Nucleon Structure from Basis Light-Front Quantization: Recent Developments and Perspectives

Chandan Mondal

Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou, China



BLFQ Collaboration

Siqi Xu, . . . , Xingbo Zhao, Yang Li and James P. Vary

Seville, Spain, September 10, 2026

Overview



Introduction

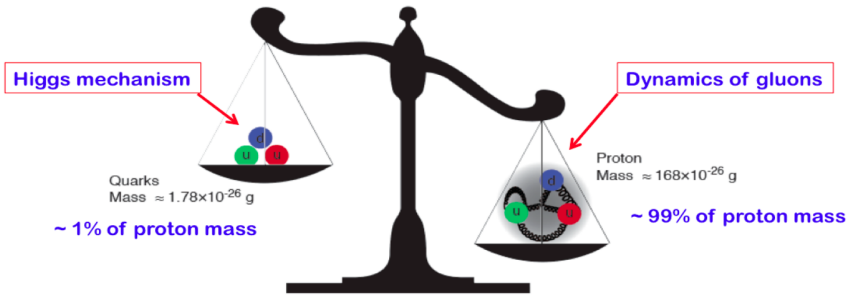
Basis Light-Front Quantization (BLFQ)

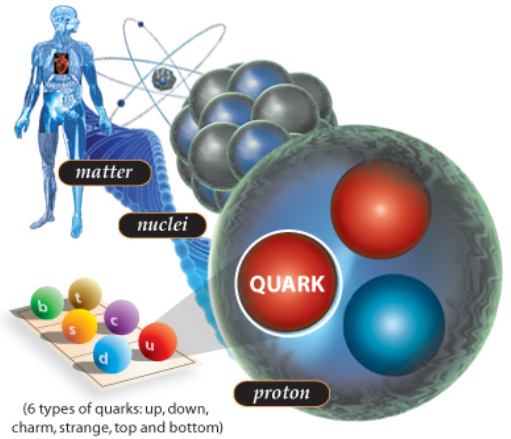
Description of Nucleon State

Conclusions

Fundamental Properties: Mass, Spin and Structure

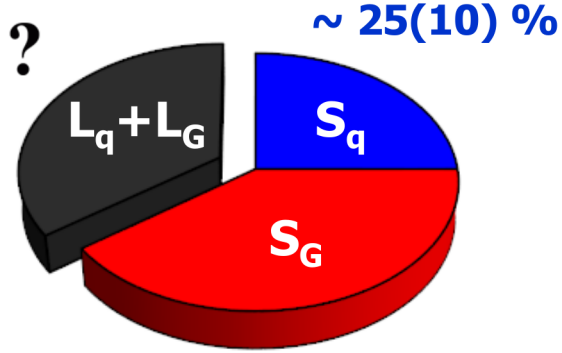
Mass





Spin

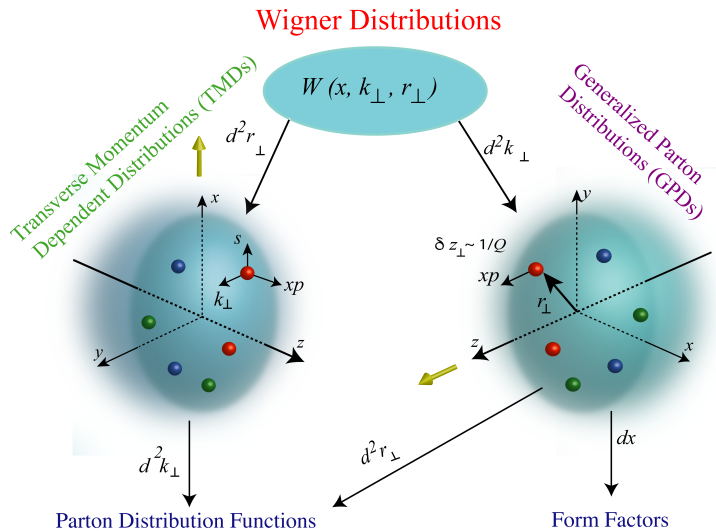
Orbital angular momentum



~ 25(10) %

~ 40(?) %

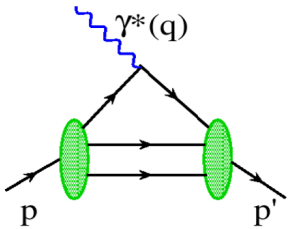
Structure



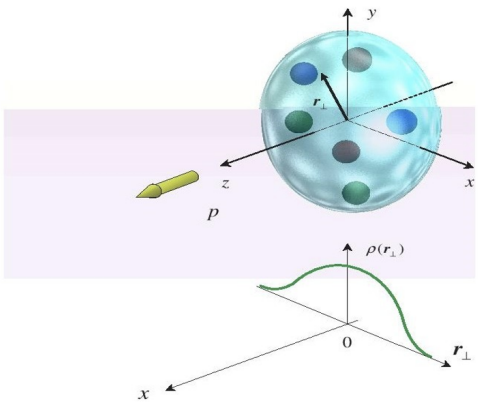
Ideal facilities : EIC & EicC

Form Factors vs PDFs vs GPDs

Elastic Scattering

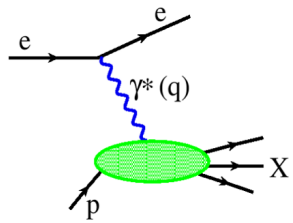


Established extended nature of nucleon

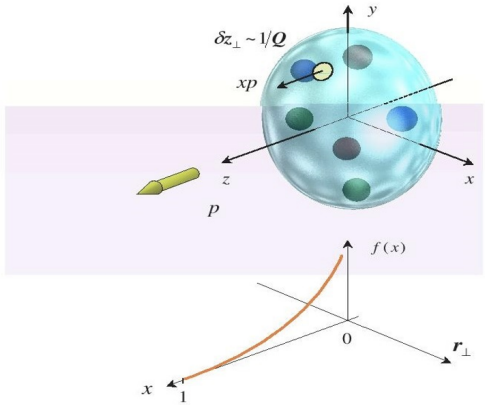


charge and magnetization distribution

Deep Inelastic Scattering

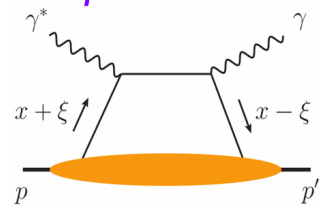


discovered the existence (quarks) inside the nucleon

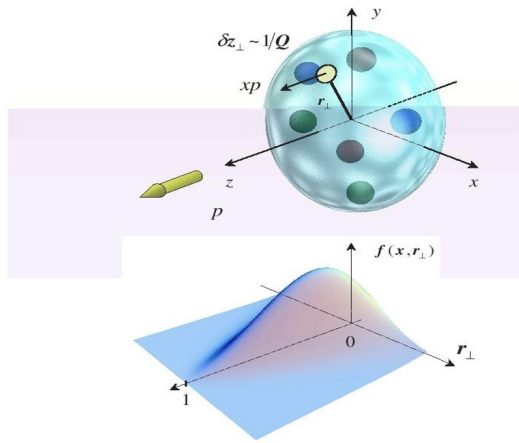


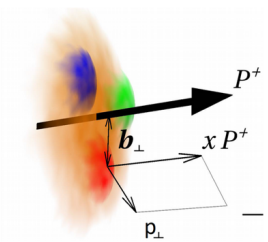
longitudinal momentum distribution

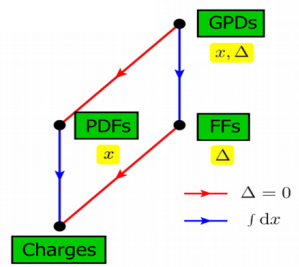
Deeply virtual Compton Scattering



provides 3D spatial structure of the nucleon

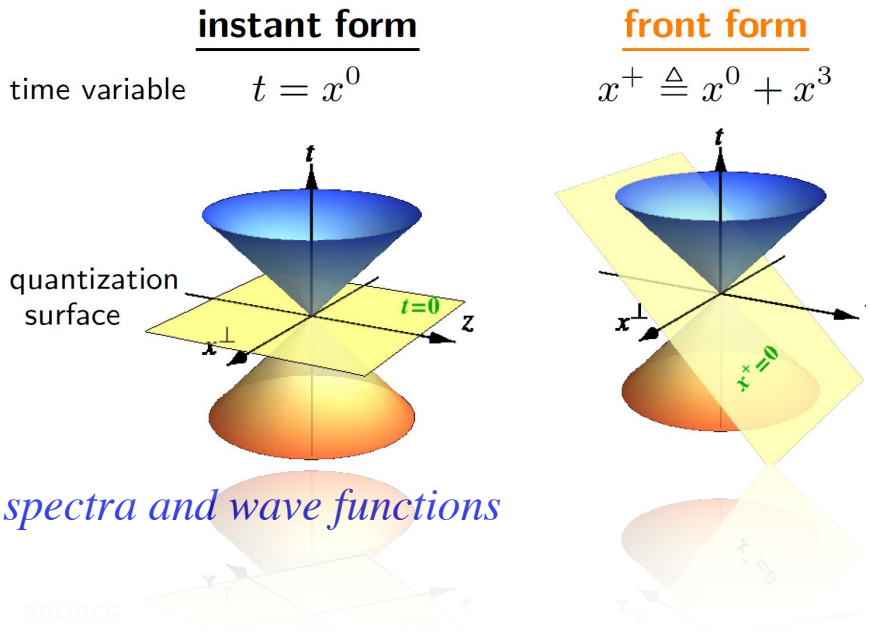






Light-Front Hamiltonian Framework

- Quantization on the light-front : $x^+ = x^0 + x^3$
- LF momenta : $P^+ = P^0 + P^3$; $\mathbf{P}_\perp$
- Choose transverse rest frame : $\mathbf{P}_\perp = 0$
- LF Hamiltonian : $H_{\rm LF} \equiv P^+P^-$
- Eigenvalue problem : $H_{\rm LF}\,|P,\Lambda\rangle = M^2\,|P,\Lambda\rangle$



Simultaneous access to mass spectra and wave functions

$$H_{\rm LF} = \sum_i \frac{m_i^2 + \boldsymbol{p}_{i\perp}^2}{x_i} + H_{\rm int}$$

$x_i = p_i^+/P^+$ → LF momentum fraction carried by *i*th parton

Applicable to hadrons as well as to nuclei

- Expansion of an eigenstate of the system in Fock space basis

$$|P,\Lambda\rangle = \sum_N \sum_{\lambda_1,\ldots,\lambda_N} \int \frac{\prod_{i=1}^N \mathrm{d}x_i \mathrm{d}^2\boldsymbol{p}_{i\perp}}{\left[2(2\pi)^N\right]^2 \sqrt{x_1 \ldots x_N}} \delta\left(1 - \sum_{i=1}^N x_i\right) \delta^2\left(\sum_{i=1}^N \boldsymbol{p}_{i\perp}\right) \psi^\Lambda_{\{\lambda_i\}_N}(\{p_i\}_N) \,|\{\lambda_i, p_i\}_N\rangle$$

$\psi^\Lambda_{\{\lambda_i\}_N}(\{p_i\}_N)$ → LF helicity amplitude

$|\{\lambda_i, p_i\}_N\rangle$ → Normalized N-parton state

Building the LF Hamiltonian matrix using BLFQ

Step-1: Expand the system's state in the Fock space

Fock state expansion of the meson, baryon and deuteron bound state

$$
\begin{aligned}
|\Psi\rangle_M &= \psi_{\{q\bar{q}\}}\,|q\bar{q}\rangle + \psi_{\{q\bar{q}g\}}\,|q\bar{q}g\rangle + \psi_{\{q\bar{q}gg\}}\,|q\bar{q}gg\rangle + \psi_{\{q\bar{q}q\bar{q}\}}\,|q\bar{q}q\bar{q}\rangle + \ldots \\
|\Psi\rangle_B &= \psi_{\{qqq\}}\,|qqq\rangle + \psi_{\{qqqg\}}\,|qqqg\rangle + \psi_{\{qqqq\bar{q}\}}\,|qqqq\bar{q}\rangle + \ldots \\
|\Psi\rangle_D &= \psi_{\{qqq\ qqq\}}\,|qqq\ qqq\rangle + \psi_{\{qqq\ qqq\ g\}}\,|qqq\ qqq\ g\rangle + \ldots
\end{aligned}
$$

Step-2: Decide how to describe each particle's motion and quantum numbers.

