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NAWI Graz
Natural Sciences

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Fundação
para a Ciência
e a Tecnologia

EXO HAD
EXOTIC HADRONS TOPICAL COLLABORATION

Few-body problems with functional methods

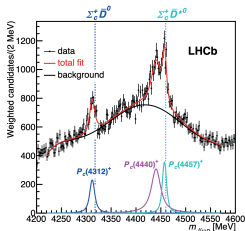
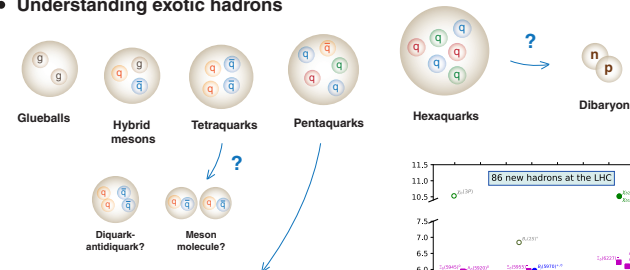
Gernot Eichmann

University of Graz

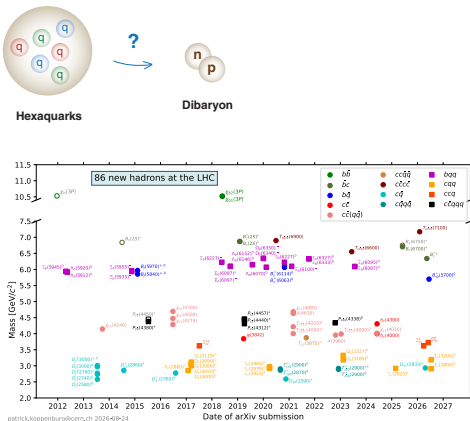
NSTAR 2026 - 15th International Workshop on the Physics of Excited Nucleons
Sevilla, Spain, Sep 9, 2026

Motivation

• Understanding exotic hadrons



LHCb Collaboration,
PRL 122 (2019) 22, 22201
PRL 131 (2023) 3, 031901

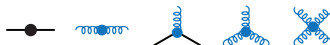


patrick.koppenburg@cern.ch 2026-09-24

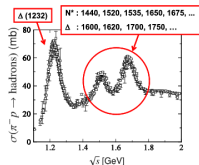
<https://www.nikhef.nl/~pkoppenb/particles.html>
<https://qwg.ph.nat.tum.de/exoticshub>

Theory tools

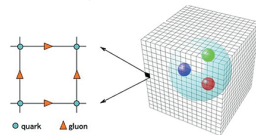
Functional methods (DSEs & BSEs, FRG, ...)



Amplitude analyses



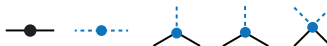
Lattice QCD



Phenomenological models

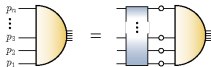


Effective theories (ChPT, ...)



Functional methods

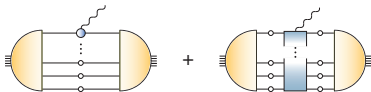
- Hadronic **bound-state equations**
(Bethe-Salpeter & Faddeev eqs)



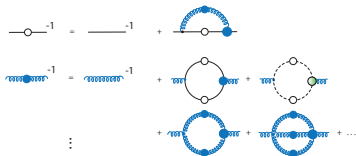
“QFT analogue of Schrödinger eq.”

- hadron masses & “wave functions”
- **spectroscopy calculations**

- Structure calculations:**
form factors, PDFs, GPDs, TMDs,
two-photon processes, ...



- Ingredients: **QCD’s n-point functions**,
computed from DSEs (quantum eqs. of motion)
or FRG (functional renormalization group)

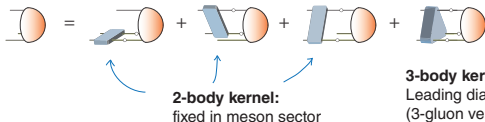


- Dynamical mass generation,
gluon mass gap, confinement,
QCD phase diagram, ...

Baryons

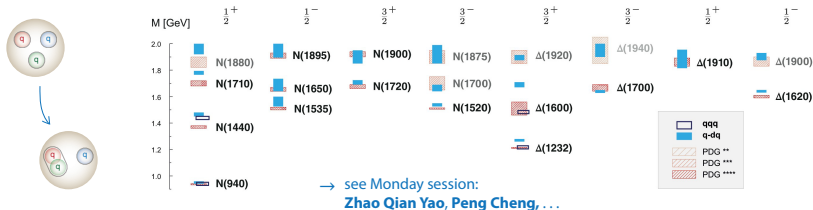
- Three-quark BSE (Faddeev equation) for baryons:**

Reviews: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016),
Burkert, GE, Klempt, PPNP 146 (2026)



- Diquark clustering in baryons?**

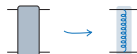
GE, Fischer, Sanchis-Alepuz, PRD 94 (2016), Barabanov et al., PPNP 116 (2021)



- Rainbow-ladder:**

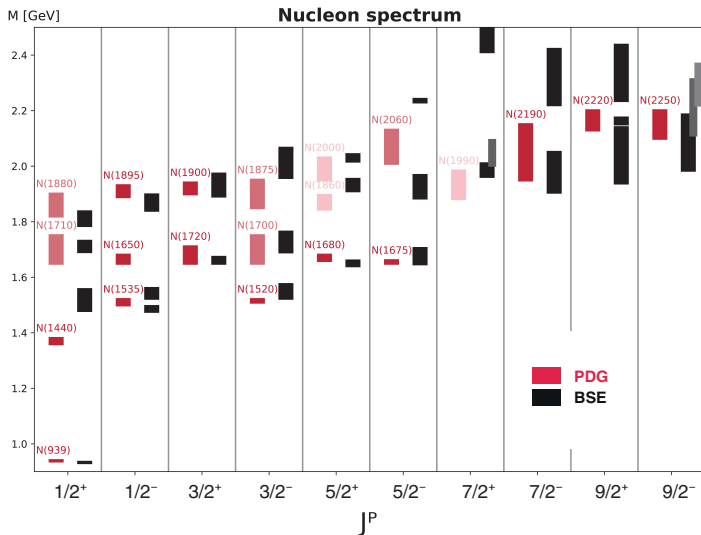
Solve quark DSE → 
2-body kernel fixed in
light meson sector (m_π, f_π):

Maris, Tandy, PRC 60 (1999),
Qin, Chang, Liu, Roberts, Wilson, PRC 84 (2011)



