

Improving our understanding of hadron structure with generalised parton distributions

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NSTAR 2026 Sevilla

OPTIMIST

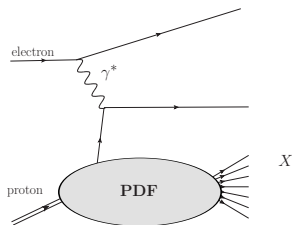


**MY GLASS
IS HALF
FULL**

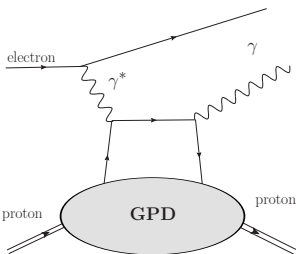
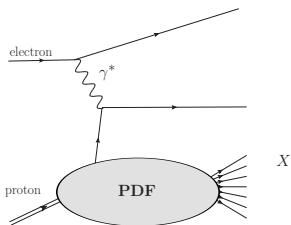
PESSIMIST



**MY GLASS
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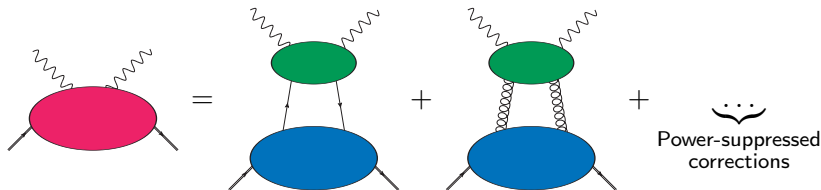


- Deep Inelastic Scattering, only the recoiled electron is characterised.
- The photon virtuality Q^2 is large compared to all other scales.
- The process is inclusive (summed over all possible final states).
- It probes the Parton Distribution Functions (PDF)



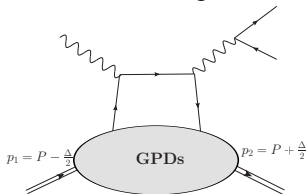
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- The process is inclusive (summed over all possible final states).
- It probes the Parton Distribution Functions (PDF)
- Exclusive processes : the final state is **fully** characterised
- Example : Deeply virtual Compton Scattering (DVCS) $ep \rightarrow ep\gamma$
- The photon virtuality Q^2 is large compared to all other scales (including t the total momentum transfer squared).
- It probes Generalised Parton Distributions (GPDs)

- When the photon is strongly virtual : $Q^2 = -q^2 \gg M^2, t$

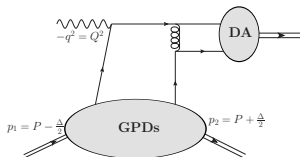


- Decomposition of DVCS between perturbative (green) and non-perturbative (blue) parts.
- Perturbative part $\rightarrow$ description of the interaction between the probe and a parton inside hadron
- Non-perturbative part : description of a parton-hadron amplitude called Generalised Parton Distributions (GPDs)
- GPDs is where the information on the hadron structure lies.

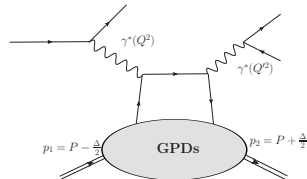
Timelike Compton Scattering



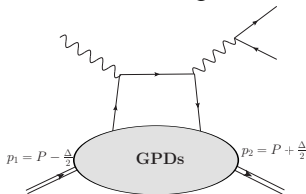
Deeply virtual meson production



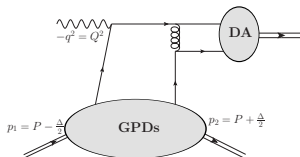
Double Deeply Virtual Compton Scattering



