Light-front transverse nucleon charge and magnetisation densities

three-body and quark + diquark pictures compared

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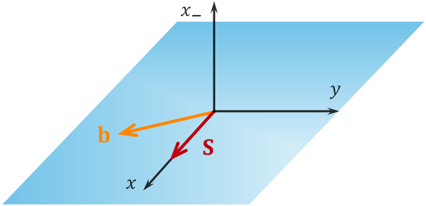
Why use light-front densities?

- **Old 3D picture:** transform G_E and G_M in the Breit frame $\Rightarrow$ a "static density"
- **Relativistic problem:** boosts change the wave function, so the transform mixes internal structure with motion. At $Q^2 \lesssim 15\,\mathrm{GeV}^2$, this cannot be ignored
- **Light-front solution:** fix light-front time in a Drell–Yan–West frame and transform only the transverse momentum
- **Gain and limit:** a well-defined density in $\boldsymbol{b}$, but no light-front-longitudinal distribution



Blue plane: transverse to the motion.

$\boldsymbol{b}$: position; $\boldsymbol{S}$: spin.

arXiv:2511.13997, Sec. I and Fig. 1.

Two nucleon pictures

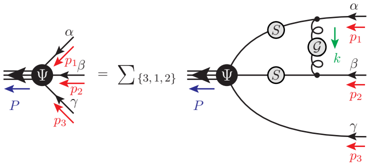
▶ Two very different descriptions; both Poincaré-covariant.

Three-body Faddeev equation

- three dressed quarks interacting directly
- detailed calculation: 128 scalar functions
- direct link to the QCD action

Quark + interacting diquark, $q(qq)$

- quark + dynamical diquark correlation
- quarks continually exchange their roles
- simpler: only 8 tensor structures



Three-body Faddeev equation.

arXiv:2511.13997, Secs. II–III and Fig. 2.

Why compare? Agreement means that the density pattern is not tied to one particular nucleon picture.

No tuning: the interaction and current-quark mass were fixed in earlier studies.

Next: use the two sets of F_1^N and F_2^N to calculate the transverse densities.

Three formulas define the transverse densities

- $F_1 \longrightarrow$ *charge*; $F_2 \longrightarrow$ *magnetisation*; transverse spin $\longrightarrow$ *sideways distortion*.

$F_1 \longrightarrow$ transverse charge density

$$\rho^t_{\rm ch}(b) = \int_0^\infty \frac{d\Delta}{2\pi}\,\Delta J_0(b\Delta)F_1^t(\Delta^2)$$

$$\rho^u_{\rm ch} = 2\rho^p_{\rm ch} + \rho^n_{\rm ch},$$

$$\rho^d_{\rm ch} = \rho^p_{\rm ch} + 2\rho^n_{\rm ch}$$

$t = p, n, u, d$; flavour densities are charge-stripped.

$F_2 \longrightarrow$ transverse magnetisation density

$$\rho^t_m(b) = \int_0^\infty \frac{d\Delta}{2\pi}\,\Delta J_0(b\Delta)F_2^t(\Delta^2)$$

$$\kappa^t = F_2^t(0) = 2\pi\int_0^\infty db\,b\,\rho^t_m(b)$$

The total transverse magnetisation gives κ^t.

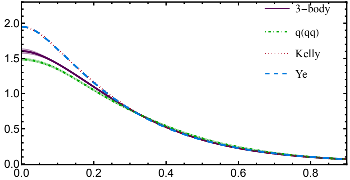
Transverse spin $\boldsymbol{S}_\perp \longrightarrow$ sideways distortion

$$\rho^t_T(\boldsymbol{b}) = \rho^t_{\rm ch}(b) + \sin(\phi_b - \phi_s)\frac{\tilde\rho^t_m(b)}{2m_N b}, \qquad \tilde\rho^t_m(b) = -b\,\frac{\partial\rho^t_m(b)}{\partial b}$$

The extra term integrates to zero: it changes the shape, not the total charge, and produces a dipole-like distortion.