Respects chiral symmetry,
satisfies chiral properties

Baryons with higher spin

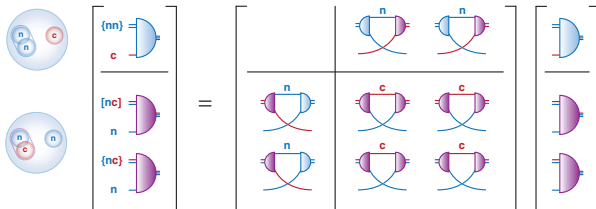


preliminary

GE, Fischer,
Obergruppenberger,
in preparation

Heavy baryons

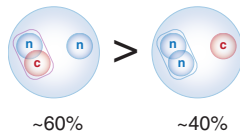
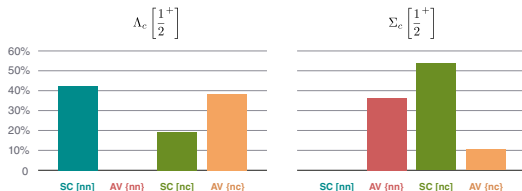
Quark-diquark BSE for Σ_c :



$n[nc]$, $n\{nc\}$ necessary, otherwise no equation. Presumably these are also dominant: cannot switch off $n[nc]$, $n\{nc\}$, but $c\{nn\}$

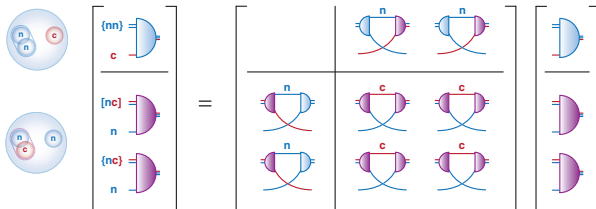
Analogous for Λ_c and hyperons

Results: **wave function contributions** [Torcato, Arriaga, GE, Peña, FBS 64 \(2023\)](#)



Heavy baryons

Quark-diquark BSE for Σ_c :



$n[nc]$, $n\{nc\}$ necessary, otherwise no equation. Presumably these are also dominant: cannot switch off $n[nc]$, $n\{nc\}$, but $c\{nn\}$

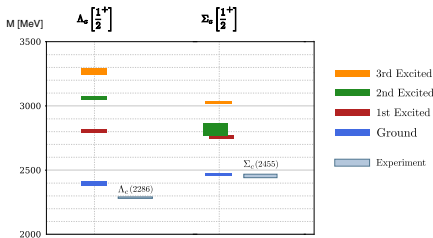
Analogous for Λ_c and hyperons

Results: **spectrum**

Torcato, Arriaga, GE, Peña, FBS 64 (2023)

see also:

Yin, Chen, Krein, Roberts, Segovia, PRD 100 (2019)



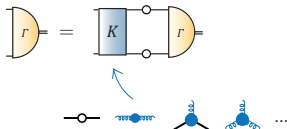
Complete calculation underway:

nnn, nns, nss, sss, nnc, ncc, ccc, SSC, SCC, nsc

André Torcato, Arriaga, GE, Peña, in preparation

Towards ab-initio

- **Goal:** go towards ab-initio calculations by calculating **higher n-point functions**



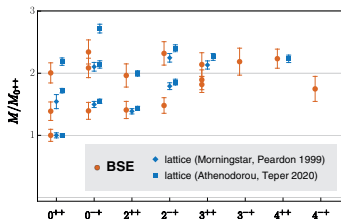
...
 Williams, Fischer, Heupel, PRD 93 (2016),
 Cyrol et al., PRD 97 (2018),
 Oliveira, Silva, Skullerud, Sternbeck, PRD 99 (2019),
 Aguilar et al., EPJ C 80 (2020),
 Huber, PRD 101 (2020),
 Qin, Roberts, Chin. Phys. Lett. 38 (2021),
 GE, Pawłowski, Silva, PRD 104 (2021),
 Miramontes, Morgado, Papavassiliou, Pawłowski, EPJ C 85 (2025)
 Huber, Fischer, Sanchis-Alepuz, EPJ C 85 (2025)
 Alkofer, Ferreira, Miramontes, Morgado, Papavassiliou, PLB 880 (2026),
 ...

truncation error:

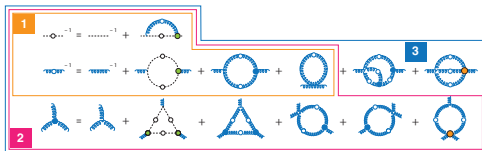
1 60% **2** 10% **3** 4%

- **Glueball spectrum** agrees with lattice QCD

Huber, Fischer, Sanchis-Alepuz, EPJ C 80 (2020), EPJ C 81 (2021)



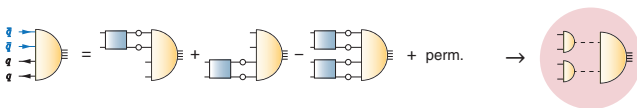
- **Coupled Yang-Mills DSEs** GE, Pawłowski, Silva, PRD 104 (2021)



Four-quark states

- **Four-quark BSE** creates internal meson and diquark poles $\Rightarrow$ resonances!

GE, Fischer, Heupel, PLB 753 (2016), Wallbott, GE, Fischer, PRD 102 (2020), **Hoffer**, GE, Fischer, PRD 109 (2024) & PRD 111 (2025)



$$\Gamma(p, q, k, P) = \sum_i f_i(p^2, q^2, k^2, \{\omega_j\}, \{\eta_j\}) \tau_i(p, q, k, P) \otimes \text{Color} \otimes \text{Flavor}$$

9 Lorentz invariants:

256 Dirac-
Lorentz tensors

2 Color
tensors

$K \psi_i = \lambda_i \psi_i$	$\dim K$	memory
Mesons	10^3	20 MB
Baryons	10^8	10^7 GB
Tetraquarks	10^{13}	10^{18} GB

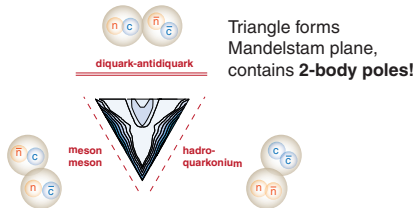
- Group momentum variables into multiplets of **permutation group S4**:
can switch off groups of variables without destroying symmetries

GE, Fischer, Heupel, PRD 92 (2015)

$$f_i(\underbrace{S_0, \nabla}_{\text{important}}, \underbrace{\Delta, \bigcirc}_{\text{not important}})$$



- Efficient approximation: $f_i(S_0) \times$ **poles**,
keep “physical” tensors: meson-meson, dq-dq, ...
4-quark BSE determines admixture dynamically!



