

Baryon structure from lattice QCD

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Seville, Sept. 10th 2026

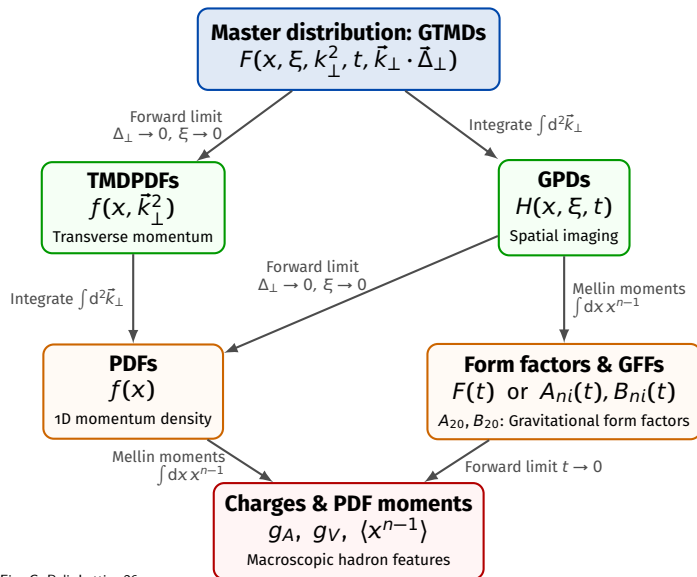


Fig: G. Bali, Lattice 26.

Aim for a comprehensive understanding of baryon structure.

Compute matrix elements on the lattice.

Local operators:

Direct determination of charges, form factors, generalised form factors (GFFs).

Equal-Euclidean time non-local operators:

Information on light-cone PDFs, GPDs, TMDs.

Obtained, e.g., via factorization through the determination of quasi-/pseudo distributions.

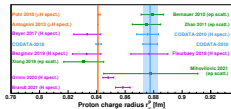
Flavour separation: individual quark and gluon contributions.

A diagram showing a particle (represented by a blue sphere) in a trap (represented by an orange oval). A vertical arrow labeled k_T points upwards from the particle. A horizontal arrow labeled xp points to the left. A dashed line labeled b_T connects the particle to the center of the trap. A red sphere is at the bottom of the trap, and a cyan sphere is to the right of the particle.

Complementary lattice calculations.

Axial form factor needed
to predict $\sigma_{\nu N}$, $\sigma_{\nu A}$.

**MAGIX, PRad-II,
ULQ2, MUSE, AMBER**



Proton charge/magnetic
radius puzzle.

Precision β decay experiments.
 $g_{S,T}$ needed to determine
size of BSM contributions.

Figure 1 is a log-log plot showing the evolution of the 50 kDa nucleosome cross-section (nm²) and the 50 kDa nucleosome density (nm⁻³) as a function of initial mass (kDa/μl). The x-axis represents initial mass (kDa/μl) on a log scale from 10⁻⁴ to 10⁻¹. The left y-axis represents the 50 kDa nucleosome cross-section (nm²) on a log scale from 10⁻⁴ to 10⁻¹. The right y-axis represents the 50 kDa nucleosome density (nm⁻³) on a log scale from 10⁻⁴ to 10⁻¹. The plot includes several curves representing different nucleosome states: COMPACT (blue), SUBCOMPACT (orange), COMPACT (green), SUBCOMPACT (red), COMPACT (purple), and SUBCOMPACT (brown). A shaded region labeled 'Nucleosome Fog' is shown at low initial mass and low density.

Direct dark matter detection:
 σ^q needed to predict the
 WIMP-nucleon cross-section.

Significant recent progress. Very active area.

Many results with all systematics under control and consistency across different determinations.

► Nucleon form factors

- Electromagnetic form factors of the proton and neutron: $G_{E,M}^{p,n}(Q^2)$.
- (Weak) axial form factor: $G_A(Q^2)$.
- Steps towards transition form factors involving resonances.

► Moments of (generalised) parton distribution functions

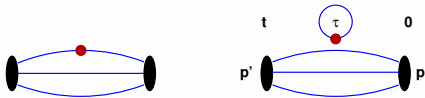
- First helicity and transversity moments and second unpolarised moment: well studied, including flavour separation.
- Push to higher moments: results up to 4th moment of PDFs with standard techniques and 5th moment with new techniques.
- Second moment (and higher) of GPDs.

Direct determination of (generalised) parton distribution functions and TMDs.

Lattice details: local operators

Steps in the analysis:

Calculate correlation functions



(Left) isovector ($u - d$) flavour combination.

(Right) isoscalar (u, d, s, c, g).

Fit

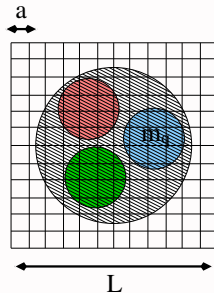
$$C_{3pt, \Gamma_i}^{\vec{p}', \vec{p}, J}(t, \tau) \xrightarrow{t, \tau \rightarrow \infty} A e^{-E_{\vec{p}'}(t-\tau)} e^{-E_{\vec{p}}\tau} \langle N(\vec{p}') | J | N(\vec{p}) \rangle^{\text{latt}}.$$

Matching from the lattice to a continuum scheme.

E.g., non-perturbative matching: lattice $\rightarrow$ RI'-(S)MOM scheme,
perturbative: RI'-(S)MOM $\rightarrow$ $\overline{\text{MS}}$.

Mixing between $q - g$ sectors, within the q sector (some actions).

Explore parameter space:



Finite volume, cut-off and quark mass dependence.

