# Nucleon chiral-invariant mass and gravitational form factors in the parity doublet model

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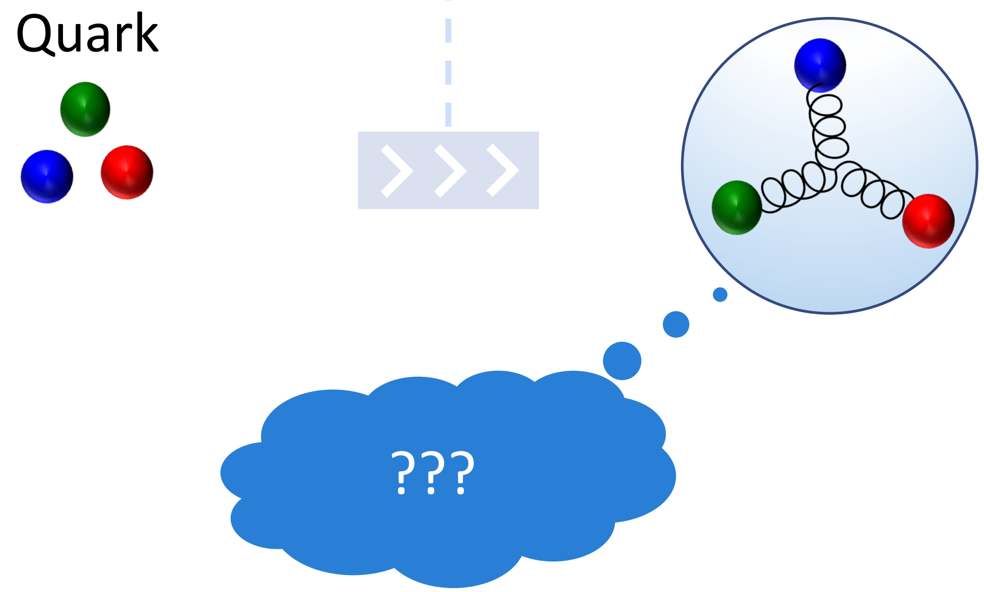
2026/9/7

M. K., M. Harada and Y. L. Ma, Phys.Lett.B 876 (2026) 140400

# Introduction: origin of the nucleon mass ($M_N$)



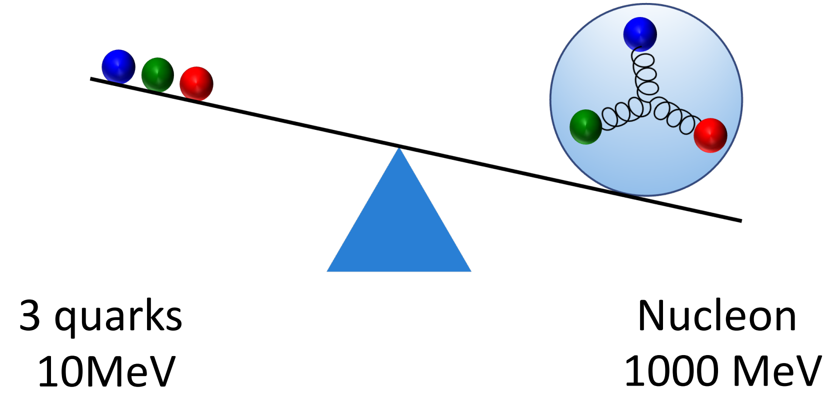
Nucleon is composed of quarks, but...




**Internal structure** is not yet fully understood.

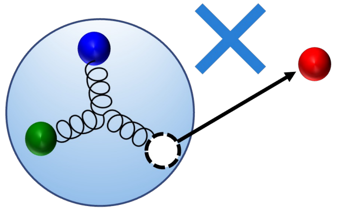
- Origin of **nucleon mass**
- **Mass** decomposition

## Mass gap problem




Nucleon mass is much heavier than sum of three quarks.

## Confinement problem



It is impossible to isolate a single quark: single quark is not observable.

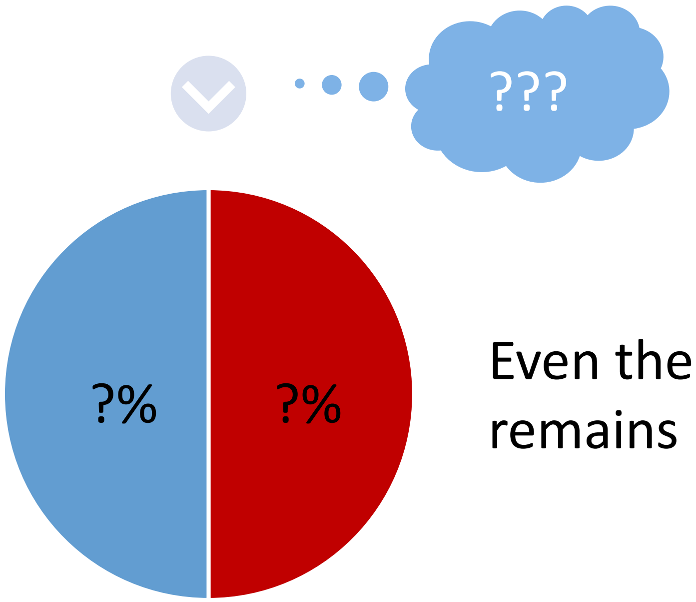
# Introduction: low-energy mass decomposition



In a low-energy description, $M_N$ can be decomposed into…

$$M_N = m_{\text{var}} + m_{\text{inv}}$$

- Contribution from chiral SSB
- Chiral invariant component




Even the rough decomposition of $M_N$ remains unclear…

# Introduction: $m_{\mathrm{inv}}$ and neutron star study



In a low-energy description, $M_N$ can be decomposed into…

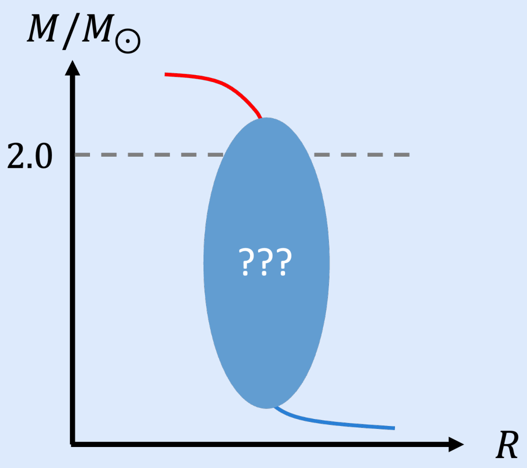
$$M_N = m_{\mathrm{var}} + m_{\mathrm{inv}}$$

- Contribution from chiral SSB
- Chiral invariant component

In the high-density region, …

- $m_{\mathrm{var}} \to 0$ (chiral symmetry restoration)
- Density independence of $m_{\mathrm{inv}}$

Affects Neutron star M-R relation.




Model analyses w/ $m_{\mathrm{inv}}$

T. Minamikawa et al., PRC (2021)
T. Minamikawa et al., Symmetry (2023)
B. Gao et al., PRC (2024)
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based on **parity doublet model**

Neutron-star constraints on $m_{\mathrm{inv}}$

$$m_{\mathrm{inv}} = 500 - 900\ \mathrm{MeV}$$

How about other observables?

