Form factors of radially excited pion within a contact interaction

Axel Ahiezer Ortiz Villaseñor

University of Sonora
University of Huelva

In collaboration with

L. Albino
A. Bashir
J. J. Cobos
J. Segovia

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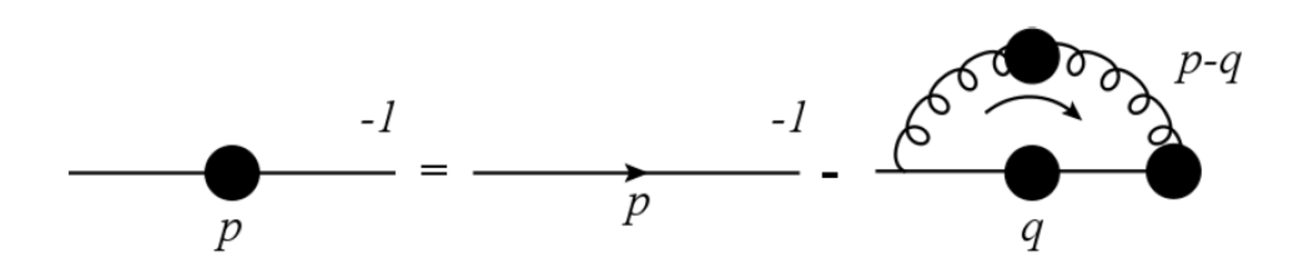
Schwinger-Dyson equations (SDEs) [1,2]

- The SDEs constitute an infinite set of coupled integral equations that determine the Green's functions of a quantum field theory.
- They provide a nonperturbative framework.
- The SDE for the quark propagator in QCD is given by

$$S^{-1}(p;\Lambda) = S_0^{-1}(p) + \Sigma(p;\Lambda) \; ,$$

$$\Sigma(p;\Lambda) = \int \frac{d^4q}{(2\pi)^4} g^2(\Lambda) D_{\mu\nu}(p-q;\Lambda) \frac{\lambda^a}{2} \gamma_\mu S(q;\Lambda) \Gamma^a_\nu(q,p;\Lambda) \; .$$



Diagrammatic representation of the Schwinger-Dyson equation for the quark propagator in QCD.

[1] Julian Schwinger, hys. Rev. **82** (1951) 664.

[2] Freeman Dyson, Phys. Rev. **75** (1949) 1736.

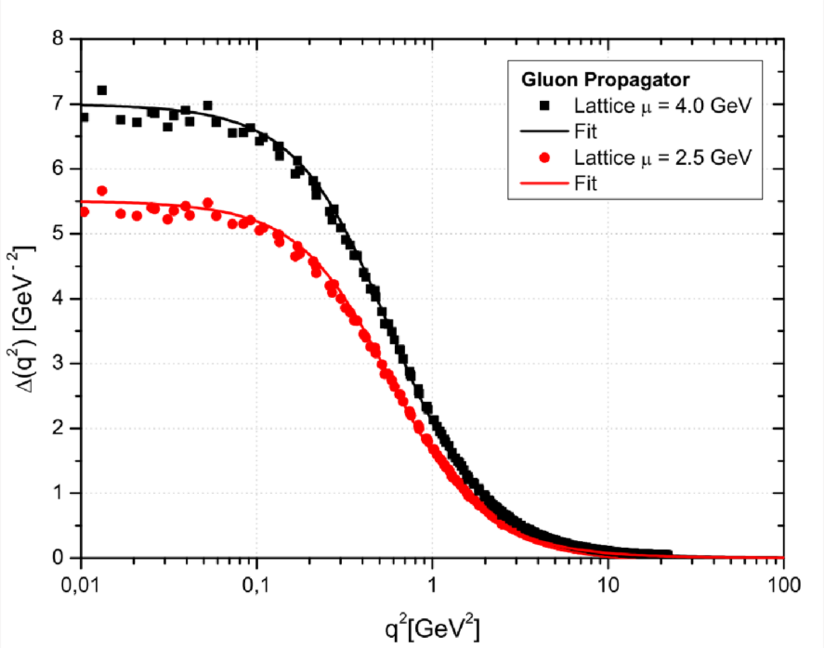
Contact interaction model (SCI) [3]

