Phenomenology of quark-gluon-quark interference inside of hadron

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Outline:

I present the extraction of ***genuine twist-three distributions*** from the analyses of the global data.

- It **the first** analyses of such kind
  - Simultanious extraction of d_2, g_2, $f_{1T}^{\perp}$ (Sivers), and $g_{1T}^{\perp}$ (wgt) functions
- It **opens new exciting perspectives** for studies of hadron structure

References

[1] AV, G.Portela, S.Rodini, *Phys.Rev.Lett. 137* (2026) 6, 061902
[2] AV, G.Portela, [2607.01866] (contains a lot of extra material, 90 pages)

What are genuine twist-three distributions?

$$\frac{1}{Q^0} : \overset{\text{twist-2}}{\boxed{q, \Delta q, \delta q}} \longrightarrow \boxed{\begin{array}{c}\text{probability densities}\\ \text{parton-"static"}\end{array}} \longleftrightarrow \boxed{\Big|\langle q|P\rangle\Big|^2}$$

What are genuine twist-three distributions?

$\frac{1}{Q^0}$: twist-2 $\boxed{q, \Delta q, \delta q}$ $\longrightarrow$ probability densities parton-"static" $\longleftrightarrow$ $\left| \langle q|P\rangle \right|^2$

$\frac{1}{Q^1}$: "dynamical" twist-3 $\boxed{g_2, g_T, h_2, e, e_T, \ldots}$ $\longrightarrow$ forces, orbital mom., ... parton-"dynamics"

$\uparrow$

"genuine" twist-3 $\boxed{T, \Delta T, H, E}$ $\longrightarrow$ parton interference $\longleftrightarrow$ $\mathrm{Re}\langle P|q|gq|P\rangle$, $\mathrm{Im}\langle P|q|gq|P\rangle$

What are genuine twist-three distributions?

twist-2

$\frac{1}{Q^0}$: $q, \Delta q, \delta q$ $\longrightarrow$ probability densities parton-"static" $\longleftrightarrow$ $\left|\langle q|P\rangle\right|^2$

"dynamical" twist-3

$\frac{1}{Q^1}$: $g_2, g_T, h_2, e, e_T, \ldots$ $\longrightarrow$ forces, orbital mom., ... parton-"dynamics"

"genuine" twist-3

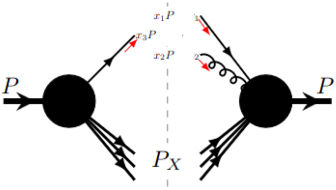
$T, \Delta T, H, E$ $\longrightarrow$ parton interference $\longleftrightarrow$ $\mathrm{Re}\langle P|q|gq|P\rangle$ $\mathrm{Im}\langle P|q|gq|P\rangle$

$$\langle p,S|g\,\bar{q}(z_1n)F^{\mu+}(z_2n)\gamma^+q(z_3n)|p,S\rangle = 2\epsilon_T^{\mu\nu}s_\nu(p^+)^2M\int[dx]\,T_f(x_1,x_2,x_3)\,e^{-ip^+x_iz_i}$$

$$\langle p,S|ig\,\bar{q}(z_1n)F^{\mu+}(z_2n)\gamma^+\gamma^5q(z_3n)|p,S\rangle = -2s_T^\mu(p^+)^2M\int[dx]\,\Delta T_f(x_1,x_2,x_3)\,e^{-ip^+x_iz_i}$$

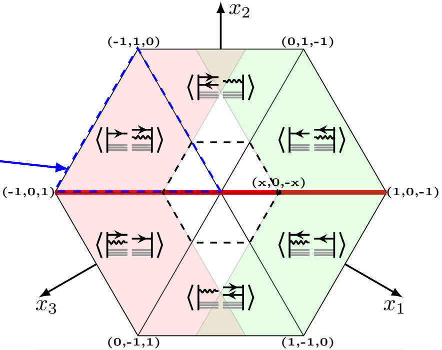
Genuine twist-3 distributions are
irreducible basis for all other twist-3 distributions