Four-quark states

Open-flavor states

Hoffer, GE, Fischer,
PRD 111 (2025)

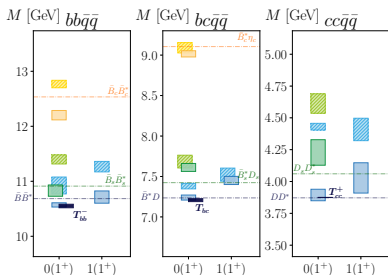


meson molecule

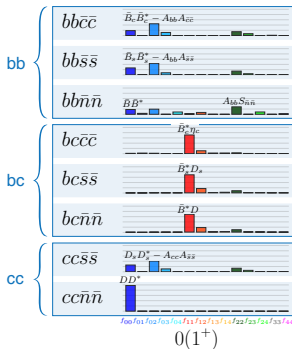


diquark-antidiquark

Can **quantify** meson-meson and diquark-antidiquark contributions!


$$bb\bar{c}\bar{c} \begin{cases} \text{excited} \\ \text{ground} \end{cases}$$
$$b\bar{c}c\bar{c} \begin{cases} \text{excited} \\ \text{ground} \end{cases}$$
$$bb\bar{s}\bar{s} \begin{cases} \text{excited} \\ \text{ground} \end{cases}$$
$$bc\bar{s}\bar{s} \begin{cases} \text{excited} \\ \text{ground} \end{cases}$$
$$cc\bar{s}\bar{s} \begin{cases} \text{excited} \\ \text{ground} \end{cases}$$
$$bb\bar{n}\bar{n} \begin{cases} \text{excited} \\ \text{ground} \end{cases}$$
$$bc\bar{n}\bar{n} \begin{cases} \text{excited} \\ \text{ground} \end{cases}$$
$$cc\bar{n}\bar{n} \begin{cases} \text{excited} \\ \text{ground} \end{cases}$$

Wave-function components:



T_{bb} is complicated mixture of molecule & diquark-antidiquark

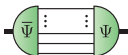
T_{cc} sits on top of
threshold, almost
pure molecule

Four-quark states

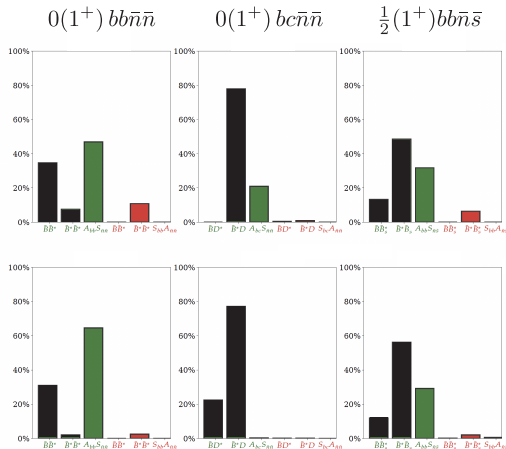
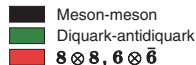
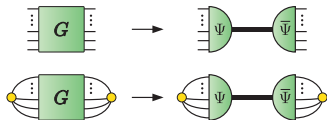
How to quantify meson-meson and diquark-antidiquark contributions?

Hoffer, GE, Fischer, Wagner, in preparation

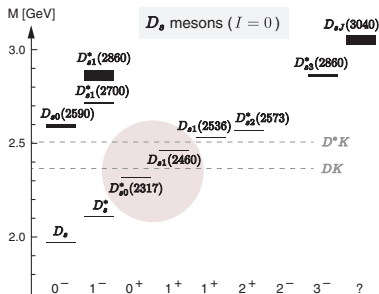
- Norm contributions



- Overlap factors (also accessible on lattice)



Single-charm states



- D_s mesons:** ~40 MeV below thresholds, masses much below quark model.

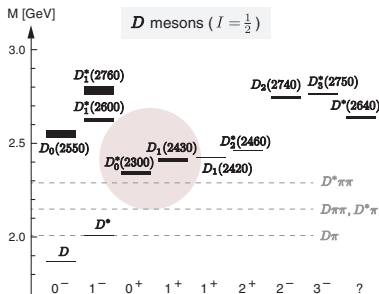
Hadronic molecules?

Guo, Hanhart, Meißner, Wang, Zhao, Zou, RMP 90 (2018)

Mixture of $c\bar{s}, (c\bar{s})(n\bar{n}), (c\bar{s})(s\bar{s}), \dots?$

- Compact diquark-antidiquark states?**

Maiani, Polosa, Riquer, PRD 100 (2019)



- D mesons:** resonances, UChPT predicts two poles for each state

Du, Albaladejo, Fernandez-Soler, Guo, Hanhart, Meißner, Nieves, Yao, PRD 98 (2018), ...

Compatible with lattice results

Gayer, Lang, Ryan, Tims, Thomas, Wilson, JHEP 123 (2021)

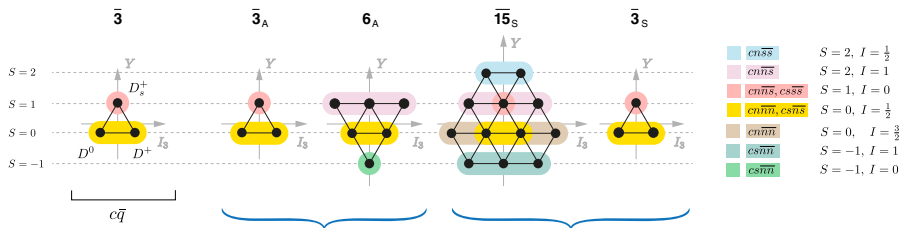
Asokan, Tang, Guo, Hanhart, Kamiya, Meißner, EPJC 83 (2023)

Luo, Wu, Shi, Du, PRD 114 (2026), Zhuang, Dominguez, Molina, 2606.03826

Mixture of $c\bar{n}, (c\bar{n})(n\bar{n}), (c\bar{n})(s\bar{s}), \dots?$

Single-charm states

SU(3) multiplets for light & strange quarks in $c\bar{q}\bar{q}q$: $\bar{3} \otimes \bar{3} \otimes 3 = \bar{3}_A \oplus 6_A \oplus \bar{15}_S \oplus \bar{3}_S$



