Diquark correlations in the proton parton distribution amplitude

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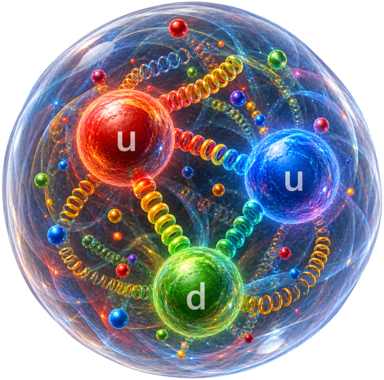
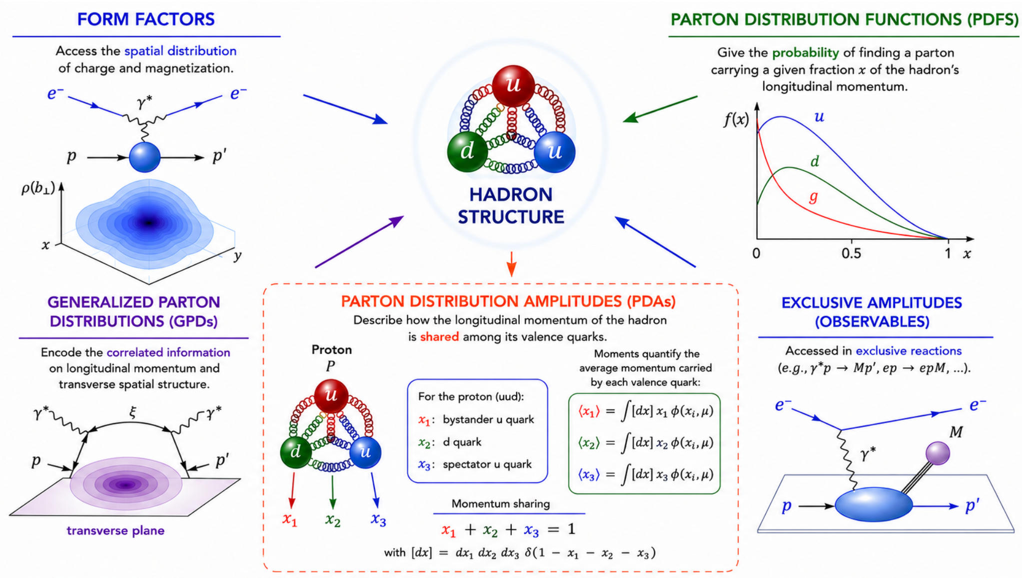
Understanding the internal structure of the proton

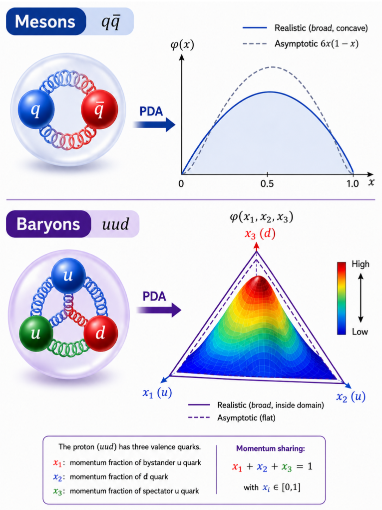
- QCD is the fundamental theory of the strong interaction.
- It successfully describes the interactions of **quarks** and **gluons**.
- However, understanding how hadron structure emerges from QCD remains a major challenge.
- **How does the proton acquire its internal structure?**




FORM FACTORS
Access the spatial distribution of charge and magnetization.
e^-
γ^*
e^-
p
p'
$\rho(b_\perp)$
x
y
HADRON STRUCTURE
PARTON DISTRIBUTION FUNCTIONS (PDFS)
Give the probability of finding a parton carrying a given fraction x of the hadron's longitudinal momentum.
$f(x)$
u
d
g
0
0.5
1
x
GENERALIZED PARTON DISTRIBUTIONS (GPDs)
Encode the correlated information on longitudinal momentum and transverse spatial structure.
γ^*
ξ
γ^*
p
p'
transverse plane
PARTON DISTRIBUTION AMPLITUDES (PDAs)
Describe how the longitudinal momentum of the hadron is shared among its valence quarks.
Proton
P
u
d
u
x_1
x_2
x_3
For the proton (uud):
x_1: bystander u quark
x_2: d quark
x_3: spectator u quark
Moments quantify the average momentum carried by each valence quark:
$\langle x_1 \rangle = \int [dx]\, x_1\, \phi(x_i, \mu)$
$\langle x_2 \rangle = \int [dx]\, x_2\, \phi(x_i, \mu)$
$\langle x_3 \rangle = \int [dx]\, x_3\, \phi(x_i, \mu)$
Momentum sharing
$x_1 + x_2 + x_3 = 1$
with $[dx] = dx_1\, dx_2\, dx_3\, \delta(1 - x_1 - x_2 - x_3)$
EXCLUSIVE AMPLITUDES (OBSERVABLES)
Accessed in exclusive reactions ($e.g., \gamma^* p \to M p', ep \to epM, \ldots$).
e^-
e^-
M
γ^*
p
p'

