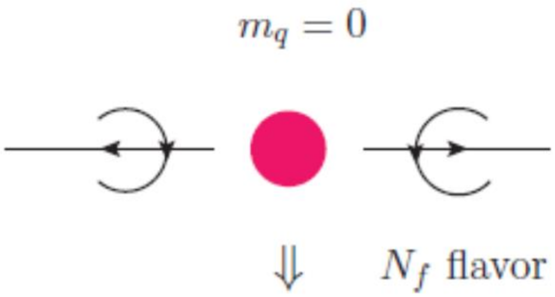
- Pion-Pion scattering
- Pion-N scattering

IV. Summary and discussion

I. Introduction

Symmetry and symmetry breaking play dominant roles in particle and nuclear physics.

- Chiral symmetry and chiral symmetry breaking controls the dynamics of light hadron physics at low energy.



$$SU(N_f)_L \times SU(N_f)_R \Rightarrow SU(N_f)_V$$

Chiral symmetry breaking ⇒ Scalar mesons are integrated out,

Pion as NGBs are left DoFs.

I. Introduction

- ChPT (Chiral EFT), anchored on the fundamental symmetriess of QCD, self-consistent power counting, achieved great successes in the description of hadron dynamics;

- S. Weinberg, Physica A 96 (1979) 1-2, 327-340; Phys.Lett.B 251 (1990) 288-292; Nucl.Phys.B 363 (1991) 3-18.
- R. Machleidt, D.R. Entem, Phys.Rept. 503 (2011) 1-75.

- ***Finite nuclei as well as infinite nuclear matter at low density*** can be fairly accurately accessed by ***nuclear EFTs***, ``pionless or pionful, (*s*EFT)" anchored on relevant symmetries and invariances .

I. Introduction

Scalar meson with $m_\sigma \cong 600$ MeV, very important in particle & nuclear physics.
Known long time ago.

- PDG for $f_0(500)$:

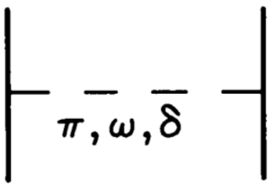
$$m = 400 \sim 550 \text{ MeV}$$
$$\Gamma = 400 \sim 700 \text{ MeV}$$

But, a local bosonic field works;

e.g., Bonn boson exchange potential;

RMF

R. Machleidt et al., The Bonn meson-exchange model for the nucleon–nucleon interaction



$$\begin{aligned}\mathcal{L} = &\bar{\psi}\left(\gamma\left(i\partial - g_\omega\omega - g_\rho\vec{\rho}\vec{\tau} - eA\right) - m - g_\sigma\sigma\right)\psi \\ &+ \frac{1}{2}(\partial\sigma)^2 - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega^2 \\ &- \frac{1}{4}\vec{\mathrm{R}}_{\mu\nu}\vec{\mathrm{R}}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^2 \\ &- \frac{1}{4}\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu},\end{aligned}$$

- B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. 16, 1 (1986)

I. Introduction

- Standard EFTs, as befits their premise, are expected to ***break down at some high density*** (and low temperature) relevant to, say, the interior of massive stars.

E.g, In sEFT, the power counting in density is $O(k_F^q)$. For the normal nuclear matter, the expansion requires going to $\sim q = 5$.

➢ J. W. Holt, M. Rho and W.Weise, 1411.6681

➢ It is interesting to include the lightest scalar meson in ChPT (a) as an explicit degree of freedom, (b) anchored on symmetries of QCD, and (c) with self-consistent power counting (errors can be estimated).

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I. Introduction

- ChPT is based on the nonlinear realization of chiral symmetry, the 4th component of the chiral four-vector is integrated out.
- How to include the sigma meson (0^{++}) field in ChPT?
- Trace anomaly in effective models：

anomaly match

Trace anomaly in QCD $\longleftrightarrow$ Trace anomaly EFT

$$\theta^{\mu}_{\mu} = \frac{\beta(g)}{2g} G_{\mu\nu} G^{\mu\nu} \propto \chi^4 \longleftrightarrow \delta_D V(\chi) \propto \chi^4$$

Dilaton compensator;
A source of scalar meson in EFT

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I. Introduction

PHYSICAL REVIEW D VOLUME 2, NUMBER 4 15 AUGUST 1970

Broken Chiral and Conformal Symmetry in an Effective-Lagrangian Formalism

C. J. Isham, Abdus Salam,* and J. Strathdee
International Atomic Energy Agency
and
UNESCO, International Centre for Theoretical Physics, Miramare, Trieste, Italy
(Received 13 April 1970)

The simultaneous breaking of conformal and chiral symmetry is investigated within the framework of nonlinear realizations and effective Lagrangians. The explicit introduction of a massless dilaton field, χ enables conformal invariance to be preserved in Lagrangians for massive matter fields. It is shown that the

7.A.2 Nuclear Physics B22 (1970) 478-492. North-Holland Publishing Company

ASPECTS OF CONFORMAL SYMMETRY AND CHIRALITY

John ELLIS
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, England

Received 15 June 1970

Abstract: Two simple Lagrangian models combining conformal symmetry and current algebra are exhibited. Either both symmetries are realized linearly, or both symmetries are realized in the Goldstone manner. The scalar isoscalar to pseudoscalar meson couplings differ from those obtained by naive low-energy theor-