Here comes the **basis** **(Heart of the BLFQ)**

Single-particle basis state → **to identify a particle**

$$\psi_{rs}(x,\vec{\kappa}^{\perp}) = \sum_{n,m,l} \langle n,m,l,r,s|\psi\rangle \times \phi_{nm}\left(\vec{\kappa}^{\perp}\right)\chi_l(x)$$

- $\langle n,m,l,r,s|\psi\rangle$: Eigenvector of the Hamiltonian
- $\phi_{nm}\left(\vec{\kappa}^{\perp}\right)$: Single particle motion in **transverse direction**
- $\chi_l(x)$: Single particle motion in **longitudinal direction**

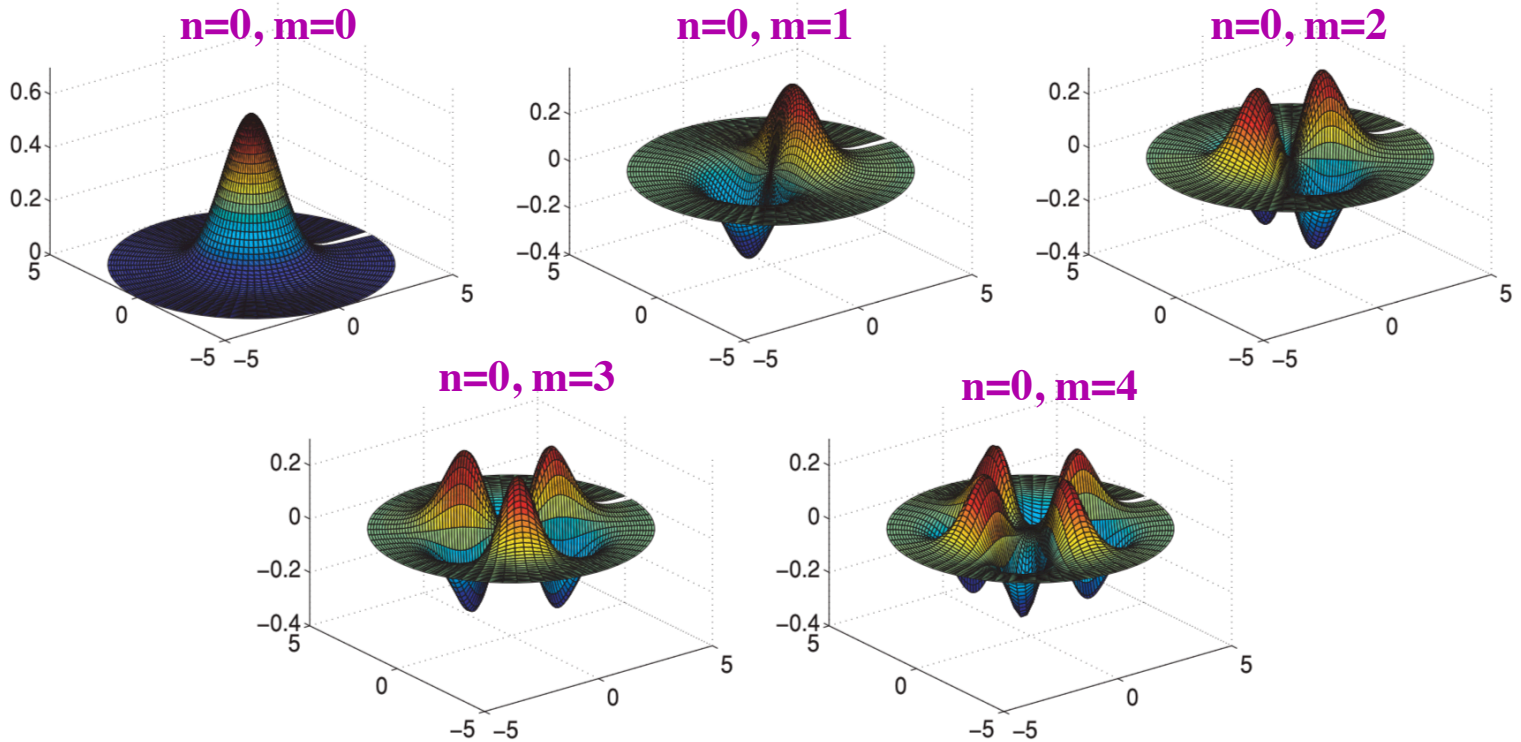
Build **many-body basis states** for each Fock sector

Step-3: Compute the Hamiltonian matrix $\langle \text{basis}_i|\, H\, |\text{basis}_j\rangle$

Hamiltonian = K.E. terms + QCD interactions

Formation of Basis: Transverse direction

2D Harmonic oscillator: $\phi_{n,m}(\mathbf{k}_\perp; b) = \frac{\sqrt{2}}{b(2\pi)^{3/2}} \sqrt{\frac{n!}{(n+|m|)!}} e^{-k_\perp^2/2b^2} \left(\frac{|k_\perp|}{b}\right)^{|m|} L_n^{|m|}\left(\frac{k_\perp^2}{b^2}\right) e^{im\theta}$



To limit the basis space

$$\sum_i (2n_i + |m_i| + 1) \leq N_{\max}$$

Basis parameter

— *J.P. Vary et al., PRC 81, 035205 (2010)*

$$\Lambda_{\rm IR} \approx b/\sqrt{N_{\max}}$$

$$\Lambda_{\rm UV} \approx b\sqrt{N_{\max}}$$

Formation of Basis: Longitudinal direction

Plain wave basis: $\psi_k(x^-) = \frac{1}{\sqrt{2L}} e^{i\frac{\pi}{L}kx^-}$ for $-L \le x^- \le L$

where $k = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \ldots$

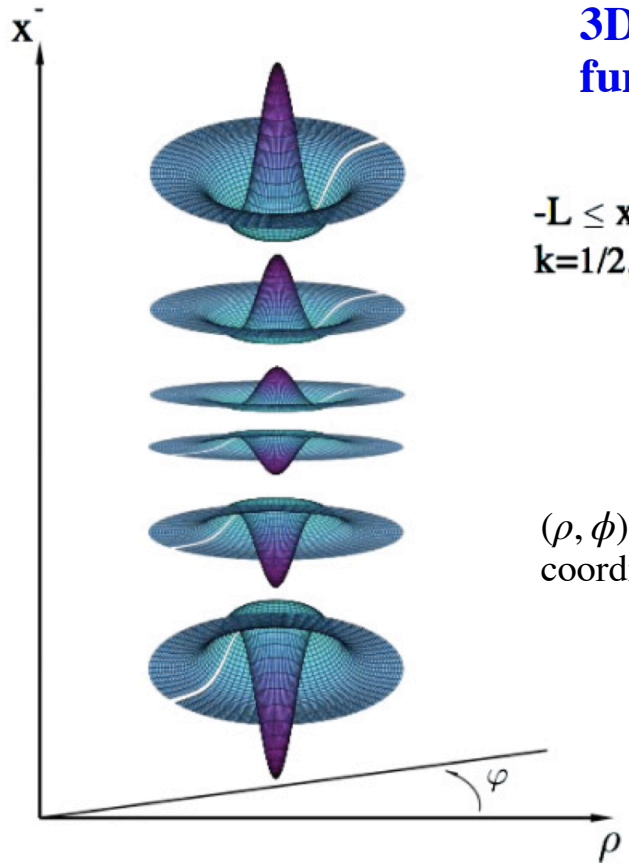
Longitudinal momentum fraction carried by parton

$$x_i = \frac{p_i^+}{P^+} = \frac{k_i}{K}$$

$$K = \sum_i k_i$$

↓

Basis parameter



3D basis function

$-L \le x^- \le +L$
k=1/2, n=1, m=0

(ρ, ϕ) are the cylindrical coordinates in (x^1, x^2)

— *J.P. Vary et al., PRC 81, 035205 (2010)*

BLFQ Algorithm Flowchart

Basis Enumeration

$|q\rangle$

Single quark/gluon Hamiltonian

$|N\rangle$

Diagonalize Hamiltonian Matrix

Input

Hamiltonian Matrix Generation

Generating mass counter term list

Diagonalize Hamiltonian Matrix

$P^-P^+|q/g\rangle = M^2_{q/g}|q/g\rangle$

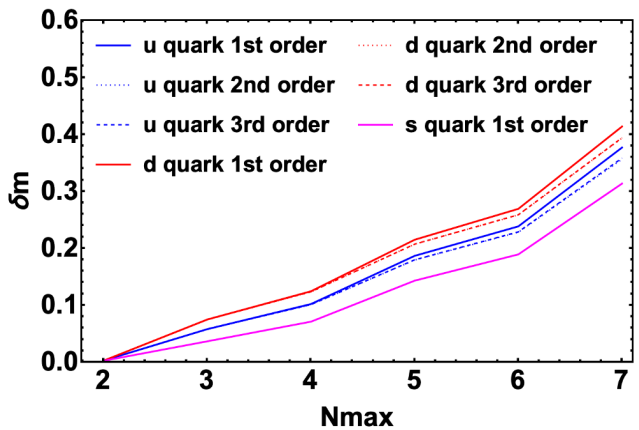
Renormalization Condition:
$M_q = m_q, M_g = 0$

Light-Front Wave Functions

$i=1$ $i=2$ $i=3$

For Quark: $|q\rangle = |q\rangle + |qg\rangle + |qq\bar{q}\rangle + |qgg\rangle + |qq\bar{q}g\rangle + |qggg\rangle$

For Gluon: $|g\rangle = |g\rangle + |q\bar{q}\rangle + |gg\rangle + |q\bar{q}g\rangle + |ggg\rangle$

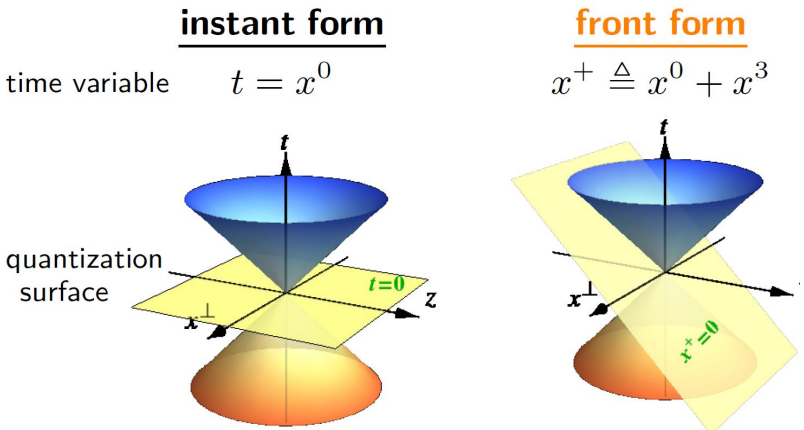


Observables: $O \equiv \left\langle N \middle| \hat{O} \middle| N \right\rangle$

Distribution Functions, Form Factors, PDFs, GPDs, TMDs…

Summary: BLFQ Framework

A computational framework for solving relativistic many-body bound state problems in quantum field theories



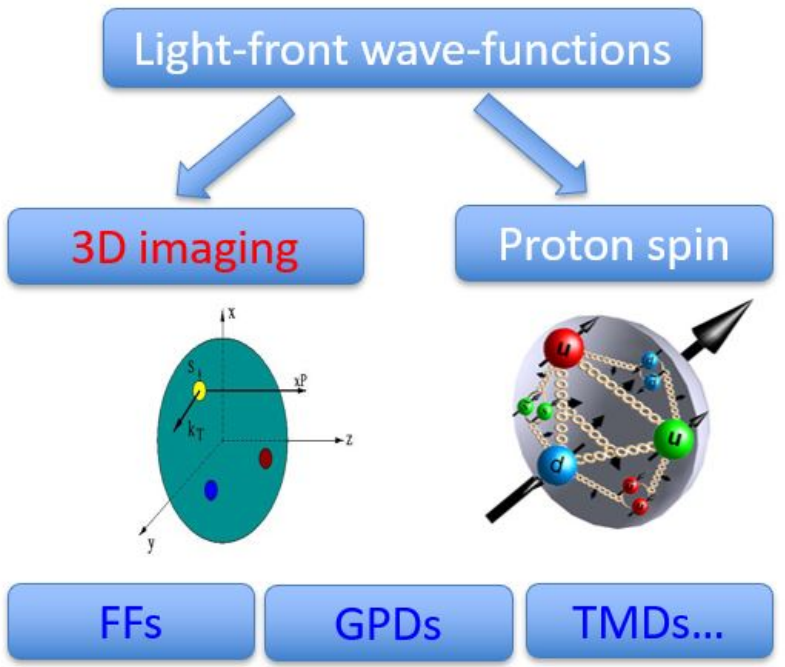
$$P^- P^+ |\Psi\rangle = M^2 |\Psi\rangle$$

- $P^\pm \equiv P^0 \pm P^3$: light-front momentum (Hamiltonian)
- First-principle / effective Hamiltonian as input
- Access to mass (M) and LFWFs

- Evaluate observables

$$O \sim \langle \Psi | \hat{O} | \Psi \rangle$$

GOAL



[1] Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).

