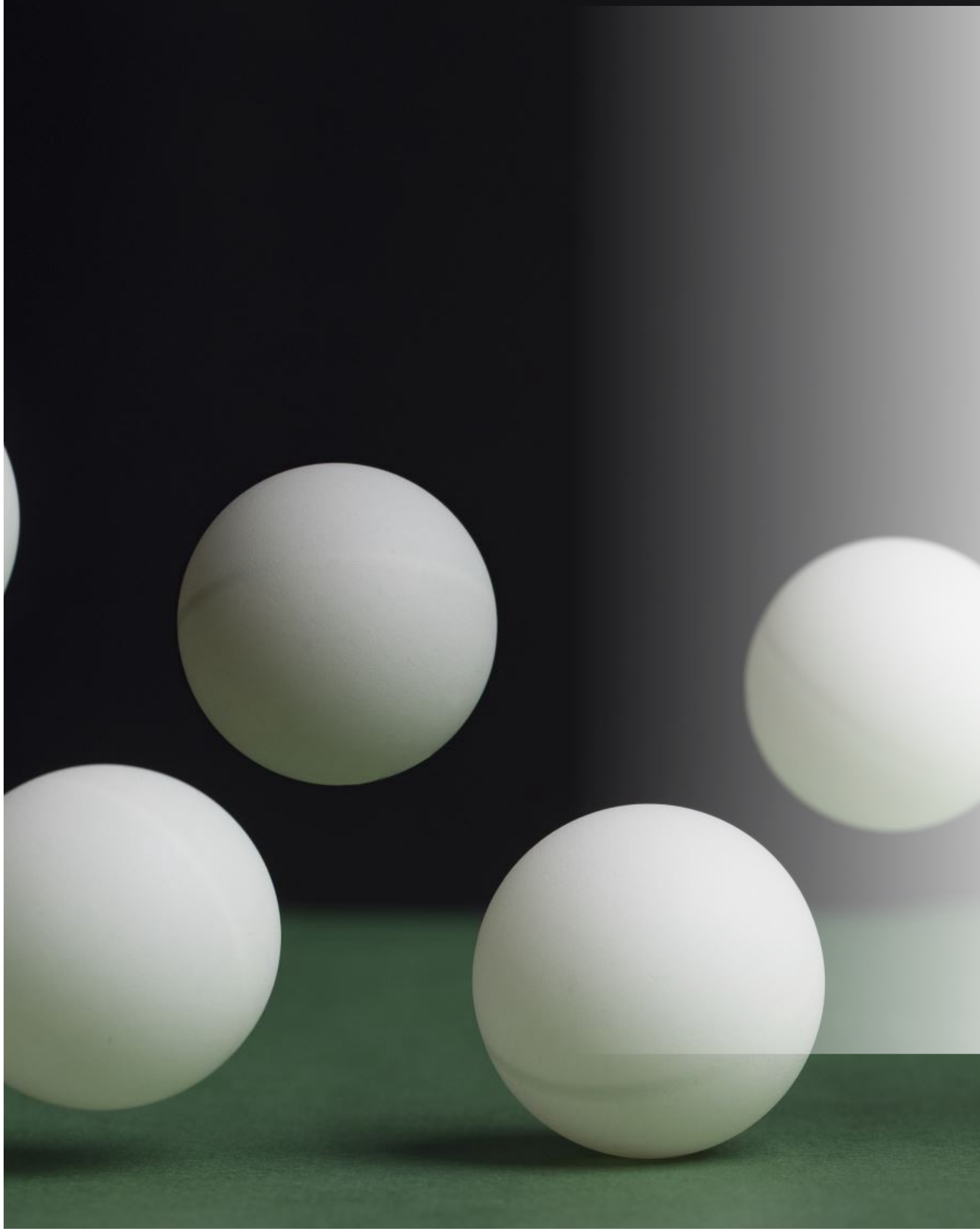

Dynamical Coupled Channel Models for Hadron Dynamics

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Thomas Jefferson National Accelerator Facility

Outline

- Overview of formalism and applications of DCC models [2505.02745/Review]
  - Meson and photon-induced production of light baryons
  - Mesons: The curious case of the K(1460) [2511.02543]
- $\boldsymbol{\Lambda}$ and $\boldsymbol{\Sigma}$ spectrum with DCC → D. Rönchen's talk
- Three-body unitary DCC calculations for excited mesons → S. Nakamura's talk
- (πN)- and three$(\pi\pi N)$-body effects in baryons (IVU) → Jinzi Wu's talk
- Baryon transition form factors → Yufei Wang's talk
- Unitarized triangle effects → Ajay Sakthivasan's talk

- Plenary talk by T. Sato on the EBAC and ANL/Osaka amplitudes

- [other non-DCC approaches for data analysis: SAID, Bonn-Gatchin, MAID, JPAC,….]

Work supported by: National Science Foundation Grant No. PHY 2310036

White paper baryon spectroscopy

Physics opportunities with meson beams [1503.07763]

William J. Briscoe, Michael Döring, Helmut Haberzettl, D. Mark Manley, Megumi Naruki, Igor I. Strakovsky and Eric S. Swanson

Eur. Phys. J. A (2015) **51**: 129 DOI 10.1140/epja/i2015-15129-5

Strange Hadron Spectroscopy with Secondary KL Beam in Hall D

KLF Collaboration • Moskov Amaryan (Old Dominion U.) Show All(152)

Aug 18, 2020 [2008.08215]

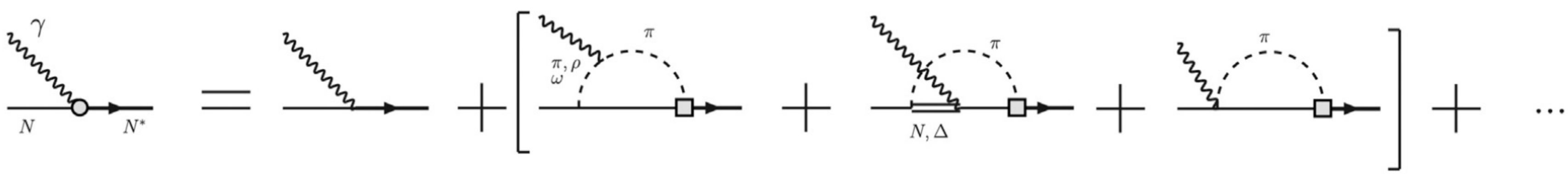
Hadron Physics Opportunities at FAIR [2025.15986]

J. G. Messchendorp[a,*], F. Nerling[a,b,c,**], P. Achenbach[d], J. Aichelin[e,f], M. Albaladejo[g], L. An[h], K. Aoki[i], G. Appagere[j],

Different DCC approaches (baryons)

[2505.02745/Review and https://collaborations.fz-juelich.de/ikp/meson-baryon/main https://www.rcnp.osaka-u.ac.jp/~anl-osk/]

- EBAC and ANL/Osaka analysis.
- Juelich-Bonn (JuBo)-Washington/JBW analysis
  - Both share many aspects of channel space and formalism but differ in some details such as coupling of the photon. ANL/Osaka:



  - ANL/Osaka has been extended to analysis of $\pi\pi$N final, KY, and neutrino-induced reactions
  - JBW analysis focused on electroproduction reactions, also extensions to charm
- Related approaches in 3-body meson physics (Nakamura, IVU) and infinite-volume physics for lattice QCD (FVU) Maxim Mai's talk

Three-body unitarity with isobars *

[1706.06118]

$$\langle q_1, q_2, q_3|(\hat{T}-\hat{T}^\dagger)|p_1, p_2, p_3\rangle = i\int_P \langle q_1, q_2, q_3|\hat{T}^\dagger|k_1, k_2, k_3\rangle\langle k_1, k_2, k_3|\hat{T}|p_1, p_2, p_3\rangle$$

$$\times \prod_{\ell=1}^{3}\left[\frac{\mathrm{d}^4 k_\ell}{(2\pi)^4}(2\pi)\boxed{\delta^+(k_\ell^2 - m^2)}\right](2\pi)^4\delta^4\left(P - \sum_{\ell=1}^{3} k_\ell\right)$$

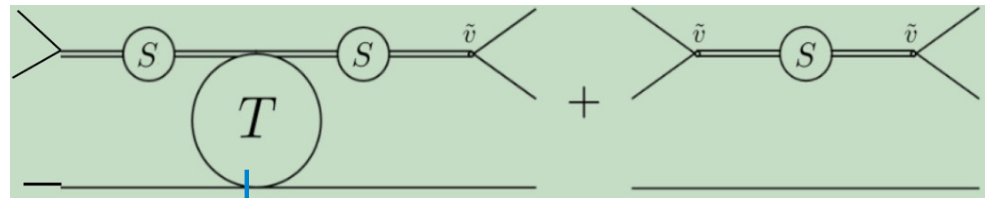
delta function sets all intermediate particles on-shell

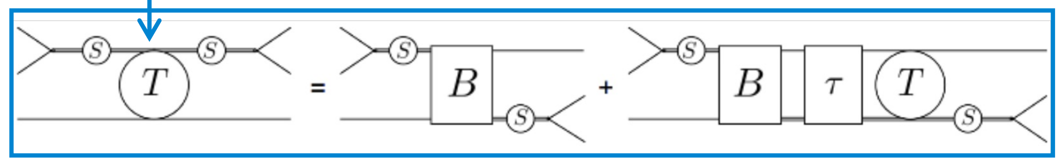
Idea: To construct a 3B amplitude, start directly from unitarity (based on ideas of 60's); match a general amplitude to it

* "Isobar" stands for two-body sub-amplitude which can be resonant or not; can be matched to CHPT expansion to one loop if desired. Isobars are re-parametrizations of full 2-body amplitudes [Bedaque] [Hammer]

Three-body unitarity

$$\langle q_1, q_2, q_3|(\hat{T}-\hat{T}^\dagger)|p_1, p_2, p_3\rangle = i\int_P \langle q_1, q_2, q_3|\hat{T}^\dagger|k_1, k_2, k_3\rangle\langle k_1, k_2, k_3|\hat{T}|p_1, p_2, p_3\rangle$$

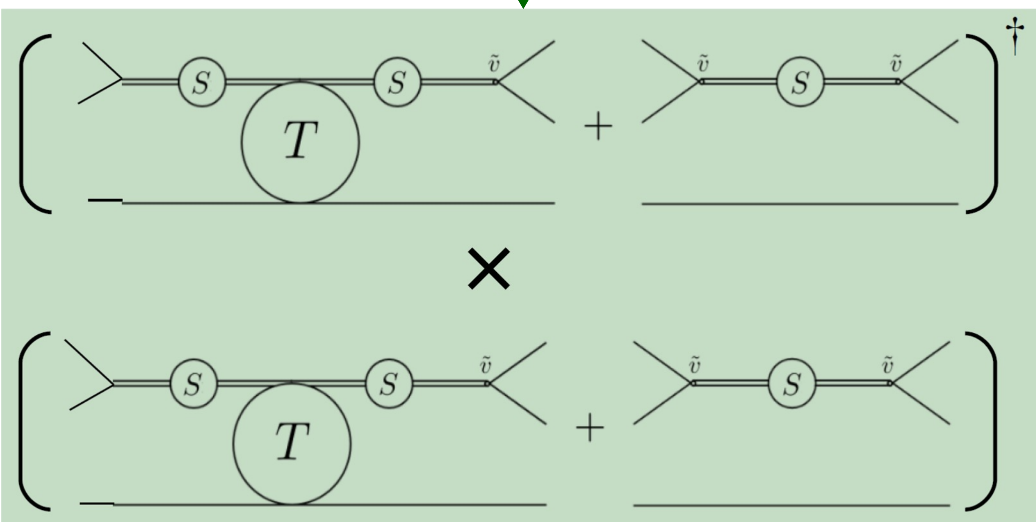




General Ansatz for the isobar-spectator interaction
→ **B & τ** are **new** unknown functions

Three-body unitarity

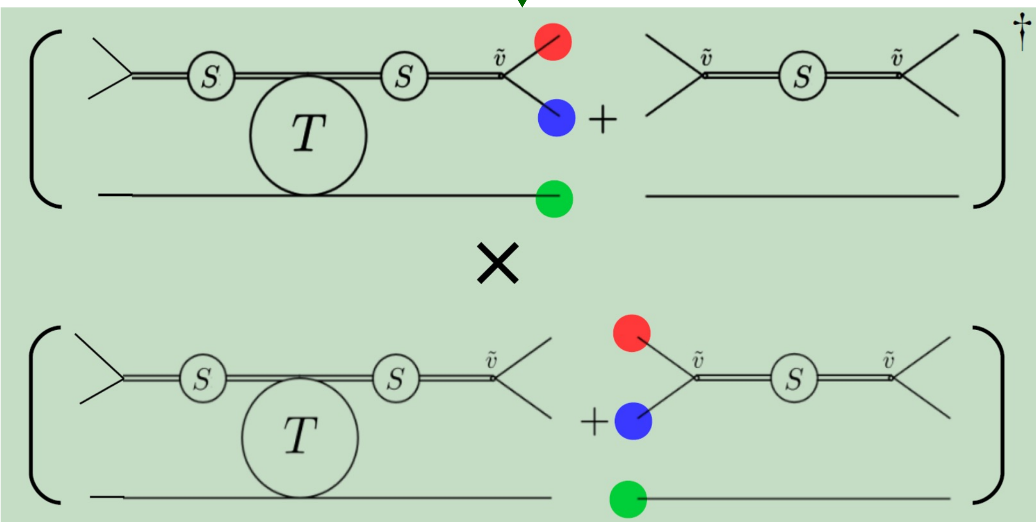
$$\langle q_1, q_2, q_3|(\hat{T}-\hat{T}^\dagger)|p_1, p_2, p_3\rangle \quad = \quad i\int_P \langle q_1, q_2, q_3|\hat{T}^\dagger|k_1, k_2, k_3\rangle\langle k_1, k_2, k_3|\hat{T}|p_1, p_2, p_3\rangle$$