Cost of HMC: $\propto 1/(a^{\geq 6} m_\pi^{\approx 7.5})$

Perform extrapolation $a \rightarrow 0, L \rightarrow \infty,$
 $m_\pi \rightarrow m_\pi^{\text{phys}}.$

Challenges in the baryon sector

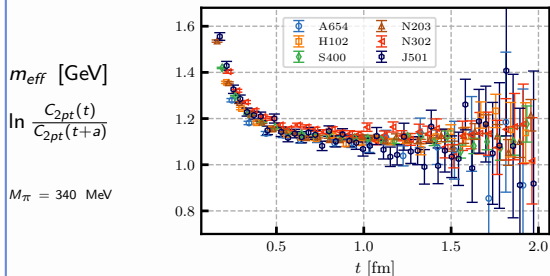
Meson sector: lattice provides (very) precise results for α_s , m_q , $q \in \{u/d, s, c, b\}$, form factors, see [FLAG 24, 2411.04268].

Statistical noise

Signal vs noise decays with $e^{-(E_N - 3M_\pi/2)t}$.

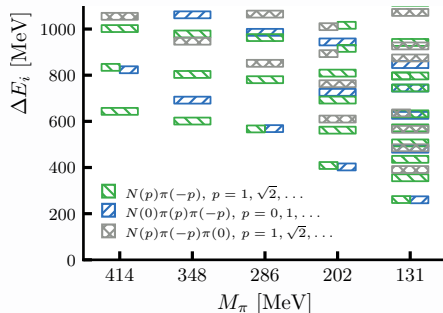


$$C_{2pt}(t) \xrightarrow{t \rightarrow \infty} Ae^{-E_N t}$$



Excited state contributions

Dense spectrum of $N\pi$, $N\pi\pi$, ... states as $M_\pi \rightarrow M_\pi^{phys}$.



Parametrizing the quark mass dependence: not clear how well ChPT describes the quark mass dependence in the range $M_\pi \sim (M_\pi^{phys} - 300 \text{ MeV})$. $\rightarrow$ Need $M_\pi \approx M_\pi^{phys}$.

Form factors of the nucleon: electromagnetic

$$\langle N(p') | J_\mu^{em} | N(p) \rangle = \bar{u}_N(p') \left[F_1^{p,n}(Q^2) \gamma_\mu + \frac{F_2^{p,n}(Q^2)}{2m_N} \sigma_{\mu\nu} Q^\nu \right] u_N(p)$$

$$N \in \{p, n\}, J_\mu^{em} = \sum_q e_q V_\mu^q, V_\mu = \bar{q} \gamma_\mu q \text{ with } q \in \{u, d, s, \dots\}.$$

Sachs FFs. $G_E^{p,n}(Q^2) = F_1^{p,n}(Q^2) - \frac{Q^2}{4m_N^2} F_2^{p,n}(Q^2), \quad G_M^{p,n}(Q^2) = F_1^{p,n}(Q^2) + F_2^{p,n}(Q^2)$

Shape at low Q^2

$$G(Q^2) = G(0) \left[1 - \frac{1}{6} \langle r^2 \rangle Q^2 + \dots \right],$$

$$G_E^{p,n}(0) = 1/0, \quad G_M^{p,n}(0) = \mu^{p,n}.$$

Difficult to achieve low $Q^2 \neq 0$, (conventionally) $\rightarrow$ large L , $p_j = (2\pi n/L)$.

Extrapolation to reach $G_M(0) \rightarrow \mu$ and $\langle r_M^2 \rangle$.

Some radii sensitive to M_π : $\langle r_{E,M}^2 \rangle^{p-n}$ diverge as $M_\pi \rightarrow 0$.

Shape at high Q^2

Difficult due to signal to noise problem, excited states and cut-off effects.

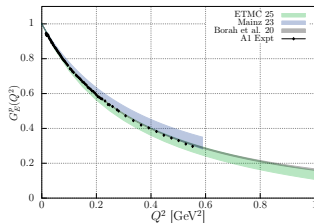
Talk: Syritsin, Thu. 12:30pm

(Most) lattice results cover the range $Q^2 \sim 0.04 - 1.00 \text{ GeV}^2$.

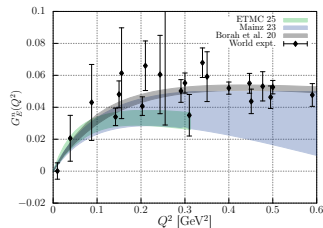
Electromagnetic form factors of the proton and neutron

[Mainz 23, 2309.06590] full systematics included, [ETMC 25, 2507.20910] fit with the highest AIC weight shown.