- Fock expansion of baryonic bound states:

$$|\text{Meson}\rangle = \psi_{(q\bar{q})}|q\bar{q}\rangle + \psi_{(q\bar{q}+1g)}|q\bar{q}g\rangle + \psi_{(q\bar{q}+q\bar{q})}|q\bar{q}q\bar{q}\rangle + \dots ,$$

$$|\text{Proton}\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle + \psi_{(3q+q\bar{q})}|qqqq\bar{q}\rangle + \dots ,$$

Solution proposed by BLFQ

Discrete basis and their direct product	**Truncation**
2D HO $\phi_{nm}(p^{\perp})$ in the transverse plane	$\sum_i (2n_i + \lvert m_i\rvert + 1) \leq N_{\max}$
Plane-wave in the longitudinal direction	$\sum_i k_i = K, \quad x_i = \frac{k_i}{K}$
Light-front helicity state for spin d.o.f.	$\sum_i (m_i + \lambda_i) = M_J$
$\alpha_i = (k_i, n_i, m_i, \lambda_i)$ $\lvert\alpha\rangle = \otimes_i \lvert\alpha_i\rangle$	Fock sector truncation

Large $N_{\max}$ and $K \rightarrow$ High UV cutoff & low IR cutoff

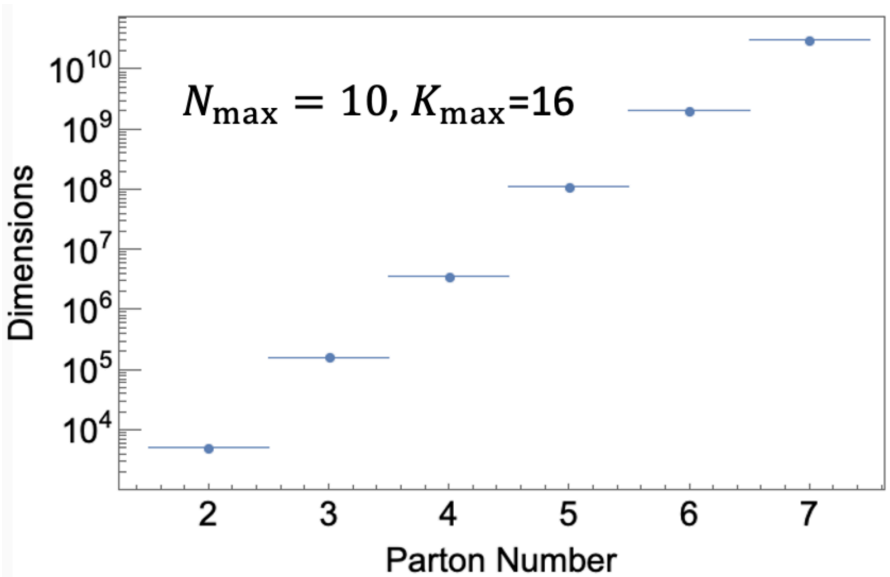
- Exact factorization between center-of-mass motion and intrinsic motion

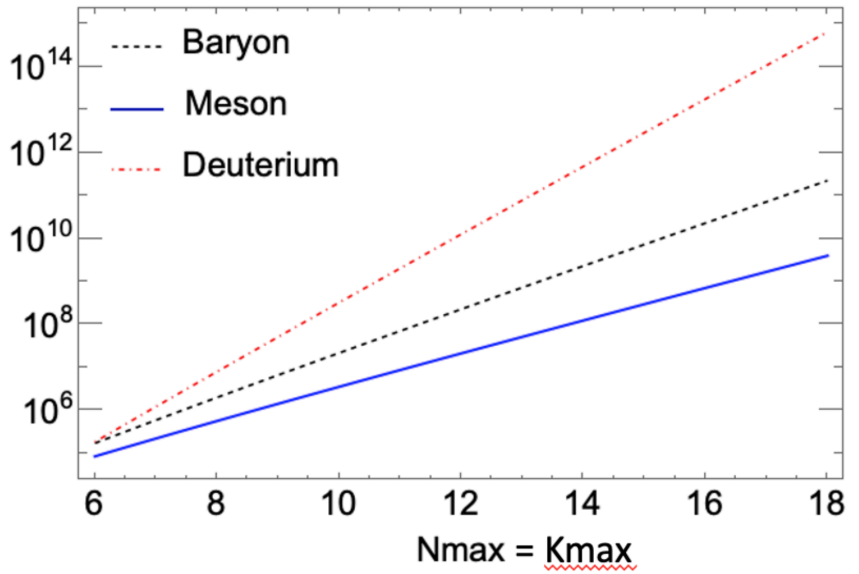
[1] Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).

Dimension of Basis States

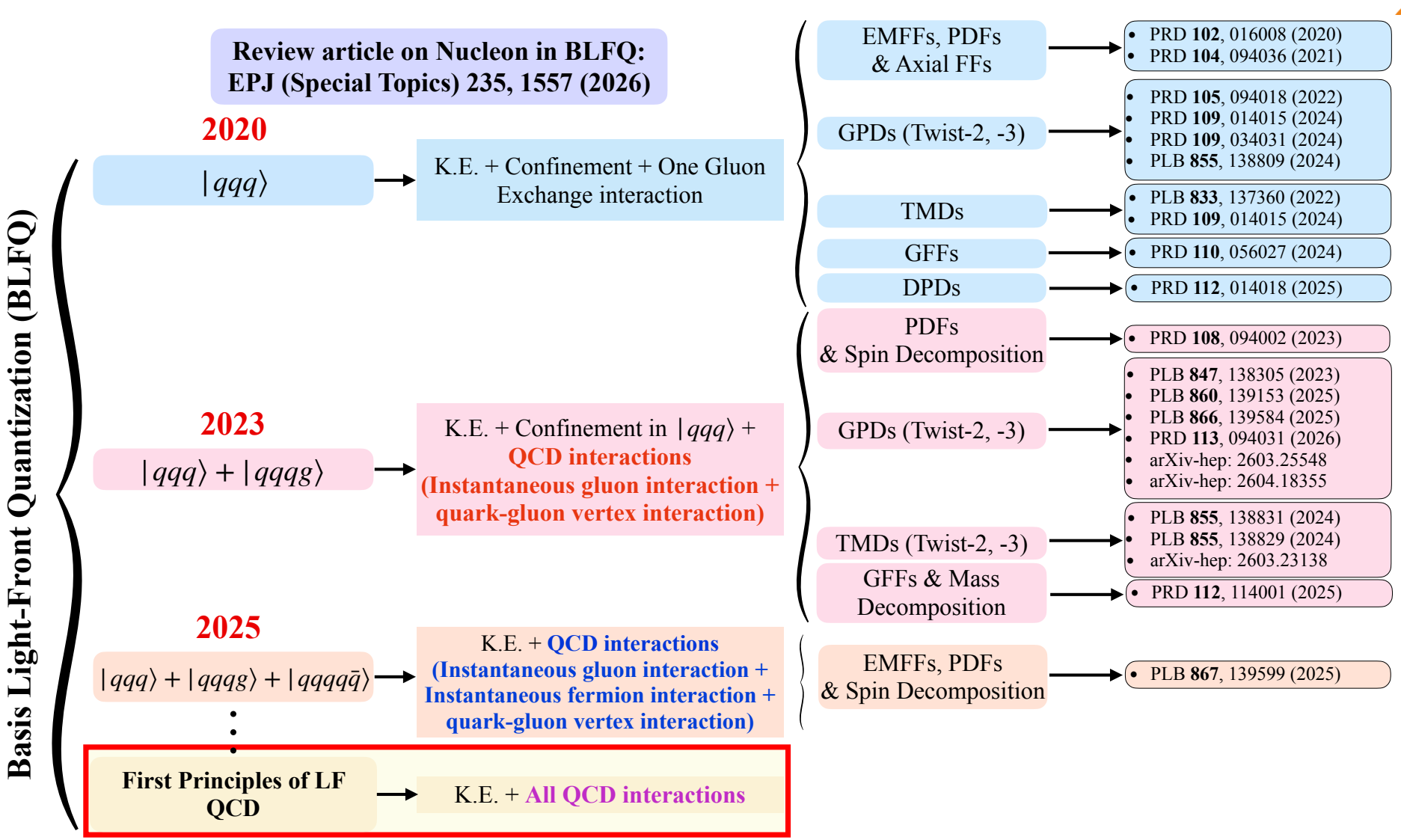
- Dimensions increase with increase in number of Fock sectors

Motivation for Quantum Computing





BLFQ Road-map toward First Principles



[0] BLFQ Collaboration

Proton with One Dynamical Gluon

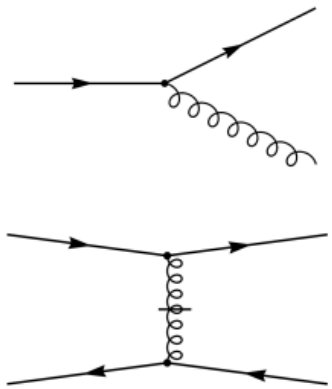
$$\{|\psi_{uud}|^2 = 44\% \quad ; \quad |\psi_{uudg}|^2 = 56\%\}$$

$$P^+P^-|\Psi\rangle = M^2|\Psi\rangle \qquad |\text{proton}\rangle = \psi_{uud}|uud\rangle + \psi_{uudg}|uudg\rangle$$

$$P^- = P^-_{\rm QCD} + P^-_C$$

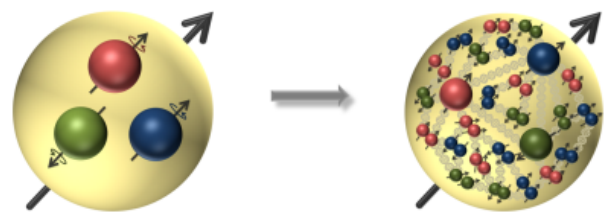
QCD Interaction:

$$P^-_{\rm QCD} = \int \mathrm{d}x^- \mathrm{d}^2x^\perp \Big\{ \frac{1}{2}\bar\psi\gamma^+ \frac{m_0^2 + (i\partial^\perp)^2}{i\partial^+}\psi - \frac{1}{2}A^i_a\left[m_g^2 + (i\partial^\perp)^2\right]A^i_a + g_s\bar\psi\gamma_\mu T^a A^\mu_a\psi + \frac{1}{2}g_s^2\bar\psi\gamma^+T^a\psi\frac{1}{(i\partial^+)^2}\bar\psi\gamma^+T^a\psi\Big\},$$



Confinement only in leading Fock:

$$P^-_C P^+ = \frac{\kappa^4}{2}\sum_{i\neq j}\left\{\vec r^{\,2}_{ij\perp} - \frac{\partial_{x_i}(x_ix_j\partial_{x_j})}{(m_i+m_j)^2}\right\}$$



Parameters:

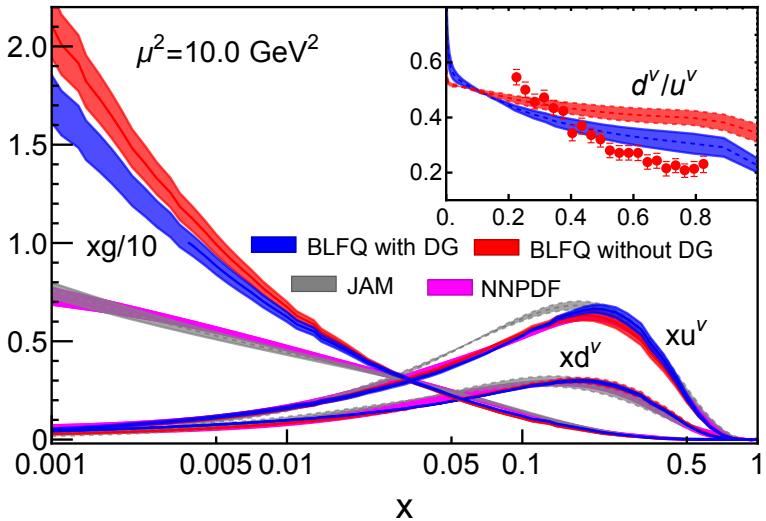
Truncation: Nmax=9, K=16.5
HO parameters: b=0.7GeV, b_{inst}=3GeV