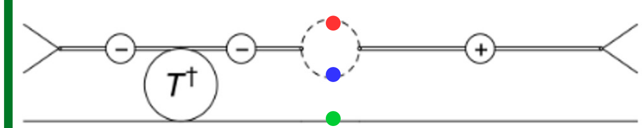


General connected-disconnected structure

Three-body unitarity

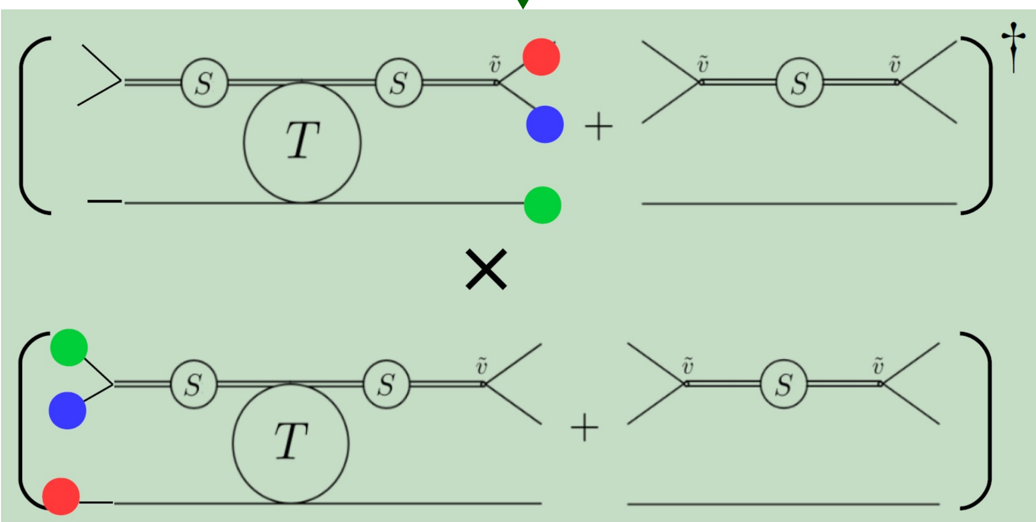
$$\langle q_1,q_2,q_3|(\hat{T}-\hat{T}^\dagger)|p_1,p_2,p_3\rangle = i\int_P \langle q_1,q_2,q_3|\hat{T}^\dagger|k_1,k_2,k_3\rangle\langle k_1,k_2,k_3|\hat{T}|p_1,p_2,p_3\rangle$$

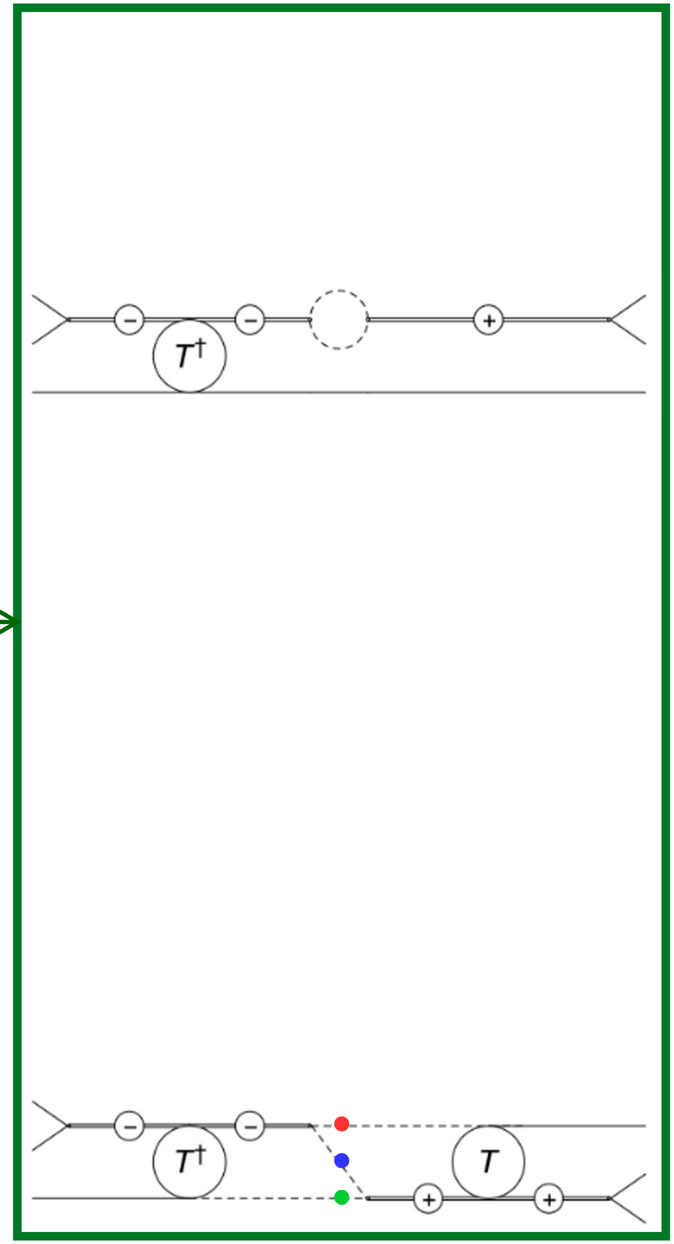




Three-body unitarity

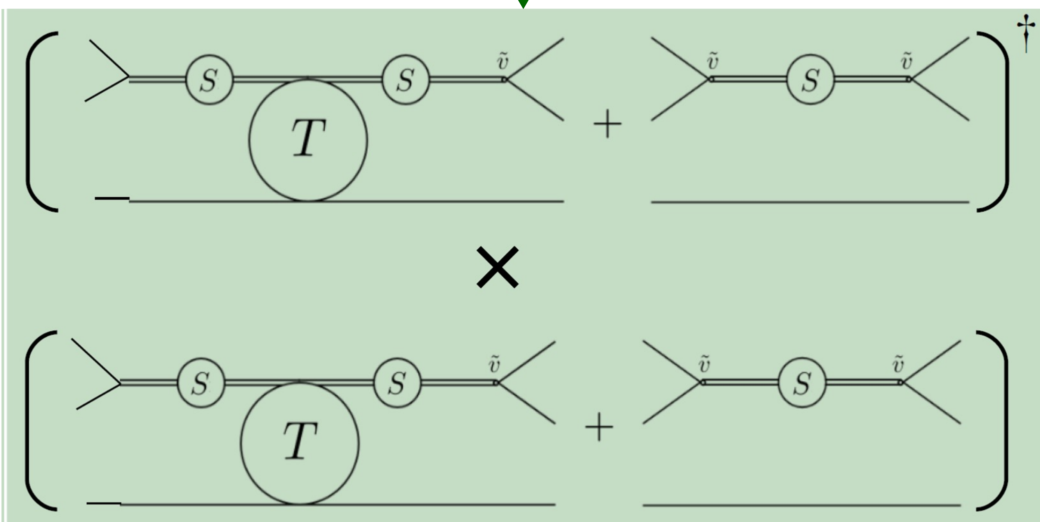
$$\langle q_1, q_2, q_3|(\hat{T}-\hat{T}^\dagger)|p_1, p_2, p_3\rangle \quad = \quad i\int_P \langle q_1, q_2, q_3|\hat{T}^\dagger|k_1, k_2, k_3\rangle\langle k_1, k_2, k_3|\hat{T}|p_1, p_2, p_3\rangle$$



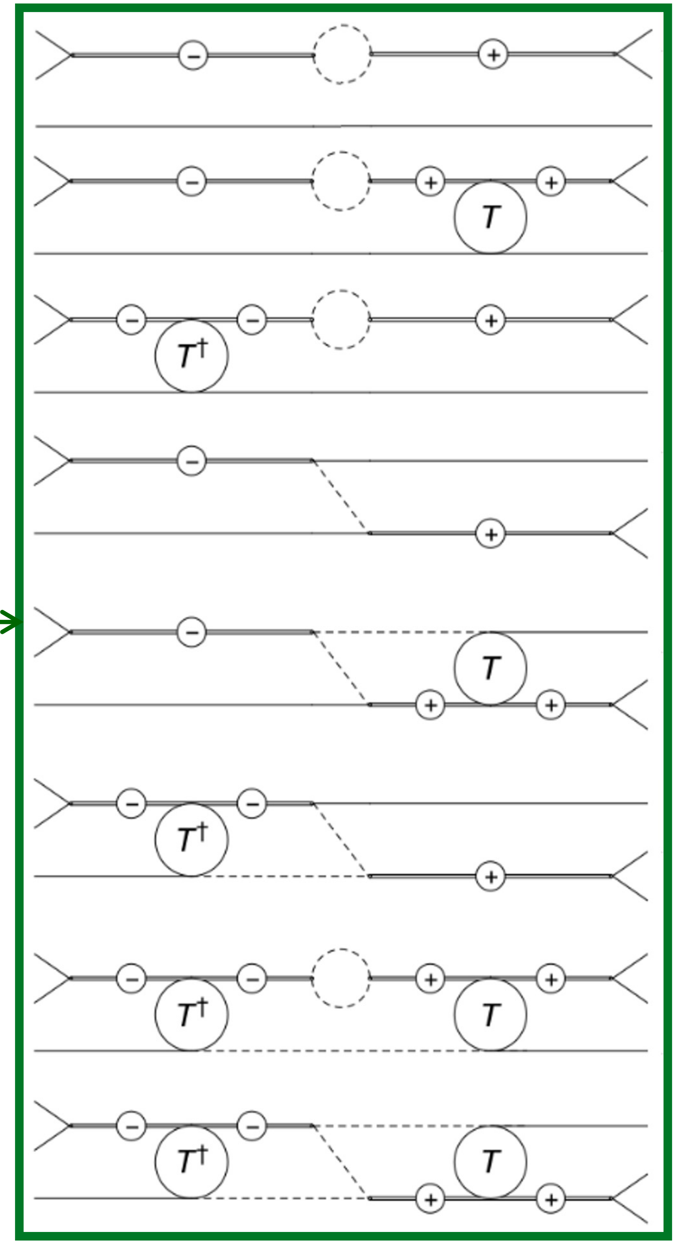


Three-body unitarity

$$\langle q_1, q_2, q_3|(\hat{T}-\hat{T}^\dagger)|p_1,p_2,p_3\rangle \quad = \quad i\int_P \langle q_1,q_2,q_3|\hat{T}^\dagger|k_1,k_2,k_3\rangle\langle k_1,k_2,k_3|\hat{T}|p_1,p_2,p_3\rangle$$

