

NUCLEON STRUCTURE FROM BASIS LIGHT-FRONT QUANTIZATION: RECENT DEVELOPMENTS AND PERSPECTIVES



Chandan Mondal

Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou, China



BLFQ Collaboration

Siqi Xu, ..., Xingbo Zhao, Yang Li and James P. Vary

Seville, Spain, September 10, 2026

Overview



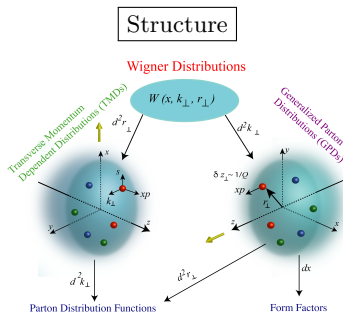
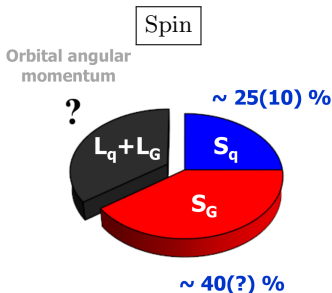
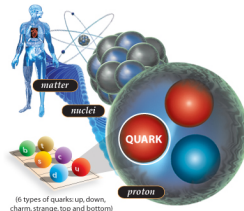
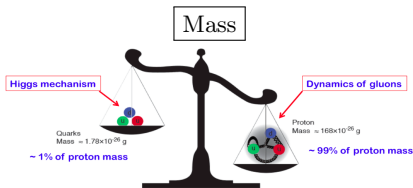
Introduction

Basis Light-Front Quantization (BLFQ)

Description of Nucleon State

Conclusions

Fundamental Properties: Mass, Spin and Structure

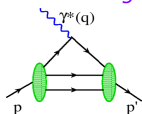


Ideal facilities : EIC & EicC

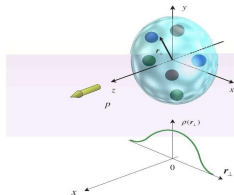
Form Factors vs PDFs vs GPDs



Elastic Scattering

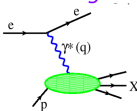


Established extended nature of nucleon

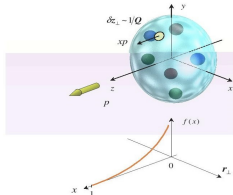


charge and
magnetization
distribution

Deep Inelastic Scattering

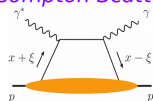


discovered the existence
(quarks) inside the nucleon

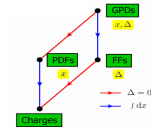
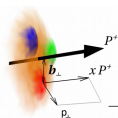
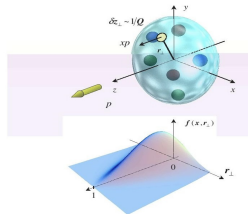


longitudinal
momentum
distribution

Deeply virtual Compton Scattering



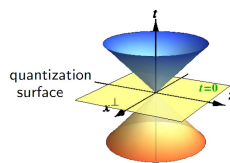
- provides 3D spatial structure of the nucleon



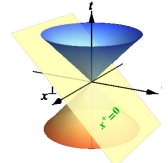
Light-Front Hamiltonian Framework

- Quantization on the light-front : $x^+ = x^0 + x^3$
- LF momenta : $P^+ = P^0 + P^3$; $\mathbf{P}_\perp$
- Choose transverse rest frame : $\mathbf{P}_\perp = 0$
- LF Hamiltonian : $H_{\text{LF}} \equiv P^+ P^-$
- Eigenvalue problem : $H_{\text{LF}} |P, \Lambda\rangle = M^2 |P, \Lambda\rangle$

instant form
time variable $t = x^0$



front form
 $x^+ \triangleq x^0 + x^3$



Simultaneous access to mass spectra and wave functions

$$H_{\text{LF}} = \sum_i \frac{m_i^2 + \mathbf{p}_{i\perp}^2}{x_i} + H_{\text{int}}$$

$x_i = p_i^+ / P^+ \rightarrow$ LF momentum fraction carried by i th parton

Applicable to hadrons as well as to nuclei

- Expansion of an eigenstate of the system in Fock space basis

$$|P, \Lambda\rangle = \sum_N \sum_{\lambda_1, \dots, \lambda_N} \int \frac{\prod_{i=1}^N dx_i d^2 \mathbf{p}_{i\perp}}{[2(2\pi)^N]^2 \sqrt{x_1 \dots x_N}} \delta\left(1 - \sum_{i=1}^N x_i\right) \delta^2\left(\sum_{i=1}^N \mathbf{p}_{i\perp}\right) \psi_{\{\lambda_i\}_N}^\Lambda(\{p_i\}_N) |\{\lambda_i, p_i\}_N\rangle$$

LF helicity
amplitude

Normalized N -
parton state

Building the LF Hamiltonian matrix using BLFQ

Step-1: Expand the system's state in the Fock space

Fock state expansion of the meson, baryon and deuteron bound state

$$|\Psi\rangle_M = \psi_{\{q\bar{q}\}} |q\bar{q}\rangle + \psi_{\{q\bar{q}g\}} |q\bar{q}g\rangle + \psi_{\{q\bar{q}gg\}} |q\bar{q}gg\rangle + \psi_{\{q\bar{q}q\bar{q}\}} |q\bar{q}q\bar{q}\rangle + \dots$$

$$|\Psi\rangle_B = \psi_{\{qqq\}} |qqq\rangle + \psi_{\{qqqg\}} |qqqg\rangle + \psi_{\{qqqq\bar{q}\}} |qqqq\bar{q}\rangle + \dots$$

$$|\Psi\rangle_D = \psi_{\{qqq\ qqq\}} |qqq\ qqq\rangle + \psi_{\{qqq\ qqq\ g\}} |qqq\ qqq\ g\rangle + \dots$$

Step-2: Decide how to describe each particle's motion and quantum numbers.