PHYSICAL REVIEW D VOLUME 21, NUMBER 12 15 JUNE 1980

Effective Lagrangian with two color-singlet gluon fields

Joseph Schechter
Physics Department, Syracuse University, Syracuse, New York 13210
(Received 10 December 1979)

An effective Lagrangian with scalar and pseudoscalar "matter" fields as well as scalar and pseudoscalar (gauge-invariant) color-singlet gluon fields is constructed. It is a representation of quantum chromodynamics in that it has the same symmetry structure as that theory. The present model is a generalization of a one-(pseudoscalar) gluon-field effective Lagrangian which compactly summarizes the results of the $1/N_c$ approximation, gives a simple picture for the "U(1) problem," and essentially reduces to a type of generalized σ model. The new feature here is that, in addition to the chiral-anomaly equation, the trace-anomaly equation is automatically fulfilled. This gives a framework for discussion of the properties of a 0^+ color-singlet gluon field. The model may be formulated either so that this field is eliminated by a constraint

- Trace anomaly in QCD

$$\partial_\mu D^\mu = \frac{\beta(\alpha_s)}{4\alpha_s} Tr\,(G_{\mu\nu}G^{\mu\nu})$$

A possible way for a scalar meson.

Anomaly matching: $\theta^\mu_\mu = -\frac{m_\sigma^2}{4f_\sigma^2}\chi^4$

How to construct an EFT with power counting mechanism like, e.g., NGB in ChPT?

II. Chiral-scale effective field theory

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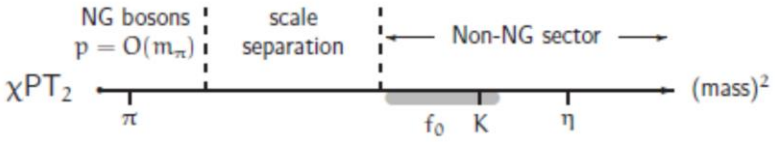
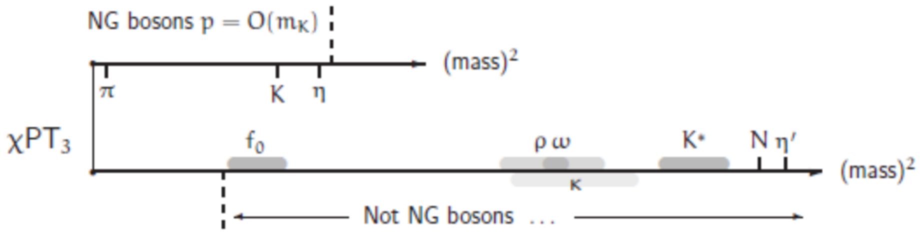
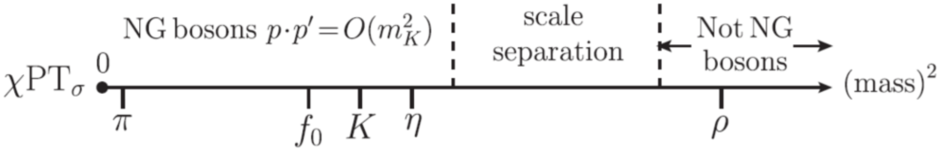
A resolution

Different types of ChPT:



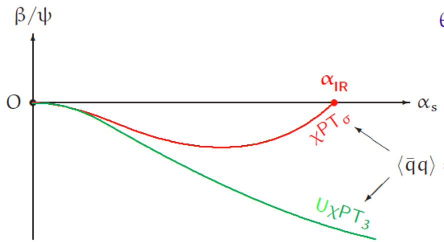
Standard χPT_2 OK: $m_s \neq 0$ stops θ^μ_μ from vanishing. Consistent with either χPT_3 or χPT_σ.





Crewther and Tunstall , PRD91(2015), 034016, e-Print: 1312.3319

Possible realization:



$$\theta^\mu_\mu = \frac{\beta(\alpha_s)}{4\alpha_s} G^2 + (1+\gamma_m) \sum_{q=u,d,s} m_q \bar{q} q$$

IR fixed point: $\beta(\alpha_{IR}) = 0$

$\chi PT_\sigma \Rightarrow$ expand in θ^μ_μ

$\alpha_s \lesssim \alpha_{IR}$, $m_{u,d,s} \sim 0$

about scale-*dependent* $|\text{vac}\rangle$

$\Rightarrow$ NG bosons π, K, η, σ.

✓ $m_\sigma^2 \propto \Delta\alpha_s \to 0, \alpha_s \to \alpha_{IR}$

✓ $m_\pi^2 \propto m_s \to 0, m_q \to 0$

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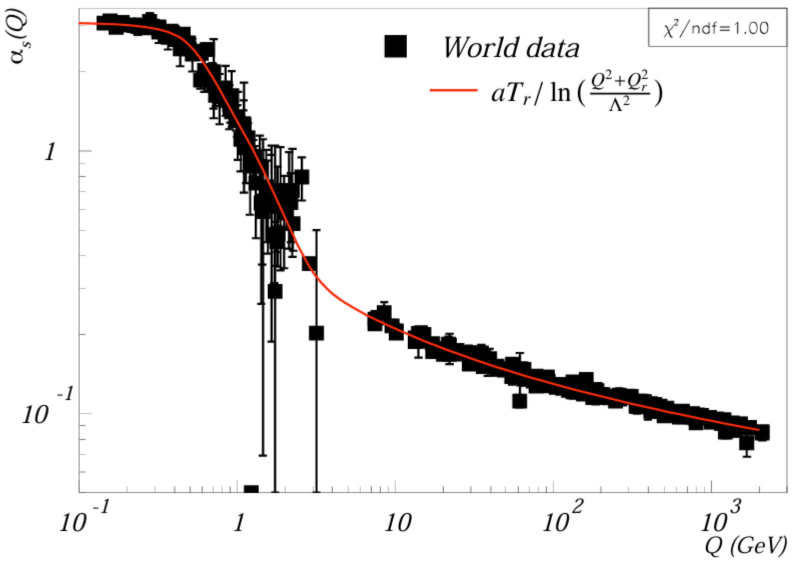
II. Chiral-scale effective field theory

➤ A. Deur, e-Print: 2502.06535 [hep-ph].