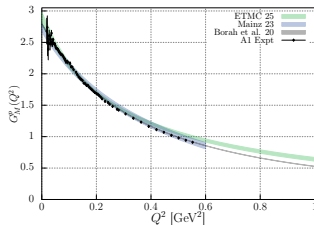
$$G_E^p(Q^2)$$



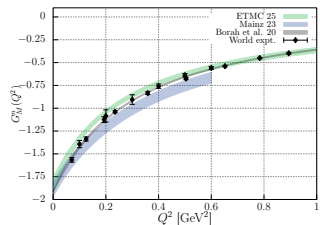
$$G_E^n(Q^2)$$



$$G_M^p(Q^2)$$



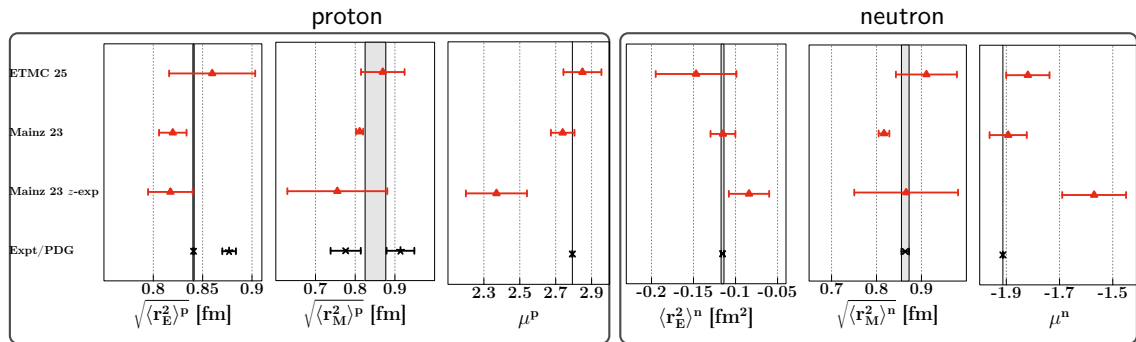
$$G_M^n(Q^2)$$



$G_{E,M}^{p,n}$ more difficult than $G_{E,M}^{p-n}$ due to noisy disconnected contributions.

[Borah et al., 2003.13640], [A1, 1307.6227], for world expt. see refs. in Borah et al..

Electromagnetic radii and magnetic moments



$\langle r_E^2 \rangle^p$: CODATA 2022 (μH), CODATA 2010 ($ep+H$ spec.).

$\langle r_M^2 \rangle^p$: (ep) [Lee et al., 1505.01489] A1 ($\square$), world data excluding A1 ($\diamond$), “puzzle”: e.g., [Lin et al., 2412.12885].

Mainz 23: fit $G_{E,M}$ together using $O(q^3)$ covariant baryon ChPT [Bauer et al., 1209.3872].

Common LECs $\rightarrow$ reduced uncertainty on μ , $\langle r_M^2 \rangle$ compared to independent z-expansion of $G_{E,M}$.

Axial form factors of the nucleon

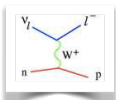
- Long baseline ν -oscillation experiments produce ν_μ and $\bar{\nu}_\mu$ with a range of energies.
- E_ν reconstructed from detected decay products of interactions in the near and far detector.

Range of interactions, nuclear effects, detector limitations, ...

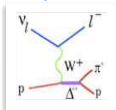
→ **Monte-Carlo simulation of the neutrino-nucleus (-nucleon) cross-section is needed.**

Neutrino-nucleon scattering cross-section

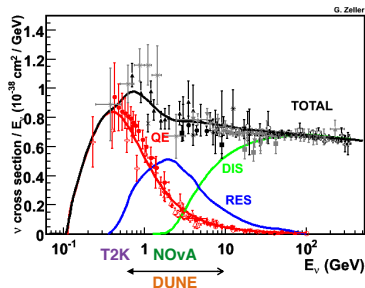
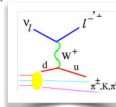
Quasi-elastic scattering (QE)



Resonance production (RES)



Deep Inelastic scattering (DIS)



J.A. Formaggio, G. Zeller, Reviews of Modern Physics, 84 (2012)

QE: $\sigma_{\nu N}$ depends on

$$\langle p(p') | \bar{u} \gamma_\mu (1 - \gamma_5) d | n(p) \rangle = \bar{u}_p (\text{EM FFs}) u_n + \bar{u}_p \left[\gamma_\mu \gamma_5 \mathbf{G}_A(Q^2) + \frac{q^\mu}{2m_N} \gamma_5 \tilde{\mathbf{G}}_P(Q^2) \right] u_n$$

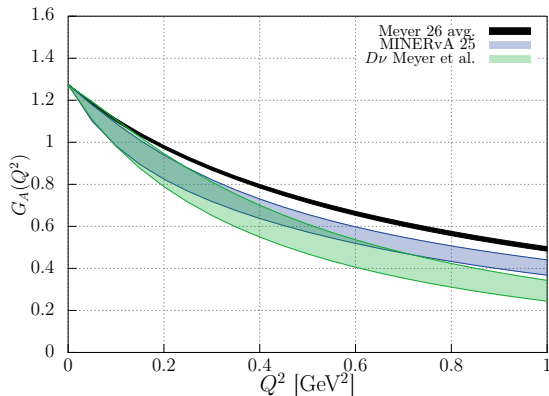
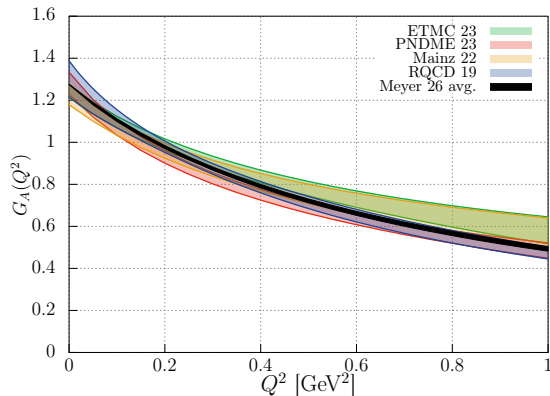
Contribution of $\tilde{\mathbf{G}}_P(Q^2)$ is suppressed.

Expt. information from scattering data:

$\bar{\nu}$ - p and ν - D , expts. with nuclear targets and $\bar{\nu}$ -H (within hydrocarbons), ...

RES: first steps towards $N \rightarrow$ resonance transition matrix elements on the lattice.

Isvector axial form factor



Lattice average: [Meyer 26, 2601.02676] ($G_A(0) = g_A$ fixed to experiment, also included [NME, 2103.05599]).