Here comes the **basis** (Heart of the BLFQ)

Single-particle basis state $\rightarrow$ to identify a particle

$$\psi_{rs}(x, \vec{\kappa}^\perp) = \sum_{n,m,l} \langle n, m, l, r, s | \psi \rangle \times$$

Eigenvector of
the Hamiltonian

Single particle motion
in **transverse direction**

$$\phi_{nm}(\vec{\kappa}^\perp) \chi_l(x)$$

Single particle
motion
in **longitudinal
direction**

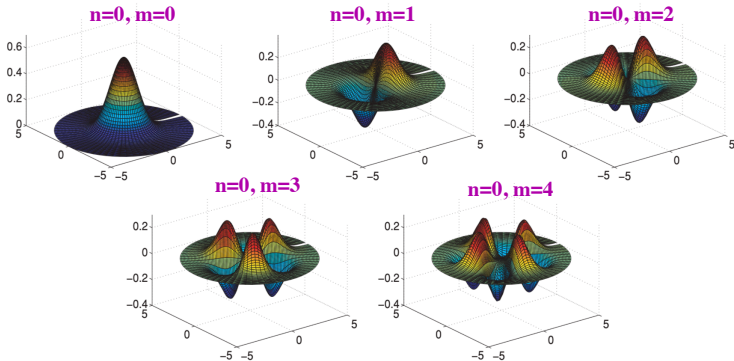
Build **many-body basis states** for each Fock sector

Step-3: Compute the Hamiltonian matrix $\langle \text{basis}_i | H | \text{basis}_j \rangle$

Hamiltonian = K.E. terms + QCD interactions

Formation of **Basis**: Transverse direction

2D Harmonic oscillator: $\phi_{n,m}(\mathbf{k}_\perp; b) = \frac{\sqrt{2}}{b(2\pi)^{3/2}} \sqrt{\frac{n!}{(n+|m|)!}} e^{-k_\perp^2/2b^2} \left(\frac{|k_\perp|}{b}\right)^{|m|} L_n^{|m|}\left(\frac{k_\perp^2}{b^2}\right) e^{im\theta}$



To limit the basis space

$$\sum_i (2n_i + |m_i| + 1) \leq N_{\max}$$



Basis parameter

— J.P. Vary et al., PRC 81, 035205 (2010)

$$\Lambda_{\text{IR}} \approx b/\sqrt{N_{\max}}$$

$$\Lambda_{\text{UV}} \approx b\sqrt{N_{\max}}$$

Formation of **Basis**: Longitudinal direction

Plain wave basis: $\psi_k(x^-) = \frac{1}{\sqrt{2L}} e^{i\frac{\pi}{L} k x^-}$ for $-L \leq x^- \leq L$

where $k = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$

Longitudinal momentum fraction carried by parton

$$x_i = \frac{p_i^+}{P^+} = \frac{k_i}{K}$$

$$K = \sum_i k_i$$



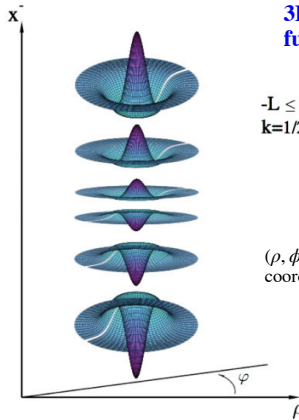
Basis parameter

**3D basis
function**

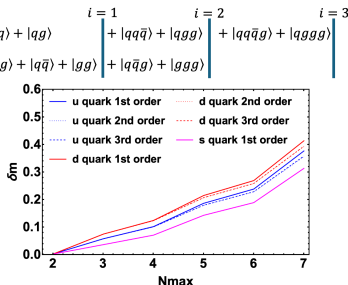
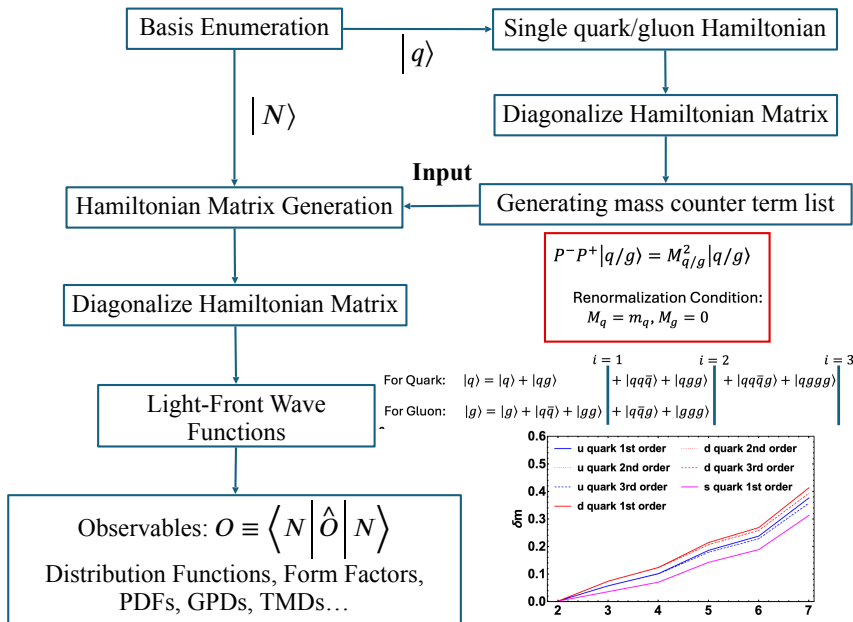
$$-L \leq x^- \leq +L$$

$$k=1/2, n=1, m=0$$

(ρ, ϕ) are the cylindrical
coordinates in (x^1, x^2)

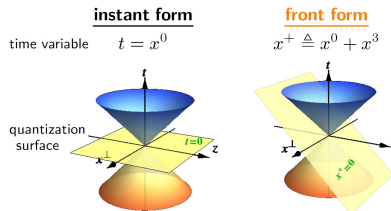


BLFQ Algorithm Flowchart



Summary: BLFQ Framework

A computational framework for solving relativistic many-body bound state problems in quantum field theories



$$P^- P^+ |\Psi\rangle = M^2 |\Psi\rangle$$

- ▶ $P^\pm \equiv P^0 \pm P^3$: light-front momentum (Hamiltonian)
- ▶ First-principle / effective Hamiltonian as input
- ▶ Access to mass (M) and LFWFs

- ▶ Evaluate observables

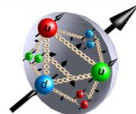
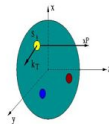
$$O \sim \langle \Psi | \hat{O} | \Psi \rangle$$

GOAL

Light-front wave-functions

3D imaging

Proton spin



FFs

GPDs

TMDs...