Antisymmetric in $\bar{q}\bar{q}$:
scalar diquarks

Symmetric in $\bar{q}\bar{q}$:
axialvector diquarks

Diquark-antidiquark
picture

- Expectation:**

Guo, Hanhart, Commun.
Theor. Phys. 77 (2025)

(strongly) attractive
attractive

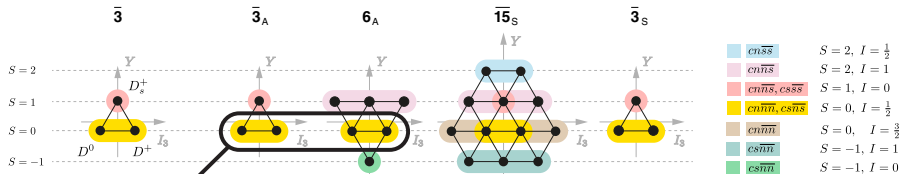
(strongly) repulsive
strongly repulsive

Meson-molecule
picture

- Similar trend on lattice Gregory, Guo, Hanhart, Krieg, Luu, EPJA 71 (2025)
- Functional methods? No preference on diquark/antidiquark or meson-meson, determined dynamically → investigate

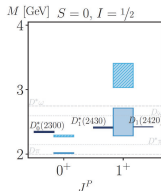
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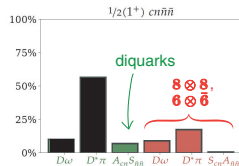
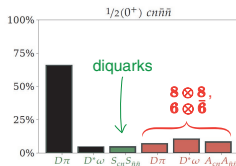


$\bar{3}_A, 6_A$:

GE, Fischer,
Hanhart,
Hoffer,
in preparation



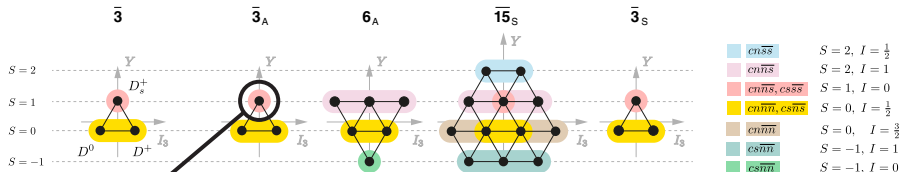
$c n n \bar{n}$ excited
 $c n n \bar{n}$ ground
experimental



- Compatible with two-pole structure (but so far no coupling to $c\bar{q}$, no axial anomaly)
- Meson-meson dominates, diquarks strongly suppressed

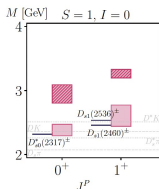
Single-charm states

SU(3) multiplets for light & strange quarks in $c\bar{q}\bar{q}q$: $\bar{3} \otimes \bar{3} \otimes 3 = \bar{3}_A \oplus 6_A \oplus \bar{15}_S \oplus \bar{3}_S$

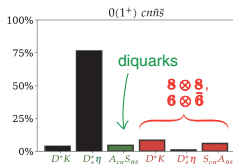
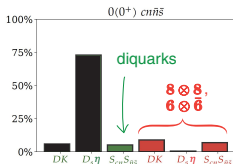


$\bar{3}_A$:

GE, Fischer,
Hanhart,
Hoffer,
in preparation

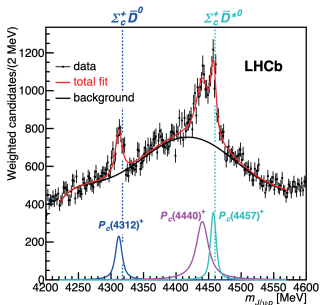


$c\bar{n}\bar{n}$ excited
 $c\bar{n}\bar{n}$ ground
experimental

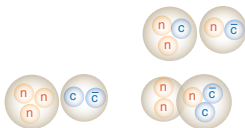


- Compatible with experiment (but so far no coupling to $c\bar{q}$, no axial anomaly)
- Meson-meson dominates, diquarks strongly suppressed

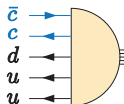
Pentaquarks?



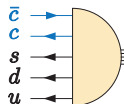
LHCb Collaboration,
PRL 122 (2019) 22, 22201
PRL 131 (2023) 3, 031901



$P_c^+(uudc\bar{c})$



$P_{cs}^0(udsc\bar{c})$



- Well-established states:

$P_c(4312)^+$
 $P_c(4440)^+$
 $P_c(4457)^+$

$P_{cs}(4338)^0$
 $P_{cs}(4459)^0$

narrow
(10... 20 MeV)

$P_c(4337)^+$... only $\sim 3\sigma$, threshold effect?
 $P_c(4380)^+$... broad (~ 200 MeV)

- Proximity to thresholds suggests molecular nature:

$\left\{ \begin{array}{l} \Sigma_c D, \\ \Lambda_c D, \\ N \eta_c (?), \\ \text{diquark-triquark } (?), \dots \end{array} \right.$

$\Xi_c D,$
 $\Lambda_c D_s,$
 $\Lambda \eta_c (?),$
Wu, Molina, Oset, Zou,
PRL 105 (2010), PRC 84
(2011), ...

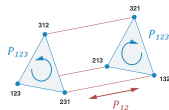
Five-body systems

- **Five-body equation**

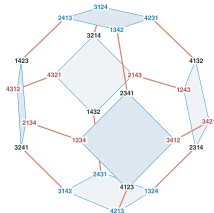
GE, **Torres**, PRD 111 (2025),
GE, Peña, **Torres**, PLB 866 (2025)

$$\Gamma = \sum_{\text{10 permutations}} \Gamma - \sum_{\text{15 permutations}} \Gamma$$

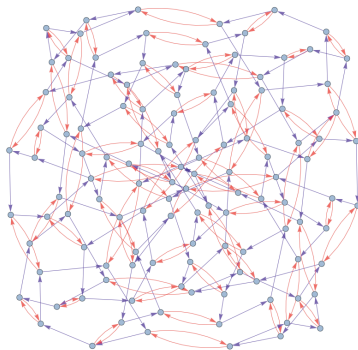
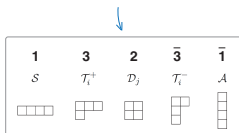
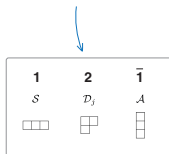
- Multiplets of permutation group S_5 ?