Proton and neutron transverse charge densities

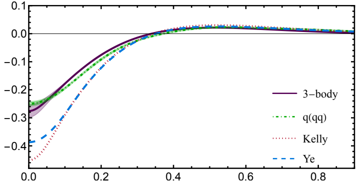
proton



Positive at every b

$\rho^p_{\mathrm{ch}}(b) > 0 \quad \longrightarrow \quad$ decreases as b grows

neutron



Changes sign

negative centre $\longrightarrow$ $b_0 \simeq 0.35\,\mathrm{fm}$ $\longrightarrow$ positive tail

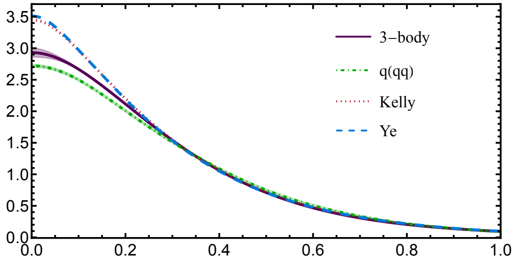
Why does the neutron density change sign?

$$\rho^n_{\mathrm{ch}}(b) = \int_0^\infty \frac{d\Delta}{2\pi}\ \underbrace{\Delta J_0(b\Delta)}_{\text{oscillating weight}}\ \underbrace{F_1^n(\Delta^2)}_{\leq 0}$$

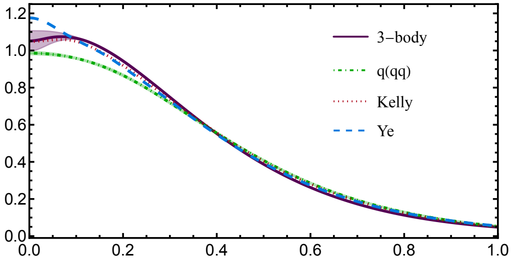
At $b = 0$: $J_0 = 1$ and $F_1^n \leq 0$, so $\rho^n_{\mathrm{ch}} < 0$.
As b grows: $J_0(b\Delta)$ oscillates and reweights the integral.
At $b_0 \simeq 0.35\,\mathrm{fm}$: the balance reverses and the density becomes positive.

Largest difference: visible mainly at small b, where the large-Q^2 form factors matter most.

u quark in the proton



d quark in the proton

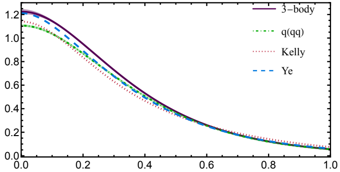


- **Central u-quark dominance:** for $b < b_0$, $\rho^u_{\rm ch} - 2\rho^d_{\rm ch} = -3\rho^n_{\rm ch} \Rightarrow \rho^u_{\rm ch} > 2\rho^d_{\rm ch}$
- **Charge-stripped densities:** the factors $+\frac{2}{3}$ and $-\frac{1}{3}$ are not included, so both curves are positive
- Same qualitative picture

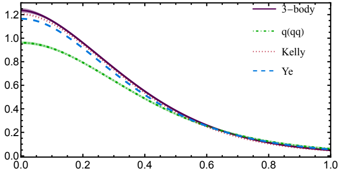
arXiv:2511.13997, Fig. 5 panels A, B. Densities in fm^{-2}, $b = |\boldsymbol{b}|$.

The valence d quark is magnetically very active

u quarks in the proton $\rho_m^u(b)/\kappa_u$



d quark in the proton $\rho_m^d(b)/\kappa_d$



Divide by each flavour's κ: compare shapes $\longrightarrow$ similar u and d profiles.

κ: sign and overall strength

	κ_p	κ_n	κ_u	κ_d
Exp.	1.75	−1.91	1.59	−2.08
3-body	1.23	−1.33	1.13	−1.43
$q(qq)$	1.80	−1.86	1.74	−1.92

κ_u: two u quarks $\quad$ κ_d: one d quark

$|\kappa_d| > |\kappa_u|$ in experiment and both calculations.

- **Orbital motion:** strong d response may be linked to greater orbital angular momentum.
- **Quark–diquark picture:** scalar diquarks alone are not enough; axial-vector correlations add spin and pairing structures.
- **Three-body counterpart:** mixed-symmetric (MS) components are important.
- **RL limitation:** 3-body reproduces the pattern, but gives $|\kappa|$ values that are too small.