¹Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).

- Fock expansion of baryonic bound states:

$$|\text{Meson}\rangle = \psi_{(q\bar{q})}|q\bar{q}\rangle + \psi_{(q\bar{q}+1g)}|q\bar{q}g\rangle + \psi_{(q\bar{q}+q\bar{q})}|q\bar{q}q\bar{q}\rangle + \dots,$$

$$|\text{Proton}\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle + \psi_{(3q+q\bar{q})}|qqqq\bar{q}\rangle + \dots,$$



Solution proposed by BLFQ

Discrete basis and their direct product	Truncation
2D HO $\phi_{nm}(p^\perp)$ in the transverse plane	$\sum_i (2n_i + m_i + 1) \leq N_{\max}$
Plane-wave in the longitudinal direction	$\sum_i k_i = K, \quad x_i = \frac{k_i}{K}$
Light-front helicity state for spin d.o.f.	$\sum_i (m_i + \lambda_i) = M_J$
$\alpha_i = (k_i, n_i, m_i, \lambda_i)$ $ \alpha\rangle = \otimes_i \alpha_i\rangle$	Fock sector truncation

Large $N_{\max}$ and $K \rightarrow$ High UV cutoff & low IR cutoff

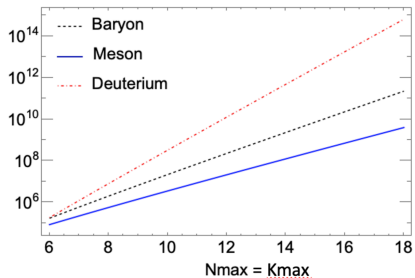
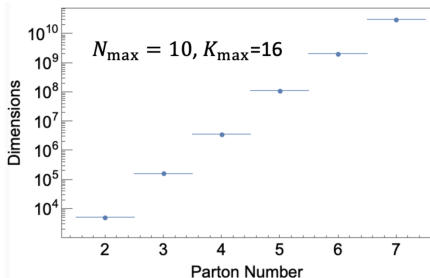
- Exact factorization between center-of-mass motion and intrinsic motion

¹Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).

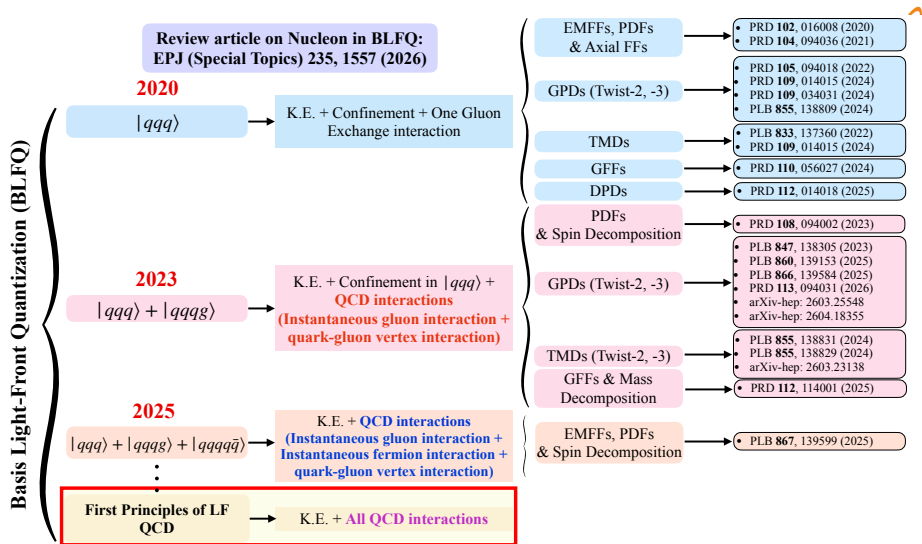
Dimension of Basis States

- Dimensions increase with increase in number of Fock sectors

➡ Motivation for Quantum Computing



BLFQ Road-map toward First Principles



Proton with One Dynamical Gluon

$$\{|\psi_{uud}|^2 = 44\% \quad ; \quad |\psi_{uudg}|^2 = 56\%\}$$

$$P^+ P^- |\Psi\rangle = M^2 |\Psi\rangle$$

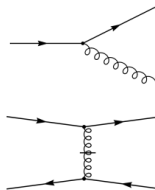
$$|\text{proton}\rangle = \psi_{uud} |uud\rangle + \psi_{uudg} |uudg\rangle$$



QCD Interaction:

$$P^- = P_{\text{QCD}}^- + P_C^-$$

$$P_{\text{QCD}}^- = \int dx^- d^2x^\perp \left\{ \frac{1}{2} \bar{\psi} \gamma^+ \frac{m_0^2 + (i\partial^\perp)^2}{i\partial^+} \psi - \frac{1}{2} A_a^i [m_g^2 + (i\partial^\perp)^2] A_a^i + g_s \bar{\psi} \gamma_\mu T^a A_a^\mu \psi + \frac{1}{2} g_s^2 \bar{\psi} \gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ T^a \psi \right\},$$



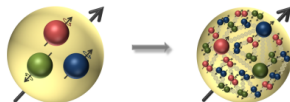
Confinement only in leading Fock:

$$P_C^- P^+ = \frac{\kappa^4}{2} \sum_{i \neq j} \left\{ \{ \vec{r}_{ij}^2 - \frac{\partial_{x_i}(x_i x_j \partial_{x_j})}{(m_i + m_j)^2} \} \right\}$$