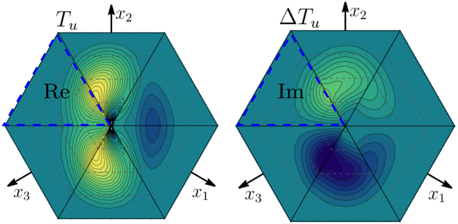
Twist-3 distribution depends on 2 variables



momentum conservation
$$x_1 + x_2 + x_3 = 0$$

$$\begin{cases} \left(\langle p|a^\dagger_{x_3}\right)\left(b^\dagger_{x_2} a_{x_1}|p\rangle\right), & x_{1,2}>0,\ x_3<0 \\ \left(\langle p|a_{x_1}\right)\left(b^\dagger_{x_2} a^\dagger_{x_3}|p\rangle\right), & x_{2,3}>0,\ x_1<0 \\ \left(\langle p|a_{x_1} b_{x_2}\right)\left(a^\dagger_{x_3}|p\rangle\right), & x_3>0,\ x_{1,2}<0 \\ \ldots \end{cases}$$





Twist-3 PDFs represent interference between different components of the proton wave function.

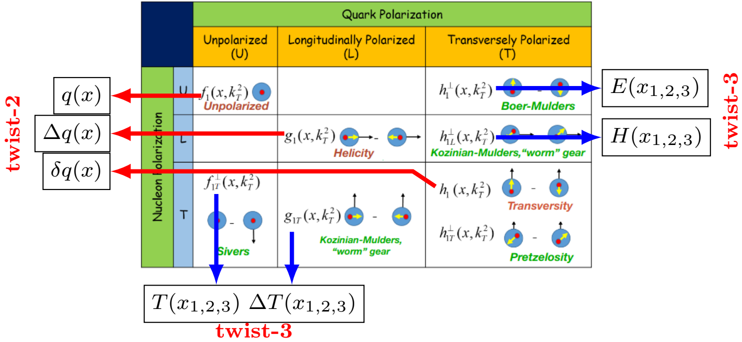
Each sector is a particular interference process.

T and $\Delta T \sim$ Re and Im
(loosely speaking)

Twist-three PDFs are everywhere in polarized physics

Most known "connections" to twist-3

- DIS structure function $g_2(x,Q)$ (accompany helicity $g_1(x,Q)$)
- d_2 moment (local $\bar{q}Fq$-operator)
- TMDs ($f_{1T}^{\perp}$, g_{1T}, etc)
- Spin-symmetries, Jet-asymmetries (A_N, ..)
- (Twist-three) GPDs
- ...

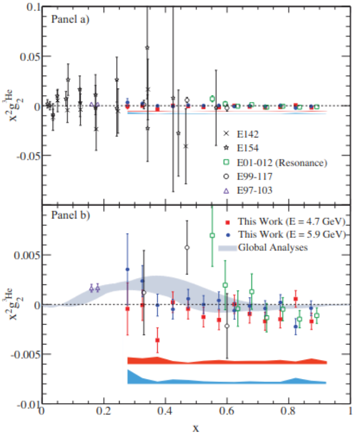


The theory of twist-3 PDFs is studied sufficiently well in 80's-00's. We know

- Symmetry relations, limiting properties, sum rules
- Evolution at LO
- Many coefficient function are computed at NLO (1-loop)
- Many model computations

However, **all this was never applied in practice** (to the real data), because

- Twist-3 PDFs are 2D-functions (too complicated object)
- Data is 1D (cross-section), i.e. it integrates 2D→1D
- Twist-3 PDFs are small
- Data is often very bad...



The main idea:
fit distinct observables altogether

- increases the number of available data points,
- fixes various parts of 2D PDFs.

