The nature of $\Sigma(1430)$ from $\Lambda_c \to \Lambda\pi^+\pi^+\pi^-$ reaction and correlation functions

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Based on: H.P. Li, C.W. Xiao, WHL, J.J. Wu, E. Wang, E. Oset, PRD 110 (2024) 114018;

Y.Y. Li, J. Song, E. Oset, WHL, R. Molina, EPJC 85 (2025) 1086.

Outline

- Introduction
- Chiral unitary approach: $\bar{K}N$ interaction in coupled channels, the $\Sigma^*(1430)(1/2^-)$ state
- Unravel the nature of the $\Sigma^*(1430)(1/2^-)$ state from correlation function
- $\Sigma^*(1430)$ production in $\Lambda_c \to \Lambda\pi^+\pi^+\pi^-$ reaction
- Summary

◆ Introduction: the $\Sigma^*(1430)(1/2^-)$ state

- The Σ^* states with $J^P = 1/2^-$ are a **benchmark** for hadron spectroscopy, requiring **large meson-baryon components** on top of the conventional qqq seed.

  A recent review on the $\Sigma*(1/2-)$ state: E. Wang, L.S. Geng, J.J. Wu, J.J. Xie, B.S. Zou, CPL41(2024)101401.

- A $\Sigma^*(1/2^-)$ state near the $\bar{K}N$ threshold ($M \sim 1430$ MeV) was predicted:

  Kaiser, Siegel, Weise, NPA 594 (1995) 325; Oset, Ramos, NPA 635 (1998) 99;
  Oller, Meißner, PLB500(2001)263; Jido, Oller, Oset, Ramos, Meißner, NPA 725 (2003) 181

  $\Downarrow$

  two-poles structure of Λ(1405) was found.

- Proposals to search for the predicted $\Sigma^*(1430)(\mathbf{1/2^-})$ state:

  $\gamma n \to K^+\Sigma^{*-}(1/2^-)$ Y.H. Lyu, H. Zhang, N.C. Wei, B.C. Ke, E. Wang, J.J. Xie, CPC 47(2023) 053108

  $\bar{\nu}_l p \to l^+\Phi B$ X.L. Ren, E. Oset, L.A. Ruso, M. J. Vicente Vacas, PRC 91 (2015) 045201;
  J.J. Wu, B.S. Zou, Few Body Syst. 56 (2015)165

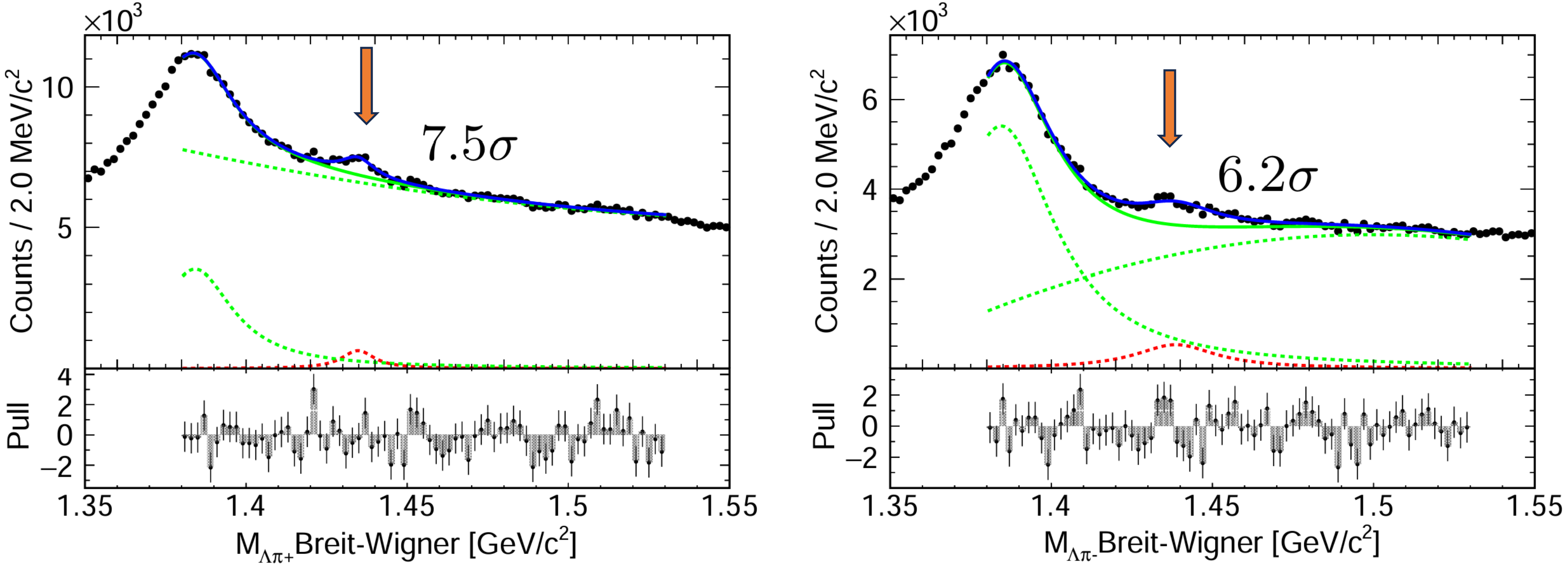
  $\chi_{c0}(1P) \to \bar{\Sigma}\Sigma\pi$ E. Wang, J.J. Xie, E. Oset, PLB 753 (2016) 526

  $\chi_{c0} \to \bar{\Lambda}\Sigma\pi$ L.J. Liu, E. Wang, J.J. Xie, K.L. Song, J.Y. Zhu, PRD 98 (2018) 114017

$\Lambda_c^+ \to \pi^+\pi^0\pi^-\Sigma^+$ $\quad$ $\Lambda_c^+ \to \pi^+\pi^+\pi^-\Lambda$ J.-J. Xie, E. Oset, PLB 792 (2019) 450

- A signal of the $\Sigma^*(1430)$ state was recently observed by **Belle Collaboration** in $\Lambda_c^+ \to \Lambda\pi^+\pi^-\pi^-$:

Y. Ma et al. [Belle], Phys. Rev. Lett. 130, 151903 (2023)



Mass: $1434.3 \pm 0.6(\text{stat.})^{+0.9}_{-0.0}(\text{syst.})$ MeV $\quad$ $1438.5 \pm 0.9(\text{stat.})^{+0.2}_{-2.5}(\text{syst.})$ MeV

Our aim: to understand further the nature of the $\Sigma^*(1430)$ state and its properties.