Parameters:

Truncation: Nmax=9, K=16.5

HO parameters: b=0.7GeV, b_{inst}=3GeV

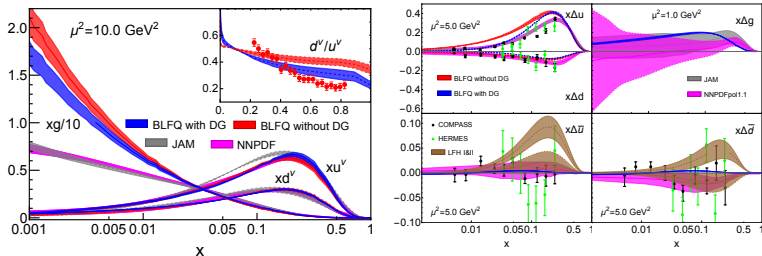


m_u	m_d	m_g	κ	m_f	g
0.31GeV	0.25GeV	0.50GeV	0.54GeV	1.80GeV	2.40

¹ BLFQ: S. Xu *et. al.*, Phys.Rev.D 108 (2023) 094002.

² Brodsky, Teramond, Dosch and Erlich, Phys. Rep. 584, 1 (2015).

³ Li, Maris, Zhao and Vary, Phys. Lett. B (2016); M. Burkardt, Phys. Rev. D 58, 096015 (1998).



Diagonalizing $H_{\text{eff}} \Rightarrow$ LF wavefunction $\Rightarrow$ Initial PDFs $\Rightarrow$ Scale evolution

$$\text{Model scale } \mu_0^2 = 0.24 \pm 0.01 \text{ GeV}^2$$

- ▶ Quark momentum : $\langle x \rangle_u = 0.261 \pm 0.005$, $\langle x \rangle_d = 0.109 \pm 0.005$ at 10 GeV^2 .
- ▶ Quark spin: $\frac{1}{2} \Sigma_u = 0.438 \pm 0.004$, $\frac{1}{2} \Delta \Sigma_d = -0.080 \pm 0.002$.
- ▶ Gluon spin: $\Delta G = 0.131 \pm 0.003$, PHENIX: $\Delta G^{[0.02, 0.3]} = 0.2 \pm 0.1$.
- ▶ Sea quarks: solely generated from the QCD evolution.

¹LFH: PRL 124 (2020), 082003; PHENIX: PRL 103 (2009) 012003.

BLFQ Predictions for Spin Decomposition

Fock expansion:

$$| \text{Proton} \rangle = a | uud \rangle + b | uudg \rangle + \dots$$



Quark and gluon helicities :

$$\Delta \Sigma_q = \int dx \Delta q(x)$$

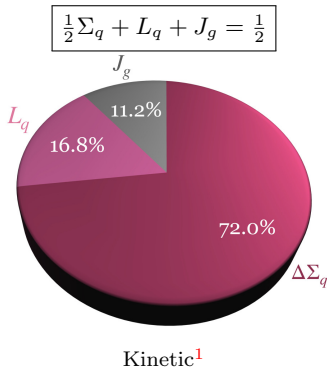
$$\Delta \Sigma_g = \int dx \Delta G(x)$$

Total AM² :

$$J_i = \frac{1}{2} \int dx x [H_i(x, 0, 0) + E_i(x, 0, 0)]$$

Kinetic OAM² :

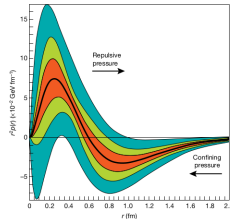
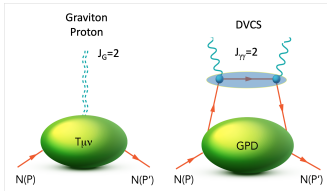
$$L_q = \frac{1}{2} \int dx [x \{H_q(x, 0, 0) + E_q(x, 0, 0)\} - \tilde{H}_q(x, 0, 0)]$$



¹BLFQ: S. Xu *et. al.*, Phys.Rev.D 108 (2023) 094002.

²X. Ji, Phys.Rev.Lett. 78, 610 (1997).

Proton Gravitational Form Factors



- Parametrization of matrix element in terms of GFFs

$$\begin{aligned} \langle P' | T_i^{\mu\nu}(0) | P \rangle = & \bar{U}' \left[-B_i(q^2) \frac{\bar{P}^\mu \bar{P}^\nu}{M} + (A_i(q^2) + B_i(q^2)) \frac{1}{2} (\gamma^\mu \bar{P}^\nu + \gamma^\nu \bar{P}^\mu) \right. \\ & \left. + C_i(q^2) \frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2) M g^{\mu\nu} \right] U \end{aligned}$$

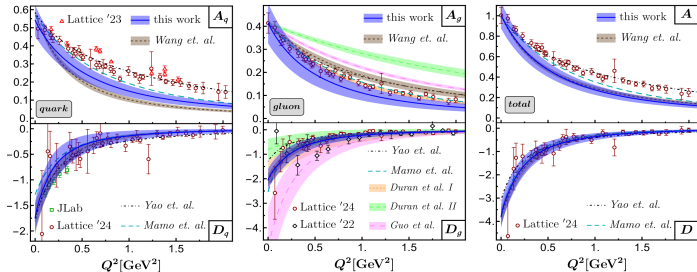
- Momentum sum rule : $\sum_i A^i(0) = 1$
- Gravitomagnetic moment sum rule : $\sum_i B^i(0) = 0$
- Spin sum rule: $J^i = \frac{1}{2} [A^i(0) + B^i(0)]$
- $4C(q^2) = D(q^2)$ provides shear forces and the pressure distributions

¹Burkert *et. al.*: Rev. Mod. Phys. 95, 041002 (2023); Ji, Phys. Rev. Lett. 78, 610 (1997)