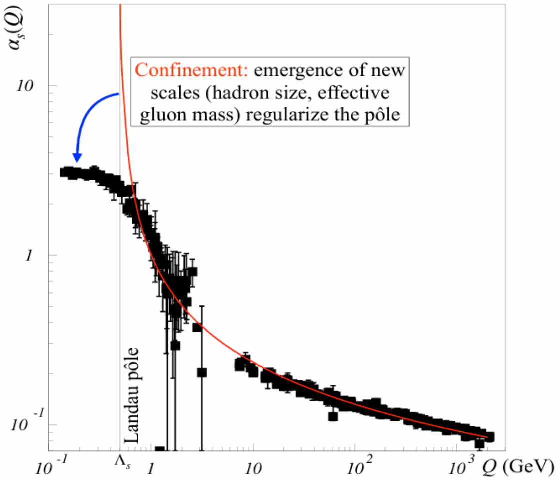


Fig. 8: **Regularization of the Landau pôle.**

➤ The absence of Q^2-dependence of α_s in the IR, i.e., the vanishing QCD β-function, is variously named the freezing of α_s, the conformal window of QCD, or the $Q^2=0$ fixed point.

II. Chiral-scale effective field theory

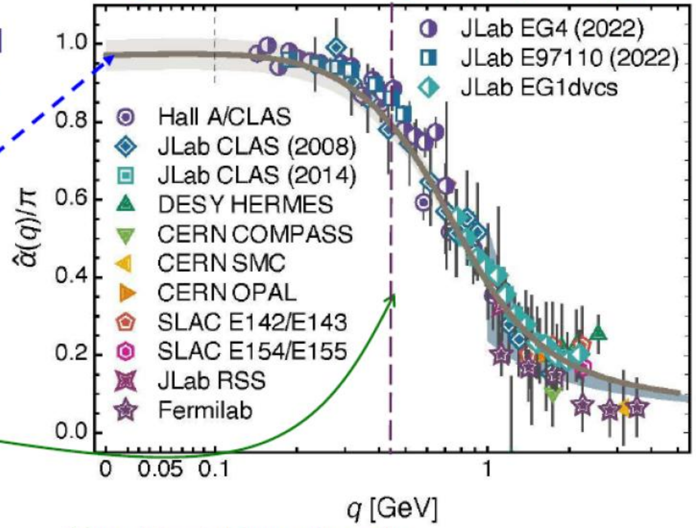
ECT*
EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

Dyson-Schwinger Equations in Modern Mathematics and Physics (DSEMP2014) Trento, Italy, September 22-26, 2014

Effective charge from lattice QCD, *Zhu-Fang Cui, Jin-Li Zhang et al.*, *NJU-INP 014/19*, arXiv:1912.08232 [hep-ph], Chin. Phys. C 44 (2020) 083102/1-10

Process independent effective charge = running coupling

- Modern theory – exploiting pinch technique and background field method – enables unique QCD analogue of
  "Gell-Mann – Low"
  running charge to be
  rigorously defined and calculated
- Analysis of QCD's gauge sector
  yields a *parameter-free prediction*
- N.B. Qualitative change in $\hat{\alpha}_{PI}(k)$ at $k \approx ½\, m_p$
- No Landau Pole
- Below $k \sim \hat{m}_0$, interactions become scale independent, just as they were in the Lagrangian; so, QCD becomes practically conformal again



✓ *Process independent strong running coupling*
Daniele Binosi *et al.*, arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7

Slide borrowed from C. Roberts

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II. Chiral-scale effective field theory

$$\mathcal{L}_{\chi PT_\sigma}^{LO} = \mathcal{L}_{inv}^{d=4} + \mathcal{L}_{anom}^{d>4} + \mathcal{L}_{mass}^{d<4},$$

$$\begin{aligned}
\mathcal{L}_{inv}^{d=4} &= c_1 \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^2 \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) \\
&+ \frac{1}{2} c_2 \partial_\mu \chi \partial^\mu \chi + c_3 \left(\frac{\chi}{f_\chi}\right)^4, \\
\mathcal{L}_{anom}^{d>4} &= (1-c_1)\frac{f_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^{2+\beta'} \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) \\
&+ \frac{1}{2}(1-c_2)\left(\frac{\chi}{f_\chi}\right)^{\beta'} \partial_\mu \chi \partial^\mu \chi \\
&+ c_4 \left(\frac{\chi}{f_\chi}\right)^{4+\beta'}, \\
\mathcal{L}_{mass}^{d<4} &= \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^{3-\gamma_m} \mathrm{Tr}\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right),
\end{aligned}$$

$$m_\sigma^2 f_\sigma^2 = f_\pi^2\left(m_K^2 + \frac{1}{2}m_\pi^2\right)(3-\gamma_m)(1+\gamma_m) - \beta'(4+\beta')$$