[ETMC 23, 2309.05774] [PNDME 23, 2305.11330] [Mainz 22, 2207.03440] [RQCD 19, 1911.13150]

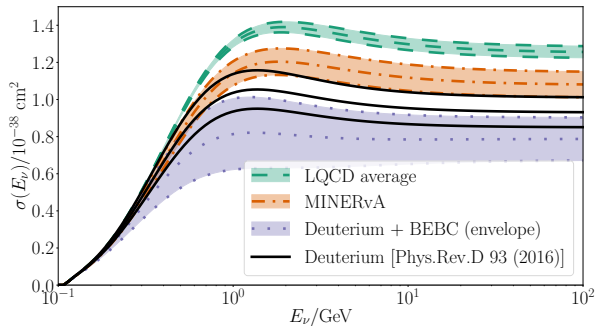
[MINERvA 25, 2512.14097] ($\bar{\nu}$ -H (in hydrocarbon molecules)) [$D\nu$ A Meyer et al, 1603.03048]. See also [O Tomalak et al, 2601.21155]

See also talk: **Alvarado, Tue 7:35pm.**

Isovector axial form factor: impact on the QE cross-section

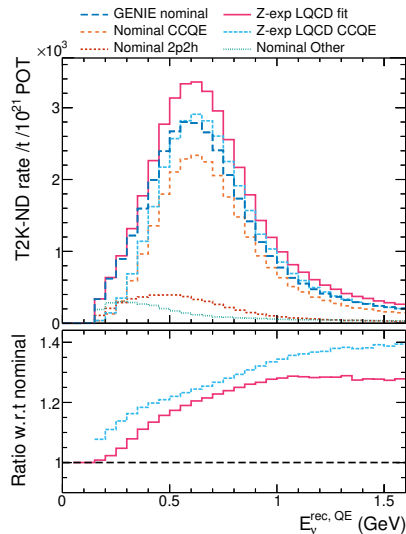
11

[MINERvA, 2512.14097]

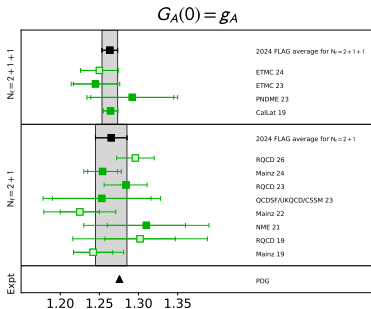


Slower fall-off of the lattice FF leads to a larger $\sigma_{\nu N}$.

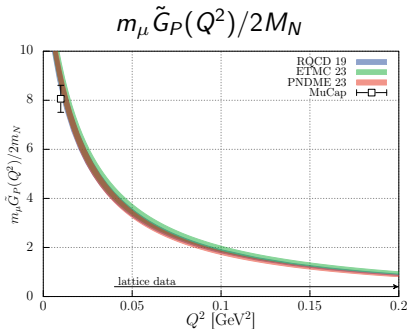
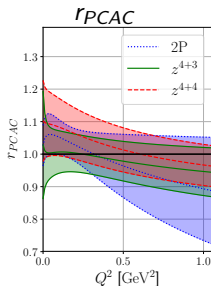
[Meyer et al., 2201.01839] GENIE simulation of ν_{μ} -H₂O charged-current QE event rates at near detector of T2K: 20% higher rate with a lattice G_A [CalLat, 2111.06333] compared to a dipole fit. $G_A(Q^2) = g_A/(1 + Q^2/M_A^2)$ with $M_A = 0.94$ GeV



Isovector axial form factor: cross-checks



[RQCD, 1911.13150]



- PCAC relation reproduced in the continuum limit.

$$m_N G_A(Q^2) \mathbf{r}_{PCAC} = m_q G_P(Q^2) + \frac{Q^2}{4m_N} \tilde{G}_P(Q^2), \quad \mathbf{r}_{PCAC} = 1 + \mathcal{O}(a^2)$$

where the **pseudoscalar form factor**: $\langle p(p') | \mathbf{P} | n(p) \rangle = \bar{u}_p i \gamma_5 \mathbf{G}_P(Q^2) u_n$.

- Induced form factor $\tilde{G}_P(Q^2)$ consistent with expt. at the muon capture point $Q^2 = 0.88 m_\mu^2$.

Transition form factors involving resonances

For example, $N \rightarrow N\pi$ transition matrix elements: $N \rightarrow \Delta(1232), N^*$ resonances for vector and axial currents.

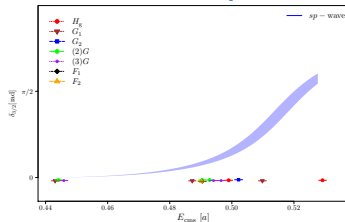
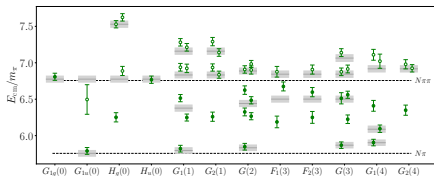
Straightforward if the resonance is stable ($m_\pi \nearrow$). $N \rightarrow \Delta$ and $N \rightarrow \text{Roper}$ previously studied.

$N \rightarrow \text{resonance}$ requires finite volume formalism for $N\pi \rightarrow N\pi$ ($2 \rightarrow 2$) scattering as well as $N \rightarrow N\pi$ ($1 + \mathcal{J} \rightarrow 2$). Meson $1 + \mathcal{J} \rightarrow 2$ transitions mostly studied so far, see, however, [Gao et al., 2310.01168, 2502.12074] $\gamma^* N \rightarrow \pi N$.

First step: spectroscopy, talks Morningstar, Mon 12pm, Bulava, Thu 3pm, Pittler, Thu 4:10pm.