Transverse radii: Dirac versus Pauli

	Dirac / charge		Pauli / magnetisation	
	λ_1^u/fm	λ_1^d/fm	λ_2^u/fm	λ_2^d/fm
3-body	0.68	0.69	0.63	0.68
$q(qq)$	0.53	0.57	0.62	0.67
Ye	0.66(1)	0.67(1)	0.71(12)	0.69(14)
Kelly	0.65	0.66	0.68	0.76

- **Dirac radii:** u and d have nearly the same transverse size within each calculation.
- **Comparing approaches:** 3-body Dirac radii are larger than $q(qq)$ and closer to the Ye and Kelly values.
- **Outer-region contribution:** 3-body densities are slightly larger at $b \gtrsim 1.1\,\mathrm{fm}$. Density farther from the centre carries more weight in the radius.
- **Pauli radii:** both calculations give similar results. The d-quark radius is 9(2)% larger than the u-quark radius.

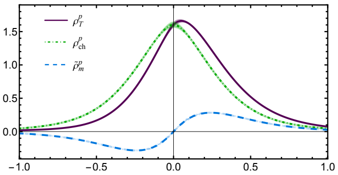
arXiv:2511.13997, Table I and Secs. IV.A–IV.B.

Transverse polarisation: a sideways distortion

- Magnetisation contributes to a sideways distortion of the charge density.

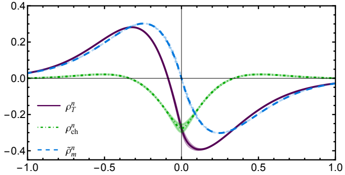
For $\boldsymbol{S} \parallel +\hat{x}$ and $b_x = 0$: $\rho_T(\boldsymbol{b}) = \rho_{\rm ch}(b) + \sin\phi_b \, \dfrac{\widetilde{\rho}_m(b)}{2m_N b}$

proton: $\kappa_p > 0$



Higher density at $+b_y$, lower at $-b_y$.

neutron: $\kappa_n < 0$



More positive at $-b_y$, more negative at $+b_y$.

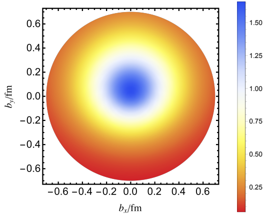
Read the curves: green = symmetric $\rho_{\rm ch}$, blue = spin-dependent term, purple = their sum ρ_T.
Polarisation: $y > 0$: $\rho_p \uparrow$, $\rho_n \downarrow \Rightarrow$ opposite distortions ($\kappa_p > 0$, $\kappa_n < 0$).
Total charge: the added term integrates to zero over the transverse plane.

3-body results shown. The $q(qq)$ calculation gives a very similar pattern.

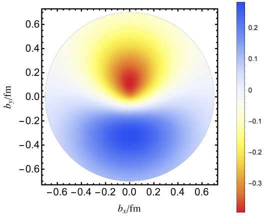
Two-dimensional maps for a $+\hat{x}$ polarised nucleon

Axes: horizontal b_x, vertical b_y **Spin:** $\boldsymbol{S} \parallel +\hat{x}$
Infinite-momentum-frame view: the nucleon moves toward the observer.

proton $\kappa_p > 0$



neutron $\kappa_n < 0$



Colours: blue = larger positive density,
red = smaller positive density, **not negative**.
Distortion: the density peak shifts slightly upward $(+b_y)$.

Colours: blue = positive, white $\simeq 0$,
red = negative.
Main regions: positive below, negative above.

More visible in the neutron: positive and negative regions make the distortion clearer.
Symmetry: no rotational symmetry, but reflection across the y axis remains $(b_x \to -b_x)$.

Summary and perspective

Main results

- **Light front:** F_1 and F_2 give well-defined transverse charge and magnetisation densities. These describe the nucleon in two dimensions.
- **Neutron:** The charge density is negative at small b. It crosses zero near 0.35 fm and becomes positive farther out. The total charge is still zero.
- **Flavour structure:** The u and d Dirac radii are nearly equal. Both calculations give a 9(2)% larger Pauli radius for d. The d flavour also has a strong magnetic response: $|\kappa_d| > |\kappa_u|$.
- **Polarisation:** Transverse spin redistributes charge sideways. The proton and neutron show opposite distortions, while their total charges remain unchanged.