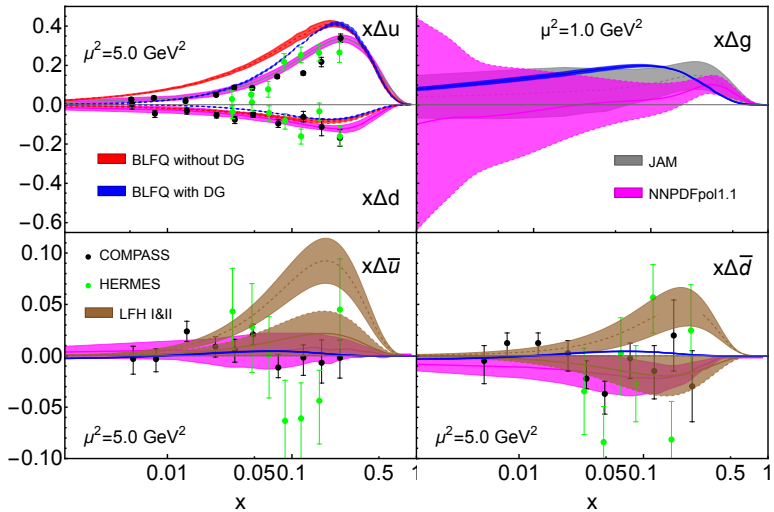
m_u	m_d	m_g	κ	m_f	g
0.31GeV	0.25GeV	0.50GeV	0.54GeV	1.80GeV	2.40

[1] BLFQ: S. Xu *et. al.*, Phys.Rev.D 108 (2023) 094002.
[2] Brodsky, Teramond, Dosch and Erlich, Phys. Rep. 584, 1 (2015).
[3] Li, Maris, Zhao and Vary, Phys. Lett. B (2016); M. Burkardt, Phys. Rev. D 58, 096015 (1998).

Unpolarized and Helicity PDFs

BLFQ: PRD 108 (2023) 094002





Diagonalizing $H_{\text{eff}} \Rightarrow$ LF wavefunction $\Rightarrow$ Initial PDFs $\Rightarrow$ Scale evolution

Model scale $\mu_0^2 = 0.24 \pm 0.01$ GeV^2

- Quark momentum : $\langle x \rangle_u = 0.261 \pm 0.005, \quad \langle x \rangle_d = 0.109 \pm 0.005$ at 10 GeV^2.
- Quark spin: $\frac{1}{2}\Sigma_u = 0.438 \pm 0.004, \quad \frac{1}{2}\Delta\Sigma_d = -0.080 \pm 0.002.$
- Gluon spin: $\Delta G = 0.131 \pm 0.003$, PHENIX: $\Delta G^{[0.02,0.3]} = 0.2 \pm 0.1$.
- Sea quarks: solely generated from the QCD evolution.

[1] LFH: PRL 124 (2020), 082003; PHENIX: PRL 103 (2009) 012003.

BLFQ Predictions for Spin Decomposition

Fock expansion:

$$|\,\text{Proton}\rangle = a\,|\,uud\rangle + b\,|\,uudg\rangle + \ldots$$

Quark and gluon helicities :

$$\Delta\Sigma_q = \int \mathrm{d}x\,\Delta q(x)$$

$$\Delta\Sigma_g = \int \mathrm{d}x\,\Delta G(x)$$

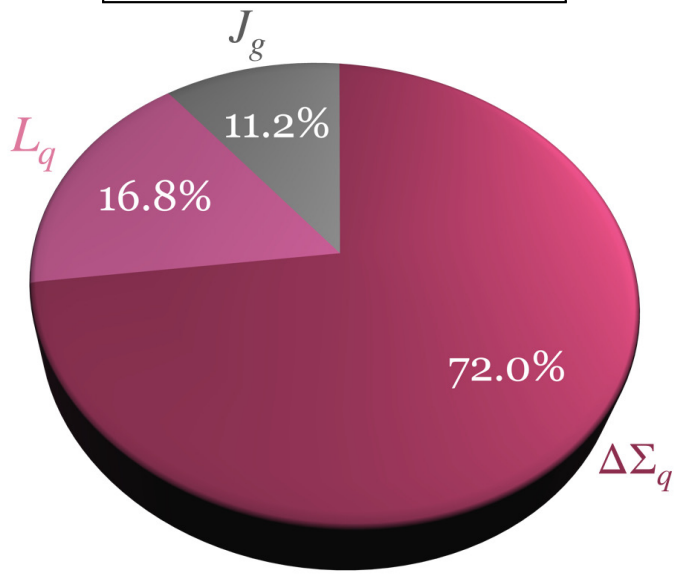
Total AM[2] :

$$J_i = \frac{1}{2}\int \mathrm{d}x\,x\,[H_i(x,0,0) + E_i(x,0,0)]$$

Kinetic OAM[2] :

$$L_q = \frac{1}{2}\int \mathrm{d}x\,[x\,\{H_q(x,0,0)+E_q(x,0,0)\}-\widetilde{H}_q(x,0,0)]$$

$\frac{1}{2}\Sigma_q + L_q + J_g = \frac{1}{2}$



Kinetic[1]

[1] BLFQ: S. Xu *et. al.*, Phys.Rev.D 108 (2023) 094002.
[2] X. Ji, Phys.Rev.Lett. 78, 610 (1997).

BLFQ Results for Spin Decomposition

Fock expansion:

$$|\,\text{Proton}\rangle = a\,|\,uud\rangle + b\,|\,uudg\rangle + \ldots$$

Quark and gluon helicities :

$$\Delta\Sigma_q = \int \mathrm{d}x\,\Delta q(x)$$

$$\Delta\Sigma_g = \int \mathrm{d}x\,\Delta G(x)$$

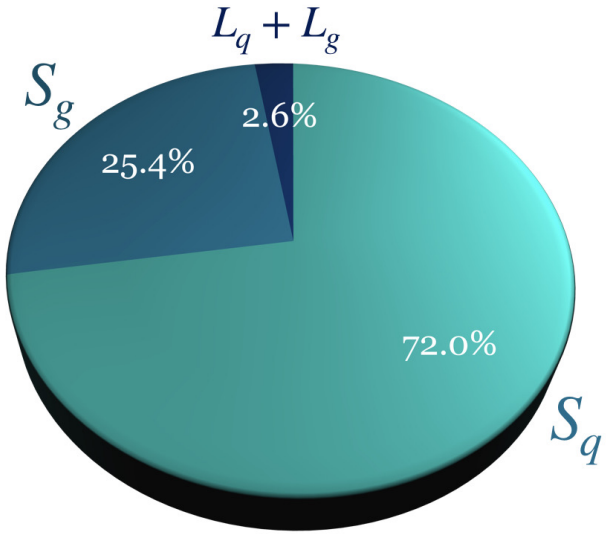
Total AM :

$$J_i = \frac{1}{2}\int \mathrm{d}x\,x\,[H_i(x,0,0) + E_i(x,0,0)]$$

Canonical OAM :

$$l_i^z = -\int \mathrm{d}x\,\mathrm{d}^2\vec{p}_\perp\,\frac{\vec{p}_\perp^{\,2}}{M^2}\,F_{1,4}^i(x,0,\vec{p}_\perp^{\,2},0,0)$$

$\frac{1}{2}\Sigma_q + \Sigma_g + l_q^z + l_g^z = \frac{1}{2}$

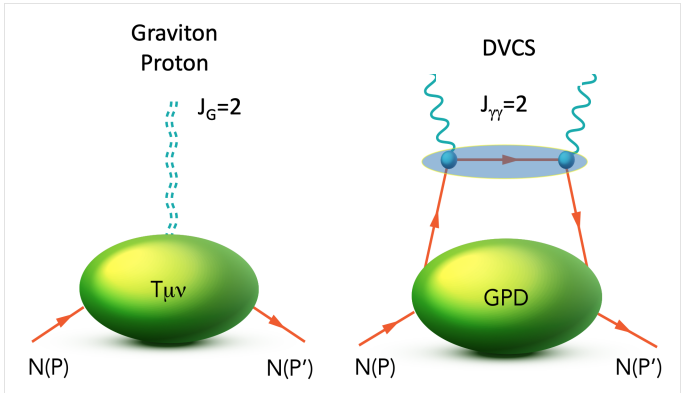


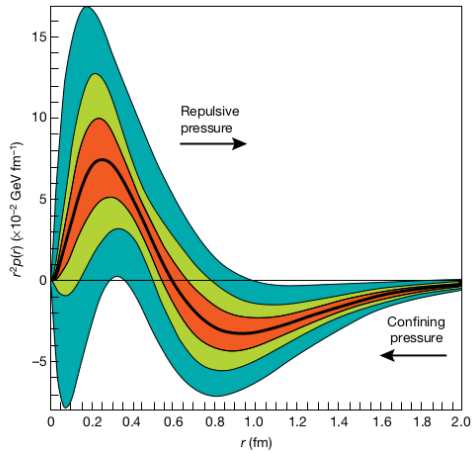
Canonical[1]

[1] BLFQ: S. Xu *et. al.*, Phys.Rev.D 108 (2023) 094002.

[2] Jaffe and Manohar, Nucl.Phys.B 337, 509–546 (1990).

Proton Gravitational Form Factors





- Parametrization of matrix element in terms of GFFs

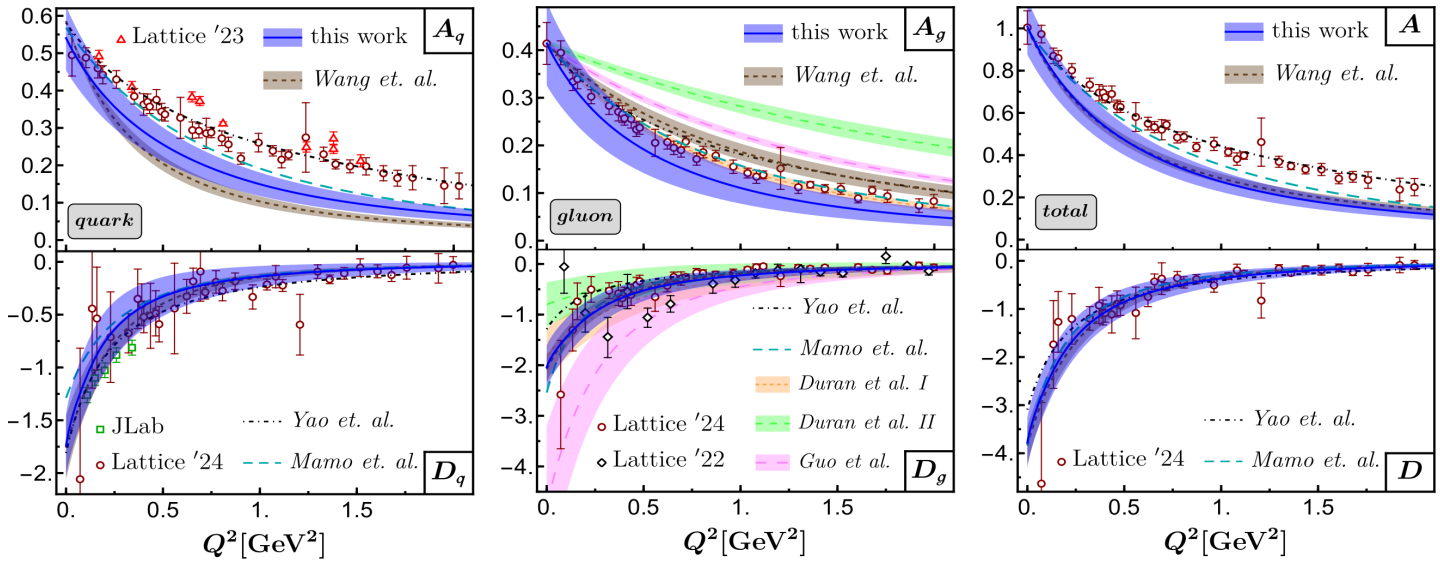
$$\langle P'|T_i^{\mu\nu}(0)|P\rangle = \bar{U}'\Big[-B_i(q^2)\frac{\bar{P}^\mu\bar{P}^\nu}{M} + (A_i(q^2)+B_i(q^2))\frac{1}{2}(\gamma^\mu\bar{P}^\nu+\gamma^\nu\bar{P}^\mu)$$
$$+C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2)Mg^{\mu\nu}\Big]U$$