Analog to GMOR relation for pion

Both γ_m and β' are evaluated at α_{IR}

Order counting

ChPT	Chiral-scale EFT
$\partial_\mu \sim m_\pi \sim O(p)$	$\partial_\mu \sim m_\pi \sim m_\sigma \sim O(p)$
$m_q \sim O(p^2)$	$m_q \sim O(p^2)$
	$\Delta\alpha_s = \alpha_{IR} - \alpha_s \sim O(p^2)$

II. Chiral-scale effective field theory

The NLO Lagrangian = the higher chiral-scale order corrections due to the current quark mass and derivatives on the NGBs + the leading terms in the expansion of γ_m and β' in $\Delta\alpha_s$.

$$\chi^{\gamma_m(\alpha_{\rm IR})} \to \chi^{\gamma_m(\alpha_{\rm IR})}\left[1+\sum_{n=1}^{\infty} C_n\left(\Delta\alpha_s\Sigma\right)^n\right],$$

$$\Delta\alpha_s \sim O(p^2).$$

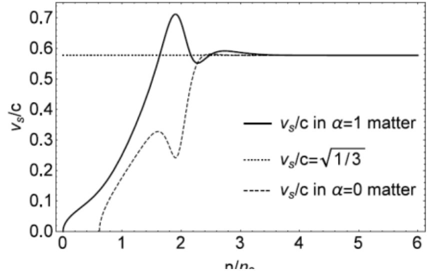
$$\mathrm{Tr}(\hat{a}_{\perp\mu}\hat{a}_{\perp}^{\mu}); \qquad \mathrm{Tr}(\hat{a}_{\|\mu}\hat{a}_{\|}^{\mu}); \qquad \frac{1}{2g^2}\mathrm{Tr}(V_{\mu\nu}V^{\mu\nu});$$

$$\partial_\mu\chi\partial^\mu\chi; \qquad \mathrm{Tr}\left(\hat{\mathcal{M}}+\hat{\mathcal{M}}^\dagger\right).$$

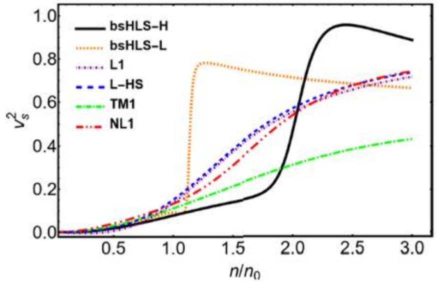
II. Chiral-scale effective field theory

- This theory has been successfully used in several aspects:

  - The $\Delta I = 1/2$ rule for nonleptonic K decays is then a consequence of χPTσ, with a $K_S\sigma$ coupling fixed by data for $\gamma\gamma \to \pi\pi$ and $K_S \to \gamma\gamma$.
    - Crewther and Tunstall , PRD91(2015), 034016, e-Print: 1312.3319
  - The pseudoconformal structure of compact star matter;
    - W. G. Paeng, T. T. S. Kuo, H. K. Lee, *YLM*, M. Rho, arXiv: 1704.02775.



  - The peak of sound velocity in nuclear matter;
    - L. Q. Zhang, Y. Ma and *YLM*, 2410.04142; 2412.19023.



  - ……

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III. Hadron-hadron scattering

- Consider the leading order Lagrangian:

$$\mathcal{L}^{\rm LO}_{\chi{\rm PT}_\sigma} = \mathcal{L}^{\rm LO}_{\chi{\rm PT}_\sigma;{\rm M}} + \mathcal{L}^{\rm LO}_{\chi{\rm PT}_\sigma;{\rm B}}$$

$$\mathcal{L}^{\rm LO}_{\chi{\rm PT}_\sigma;{\rm M}} = \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\sigma}\right)^2 {\rm Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\sigma}\right)^{3-\gamma_m} {\rm Tr}\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V_\chi(\chi)$$

$$\mathcal{L}^{\rm LO}_{\chi{\rm PT}_\sigma;{\rm B}} = \bar{\psi}\left(i\not{D} - g_\chi m \frac{\chi}{f_\sigma} + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu\right)\psi$$

- ✓ $\sigma\pi\pi$ coupling includes both derivative (different from LσM) and no-derivative sectors;
- ✓ σNN coupling is non-derivative $g_{\sigma NN} = \frac{m_N}{f_\sigma}$~(3~9).

$$U(x) = e^{i\pi/f_\pi}$$
$$\chi(x) = e^{\sigma/f_\sigma}$$
$$M = diag(m_\pi^2, m_\pi^2)$$
$$\gamma_m \approx 1$$
$$g_\chi m = m_N$$