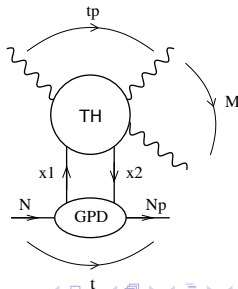
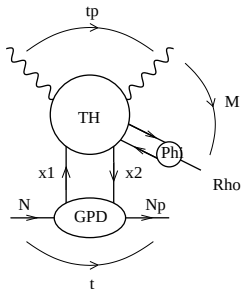
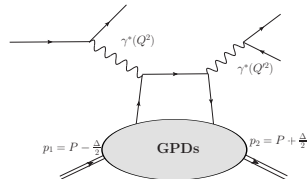
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Double Deeply Virtual Compton Scattering



Generalised Parton Distributions

$$\begin{aligned} & \frac{1}{2} \int \frac{e^{ixP^+z^-}}{2\pi} \langle P + \frac{\Delta}{2} | \bar{\psi}^q(-\frac{z}{2}) \gamma^+ \psi^q(\frac{z}{2}) | P - \frac{\Delta}{2} \rangle dz^- |_{z^+=0, z=0} \\ &= \frac{1}{2P^+} \left[H^q(x, \xi, t) \bar{u} \gamma^+ u + E^q(x, \xi, t) \bar{u} \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u \right]. \end{aligned}$$

D. Müller *et al.*, Fortschr. Phys. 42 101 (1994)

X. Ji, Phys. Rev. Lett. 78, 610 (1997)

A. Radyushkin, Phys. Lett. B380, 417 (1996)

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- $t = \Delta^2$: the Mandelstam variable
- Caveat! In gauges other than the lightcone one, a Wilson line is necessary to make the GPDs gauge invariant

- Forward limit:

$$H(x, \xi = 0, t = 0) = q(x)\Theta(x) - \bar{q}(-x)\Theta(-x)$$

Optical Theorem

- Forward limit:

Optical Theorem

- Polynomiality Property:

$$\int_{-1}^1 dx x^m H^q(x, \xi, t; \mu) = \sum_{j=0}^{\left[\frac{m}{2}\right]} \xi^{2j} C_{2j}^q(t; \mu) + \text{mod}(m, 2) \xi^{m+1} C_{m+1}^q(t; \mu)$$

X. Ji, J.Phys.G 24 (1998) 1181-1205
A. Radyushkin, Phys.Lett.B 449 (1999) 81-88

Special case :

$$\int_{-1}^1 dx H^q(x, \xi, t; \mu) = F_1^q(t) \quad \int_{-1}^1 dx E^q(x, \xi, t; \mu) = F_2^q(t)$$

Lorentz Covariance

- Forward limit:

Optical Theorem

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

$$\left| H^q(x, \xi, t) - \frac{\xi^2}{1 - \xi^2} E^q(x, \xi, t) \right| \leq \sqrt{\frac{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}{1 - \xi^2}}$$

A. Radyushkin, Phys. Rev. D **59**, 014030 (1999)

B. Pire *et al.*, Eur. Phys. J. C **8**, 103 (1999)

M. Diehl *et al.*, Nucl. Phys. B **596**, 33 (2001)

P.V. Pobilitza, Phys. Rev. D **65**, 114015 (2002)

Positivity of Hilbert space norm

- Forward limit:
- Polynomiality Property:
- Positivity property:
- Support property:

Optical Theorem

Lorentz Covariance

Positivity of Hilbert space norm

$$x \in [-1; 1]$$

M. Diehl and T. Gousset, Phys. Lett. B428, 359 (1998)

Relativistic quantum mechanics

- Forward limit:

Optical Theorem

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

Relativistic quantum mechanics

- Continuity at the crossover lines

→ GPDs are continuous albeit non-analytical at $x = \pm\xi$

J. Collins and A. Freund, PRD 59 074009 (1999)

Factorisation theorem

- Forward limit:

Optical Theorem

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

Relativistic quantum mechanics

- Continuity at the crossover lines

Factorisation theorem

- Scale evolution property

→ generalization of DGLAP and ERBL evolution equations

D. Müller *et al.*, Fortschr. Phys. 42, 101 (1994)

Renormalization

- Forward limit:

Optical Theorem

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

Relativistic quantum mechanics

- Continuity at the crossover lines

Factorisation theorem

- Scale evolution property

Renormalization

Problem

- There is hardly any model fulfilling all these constraints *a priori*.
- Lattice QCD computations remain very challenging.