- Momentum sum rule : $\sum_i A^i(0) = 1$
- Gravitomagnetic moment sum rule : $\sum_i B^i(0) = 0$
- Spin sum rule: $J^i = \frac{1}{2}\left[A^i(0)+B^i(0)\right]$
- $4C(q^2) = D(q^2)$ provides shear forces and the pressure distributions

[1] Burkert *et. al.*: Rev. Mod. Phys. 95, 041002 (2023); Ji, Phys. Rev. Lett. 78, 610 (1997)

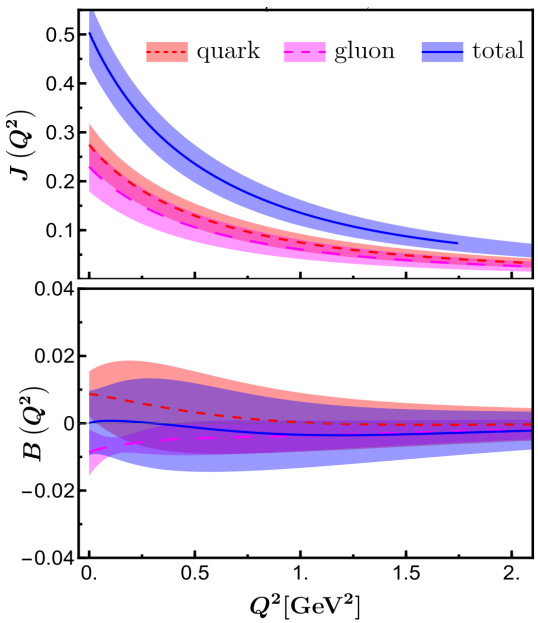
BLFQ Results for Proton GFFs

$$|\text{Nucleon}\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle$$



- QCD EMT : $T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}$, where
  $T_q^{\mu\nu} = i\overline{\psi}\gamma^{\{\mu}\mathcal{D}^{\nu\}}\psi$ and
  $T_g^{\mu\nu} = -F^{\mu\lambda a}F^{\nu}_{\lambda a} + \frac{1}{4}g^{\mu\nu}\left(F_{\lambda\sigma a}\right)^2$
- $A(Q^2)$ and $B(Q^2)$: T^{++} component
- $D(Q^2) = 4C(Q^2)$: T^{ij} components

$$\boxed{\textstyle\sum_i A^i(0) = 1 \text{ and } \sum_i B^i(0) = 0}$$



[1] BLFQ: S. Nair *et. al.*, Phys.Rev.D 112, 114001 (2025)

Quark TMDs

Leading Twist TMDs

Nucleon Spin Quark Spin

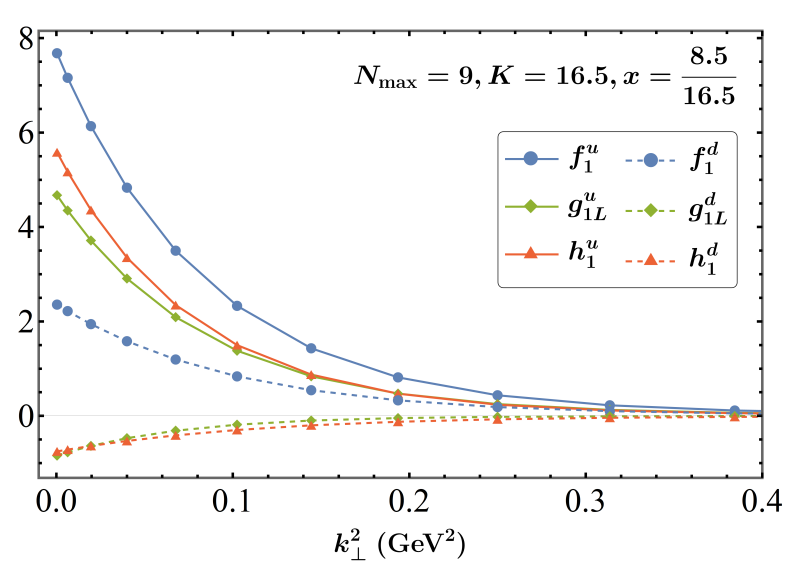
Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	f_1 =		$h_1^\perp$ = − Boer-Mulders
L		g_{1L} = − Helicity	$h_{1L}^\perp$ = −
T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = −	h_1 = − Transversity $h_{1T}^\perp$ = −

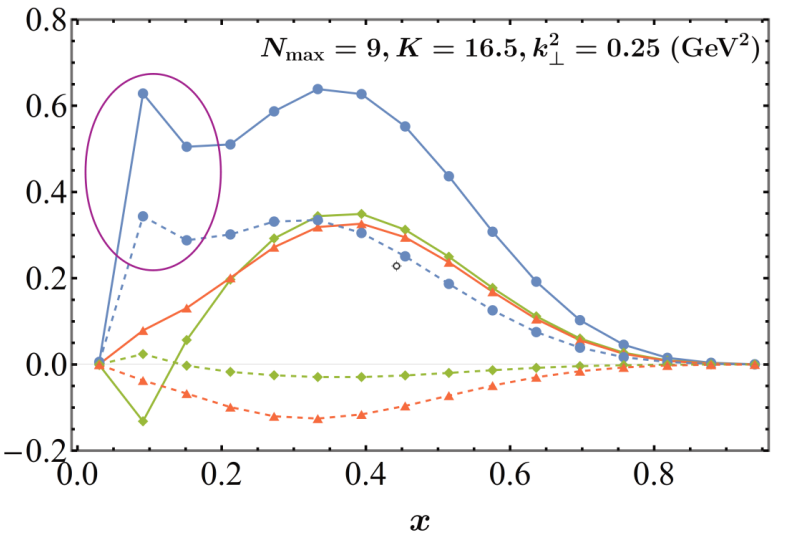
- Positivity bounds

$$f_1^q(x,\boldsymbol{k}_\perp^2) > 0, \quad f_1^q(x,\boldsymbol{k}_\perp^2) \geq |g_{1L}^q(x,\boldsymbol{k}_\perp^2)|,$$

$$f_1^q(x,\boldsymbol{k}_\perp^2) \geq \frac{|\boldsymbol{k}_\perp|}{M}|g_{1T}^q(x,\boldsymbol{k}_\perp^2)|,$$

$$f_1^q(x,\boldsymbol{k}_\perp^2) \geq \frac{|\boldsymbol{k}_\perp|^2}{2M^2}|h_1^{\perp q}(x,\boldsymbol{k}_\perp^2)|$$





[1] BLFQ: Hongyao Yu, *et. al.* coming soon...

[2] A. Accardi *et al.*, Eur.Phys.J.A 52 (2016) 9, 268.

Gaussian Ansatz Compatibility?

- To check compatibility of BLFQ results with the Gaussian ansatz :

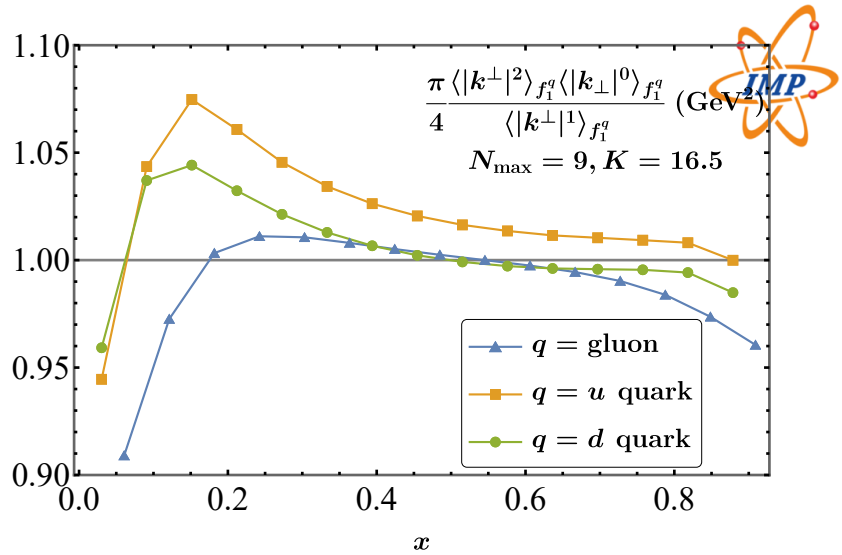
$$f_1^i(x, k_\perp^2) \approx a \frac{\exp\left(-\frac{|k_\perp|^2}{r}\right)}{\pi r}$$

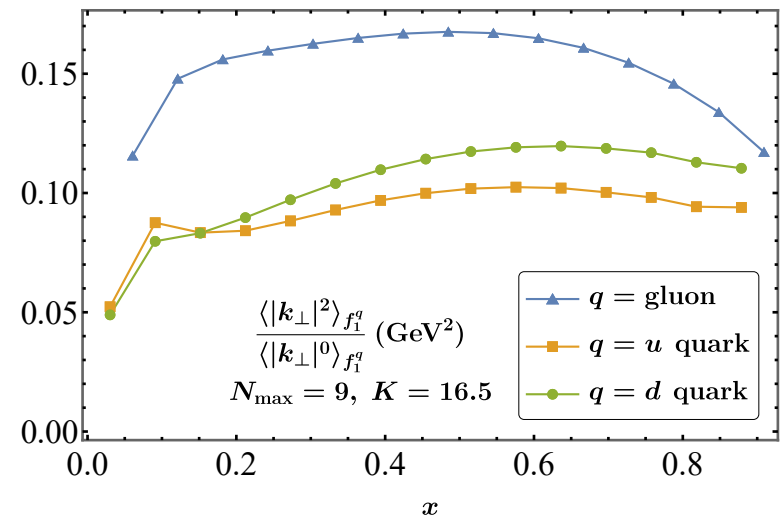
where $a = \langle |k_\perp|^0 \rangle_{f_1^i}$ and $r = \langle |k_\perp|^2 \rangle_{f_1^i}$

- If the Gaussian ansatz holds :

$$\frac{\langle |k_\perp|^2 \rangle_{f_1^i} \times \langle |k_\perp|^0 \rangle_{f_1^i}}{(\langle |k_\perp|^1 \rangle_{f_1^i})^2} \times \frac{\pi}{4} = 1$$

BLFQ results do not support Gaussian ansatz





[1] Hongyao Yu, *et. al.* in preparation

[2] BLFQ: Hongyao Yu, *et. al.* Phys.Lett.B 855, 138831 (2024)

Efremov-Teryaev-Qiu-Sterman (ETQS) Function

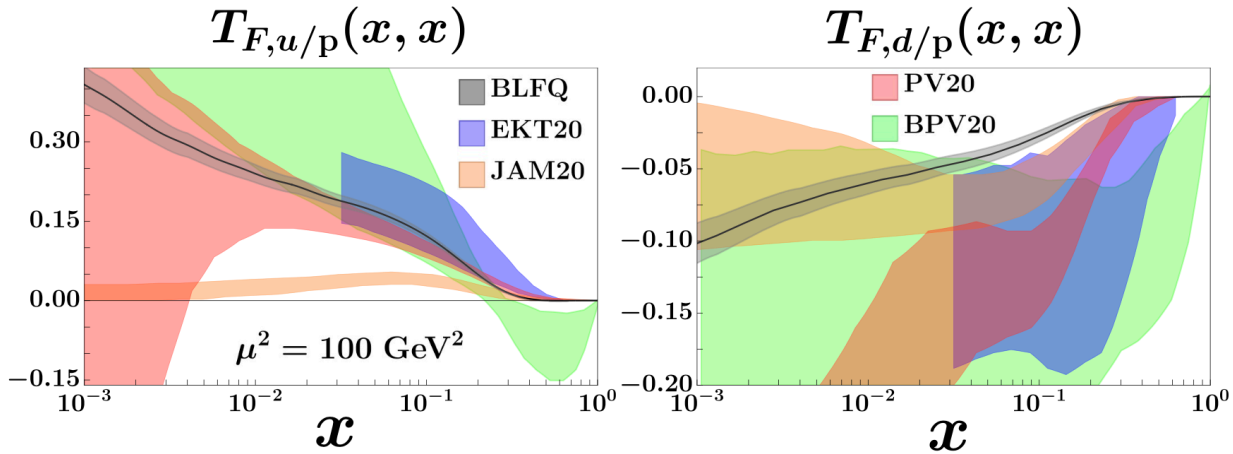
$$\hat{M}^{\mu}_{F,\alpha\beta}(x_1,x_2) = \int \frac{\mathrm{d}y_1^- \mathrm{d}y_2^-}{(2\pi)^2} e^{i\frac{y_1^-}{2}(x_1+x_2)P^+} e^{iy_2^-(x_2-x_1)P^+} \times \langle P,S|\bar{\psi}_\beta(-\tfrac{y_1^-}{2}) g F^{+\mu}\left(y_2^-\right) \psi_\alpha(\tfrac{y_1^-}{2})|P,S\rangle,$$

$$\hat{M}^{\mu}_{F}(x_1,x_2) = \frac{M}{2}\Big[T_F \epsilon_\perp^{\nu\mu} S_{\perp\nu}\gamma^- + \tilde{T}_F i S_\perp^\mu \gamma_5\gamma^- + \tilde{T}_F^{(\sigma)} i\Lambda\gamma_5\gamma_\perp^\mu\gamma^- + T_F^{(\sigma)} i\gamma_\perp^\mu\gamma^-\Big]$$

Zhu *et al.*, (BLFQ Collaboration)
arXiv-hep: 2603.23138

ETQS function after evolutions:

- Qualitative agreement with the extractions.