III. Hadron-hadron scattering

- Pion-Pion scattering

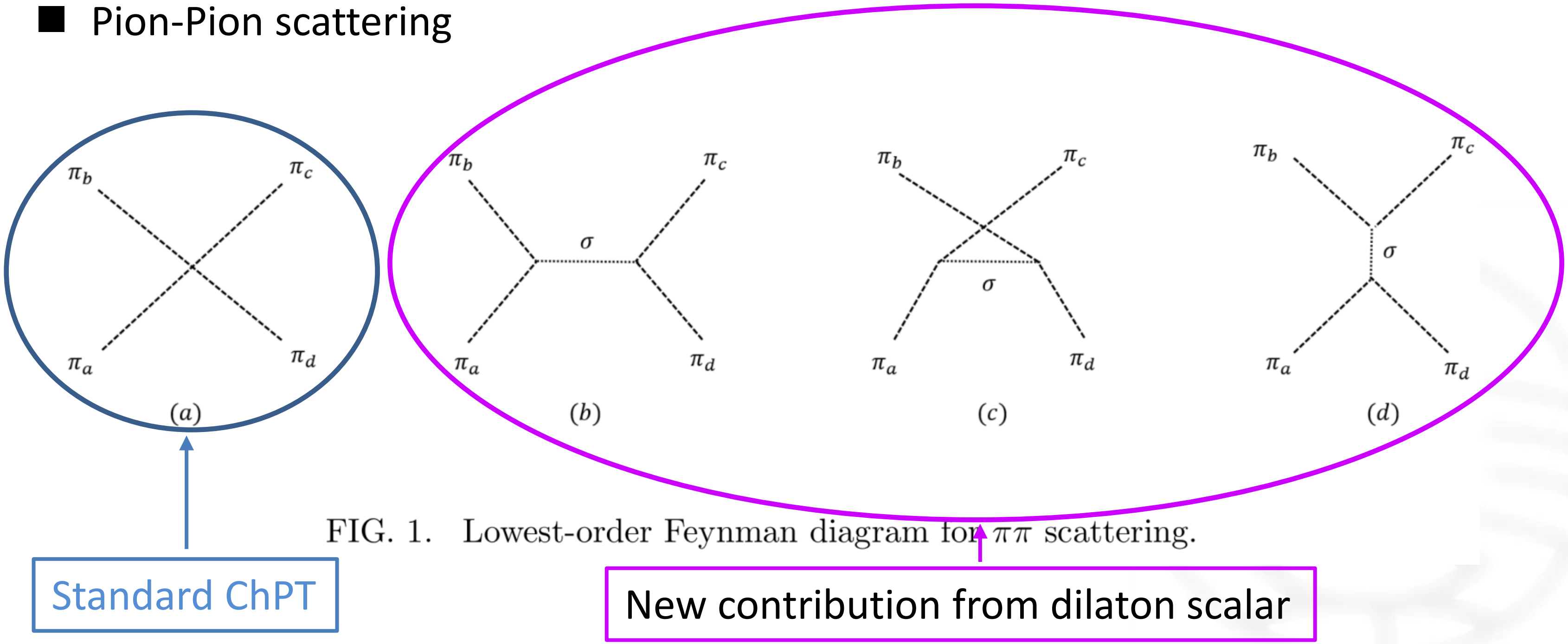


FIG. 1. Lowest-order Feynman diagram for $\pi\pi$ scattering.

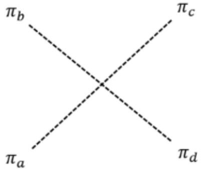
III. Hadron-hadron scattering

$$
\begin{aligned}
i\mathcal{M}_{(a)} = & \; i\left(\delta_{ab}\delta_{cd}\frac{s-m_\pi^2}{f_\pi^2} + \delta_{ac}\delta_{bd}\frac{t-m_\pi^2}{f_\pi^2} + \delta_{ad}\delta_{bc}\frac{u-m_\pi^2}{f_\pi^2}\right) \\
& - \frac{i}{3f_\pi^2}\left(\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}\right)\left(\Lambda_a + \Lambda_b + \Lambda_c + \Lambda_d\right)
\end{aligned}
$$

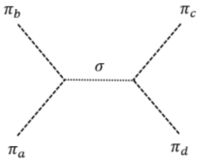
$$
\begin{aligned}
i\mathcal{M}_{(b)} = & \; \frac{i}{f_\sigma^2}\left[(s-m_\pi^2)\left(1+\frac{m_\sigma^2-m_\pi^2}{s-m_\sigma^2}\right) + (\Lambda_a+\Lambda_b-m_\pi^2)(\Lambda_c+\Lambda_d-m_\pi^2)\frac{1}{s-m_\sigma^2}\right. \\
& \left. - \frac{s-m_\pi^2}{s-m_\sigma^2}\left(\Lambda_a+\Lambda_b+\Lambda_c+\Lambda_d-2m_\pi^2\right)\right]\delta_{ab}\delta_{cd},
\end{aligned}
$$

$$
\begin{aligned}
i\mathcal{M}_{(c)} = & \; \frac{i}{f_\sigma^2}\left[(t-m_\pi^2)\left(1+\frac{m_\sigma^2-m_\pi^2}{t-m_\sigma^2}\right) + (\Lambda_a+\Lambda_c-m_\pi^2)(\Lambda_b+\Lambda_d-m_\pi^2)\frac{1}{t-m_\sigma^2}\right. \\
& \left. - \frac{t-m_\pi^2}{t-m_\sigma^2}\left(\Lambda_a+\Lambda_b+\Lambda_c+\Lambda_d-2m_\pi^2\right)\right]\delta_{ac}\delta_{bd},
\end{aligned}
$$