◆ Chiral unitary approach: $\bar{K}N$ interaction in coupled channels, the $\Sigma^*(1430)(1/2^-)$ state

Coupled channels : $\bar{K}^0 p$, $\pi^+\Sigma^0$, $\pi^0\Sigma^+$, $\pi^+\Lambda$, and $\eta\Sigma^+$ ($I_3 = 1$, without the Coulomb interaction)

The transition potentials of the coupled channels can be derived from the leading-order chiral Lagrangian:

$$V_{ij} = -\frac{1}{4f^2} C_{ij}(k_i^0 + k_j^0), \qquad f = 93 \text{ MeV}, \qquad k_i^0 = \frac{s + m_i^2 - M_i^2}{2\sqrt{s}},$$

C_{ij}	$\bar{K}^0 p$	$\pi^+\Sigma^0$	$\pi^0\Sigma^+$	$\pi^+\Lambda$	$\eta\Sigma^+$
$\bar{K}^0 p$	1	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{2}}$	$\sqrt{\frac{3}{2}}$
$\pi^+\Sigma^0$		0	-2	0	0
$\pi^0\Sigma^+$			0	0	0
$\pi^+\Lambda$				0	0
$\eta\Sigma^+$					0

The scattering matrix T follows the Bethe-Salpeter equation in coupled channels,

$$T = [I - VG]^{-1}V,$$

where G is a diagonal matrix with its elements Gi being the meson-baryon loop function regularized with a cutoff,

$$G_i = 2M_i \int_{|\vec{q}|<q_{\max}} \frac{\mathrm{d}^3q}{(2\pi)^3} \frac{w_1(\vec{q}\,) + w_2(\vec{q}\,)}{2\,w_1(\vec{q}\,)\,w_2(\vec{q}\,)} \frac{1}{s - [w_1(\vec{q}\,) + w_2(\vec{q}\,)]^2 + i\epsilon},$$

$$w_1(\vec{q}\,) = \sqrt{\vec{q}^{\,2} + m_i^2},\ w_2(\vec{q}\,) = \sqrt{\vec{q}^{\,2} + M_i^2}$$

G function in the 2nd Riemann sheet:

$$G_i^{II}(\sqrt{s}) = G_i(\sqrt{s}) + i\,\frac{M}{2\pi\sqrt{s}}\,q_{\rm on}, \qquad \text{for } \ \mathrm{Re}(\sqrt{s}) > \sqrt{s}_{\rm th},$$

with

$$q_{\rm on} = \lambda^{1/2}(s, m_i^2, M_i^2)/2\sqrt{s}$$

⇨ **A pole of T matrix:**

$$\sqrt{s_p} = 1431.83 - i\ 104.75\ \mathrm{MeV} \qquad (\text{with } q_{\rm max}\text{=630 MeV })$$

Oset, Ramos, NPA 635 (1998) 99.

corresponding to $\Sigma^*(1430)(1/2^-)$ state.

⇩

a dynamically generated state from $\bar{K}^0 p$, $\pi^+\Sigma^0$, $\pi^0\Sigma^+$, $\pi^+\Lambda$, and $\eta\Sigma^+$ interactions in S-wave.

◆ Unravel the nature of the $\Sigma^*(1430)(1/2^-)$ from correlation functions

- **Femtoscopic correlation functions (CFs):**

wave function of the hadron pair

$$C(p_a, p_b) = \frac{P(p_a, p_b)}{P(p_a)P(p_b)}, \quad \Longrightarrow \quad C(p) = \int \mathrm{d}^3 r S_{12}(\vec{r})|\psi(\vec{r}, p)|^2,$$

Source function

$$S_{12}(\vec{r}) = \frac{1}{(\sqrt{4\pi}R)^3}\exp\left(-\frac{\vec{r}^2}{4R^2}\right).$$ (in Gaussian with radius R)

CFs emerge a relevant experimental tool, showing the potential to provide information on **the interaction of hadrons**, **scattering parameters**, and the **nature of hadronic resonances**.

- **CFs for coupled channels related to the $\Sigma^*(1430)(1/2^-)$ state**

Coupled channels : $\bar{K}^0 p$, $\pi^+\Sigma^0$, $\pi^0\Sigma^+$, $\pi^+\Lambda$, and $\eta\Sigma^+$ ($I_3 = 1$, without the Coulomb interaction)

$$
\begin{aligned}
C_{\bar{K}^0 p}(p_{\bar{K}^0}) = 1 + 4\pi\theta(q_{\max} - p_{\bar{K}^0}) \int \mathrm{d}r r^2 S_{12}(r) \\
\times \{|j_0(p_{\bar{K}^0} r) + T_{11}(E)\tilde{G}_1(r,E)|^2 \\
+ |T_{21}(E)\tilde{G}_2(r,E)|^2 + |T_{31}(E)\tilde{G}_3(r,E)|^2 \\
+ |T_{41}(E)\tilde{G}_4(r,E)|^2 + |T_{51}(E)\tilde{G}_5(r,E)|^2 \\
- j_0^2(p_{\bar{K}^0} r)\},
\end{aligned}
$$