Cayley graph of S_3



Cayley graph of S_4



Cayley graph of S_5

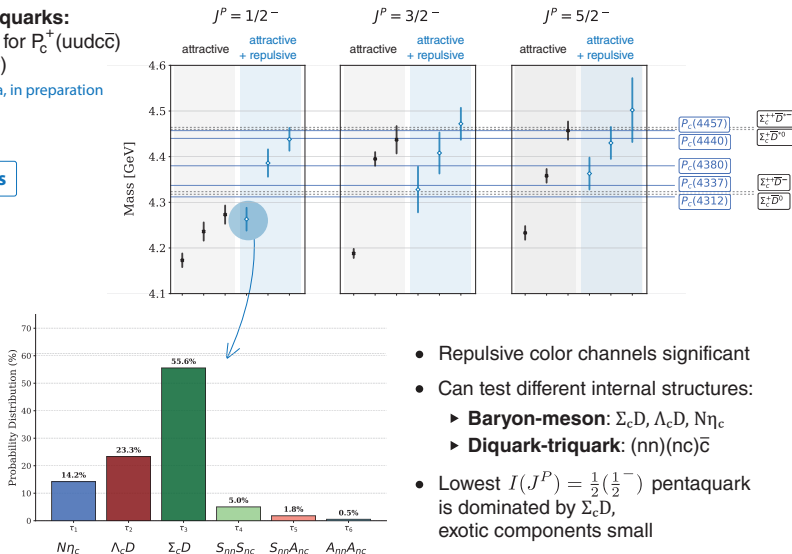
LHCb pentaquarks

QCD pentaquarks:

First results for $P_c^+(uudc\bar{c})$
(preliminary)

Torres, GE, Peña, in preparation

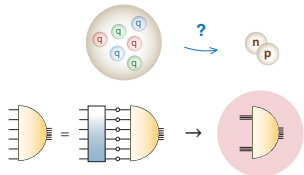
Raúl Torres



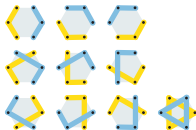
Six-body systems

- 6-body equations: Transition from quarks & gluons to **light nuclei**

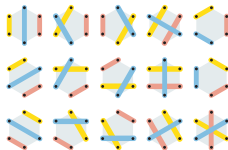
GE, **Hoffer**, in preparation



- 10 possible 3-body configurations:
(123)(456), ...
→ **baryon clusters**



- 15 possible 2-body configurations:
(12)(34)(56), ...
→ **diquark clusters**



$$\Gamma^{(\mu)}(p, q, k, l, r, P) = \sum_i f_i(\underbrace{18 \text{ variables}}_{\substack{1 + 1 + 5 + 5 + 9 \\ (+ 3 \text{ constraints})}}) \tau_i^{(\mu)}(p, q, k, l, r, P) \otimes \underbrace{\text{Flavor}}_{\substack{\begin{smallmatrix} \square & \square \\ \square & \square \end{smallmatrix} \\ 5 \\ \text{(deuteron)}}} \otimes \underbrace{\text{Color}}_{\substack{\begin{smallmatrix} \square & \square \\ \square & \square \end{smallmatrix} \\ \bar{5}}} \\ \approx f_i(S_0) \times \text{poles}$$

J = 0: 4096 tensors (320 s waves)
J = 1: 12288 tensors (576 s waves)

$\bar{1}, \bar{5}, \bar{9}, \bar{10}$

Joshua Hoffer

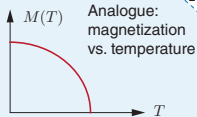
Summary

- Solving **multiquark BSEs** has become possible with new permutation-group tools
 - ▶ **4-quark BSE:** 4-quark states form meson-meson (+ diquark-antidiquark) clusters
[Hoffer, GE, Fischer, PRD 109 \(2024\)](#), [PRD 111 \(2025\)](#)
 - ▶ **5-quark BSE:** solved for scalar model, results for QCD pentaquarks underway
[GE, Torres, PRD 111 \(2025\)](#), [GE, Peña, Torres, PLB 866 \(2025\)](#) → [Raúl Torres](#)
 - ▶ **6-quark BSE:** under construction → [Joshua Hoffer](#)
- “Beyond rainbow-ladder” $\Leftrightarrow$ solve coupled DSEs including gluons
→ [Markus Huber](#), [Ángel Miramontes](#) and Tuesday session:
[Joannis Papavassiliou](#), [Mauricio Ferreira](#), [Jose Manuel Morgado](#), [Jonas Wessely](#), ...

Thank you!

Backup slides

Functional methods



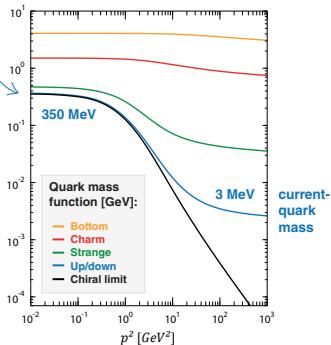
- Ingredients: **QCD's n-point functions**,
Satisfy quantum eqs. of motion (DSEs)

Quark mass

$$\begin{aligned}
 \text{---} \circ^{-1} &= \text{---}^{-1} + \text{---} \circ \text{---} \\
 \text{---} \bullet \text{---}^{-1} &= \text{---} \bullet \text{---}^{-1} + \text{---} \circ \text{---} + \text{---} \bullet \text{---} \\
 &\vdots
 \end{aligned}$$

The diagrams represent quark self-energy corrections. The first equation shows a bare quark line with a self-energy loop. The second equation shows a quark line with a self-energy loop and a gluon exchange loop. The third equation shows a quark line with a self-energy loop and a gluon exchange loop, plus a gluon exchange loop with a ghost loop.

"constituent-quark mass"

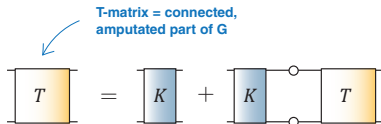


→ Dynamical mass generation,
gluon mass gap, confinement, ...