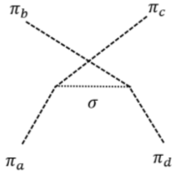
$$
\begin{aligned}
i\mathcal{M}_{(d)} = & \; \frac{i}{f_\sigma^2}\left[(u-m_\pi^2)\left(1+\frac{m_\sigma^2-m_\pi^2}{u-m_\sigma^2}\right) + (\Lambda_a+\Lambda_d-m_\pi^2)(\Lambda_b+\Lambda_c-m_\pi^2)\frac{1}{u-m_\sigma^2}\right. \\
& \left. - \frac{u-m_\pi^2}{u-m_\sigma^2}\left(\Lambda_a+\Lambda_b+\Lambda_c+\Lambda_d-2m_\pi^2\right)\right]\delta_{ac}\delta_{bd}.
\end{aligned}
$$



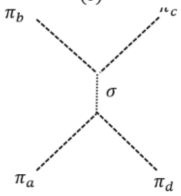
(a)



(b)



(c)



(d)

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III. Hadron-hadron scattering

- In general, the T-matrix element for this scattering process can be parameterized as

$$T^{ab;cd}(p_a, p_b; p_c, p_d) = \delta_{ab}\delta_{cd}A(s,t,u) + \delta_{ac}\delta_{bd}A(t,u,s) + \delta_{ad}\delta_{bc}A(u,s,t).$$

At threshold, one has $s = 4m_\pi^2, t = u = 0$, one has

$$\begin{aligned}
A(s,t,u) &= \frac{4m_\pi^2 - m_\pi^2}{f_\pi^2} + \frac{1}{f_\sigma^2}\left[(4m_\pi^2 - m_\pi^2)\left(1 + \frac{m_\sigma^2}{4m_\pi^2 - m_\sigma^2}\right) + m_\pi^2\left(1 + \frac{m_\sigma^2}{4m_\pi^2 - m_\sigma^2}\right)\right] \\
&= \frac{3m_\pi^2}{f_\pi^2} + \frac{4m_\pi^2}{f_\sigma^2}\left(1 + \frac{m_\sigma^2}{4m_\pi^2 - m_\sigma^2}\right), \\
A(t,u,s) &= \frac{-m_\pi^2}{f_\pi^2} + \frac{1}{f_\sigma^2}\left[(-m_\pi^2)\left(1 + \frac{m_\sigma^2}{-m_\sigma^2}\right) + m_\pi^2\left(1 + \frac{m_\sigma^2}{-m_\sigma^2}\right)\right] \\
&= -\frac{m_\pi^2}{f_\pi^2}, \\
A(u,s,t) &= \frac{-m_\pi^2}{f_\pi^2} + \frac{1}{f_\sigma^2}\left[(-m_\pi^2)\left(1 + \frac{m_\sigma^2}{-m_\sigma^2}\right) + m_\pi^2\left(1 + \frac{m_\sigma^2}{-m_\sigma^2}\right)\right] \\
&= -\frac{m_\pi^2}{f_\pi^2}.
\end{aligned}$$

(

- Only $A(s,t,u)$ is changed by the including of dilaton scalar.

III. Hadron-hadron scattering

- We finally obtain :

$$T_{\rm thr}^{I=0} = 3A(s,t,u) + A(t,u,s) + A(u,s,t) = \frac{7m_\pi^2}{f_\pi^2} + \frac{12m_\pi^2}{f_\sigma^2}\left(1 + \frac{m_\sigma^2}{4m_\pi^2 - m_\sigma^2}\right),$$

$$T_{\rm thr}^{I=1} = A(t,u,s) - A(u,s,t) = 0,$$

$$T_{\rm thr}^{I=2} = A(t,u,s) + A(u,s,t) = -\frac{2m_\pi^2}{f_\pi^2}.$$

$$a_0^0 = \frac{1}{32\pi} T_{\rm thr}^{I=0} = \frac{7m_\pi^2}{32\pi f_\pi^2} + \frac{3m_\pi^2}{8\pi f_\sigma^2}\left(1 + \frac{m_\sigma^2}{4m_\pi^2 - m_\sigma^2}\right)$$

- Only the $T_{thr}^{I=0}$ is modified by the scalar exchange.

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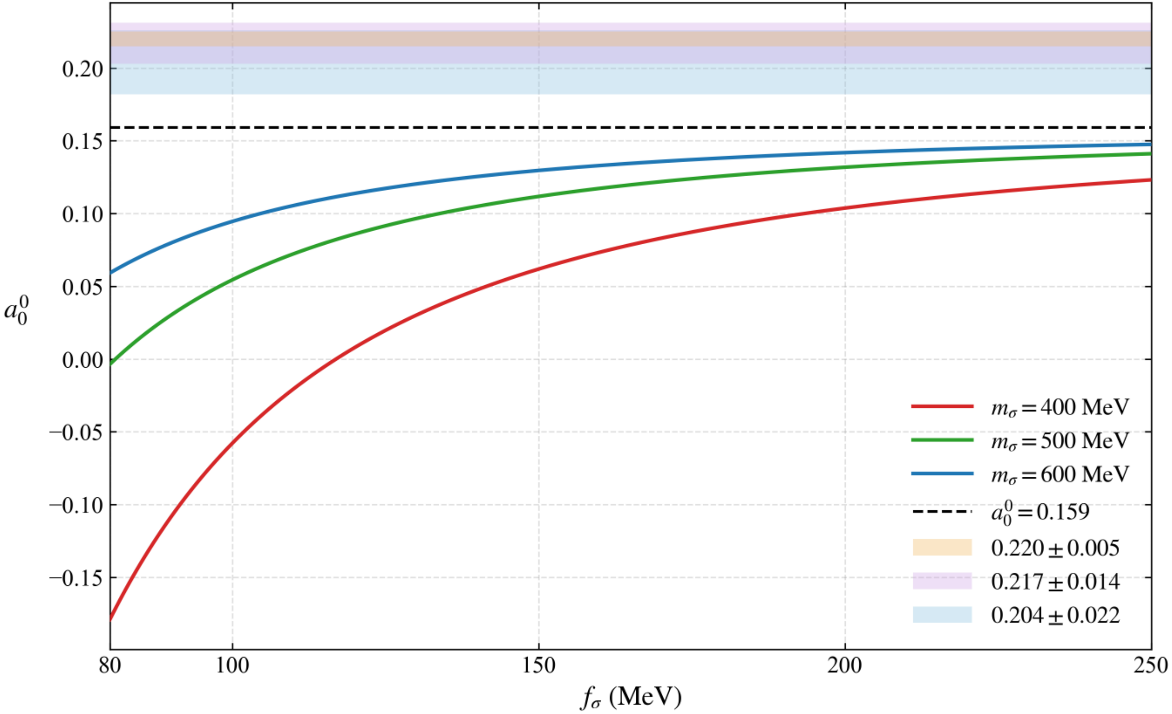
III. Hadron-hadron scattering