# Gravitational form factors

Average momentum: $\bar{P} = (p + p')/2$
Momentum transfer: $q = p' - p \ \ (t = -q^2)$



$$\langle N(p',s')|\Theta^{\mu\nu}|N(p,s)\rangle$$
$$= \bar{u}^{s'}(p')\left[\boxed{A_N(t)}\frac{\bar{P}^\mu \bar{P}^\nu}{m_N} + \boxed{J_N(t)}\frac{i\sigma^{\mu\rho}q_\rho \bar{P}^\nu + i\sigma^{\nu\rho}q_\rho \bar{P}^\mu}{2m_N} + \boxed{D_N(t)}\frac{(q^\mu q^\nu - g^{\mu\nu}q^2)}{4m_N}\right]u^s(p)$$

**Gravitational form factors (GFFs)**

Related to **nucleon mass decomposition**

# Gravitational form factors

Average momentum: $\bar{P} = (p + p')/2$
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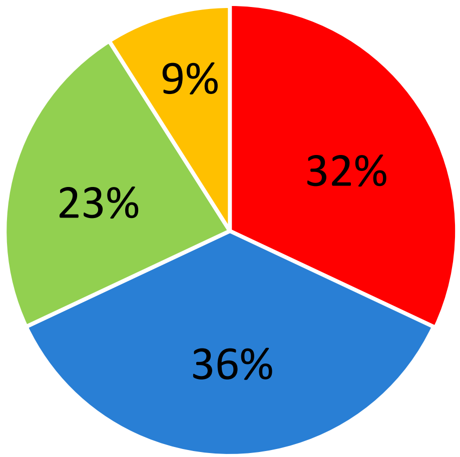
$A(t)$ : Mass $m_N$
$\rightarrow A(0) = 1$

$\langle p|\Theta^{00}|p\rangle = 2m_N$
$(m_N = 938$ MeV)

Proton mass decomposition




- Quark mass
- Quark energy
- Gluon energy
- Trace anomaly

Lattice simulation:
Yi-Bo Yang et al. PRL (2018)

Based on Ji's decomposition $\Theta^{00} = H_m + H_E + H_g + H_a/4$

- One of decomposition methods
- How much of $m_N$ arises **from chiral SSB**?

# Gravitational form factors

Average momentum: $\bar{P} = (p + p')/2$
Momentum transfer: $q = p' - p \;\; (t = -q^2)$



$$\langle N(p', s')|\Theta^{\mu\nu}|N(p, s)\rangle$$

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$A(t)$ : Mass $m_N$
$\rightarrow A(0) = 1$

$J(t)$ : Angular momentum
$\rightarrow J(0) = 1/2$

$D(t)$: Pressure $p$
$\rightarrow D(0) = ???$

$\langle p|\Theta^{00}|p\rangle = 2m_N$
($m_N = 938$ MeV)

Proton mass decomposition

# Gravitational form factors



$$\langle N(p',s')|\Theta^{\mu\nu}|N(p,s)\rangle$$
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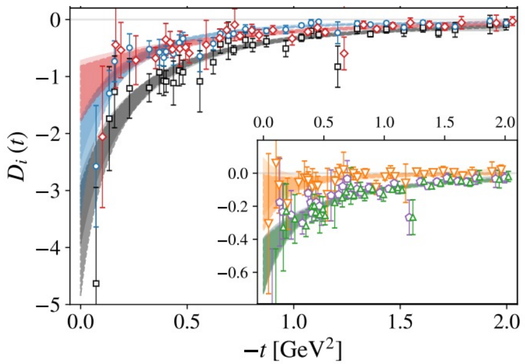
$\boxed{D(t)\text{: Pressure } p \to D(0) = ???}$

What information can be extracted from $D(t)$?

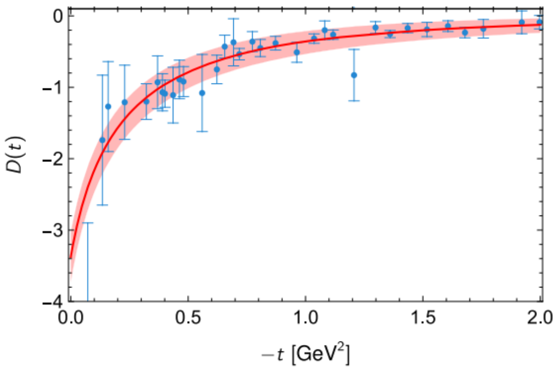
$D(t)$ has already been studied in …

Lattice QCD simulation




D. C. Hackett, et al., PRL(2024)

Dispersive method

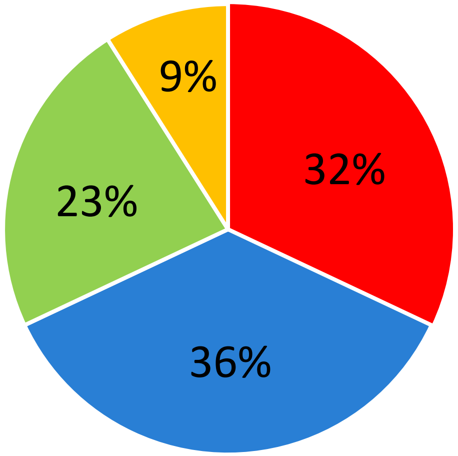



X. H. Cao, et al., Nature Commun(2025)

# Motivation for our study



Proton mass decomposition




- Quark mass
- Quark energy
- Gluon energy
- Trace anomaly

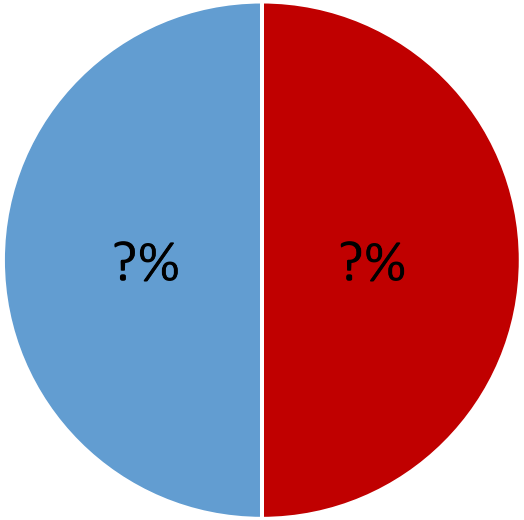
Lattice simulation:
Yi-Bo Yang et al. PRL (2018)

Based on Ji's decomposition

$$\Theta^{00} = H_m + H_E + H_g + H_a/4$$

It is unclear
how much chiral SSB contributes.

**Alternative decomposition at low energies**




$$M_N = m_{\rm var} + m_{\rm inv}$$

- Chiral variant component (Chiral SSB)
- Chiral invariant component

Does the ***D*-form factor** provide any insight?

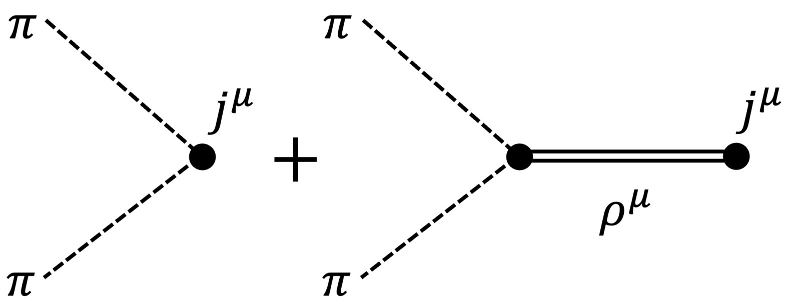
# Meson dominance in form factors

Before considering the nucleon D-form factor, let us recall...