[Bulava et al., 2208.03867] $m_\pi = 200$ MeV

[ETMC, 2307.12846] m_π^{phys}



Computing $N \rightarrow N\pi$ matrix elements: [Barca et al., 2211.12278], [Barca, 2508.09006], [ETMC, 2408.03893], . . .

Mellin moments of PDFs

Example of n -th moment of the unpolarised PDF:

$$\langle x^{n-1}(\mu) \rangle_{q(-)^n} = \int_0^1 dx x^{n-1} [q(x, \mu) + (-)^n \bar{q}(x, \mu)]$$

Notation: $q^- := q - \bar{q} (= q_v)$, $q^+ := q + \bar{q} (= q_v + 2\bar{q})$.

OPE connects the moments to **matrix elements of local operators**

$$\langle N(p) | \mathcal{O}_{\mu_1 \dots \mu_n}^n | N(p) \rangle \sim \langle x^{n-1} \rangle_{q(-)^n} ((p_{\mu_1} \dots p_{\mu_n}) - \text{traces})$$

$\mathcal{O}_{\mu_1 \dots \mu_n}^n = \bar{q} \gamma_{(\mu_1} i \overleftrightarrow{D}_{\mu_2} \dots i \overleftrightarrow{D}_{\mu_n)} q$ where $(\mu_1 \dots)$: symmetrized, traceless.

$n = 1, 2, 3, \dots$ operators are dimension 3, 4, 5, 6
and have 0, 1, 2, 3... derivatives.

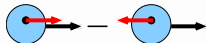
Unpolarised

$$\langle x^{n-1} \rangle_q$$



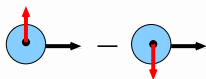
Helicity

$$\langle x^{n-1} \rangle_{\Delta q}$$



Transversity

$$\langle x^{n-1} \rangle_{\delta q}$$



Mellin moments of PDFs

Information on the PDFs through moments:

- In principle, with knowledge of moments to high enough n , PDF can be reconstructed.
→ High n more important for large x .
- Lower moments can be used as constraints in phenomenological analyses.
- Determination of different flavour combinations is possible, also gluonic operators.
- There are no higher twist corrections. Matrix elements of higher twist operators computable.

However,

- Signal to noise deteriorates as n increases.
- Renormalisation: reduced symmetry on the lattice.
If mixing with lower $n' = n - \ell$ is not prohibited by symmetry $\Rightarrow a^{-\ell}$ power divergencies.

Standard methods: fourth moment possible (e.g. in a boosted frame to avoid mixing).

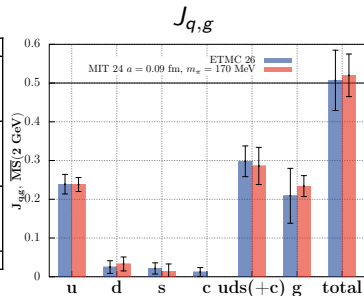
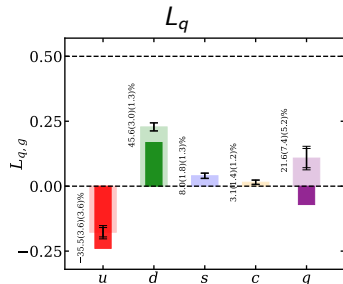
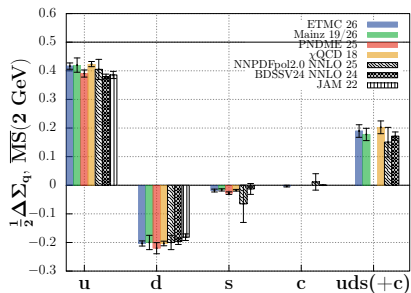
New developments [Shindler, 2311.18704]: gradient flow approach gives improved signal and operator mixing like in the continuum.

First helicity moment $\langle 1 \rangle_{\Delta q^+}$, $\overline{\text{MS}}$, $\mu = 2 \text{ GeV}$

Gauge invariant Ji spin decomposition of the nucleon

$$\frac{1}{2} = J_g + \sum_q J_{q^+} = J_g + \sum_q \left(\frac{1}{2} \Delta \Sigma_{q^+} + L_{q^+} \right),$$

$$\Delta \Sigma_{q^+} = \langle 1 \rangle_{\Delta q^+}$$

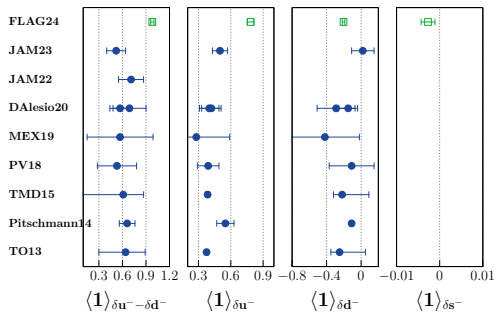


$\left(\frac{1}{2} \Sigma_{q^+} \right)$ [ETMC, 2607.21230], [Mainz 26, 2605.06559], [Mainz 19, 1911.01177], [PNDME 25, 2503.07100], [χ QCD 18, 1806.08366], [NNPDFpol2.0NNLO, 2503.11814], [BDSSV24-NNLO, 2407.11635], [JAM 22, 2202.03372].