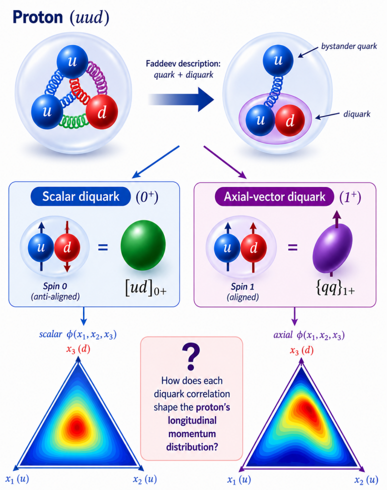
From Mesons to Baryons

- Meson PDAs have been extensively studied using continuum approaches and lattice QCD.
- At experimentally accessible scales, they are typically **broad and concave**, differing significantly from their asymptotic forms.
- In contrast, considerably less is known about **baryon PDAs**.
- In particular, the dynamical origin of the **momentum asymmetry** of the proton PDA remains incompletely understood.



What Shapes the Proton PDA?

- The proton is a genuine **three-body bound state**, making its PDA considerably more complex than in mesons.
- Strong nonperturbative quark–quark correlations (diquarks) naturally emerge within a covariant Faddeev description.
- The dominant correlations are scalar diquarks (0^+), and axial-vector diquarks (1^+).
- **How does each diquark correlation shape the proton's longitudinal momentum distribution?**



The Proton as a Quark–Diquark System

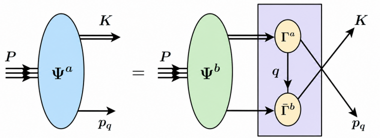
- The nucleon is described through a **Poincaré-covariant Faddeev equation**.
- Strong quark–quark correlations allow the three-body problem to be reorganized in terms of **quark–diquark configurations**.
- The Faddeev wave function contains three components, corresponding to each possible choice of the **bystander quark**:

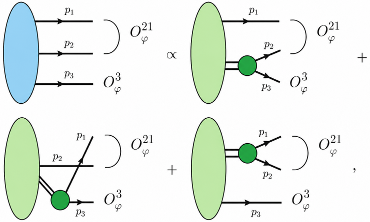
$$\Psi = \psi_1 + \psi_2 + \psi_3.$$

- Each component contains both

$$J^P = 0^+ \quad \text{and} \quad J^P = 1^+$$

diquark correlations.

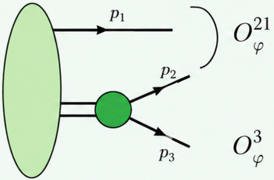




Scalar Diquark Contribution (0^+)

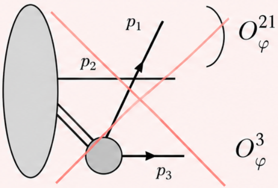
Leading-twist projection
(Mellin moments)

(i) Bystander u with $[ud]_{0+}$ diquark
Survives at leading twist



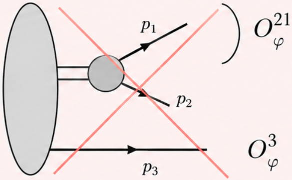
- The scalar $[ud]$ diquark correlation contributes to the proton PDA at **leading twist**.

(ii) Permutation 1
Does not contribute at leading twist
(requires nonzero orbital angular momentum)



- This contribution is suppressed at **leading twist** and **requires nonzero orbital** angular momentum.

(iii) Permutation 2
Vanishes
($[uu]_{0+} = 0$, *isoscalar diquark required)*



- This **contribution vanishes** because **a scalar** $[uu]$ diquark correlation is zero.

Contributes at leading twist | Does not contribute at leading twist | Forbidden (vanishes or higher twist)

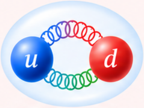
Only the configuration with a bystander u quark and a scalar $[ud]$ diquark contributes to the **proton** PDA at **leading twist.**

From the diquark PDA to the proton PDA

In the quark–diquark picture, the proton PDA is built in two steps:

1 Diquark distribution amplitude

Internal momentum sharing (two quarks)

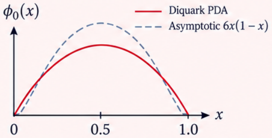


Scalar diquark (ud)

Compute its Mellin moments:

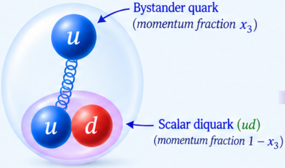
$$\langle x^n \rangle_{[ud]} = \int_0^1 dx\, x^n \phi_0(x)$$