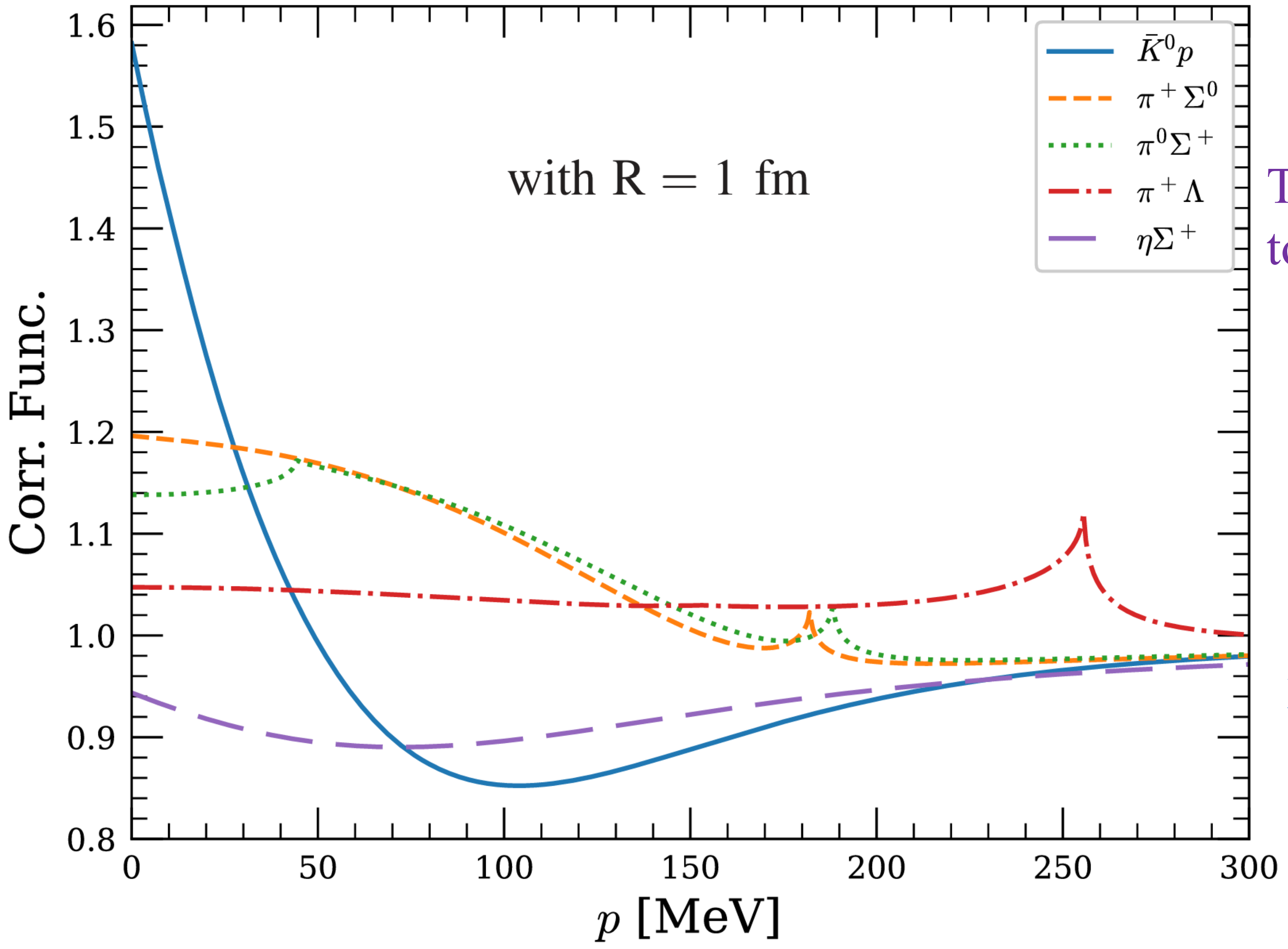
$$
\begin{aligned}
C_{\pi^+\Sigma^0}(p_{\pi^+}) = 1 + 4\pi\theta(q_{\max} - p_{\pi^+}) \int \mathrm{d}r r^2 S_{12}(r) \\
\times \{|j_0(p_{\pi^+} r) + T_{22}(E)\tilde{G}_2(r,E)|^2 \\
+ |T_{12}(E)\tilde{G}_1(r,E)|^2 + |T_{32}(E)\tilde{G}_3(r,E)|^2 \\
+ |T_{42}(E)\tilde{G}_4(r,E)|^2 + |T_{52}(E)\tilde{G}_5(r,E)|^2 \\
- j_0^2(p_{\pi^+} r)\},
\end{aligned}
$$

$$
\tilde{G}_i = 2M_i \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{w_1(\vec{q}) + w_2(\vec{q})}{2w_1(\vec{q})w_2(\vec{q})} \times \frac{j_0(|\vec{q}|r)}{s - [w_1(\vec{q}) + w_2(\vec{q})]^2 + i\epsilon},
$$

j_0: the zeroth order spherical Bessel function;

$T_{ij}(E)$: the two-body scattering amplitudes.

Correlation functions (using the chiral unitary approach above, with cutoff q_{max} =630MeV)