$$a_0^0 = \frac{1}{32\pi} T_{\rm thr}^{I=0} = \frac{7m_\pi^2}{32\pi f_\pi^2} + \frac{3m_\pi^2}{8\pi f_\sigma^2}\left(1 + \frac{m_\sigma^2}{4m_\pi^2 - m_\sigma^2}\right) \quad < 0$$



$a_0^0 \leq 0.147$
$<$ 0.159(LO ChPT) $<$
empirical values

G. Colangelo, et al., Phys. Lett. B 488 (2000) 261268; G. Colangelo, et al., Nuclear Phys. B 603 (2001) 125,179.
G. Colangelo, et al., Eur. Phys. J. C 59 (2009) 777793.
M. Kermani, et al., (CHAOS Collaboration), Phys. Rev. C 58 (1998) 34313441.

III. Hadron-hadron scattering

- Pion-Nucleon scattering

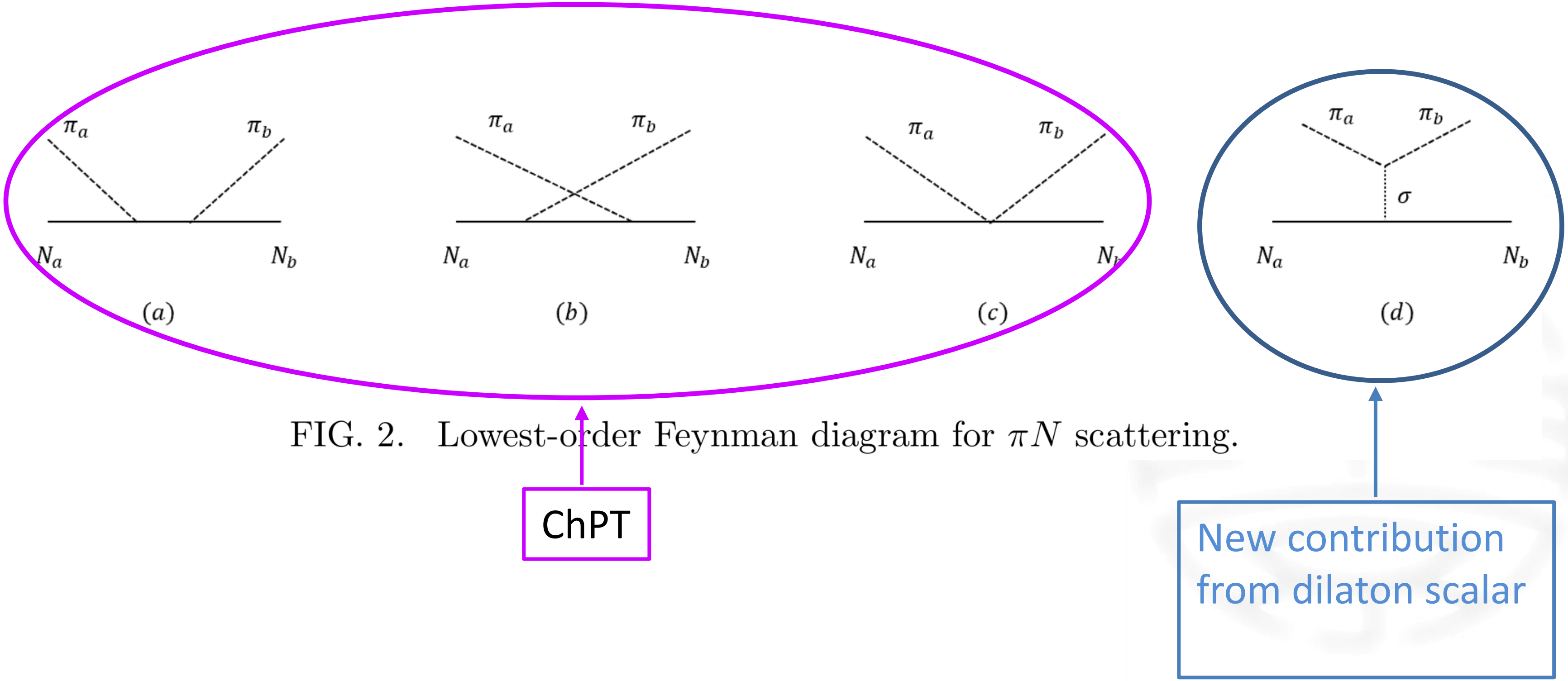


FIG. 2. Lowest-order Feynman diagram for πN scattering.