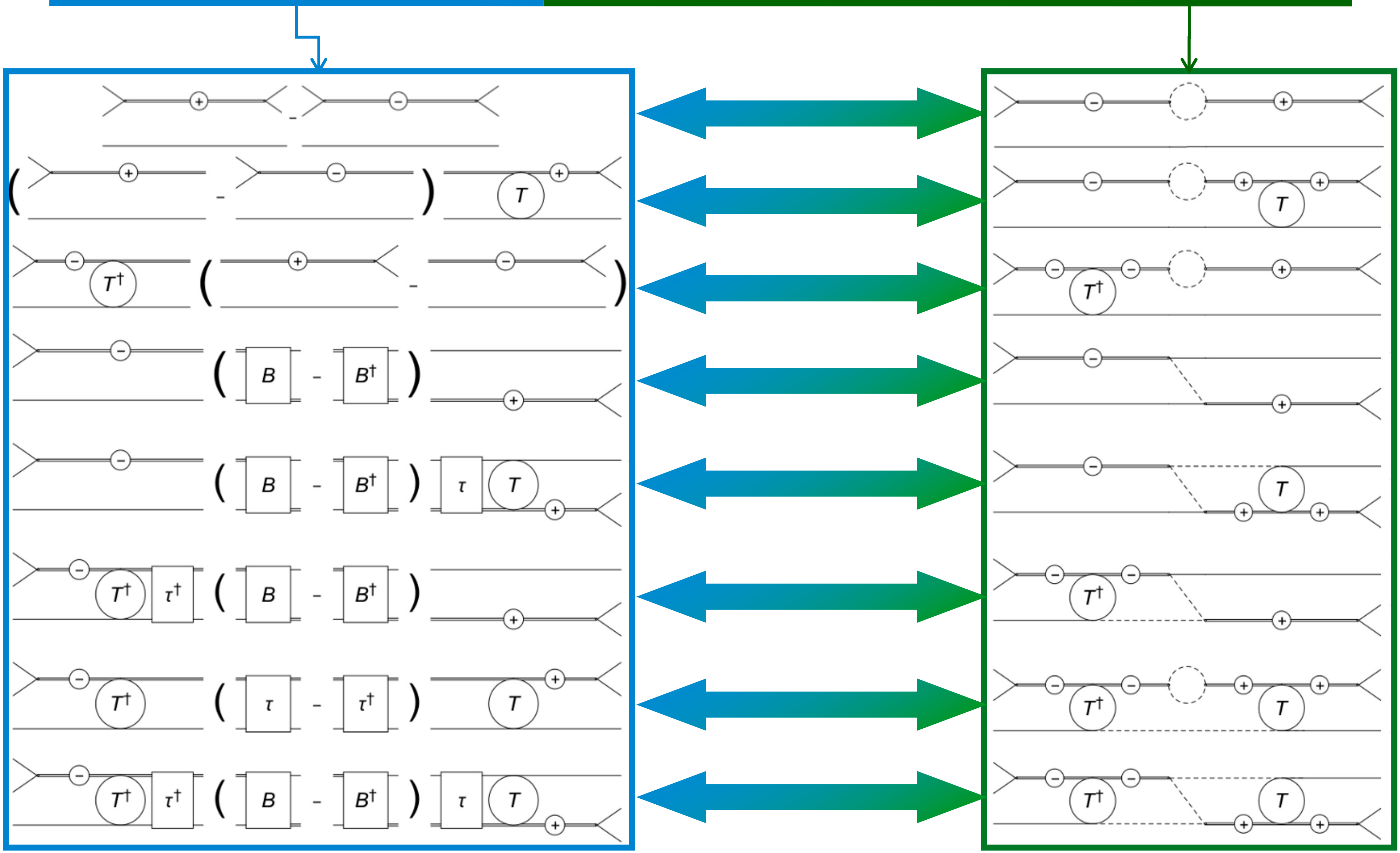


8 topologies



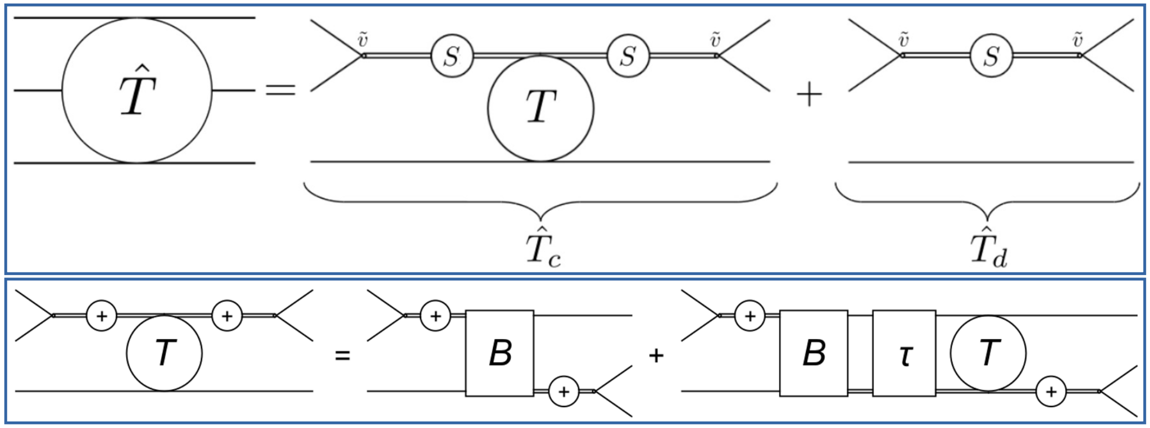
Three-body unitarity

$$\langle q_1,q_2,q_3|(\hat{T}-\hat{T}^\dagger)|p_1,p_2,p_3\rangle \;=\; i\int_P \langle q_1,q_2,q_3|\hat{T}^\dagger|k_1,k_2,k_3\rangle\langle k_1,k_2,k_3|\hat{T}|p_1,p_2,p_3\rangle$$



Scattering amplitude

$3 \to 3$ scattering amplitude is a 3-dimensional integral equation



LS-type

- Imaginary parts of ***B, S*** are fixed by **unitarity/matching**
- B, S are determined **consistently** through 8 different relations

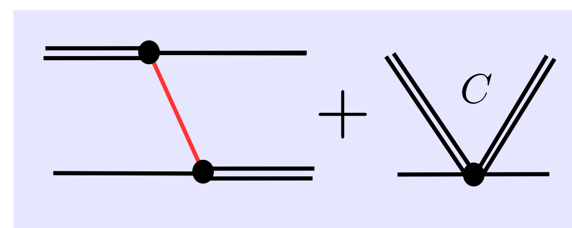
Matching → $\mathrm{Disc}\, B(u) = 2\pi i \lambda^2 \dfrac{\delta\left(E_Q - \sqrt{m^2+\mathbf{Q}^2}\right)}{2\sqrt{m^2+\mathbf{Q}^2}}$

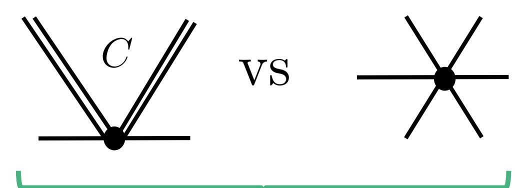
- un-subtracted dispersion relation

$$\langle q|B(s)|p\rangle = -\frac{\lambda^2}{2\sqrt{m^2+\mathbf{Q}^2}\left(E_Q-\sqrt{m^2+\mathbf{Q}^2}+i\epsilon\right)} + C$$

- one-**π** exchange in TOPT ⟶ *RESULT, NOT INPUT !*

- One <u>can</u> map to field theory but does not have to. Result is a-priori dispersive.

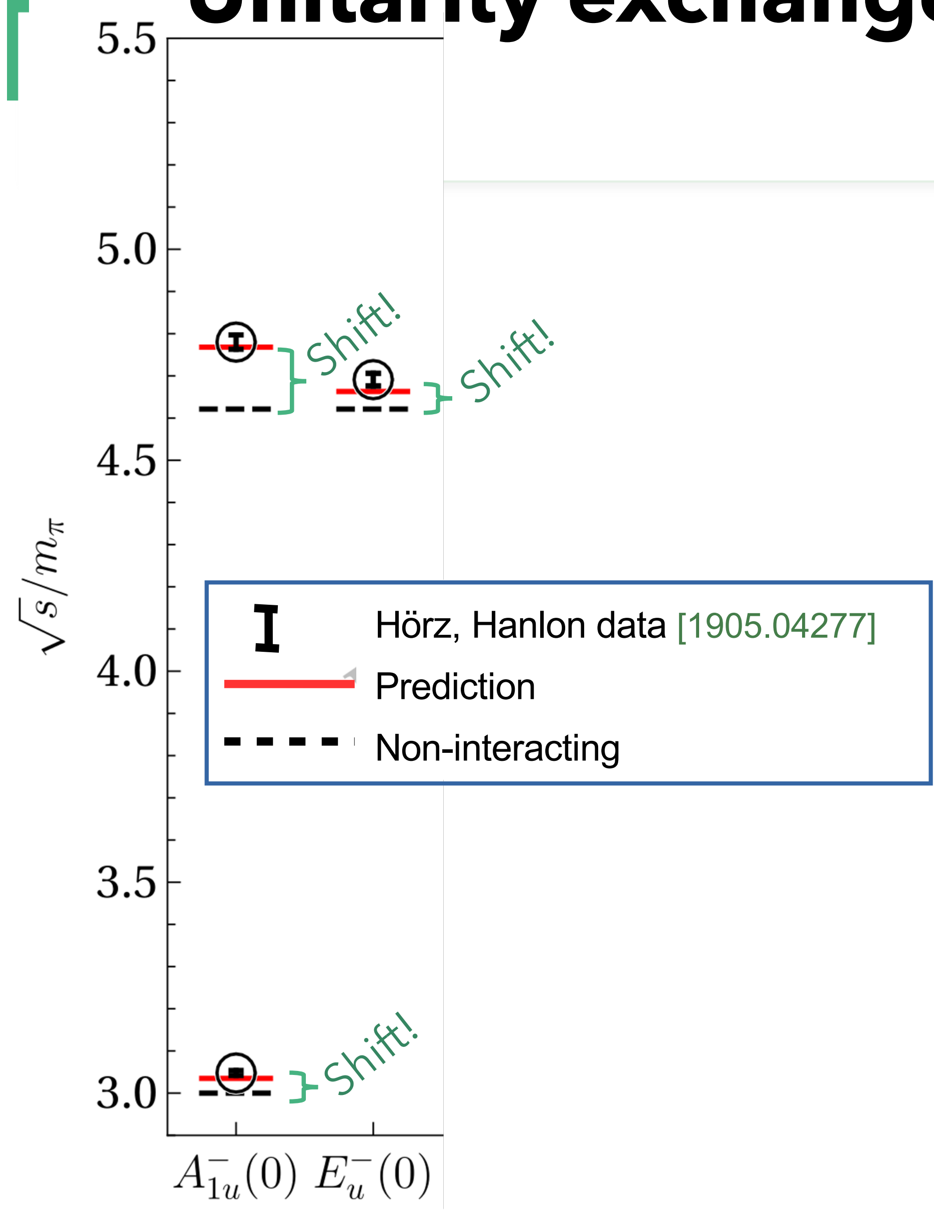




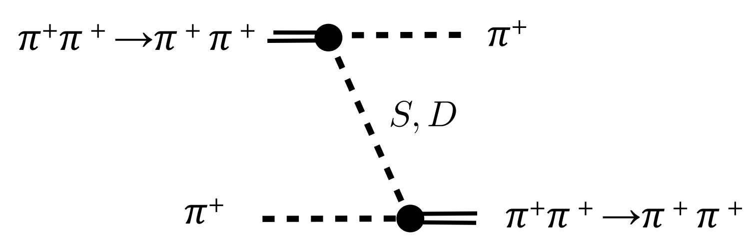
Add. Steps to map to theory might be needed [Brett (2021)]

Unitarity exchanges visible in lattice: $3\pi^+$

[1911.09047, GWQCD/Culver et al.]



S **D** (lowest participating wave)

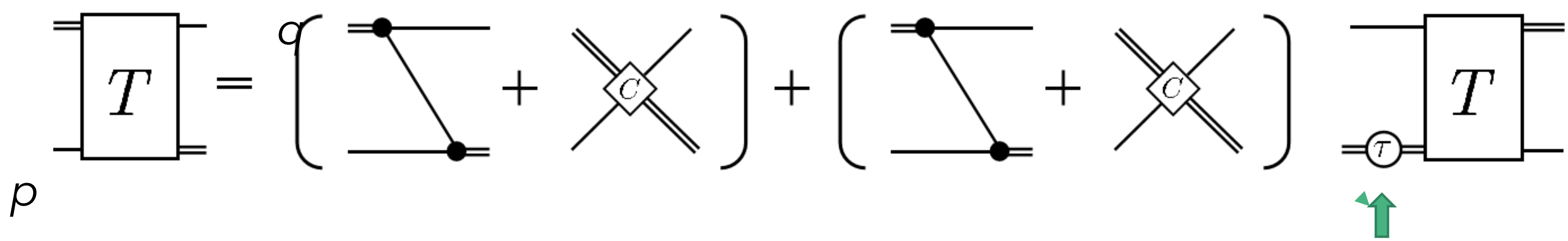


Particle exchange quantitatively predicts energy eigenvalues

Scattering equation

- 4-dim BSE requires 3D reduction

$$\langle q|\,T(s)\,|p\rangle = \langle q|\,B(s)\,|p\rangle + \langle q|\,C(s)\,|p\rangle + \int \frac{d^4k}{(2\pi)^4}\,\langle q|\,(B(s)+C(s))\,|k\rangle\,\tau(\sigma(k))\,\langle k|\,T(s)\,|p\rangle\ ,$$



Exchange:
- Complex
- Required by unitarity

Contact term:
- Does not destroy unitarity
- Free parametrization: fit to data

Isobar-spectator Green's functions

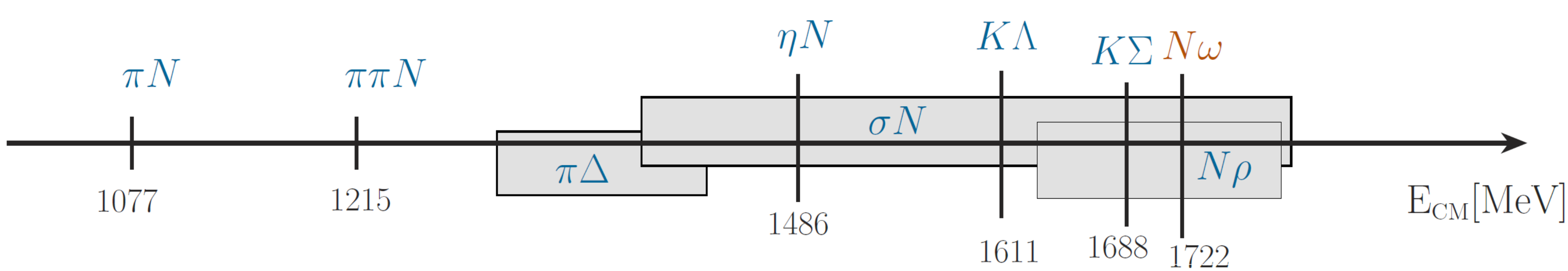
JBW DCC approach (Jülich-Bonn-Washington)

Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

The scattering equation in partial-wave basis

$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty dq \quad q^2 \quad \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle \frac{1}{E-E_\gamma(q)+i\epsilon} \langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$