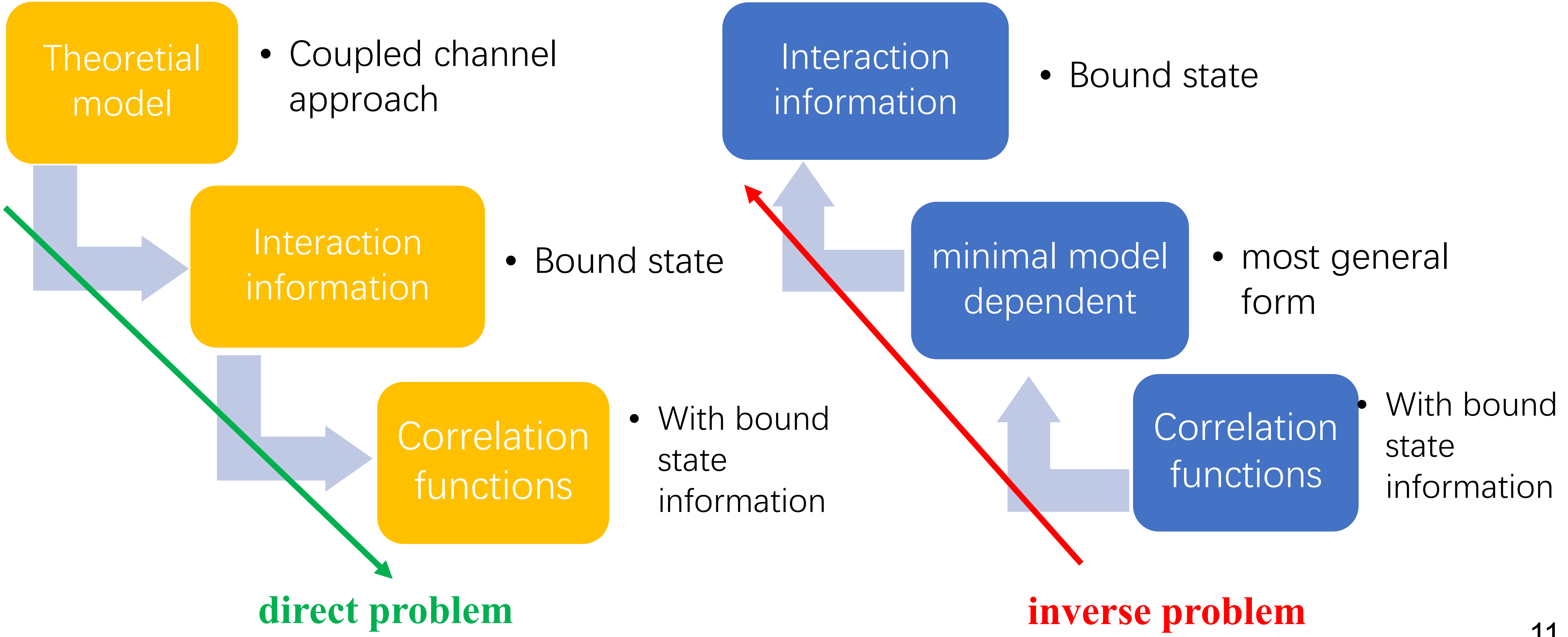


The observed cusps correspond to the opening ofs ome channels.

For inverse problem: to generate pseudo data, assuming ±0.02 error.

- **Inverse problem:** to see which information we can get from the correlation functions

(if we can get evidence of the existence of this $\Sigma^*(1430)(1/2^-)$ state from CFs?)



✓ How to do the inverse problem:

1) Use bootstrap or resampling method to generate pseudo data of CFs, assuming ±0.02 error. (~50 sets,)

Generating random centroids of the data

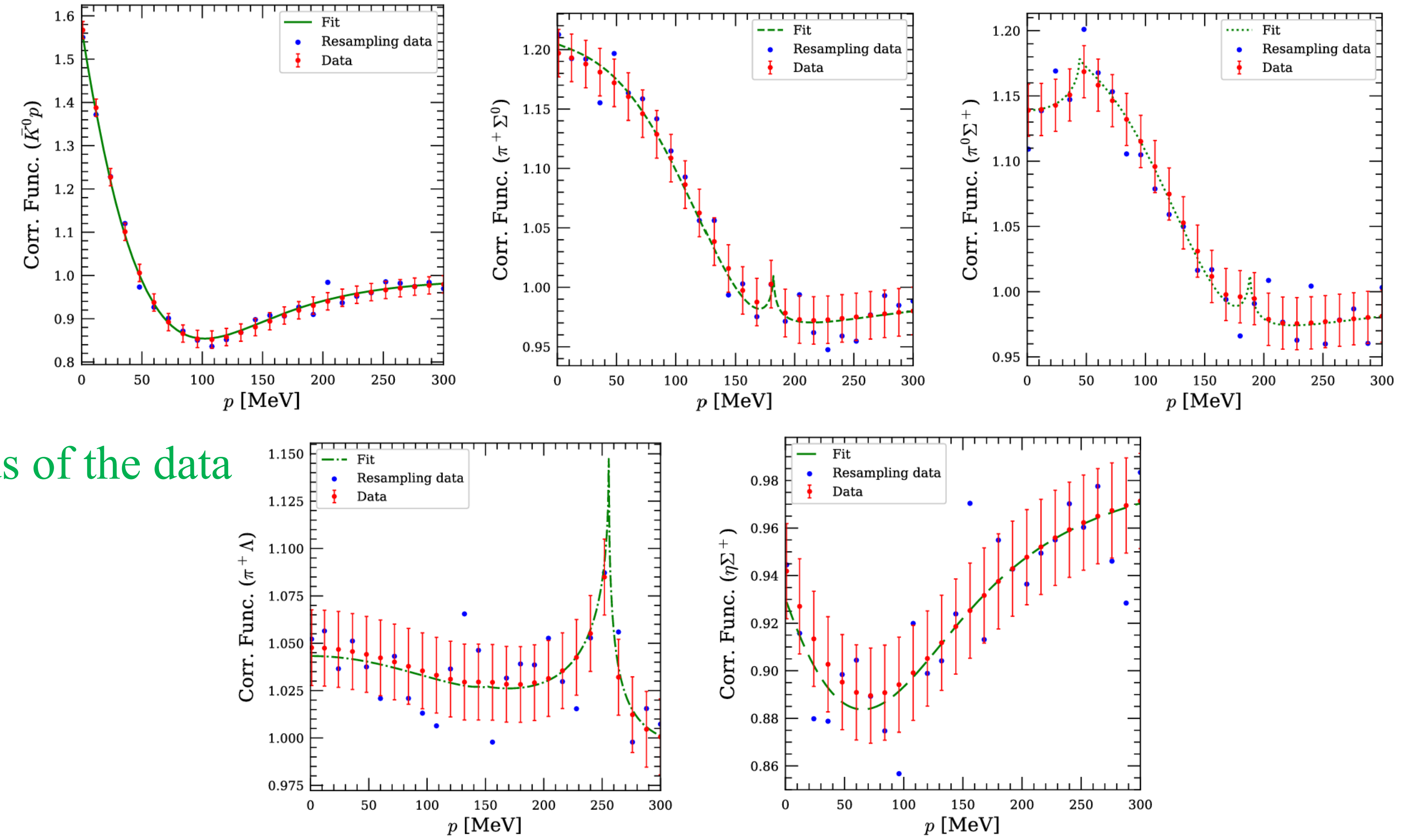


FIG. 3. Correlation functions for different channels, with 26 points in each curve with an error of ±0.02. The centroids of the red data follow the theoretical curve. The blue point is obtained from one of the resampling runs, with a random Gaussian generation of the centroids of each point.

2) Assume an energy dependence interaction potential

$$V_{ij} = -\frac{1}{4f^2}\tilde{C}_{ij}\underbrace{(k_i^0 + k_j^0)}_{}$$ (A general interaction)

(being practically constant, at the energy around 1430 MeV.)