**Pion electromagnetic form factor:** $\langle \pi(p')|j_\mu(0)|\pi(p)\rangle = (p'_\mu + p_\mu)F_V^\pi(t)$

It is well described by **vector meson dominance:**
Experimental data of pion charge radius can be reproduced by **vector meson dominance**.

$\langle \pi(p')|j_\mu(0)|\pi(p)\rangle \simeq$




(This picture can be naturally formulated within the framework of hidden local symmetry.)

Expect **meson dominance** to be realized in the GFFs as well.

# Meson dominance in GFFs

EMT ($\Theta^{\mu\nu}$) can couple to spin-0 and spin-2 mesons (isosinglet).

- Spin-0 isosinglet meson: $f_0(500)$, $f_0(980)$, …
- Spin-2 isosinglet meson: $f_2(1270)$, …

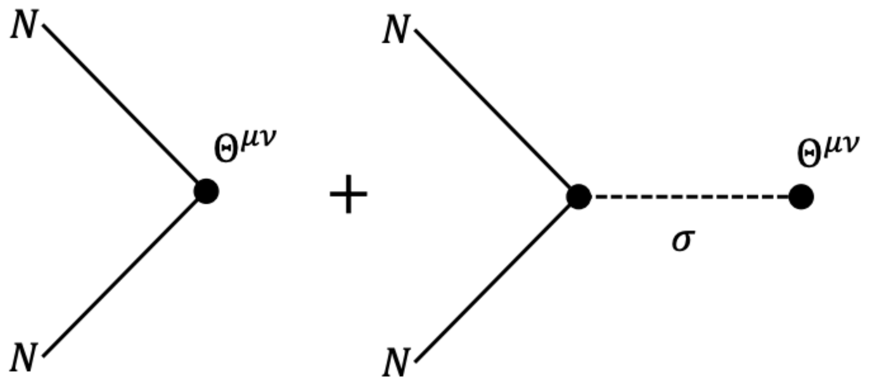
Meson dominance picture has also been considered in

W. Broniowski and E. Ruiz Arriola, PLB (2024)
W. Broniowski and E. Ruiz Arriola, PRD (2025)
R. Stegeman and R. Zwicky, JHEP (2026)
R. Stegeman and R. Zwicky, JHEP (2026)

We assume…

Isosinglet scalar meson dominance (**sigma meson dominance**)

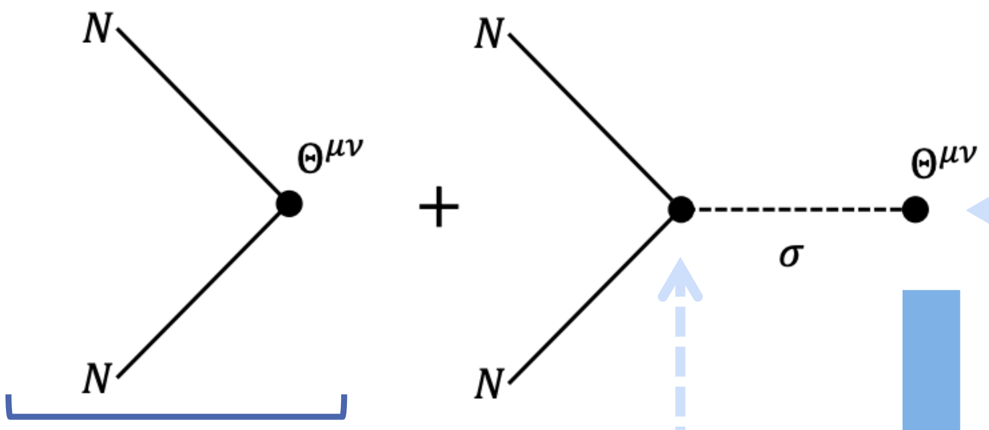
$$\langle N(p')|\Theta_{\mu\nu}(0)|N(p)\rangle \simeq$$




Evaluate $D(t)$ by using model.

# $D(t)$ from Linear Sigma Model (LSM)






No contribution to $D(t)$

Coupling between $\sigma$ meson and EMT that satisfies $\Theta^{\mu}_{\mu} = \partial_\mu j^{\mu}_{D}$:

$$\langle 0|\Theta^{\mu\nu}|\sigma(p)\rangle = \frac{\sigma_0}{3}(p^\mu p^\nu - g^{\mu\nu} m_\sigma^2)e^{-ip\cdot x}$$

Yukawa coupling

$$g_{NN\sigma} = \frac{m_N}{f_\pi}$$

$m_{\text{inv}}$=0

LSM ($\mathcal{L}_{\text{LSM}} = \mathcal{L}_N + \mathcal{L}_M$)
Nucleon: $N$
Mesons: $M = \sigma + i\pi^a\tau^a$

$$\mathcal{L}_N = \bar{N}_r i\gamma^\mu\partial_\mu N_r + \bar{N}_l i\gamma^\mu\partial_\mu N_l - g\left(\bar{N}_r M^\dagger N_l + \bar{N}_l M N_r\right),$$

$$\mathcal{L}_M = \frac{1}{4}\operatorname{tr}\left[\partial^\mu M^\dagger \partial_\mu M\right] - V(M)$$

$$D^{(\text{LSM})}(t) = -\frac{4m_N^2}{3}\frac{1}{m_\sigma^2 + t}$$

Is described by

- $m_\sigma$ (sigma meson mass)
- $m_N \propto \langle 0|\sigma|0\rangle$

$m_{\text{inv}}$=0

R. Stegeman and R. Zwicky, (2025)
M. K., M. Harada, Y.-L. Ma, (2026)

# $D(t)$ from Linear Sigma Model (LSM)



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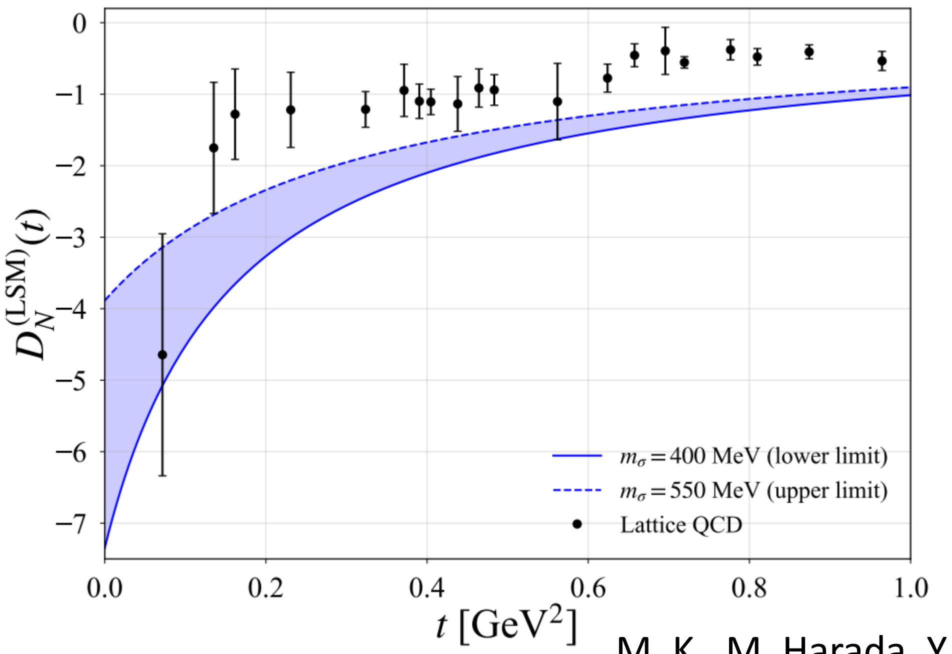


M. K., M. Harada, Y.-L. Ma, PLB (2026)

- Compared with lattice results.
  (D. C. Hackett, et al., PRL(2024))
- Consider the range for $m_\sigma$:
  $m_\sigma = 400 - 550\ MeV$.
  (These values are listed in PDG)

LSM model result is deviated from lattice observation...