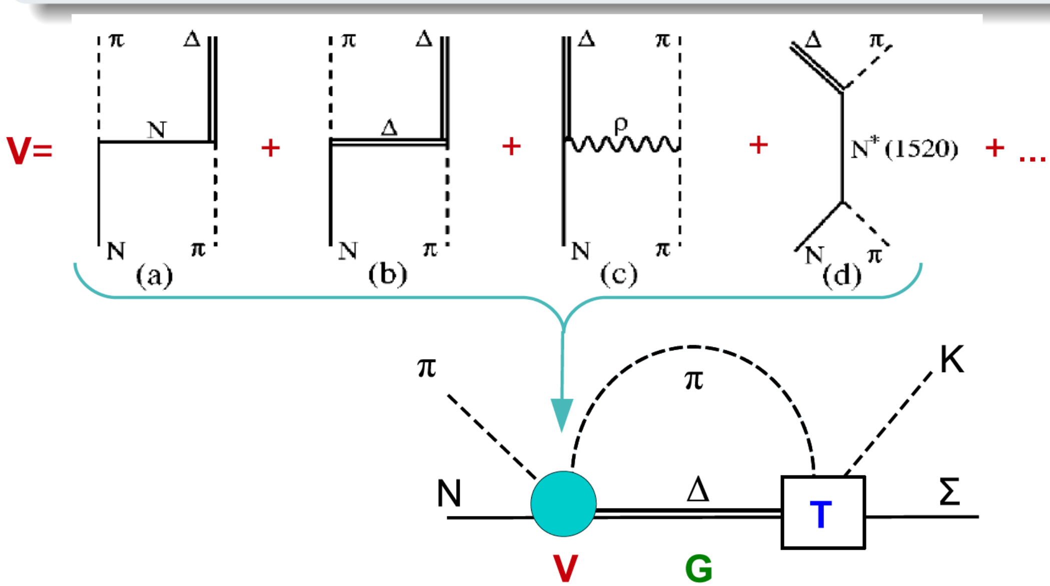
- channels ν, μ, γ:



JBW DCC approach (Jülich-Bonn-Washington)

The scattering equation in partial-wave basis

$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty dq \quad q^2 \quad \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle \frac{1}{E-E_\gamma(q)+i\epsilon} \langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$



- potentials V constructed from effective $\mathcal{L}$
- s-channel diagrams: T^P

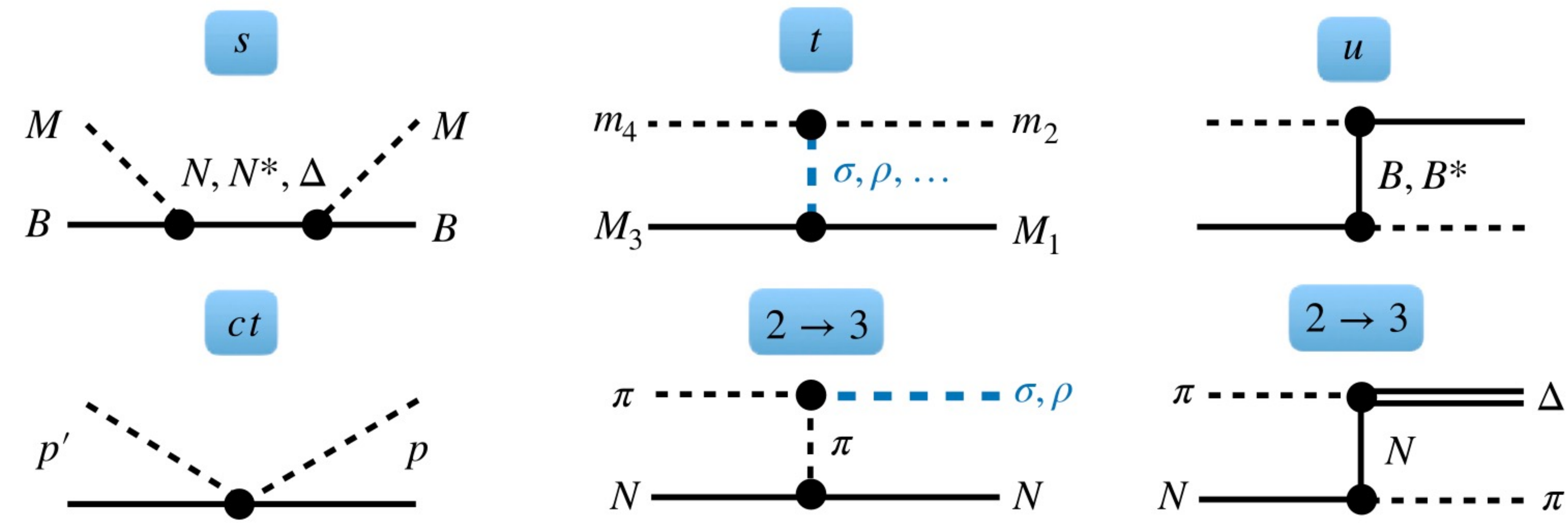
  genuine resonance states
- t- and u-channel: T^{NP}

  dynamical generation of poles

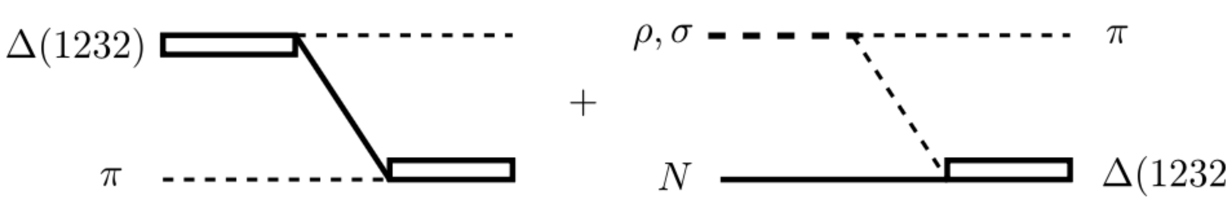
  partial waves strongly correlated
- contact terms

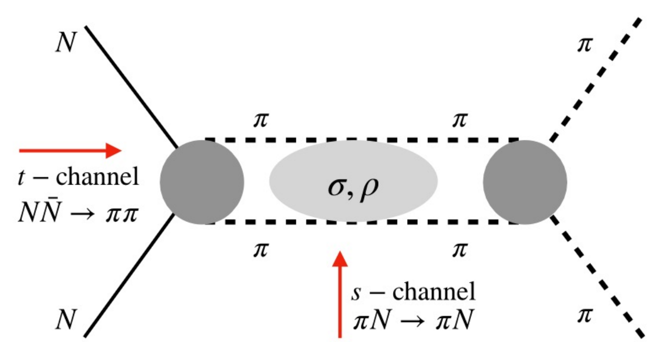
Details channel transitions

- 2→2 and 2→3 transitions



- 3→3 transitions
- crossed channels





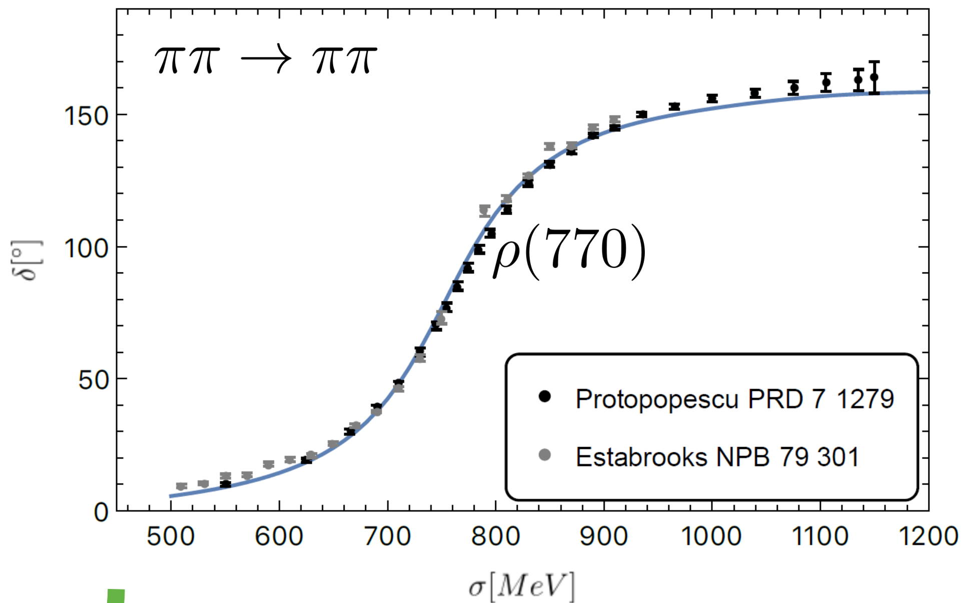
Transitions in s, t, and u-channels

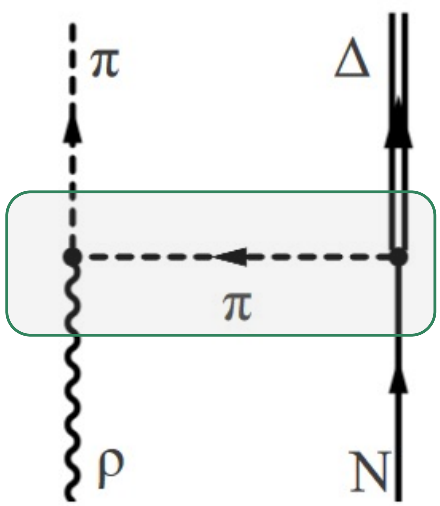
- 21 s-channel exchanges (resonance)
- Contact terms
- t and u-channel exchanges (ANL/Osaka & JuBo/JBW):

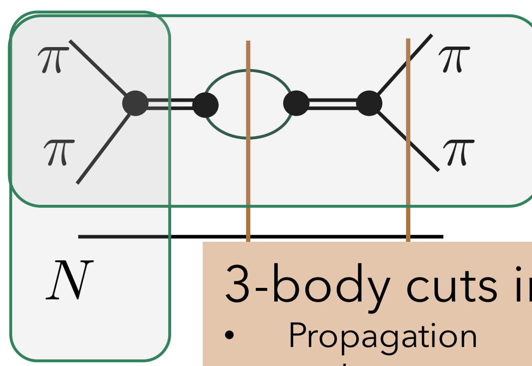
μ	πN	ηN	$K\Lambda$	$K\Sigma$	ωN	$\pi\Delta$	σN	ρN
πN	N, Δ, $(\pi\pi)_\sigma$, σ, f_0 $(\pi\pi)_\rho$, ρ	a_0, N	K^*, Σ, Σ^* , κ	K^*, Λ, Σ, Σ^* , κ	ρ, N	ρ, N, Δ	π, N	π, ω, a_1, N , Δ, C
ηN		N, f_0	K^*, Λ, κ	K^*, Σ , Σ^*, κ	ω, N		N	N
$K\Lambda$			ω, f_0, ϕ, Ξ, Ξ^*	ρ, a_0, Ξ, Ξ^*	K, K^*, Λ			
$K\Sigma$				ρ, ω, ϕ, f_0, a_0, Ξ, Ξ^*	K, K^*, Σ, Σ^*			
ωN					σ, N			
$\pi\Delta$						ρ, N, Δ	π	π, N
σN							σ, N	N
ρN								ρ, N, Δ, C

Three-body channels $\sigma N, \pi\Delta, \rho N$

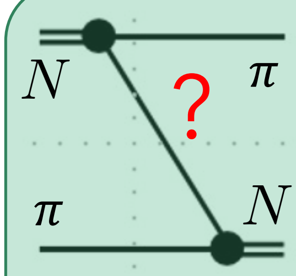
- Resonant sub-channels
- Fit 2→2 amplitude to 2→ 2 scattering data
- Include as sub-channel in 3-body amplitude:
- 3-body unitarity: Requires, e.g.







3-body cuts in
- Propagation
- exchange



J. Wu's talk

JuBo: Data base

- $\pi N \to X$: > **7,000** data points ($\pi N \to \pi N$: GW-SAID WI08 (ED solution))
- $\gamma N \to X$:

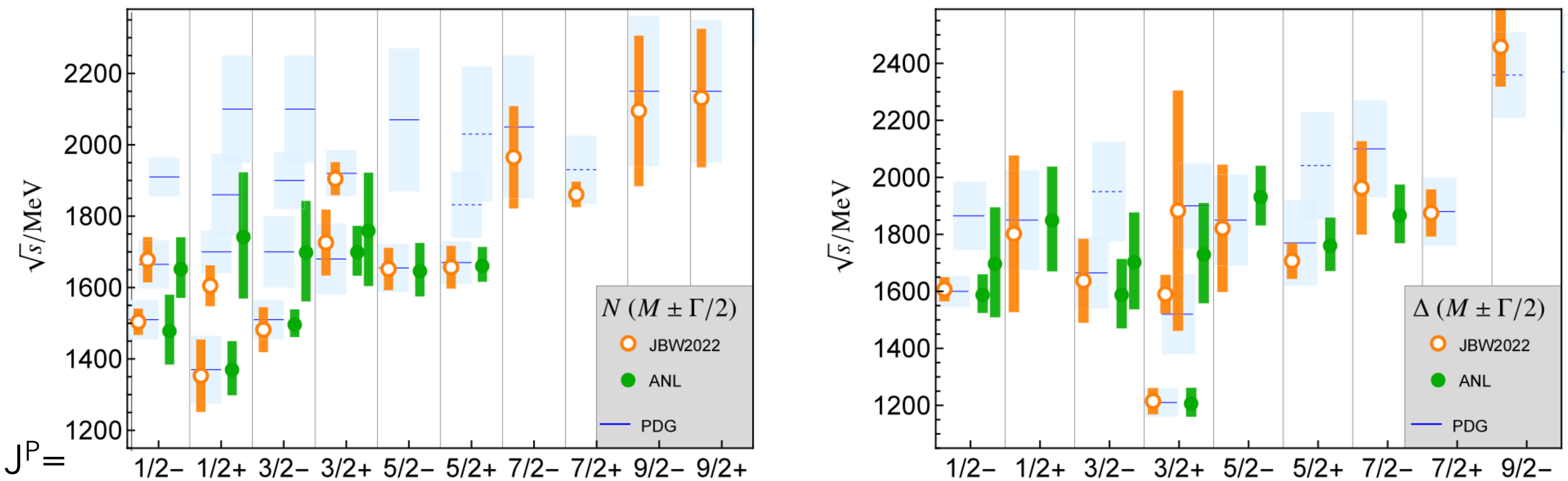
update: $\pi N \to \omega N$ [2208.03061/Wang]
Upcoming data from JParc

Reaction	Observables (# data points)	p./channel
$\gamma p \to \pi^0 p$	$d\sigma/d\Omega$ (18721), Σ (2927), P (768), T (1404), $\Delta\sigma_{31}$ (140),	
	G (393), H (225), E (467), F (397), $C_{x'_L}$ (74), $C_{z'_L}$ (26)	25,542
$\gamma p \to \pi^+ n$	$d\sigma/d\Omega$ (5961), Σ (1456), P (265), T (718), $\Delta\sigma_{31}$ (231),	
	G (86), H (128), E (903)	9,748
$\gamma p \to \eta p$	$d\sigma/d\Omega$ (9112), Σ (403), P (7), T (144), F (144), E (129)	9,939
$\gamma p \to K^+\Lambda$	$d\sigma/d\Omega$ (2478), P (1612), Σ (459), T (383),	
	$C_{x'}$ (121), $C_{z'}$ (123), $O_{x'}$ (66), $O_{z'}$ (66), O_x (314), O_z (314),	5,936
$\gamma p \to K^+\Sigma^0$	$d\sigma/d\Omega$ (4271), P (422), Σ (280), T (127), $C_{x',z'}$ (188), $O_{x,z}$ (254)	5,542
$\gamma p \to K^0\Sigma^+$	$d\sigma/d\Omega$ (242), P (78)	320
	in total	**57,027**

Roenchen et al.