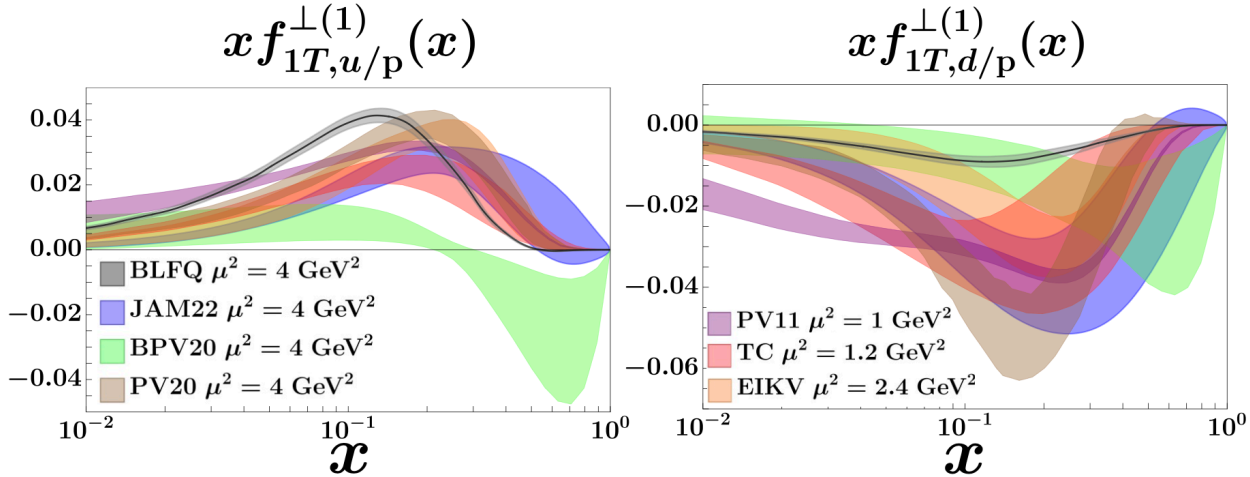


1st moment of the ETQS function:

$$\pi T_F(x,x) = f_{1T}^{\perp(1)}(x)$$

- 1st moment provides Sivers functions

— NPB **667**, 201 (2003)
— PRD **59**, 014004 (1999)



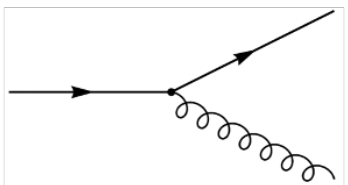
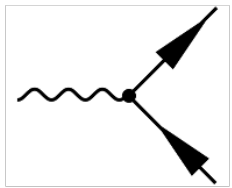
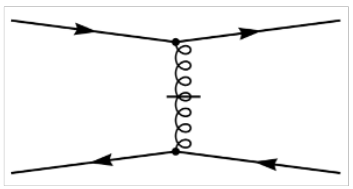
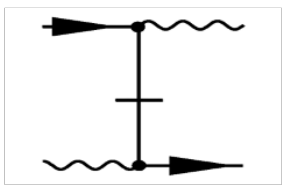
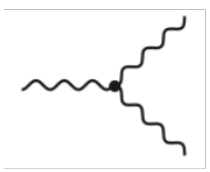
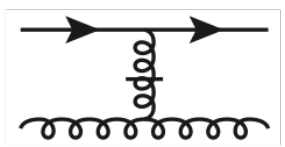
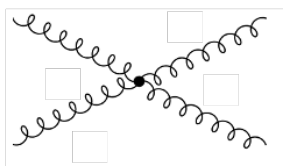
Hamiltonian with sea quarks and upto three gluons

Fock expansion: $|N\rangle \rightarrow |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$

QCD light-front Hamiltonian from QCD Lagrangian: [S. Brodsky, H-C Pauli, S. Pinsky, '97]

$$\mathscr{L}_{QCD} = \bar{\psi}(\not{D} - m)\psi - \frac{1}{4}G^{\alpha}_{\mu\nu}G^{\mu\nu}_{\alpha} \Longrightarrow P^{-}_{QCD} = H_K + H_I$$

$A^+ = 0$

$$H_K = \frac{1}{2}\int \mathrm{d}^3x\, \bar{\psi}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\psi - \frac{1}{2}\int \mathrm{d}^3x\, A^i_a (\mathrm{i}\partial^\perp)^2 A^i{}_a$$

$$\begin{aligned}
H_I = &+ g\int \mathrm{d}^3x\, \bar{\psi}\gamma_\mu A^\mu \psi \\
&+ \tfrac{1}{2}g^2 \int \mathrm{d}^3x\, \bar{\psi}\gamma_\mu A^\mu \frac{\gamma^+}{\mathrm{i}\partial^+}\gamma_\nu A^\nu \psi \\
&- \mathrm{i}g^2 \int \mathrm{d}^3x\, f^{abc}\bar{\psi}\gamma^+ T^c \psi \frac{1}{(\mathrm{i}\partial^+)^2}\left(\mathrm{i}\partial^+ A^\mu_a A_{\mu b}\right) \\
&+ \tfrac{1}{2}g^2 \int \mathrm{d}^3x\, \bar{\psi}\gamma^+ T^a \psi \frac{1}{(\mathrm{i}\partial^+)^2}\bar{\psi}\gamma^+ T^a \psi \\
&+ \mathrm{i}g \int \mathrm{d}^3x\, f^{abc}\mathrm{i}\partial^\mu A^{\nu a} A^b_\mu A^c_\nu \\
&- \tfrac{1}{2}g^2 \int \mathrm{d}^3x\, f^{abc} f^{ade}\mathrm{i}\partial^+ A^\mu_b A_{\mu c}\frac{1}{(\mathrm{i}\partial^+)^2}\left(\mathrm{i}\partial^+ A^\nu_d A_{\nu e}\right) \\
&+ \tfrac{1}{4}g^2 \int \mathrm{d}^3x\, f^{abc} f^{ade} A^\mu_b A^\nu_c A_{\mu d} A_{\nu e}.
\end{aligned}$$

ψ: quark field operator

A^a_μ: gluon field operator

7 terms in H_I

Dimensions of the Basis States

$$|N\rangle \to |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$$

$N_{max} = 7,\ K_{max} = 16$

	$\|qqq\rangle$	$\|qqqg\rangle$	$\|qqq\ q\bar{q}\rangle$	$\|qqq\ gg\rangle$	$\|qqq\ ggg\rangle$	$\|qqq\ q\bar{q}\ g\rangle$
Dimension	35,088	592,960	3,901,500	5,169,360	19,603,584	7,128,576
Color Config.	1	2	3	6	22	8

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\ u\bar{u}\rangle + |qqq\ d\bar{d}\rangle + |qqq\ s\bar{s}\rangle$$

Basis Dimension= 12,332,548

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\ u\bar{u}\rangle + |qqq\ d\bar{d}\rangle + |qqq\ s\bar{s}\rangle + |qqq\ gg\rangle$$

Basis Dimension= 17,501,908

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\ q\bar{q}\rangle + |qqq\ gg\rangle + |qqq\ ggg\rangle$$

Basis Dimension= 37,105,492

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\ q\bar{q}\rangle + |qqq\ gg\rangle + |qqq\ ggg\rangle + |qqq\ q\bar{q}\ g\rangle$$

Basis Dimension= 58,491,220

Input Parameters in Hamiltonian

- Truncation Parameters: up to 6-parton Fock sectors with $N_{\max} = 7$ & $K = 10$

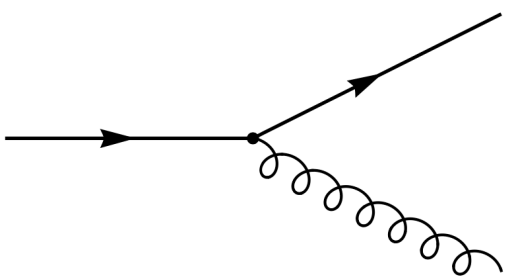
$$|\mathrm{N}\rangle \to |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$$

- Input Parameters:

m_u	m_d	m_s	m_f	g	b_{inst}	b
0.45 GeV	0.449 GeV	0.6 GeV	2.0 GeV	2.6	3.0 GeV	0.6 GeV

$M_p = 0.93$ GeV

- determined by fitting electromagnetic Form Factors and proton mass
- separate quark mass in the quark-gluon vertex m_f is needed