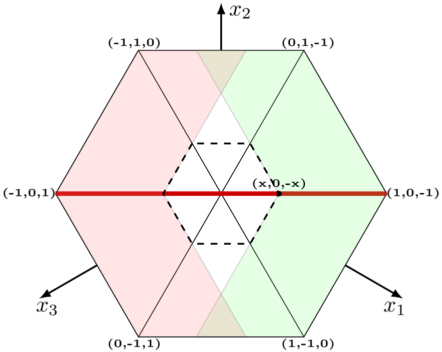
Should be done at least at LO (one-loop)

List of observables:

- g_2 (DIS structure function)
- d_2 (moment)
- $f_{1T}^{\perp}$ (Sivers TMD distribution)
- $g_{1T}^{\perp}$ (worm-gear-T TMD distribution)

- A_N (SSA)
- ...

Different observables are sensitive to different parts of the hexagon

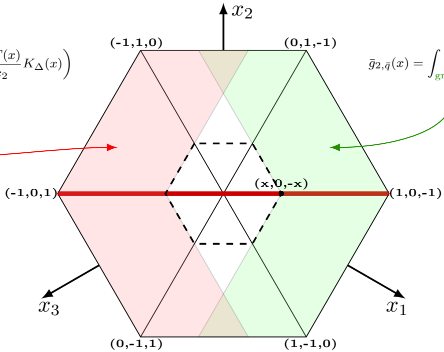


Different observables are sensitive to different parts of the hexagon

NLO
[Braun,Korechemsky, Manashov, 00-01]
[Kodaira, Tanaka, et al, 96-97]
[Geyer, Mueller, Robaschik, 96]
etc

$$\bar{g}_{2,q}(x) = \int_{\text{red}} [dx] \left(\frac{dT(x)}{dx_2} K(x) + \frac{d\Delta T(x)}{dx_2} K_\Delta(x) \right)$$

$$\bar{g}_{2,\bar{q}}(x) = \int_{\text{green}} [dx] \left(\frac{dT(x)}{dx_2} K(x) + \frac{d\Delta T(x)}{dx_2} K_\Delta(x) \right)$$



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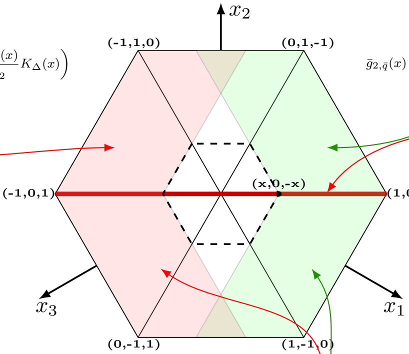
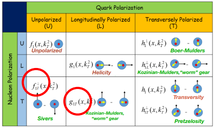


QS function $= T(-x,0,x)$



$$\int [dx] \left(T(x) K_T(x) + \Delta T(x) K_{T\Delta}(x)\right)$$

NLO

[Scimemi, Tarasov, AV, 1901.04519]
[Rein, Rodini, AV, 2209.00962]

Different observables are sensitive to different parts of the hexagon

$$\bar{g}_{2,q}(x) = \int_{\text{red}} [dx] \left(\frac{dT(x)}{dx_2} K(x) + \frac{d\Delta T(x)}{dx_2} K_{\Delta}(x) \right)$$

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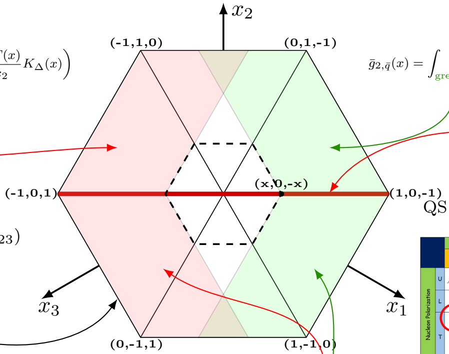
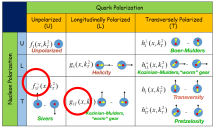