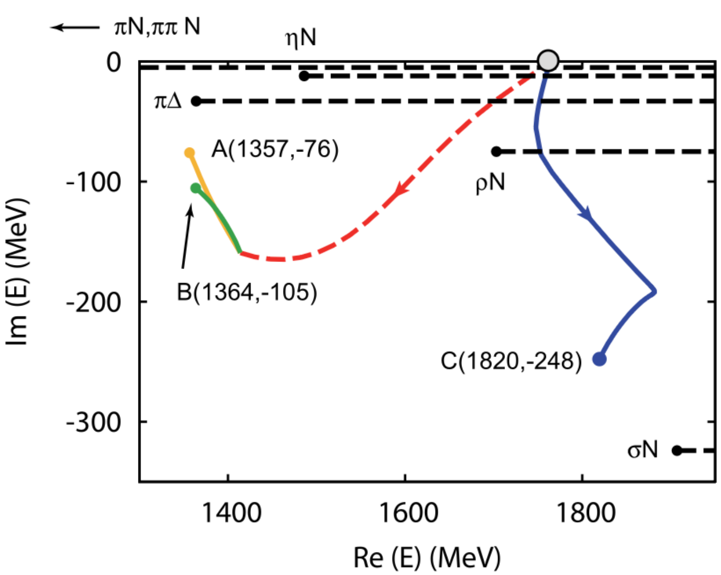
New interface [https://jbw.phys.gwu.edu/]

Results: Resonance spectrum

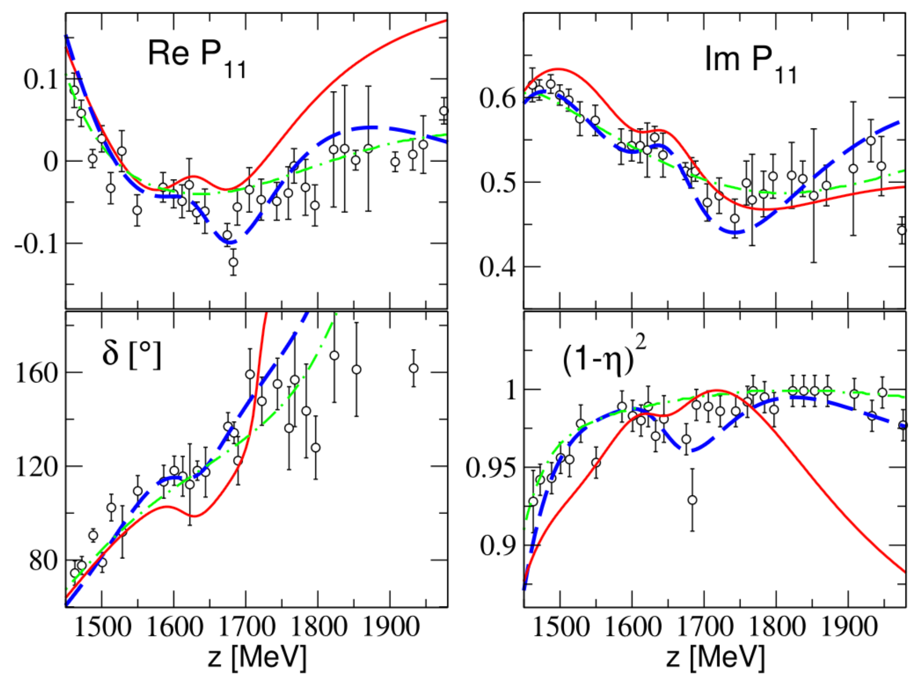


Highlight: P11

- **ANL/Osaka** [0909.1356/nucl-th]
- Roper and N(1710) (?) in ANL/Osaka from one genuine s-channel state & channel dynamics



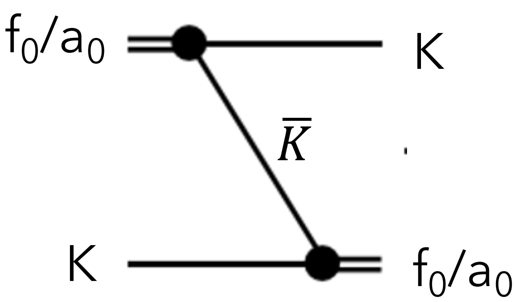
- **JuBo** [1211.6998/nucl-th]
- Roper generated from ρ t-channel but also strong coupling to σN channel [9911080/nucl-th]
- But genuine pole in IVU/Wu's talk
- N(1710) comes from fitting K**Λ**, and appears right in elastic $\boldsymbol{\pi}$N (!) → not entirely stable result across all analyses

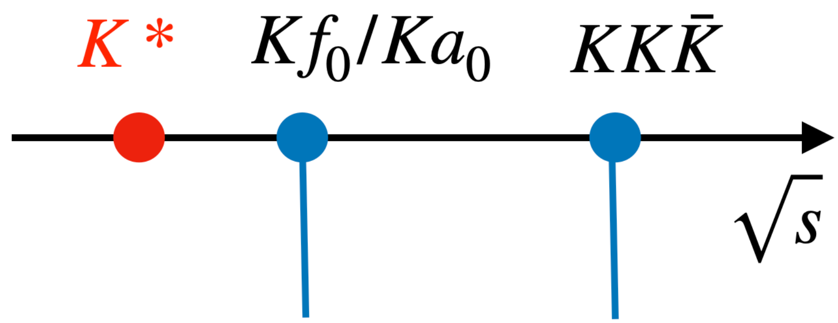


[See also talk by S. Nakamura]

DCC for mesons: the K(1460)

[2511.02543 ,
MD, K. Khemchandani, A. Martinez

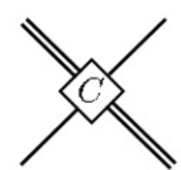




- Hadronic molecule hypothesis (stable, mass-degenerate f_0 and a_0):

Zhang, Hanhart et al. [Eur. Phys. J A 58,22 (2022)] predict binding energy in MeV:

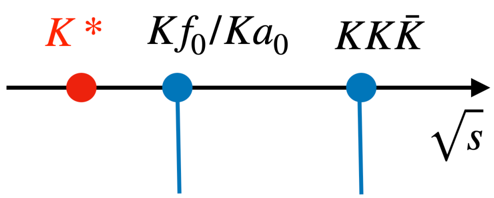
α	Λ [GeV]	$t(a)$	$+t(b)$	$+s(a)$	$+s(b)$	*+streched boxes*	*+crossed boxes*
0	0.5	0.51 (1)	0.66 (1.29)	0.83 (1.63)	0.52 (1.02)	0.58 (1.14)	0.68 (1.33)
1	0.5	1.63 (1)	2.51 (1.54)	3.93 (2.41)	**1.63** (1)	1.86 (1.14)	2.21 (1.36)
$1^\dagger$	0.5	1.58 (1)	2.44 (1.54)	3.81 (2.41)	1.59 (1.01)	1.81 (1.15)	**2.16** (1.37)

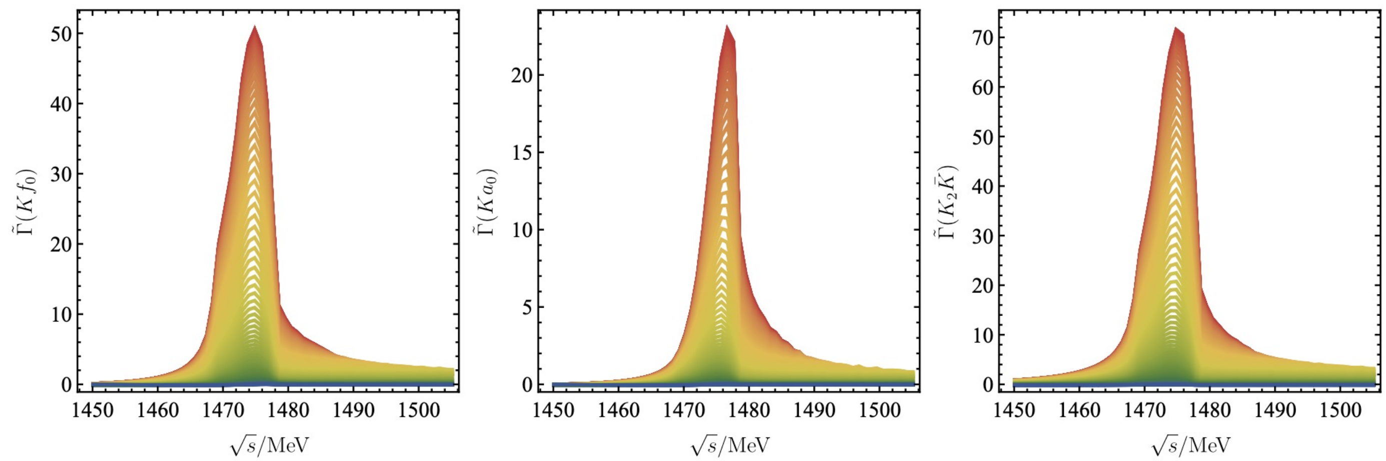


=0

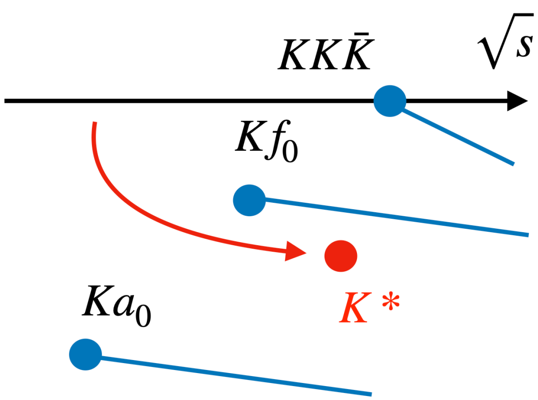
Pole trajectory

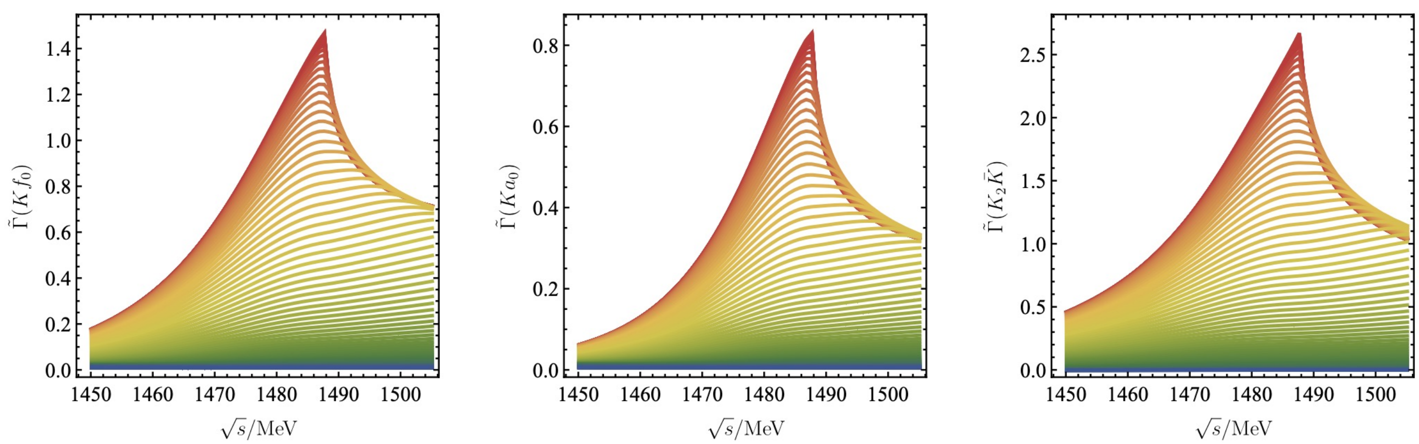
Stable isobars (slightly different model)