Within the SCI model, we have a momentum independent interaction where the gluon propagator is approximated as a constant:

$$g^2 D_{\mu\nu}(k) \to \frac{4\pi\alpha_{IR}^{(n)}}{m_g^2}\delta_{\mu\nu}\ , \ \ m_g = 500\ \mathrm{MeV}\ .$$

It retains key features of QCD such as:

- Preserves symmetries
- Low energy relations
- Confinement
- Chiral symmetry breaking



Gluon propagator dressing function [4].

Model parameters

Where $\alpha_{IR}^{(n)}$ is an infrared effective coupling that evolves as [5,6]

$$\alpha(\Lambda) = \alpha_{IR}^{(0)} \left(\frac{\Lambda_{UV}^{(0)}}{\Lambda} \right)^2 \log^{-1} \left(\frac{\Lambda}{\Lambda_0} \right)^{\beta} , \quad \Lambda_0 = 0.357 \text{ GeV} \, , \; \beta = 1.075 \, .$$

Where the values of the ultraviolet cutoff $\Lambda_{UV}^{(n)}$ and the infrared effective coupling $\alpha_{IR}^{(n)}$ are

n	$\Lambda_{\mathrm{UV}}^{(n)}/\mathrm{GeV}$	$\alpha_{\mathrm{IR}}^{(n)}/\pi$
0	0.905	0.363
1	1.200	0.159
2	1.600	0.072

[5] P.-L. Yin et al, Phys. Rev. D **100**, 034008 (2019).

[6] M. A. Bedolla et al, Phys. Rev. D **93**, 094025 (2016).

Bethe-Salpeter equation (BSE)

The BSE provides a relativistic covariant description of quark-antiquark bound states.

Within the SCI, the standard homogeneous BSE for the n-th radially excited state is given by

$$\lambda_n \Gamma_n(P) = -\frac{4\pi\alpha_{IR}^{(n)} C_F}{m_g^2} \int \frac{d^4k}{(2\pi)^4} \Pi_n(k) \gamma_\mu S(k+P) \Gamma_n(P) S(k) \gamma_\mu \ , \quad C_F = \frac{4}{3} \ .$$

The nodal structure in the BSE is introduced through [7,8]

$$\Pi_n(k) = \prod_{i=1}^{n} (1 - d_n^{(i)} k^2) \ ,$$

where k denotes the integration momentum and $\{d_n^{(1)}, d_n^{(2)}, \ldots, d_n^{(n)}\}$ are the set of n nodes.

[7] H. L. L. Roberts et al, Few Body Syst. **51**, 1 (2011). [8] M.K. Volkov, Phys. Atom. Nucl. **60**, 1920-1929 (1997).

Bethe-Salpeter amplitude (BSA)

The most general Lorentz invariant decomposition for the BSA of a pseudoscalar mesons is given by

$$\Gamma_n(p,P) = \gamma_5 \left[iE_n(p,P) + (\gamma \cdot P)\, F_n(p,P) + (\gamma \cdot p)\,(p \cdot P) G_n(p,P) \right.$$
$$\left. + \sigma_{\mu\nu} P_\mu p_\nu H_n(p,P) \right] \; .$$

Within the SCI model, the BSA is simplified to

$$\Gamma_n(P) = \gamma_5 \left(iE_n + \frac{\gamma \cdot P}{M} F_n \right) \; .$$

The values of the nodes $d_n^{(i)}$ are:

n	$d_n^{(1)}/\mathrm{GeV}^{-2}$	$d_n^{(2)}/\mathrm{GeV}^{-2}$
1	1.7891	-
2	0.2000	7.73