- In the limit $\xi \rightarrow 0$, one recovers a density interpretation:
 - ▶ 1D in momentum space (x)
 - ▶ 2D in coordinate space $\vec{b}_\perp$ (related to t)

M. Burkardt, Phys. Rev. D62, 071503 (2000)

Interpretation of GPDs I

2+1D structure of the nucleon



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- Possibility to extract density from experimental data

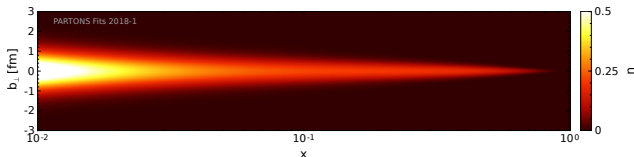


figure from H. Moutarde *et al.*, EPJC 78 (2018) 890

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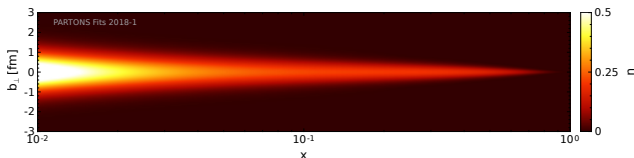


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- Correlation between x and $b_\perp \rightarrow$ going beyond PDFs and FFs.

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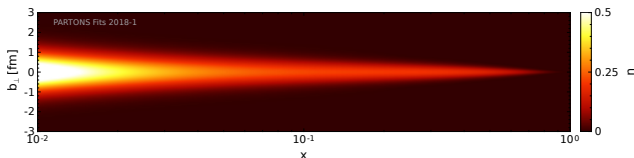
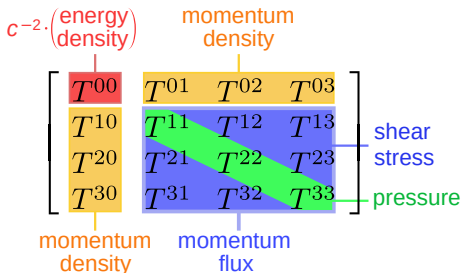


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- Correlation between x and $b_\perp \rightarrow$ going beyond PDFs and FFs.
- Caveat: no experimental data at $\xi = 0$
 $\rightarrow$ extrapolations (and thus model-dependence) are necessary

Interpretation of GPDs II

Connection to the Energy-Momentum Tensor



How energy, momentum, pressure are shared between quarks and gluons

Caveat: renormalization scheme and scale dependence

C. Lorcé et al., PLB 776 (2018) 38-47,

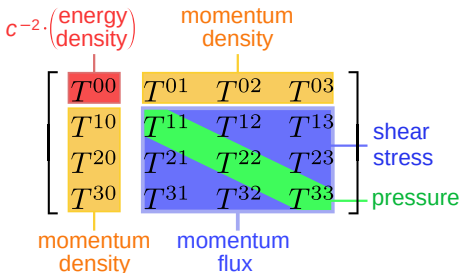
M. Polyakov and P. Schweitzer,

IJMPA 33 (2018) 26, 1830025

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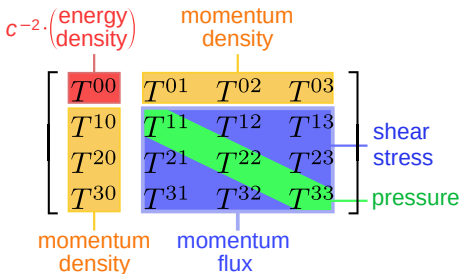
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$$\begin{aligned} \langle p', s' | T_{q,g}^{\mu\nu} | p, s \rangle = & \bar{u} \left[P^{\{\mu} \gamma^{\nu\}} A_{q,g}(t; \mu) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{M} C_{q,g}(t; \mu) \right. \\ & \left. + M g^{\mu\nu} \bar{C}_{q,g}(t; \mu) + \frac{P^{\{\mu} i \sigma^{\nu\}} \Delta}{2M} B_{q,g}(t; \mu) + \frac{P^{[\mu} i \sigma^{\nu]} \Delta}{2M} D_{q,g}(t; \mu) \right] u \end{aligned}$$