How important is the missing contribution from $m_{\mathrm{inv}}$?

# Chiral effective model with $m_{\mathrm{inv}}$



$m_{\mathrm{inv}}$ ($m_0$) is introduced within the framework of Parity Doublet Model (PDM)

$$\mathcal{L}_N^{(\mathrm{PDM})} = \bar{\psi}_{1r} i\gamma^\mu \partial_\mu \psi_{1r} + \bar{\psi}_{1l} i\gamma^\mu \partial_\mu \psi_{1l} + \bar{\psi}_{2r} i\gamma^\mu \partial_\mu \psi_{2r} + \bar{\psi}_{2l} i\gamma^\mu \partial_\mu \psi_{2l}$$
$$- m_0 \left[\bar{\psi}_{1l}\psi_{2r} - \bar{\psi}_{1r}\psi_{2l} - \bar{\psi}_{2l}\psi_{1r} + \bar{\psi}_{2r}\psi_{1l}\right]$$
$$- g_1 \left[\bar{\psi}_{1r} M^\dagger \psi_{1l} + \bar{\psi}_{1l} M \psi_{1r}\right]$$
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C. E. Detar and T. Kunihiro, PRD (1989)
D. Jido, M. Oka, A. Hosaka, PTP (2001)

- Chiral invariant mass: $m_0$
- **Positive-parity nucleon** ($\psi_1$): N(939)
- **Negative-parity nucleon** ($\psi_2$): $N^*(1535)$

✓ $N^*(1535)$ is …

- incorporated as an explicit degree of freedom in model (simply treated as a single-particle state)
- identified as parity partner of N(939)

✓ PDM has been studied in…

- vacuum properties (decays)
- neutron star matter

Transformation properties of nucleons

**Parity transformation:**

$$\psi_1(t,\vec{x}) \to \gamma^0 \psi_1(t,-\vec{x}),$$
$$\psi_2(t,\vec{x}) \to -\gamma^0 \psi_1(t,-\vec{x})$$

**Chiral $SU(2)_L \times SU(2)_R$ transformations:**

$$\psi_{1r} \to g_R \psi_{1r}, \quad \psi_{1l} \to g_L \psi_{1l},$$
$$\psi_{2r} \to g_L \psi_{2r}, \quad \psi_{2l} \to g_R \psi_{2l} \quad \text{(Mirror assignment)}$$

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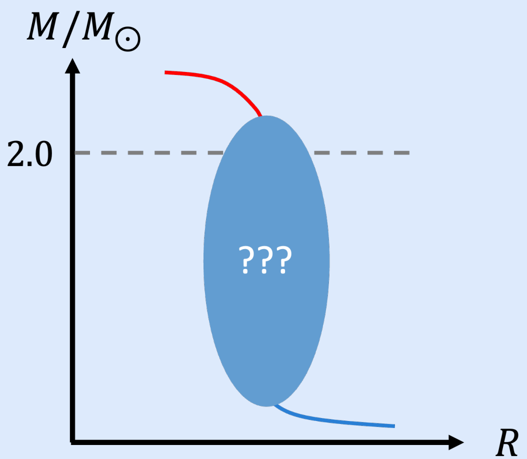
Neutron star study based on PDM




T. Minamikawa et al., PRC (2021)
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Neutron-star constraints on $m_{\text{inv}}$

$m_{\text{inv}} = 500 - 900$ MeV

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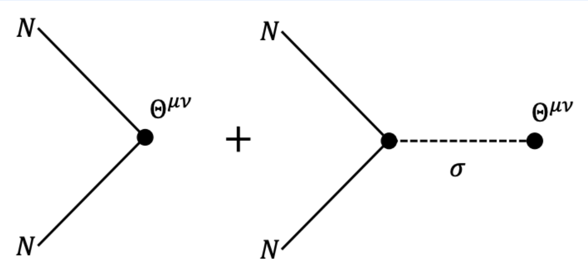
In this study, we evaluate $D(t)$ based on PDM.

Adopt the scalar meson dominance…




$D^{(\text{PDM})}(t) = ???$

$D(t)$ constraints on $m_{\text{inv}}$

$m_{\text{inv}} = ???$

# Chiral effective model with $m_{\text{inv}}$



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- Chiral invariant mass: $m_0$
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Nucleon mass:

$$m_N = \frac{1}{2}\left((g_1 - g_2) f_\pi + \sqrt{f_\pi^2 (g_1 + g_2)^2 + 4m_0^2}\right)$$

Yukawa coupling:

$$g_{NN\sigma} = \frac{1}{f_\pi}\left(m_N - \frac{2m_0^2}{m_N + m_{N^*}}\right)$$

$(m_{N^*} = 1535\ \text{MeV})$

$m_{\text{inv}}$ appears in $m_N$ and $g_{NN\sigma}$.

$g_{NN\sigma}$ is reduced by $m_{\text{inv}}$.

# $D(t)$ from Parity Doublet Model (PDM)

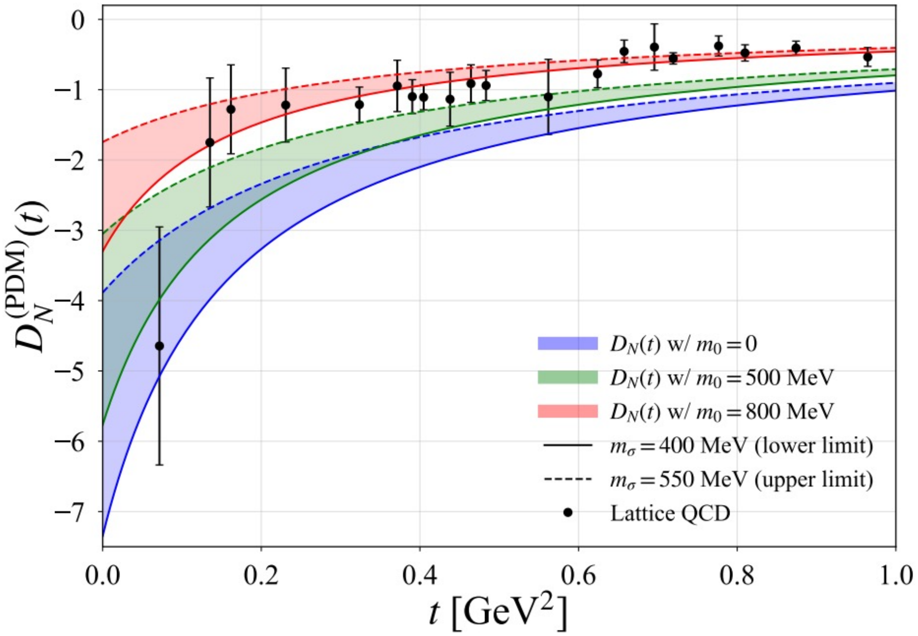


M. K., M. Harada, Y.-L. Ma, PLB (2026)

$$D^{(\mathrm{PDM})}(t) = \left(1 - \frac{2m_0^2}{m_N\left(m_N + m_{N^*}\right)}\right) D^{(\mathrm{LSM})}(t)$$

Chiral invariant mass $m_{\mathrm{inv}}$ $(m_0)$ suppresses $D(t)$.