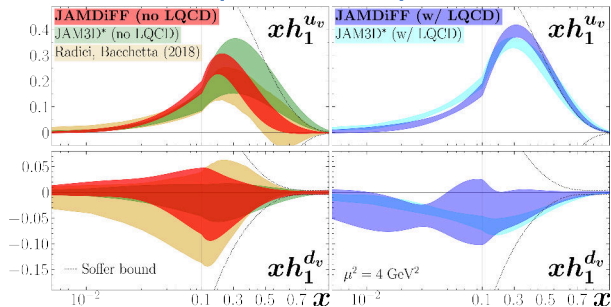
$(J_{q,g})$ [MIT 24, 2310.08484], [ETMC, 2607.21230] from moments of GPDs: generalised FFs $A_{20}^{q,g}(Q^2)$, $B_{20}^{q,g}(Q^2)$, $J_{q,g} = \frac{1}{2} [A_{20}^{q,g}(0) + B_{20}^{q,g}(0)]$, $L_{q^+} = J_{q^+} - \frac{1}{2} \Sigma_{q^+}$.

First transversity moment $\langle 1 \rangle_{\delta q^-}$, $\overline{\text{MS}}$, $\mu = 2 \text{ GeV}$

[FLAG 24, 2411.04268]



[JAM 23, 2306.12998]



JAM 23: valence transversity PDFs $h_1^{u_v, d_v} \equiv h_1^{u, d} - h_1^{\bar{u}, \bar{d}}$ from global analysis of dihadron production.

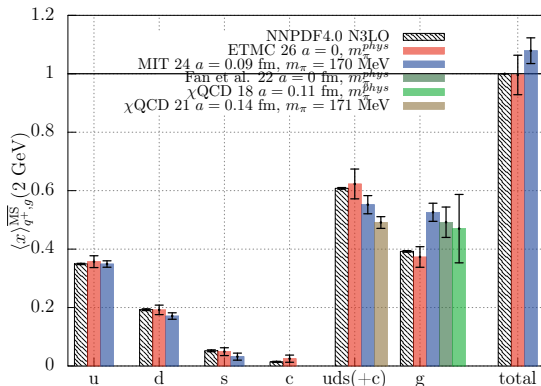
Constraints: [ETMC, 1909.00485] and [PNDME, 1808.07597] results for $\langle 1 \rangle_{\delta u^-}$ and $\langle 1 \rangle_{\delta d^-}$.

Soffer bound applied: (parton model) [Soffer, hep-ph/9409254] $|h_1^q(x)| \leq (q(x) + \Delta q(x))/2$.

FLAG 24: $\langle 1 \rangle_{\delta u^-, \delta d^-, \delta s^-}$ from [PNDME, 1808.07597].

Unpolarised second moment $\langle x \rangle_{q^+,g}$, $\overline{\text{MS}}$, $\mu = 2 \text{ GeV}$

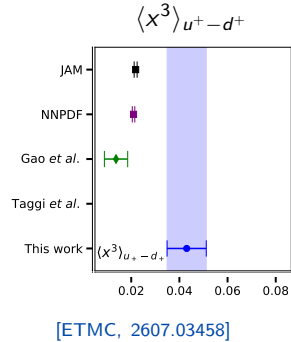
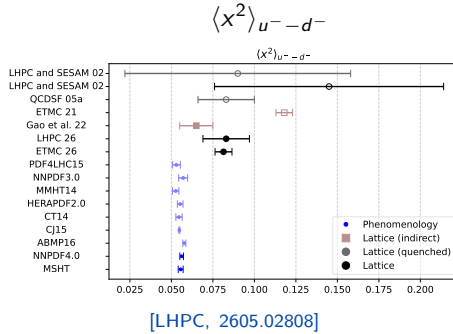
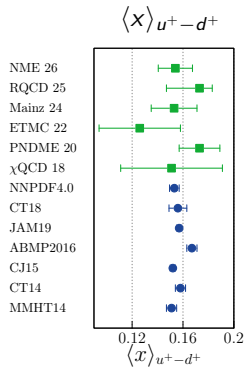
Quark and gluon momentum fractions: $\sum_q \langle x \rangle_{q^+} + \langle x \rangle_g = 1$.



[NNPDF4.0N3LO, 2402.18635] (moments computed using [LHAPDF, 1412.7420]).

[ETMC, 2607.21230], [MIT 24, 2310.08484], [Fan et al. 22, 2208.00980], [χQCD 18, 1805.00531], [χQCD 21, 2111.09329].

Isvector unpolarised moments $\langle x^{n-1} \rangle$ up to $n = 4$, $\overline{\text{MS}}$, $\mu = 2 \text{ GeV}$ ¹⁹



[LHPC, 2605.02808] (Fig. E. Taggi): $a \approx 0.116 \text{ fm}$ and $a \approx 0.093 \text{ fm}$, $M_\pi \approx 135 \text{ MeV}$.

Also $\langle x^2 \rangle_{\Delta u^+ - \Delta d^+}$ and $\langle x^2 \rangle_{\delta u^- - \delta d^-}$.

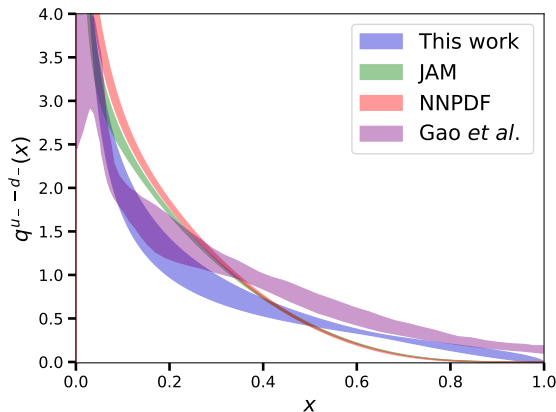
[ETMC, 2607.03458] $a \approx 0.08 \text{ fm}$, $M_\pi \approx 139 \text{ MeV}$. Also $u + d$, s and d flavour combinations (disconnected contributions neglected) and off-forward limit (moments of GPDs).