BLFQ Results for Proton GFFs

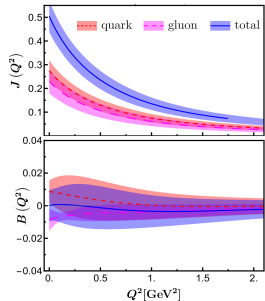


$$|\text{Nucleon}\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle$$



- QCD EMT : $T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}$, where
 $T_q^{\mu\nu} = i\bar{\psi}\gamma^{\{\mu}\mathcal{D}^{\nu\}}\psi$ and
 $T_g^{\mu\nu} = -F^{\mu\lambda}F_{\lambda}^{\nu} + \frac{1}{4}g^{\mu\nu}(F_{\lambda\sigma}F_{\sigma\lambda})^2$
- $A(Q^2)$ and $B(Q^2)$: T^{++} component
- $D(Q^2) = 4C(Q^2)$: T^{ij} components

$$\sum_i A^i(0) = 1 \text{ and } \sum_i B^i(0) = 0$$



¹BLFQ: S. Nair *et. al.*, Phys.Rev.D 112, 114001 (2025)

Efremov-Teryaev-Qiu-Sterman (ETQS) Function

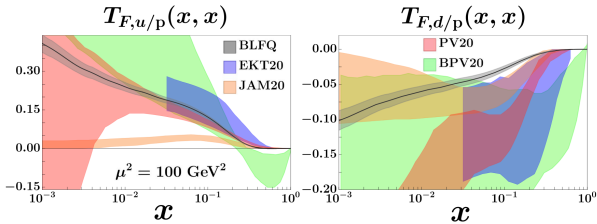
$$\hat{M}_{F,\alpha\beta}^{\mu}(x_1, x_2) = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{i\frac{y_1^-}{2}(x_1+x_2)P^+} e^{iy_2^-(x_2-x_1)P^+} \\ \times \langle P, S | \bar{\psi}_{\beta}(-\frac{y_1^-}{2}) g F^{+\mu}(y_2^-) \psi_{\alpha}(\frac{y_1^-}{2}) | P, S \rangle,$$

$$\hat{M}_F^{\mu}(x_1, x_2) = \frac{M}{2} \left[\tilde{T}_F^{\nu\mu} S_{\perp\nu} \gamma^- + \tilde{T}_F^{\mu} i S_{\perp}^{\mu} \gamma_5 \gamma^- \right. \\ \left. + \tilde{T}_F^{(\sigma)} i \Lambda \gamma_5 \gamma_{\perp}^{\mu} \gamma^- + \tilde{T}_F^{(\sigma)} i \gamma_{\perp}^{\mu} \gamma^- \right]$$

Zhu et al., (BLFQ Collaboration)
arXiv-hep: 2603.23138

ETQS function after evolutions:

- Qualitative agreement with the extractions.

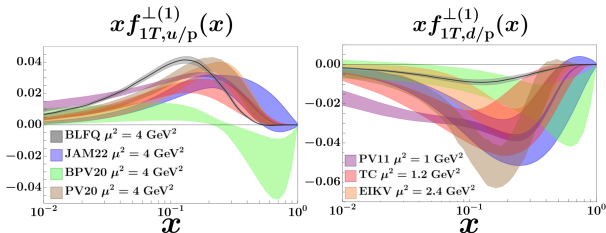


1st moment of the ETQS function:

$$\pi T_F(x, x) = f_{1T}^{\perp(1)}(x)$$

- 1st moment provides
Sivers functions

— NPB **667**, 201 (2003)
— PRD **59**, 014004 (1999)



Hamiltonian with sea quarks and upto three gluons

Fock expansion:

$$|N\rangle \rightarrow |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle \\ + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$$

QCD light-front Hamiltonian from QCD Lagrangian:

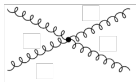
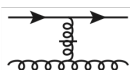
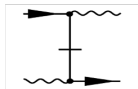
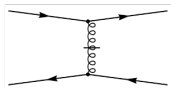
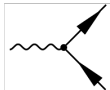
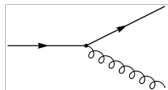
[S. Brodsky, H-C Pauli, S. Pinsky, '97]

$$\mathcal{L}_{QCD} = \bar{\psi}(iD - m)\psi - \frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu}$$



$$P_{QCD} = H_K + H_I$$

$$A^+ = 0$$



$$H_K = \frac{1}{2} \int d^3x \bar{\psi} \gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+} \psi - \frac{1}{2} \int d^3x A_a^i (i\partial^\perp)^2 A_a^i$$

$$H_I = +g \int d^3x \bar{\psi} \gamma_\mu A^\mu \psi \\ + \frac{1}{2} g^2 \int d^3x \bar{\psi} \gamma_\mu A^\mu \frac{\gamma^+}{i\partial^+} \gamma_\nu A^\nu \psi \\ - i g^2 \int d^3x f^{abc} \bar{\psi} \gamma^+ T^c \psi \frac{1}{(i\partial^+)^2} (i\partial^+ A_a^\mu A_{\mu b}) \\ + \frac{1}{2} g^2 \int d^3x \bar{\psi} \gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ T^a \psi \\ + i g \int d^3x f^{abc} i\partial^\mu A^{\nu a} A_\mu^b A_\nu^c \\ - \frac{1}{2} g^2 \int d^3x f^{abc} f^{ade} i\partial^+ A_b^\mu A_{\mu c} \frac{1}{(i\partial^+)^2} (i\partial^+ A_d^\nu A_{\nu e}) \\ + \frac{1}{4} g^2 \int d^3x f^{abc} f^{ade} A_b^\mu A_c^\nu A_{\mu d} A_{\nu e}.$$