Standard observables

Standard SCI solutions for the pion radial spectrum. The table list the BSAs, canonical normalization constants N_n^s, masses m_n and decay constants $f_{\pi_n}^s$ for the ground-state pion and its first two radial excitation.

n	E_n^{s}	F_n^{s}	m_n/GeV	N_n^{s}	$f_{\pi_n}^{\mathrm{s}}/\mathrm{GeV}$
0	0.9914	0.1309	0.139	0.2759	0.101
1	0.9768	0.2144	1.300	1.4065	0.075
2	0.99998	-0.00675	1.803	39.752	-0.0002

Electromagnetic current

The most general Lorentz-covariant decomposition of the vector current describing the EM coupling of a photon to an incoming pseudoscalar meson can be written as [9]

$$\mathcal{J}_\mu^{n\to n'}(p_f,p_i) = F_+^{n\to n'} J_\mu^+ + F_-^{n\to n'} J_\mu^- \ , \quad K = \frac{p_f+p_i}{2} \ , Q = p_f - p_i \ ,$$

$$J_\mu^+ \equiv J_\mu^+(p_f,p_i) = 2K_\mu + 2\delta_-\left(Q^2\right) Q_\mu \ ,$$
$$J_\mu^- \equiv J_\mu^-(p_f,p_i) = 2\delta_-\left(Q^2\right) K_\mu - \left[1 + 2\delta_+\left(Q^2\right)\right] Q_\mu \ , \quad \delta_\pm\left(Q^2\right) = \frac{\left(m_f^2 \pm m_i^2\right)}{Q^2} \ .$$

The transverse and longitudinal structure are given by $F_+^{n\to n'}\left(Q^2\right)$ and $F_-^{n\to n'}\left(Q^2\right)$ respectively.

These form factors can be projected as $F_\pm^{n\to n'}\left(Q^2\right) = \frac{\left(p_f \pm p_i\right)_\mu \mathcal{J}_\mu^{n\to n'}(p_f,p_i)}{\mathcal{D}\left(Q^2\right)}$,

with

$$\mathcal{D}\left(Q^2\right) \equiv -\frac{\left(\left(m_f - m_i\right)^2 + Q^2\right)\left(\left(m_f + m_i\right)^2 + Q^2\right)}{Q^2} \ .$$

[9] J. Carbonell et al, Phys. Rev. D **91**, 076010 (2015).

Electromagnetic current

Within the impulse approximation, the transition current associated with the process $\gamma + \pi_n \to \pi_{n'}$ is given by

$$\mathcal{J}_\mu^{n\to n'}(p_f, p_i) = \mathrm{Tr}\int_{dk} \Gamma_{n'}(-p_f)\, S(k_f)\Lambda_\mu(Q)S(k_i)\Gamma_n(p_i)S(k)\ ,$$
$$k_{i(f)} = k + p_{i(f)}\ .$$

Within the SCI model, the impulse approximation ensures electromagnetic current conservation in elastic processes, that is

$$Q_\mu \mathcal{J}_\mu^{n\to n}(p_f, p_i) = 0 \qquad \Rightarrow \qquad F_-^{n\to n}(Q^2) = 0$$

Any deviation from this condition therefore signals the presence of spurious longitudinal contributions to the electromagnetic current.

The charge normalization together with the orthonormality between different radial states can be expressed as $F_+^{n\to n'}(0) = \delta_{nn'}$.