QS function $= T(-x,0,x)$

$$d_2 = \int_{\text{hexagon}} [dx] T(x_{123})$$



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Different observables are sensitive to different parts of the hexagon

$$\bar{g}_{2,q}(x) = \int_{\text{red}} [dx] \left(\frac{dT(x)}{dx_2} K(x) + \frac{d\Delta T(x)}{dx_2} K_\Delta(x) \right)$$

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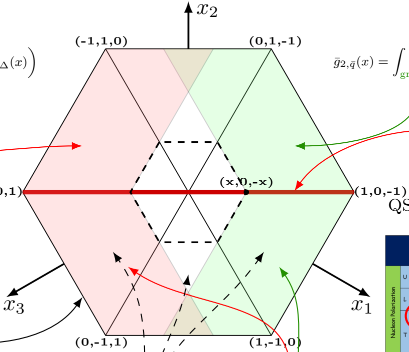

QS function $= T(-x,0,x)$

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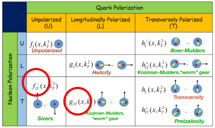


$$\int [dx] \left(T(x) K_T(x) + \Delta T(x) K_{T\Delta}(x) \right)$$

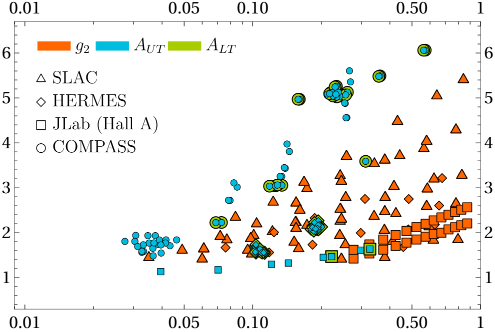
Evolution: $$\frac{dT(x_{123},\mu)}{d\ln\mu} = \int_{\text{outer hex}} [dy] \left[P(x,y) T(y,\mu) + P_\Delta(x,y) \Delta T(y,\mu) \right]$$

[Bukhvostov,Frolov,Lipatov,Kuraev, 85] ... [Braun,Manashov,Pirnay, 0909.3410]

Data

d_2
SANE
...
RQCD
$N_{\text{pt}} = 13$

g_2
SLAC
JLab
HERMES
$N_{\text{pt}} = 168$



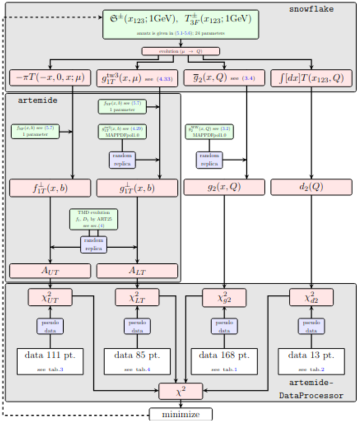
$f_{1T}^{\perp}$
HERMES
COMPASS
JLab
$N_{\text{pt}} = 117$