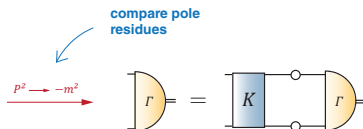


Bethe-Salpeter equations

Write down **inhomogeneous BSE**:

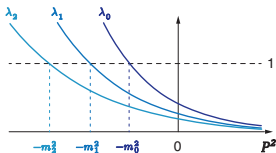


Homogeneous BSE at pole:



BSE = eigenvalue equation,
pole in T $\Leftrightarrow$ **eigenvalue = 1**

$$KG_0 \Gamma_i = \lambda_i \Gamma_i$$



Explicitly:

$$\Gamma(p, P) = \int \frac{d^4 q}{(2\pi)^4} K_{\alpha\gamma, \delta\beta}(p, q, P) [S(q_+) \Gamma(q, P) S(q_-)]_{\gamma\delta}$$

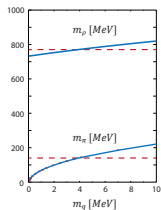
Basis decomposition:

$$\Gamma(p, P) = \sum_{i=1}^n f_i(p^2, \hat{p} \cdot \hat{P}, P^2) \tau_i(p, P)$$

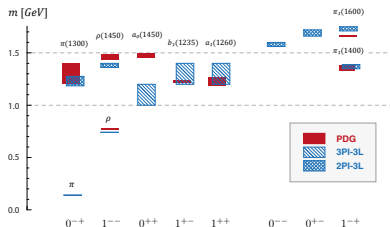
$\Rightarrow$ Coupled Lorentz-invariant equations
for the dressing functions f_i

Mesons

- Pion is **Goldstone boson**: $m_\pi^2 \sim m_q$



- Light meson spectrum** beyond rainbow-ladder

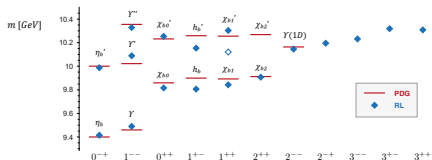


Williams, Fischer, Heupel,
PRD 93 (2016)

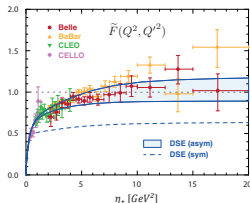
GE, Sanchis-Alepuz, Williams,
Alkofer, Fischer, PPNP 91 (2016)

- Bottomonium spectrum**

Fischer, Kubrak, Williams, EPJ A 51 (2015)



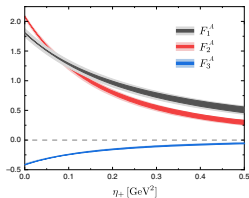
- Pion transition form factor**



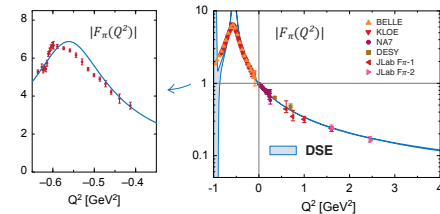
GE, Fischer, Weil, Williams,
PLB 774 (2017)

Form factors

- **Axial & scalar $\rightarrow 2\gamma$ FFs** for muon g-2
GE, Fischer, Häuser, Regenfelder, EPJC 85 (2025)

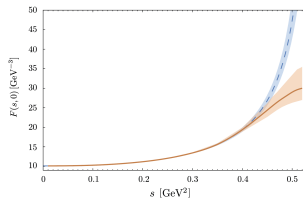
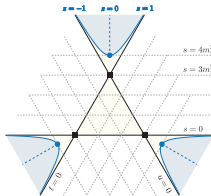
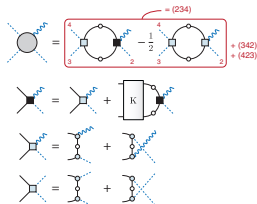


- **Timelike FFs** dynamically develop ρ pole with $\pi\pi$ decay channel
Miramontes, Sanchis-Alepuz, Alkofer, PRD 103 (2021)



GE, Fischer,
Williams,
PRD 101
(2020)

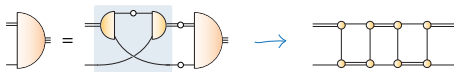
- **Timelike form factor $\gamma\pi \rightarrow \pi\pi$**
Miramontes, GE, Alkofer, 2504.20899



Diquark correlations

- Quark-diquark (two-body) equation**

Oettel et al., PRC 58 (1998), GE et al., Ann. Phys. 323 (2008), Cloet et al., FBS 46 (2009), Segovia et al., PRL 115 (2015), Chen et al., PRD 97 (2018)



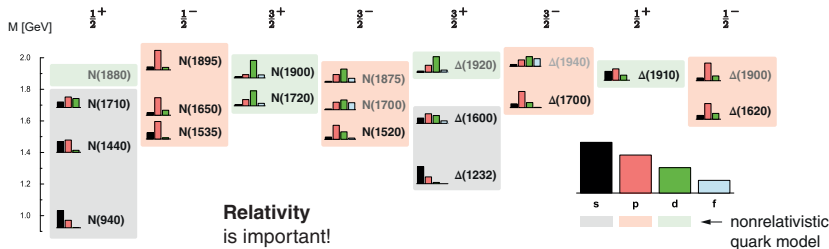
Diquark clustering in baryons?

Barabanov et al., Prog. Part. Nucl. Phys. 116 (2021)



- Three-quark and quark-diquark results very similar**

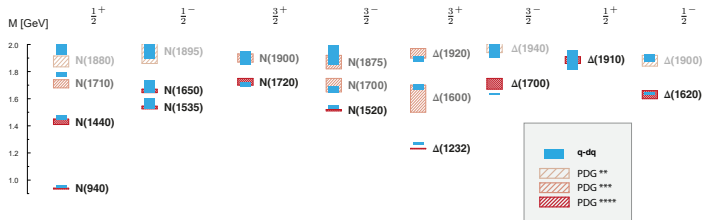
GE, Fischer, Sanchis-Alepuz, PRD 94 (2016), GE, FBS 63 (2022)



Diquark correlations

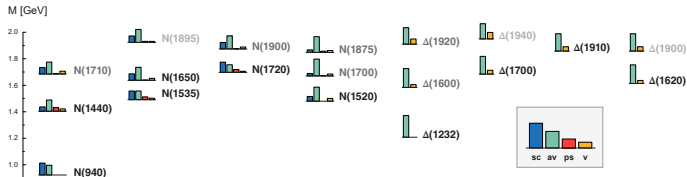
Light baryon spectrum

GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

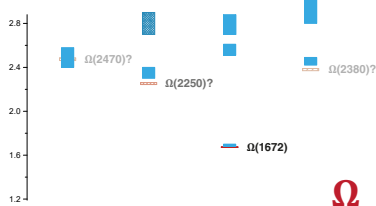
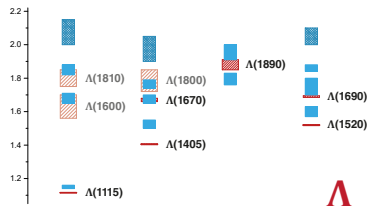
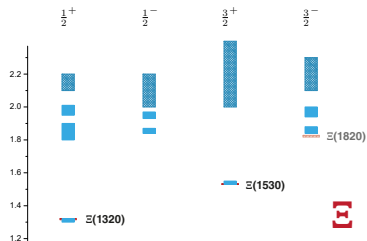
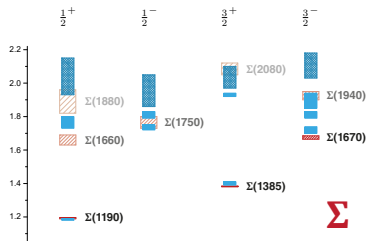