- Compared with lattice results.
  (D. C. Hackett, et al., PRL(2024))
- Consider the range for $m_\sigma$:
  $m_\sigma = 400 - 550$ MeV.
  (These values are listed in PDG)
- Consider several values of $m_0$ :
  $m_0 = 0, 500, 800$ MeV




Large $m_{\mathrm{inv}}$ leads to consistency with…
- ✓ recent lattice data.
- ✓ recent M-R studies based on PDM.

# Summary and discussion



Suggests that GFFs are the key to the origin of mass.

M. K., M. Harada, Y.-L. Ma, PLB (2026)

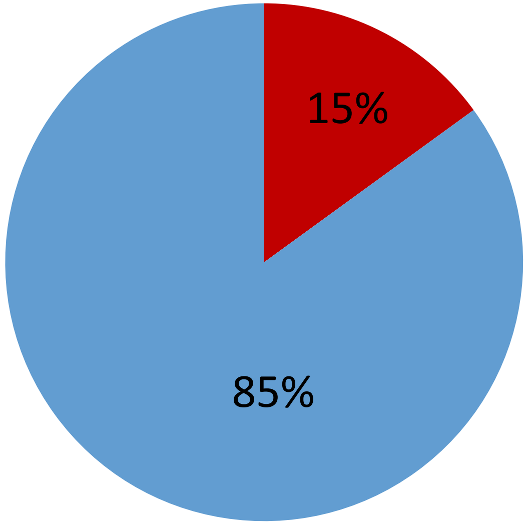
We have evaluated D-form factor under the sigma meson dominance. ››› Chiral invariant mass dominates $m_N$ (based on lattice data comparison).

Comparison w/ recent lattice data:
$m_{\text{inv}} = 800\ \text{MeV}$

Seem to suggest... ›››

Neutron-star constraints on $m_{\text{inv}}$
$m_{\text{inv}} = 500\ - 900\ \text{MeV}$

$m_N\ = 938$ MeV decomposition




- Chiral variant component
- Chiral invariant component ($m_0\ = 800\ \text{MeV}$)

# Summary and discussion



## 1. Consistency with other form factors (not GFFs)

For the axial form factor, the axial charge in PMD is given by

$$g_A^{(\mathrm{PDM})} = \cos 2\theta$$

$\theta$ is mixing angle between $\psi_1$ and $\psi_2$: $\tan 2\theta = 2m_0 / \sqrt{(m_N + m_{N^*})^2 - 4m_0^2}$

PDM: $g_A^{(\mathrm{PDM})} < g_A^{(\mathrm{exp.})} = 1.27$

→ **Additional interaction terms** can improve.
(derivative interactions) Yamazaki and Harada (2019)

## 2. What is the sigma meson?

In our study, $\sigma$ is treated as chiral partner of $\pi$.

**Possible interpretations/components**

Glueball, dilatonic particle,
four-quark state, meson-meson resonance state...

## 3. Contributions beyond $\sigma$ dominance

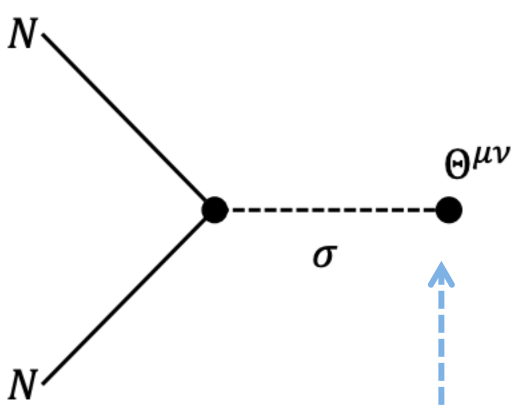
Other exchange may also contribute to D(t)

- **Heavier iso-singlet scalar mesons**
- **Tensor mesons**

→ Toward a more precise evaluation

Thank you

# Improved EMT and EMT-sigma coupling




LSM ($\mathcal{L}_{\mathrm{LSM}} = \mathcal{L}_N + \mathcal{L}_M$)
Nucleon: $N$
Mesons: $M = \sigma + i\pi^a\tau^a$

$$T^{\mu\nu} = \frac{1}{2}\bar{N}_r i(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu)N_r + \frac{1}{2}\bar{N}_l i(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu)N_l + \frac{1}{2}\,\mathrm{tr}\left[\partial^\mu M^\dagger\,\partial^\nu M\right] - g^{\mu\nu}\mathcal{L}$$

$$T^\mu_{\ \mu} \neq \partial_\mu j^\mu_D$$

Improve EMT to satisfy $\Theta^\mu_{\ \mu} = \partial_\mu j^\mu_D$

$$\Theta^{\mu\nu} = \frac{1}{2}\bar{N}_r i(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu)N_r + \frac{1}{2}\bar{N}_l i(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu)N_l + \frac{1}{2}\,\mathrm{tr}\left[\partial^\mu M^\dagger\,\partial^\nu M\right] - g^{\mu\nu}\mathcal{L} - \frac{1}{12}(\partial^\mu\partial^\nu - g^{\mu\nu}\partial^2)\mathrm{tr}[MM^\dagger]$$

$$\Theta^\mu_{\ \mu} = \partial_\mu j^\mu_D$$

Sigma meson couples to EMT.

$$\langle 0|\Theta^{\mu\nu}|\sigma(p)\rangle = \frac{f_\pi}{3}(p^\mu p^\nu - g^{\mu\nu}m_\sigma^2)e^{-ip\cdot x}$$

This is similar to dilaton framework:

$$\langle 0|\Theta^{\mu\nu}|\phi(p)\rangle = \frac{f_\phi}{3}(p^\mu p^\nu - g^{\mu\nu}m_\phi^2)e^{-ip\cdot x}$$

By adopting the nonlinear representation of M,
scalar meson field can be interpreted as behaving in a dilaton-like manner.
In this sense, the structure of the LSM is closely related to dilaton framework,
allowing the scalar field to couple to the EMT.

# Chiral effective model with $m_{\text{inv}}$

$m_{\text{inv}}$ ($m_0$) is introduced within the framework of Parity Doublet Model (PDM)

$$\mathcal{L}_N^{(\text{PDM})} = \bar\psi_{1r} i\gamma^\mu \partial_\mu \psi_{1r} + \bar\psi_{1l} i\gamma^\mu \partial_\mu \psi_{1l} + \bar\psi_{2r} i\gamma^\mu \partial_\mu \psi_{2r} + \bar\psi_{2l} i\gamma^\mu \partial_\mu \psi_{2l}$$
$$- m_0 \left[\bar\psi_{1l}\psi_{2r} - \bar\psi_{1r}\psi_{2l} - \bar\psi_{2l}\psi_{1r} + \bar\psi_{2r}\psi_{1l}\right]$$
$$- g_1 \left[\bar\psi_{1r} M^\dagger \psi_{1l} + \bar\psi_{1l} M \psi_{1r}\right]$$
$$- g_2 \left[\bar\psi_{2r} M \psi_{2l} + \bar\psi_{2l} M^\dagger \psi_{2r}\right]$$

C. E. Detar and T. Kunihiro, PRD (1989)
D. Jido, M. Oka, A. Hosaka, PTP (2001)

- Chiral invariant mass: $m_0$
- Positive-parity nucleon ($\psi_1$): N(939)
- Negative-parity nucleon ($\psi_2$): $N^*(1535)$