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$$\int_{-1}^1 dx x H_q(x, \xi, t; \mu) = A_q(t; \mu) + (2\xi)^2 C_q(t; \mu)$$

$$\int_{-1}^1 dx x E_q(x, \xi, t; \mu) = B_q(t; \mu) - (2\xi)^2 C_q(t; \mu)$$

- Ji sum rule

- Fluid mechanics analogy

X. Ji, PRL 78, 610-613 (1997)

M.V. Polyakov PLB 555, 57-62 (2003)

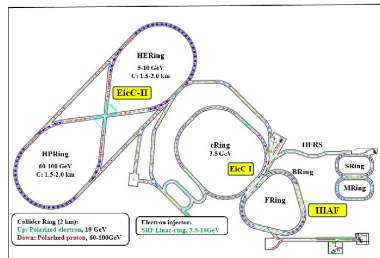
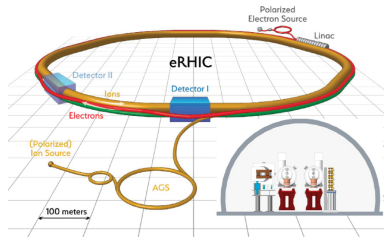
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- Current experimental facilities: fixed targets



- Future facilities: colliders



How are experimental data connected to the 3D structure of the nucleon ?

$$\Im \mathcal{H}(\xi, t, Q^2) = 2 \int_{\xi}^1 \frac{dx}{\xi} \Im C\left(\frac{x}{\xi}, \frac{t}{Q^2}, \alpha_s(Q^2); \partial_{\xi}^{(n)}\right) \\ \int_x^1 \frac{dy}{y} K\left(y, \frac{\xi}{x}; Q^2, \mu^2\right) H\left(\frac{x}{y}, \xi, t, \mu^2\right)$$

DVCS Amplitude extracted
from experimental data


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Targeted GPD function
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Description of probe-parton interaction computed in pQCD

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Description of probe-parton interaction computed in pQCD

Evolution kernel providing a scale resolution tuning

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- **S. Piloneta** will enlarge the scope of the session by explaining us how the kinematic power corrections are also important for TMD physics.

Experimental data

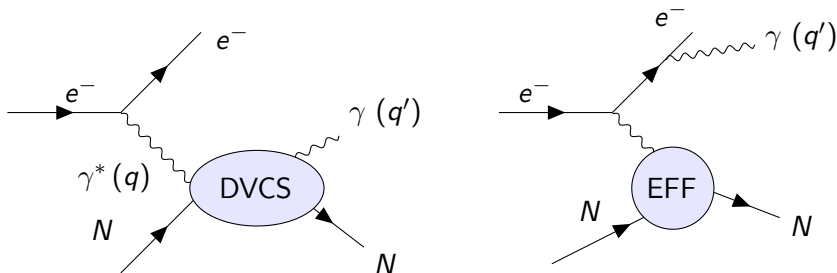
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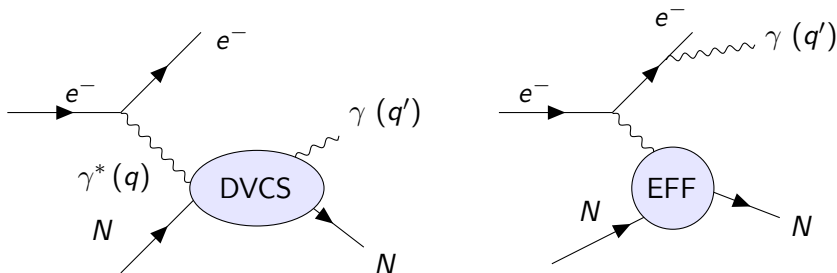
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$$ep \rightarrow ep\gamma$$