Reconstruct the PDA:



2 Quark–diquark structure

Momentum sharing inside the proton (quark + diquark)



Compute the Mellin moments
of the quark–diquark amplitude:

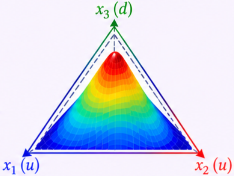
$$\langle x_3^m (1-x_3)^n \rangle = \int_0^1 dx_3\, x_3^m (1-x_3)^n \Phi(x_3)$$

Convolution

Combine both levels of momentum sharing

3 Proton PDA

Three-quark momentum sharing



Proton PDA from the convolution:

$$\phi_p^S(x_1, x_2, x_3) = \int_0^1 dz\, \phi_0(z)\, \Phi(x_1, x_2, x_3; z)$$

Equivalently, in terms of Mellin moments:

$$\langle x_1^l x_2^m x_3^n \rangle = \sum_{k=0}^{n} C_{lmn}^k \langle z^k \rangle_{[ud]} \langle x_3^{n-k} \rangle_{q-dq}$$

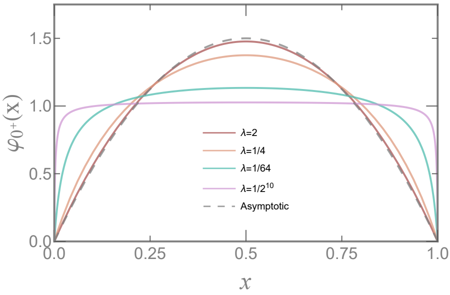
By first determining the diquark PDA and then convoluting it with the quark–diquark structure,
we obtain the **scalar–diquark contribution** to the proton PDA.

Scalar-Diquark Distribution Amplitude

- The scalar-diquark PDA is obtained from the Mellin moments

$$\langle x^m \rangle_{0^+} = \int_0^1 dx\, x^m\, \phi_0(x)$$

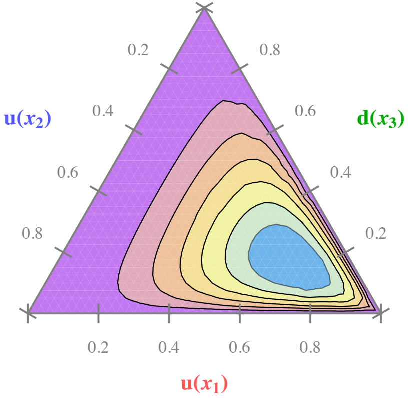
- The PDA depends explicitly on the diquark virtuality through $\lambda = \frac{M^2}{K^2}$.
- As the diquark virtuality increases, the PDA becomes **progressively broader** than the asymptotic form.



$$\phi_0(x,\lambda) = \lambda \left[1 - \frac{\lambda}{x(1-x)} \ln\left(1 + \frac{x(1-x)}{\lambda}\right)\right].$$

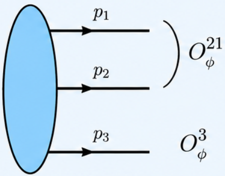
Proton PDA: Scalar-Diquark Approximation

- The proton PDA combines two momentum scales:
    - momentum sharing inside the $[ud]_{0^+}$ diquark,
    - momentum sharing between the diquark and the bystander u quark.
- The resulting PDA is **broader and more asymmetric** than the conformal-limit distribution.
- It is skewed toward large x_1: the **bystander u-quark carries, on average, more longitudinal momentum**.

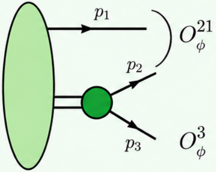


Axial–Vector Diquark Contribution (1^+)

Full projection (Mellin moments)

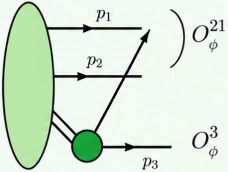


(i) Bystander u with $\{ud\}_{1+}$ (axial–vector diquark)



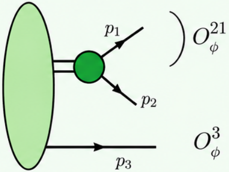
- This configuration contributes at leading twist.

(ii) Bystander d with $\{uu\}_{1+}$ (axial–vector diquark)



- This configuration contributes at leading twist.

(iii) Bystander u with $\{ud\}_{1+}$ (axial–vector diquark)



- This configuration contributes at leading twist.

Contributes at leading twist — Does not contribute at leading twist — Forbidden (vanishes or higher twist)

All three permutations contribute to the proton PDA at leading twist for the axial–vector diquark.