Reconstruction of the isovector valence unpolarised PDF

$$[u(x) - \bar{u}(x)] - [d(x) - \bar{d}(x)] \text{ from } \langle x^{n-1}(\mu) \rangle_{q(-)^n} = \int_0^1 dx x^{n-1} [q(x, \mu) + (-)^n \bar{q}(x, \mu)], n \leq 4.$$

[ETMC, 2607.03458] $a \approx 0.08$ fm, $M_\pi \approx 139$ MeV

[Gao et al., 2212.12569] $a = 0.08$ fm, m_π^{phys} , LaMET.



Use $q(x) = Nx^\alpha(1-x)^\beta$, (even or odd moments)

Use both, for even n : $\langle x^{n-1} \rangle_{q_-} = \langle x^{n-1} \rangle_{q_+} - 2\langle x^{n-1} \rangle_{\bar{q}}$ with $\langle x^{n-1} \rangle_{\bar{q}}$ from [JAM, 2202.03372].

Higher moments with the gradient flow

Gradient flow: diffusion-like evolution in field space in auxiliary “flow” time t .

Smoothing over a range $\sqrt{8t}$.

Flowed theory: same symmetries as in the continuum.

No mixing with lower-dimensional lattice operators.

Improved signal to noise (no boost necessary).

Continuum limit

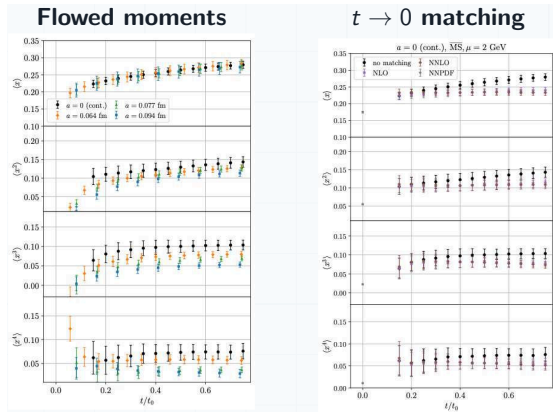
$$\langle x^{n-1} \rangle^{\text{flow}}(t) = \langle x^{n-1} \rangle^{\text{flow}}(t, a) + \underbrace{c_2 \frac{a^2}{t} + \text{other } a^2 + \dots}_{\text{extrapolate } a \rightarrow 0}$$

Small flow time expansion

$$\langle x^{n-1} \rangle^{\overline{\text{MS}}}(\mu) = C_0^{(n)}(\alpha_s(\mu)) \langle x^{n-1} \rangle^{\text{flow}}(t) + \underbrace{\sum_i c_{2i}^{(n)} \langle O_{n,2i}^{\text{flow}} \rangle t + \dots}_{\text{extrapolate } t \rightarrow 0}$$

Figure: A. Shindler (OpenLat) Lattice 26 talk.

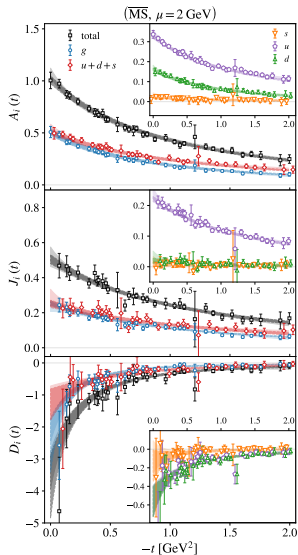
$\langle x^{n-1} \rangle_{u-d}$, $m_\pi = 300 \text{ MeV}$, $a = 0.09 - 0.06 \text{ fm}$.
Low statistics



Talk: Shindler, Tues. 7:15pm

Moments of GPDs: generalised form factors, $\overline{\text{MS}}, \mu = 2 \text{ GeV}$

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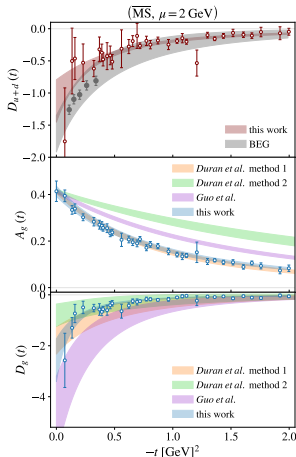


[MIT 23, 2310.08484]

$m_\pi = 170 \text{ MeV}$, $a = 0.09 \text{ fm}$.

Middle: $A_g(t)$ rescaled to give $A_g(0)$ of Duran and Guo.

[ETMC, 2607.20337]: continuum limit, m_π^{phys} .



Matrix element of the traceless symmetric energy-momentum tensor $\langle N(\vec{p}', s') | \hat{T}^{\mu\nu} | N(\vec{p}, s) \rangle$

depends on the GFFs:

$$A_{q,g}(-t) = A_{20}^{q,g}(Q^2), \quad A(0) = \langle x \rangle,$$

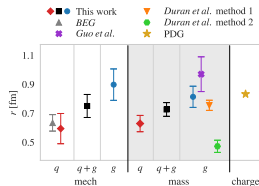
$$J_{q,g}(t) = \frac{1}{2} [A_{20}^{q,g}(t) + B_{20}^{q,g}(t)],$$

and $D_{q,g}(-t) = C_{20}^{q,g}(Q^2)$.

2nd moments of unpolarised GPD:
 $H \rightarrow A_{20}$ and C_{20} .

$E \rightarrow B_{20}$ and C_{20} .

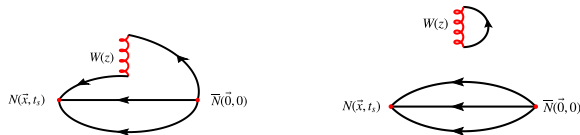
Radii of longitudinal force and energy densities.



Position space lattice methods

Calculate 4-point function with a non-local (equal time) current.