ψ : quark field operator

A_μ^a : gluon field operator

7 terms in H_I

Dimensions of the Basis States



$$|N\rangle \rightarrow |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle \\ + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$$

$$N_{max} = 7, K_{max} = 16$$

	$ qqq\rangle$	$ qqqg\rangle$	$ qqq\ q\bar{q}\rangle$	$ qqq\ gg\rangle$	$ qqq\ ggg\rangle$	$ qqq\ q\bar{q}\ g\rangle$
Dimension	35,088	592,960	3,901,500	5,169,360	19,603,584	7,128,576
Color Config.	1	2	3	6	22	8

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\ u\bar{u}\rangle + |qqq\ d\bar{d}\rangle + |qqq\ s\bar{s}\rangle$$

Basis Dimension= 12,332,548

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\ u\bar{u}\rangle + |qqq\ d\bar{d}\rangle + |qqq\ s\bar{s}\rangle + |qqq\ gg\rangle$$

Basis Dimension= 17,501,908

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\ q\bar{q}\rangle + |qqq\ gg\rangle + |qqq\ ggg\rangle$$

Basis Dimension= 37,105,492

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\ q\bar{q}\rangle + |qqq\ gg\rangle + |qqq\ ggg\rangle + |qqq\ q\bar{q}\ g\rangle$$

Basis Dimension= 58,491,220

Input Parameters in Hamiltonian



- Truncation Parameters: up to 6-parton Fock sectors with $N_{\max} = 7$ & $K = 10$

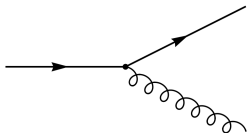
$$|N\rangle \rightarrow |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle \\ + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$$

- Input Parameters:

m_u	m_d	m_s	m_f	g	b_{inst}	b
0.45 GeV	0.449 GeV	0.6 GeV	2.0 GeV	2.6	3.0 GeV	0.6 GeV

$$M_p = 0.93 \text{ GeV}$$

- determined by fitting electromagnetic Form Factors and proton mass
- separate quark mass in the quark-gluon vertex m_f is needed



$$\propto gb + \dots \quad \text{Spin no flip}$$

$$\propto gm_f + \dots \quad \text{Spin flip}$$

Fock Sector Decomposition



$$|N\rangle \rightarrow |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle \\ + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$$

Valence Fock sector

$$|qqq\rangle \sim 45.63\%$$

Sea quark & gluon
Fock sector

$$|qqq u\bar{u}g\rangle \sim 0.11\%$$

$$|qqq d\bar{d}g\rangle \sim 0.11\%$$

$$|qqq s\bar{s}g\rangle \sim 0.08\%$$

Sea quark Fock sector

$$|qqq u\bar{u}\rangle \sim 0.19\%$$

$$|qqq d\bar{d}\rangle \sim 0.20\%$$

$$|qqq s\bar{s}\rangle \sim 0.09\%$$

Dynamical gluon Fock
sector

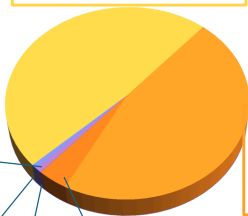
$$|qqqg\rangle \sim 44.44\%$$

Dynamical 2-gluon Fock
sector

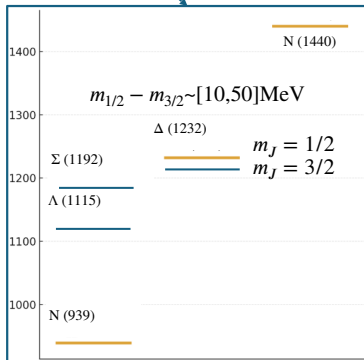
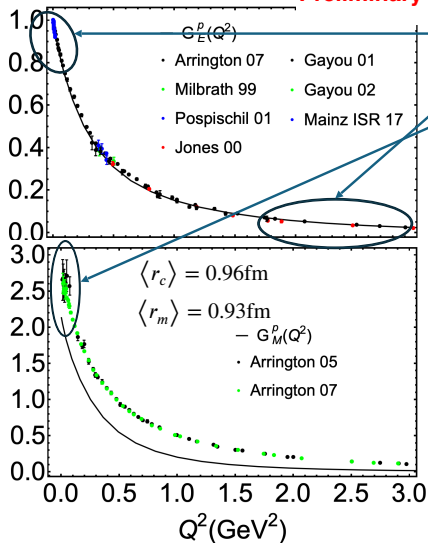
$$|qqq gg\rangle \sim 7.95\%$$

Dynamical 3-gluon Fock
sector

$$|qqq ggg\rangle \sim 1.22\%$$



Preliminary

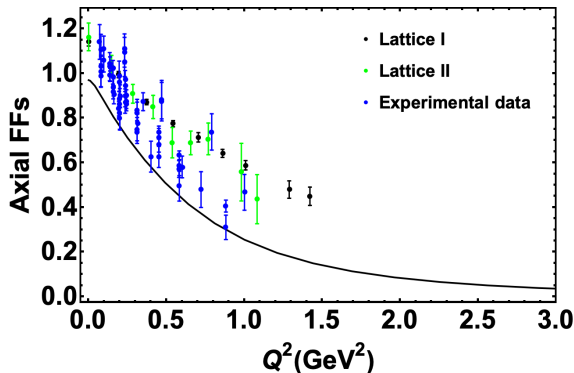


Axial Form Factor

$$\langle N(p') | A_\mu^a | N(p) \rangle = \bar{u}(p') \left[\gamma_\mu G_A(t) + \frac{(p' - p)_\mu}{2m} G_P(t) \right] \gamma_5 \frac{\tau^a}{2} u(p)$$