Chiral transformations:

$$\psi_{1r} \to g_R \psi_{1r}, \quad \psi_{1l} \to g_L \psi_{1l},$$
$$\psi_{2r} \to g_L \psi_{2r}, \quad \psi_{2l} \to g_R \psi_{2l}$$

Parity transformation:

$$\psi_1(t,\vec{x}) \to \gamma^0 \psi_1(t,-\vec{x}),$$
$$\psi_2(t,\vec{x}) \to -\gamma^0 \psi_1(t,-\vec{x})$$

Mass term:

$$(\bar\psi_1, \quad \bar\psi_2) \begin{pmatrix} g_1 f_\pi & m_0\gamma_5 \\ -m_0\gamma_5 & g_2 f_\pi \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$
$$= (\bar N_+, \quad \bar N_-) \begin{pmatrix} m_{N+} & 0 \\ 0 & m_{N-} \end{pmatrix} \begin{pmatrix} N_+ \\ N_- \end{pmatrix}$$

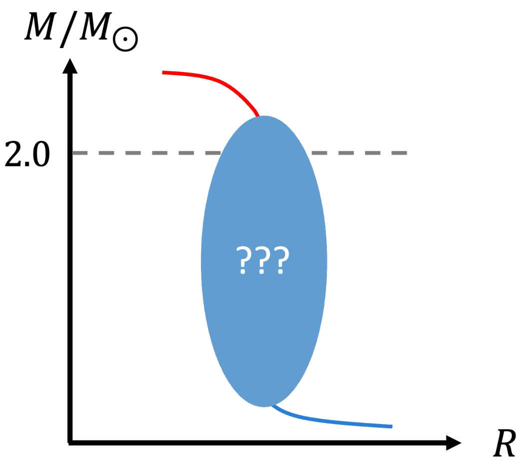
Mass eigenstate:

$$\begin{pmatrix} N_+ \\ N_- \end{pmatrix} = \begin{pmatrix} \cos\theta_N & \gamma_5 \sin\theta_N \\ -\gamma_5 \sin\theta_N & \cos\theta_N \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

Masses:

$$m_{N+} = \frac{1}{2}\left((g_1 - g_2) f_\pi + \sqrt{f_\pi^2 (g_1+g_2)^2 + 4m_0}\right),$$
$$m_{N-} = \frac{1}{2}\left((g_2 - g_1) f_\pi + \sqrt{f_\pi^2 (g_1+g_2)^2 + 4m_0}\right)$$

# If we use a larger $m_\sigma$…

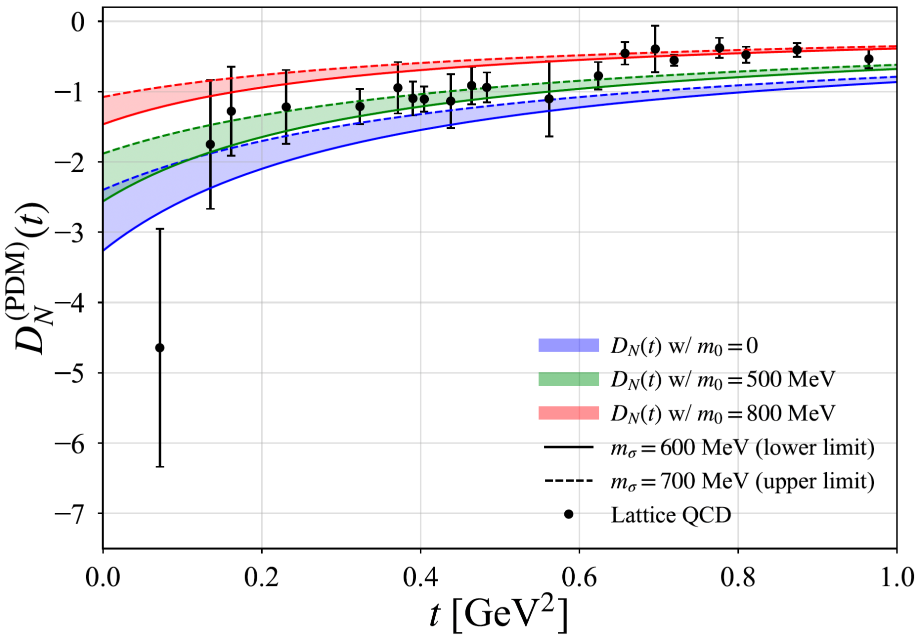



Model analyses w/ $m_{\rm inv}$

T. Minamikawa et al., PRC (2021)
T. Minamikawa et al., Symmetry (2023)
B. Gao et al., PRC (2024)
B. Gao et al., PRD (2025)
B. Gao et al., CPMA (2026)

These studies use larger $m_\sigma$ ($m_\sigma \sim 600\text{MeV}$)

- Consider the range for $m_\sigma$:
  $\boldsymbol{m_\sigma = 600 - 700}$ **MeV.**
- Consider several values of $m_0$ :
  $m_0 = 0, 500, 800$ MeV.



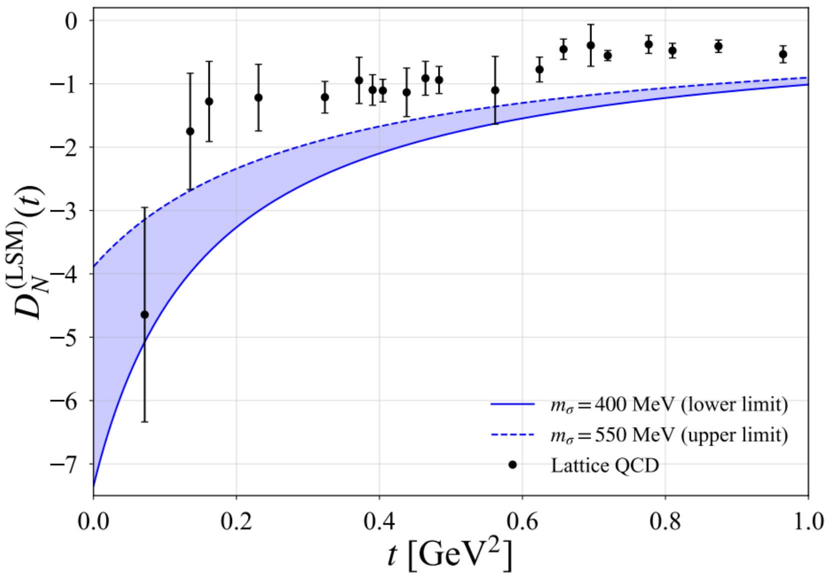

✓ A larger $m_\sigma$ improves the agreement with the lattice data even without $m_{\rm inv}$.
✓ $m_{\rm inv} \sim 500\text{MeV}$ further improves it.

More precise lattice data may be needed.
(precise sigma meson mass is also needed.)

# Numerical result and comparison

M. K., M. Harada and Y. L. Ma,
arXiv:2512.23937

## Numerical results




## Model result

$$D^{(\mathrm{LSM})}(t) = -\frac{4m_N^2}{3}\frac{1}{m_\sigma^2 + t}$$

Is described by

- sigma meson mass $m_\sigma$
- nucleon mass $m_N \propto \langle 0|\sigma|0\rangle$

Comparison and numerical setup

- Compared with lattice results.
  (D. C. Hackett, et al., PRL(2024))
- Consider the range $m_\sigma = 400 - 550$ MeV.
  (These values are listed in PDG)

✓ Model result is deviated from lattice observation...

✓ Evaluated $D(0)$ is also deviated from $D(0) \sim -3$...