Realistic, coupled-channels isobars





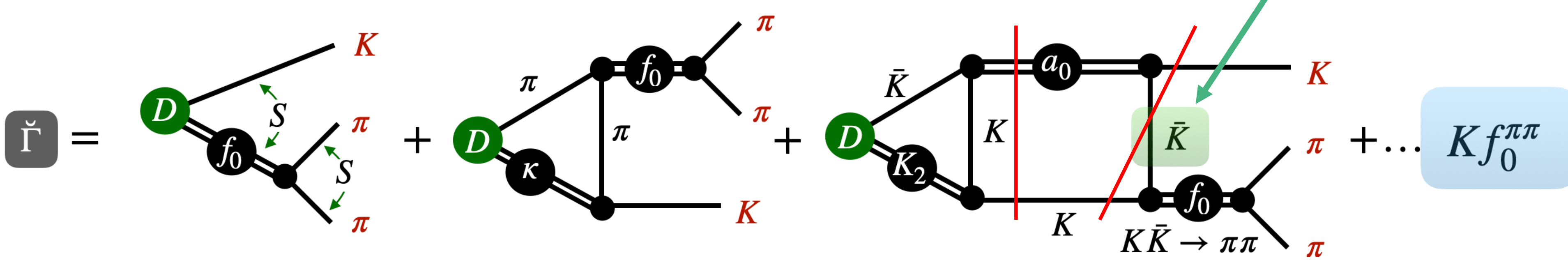
Channels:

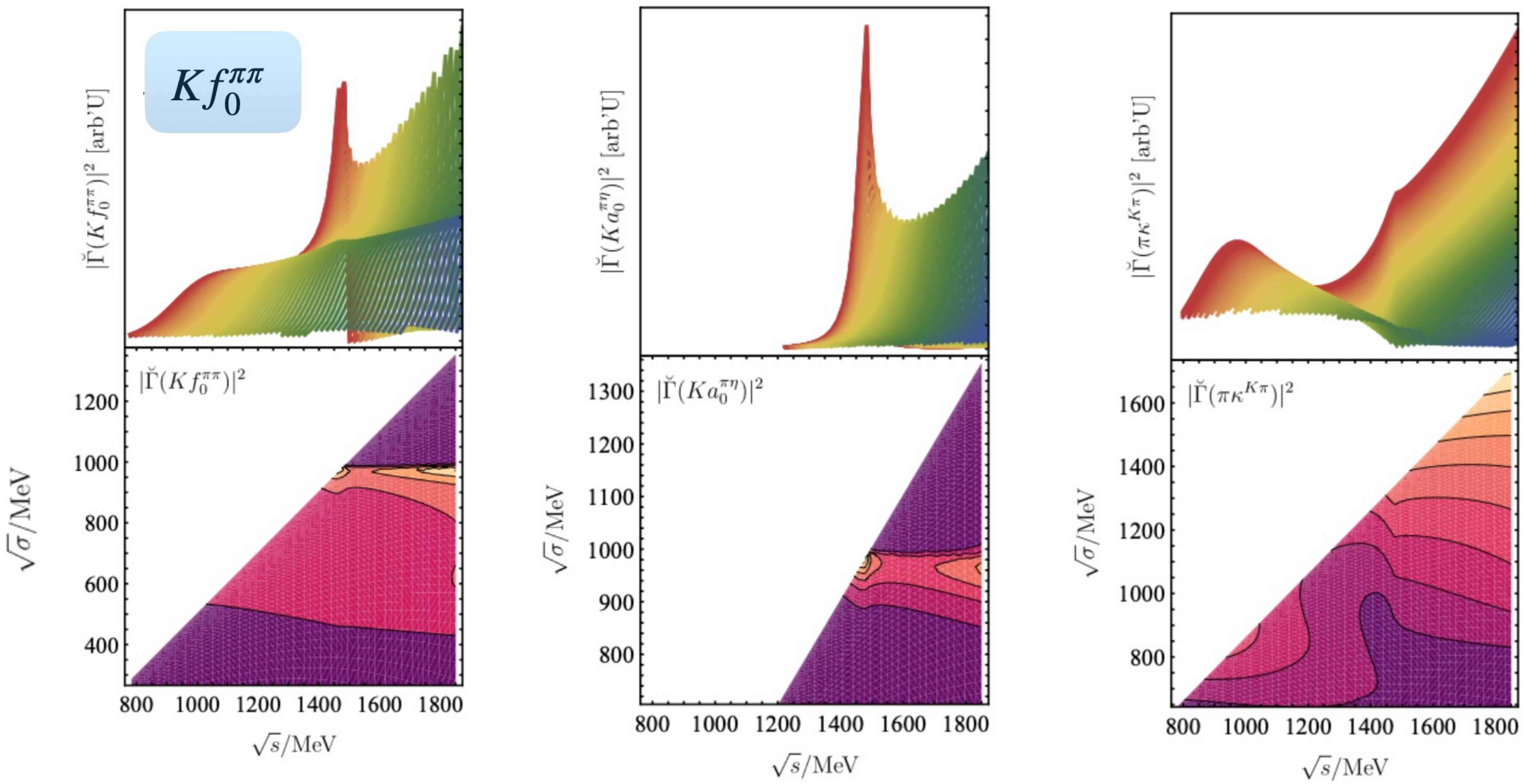
$$Kf_0^{K\bar{K}} \quad Kf_0^{\pi\pi} \quad Ka_0^{K\bar{K}} \quad Ka_0^{\pi\eta} \quad \bar{K}K_2^{KK} \quad \pi\kappa^{K\pi} \quad \eta\kappa^{K\pi}$$

All that remains is an enhanced cusp

Production reactions

Almost on-shell kaon exchange → triangle!





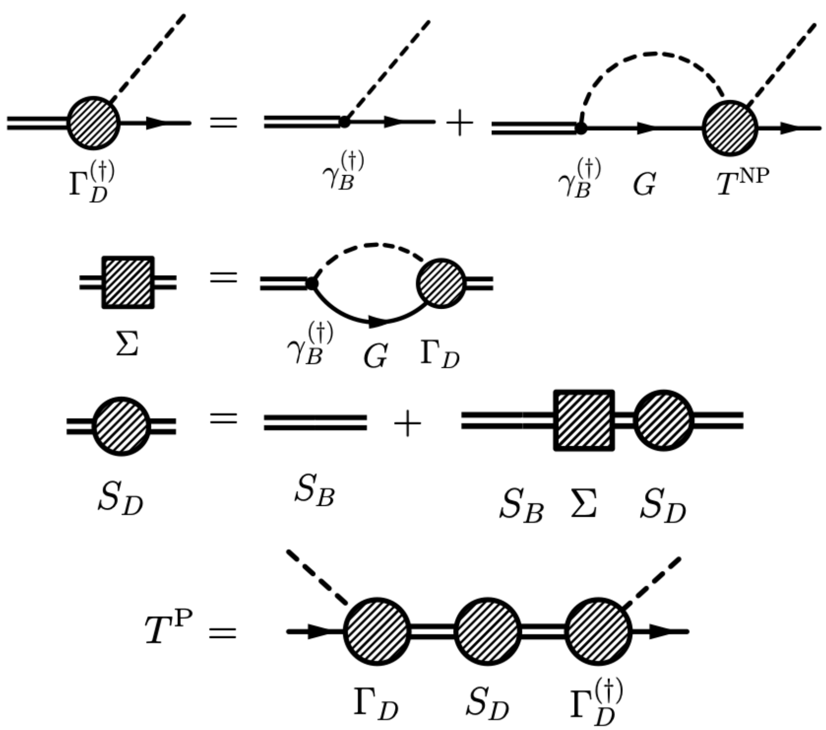
Summary

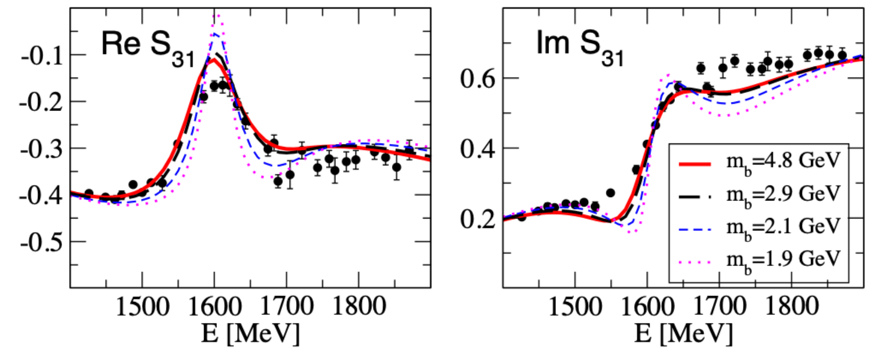
- Dynamical coupled channel models can capture many S-matrix principles
  - Multi-channel dynamics for two-body and three-body states with true unitarity
  - Complex thresholds, triangle singularities (TS) automatically contained
- Useful for phenomenological analyses of
  - Meson and photon-induced baryon production with two and more final states.
- Resonance usually included as genuine states but sometimes states arise generated from meson-baryon dynamics or are explained as kinematic effects (TS)
- Related approaches for finite-volume applications (FVU) and conceptual advances (IVU)

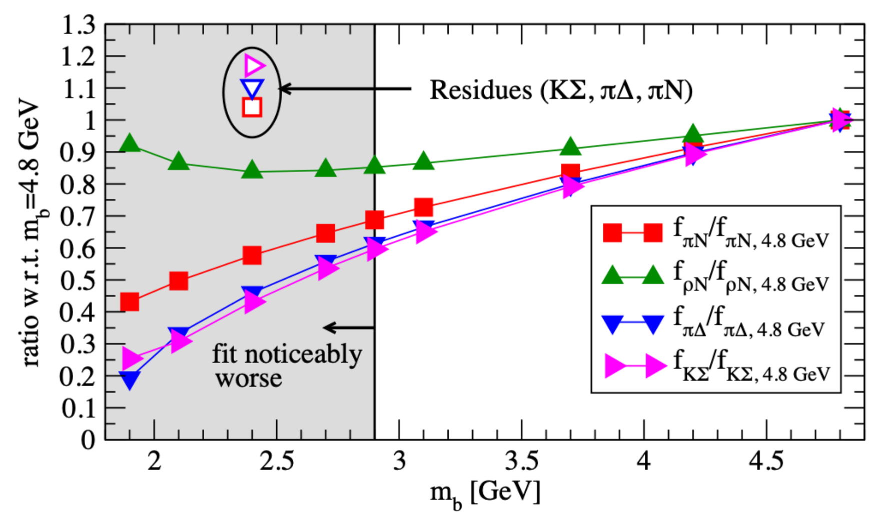
Spare slides

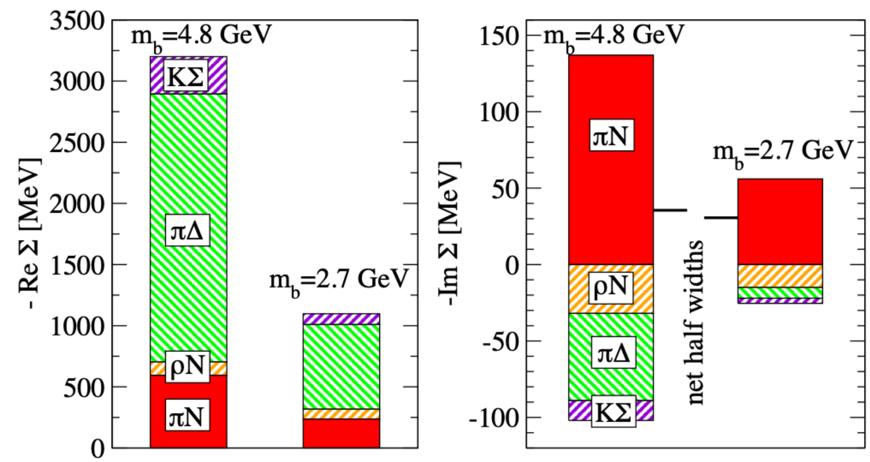
Bare states & resonances

- The bare state masses heavily depend on
  - Regularization
  - Included channels
  - Indirectly, also on the non-resonant interactions