$g_{1T}^{\perp}$
HERMES
COMPASS
JLab
$N_{\text{pt}} = 74$

$N_{\text{tot}} = 372$

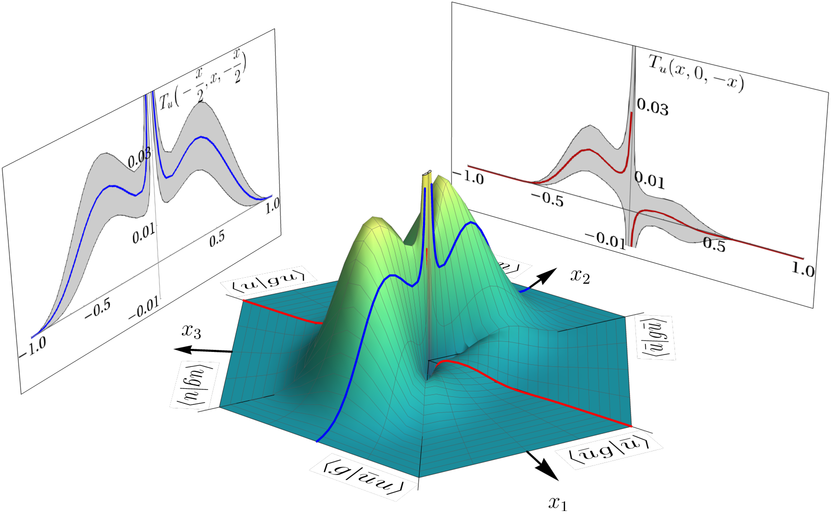
[AV,G.Protela, 2607.01866]

The largerst collection of tw3 data ever considered
The first simultanious extraction of TMD and PDF

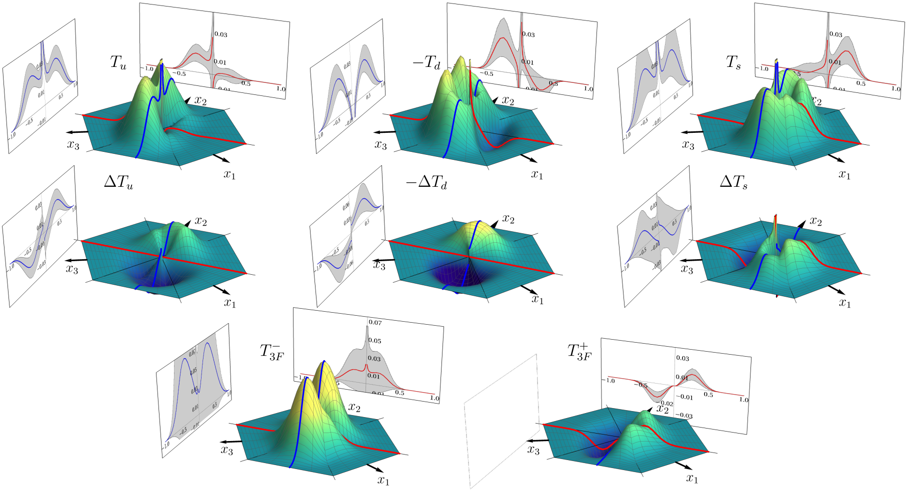


Technical details

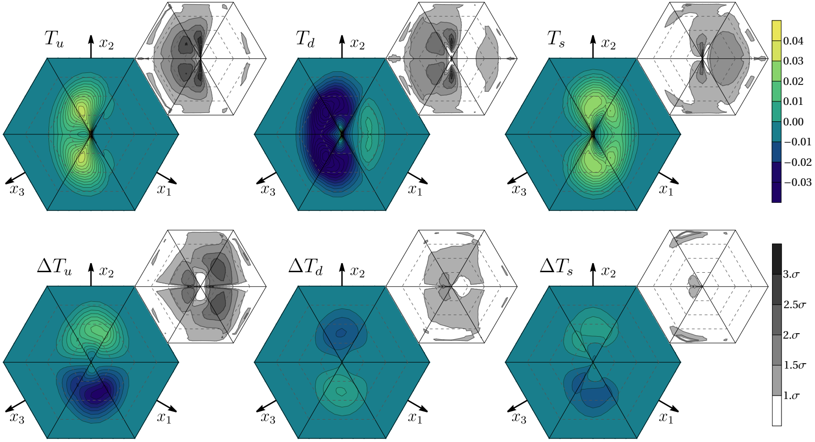
- **artemide+snowflake**
- Complete **LO** twist-3 part (i.e. LO evolution) [Rodini, Rossi, AV, 2405.01162]
- N^3LO TMD-evolution
- State-of-the-art other inputs
  - TMD part ⇒ ART25
  - unpol part ⇒ MSHT20
  - WW-helicity part ⇒ MAPPDFpol
- Uncertainly propagation
  - From data
  - From theory input