Under the isospin constrain

$\tilde{C}_{ij}$	$\bar{K}^0 p$	$\pi^+\Sigma^0$	$\pi^0\Sigma^+$	$\pi^+\Lambda$	$\eta\Sigma^+$
$\bar{K}^0 p$	$\tilde{C}_{11}$	$-\frac{1}{\sqrt{2}}\tilde{C}_{12}$	$\frac{1}{\sqrt{2}}\tilde{C}_{12}$	$-\tilde{C}_{14}$	$-\tilde{C}_{15}$
$\pi^+\Sigma^0$		$\frac{1}{2}(\tilde{C}_{22}+\tilde{C}'_{22})$	$\frac{1}{2}(-\tilde{C}_{22}+\tilde{C}'_{22})$	$\frac{1}{\sqrt{2}}\tilde{C}_{24}$	$\frac{1}{\sqrt{2}}\tilde{C}_{25}$
$\pi^0\Sigma^+$			$\frac{1}{2}(\tilde{C}_{22}+\tilde{C}'_{22})$	$-\frac{1}{\sqrt{2}}\tilde{C}_{24}$	$-\frac{1}{\sqrt{2}}\tilde{C}_{25}$
$\pi^+\Lambda$				$\tilde{C}_{44}$	$\tilde{C}_{45}$
$\eta\Sigma^+$					$\tilde{C}_{55}$

✓ **Minimal model-dependent** approach:

11 parameters + $q_{\max}$ + R, in total 13 parameters.

✓ Results for inverse problem:

We carry fits to the pseudo data of CFs (~50 sets) with a minimal model-dependent approach.

Each fit ⟹ a set of parameters ⟹ T_{ij} amplitudes and pole

⟹ **Average and dispersion of the fitted parameters, T_{ij} amplitudes and pole position.**

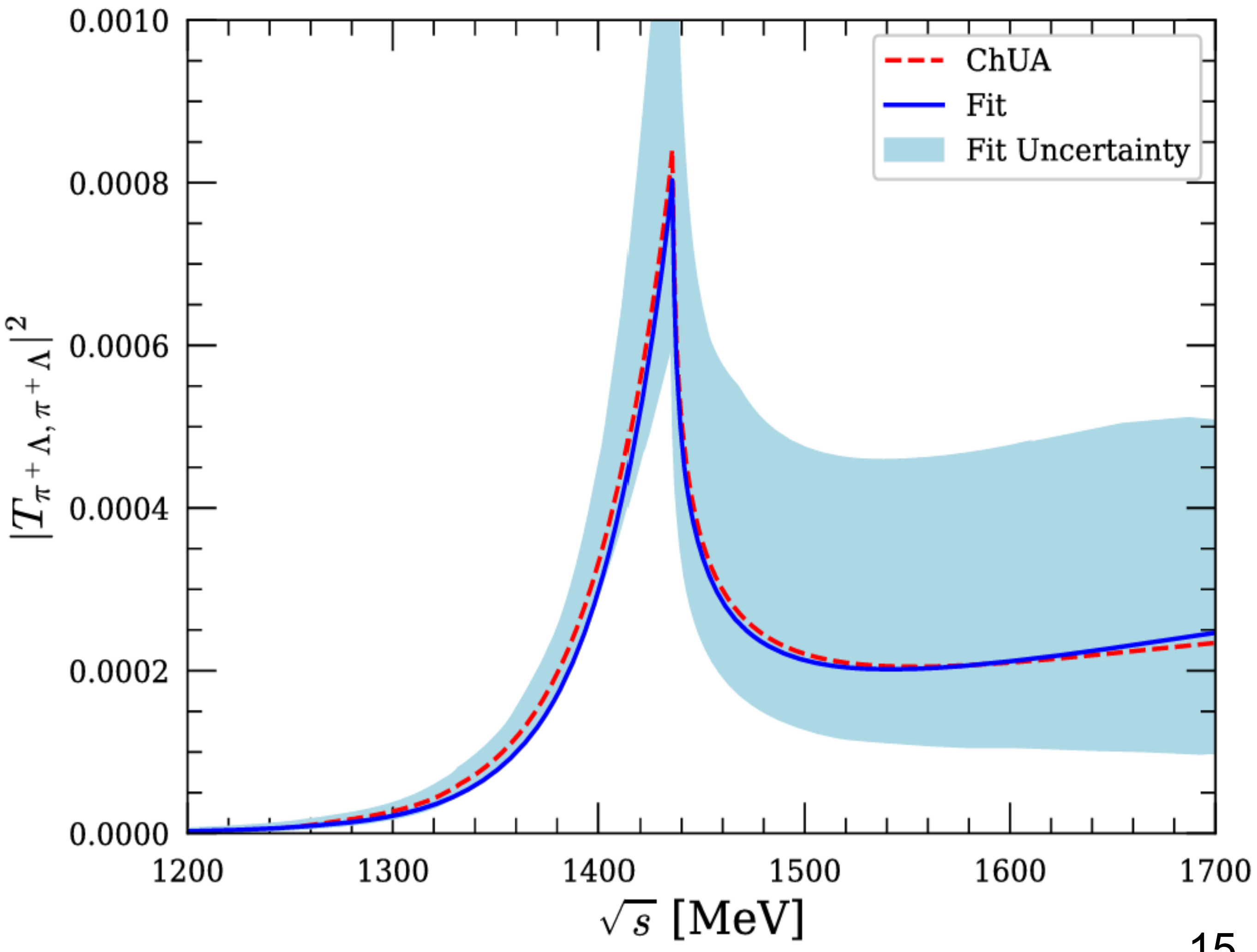
Average and dispersion of the fitted parameters:

$\tilde{C}_{11}$	$\tilde{C}_{12}$	$\tilde{C}_{14}$	$\tilde{C}_{15}$	$\tilde{C}_{22}$
1.036 ± 0.261	-0.985 ± 0.138	-1.204 ± 0.220	-0.829 ± 0.406	1.924 ± 0.147
$\tilde{C}'_{22}$	$\tilde{C}_{24}$	$\tilde{C}_{25}$	$\tilde{C}_{44}$	$\tilde{C}_{45}$
-2.136 ± 0.465	-0.057 ± 0.342	-0.028 ± 0.571	-0.053 ± 0.141	-0.066 ± 0.706
$\tilde{C}_{55}$	$q_{\max}$ (MeV)	R (fm)		
0.043 ± 0.447	653.468 ± 63.802	0.995 ± 0.029		

Pole position: $\sqrt{s_p} = (1420 \pm 10) - i(101 \pm 19)$ MeV.