- All processes with the same initial and final state **do** contribute to our measurement



- You need to find a way to disentangle both

DVCS Beam Spin Asymmetry

Exploiting interference



$$\frac{d\sigma}{d\Omega} \propto |BH + DVCS|^2 \propto |BH|^2 + |DVCS|^2 + \underbrace{\text{Interference}}_{\text{our target}}$$

- BH amplitude is well-known $\rightarrow$ access to DVCS amplitude
- BH amplitude is much larger than DVCS one $\rightarrow$ interference term is significantly larger than the pure DVCS one

DVCS Beam Spin Asymmetry

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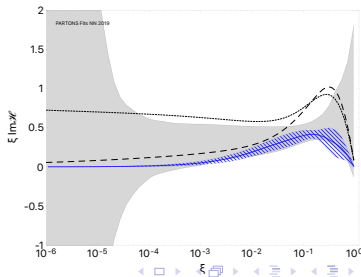
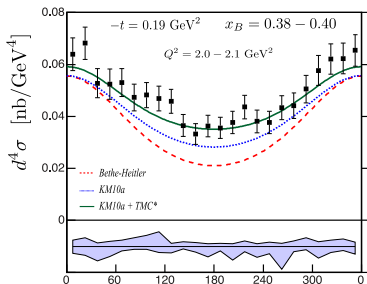


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$$A_{LU} = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}} \propto \Im \mathcal{H}$$

figs. from M. Defurne *et al.*, PRC 92 (2015) 5, 055202, and P. Sznajder *et al.*, EPJC 80 (2020) 2, 171



- Despite mixing with BH and being a rare event, DVCS *is* measurable
- One can build observables directly sensitive to the amplitude by taking advantage of the interference term.
- Good experimental signal for $\mathfrak{H}$ thanks to JLab experiments in the valence region.

However !

$$\begin{aligned}\Im \mathcal{H}(\xi, t, Q^2) &= 2 \int_{\xi}^1 \frac{dx}{\xi} \Im C\left(\frac{x}{\xi}, \frac{t}{Q^2}, \alpha_s(Q^2); \partial_{\xi}^{(n)}\right) \int_x^{\infty} \frac{dy}{y} K\left(y, \frac{\xi}{x}; Q^2, \mu^2\right) \\ &\times H\left(\frac{x}{y}, \xi, t, \mu^2\right)\end{aligned}$$

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Problem 1: Limited kinematic sensitivity

$\Im \mathcal{H}$ is generated only when $|x| \geq |\xi|$ in the convolution integral.
How do we recover the inner $-\xi \leq x \leq \xi$ region?

$$\Im \mathcal{H}(\xi, t, Q^2) = 2 \int_{\xi}^1 \frac{dx}{\xi} \Im C\left(\frac{x}{\xi}, \frac{t}{Q^2}, \alpha_s(Q^2); \partial_{\xi}^{(n)}\right) \int_x^{\infty} \frac{dy}{y} K\left(y, \frac{\xi}{x}; Q^2, \mu^2\right) \\ \times \left[H\left(\frac{x}{y}, \xi, t, \mu^2\right) + H_{\text{shadow}}\left(\frac{x}{y}, \xi, t, \mu^2\right) \right]$$

Problem 1: Limited kinematic sensitivity

$\Im \mathcal{H}$ is generated only when $|x| \geq |\xi|$ in the convolution integral.
How do we recover the inner $-\xi \leq x \leq \xi$ region?

Problem 2 : Uniqueness of the solution

Are we sure that the solution is unique? In other words, for a linear problem, do we have to handle vanishing eigenvalues?