$T_u(-\frac{x}{2}, x, -\frac{x}{2})$
0.03
0.01
−0.01
−1.0
−0.5
0.5
1.0
$T_u(x, 0, -x)$
0.03
0.01
−0.01
−1.0
−0.5
0.5
1.0
x_1
x_2
x_3
$\langle u|gu\rangle$
$\langle ug|u\rangle$
$\langle g|\bar{u}u\rangle$
$\langle \bar{u}g|\bar{u}\rangle$
$\langle \bar{u}|g\bar{u}\rangle$

Early days of twist-three phenomenology



- General agreement with earlier fits/models
- Uncertainty are reasonable, and larger than by other groups
- Evolution is important (especially for TMDs)

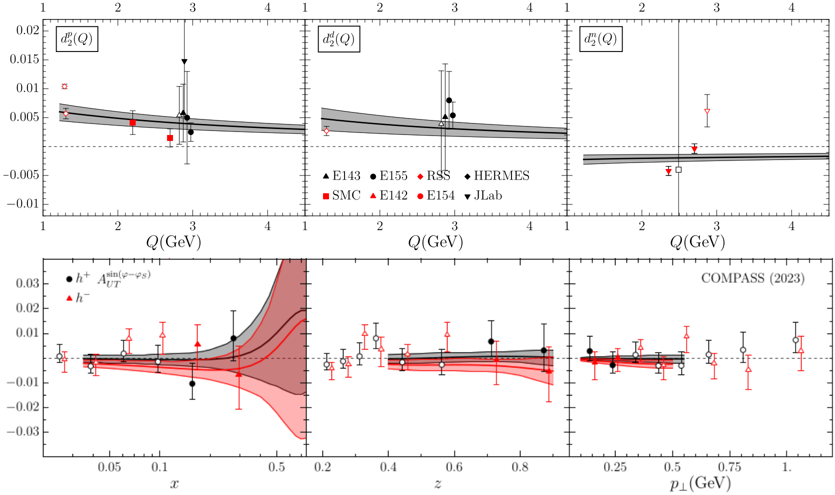


Sources of uncertanties:

- Experimental
- Theoretical (PDF = MSHT20, MAPpol1.0; TMD=ART25)

We observe tw3 distribution with 2-3σ significance.

Example of data



Many supporting details/observations in [2607.01866]

- Evolution effects for g_2, g_T, Sivers
- Cross-validation
  - Various fit configurations
  - Comparison with other groups
- Test of Sivers sign-change (based on DIS data)
- Sum rules
  - Burkhardt-Cottingham sum rule
  - Efremov-Leader-Teryaev sum rule
  - Burkardt sum rule
  - Positivity constraints
- Detailed information about hadron
  - Color forces acting on quark
  - Average transverse momentum
  - Tomography in momentum space

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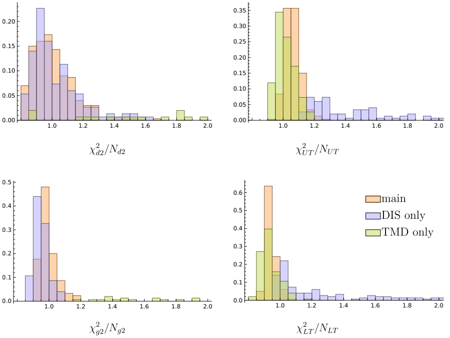
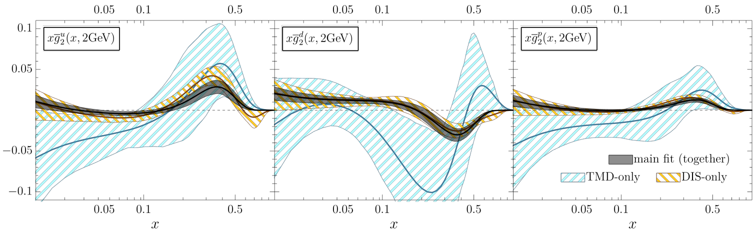
It is not possible to present all this material in a single presentation.
I will make just two short "teasers"