$$A_\mu^a = \bar{q} \gamma_\mu \gamma_5 T^a q$$

$$G_A(Q^2) = G_u(Q^2) - G_d(Q^2)$$



$$\langle r_A \rangle = 0.59 \text{ fm}$$

$$\Delta \Sigma_u = 0.78 \text{ (0.82)}$$

$$\Delta \Sigma_d = -0.19 \text{ (-0.45)}$$

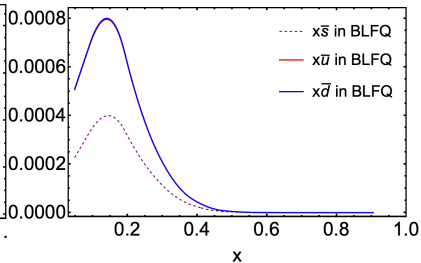
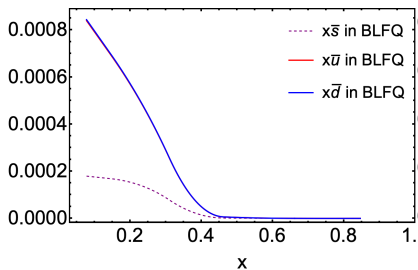
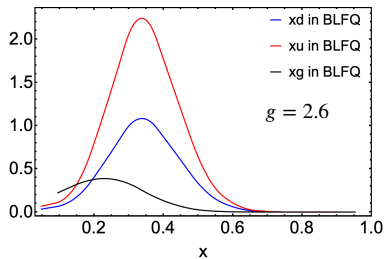
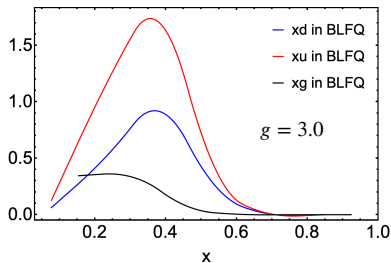
$$\Delta \Sigma_{u-d} = 0.97 \text{ (1.27)}$$

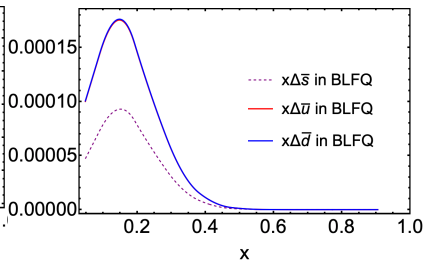
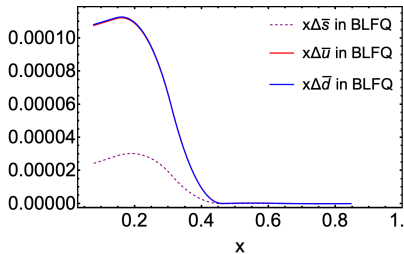
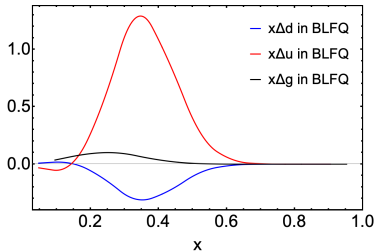
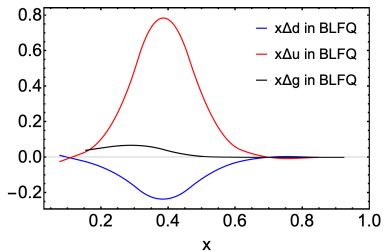
$$\Delta \Sigma_{u+d} = 0.59$$

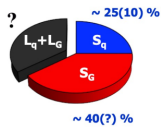
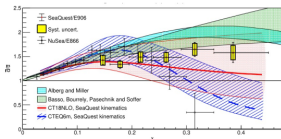
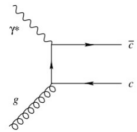
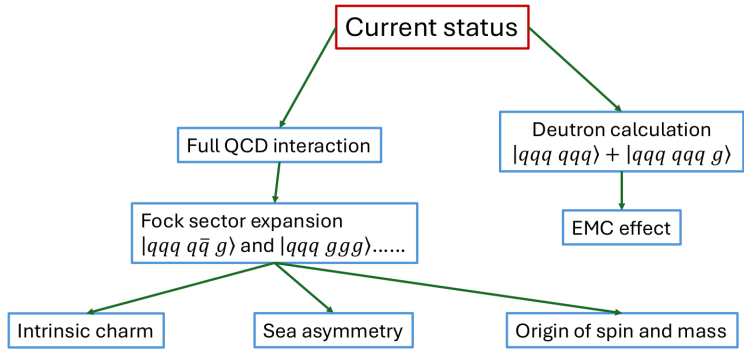
$$\Delta G = 0.13$$

- Black line: valence quark
- Qualitatively agree with the experimental data

Unpolarized PDF









- ▶ Basis Light-front Quantization : A non-perturbative approach based on light-front QCD Hamiltonian
- ▶ **LF Hamiltonian** $\Rightarrow$ **Wavefunctions** $\Rightarrow$ **Observables**.
- ▶ Calculate proton structure from QCD first principles.
- ▶ Spin structure qualitatively agree with expectation.
- ▶ Provides good description of data/global fits for various observables.

Thank You

Many-body wavefunction is built from single-particle basis states for q and g .

- A big plus:**
- Treats all particles in the Fock space equivalently
 - Facilitates handling symmetry operators for identical particles

Concerns: The resulting many-body wf includes:

- Entangled
- Intrinsic motion of quarks relative to each other
 - Overall transverse center-of-mass motion of the whole system

Solution: Add a constraint term into the Hamiltonian

$$\boxed{H' = \lambda_L (H_{\text{c.m.}} - 2b^2 I)} \quad H_{\text{c.m.}} = \left(\sum_i \vec{p}_{i\perp} \right)^2 + b^4 \left(\sum_i x_i \vec{r}_{i\perp} \right)^2$$

$H_{\text{c.m.}}$: the transverse c.m. Hamiltonian (harmonic oscillator)

$2b^2$: zero-point energy

λ_L : a large Lagrange multiplier

The modified Hamiltonian:

$$H'_{\text{eff}} = H_{\text{eff}} - \left(\sum_i \vec{p}_{i\perp} \right)^2 + \lambda_L (H_{\text{c.m.}} - 2b^2 I).$$

The resulting LFWFs are boost invariant



$$|N\rangle \rightarrow |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle \\ + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$$