Quark-photon vertex (QPV)

The QPV is constrained by the vector Ward-Green-Fradkin-Takahashi identity (VWGFTI) :

$$iQ_\mu \Lambda_\mu(Q) = S^{-1}(k_f) - S^{-1}(k_i) \, .$$

The QPV can be decomposed into longitudinal and transverse contributions:

$$\Lambda_\mu(Q) = \Lambda^L_\mu(Q) + \Lambda^T_\mu(Q) \, ,$$

where [10]

$$\Lambda^T_\mu(Q) = P_T\left(Q^2\right)\left[\gamma_\mu - \frac{(\gamma \cdot Q)Q_\mu}{Q^2}\right] \, , \quad \Lambda^L_\mu\left(Q^2\right) = \frac{(\gamma \cdot Q)Q_\mu}{Q^2} \, ,$$

with

$$P_T^{-1}\left(Q^2\right) = 1 + \frac{4\alpha_{IR}}{3\pi m_g^2} \int_0^1 d\alpha \; \alpha(1-\alpha) Q^2 \mathcal{C}_2(\omega) \, ,$$

$$\omega \equiv \omega\left(\alpha, Q^2\right) = M^2 + \alpha(1-\alpha)Q^2 \, , \quad \mathcal{C}_\beta(\omega) = \frac{\omega^{2-\beta}}{\Gamma[\beta]} \Gamma\left[\beta - 2, \omega\, \tau_{UV}^2, \omega\, \tau_{IR}^2\right] \, .$$

[10] X. Gutierrez et al, Phys. Rev. C **81**, 065202 (2010) .

Electromagnetic current conservation

The standard BSA solutions do not ensure current conservation for transitions $\gamma + \pi_n \to \pi_{n'}$, that is,

$$Q_\mu \mathcal{J}_\mu^{n\to n'} \sim m_{n'}^2 - m_n^2 \, .$$

This deficiency can be remedied through an orthogonalization procedure applied to the standard BSA, given by

$$\Gamma_n^\perp(P) = \Gamma_n^{\mathrm{s}} - \sum_{i=0}^{n-1} \sum_{j=1}^{2} \kappa_i^{(j)} \lambda_i^{(j)} \Gamma_{i(j)}^c(P) \, ,$$

where $\Gamma_{i(j)}^c$ are complementary amplitudes that retain the Lorentz structure and correspond to off-shell continuation of lower-lying states.

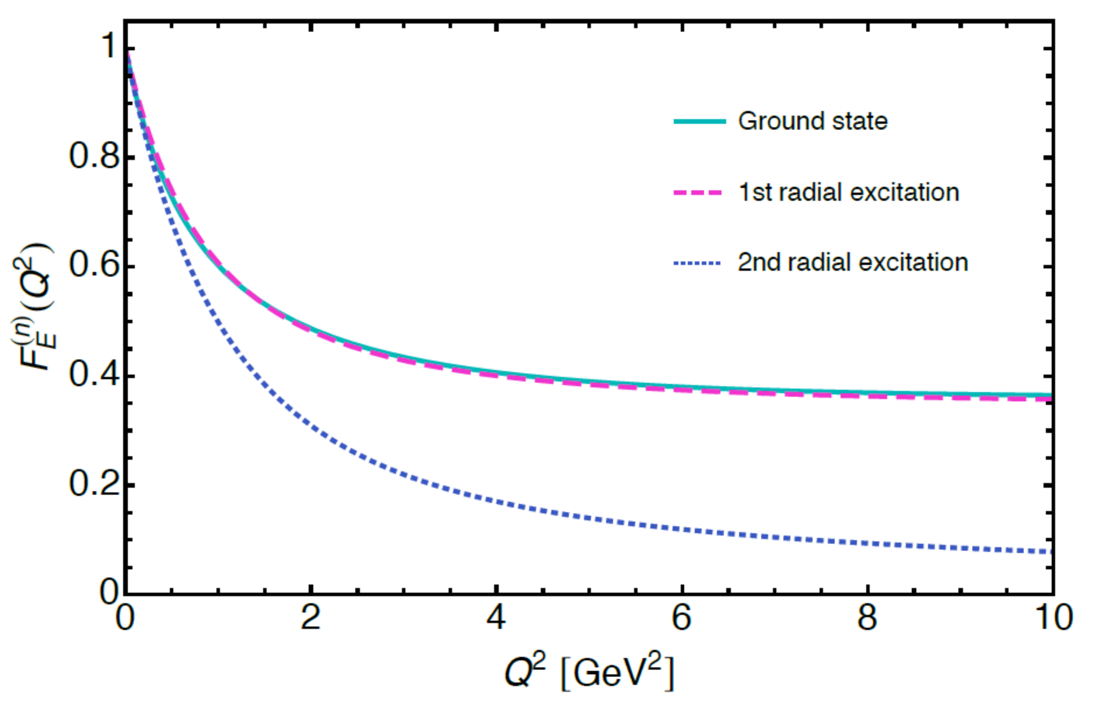
$$\lambda_i^{(j)} \equiv \lambda_i^{(j)}\left(-m_n^2\right) \neq 1 \, , \qquad \Pi_i(k) \to \Pi_i(k)/\lambda_i^{(j)} \, .$$