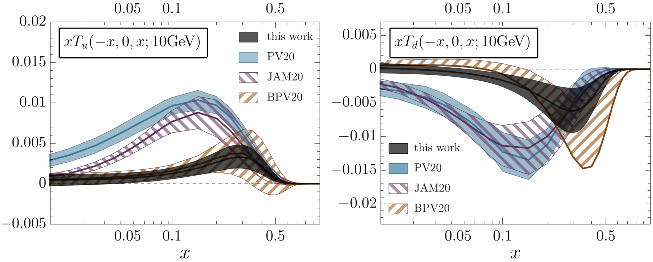


Different data support each other!

One can detemine g_2 from TMD data, and TMD from g_2 data (although with reduced precision)

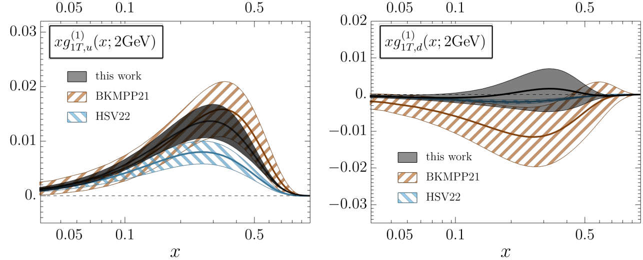
Demonstration of universality of tw3 distributions



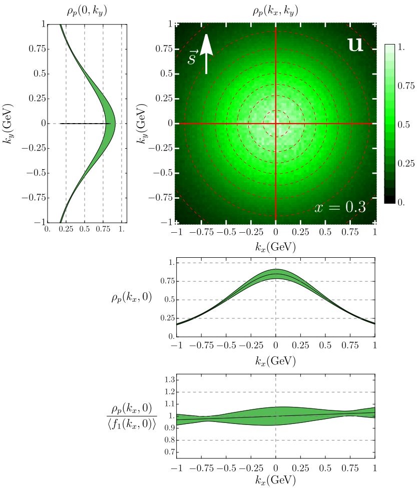


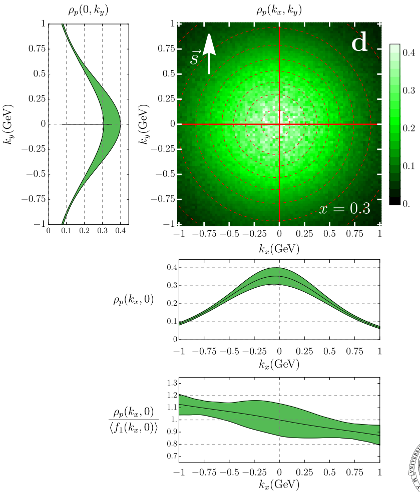
Very good agreement with BPV20 fit of Sivers (no so much with other evolution-less fits)

Wgt-function just in-between BKMPP21 and HSV22



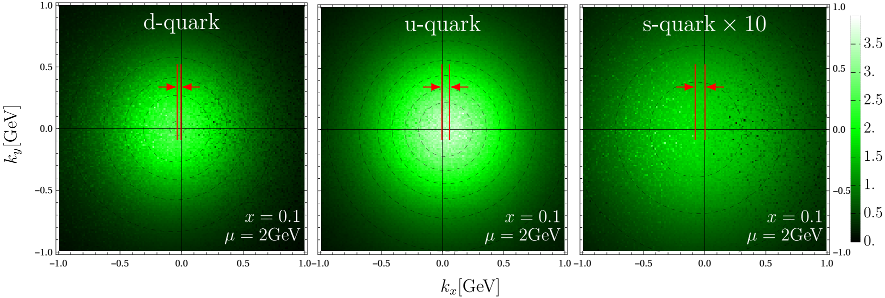
Updated tomography of the nucleon in momentum space.
Now, using N^{4}LL extractions and on a much larger data sets (ART25+SnowART26)