Solving Problem 1

For experts:

Existence and uniqueness of kinematic completion
from DGLAP to ERBL regions

N. Chouika *et al.*, Eur.Phys.J.C 77 (2017) 12, 906

N. Chouika *et al.*, Phys.Lett.B 780 (2018) 287-293

J.-M. Morgado Chavez *et al.*, Phys.Rev.D 105 (2022) 9, 094012

P. Dall'Olio *et al.*, Phys.Rev.D 109 (2024) 9, 096013

A. Castro *et al.*, Phys.Rev.D 112 (2025) 3, 034009

- (x, ξ) are not two independent variables.
- This is better seen through the polynomiality condition:

$$\int_{-1}^1 dx \, x^m H^q(x, \xi, t; \mu) = \sum_{j=0}^{\left[\frac{m}{2}\right]} \xi^{2j} C_{2j}^q(t; \mu) + \text{mod}(m, 2) \xi^{m+1} C_{m+1}^q(t; \mu)$$

- In mathematics, this is known as the Ludwig-Helgason consistency condition, indicating that the function H is in the range of the Radon transform.

- The connection between GPDs and DDs is given through:

$$H(x, \xi, t) - \Theta(-\xi \leq x \leq \xi) D\left(\frac{x}{\xi}, t\right) = \int_{\Omega} d\beta d\alpha \delta(x - \beta - \alpha\xi) F(\beta, \alpha, t)$$

A. Radyushkin, PRD 56 (1997) 5524-5557

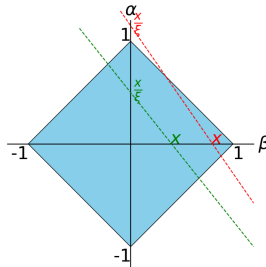
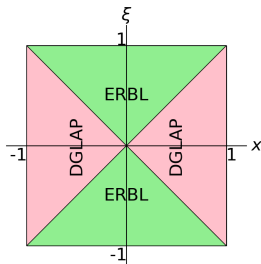
D. Müller *et al.*, Fortschr. Phys. 42 101 (1994)

- The D -term can be reabsorbed as:

$$H(x, \xi, t) = \int_{\Omega} d\beta d\alpha \delta(x - \beta - \alpha\xi) [F(\beta, \alpha, t) + \xi\delta(\beta)D(\alpha, t)]$$

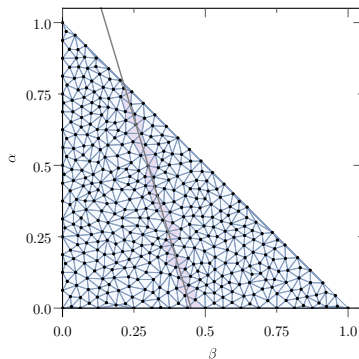
M. Polyakov and C. Weiss, PRD60 114017 (1999)

- The properties of the DD guarantee those of the GPD

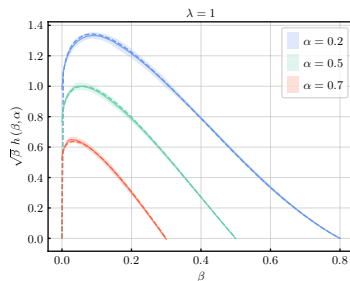
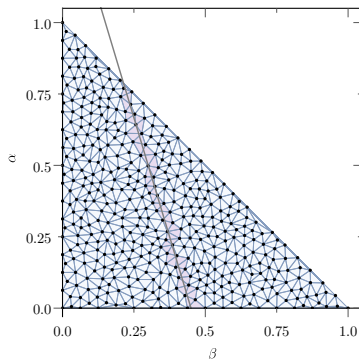


$$\begin{aligned}\Im \mathcal{H}(\xi, t, Q^2) &= 2 \int_{\xi}^1 \frac{dx}{\xi} \Im C\left(\frac{x}{\xi}, \frac{t}{Q^2}, \alpha_s(Q^2); \partial_{\xi}^{(n)}\right) \int_x^{\infty} \frac{dy}{y} K\left(y, \frac{\xi}{x}; Q^2, \mu^2\right) \\ &\quad \times \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta\left(\frac{x}{y} - \beta - \alpha\xi\right) F(\beta, \alpha, t, \mu^2)\end{aligned}$$