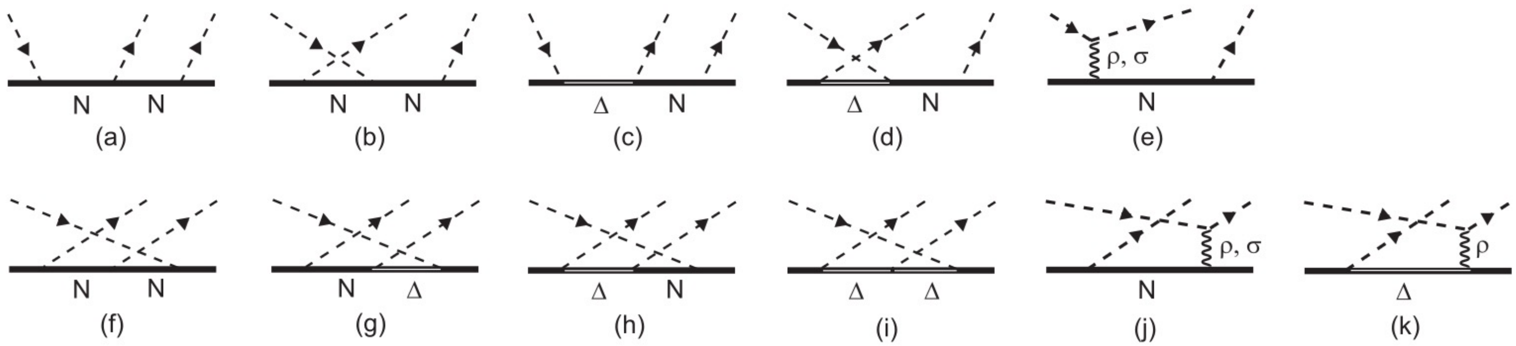




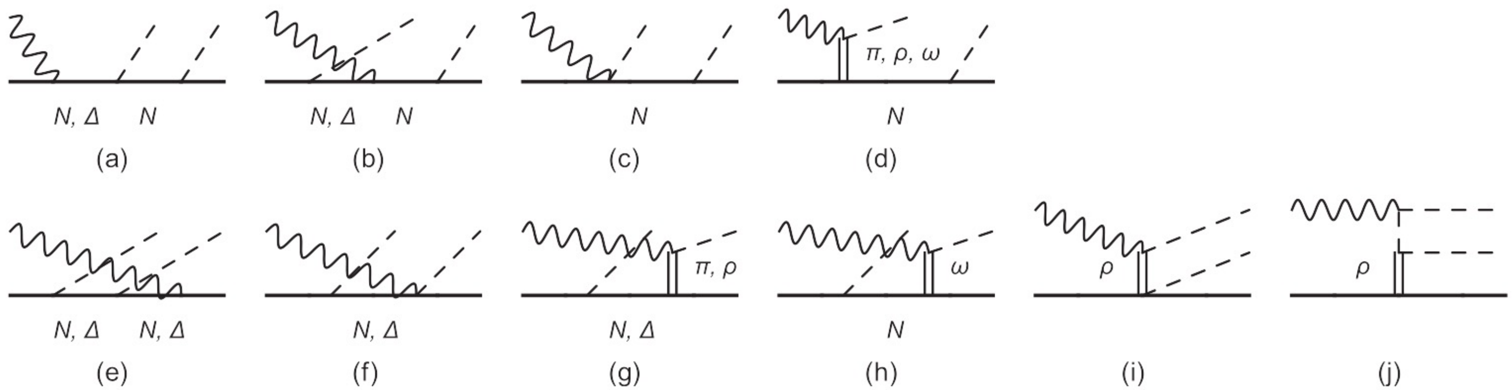


EBAC: $\pi\pi N$ final states

$\pi N \to \pi\pi N$



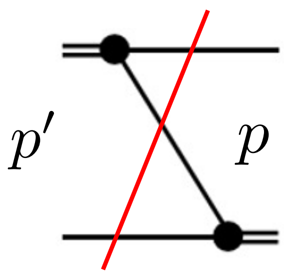
$\gamma N \to \pi\pi N$



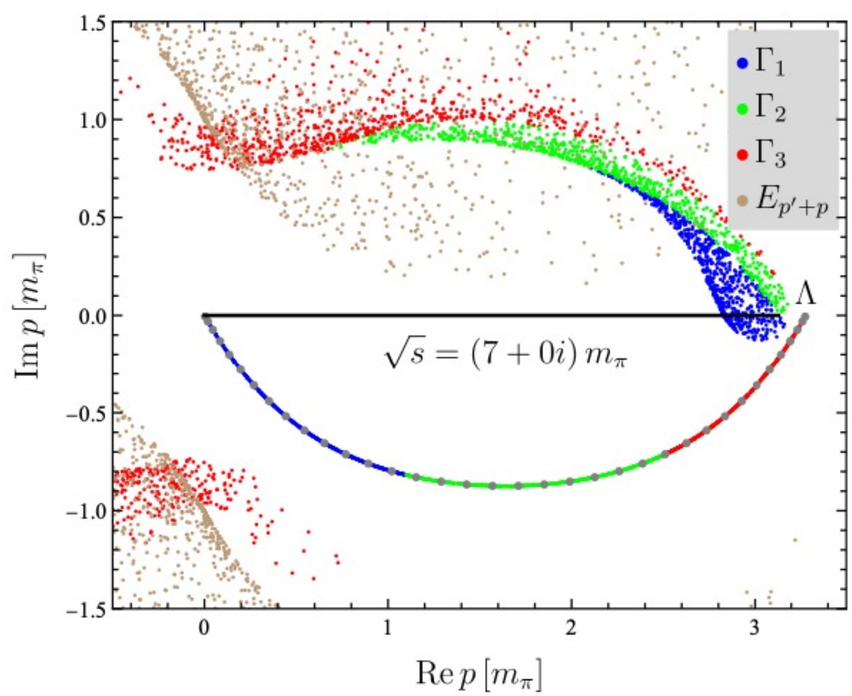
How to solve the scattering equation

$$\tilde{T}_{ji}(s,p',p) = \boxed{\tilde{B}_{ji}}(s,p',p) + \tilde{C}_{ji}(s,p',p) + \int_0^\Lambda \frac{\mathrm{d}l\, l^2}{(2\pi)^3\, 2E_l} \left(\tilde{B}_{jk}(s,p',l) + \tilde{C}_{jk}(s,p',l)\right) \tilde{\tau}_k(\sigma_l)\, \tilde{T}_{kj}(s,l,p)$$

- **Three-body cuts**



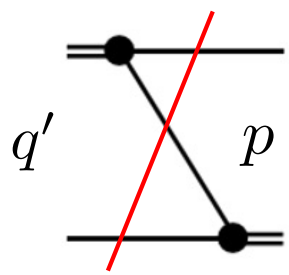
$$\tilde{B}_{ji}(s,\boldsymbol{p}',\boldsymbol{p}) = \frac{(\tilde{\boldsymbol{I}}_F)_{ji}\, v_j^*(p, P-p-p')v_i(p', P-p-p')}{2E_{p'+p}(\sqrt{s} - E_p - E_{p'} - E_{p'+p}) + i\epsilon}$$

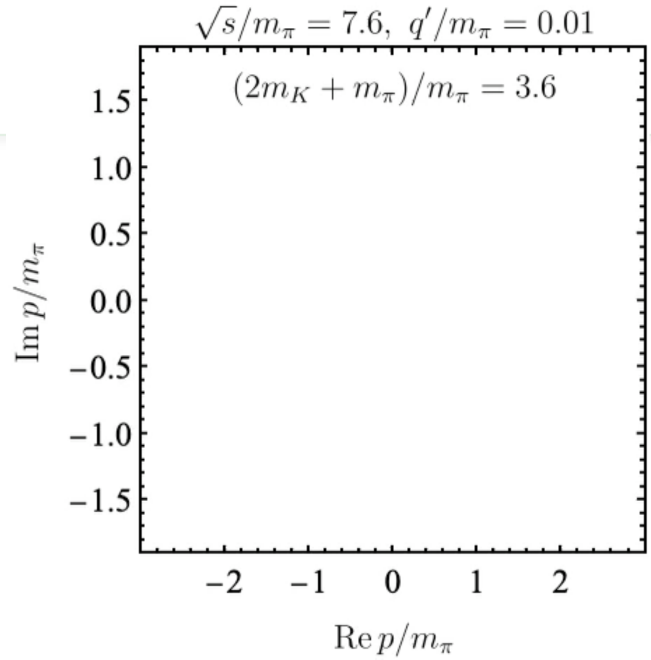


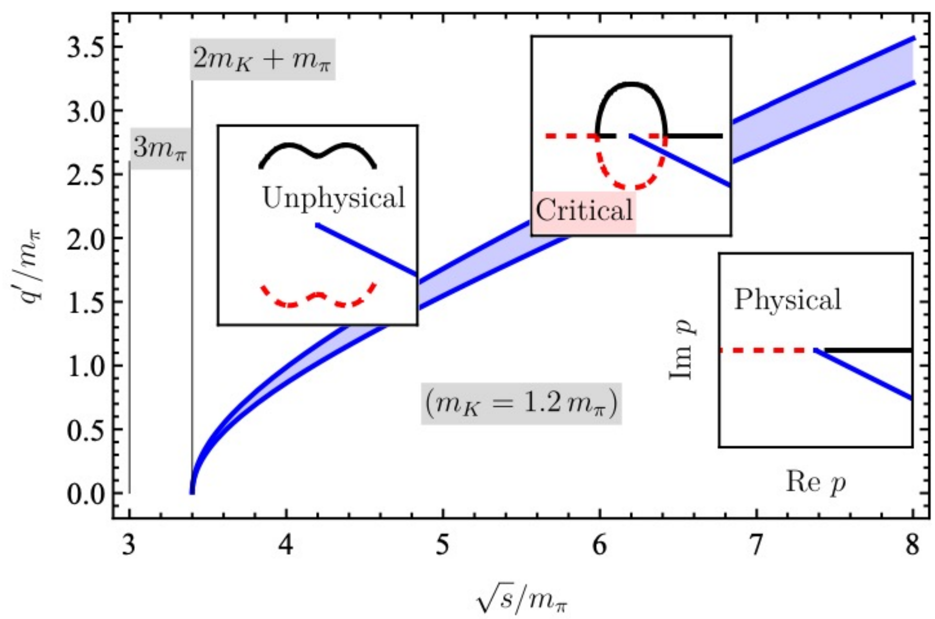
- Angle, energy dependent
- Depend also on *p′* and *p*
- Solve LSE for complex momenta on a deformed contour

How to get the solution for real, physical momenta

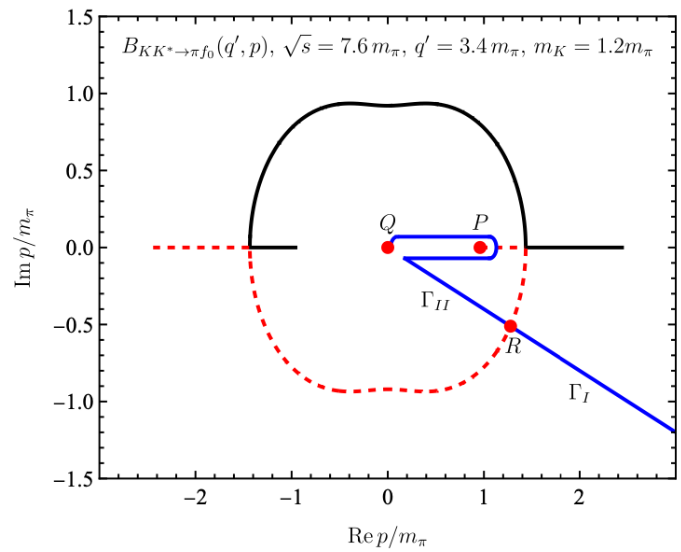
- No solution for real momenta in"critical region"







Solution 1: Contour deformation [Cahill & Sloane]



How to get the solution for real, physical momenta

Solution 2: Direct inversion [Ziegelmann et al.]

Production amplitude
$$\tilde{\Gamma}_j^T(s,q') = D_j(s,q') + \int_0^\Lambda \frac{dq\, q^2}{(2\pi)^3 2E_q}\, \tilde{B}ji(s,q',q)\, \tilde{\tau}_i(\sigma(q))\, \tilde{\Gamma}_i^T(s,q)$$

Ansatz
$$\tilde{\Gamma}^T(q) \approx \sum_{i=1}^{N} \tilde{\Gamma}^T(q_i)\, H_i(q)$$
with Lagrange polynomials
$$H_i(q) = \frac{\prod_{j\neq i}^N (q-q_j)}{\prod_{j\neq i}^N (q_i-q_j)}$$

Makes integral equation a matrix equation
$$\tilde{\Gamma}^T(q_j) = D(q_j) + A_{ji}\tilde{\Gamma}^T(q_i)$$

With singular integrals
$$A_{ji} = \int_0^\Lambda \frac{dq\, q^2}{(2\pi)^3 2E_q}\, \tilde{B}(q_j,q)\, \tilde{\tau}(\sigma(q))\, H_i(q)$$

… for which many established algorithms exist

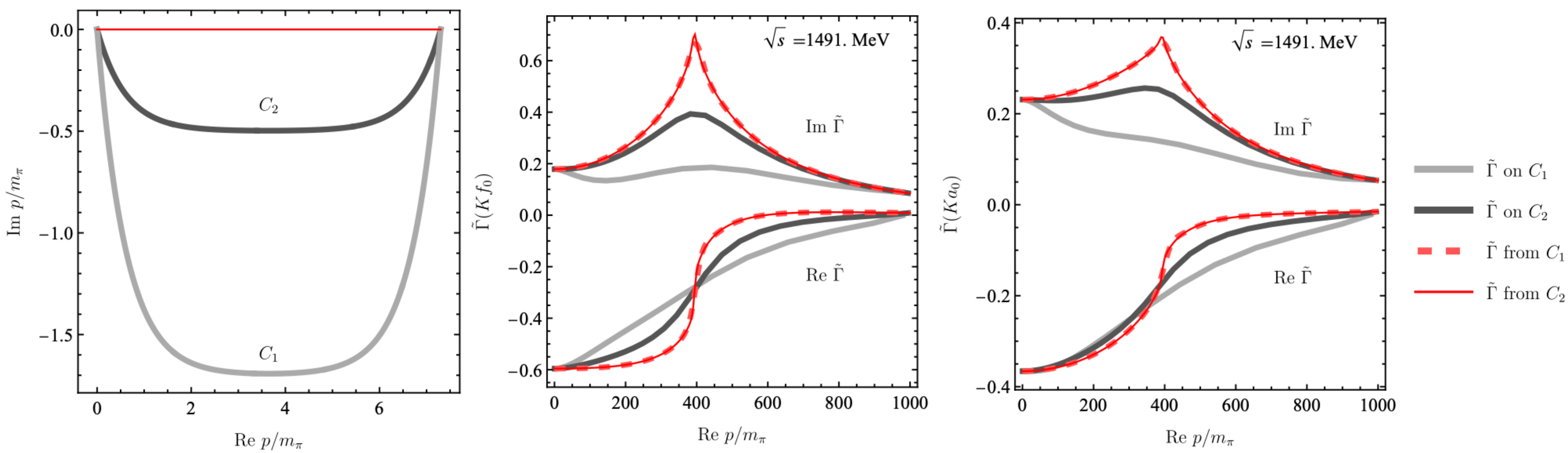
How to get the solution for real, physical momenta