The $\kappa_i^{(j)}$ are the fixed by the requirement of EM current conservation for the process.

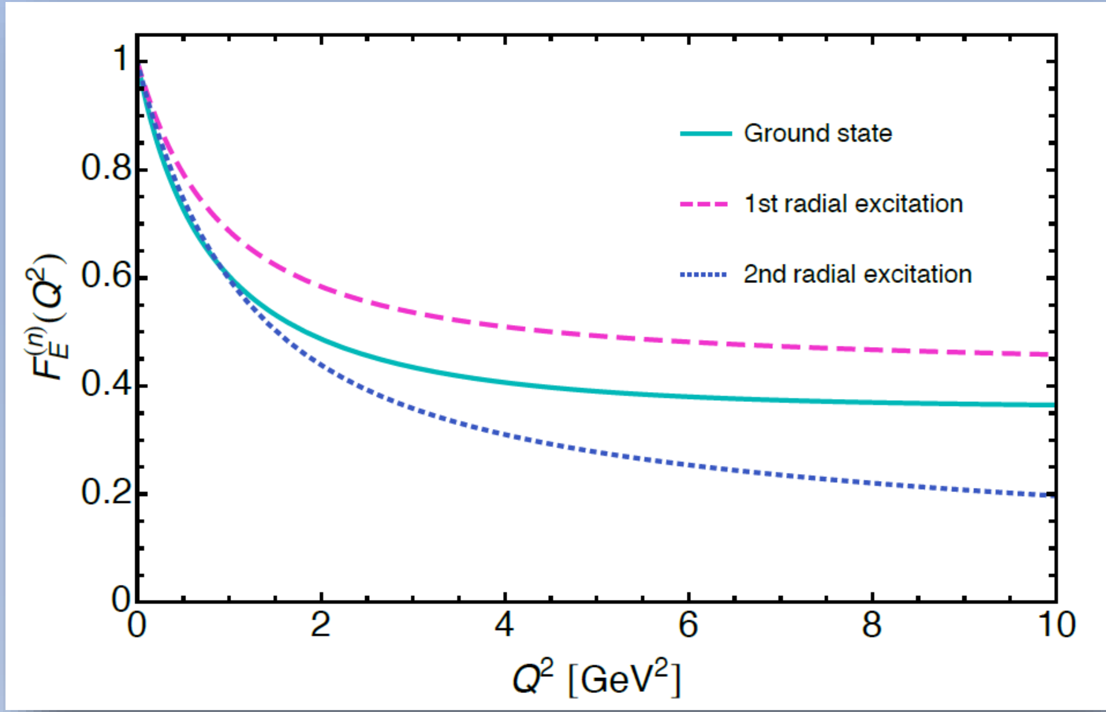
Elastic form factors (EFFs)

We define $F_E^{(n)}(Q^2) \equiv F_+^{n\to n}(Q^2)$.

EFFs obtained with the standard BSAs.



EFFs obtained with the orthogonalized BSAs.