Why it matters

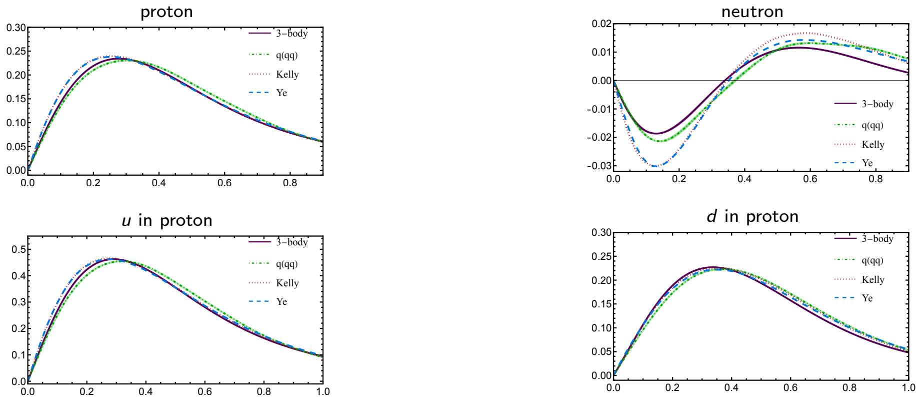
- Two different descriptions give the same main spatial picture.
- Both reflect emergent hadron mass in the quark–quark interaction.

Next steps

- Test these predictions with future lattice-QCD calculations.
- Extend the study to transitions from the nucleon to excited baryons.


Thank you!

Backup: the weighted densities $b\,\rho_{\rm ch}(b)$



- the 2D measure is $2\pi\, b\, db$, so $b\,\rho_{\rm ch}(b)$ is what integrates to the charge
- for the neutron the negative and positive areas visibly cancel
- the 3-body / $q(qq)$ radius difference lives on $b \gtrsim 1.1$ fm — not obvious even here

arXiv:2511.13997, Figs. 4 and 5, panels C and D.

Backup: interaction and numerical input

RL interaction

$$\tilde{\mathcal{G}}(y) = \frac{8\pi^2}{\omega^4} D\, e^{-y/\omega^2} + \frac{8\pi^2 \gamma_m \mathcal{F}(y)}{\ln\left[\tau + (1 + y/\Lambda_{\rm QCD}^2)^2\right]}$$

$\gamma_m = 12/25$, $\Lambda_{\rm QCD} = 0.234\,\text{GeV}$, $\tau = \text{e}^2 - 1$, $\Lambda_I = 1\,\text{GeV}$, $\omega = 0.8\,\text{GeV}$, $\omega D = 0.8\,\text{GeV}^3$.

Fixed input and outcome

- $\hat{m}_u = \hat{m}_d = 6.04\,\text{MeV} \to 4.19\,\text{MeV}$ at $\zeta = 2\,\text{GeV}$
- $m_\pi = 0.14$, $m_N = 0.94$, $f_\pi = 0.094\,\text{GeV}$
- $\alpha_{\mathcal{G}}(0)/\pi = 1.45$, $m_{\mathcal{G}} = 0.54\,\text{GeV}$
- renormalisation at $\zeta_{19} = 19\,\text{GeV}$

Observables are stable under $\omega \to (1 \pm 0.2)\omega$ at fixed ωD.

arXiv:2511.13997, Eqs. (1)–(4) and Sec. II.A.

Backup: currents and flavour separation

Nucleon currents

$$J_\mu^p = 2\,[J_{00}^{(3)}]_\mu\,, \qquad J_\mu^n = [J_{11}^{(3)} - J_{00}^{(3)}]_\mu$$

MS pieces of the amplitude do not contribute to the *charge* of a spin-half baryon.

Flavour separation

$$u:\;\; 2J_\mu^p + J_\mu^n = 3[J_{00}^{(3)}]_\mu + [J_{11}^{(3)}]_\mu$$

$$d:\;\; J_\mu^p + 2J_\mu^n = 2[J_{11}^{(3)}]_\mu$$

In-proton d-quark form factors receive no MA contribution.

Sachs form factors

$$G_E^N = \frac{1}{2i\sqrt{1+\tau}}\,\mathrm{tr}\left[J_\mu^N \hat{P}_\mu\right], \qquad G_M^N = \frac{i}{4\tau}\,\mathrm{tr}\left[J_\mu^N \gamma_\mu^T\right]$$

$$G_E^N = F_1^N - \tau F_2^N, \qquad G_M^N = F_1^N + F_2^N$$

with $\tau = Q^2/[4m_N^2]$.

Normalisation is fixed by $G_E^p(0) = 1$; current conservation gives $G_E^n(0) \equiv 0$.

arXiv:2511.13997, Eqs. (16)–(26).