Solution 3: Continued fractions

- Not rigorous as methods 1 and 2, but very good extrapolation properties
- Sample the solution on complex contour and evaluate the continued fraction at real momenta

$$f(x) = f(x_1) + \cfrac{x - x_1}{\rho(x_1, x_2) + \cfrac{x - x_2}{\rho_2(x_1, x_2, x_3) - f(x_1) + \cfrac{x - x_3}{\rho_3(x_1, x_2, x_3, x_4) - \rho(x_1, x_2) + \cdots}}}$$

[example: 2511.02543 ,
MD, K. Khemchandani, A. Martinez



Partial-wave decomposition

- Plane-wave basis

$$T_{\lambda'\lambda}(\boldsymbol{p},\boldsymbol{q}_1) = (B_{\lambda'\lambda}(\boldsymbol{p},\boldsymbol{q}_1) + C) + \sum_{\lambda''} \int \frac{d^3\boldsymbol{l}}{(2\pi)^3 2E_l} (B_{\lambda'\lambda''}(\boldsymbol{p},\boldsymbol{l}) + C)\,\tau(\sigma(l)) T_{\lambda''\lambda}(\boldsymbol{l},\mathbf{q}_1)$$

$$B^J_{\lambda\lambda'}(q_1,p) = 2\pi \int_{-1}^{+1} \mathrm{d}x\, d^J_{\lambda\lambda'}(x) B_{\lambda\lambda'}(\boldsymbol{q}_1,\boldsymbol{p}) \qquad B^J_{LL'}(q_1,p) = U_{L\lambda} B^J_{\lambda\lambda'}(q_1,p) U_{\lambda' L'}$$

- JLS basis:

$$T^J_{LL'}(q_1,p) = \left(B^J_{LL'}(q_1,p) + C_{LL'}(q_1,p)\right) + \int_0^\Lambda \frac{\mathrm{d}l\, l^2}{(2\pi)^3 2E_l} \left(B^J_{LL''}(q_1,l) + C_{LL''}(q_1,l)\right) \tau(\sigma(l)) T^J_{L''L'}(l,p)$$

Analytic cont. 3-body

[Sadasivan (2021)]
[Doering (2009)]

SMC

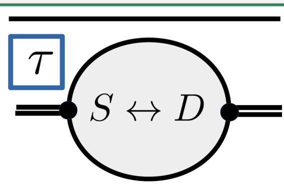
$$T^J_{LL'}(q_1,p) = \left(B^J_{LL'}(q_1,p) + C_{LL'}(q_1,p)\right) + \int_0^\Lambda \frac{\boxed{dl}\, l^2}{(2\pi)^3 2E_l} \left(B^J_{LL''}(q_1,l) + C_{LL''}(q_1,l)\right) \tau(\sigma(l)) T^J_{L''L'}(l,p)$$

$$\tau^{-1}(\sigma) = K^{-1} - \Sigma\,,$$

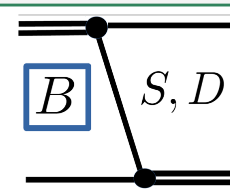
$$\Sigma = \int_0^\infty \frac{\boxed{dk}\, k^2}{(2\pi)^3} \frac{1}{2E_k} \frac{\sigma^2}{\sigma'^2} \frac{\tilde{v}(k)^* \tilde{v}(k)}{\boxed{\sigma - 4E_k^2 + i\epsilon}}$$

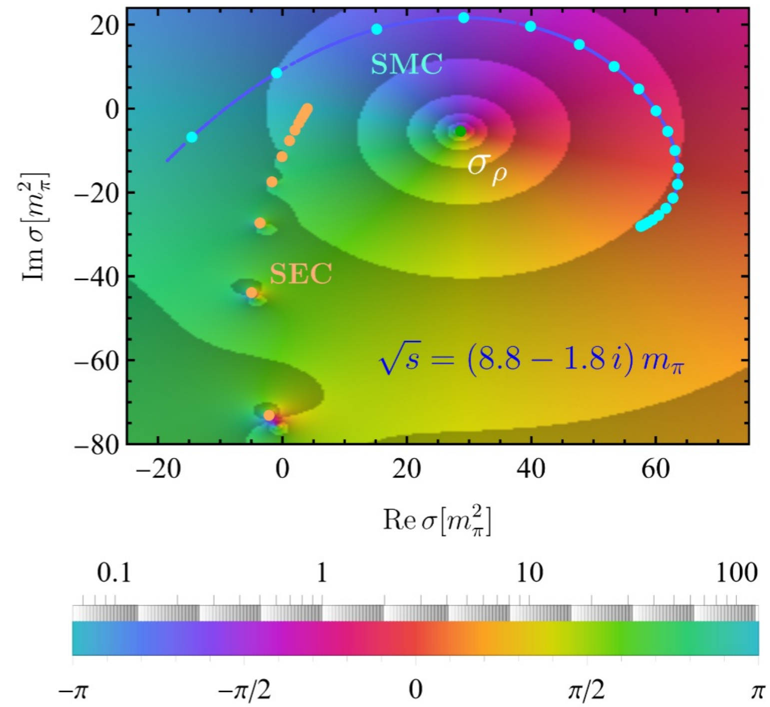
$$B_{\lambda\lambda'}(\boldsymbol{p},\boldsymbol{p}') = \frac{v^*_\lambda(P-p-p',p)\, v_{\lambda'}(P-p-p',p')}{2E_{p'+p}\boxed{\left(\sqrt{s} - E_p - E_{p'} - E_{p'+p} + i\epsilon\right)}}$$

SEC



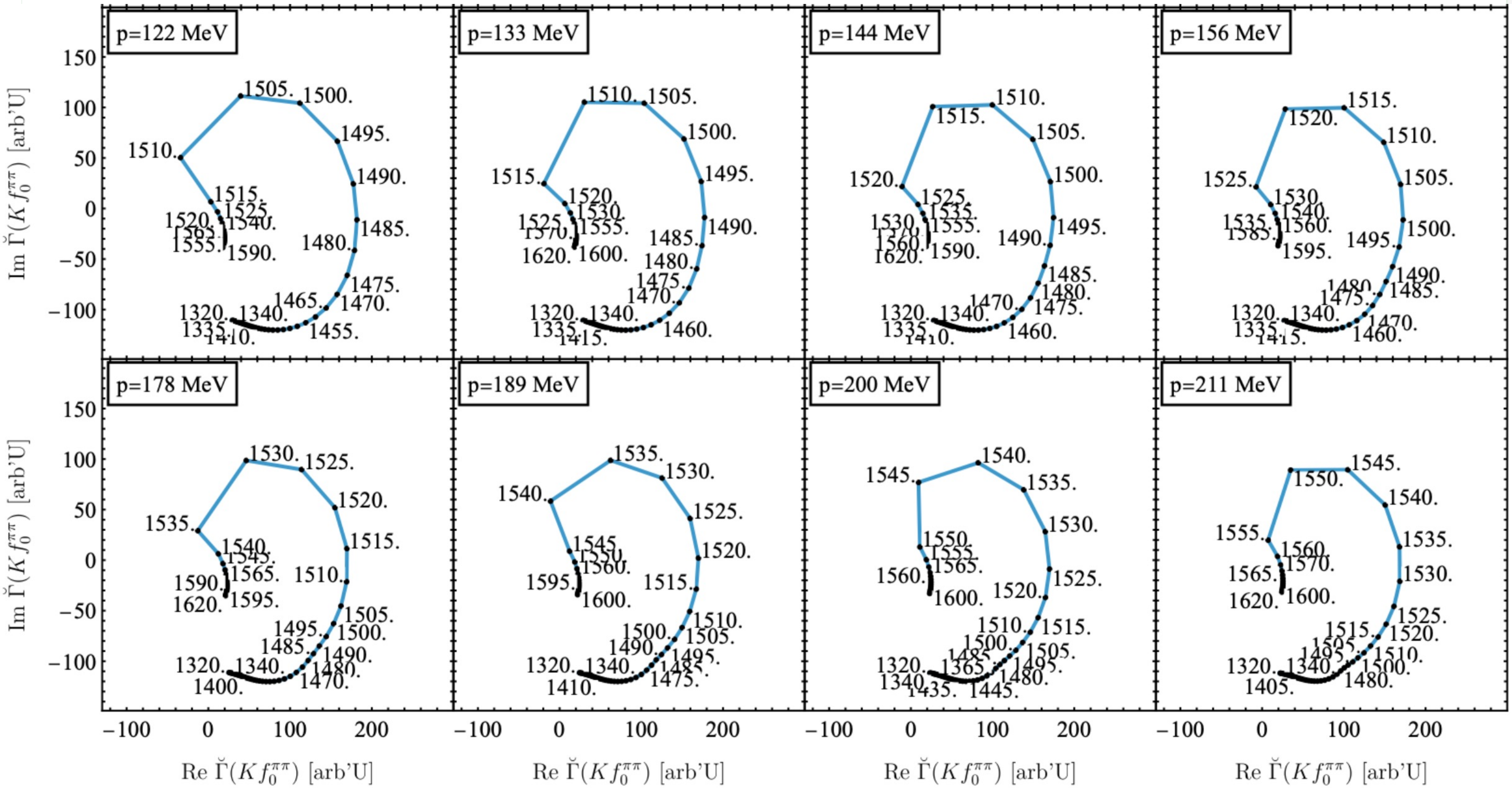
Singularities





- Two contours (SMC and SEC)
- Deform both "adiabatically" to go to complex s
- Set of rules:
  - Contours cannot intersect with each others
  - Contours cannot intersect with (3-body) cuts
- Passing singularities left or right determines sheet

Phase motion

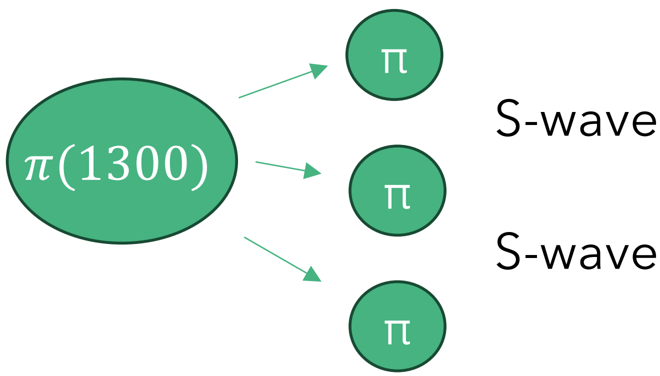


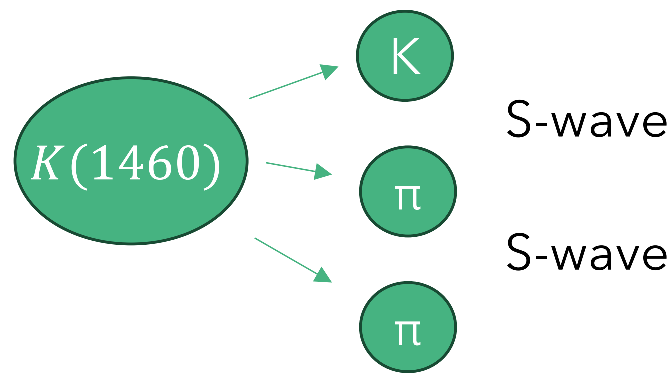
Resonance-like phase motion in the Argand plot (fix spectator momenta)

Excited pseudoscalar mesons

- Adding $J^P=0^+$ quantum numbers to pseudoscalar mesons (π, η, K) in S-wave

  → Excited pseudoscalar mesons





- Found in CLEO [JHEP 05 (2017)]
- Not in Compass
- State found in lattice QCD [Dudek et al., PRD 82, 2010]
- Dynamically generated from πf_0? [Martinez et al. PRD 84, 2011]
- New evidence from lattice QCD [Yan, Mai, Y. Feng, MD et al.2510.09476 [hep-lat]

- Experimentally well established
- Hadronic molecule Kf_0-Ka_0? [Albaladejo PRD82 (2010) Martinez PRC 83 (2011); Filikhin PRD 102 (2020), Zhang, Hanhart EPJA 58 (2022)]

A new model for strange mesons

- Two-body input from experiment when available, chiral models if not: Full coupled-channel amplitudes
- Channels: $Kf_0^{K\bar{K}}$ $Kf_0^{\pi\pi}$ $Ka_0^{K\bar{K}}$ $Ka_0^{\pi\eta}$ $\bar{K}K_2^{KK}$ $\pi\kappa^{K\pi}$ $\eta\kappa^{K\pi}$
- Sub-channels have $f_0(\pi\pi, K\bar{K}), a_0(\pi\eta, K\bar{K})$

Strangess-2 kaon cluster

- Hanhart result reproduced: With stable isobars, one gets very shallow 3-body bound state in $Kf_0^{K\bar{K}}$ $Ka_0^{K\bar{K}}$
- Slowly switch on all channels and full cc two-body amplitudes

State	$M_R - i\Gamma_R/2$
$f_0(500)$	$471.97 - i189.07$
$f_0(980)$	$988.08 - i17.36$
$a_0(980)$	$982.32 - i53.63$
$K_0^*(700)$	$832.57 - i230.70$

Two-body resonances