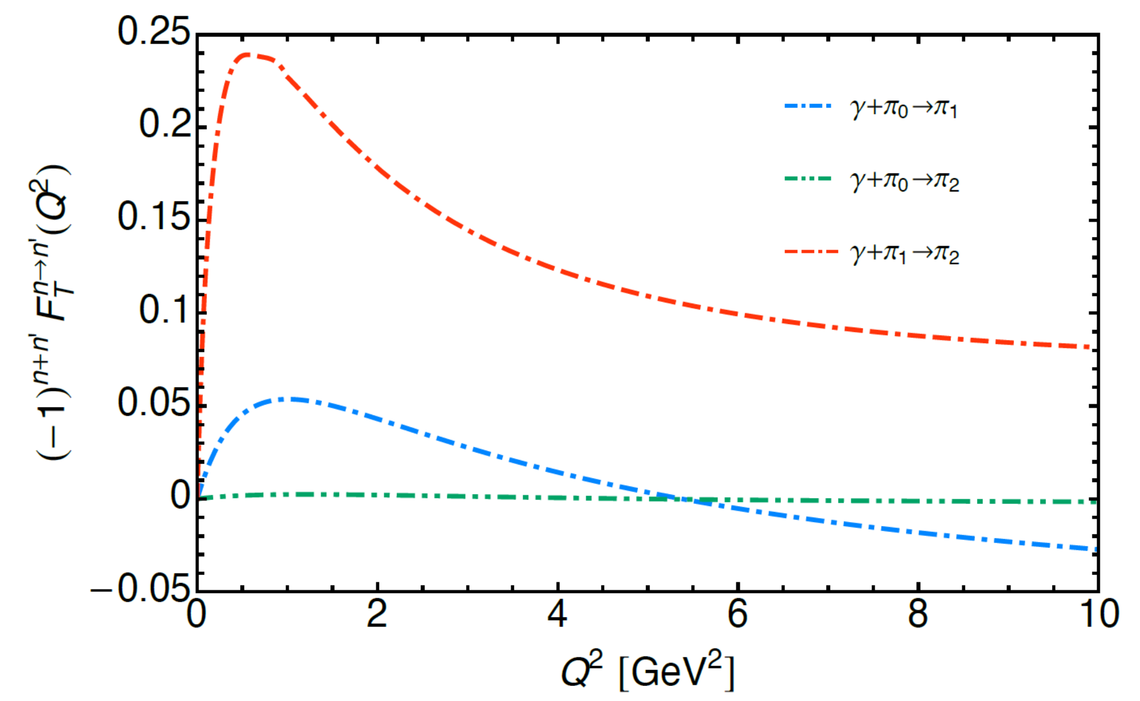
The standard construction exhibits the most rapidly decreasing EFF, whereas the first radial excitation is nearly indistinguishable from the ground state.

The orthonormalized construction modifies the overall hierarchy by broadening the EFFs of both excited states relative to their standard counterpart.