Axial–vector diquark: isotriplet ($\{ud\}_{1+}$, $\{uu\}_{1+}$)

Axial-Vector Diquark PDAs

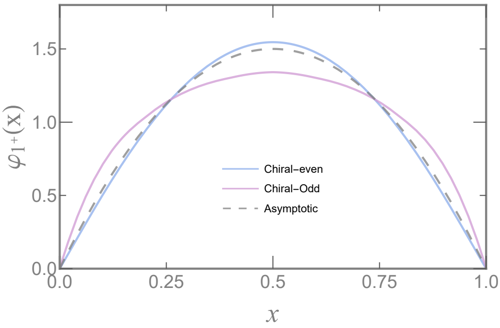
- The spin–1 structure generates two independent leading-twist projections:

$$\phi_{1^+}^{\text{even}}(x), \qquad \phi_{1^+}^{\text{odd}}(x).$$

- Both distributions are symmetric under

$$x \leftrightarrow 1 - x,$$

  but exhibit different shapes.
- The chiral-odd PDA shows a stronger **nonperturbative broadening**.



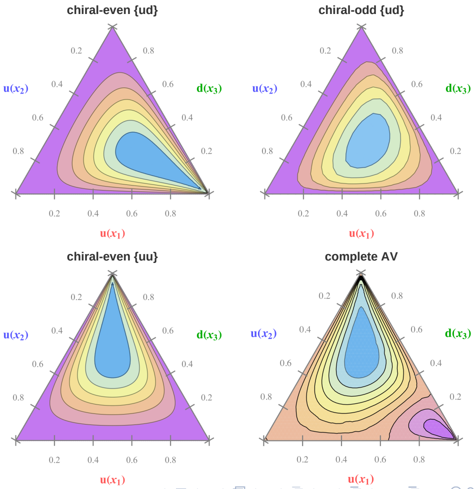
Proton PDA: Axial-Vector Diquark Approximation

- The three axial-vector spectator configurations are combined according to their spin–isospin weights:

$$\phi_{\mathrm{AV}} = \frac{1}{2}\left(-\phi^1_{\mathrm{AV}} + \phi^2_{\mathrm{AV}} + 2\phi^3_{\mathrm{AV}}\right).$$

- The resulting PDA is **broader and less asymmetric** than in the scalar-diquark approximation.

- Localized **negative regions** arise from destructive interference among the different spectator contributions.



Complete Proton PDA

- The complete PDA combines both correlations:

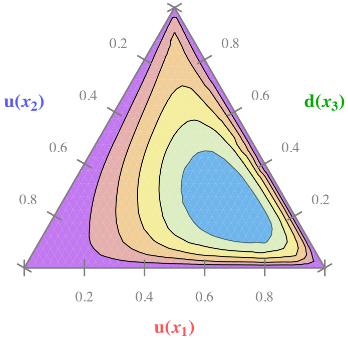
$$\phi = \mathcal{N}\left(\phi_{\mathrm{S}} + \phi_{\mathrm{AV}}\right).$$

- AV correlations **redistribute longitudinal momentum** and reduce the strong asymmetry generated by the scalar sector.
- The first Mellin moments are

$$\boxed{\langle x_1 \rangle = 0.412, \quad \langle x_2 \rangle = 0.265, \quad \langle x_3 \rangle = 0.323}$$

- The maximum remains far from the conformal limit:

$$(x_1, x_2, x_3)_{\max} \simeq (0.56,\, 0.24,\, 0.20).$$



Comparison with Other Approaches

- All nonperturbative approaches predict **asymmetric momentum sharing** in the proton.
- The scalar-diquark contribution produces a **strong enhancement of** $\langle x_1 \rangle$.
- Including axial-vector correlations **redistributes the longitudinal momentum**, bringing the first moments closer to LQCD and previous continuum results.

	$\langle x_1 \rangle_u$	$\langle x_2 \rangle_u$	$\langle x_3 \rangle_d$
Conformal	0.333	0.333	0.333
LQCD [17]	0.372	0.314	0.314
LQCD [18]	0.358	0.319	0.323
Ref. [38]	0.392	0.291	0.316
scalar only	**0.498**	**0.250**	**0.252**
scalar + AV	**0.412**	**0.265**	**0.323**

The proton PDA remains clearly asymmetric.

Summary

- We constructed the leading-twist proton PDA within a **Poincaré-covariant quark–diquark framework**.
- **Scalar-diquark correlations** generate the dominant asymmetry, favoring configurations in which the bystander quark carries a large fraction of the longitudinal momentum.
- **Axial-vector correlations** introduce additional spin–momentum structures, redistribute longitudinal momentum, and **reduce the scalar-induced asymmetry**.
- The complete proton PDA remains **broad and strongly asymmetric** with respect to the conformal limit, revealing the persistence of nonperturbative quark–quark correlations.
- The resulting momentum-sharing pattern is **qualitatively consistent with previous studies**.

Thank you.