Comparison of the fit result with the one of the chiral unitary approach taking q_{max} =630 MeV:

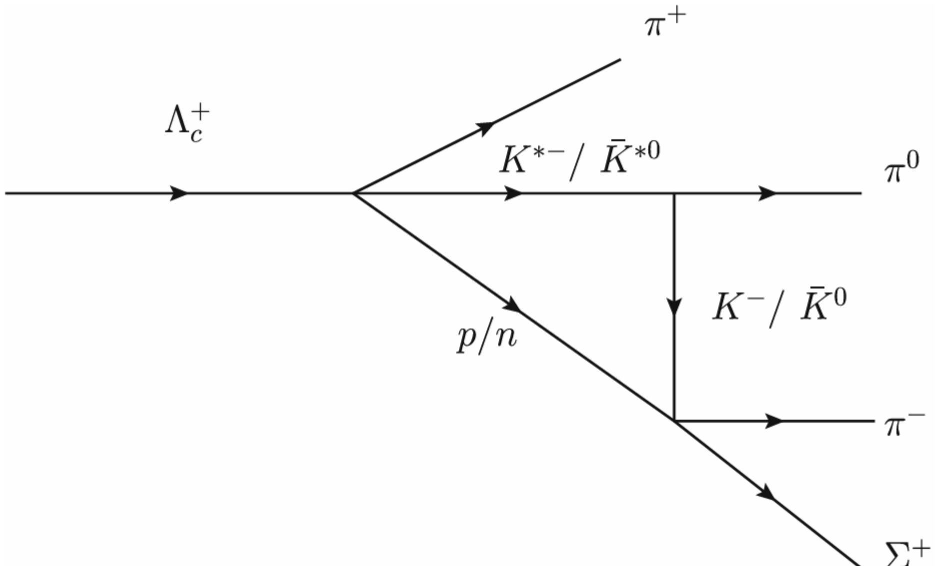
We can obtain the pole position from the correlation functions, with an uncertainty of about 10 MeV in the position and around 40 MeV in the width, with the assumed uncertainty in the correlation functions.



A clear cusp-like structure corresponding to the $\Sigma^*(1430)(1/2^-)$ state in the different amplitudes at the KbarN threshold,
with an apparent width of 30–50 MeV.

◆ $\Sigma^*(1430)(1/2^-)$ production in $\Lambda_c \to \Lambda\pi^+\pi^+\pi^-$ reaction

- Belle observed $\Sigma^*(1430)$ in $\Lambda_c^+ \to \Lambda\pi^+\pi^+\pi^-$
- The $\bar{K}N \to \pi\Lambda$ amplitudes are **small**
- Need a mechanism to **enhance** the signal
- Enter: **Triangle Singularity (TS)**
- TS can significantly enhance resonance production



$$\Lambda_c^+ \to \pi^+ K^{*-} p, \quad K^{*-} \to \pi^- \bar{K}^0, \quad \bar{K}^0 p \to \pi^+ \Lambda,$$

TS occurs when all three intermediate particles are **simultaneously on-shell,** leading to a sharp peak at $M_{\pi^+\Lambda} \sim 1434$ MeV.

Decay amplitude:

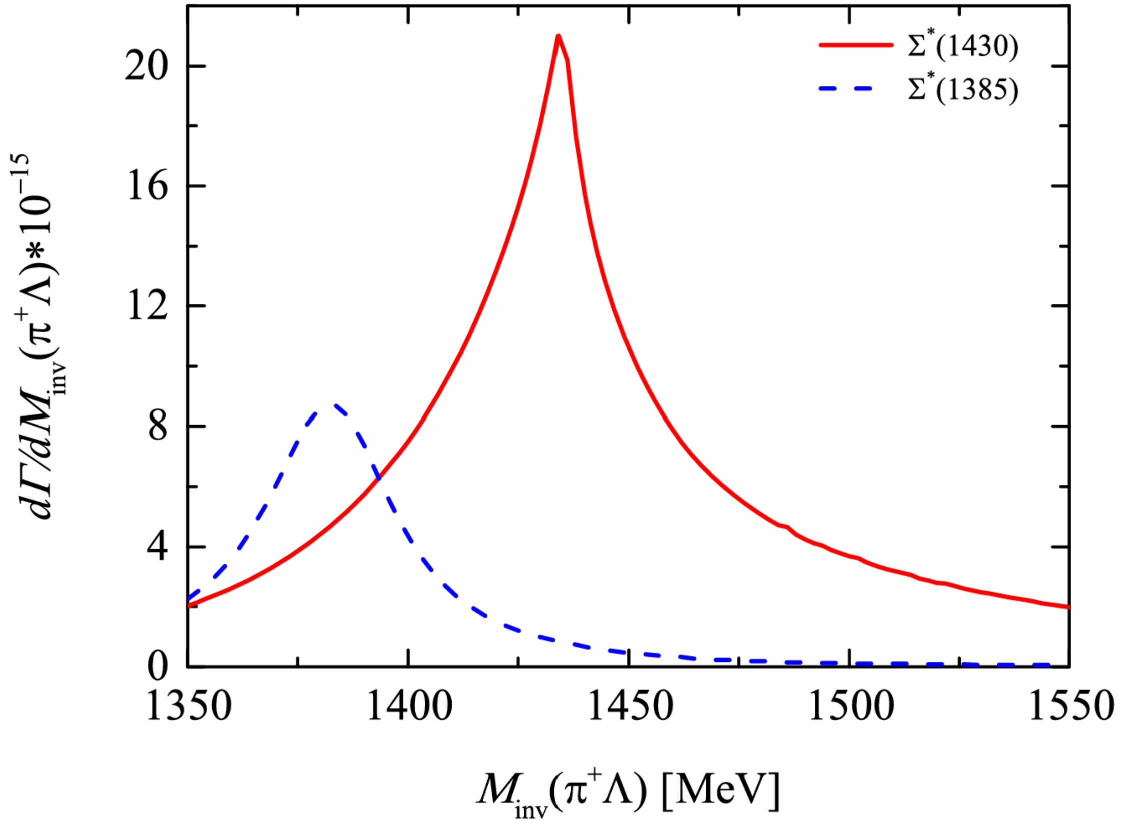
$$-iT = \sum_{\text{pol of } \bar{K}^*} \int \frac{d^4 q}{(2\pi)^4} 2M_N \frac{t_{\Lambda_c^+ \to \pi^+ K^{*-} p}}{q^2 - M_N^2 + i\epsilon} \times \frac{t_{K^{*-} \to \pi^- \bar{K}^0}}{(P-q)^2 - m_{\bar{K}^*}^2 + i\epsilon} \frac{t_{\bar{K}^0 p \to \pi^+ \Lambda}}{(P-q-k)^2 - m_{\bar{K}}^2 + i\epsilon},$$