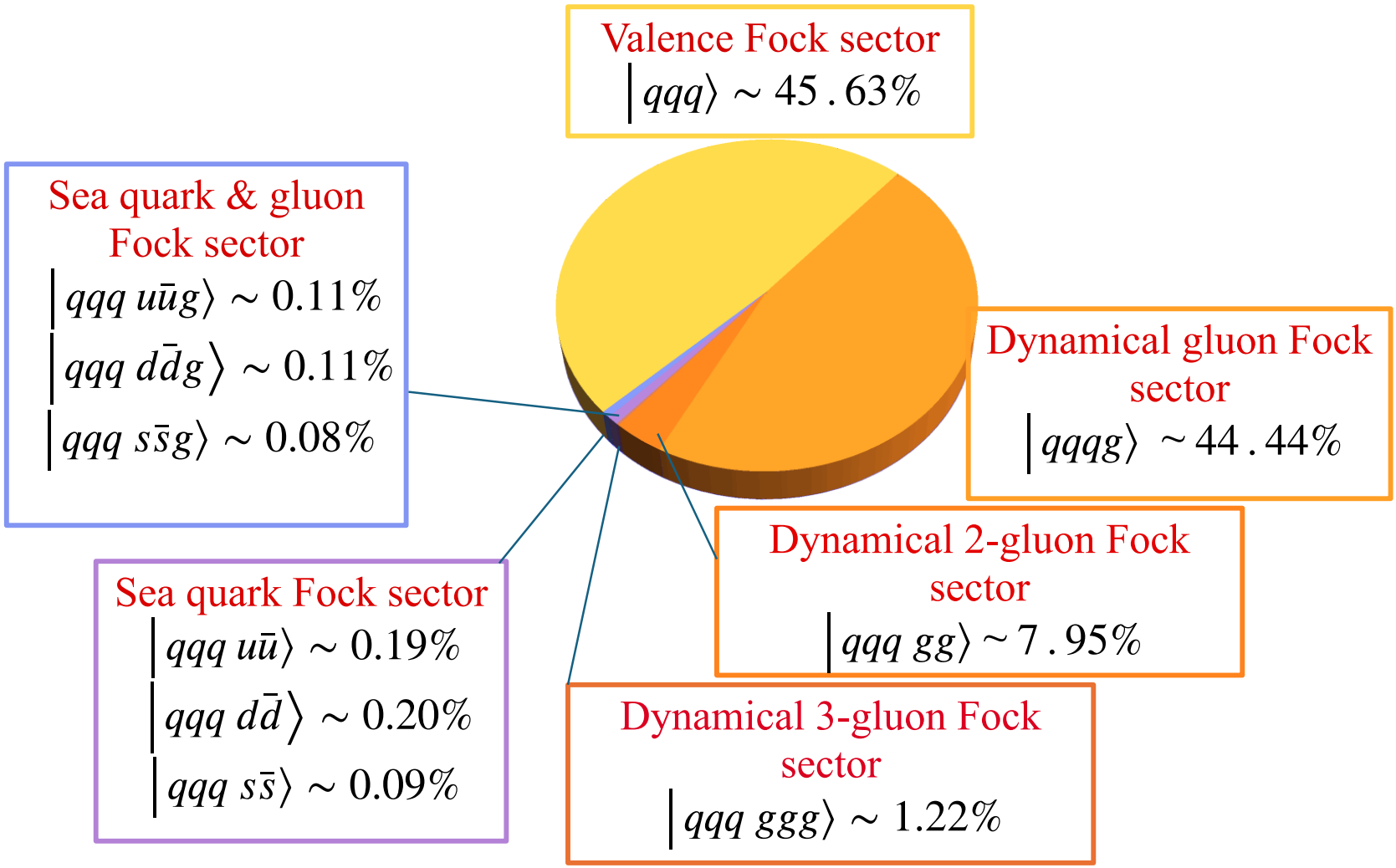


$\propto gb + \ldots$ Spin no flip

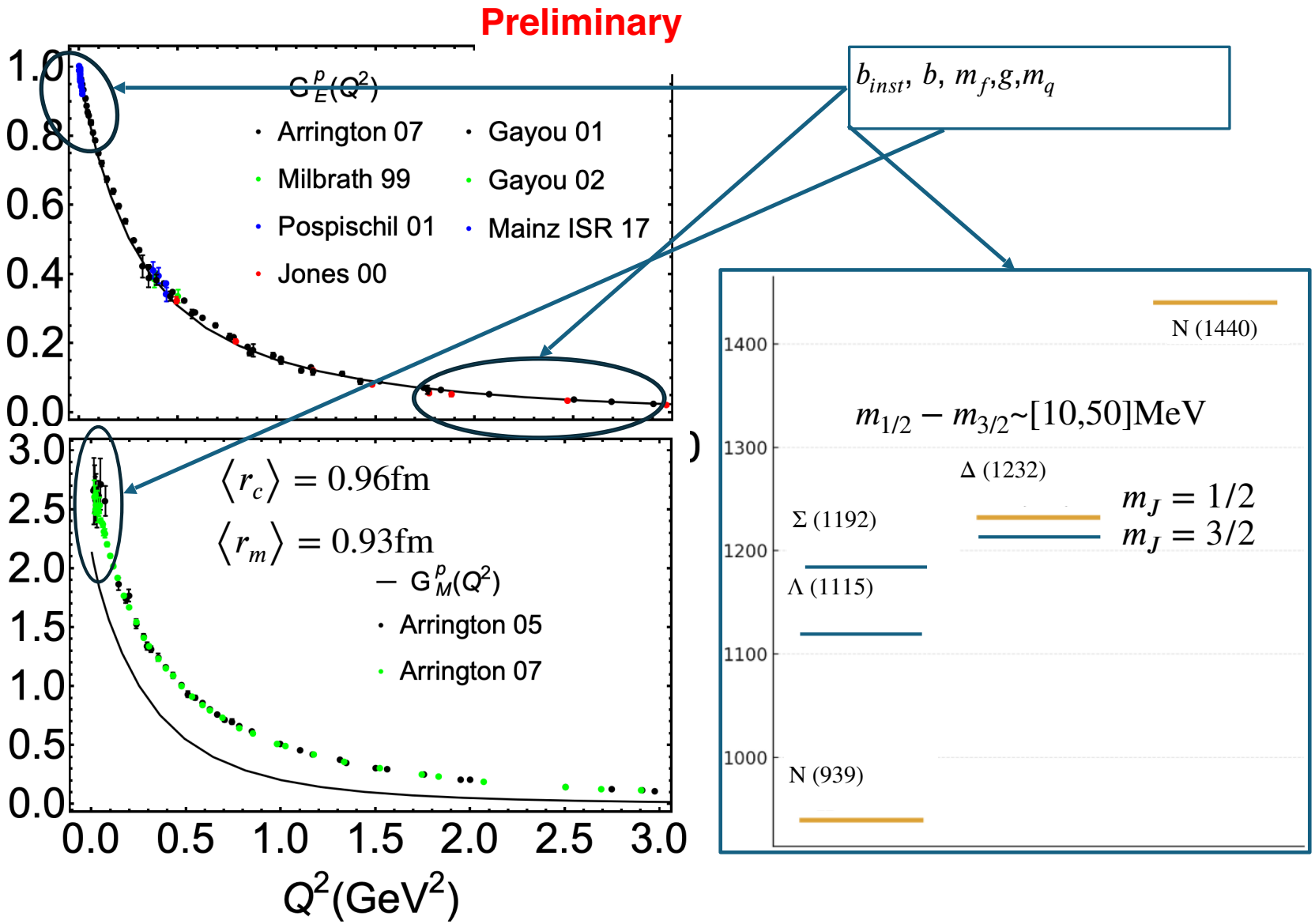
$\propto gm_f + \ldots$ Spin flip

Fock Sector Decomposition

$$|N\rangle \to |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$$



Electromagnetic FFs and Mass Spectra

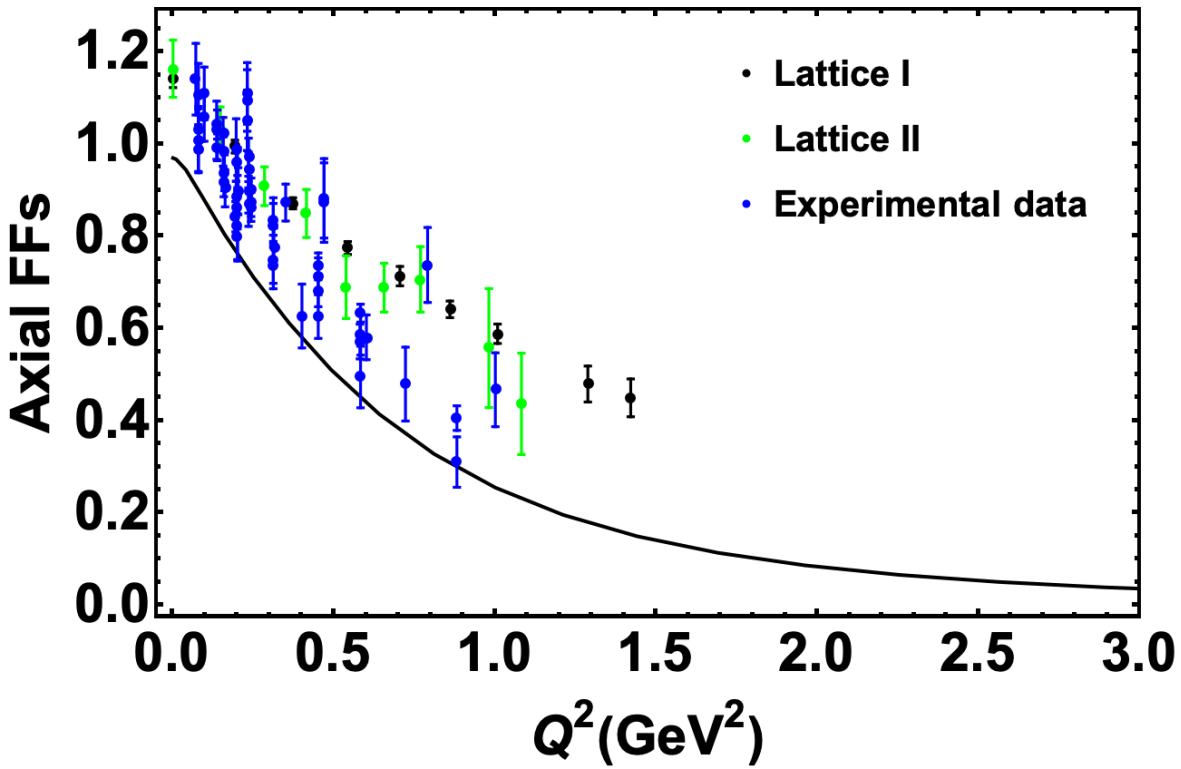


Axial Form Factor

$$\langle N(p')|A^a_\mu|N(p)\rangle = \bar{u}(p')\left[\gamma_\mu G_A(t) + \frac{(p'-p)_\mu}{2m} G_P(t)\right]\gamma_5 \frac{\tau^a}{2} u(p)$$

$$A^a_\mu = \bar{q}\gamma_\mu\gamma_5 T^a q$$

$$G_A(Q^2) = G_u(Q^2) - G_d(Q^2)$$



$\langle r_A \rangle = 0.59$ fm

$\Delta\Sigma_u = 0.78 \quad (0.82)$

$\Delta\Sigma_d = -0.19 \quad (-0.45)$

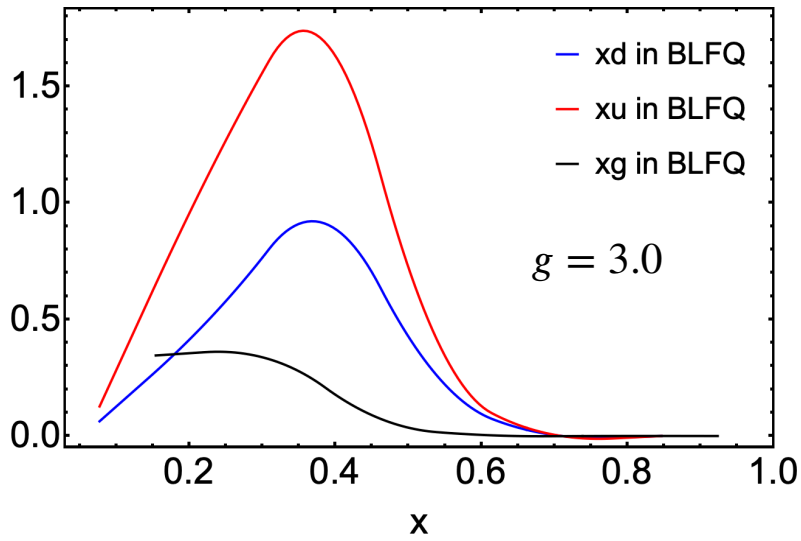
$\Delta\Sigma_{u-d} = 0.97 \quad (1.27)$