$$\begin{aligned} \Im \mathcal{H}(\xi, t, Q^2) = & 2 \int_{\xi}^1 \frac{dx}{\xi} \Im C\left(\frac{x}{\xi}, \frac{t}{Q^2}, \alpha_s(Q^2); \partial_{\xi}^{(n)}\right) \int_x^{\infty} \frac{dy}{y} K\left(y, \frac{\xi}{x}; Q^2, \mu^2\right) \\ & \times \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta\left(\frac{x}{y} - \beta - \alpha\xi\right) F(\beta, \alpha, t, \mu^2) \end{aligned}$$



$$\begin{aligned} \Im \mathcal{H}(\xi, t, Q^2) = & 2 \int_{\xi}^1 \frac{dx}{\xi} \Im C\left(\frac{x}{\xi}, \frac{t}{Q^2}, \alpha_s(Q^2); \partial_{\xi}^{(n)}\right) \int_x^{\infty} \frac{dy}{y} K(y, \frac{\xi}{x}; Q^2, \mu^2) \\ & \times \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta\left(\frac{x}{y} - \beta - \alpha\xi\right) F(\beta, \alpha, t, \mu^2) \end{aligned}$$



Solving Problem 2

For experts:
Characterising the null space
of the coefficient function

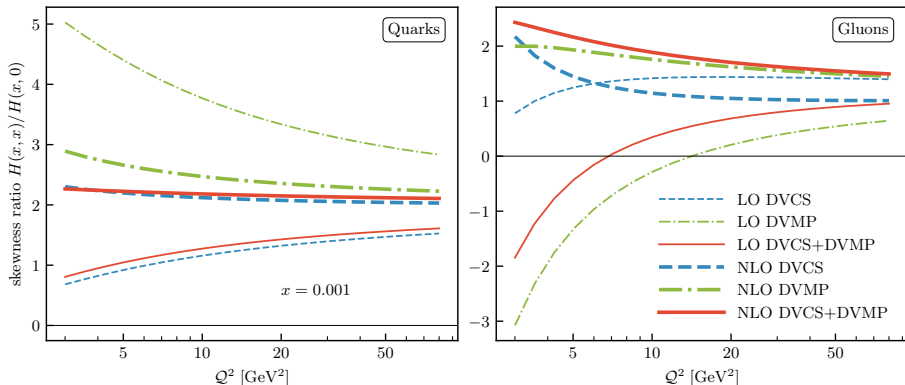
H. Dutrieux *et al.*, Phys.Rev.D 103 (2021) 11, 114019
E. Moffat *et al.*, Phys.Rev.D 108 (2023) 3, 036027
H. Dutrieux *et al.*, Eur.Phys.J.C 82 (2022) 3, 252
M. Riberdy *et al.*, Eur.Phys.J.C 84 (2024) 2, 201
M. Cuic *et al.*, in preparation

Solving Problem 2

For experts:
Characterising the null space
of the coefficient function

H. Dutrieux *et al.*, Phys.Rev.D 103 (2021) 11, 114019
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M. Cuic *et al.*, in preparation

This problem is open but mitigations are possible
See M. Cuic talk.



M. Cuic *et al.*, JHEP 12 (2023) 192

- Extraction of quark and gluon GPDs from NLO-DVCS and NLO-DVMP
- Inverse problem regularised through moments parametrisation and truncation

Reasons to be pessimistic

- Small experimental uncertainties on observables inflates when extracting DVCS amplitudes.
- The deconvolution problem is here to stay inflating even more the uncertainties on GPDs.
- Pure data driven extraction of GPDs is out of reach in a foreseeable future.

Reasons to be optimistic

- There are many theoretical constraints that we should exploit.
- The deconvolution problem can be mitigated.
- Theory precision is improving.
- Multichannel analyses have started and seem to confirm the universality of GPDs.

Take away message

- Significant progress has been done both experimentally and theoretically.
- Nevertheless, the problem of extracting GPDs is challenging, with propagation and inflation of uncertainties being the key challenge.

Thank you for your attention and enjoy your half glass

OPTIMIST



**MY GLASS
IS HALF
FULL**

PESSIMIST



**MY GLASS
IS HALF
EMPTY**