$$t_{TS}^{\Sigma^*(1430)} = g_{\Sigma^*(1430), \bar{K}^0 p} \tilde{t}_{TS}.$$

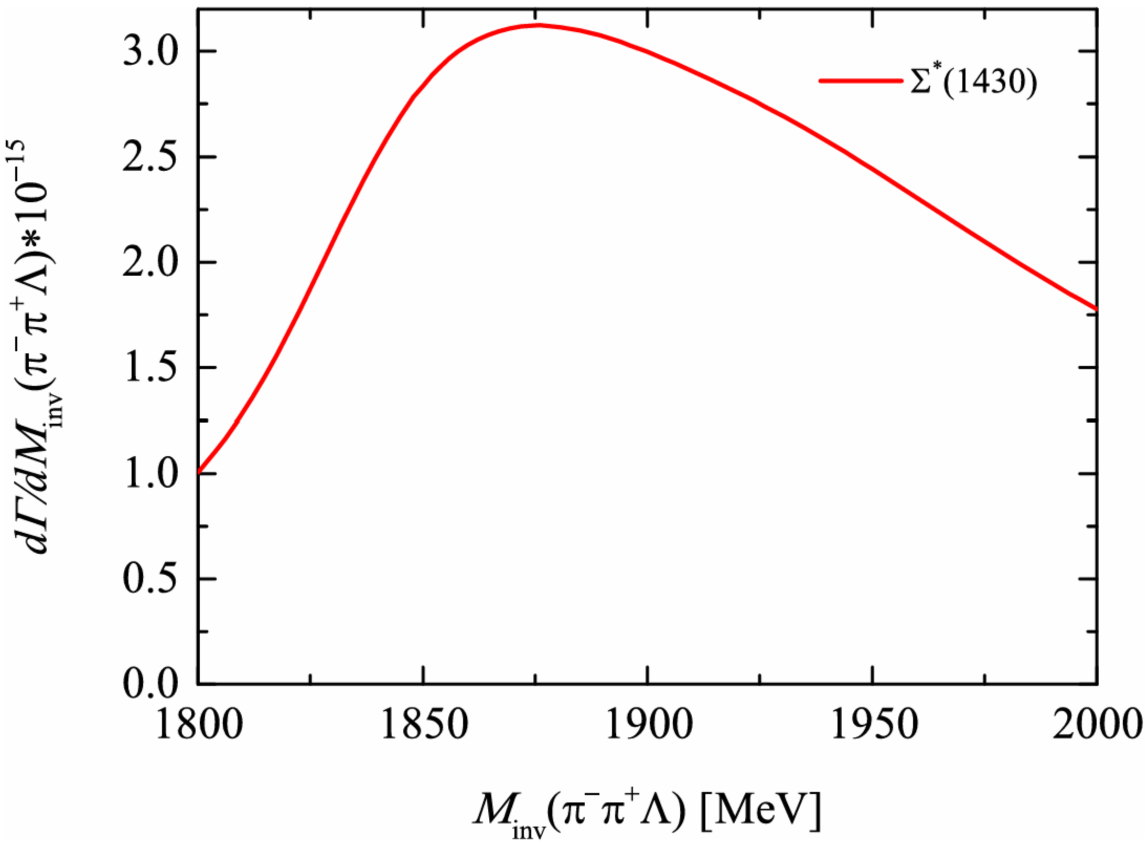
$$\tilde{t}_{\mathrm{TS}} = g A \sigma^j k^j \int \frac{d^3 q}{(2\pi)^3} \frac{2M_N}{2E_N(\vec{q}\,)\, 2\omega_{K^{*-}}(\vec{q}\,)\, 2\omega_{\bar{K}^0}(\vec{q}+\vec{k})} \times \left(2 + \frac{\vec{q}\cdot\vec{k}}{|\vec{k}|^2}\right) \frac{1}{P^0 - E_N(\vec{q}\,) - \omega_{K^{*-}}(\vec{q}\,) + i\frac{\Gamma_{K^{*-}}}{2}} \times \frac{1}{P^0 - E_N(\vec{q}\,) - k^0 - \omega_{\bar{K}^0}(\vec{q}+\vec{k}) + i\epsilon} \times \Theta(q_{\max} - |\vec{q}^{\,*}_{\mathrm{cm}}|).$$

$\pi^+\Lambda$ invariant mass distribution and $\pi^-\Sigma^*(1430)$ invariant mass distribution:

$$\frac{d\Gamma}{dM_{\text{inv}}(\pi^+\Lambda)dM_{\text{inv}}(\pi^- R)} = -\frac{1}{\pi}\text{Im}(t_{\bar{K}^0 p,\bar{K}^0 p})\frac{\Gamma_{\pi^+\Lambda}}{\Gamma_{\text{tot}}}\frac{1}{(2\pi)^3}\frac{2M_{\Lambda_c^+}2M_\Lambda}{4M_{\Lambda_c^+}^2}p_{\pi^+}\tilde{p}_{\pi^-} \times \overline{\sum_{\substack{\text{in}\\\text{pol}}}}\sum_{\substack{\text{fin}\\\text{pol}}}|\tilde{t}_{\text{TS}}|^2,$$



One can obtain the properties of the Belle peak and its strength.