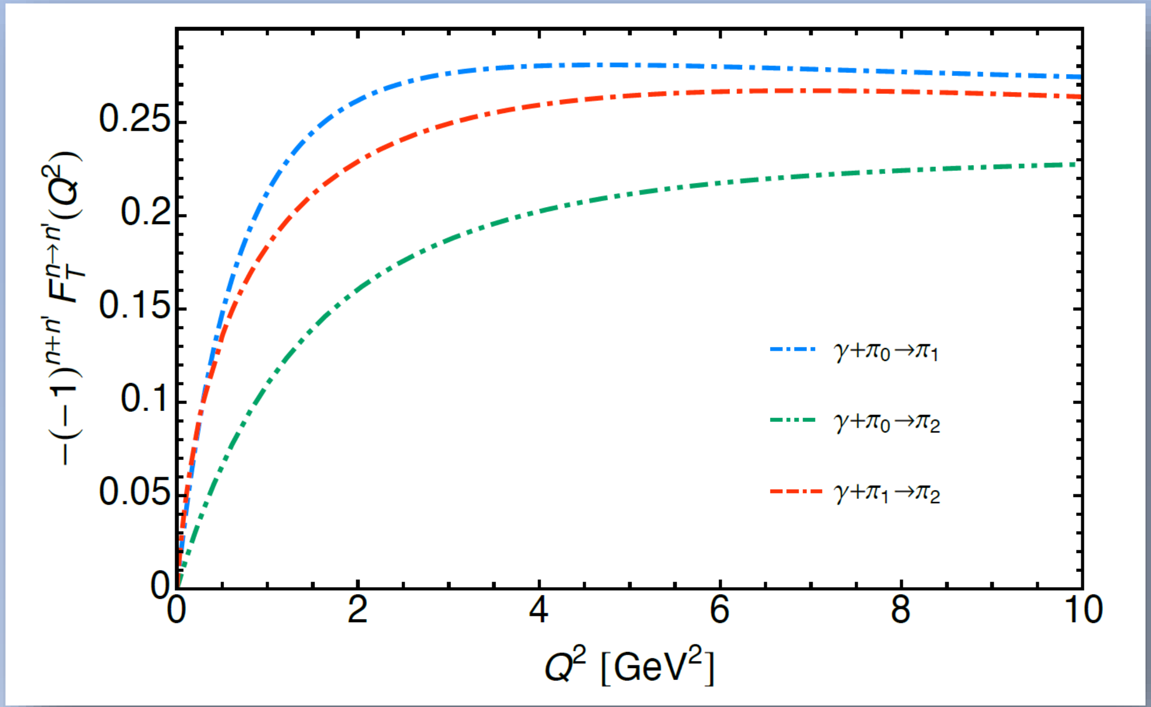
Transition form factors (TFFs)

We define $F_T^{n\to n'}(Q^2) \equiv F_+^{n\to n'}(Q^2)$.

TFFs obtained with the standard BSAs



TFFs obtained with the orthogonalized BSAs



Within the standard construction, the TFFs develop pronounced local maxima or minima around $Q^2 \sim 1\ \mathrm{GeV}$ with magnitudes that depend strongly in the particular transition.

Within the orthogonalized construction, the magnitude of the TFFs increase monotonically and the three transitions acquire comparable overall magnitudes.

Charge radii

The charge radius can be obtained through the following expression

$$\langle r_n^2 \rangle = -6 \frac{dF_E^{(n)}(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

Charge radius for the ground state of pion and its two first radial excitations, for the standard and orthonormalized solutions.

n	$r^s_{\pi_n}$ / fm	$r^\perp_{\pi_n}$ /fm
0	0.450	–
1	0.421	0.376
2	0.447	0.398

$$\frac{r^s_{\pi_1}}{r_{\pi_0}} = 0.936\,, \quad \frac{r^\perp_{\pi_1}}{r_{\pi_0}} = 0.836\,,$$

$$\frac{r^s_{\pi_2}}{r_{\pi_0}} = 0.993\,, \quad \frac{r^\perp_{\pi_2}}{r_{\pi_0}} = 0.884\,.$$

$$r_{\pi_0} \simeq r^s_{\pi_2} > r^s_{\pi_1}\,,$$

$$r_{\pi_0} > r^\perp_{\pi_2} > r^\perp_{\pi_1}\,.$$

Studies employing momentum-dependent interactions obtain the ratio [11, 12] :

$$\frac{r_{\pi_1}}{r_{\pi_0}} \approx \{0.88, 1.40\}$$

[11] A. S. Miramontes et al, Chin. Phys. **49**, 083108 (2025).

[12] A. Holl et al, Phys. Rev. C **71**, 065204 (2005) .

Summary

- In this work we investigated the electromagnetic properties of the ground state pion together with its first two radial excitations within the symmetry-preserving contact interaction.
- Although the standard SCI construction correctly reproduces the pion spectrum, charge normalization and the orthogonality of radial BSAs, we showed that transition currents involving excited states violate electromagnetic current conservation within the impulse approximation.
- This deficiency originates from the momentum independent nature of the interaction, where the implementation of radial excitations through manually introduced nodes is not sufficient to guarantee the VWGFTI for transition processes.
- To restore electromagnetic current conservation, we constructed the orthogonalized BSAs by incorporating complementary off-shell contributions from lower-lying states.
- The modified amplitudes produce harder elastic form factors and consequently the charge radii associated are smaller.