Recent other theoretical estimation
Z. Q. Yao, et al., EPJA (2025)
X.-H. Cao, et al., Nature Commun. (2025)
W. Broniowski and E. Ruiz Arriola, PRD (2025)

# GFF and "pressure" (?)

$$\langle p', s'|\Theta_{\mu\nu}(x)|p, s\rangle = \bar{u}'\left[A(t)\frac{P_\mu P_\nu}{M_N} + J(t)\frac{i\,(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\,\Delta^\rho}{2M_N} + \boxed{D(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N}}\right]u e^{i(p'-p)x}$$

$$\begin{pmatrix} \sigma_{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu], \\ P^\mu = (p^\mu + p'^\mu)/2, \\ \Delta^\mu = p'^\mu - p^\mu, \\ t = \Delta^2 \end{pmatrix} \begin{pmatrix} \text{Breit frame} \\ P^\mu = (E, \vec{0}) \\ \Delta^\mu = (0, \vec{\Delta}) \end{pmatrix}$$

t: momentum transfer

Fourier transformation

$$\Theta^{\text{static}}_{\mu\nu}(\vec{r}, \vec{s}) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{r}\cdot\vec{\Delta}} \frac{\langle p', s'|\Theta_{\mu\nu}(x=0)|p, s\rangle}{\bar{u}(p')u(p)}$$

Pressure and shear force ($\Theta^{ij}$)

$D(t)$: Pressure $p$

$\rightarrow D(0) = ???$

"Pressure" can be defined...

$$p(r) = \Theta^{\text{static}}_{ii}(r)/3$$

This Fourier-transformed quantity is interpreted as "pressure" inside nucleon.
M.V. Polyakov, PLB (2003)

# GFF and “pressure” (?)

$$\langle p', s'|\Theta_{\mu\nu}(x)|p, s\rangle$$
$$= \bar{u}'\left[A(t)\frac{P_\mu P_\nu}{M_N} + J(t)\frac{i\left(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho}\right)\Delta^\rho}{2M_N} + \boxed{D(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N}}\right]ue^{i(p'-p)x}$$

$$\left(\begin{aligned} \sigma_{\mu\nu} &= \frac{i}{2}[\gamma^\mu, \gamma^\nu], \\ P^\mu &= (p^\mu + p'^\mu)/2, \\ \Delta^\mu &= p'^\mu - p^\mu, \\ t &= \Delta^2 \end{aligned}\right)$$

Breit frame

$$P^\mu = (E, \vec{0})$$
$$\Delta^\mu = (0, \vec{\Delta})$$

t: momentum transfer

Fourier transformation

$$\Theta^{\text{static}}_{\mu\nu}(\vec{r}, \vec{s}) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{r}\cdot\vec{\Delta}} \frac{\langle p', s'|\Theta_{\mu\nu}(x=0)|p, s\rangle}{\bar{u}(p')u(p)}$$

“Pressure” can be defined…

$$p(r) = \Theta^{\text{static}}_{ii}(r)/3$$

Pressure and shear force ($\Theta^{ij}$)

$D(t)$: Pressure $p$

$\rightarrow D(0) = ???$

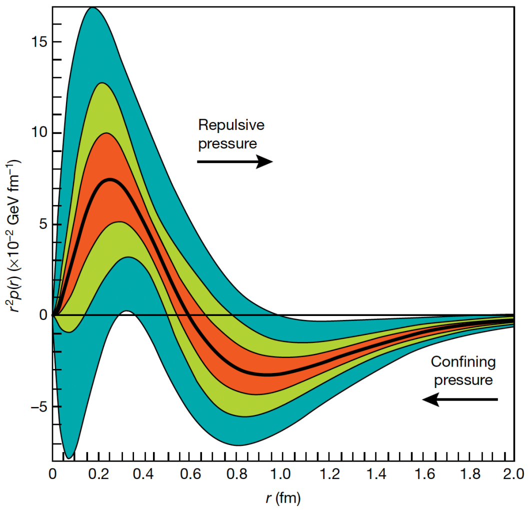
Note…

- Diagonal components are simply referred to as "pressure”.
- Different from “thermodynamic pressure”.
- Its physical interpretation remains under debate.

# Pressure and its role

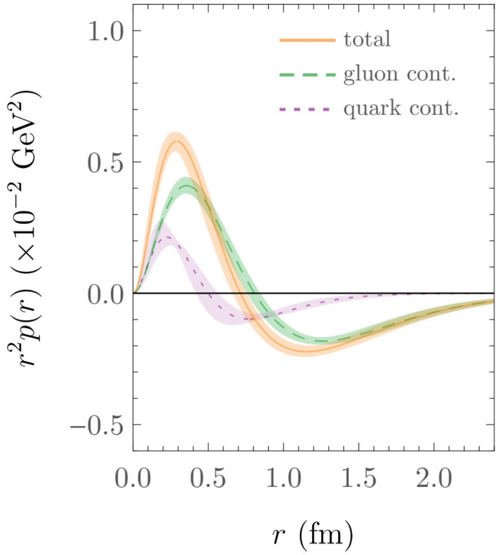
## Pressure in proton

### Experiment




V. D. Burkert et al.
Nature (2018)

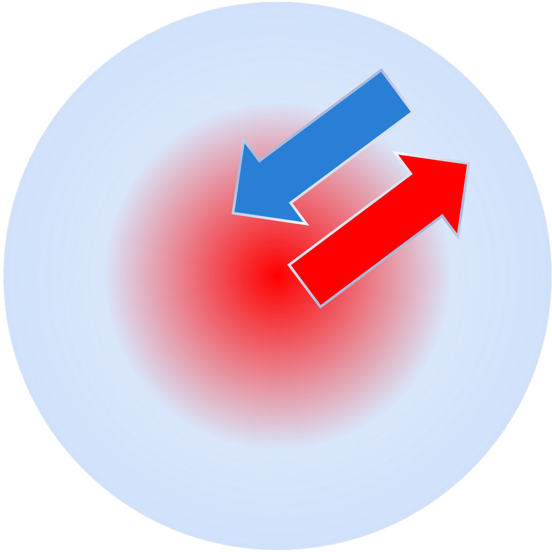
### Lattice




P.E. Shanahan,
W. Detmold PRL (2019)

Around the center of proton:
Pressure is positive. → Repulsive pressure

Toward the outer region:
Pressure becomes negative. → Confining pressure



Pressures sum to zero.

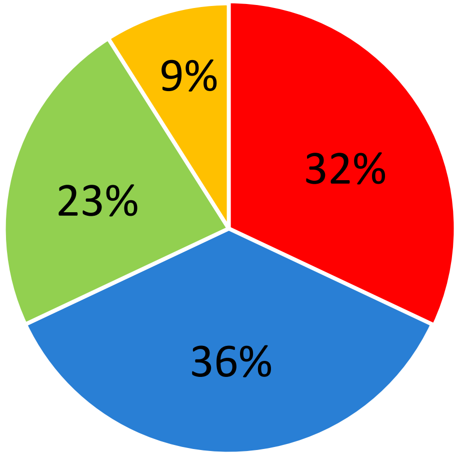
Related to confinement…?

Proton is localized and stabilized.