Avarage transverse momentum shift

$$\langle \mathbf{k}_\mu \rangle = -\pi \epsilon_{\mu\nu} s_T^\nu M \int_0^1 dx T(-x,0,x)$$



$$\langle k_x \rangle_u = 7.9^{+4.0}_{-3.5}\text{MeV}, \qquad \langle k_x \rangle_{\bar u} = 0.4^{+2.9}_{-2.9}\text{MeV},$$

$$\langle k_x \rangle_d = -12.6^{+7.4}_{-7.5}\text{MeV}, \qquad \langle k_x \rangle_{\bar d} = 2.5^{+5.8}_{-5.3}\text{MeV},$$

Budkard sum rule

[M.Budkardt, hep-ph/0311013]

$$\sum_{f=q,\bar q,g} \langle k_x \rangle_f = -25.^{+13.}_{-13.}\text{MeV}$$

Large uncontroled gluon contribution

Conclusion

The first determination of *genuine* twist-three distribution
and (by-product) of d_2, g_2, $f_{1T}^{\perp}$, $g_{1T}^{\perp}$

Novel direction of studies

- "Unification" of data from multiple polarized experiments
  (SLAC, JLab, HERMES, COMPASS, lattice, ...)
  - "first" joined fit of TMD and collinear distributions
- Exceptional test of QCD universality
  - "first" implimentation and test of proper twist-3 evolution
- Novel "vector-like" information about hadron structure
  (forces, transverse momentum, various sum rules)

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NewScientist

Physics

The proton is more quantum than we thought

Backup

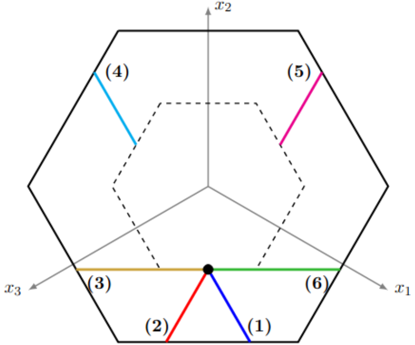
Evolution

$$\mu^2 \frac{\partial}{\partial \mu^2} \mathfrak{S}^{\pm}_{\mathrm{NS}_i} = -a_s(\mu) \mathbb{H}_{\mathrm{NS}} \mathfrak{S}^{\pm}_{\mathrm{NS}_i} \, ,$$

$$\mu^2 \frac{\partial}{\partial \mu^2} \begin{pmatrix} \mathfrak{S}^{\pm}_S \\ \mathfrak{F}^{\pm} \end{pmatrix} = -a_s(\mu) \begin{pmatrix} \mathbb{H}^{\pm}_{qq} & \mathbb{H}^{\pm}_{qg} \\ \mathbb{H}^{\pm}_{gq} & \mathbb{H}^{\pm}_{gg} \end{pmatrix} \begin{pmatrix} \mathfrak{S}^{\pm}_S \\ \mathfrak{F}^{\pm} \end{pmatrix} ,$$

Compare to DGLAP

$$\mu^2 \frac{d}{d\mu^2} q(x) = \int_x^1 \frac{dy}{y} P(\frac{x}{y}) q(y)$$



- Evolution involves all area with $> ||x||$
- Mixes $(T, \Delta T, T^{+}_{3F}, T^{-}_{3F})$ and flavors
- Two independent implementations
  `snowflake`:
  https://github.com/VladimirovAlexey/Snowflake
  `honeycomb`:
  https://github.com/slrodini/honeycomb2

Evolution comparison with unpolarized