Backup

n	$E_n^{\rm s}$	$F_n^{\rm s}$	m_n/GeV	$N_n^{\rm s}$	$f_{\pi_n}^{\rm s}/\mathrm{GeV}$	$N_n^{\perp}$	$f_{\pi_n}^{\perp}/\mathrm{GeV}$
0	0.9914	0.1309	0.139	0.2759	0.101	–	–
1	0.9768	0.2144	1.300	1.4065	0.075	0.7350	-0.0567
2	0.99998	-0.00675	1.803	39.752	-0.0002	-15.500	-0.0363

The table list the BSAs, canonical normalization constants, masses and decay constants for the ground-state pion and its first two radial excitation for the orthogonal construction.

n	$\mathbf{f}_{0(1)}^{\rm c}$	$\lambda_0^{(1)}$	$\mathbf{f}_{0(2)}^{\rm c}$	$\lambda_0^{(2)}$	$\mathbf{f}_{1(1)}^{\rm c}$	$\lambda_1^{(1)}$	$\mathbf{f}_{1(2)}^{\rm c}$	$\lambda_1^{(2)}$
1	$\{0.9973, 0.0733\}$	11.201	$\{0.8787, 0.4774\}$	-0.1539	-	-	-	-
2	$\{0.9991, 0.0415\}$	1735.71	$\{0.8943, 0.4474\}$	-0.0731	$\{0.9991, 0.0416\}$	1318.42	$\{0.8949, 0.4463\}$	0.2972

The table list the off-Shell eigenvectors $\mathbf{f}_{i(j)}^{c} = \left\{ E_{i(j)^c, F_{i(j)}^c} \right\}$ and associated eigenvalues $\lambda_i^{(j)}$.

Backup

The normalization constant for the pseudoscalar mesons is given by

$$N_n^2 = 3\frac{d}{dP^2}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}_D\left[\Gamma_n(-Q)S(q+P)\Gamma(Q)S(q)\right]\Big|_{Q^2=P^2} .$$

The leptonic decay constant for pseudoscalar mesons is given by

$$P_\mu f_{\pi_n} = 3\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}_D\left[\gamma_5\gamma_\mu S(q+P)\Gamma_n(P)S(q)\right] .$$

Backup

The regularization scheme employed through this work is the proper time regularization scheme, by introducing an exponential damping factor

$$\frac{1}{A^n} = \frac{1}{\Gamma(n)} \int_0^\infty d\tau \tau^{n-1} e^{-\tau A} \to \frac{1}{\Gamma(n)} \int_{\tau_{IR}^2}^{\tau_{UV}^2} d\tau \tau^{n-1} e^{-\tau A} \,,$$

Where $A > 0$ and $\tau_{IR}^2 = 1/\Lambda_{IR}^2$, $\tau_{UV}^2 = 1/\Lambda_{UV}^2$ are respectively infrared and ultraviolet regulators and τ_{IR} implements confinement by ensuring the absence of quark production threshold [13].

For integers $\beta \geq 0$ and $\alpha \leq \beta$ we have:

$$\mathcal{I}_\beta^\alpha(\omega) = \int \frac{d^4q}{(2\pi)^4} \frac{(q^2)^\alpha}{(q^2+\omega)^{1+\beta}} \to \frac{\omega^{1+\alpha-\beta}}{16\pi^2} \frac{\Gamma(2+\alpha)}{\Gamma(1+\beta)} \Gamma(\beta-\alpha-1, \tau_{UV}^2\,\omega, \tau_{IR}^2\,\omega) \,.$$

[13] D. Ebert et al, Phys. Lett. B **388**, 154 (1996).