Diquark content:

Barabanov et al., PNP 116 (2021)

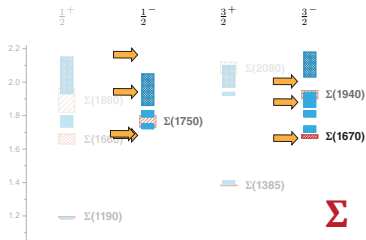


Strange baryons



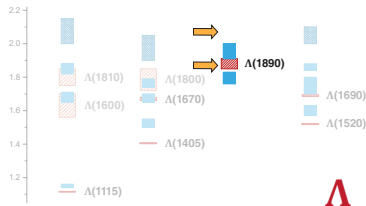
GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017

Strange baryons



New states from Bonn-Gatchina

Sarantsev et al., 1907.13387 [nucl-ex]



GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017

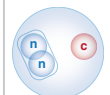
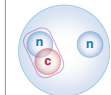
Heavy baryons

Quark content	q-dq	Isospin	contributes to
nnc	n [nc]	0, 1	Λ_c, Σ_c
$\uparrow$	n {nc}	0, 1	Λ_c, Σ_c
$n = u, d$	c [nn]	0	Λ_c
	c {nn}	1	Σ_c

$\Rightarrow \Lambda_c:$
 $\Sigma_c:$

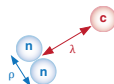
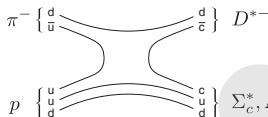
n [nc], n {nc},
n [nc], n {nc},

c [nn]
c {nn}



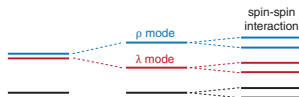
These are often assumed as dominant components

Kim, Hosaka, Kim, Noumi, Shirotori, PTEP 2014 (2014), 10, 103D01,
Shim, Hosaka, Kim, PTEP 2020 (2020) 5, 053D01



Decay of ρ mode $\rightarrow \Sigma_c \pi$
Decay of λ mode $\rightarrow p D$

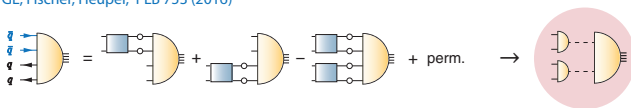
Assumptions
on spectrum &
production rates



Four-quark states

- Light scalar mesons (σ , κ , a_0 , f_0) as **four-quark states**:

GE, Fischer, Heupel, PLB 753 (2016)



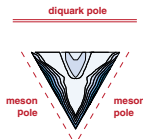
- BSE dynamically generates **meson poles** in BS amplitude:

$$f_i(S_0, \nabla, \triangle, \circ) \rightarrow 1500 \text{ MeV}$$

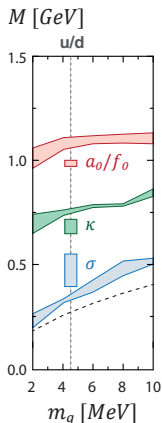
$$f_i(S_0, \nabla, \triangle, \circ) \rightarrow 1500 \text{ MeV}$$

$$f_i(S_0, \nabla, \triangle, \circ) \rightarrow 1200 \text{ MeV}$$

$$f_i(S_0, \nabla, \triangle, \circ) \rightarrow \mathbf{350 \text{ MeV !}}$$



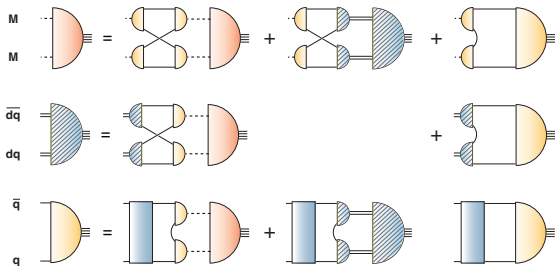
- “Light scalar mesons” look like **meson molecules**,
diquark-antidiquark components almost negligible.
Lightness is inherited from pseudoscalar Goldstone bosons!



Four-quark states

Two-body formulation: **meson-meson / diquark-antidiquark**, follows from four-quark eq. (analogue of quark-diquark for baryons)

Heupel, GE, Fischer, PLB 718 (2012)



Include **mixing with $q\bar{q}$** :
 $\pi\pi$ still dominant

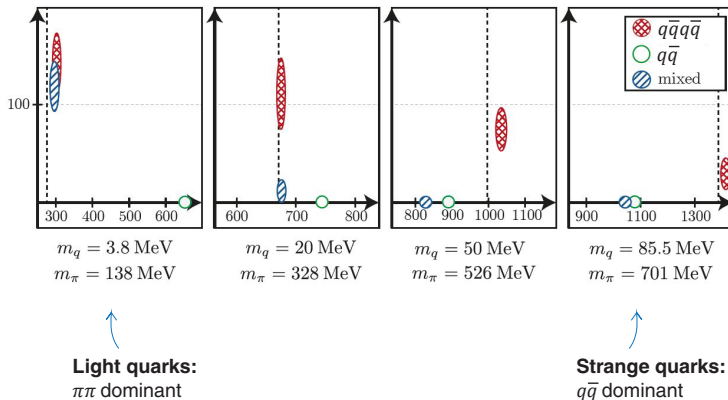
Santowsky, GE, Fischer, Wallbott,
 Williams, PRD 102 (2020)

[MeV]	ground state mass	first excitation
$\pi\pi$	416 ± 26	970 ± 130
$\pi\pi + 0^+0^+$	416 ± 26	970 ± 130
$q\bar{q}$	667 ± 2	1036 ± 8
$\pi\pi + q\bar{q}$	472 ± 22	1080 ± 280
$\pi\pi + 0^+0^+ + q\bar{q}$	456 ± 24	1110 ± 110

Four-quark states

Four-quark vs. $q\bar{q}$ dominance

Santowsky, Fischer, PRD 105 (2022)

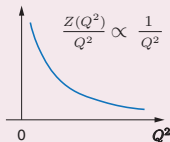


Towards ab-initio

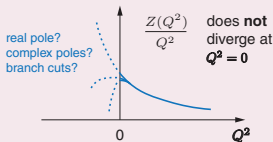
Gluon propagator:

$$D^{\mu\nu}(Q) = \underbrace{\frac{Z(Q^2)}{Q^2}}_{\text{transverse dressing}} \left(\delta^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) + \xi \underbrace{\frac{L(Q^2)}{Q^2}}_{\text{longitudinal dressing} = 1} \frac{Q^\mu Q^\nu}{Q^2}$$

- **Perturbation theory:**
Massless gluon pole



- **Nonperturbative calculations:**
Massless pole disappears!