Invariant mass distribution of the π^- $\Sigma*(1430)$ ($\Sigma*(1430) \rightarrow \pi^+\Lambda$) system in the decay $\Lambda c \rightarrow \pi^+\pi^+\pi^-\Lambda$.

A clear bump appears around 1875 MeV, interpreted as aconsequence of atriangle singularity.

Branching ratio estimation:

We estimate the branching ratio for the decay $\Lambda_c^+ \to \pi^+\pi^-\Sigma^*(1430)$ ($\Sigma^*(1430) \to \pi^+\Lambda$) by integrating the invariant mass distributions shown in Fig. 6 between 1375 MeV and 1500 MeV. By considering the Λ_c^+ width of 3.25×10^{-9} MeV,

$$\mathrm{Br}[\Lambda_c^+ \to \pi^+\pi^-\Sigma^*(1430) \to \pi^+\pi^-\pi^+\Lambda] \approx 3.5 \times 10^{-4}$$

The obtained result provides auseful prediction that can be tested in future experiments at facilities such as BESIII, Belle II, or LHCb.

Summary

- $\Sigma^*(1430)(1/2^-)$ is a dynamically generated state from $\bar{K}N$ interaction in chiral unitary approach;

- Correlation functions for the related coupled channels encode clear information about the $\Sigma^*(1430)(1/2^-)$ state;

- Inverse problem of correlation functions (resampling) can extract scattering parameters, pole position for $\Sigma^*(1430)(1/2^-)$ state;

- Production of $\Sigma^*(1430)(1/2^-)$ state in $\Lambda_c^+ \to \Lambda\pi^+\pi^+\pi^-$ decay is enhanced by triangle singularity;

- Predictions: Branching ratio $\sim 3.5 \times 10^{-4}$

  Bump at $\sim$ 1875 MeV in $\pi^-\Sigma^*(1430)$

- The nature of the $\Sigma^*(1430)$ can be further tested by measuring the correlation functions and the Br of $\Lambda_c^+ \to \Lambda\pi^+\pi^+\pi^-$ decay.

Thank you for your attention!

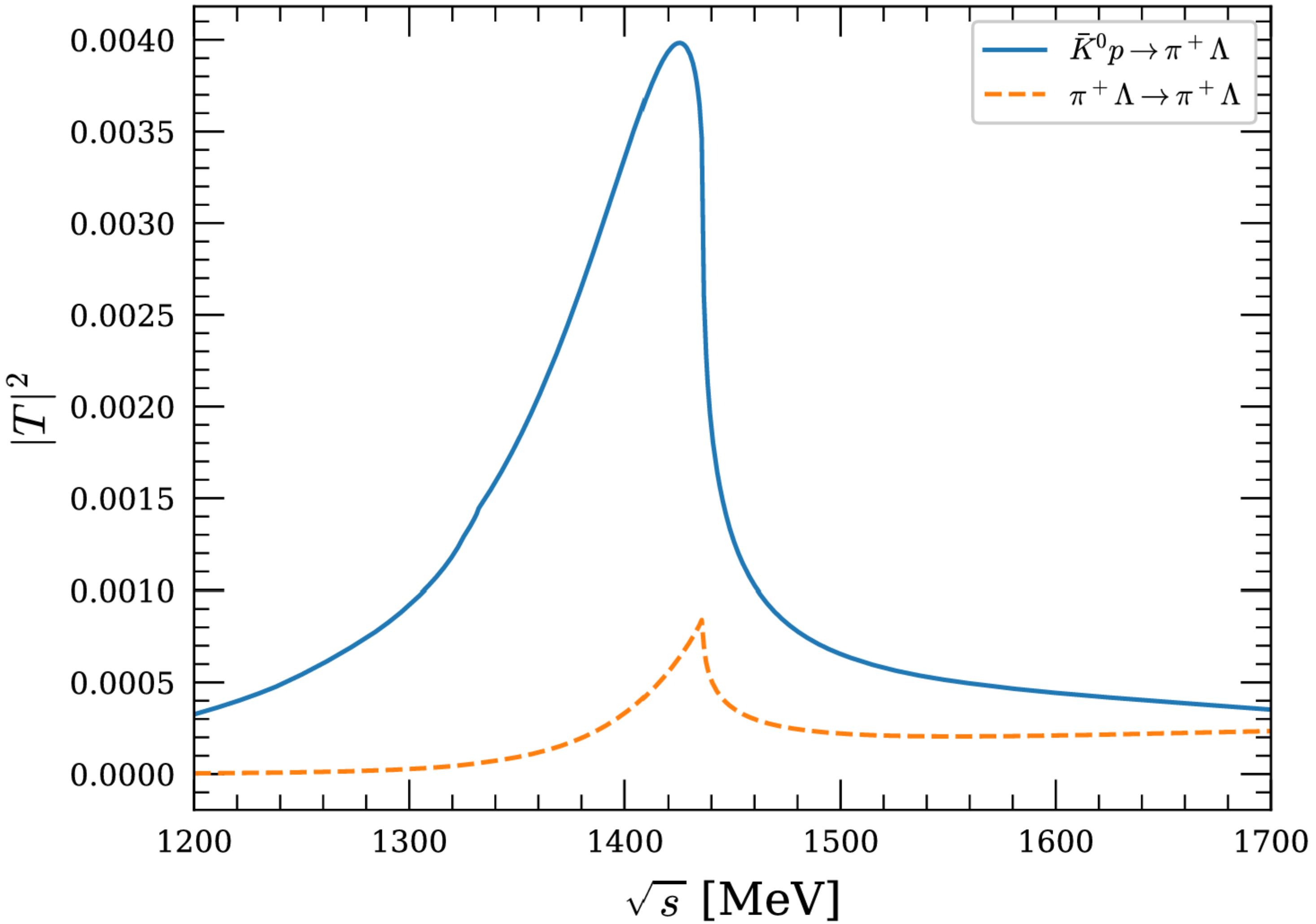


FIG. 1. Absolute squared values of T in $\bar{K}^0 p \to \pi^+ \Lambda$ and $\pi^+ \Lambda \to \pi^+ \Lambda$ of the chiral unitary approach taking $q_{\rm max} = 630$ MeV.