Extract matrix element, e.g. unpolarized PDF.



$$M^0(z, P_3) = \langle P | \bar{\psi}(z) \gamma^0 W(z) \psi(0) | P \rangle, \quad P = (P_0, 0, 0, P_3), \quad z = (0, 0, 0, z_3)$$

Collinear factorisation used to extract information on the PDFs.

Quasi-PDFs and large-momentum effective theory (LaMET) [\[Ji,1305.1539\]](#):

$$\tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-iz_3 x P_3} M^0(z, P_3),$$

$$\tilde{q}(x, P_3) = \int_{-1}^1 \frac{dy}{|y|} C^{(\tilde{q})} \left(\frac{x}{y}, \frac{y P_3}{\mu} \right) q(y, \mu^2) + \mathcal{O} \left(\frac{\Lambda_{QCD}^2}{x^2 P_3^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_3^2} \right)$$

Off light-cone $|x| > 1$ possible, x no longer Bjorken- x .

Higher twist effects at small and large x (range depends on P_3).

Position space lattice methods

Other approaches, e.g. pseudo-PDFs (SDF) [Radyushkin, 1612.05170], lattice cross-sections [Ma and Qiu, 1404.6860], hadronic tensor method [Liu et al., hep-ph/9910306, 1906.05312], current-current correlations [Braun and Müller, 0709.1348], ...

Challenges, see reviews [Constantinou, Cichy, 2608.00251], [Alexandrou, 2603.28604], [Alexandrou et al., 2604.04765], [Lin, 2506.05025], ...

- ▶ Achieving large momentum: signal to noise problem and discretisation effects. Signals up to $P_3 \lesssim 2 - 3$ GeV are achieved.
- ▶ Renormalisation: bare (extended) operator $\bar{\psi}(z)\gamma^0 W(z)\psi(0)$ has power divergences. Non-trivial to renormalise if z is not small.
- ▶ Inverse problem.

Cross-checks:

- ▶ Comparison between lattice methods and phenomenological determinations of the PDFs.
- ▶ Comparison with Mellin moments.
- ▶ Closure tests with artificial data.

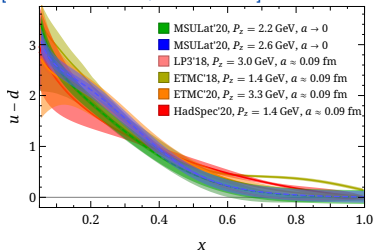
So far studies of isovector and flavour singlet unpolarised, helicity, transversity PDFs, gluon PDFs. Also higher twist PDFs and GPDs, TMDs, ...

Position space lattice methods: isovector PDFs

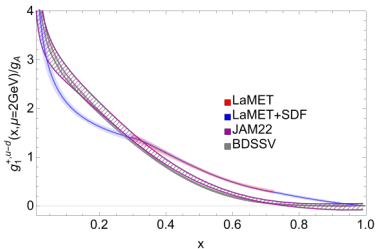
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Unpolarised

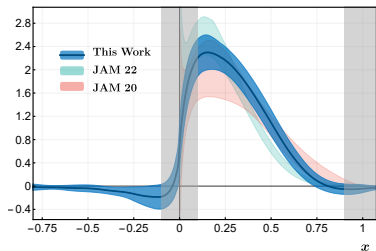
[Alexandrou et al., 2604.04765]



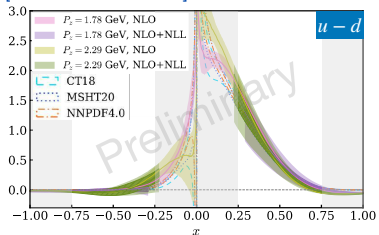
Helicity



Transversity



[Shi et al., Lattice 26]



[Gao et al., 2604.00143]

$a = 0.08$ fm, m_π^{phys} , $P_z \leq 1.5$ GeV.

LaMET+SDF, NNLO matching.

(Left) LaMET, NLO matching.

$a = 0.08$ fm, m_π^{phys} , $P_z \leq 2.3$ GeV.

[LPC, 2208.08008]

$a = 0.10 - 0.05$ fm,

$m_\pi = 220 - 350$ MeV,

$P_z \leq 2.8$ GeV.

LaMET, NLO matching.

Major effort to determine hadron structure observables on the lattice relevant (and complementary) for current and future experiments.

Flavour decomposition provided: u , d , s , c , g .

Continuing work on tackling/improving signal-to-noise problem, excited state contamination, matching, . . . , with new methods.

Consistency between results with all systematics under control for many observables.

Precision of individual results will continue to improve, also community averages can be provided (statistical/systematic error).

Summary and outlook

Nucleon form factors

- Electromagnetic: overall agreement with expt..
- Axial: lattice results fall off slower than experimental determinations. Cross-checks made. Lattice provides the most precise determination of $G_A(Q^2)$. Lattice average should be considered as input for Monte-Carlo simulation of QE ν -N cross-section.
- Also results for the strangeness EM and axial form factors.
- Future: transitions involving resonances, e.g. $N \rightarrow \Delta$. Spectroscopy studies are the first step.

Moments of PDFs (GPDs)

- Consistent results for decomposition of the nucleon spin, and parton momentum fraction. J_{tot} and $\langle x \rangle_{tot}$ consistent with $\frac{1}{2}, 1$. (Most) precise results for (some) $\frac{1}{2}\Sigma_q$ and $\langle 1 \rangle_q$.
- Feasible to compute up to 4th moments using conventional methods. New gradient flow method will give improved precision for higher moments.
- Using lattice moments can provide constraints in global phenomenological analyses. Impact, in particular, for helicity and transversity PDFs and also GPDs.