III. Hadron-hadron scattering

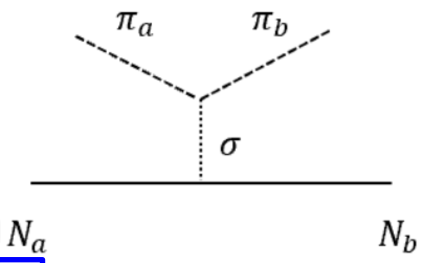
- Matrix elements:

$$i\mathcal{M}_{(a)} = -i\left(\frac{g_A}{2f_\pi}\right)^2 \bar{N}(p_2)\tau^b \not{q}_2\gamma_5 \frac{1}{\not{p}_1+\not{q}_1-m_N}\tau^a\not{q}_1\gamma_5 N(p_1),$$

$$i\mathcal{M}_{(b)} = -i\left(\frac{g_A}{2f_\pi}\right)^2 \bar{N}(p_2)\tau^a \not{q}_1\gamma_5 \frac{1}{\not{p}_1-\not{q}_2-m_N}\tau^b\not{q}_2\gamma_5 N(p_1),$$

$$i\mathcal{M}_{(c)} = -\frac{1}{4f_\pi^2}\bar{N}(p_2)\left(\not{q}_1+\not{q}_2\right)\epsilon^{abc}\tau^c N(p_1),$$

$$i\mathcal{M}_{(d)} = -i\frac{2m_N}{f_\chi^2}\bar{N}(p_2)N(p_1)\frac{1}{(q_1-q_2)^2-m_\sigma^2}(-q_1\cdot q_2+m_\pi^2)\delta^{ab}.$$



(d)

Linear sigma model

$$+\, g_{\sigma NN}\, g_{\sigma\pi\pi}\overline{N}(p_2)\frac{-\delta^{ab}}{(q_1-q_2)^2-m_\sigma^2}N(p_1)$$

$$g_{\sigma NN} = g_{\pi NN},$$
$$g_{\sigma\pi\pi} = \frac{g_{\pi NN}}{2m_N}(m_\sigma^2 - m_\pi^2)$$

III. Hadron-hadron scattering

$$\mathcal{M}^{ab}(p_1,q_1;p_2,q_2)\Big|_{\rm thr} = 2m_N\chi^\dagger\Big|_{p_2=m_N}\Big\{\delta^{ab}\left[A^+(\nu,\nu_B)+m_\pi B^+(\nu,\nu_B)\right] - i\epsilon^{abc}\tau^c\left[A^-(\nu,\nu_B)+m_\pi B^-(\nu,\nu_B)\right]\Big\}\chi\Big|_{p_1=m_N}$$

$$A^+(\nu,\nu_B) = 4m_N\left(\frac{g_A}{2f_\pi}\right)^2 - \frac{2m_N}{f_\chi^2}\frac{1}{4m_N\nu_B+2m_\pi^2-m_\sigma^2}(2m_N\nu_B+m_\pi^2), \quad \boxed{+\,g_{\sigma NN}g_{\sigma\pi\pi}\frac{-1}{t-m_\sigma^2}}$$

$$A^-(\nu,\nu_B) = 0,$$

$$B^+(\nu,\nu_B) = -\left(\frac{g_A}{2f_\pi}\right)^2\frac{4m_N\nu}{\nu^2-\nu_B^2},$$

$$B^-(\nu,\nu_B) = -\left(\frac{g_A}{2f_\pi}\right)^2\left(2+\frac{4m_N}{\nu^2-\nu_B^2}\nu_B\right)-\frac{1}{2f_\pi^2}.$$

- Only the $A^+(\nu,\nu_B)$ is modified by the scalar exchange.

$$\nu = \frac{s-u}{m_N} = \frac{(p_1+p_2)\cdot q_1}{2m_N} = \frac{(p_1+p_2)\cdot q_2}{2m_N},$$

$$\nu_B = -\frac{q_1\cdot q_2}{2m_N} = \frac{t-2m_\pi^2}{4m_N},$$

III. Hadron-hadron scattering

The scattering length:

$$a_{0+}^{\pm} = \frac{1}{8\pi(m_N + m_\pi)} T^{\pm}\Big|_{\rm thr} = \frac{1}{2\pi(1+\mu)} \left[A^{\pm} + m_\pi B^{\pm}\right]_{\rm thr}$$

Since the contribution from the dilaton scalar is proportional to $(2m_N v_B + m_\pi^2) \to 0$ for the threshold process, the scattering length at LO of ChPT is intact. **In agreement with W-T relation**

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IV. Summary and discussion

- ✓ The dilaton implemented chiral EFT---chiral-scale EFT---provides a (possible) framework for hadron processes including the lightest scalar meson.
- ✓ Several phenomena have been discussed in the literature (pseudoconformal structure, peak of SV);
- ✓ At LO, hadron-hadron scatterings are discussed, the iso-scalar channel of pion-pion scattering is improved, no contradiction is found for other processes;
- ✓ Just started, further analysis including higher/loop orders, nonperturbative calculations, ……, will be done later.

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Thank you for your attention!