Family of “**decoupling**” solutions, also seen in lattice QCD

Cucchieri, Maas, Mendes, PRD 77 (2008)
Boucaud et al., JHEP 06 (2008)
Bogolubsky et al., PLB 676 (2009)
Fischer, Maas, Pawłowski, Ann. Phys. 324 (2009)
Duarte, Oliveira, Silva, PRD 94 (2016)
Aguilar et al., EPJ C 80 (2020)

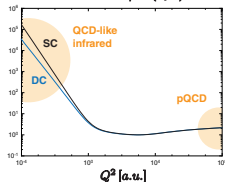
Endpoint is “**scaling**” solution, confinement manifest

Lerche, Smekal, PRD 65 (2002)
Fischer, Alkofer, PLB 536 (2002)
Alkofer, Fischer, Llanes-Estrada, MPLA 23 (2008)

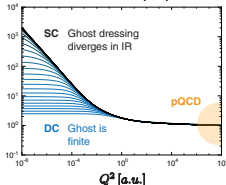
All solutions show **gluon mass gap**

$$\lim_{r \rightarrow \infty} \int \frac{d^3 Q}{(2\pi)^3} \frac{Z(Q^2)}{Q^2} e^{i \mathbf{x} \cdot \mathbf{Q}} \propto e^{-m_{\text{gap}} r}$$

Gluon $1/Z(Q^2)$



Ghost $G(Q^2)$



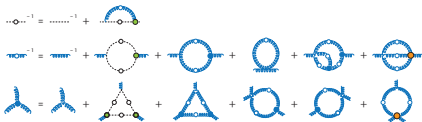
Gluon mass gap

Gluon propagator:

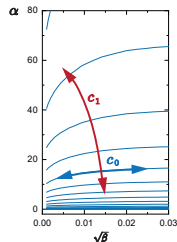
$$D^{\mu\nu}(Q) = \underbrace{\frac{Z(Q^2)}{Q^2}}_{\text{transverse dressing}} \left(\delta^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) + \xi \underbrace{\frac{L(Q^2)}{Q^2}}_{\text{longitudinal dressing} = 1} \frac{Q^\mu Q^\nu}{Q^2}$$

Coupled Yang-Mills DSEs

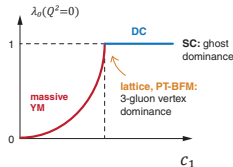
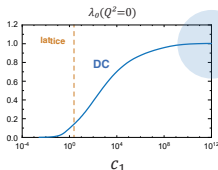
Huber, PRD 101 (2020), GE, Pawłowski, Silva, PRD 104 (2021)



- SC & DC solutions distinguished by c_1 (combination of coupling α and mass parameter β)
- Scaling solution: $c_1 \rightarrow \infty$



Gauge consistency ($L = 1$) satisfied only by SC solution



Possible scenario for consistency of DC solutions:

Then all massless (SC, DC) solutions would be physically equivalent; distinguished from massive solutions

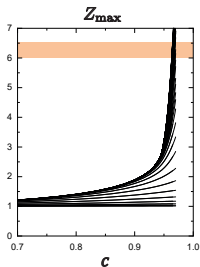
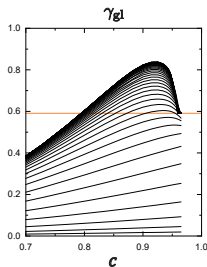
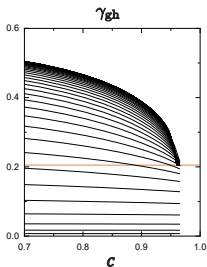
Truncation error

- Set $Z_{3g} \rightarrow c Z_{3g}$... quantifies deviation from STI
(without truncation: $c = 1$)

- Anomalous dimensions reproduced for

1	$c \sim 0.4$
2	$c \sim 0.9$
3	$c \sim 0.96$

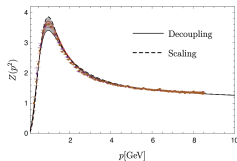
 } $\Rightarrow$ identifies “physical point”
for each truncation
[GE, Pawłowski, Silva, PRD 104 \(2021\)](#)



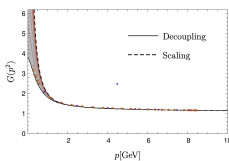
Towards precision

Markus Huber, ACHT 2025; Phys. Rept. 879 (2020) & PRD 101 (2020)

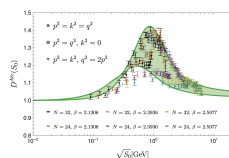
- Gluon propagator



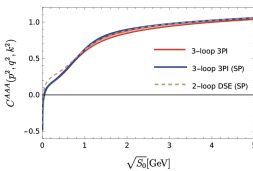
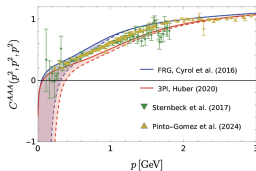
- Ghost propagator



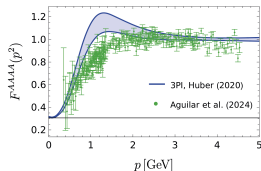
- Ghost-gluon vertex



- Three-gluon vertex: lattice vs. FRG vs. DSE



- Four-gluon vertex



Lattice: Pinto-Gómez, de Soto, Rodríguez-Quintero, PLB 838 (2023), Sternbeck et al., Pos LATTICE 2016; **FRG:** Cyrol et al., PRD 94 (2016); **DSE:** Huber, PRD 101 (2020), Huber, Fischer, Sanchis-Alepuz, 2503.03821

Lattice: Aguilar et al., PLB 858 (2024)
DSE: Huber, PRD 101 (2020)