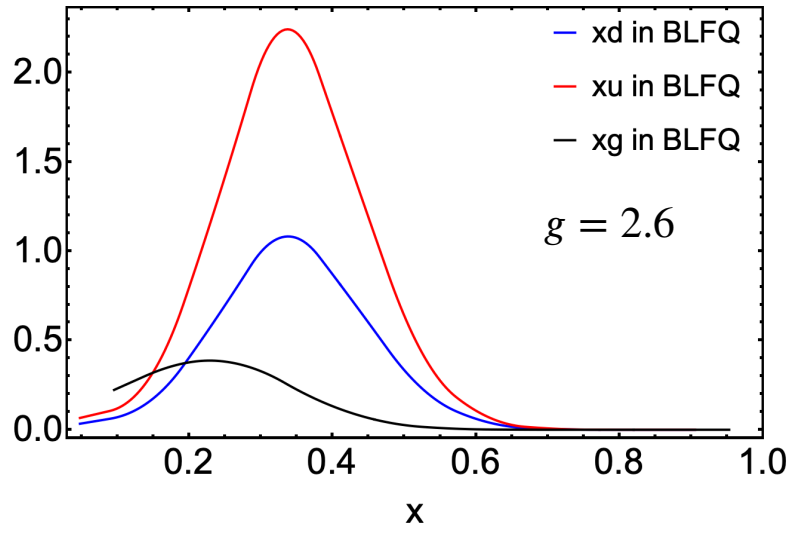
$\Delta\Sigma_{u+d} = 0.59$

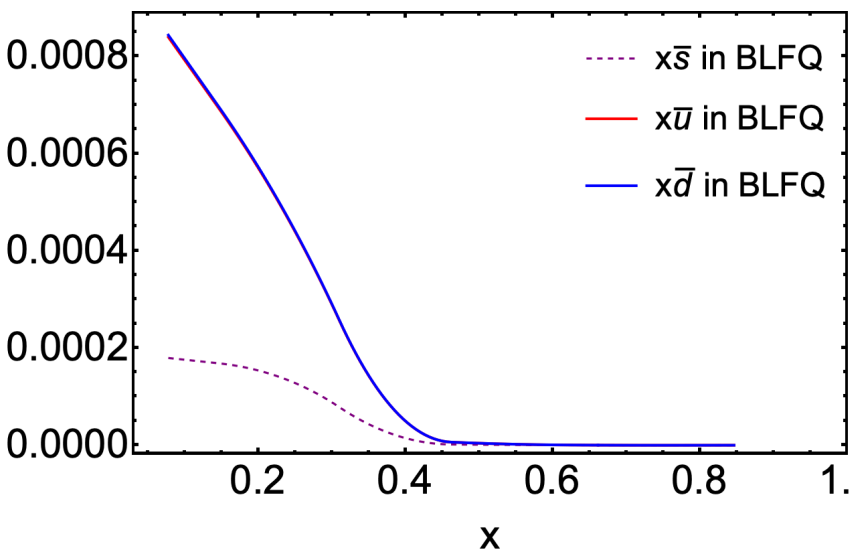
$\Delta G = 0.13$

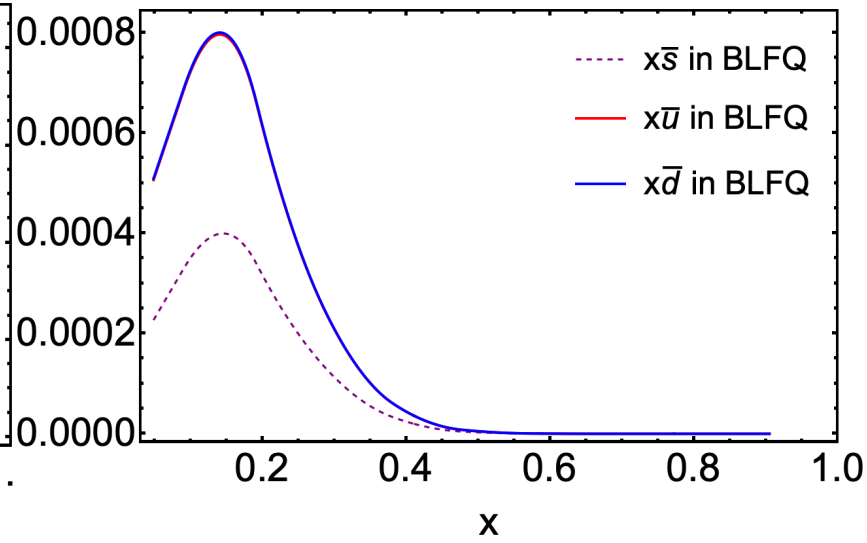
- Black line: valence quark
- Qualitatively agree with the experimental data

Unpolarized PDF

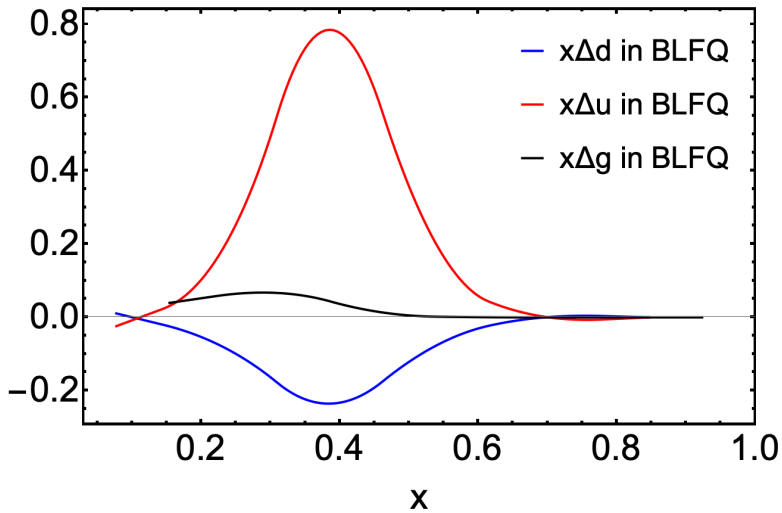


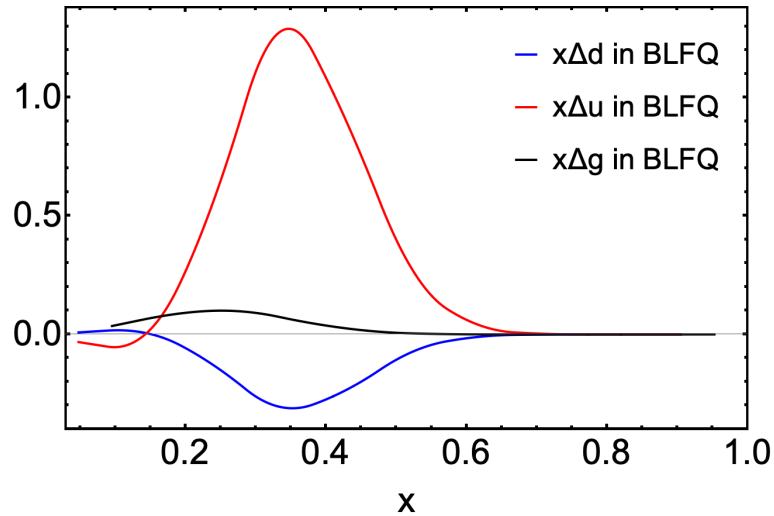


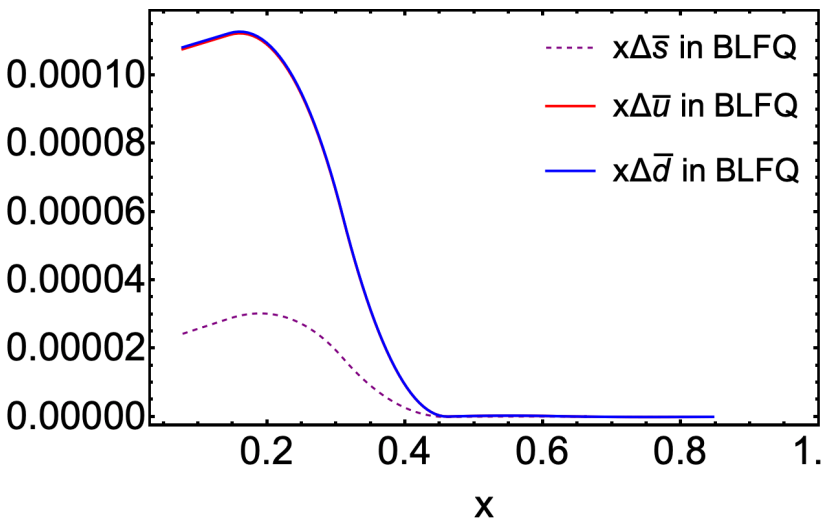


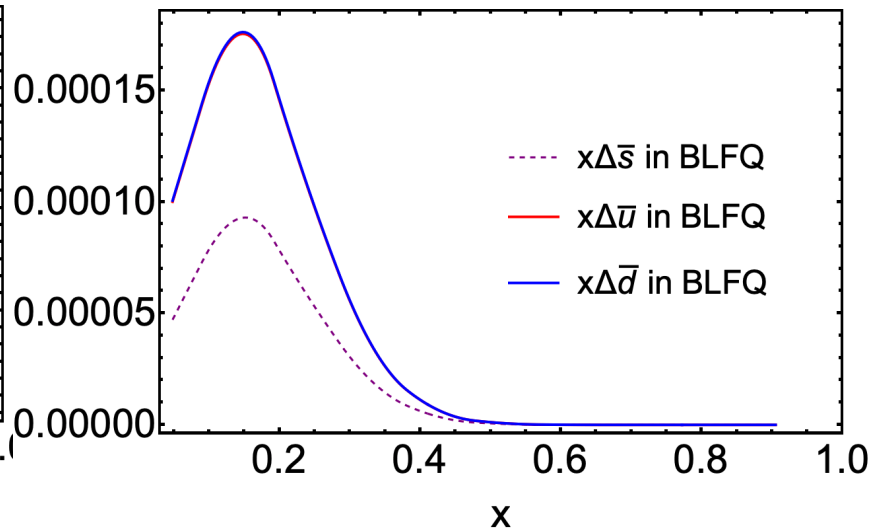


Helicity PDF









Outlook

Current status

Full QCD interaction

Deutron calculation
$|qqq\ qqq\rangle + |qqq\ qqq\ g\rangle$

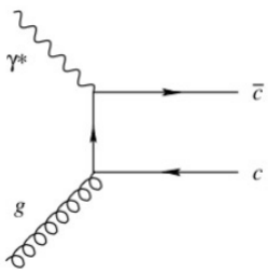
Fock sector expansion
$|qqq\ q\bar{q}\ g\rangle$ and $|qqq\ ggg\rangle$.....

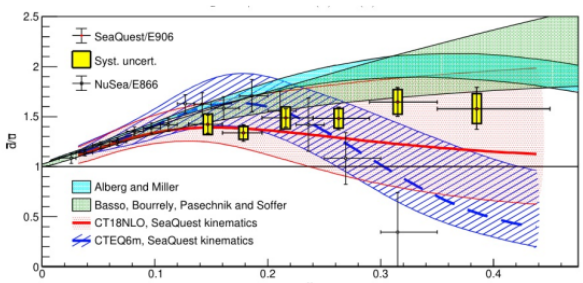
EMC effect

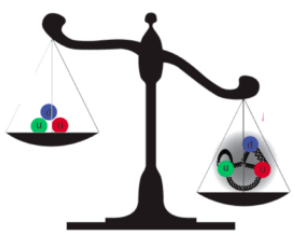
Intrinsic charm

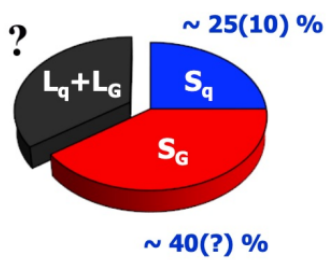
Sea asymmetry

Origin of spin and mass









Conclusions



- Basis Light-front Quantization : A non-perturbative approach based on light-front QCD Hamiltonian
- LF Hamiltonian ⇒ Wavefunctions ⇒ Observables.
- Calculate proton structure from QCD first principles.
- Spin structure qualitatively agree with expectation.
- Provides good description of data/global fits for various observables.

Thank You

Many-body wavefunction is built from single-particle basis states for q and g.

A big plus:
- Treats all particles in the Fock space equivalently
- Facilitates handling symmetry operators for identical particles

Concerns: The resulting many-body wf includes:

Entangled
- Intrinsic motion of quarks relative to each other
- Overall transverse center-of-mass motion of the whole system

Solution: Add a constraint term into the Hamiltonian

$$\boxed{H' = \lambda_L (H_{\rm c.m.} - 2b^2 I)} \qquad H_{\rm c.m.} = \left(\sum_i \vec{p}_{i\perp}\right)^2 + b^4 \left(\sum_i x_i \vec{r}_{i\perp}\right)^2$$

$H_{\rm c.m.}$: the transverse c.m. Hamiltonian (harmonic oscillator)

$2b^2$: zero-point energy

λ_L : a large Lagrange multiplier

The modified Hamiltonian:

$$H'_{\rm eff} = H_{\rm eff} - \left(\sum_i \vec{p}_{i\perp}\right)^2 + \lambda_L (H_{\rm c.m.} - 2b^2 I).$$

The resulting LFWFs are boost invariant

Basis Enumeration

$$|N\rangle \to |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$$

$|qqq\rangle \sim$ 1 color singlet state

$|qqq\ g\rangle \sim$ 2 color singlet state

2 octet $\otimes$ octet

$|qqq\ q\bar{q}\rangle \sim$ 3 color singlet states

1 singlet $\otimes$ singlet

2 octet $\otimes$ octet

$|qqq\ gg\rangle \sim$ 6 color singlet states

1 singlet $\otimes$ singlet

4 octet $\otimes$ octet

1 decuplet $\otimes$ octet $\otimes$ octet

$|qqq\ q\bar{q}\ g\rangle \sim$ 8 color singlet states

3 singlet $\otimes$ octet $\otimes$ octet

4 octet $\otimes$ octet $\otimes$ octet

1 decuplet $\otimes$ octet $\otimes$ octet

$|qqq\ ggg\rangle \sim$ 22 color singlet states

2 singlet $\otimes$ singlet

16 octet $\otimes$ octet

4 decuplet $\otimes$ octet $\otimes$ octet $\otimes$ octet