$|qqq\rangle \sim 1$ color singlet state

$|qqq\ g\rangle \sim 2$ color singlet state

2 octet $\otimes$ octet

$|qqq\ q\bar{q}\rangle \sim 3$ color singlet states

1 singlet $\otimes$ singlet

2 octet $\otimes$ octet

$|qqq\ gg\rangle \sim 6$ color singlet states

1 singlet $\otimes$ singlet

4 octet $\otimes$ octet

1 decuplet $\otimes$ octet $\otimes$ octet

$|qqq\ q\bar{q}\ g\rangle \sim 8$ color singlet states

3 singlet $\otimes$ octet $\otimes$ octet

4 octet $\otimes$ octet $\otimes$ octet

1 decuplet $\otimes$ octet $\otimes$ octet

$|qqq\ ggg\rangle \sim 22$ color singlet states

2 singlet $\otimes$ singlet

16 octet $\otimes$ octet

4 decuplet $\otimes$ octet $\otimes$ octet $\otimes$ octet

Quark TMDs

Leading Twist TMDs



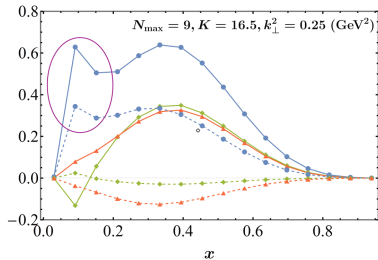
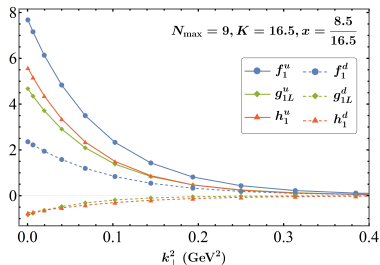
		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \odot$		$h_1^\perp = \uparrow - \downarrow$ Boer-Mulders
	L		$g_{1L} = \rightarrow - \leftarrow$ Helicity	$h_{1L}^\perp = \rightarrow - \leftarrow$
	T	$f_{1T}^\perp = \uparrow - \downarrow$ Sivers	$g_{1T}^\perp = \rightarrow - \leftarrow$	$h_1 = \uparrow - \downarrow$ Transversity $h_{1T}^\perp = \rightarrow - \leftarrow$

► Positivity bounds

$$f_1^q(x, \mathbf{k}_\perp^2) > 0, \quad f_1^q(x, \mathbf{k}_\perp^2) \geq |g_{1L}^q(x, \mathbf{k}_\perp^2)|,$$

$$f_1^q(x, \mathbf{k}_\perp^2) \geq \frac{|\mathbf{k}_\perp|}{M} |g_{1T}^q(x, \mathbf{k}_\perp^2)|,$$

$$f_1^q(x, \mathbf{k}_\perp^2) \geq \frac{|\mathbf{k}_\perp|^2}{2M^2} |h_1^{\perp q}(x, \mathbf{k}_\perp^2)|$$



¹ BLFQ: Hongyao Yu, *et. al.* coming soon...

² A. Accardi *et al.*, Eur.Phys.J.A 52 (2016) 9, 268.

Gaussian Ansatz Compatibility?

- To check compatibility of BLFQ results with the Gaussian ansatz :

$$f_1^i(x, k_\perp^2) \approx a \frac{\exp\left(-\frac{|k_\perp|^2}{r}\right)}{\pi r}$$

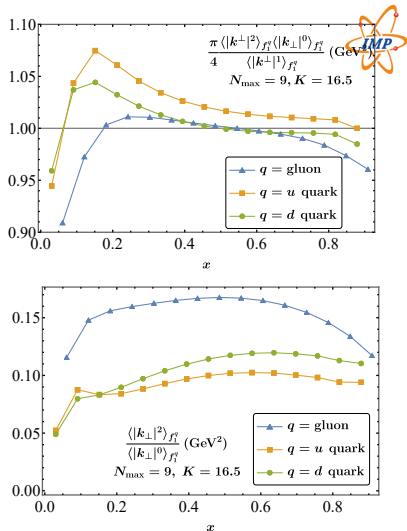
where $a = \langle |k_\perp|^0 \rangle_{f_1^i}$ and

$r = \langle |k_\perp|^2 \rangle_{f_1^i}$

- If the Gaussian ansatz holds :

$$\frac{\langle |k_\perp|^2 \rangle_{f_1^i} \times \langle |k_\perp|^0 \rangle_{f_1^i}}{(\langle |k_\perp|^1 \rangle_{f_1^i})^2} \times \frac{\pi}{4} = 1$$

BLFQ results do not support Gaussian ansatz



¹ Hongyao Yu, *et. al.* in preparation

² BLFQ: Hongyao Yu, *et. al.* Phys.Lett.B 855, 138831 (2024)

BLFQ Results for Spin Decomposition

Fock expansion:

$$| \text{Proton} \rangle = a | uud \rangle + b | uudg \rangle + \dots$$



Quark and gluon helicities :

$$\Delta \Sigma_q = \int dx \Delta q(x)$$

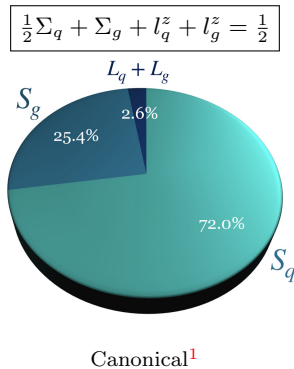
$$\Delta \Sigma_g = \int dx \Delta G(x)$$

Total AM :

$$J_i = \frac{1}{2} \int dx x [H_i(x, 0, 0) + E_i(x, 0, 0)]$$

Canonical OAM :

$$l_i^z = - \int dx d^2 \vec{p}_\perp \frac{\vec{p}_\perp^2}{M^2} F_{1,4}^i(x, 0, \vec{p}_\perp^2, 0, 0)$$



¹ BLFQ: S. Xu *et. al.*, Phys.Rev.D 108 (2023) 094002.

² Jaffe and Manohar, Nucl.Phys.B 337, 509–546 (1990).