*For an intuitive understanding of pressure,
it is linked to force density: $\mathcal{F}^j(\vec{r}) \equiv \partial^i \Theta_{ij}^{\text{static}}(\vec{r})$

# Lattice results: Yi-Bo Yang et al. PRL (2018)

Proton mass decomposition




- Quark mass
- Quark energy
- Gluon energy
- Trace anomaly

Lattice simulation:
Yi-Bo Yang et al. PRL (2018)

$$\Theta^{00} = \boxed{H_m} + \boxed{H_E} + \boxed{H_g} + \boxed{H_a/4}$$

X.-D. Ji, PRL (1995)

$$\boxed{H_m} = \sum_{u,d,s\cdots} \int d^3x\, m\bar{\psi}\psi,$$

$$\boxed{H_E} = \sum_{u,d,s\ldots} \int d^3x\, \bar{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

$$\boxed{H_g} = \int d^3x \frac{1}{2}(B^2 - E^2).$$

$$\boxed{H_a} = H_g^a + H_m^\gamma,$$

$$H_g^a = \int d^3x \frac{-\beta(g)}{g}(E^2 + B^2),$$

$$H_m^\gamma = \sum_{u,d,s\cdots} \int d^3x\, \gamma_m m\bar{\psi}\psi.$$

- Overlap valence fermions on four ensembles of Nf=2+1 domain wall fermion configurations

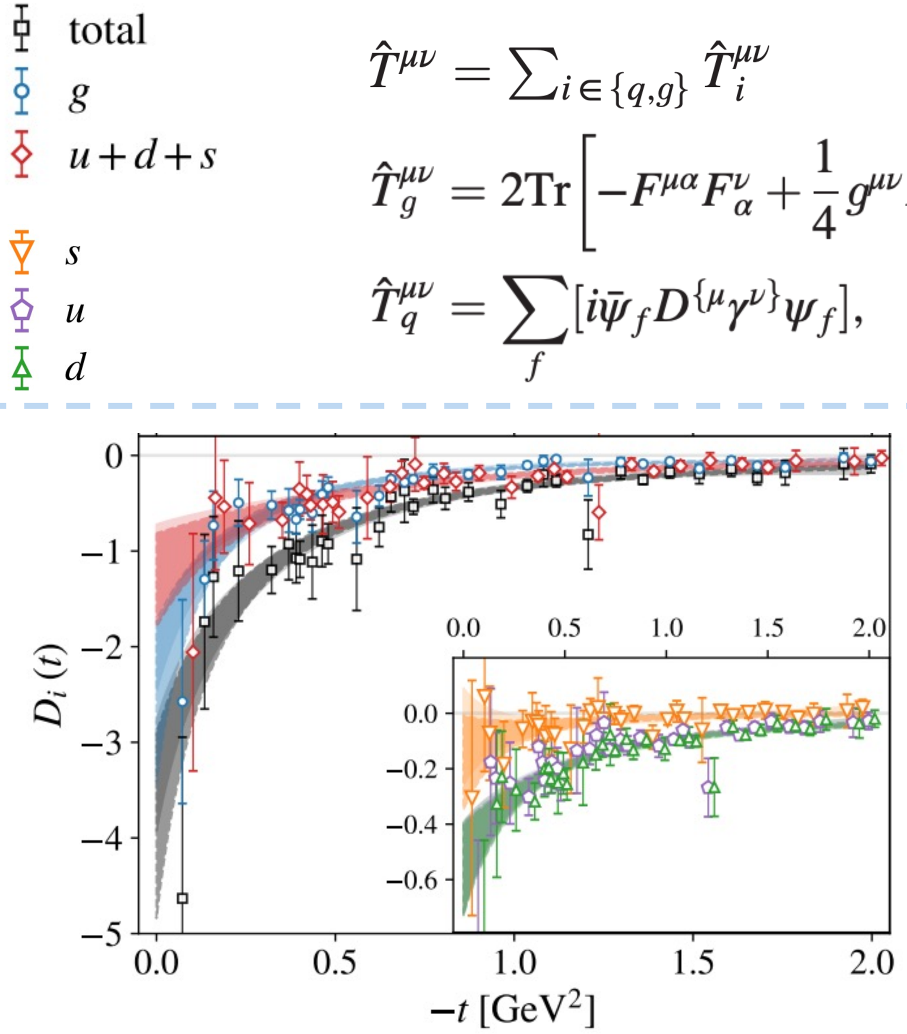
| Symbol | $L^3 \times T$ | a (fm) | $m_s^{(s)}$ (MeV) | $m_\pi$ (MeV) |
|---|---|---|---|---|
| 32ID | $32^3 \times 64$ | 0.1431(7) | 89.4 | 171 |
| 24I | $24^3 \times 64$ | 0.1105(3) | 120 | 330 |
| 48I | $48^3 \times 96$ | 0.1141(2) | 94.9 | 139 |
| 32I | $32^3 \times 64$ | 0.0828(3) | 110 | 300 |

# Lattice results: D. C. Hackett, et al., PRL(2024)

$$\hat{T}^{\mu\nu} = \sum\nolimits_{i\in\{q,g\}} \hat{T}^{\mu\nu}_i$$

$$\hat{T}^{\mu\nu}_g = 2\mathrm{Tr}\left[-F^{\mu\alpha}F^{\nu}_{\ \alpha} + \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right],$$

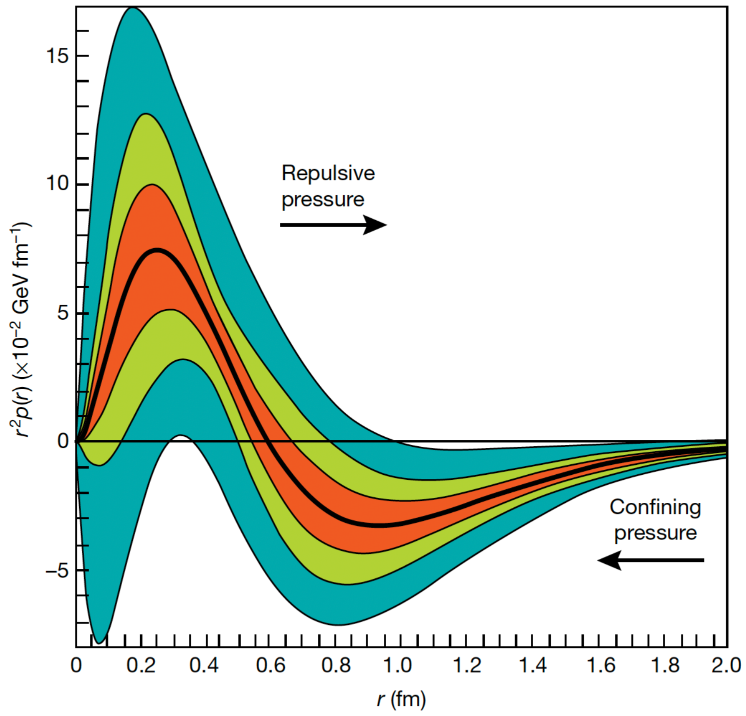
$$\hat{T}^{\mu\nu}_q = \sum_f [i\bar{\psi}_f D^{\{\mu}\gamma^{\nu\}}\psi_f],$$




- $N_f$=2+1 flavors
- Clover-improved Wilson quarks
- pion mass≅ 170MeV
- lattice spacing ≅ 0.091fm (continuum limit is not taken.)
- Volume: $L^3 \times T = 48^3 \times 96$

# Experimental results: V. D. Burkert et al. Nature 557 (2018)

Quark pressure distribution in the proton




From Deeply Virtual Compton Scattering (DVCS)

- Black solid line: extracted from D-term fitted to 6 GeV DVCS.
- Light-green band: represents uncertainties.
- Blue band: uncertainties from previous analyses.
- Red band: projected results from future 12 GeV experiments.