NSTAR2026: The 15th International Workshop on the Physics of Excited Nucleons (Sevilla, Spain)

Femtoscopy as a probe of hadron–hadron interactions

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Outline

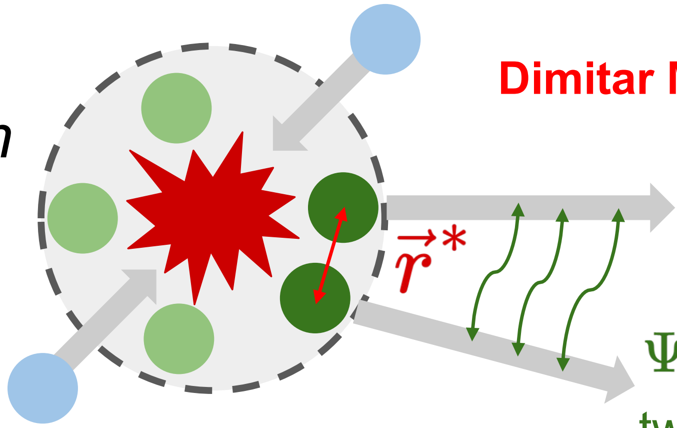
- Invariant mass distributions *versus* Koonin-Pratt CFs
- Limitations of the Koonin-Pratt formalism
- Finite-range improved Lednicky–Lyuboshits approximation for correlation functions
  - ✓ proton-proton CFs
- Open problems: $\pi^{\pm}K_S$ CFs and $\kappa/K_0^*(700)$, πD CFs and the two-pole structure of the $D_0^*(2300)$
- Conclusions

Femtoscopy

Koonin-Pratt equation

Dimitar Mihaylov @ HIN 2025

quotient of the number of pairs of particles with the same relative momentum produced in the same collision event over the reference distribution of pairs originated from mixed events (pairs originated in different collisions)



$\Psi(\vec{k}^*, \vec{r}^*)$

two-particle wave function

$$C(k^*) = \frac{N_{\mathrm{SE}}(k^*)}{N_{\mathrm{ME}}(k^*)} = \int S(r^*) \left| \Psi(\vec{k}^*, \vec{r}^*) \right|^2 d^3 r^* \xrightarrow{k^* \to \infty} 1$$

Lisa et al. *Ann.Rev.Nucl.Part.Sci.55:357-402, 2005*

Relative distance and ½ relative momentum evaluated in the pair rest frame

- Measure C(k*), fix S(r*), study the interaction.
- Detailed studies of the source in pp:
  ALICE Coll. *Phys.Lett.B 811 (2020) 135849*
  Mihaylov and Gonzalez Gonzalez, *EPJC* 83 (2023) 7, 590
- CATS framework to evaluate the above integral Mihaylov et al. *EPJC* 78 (2018) 5, 394

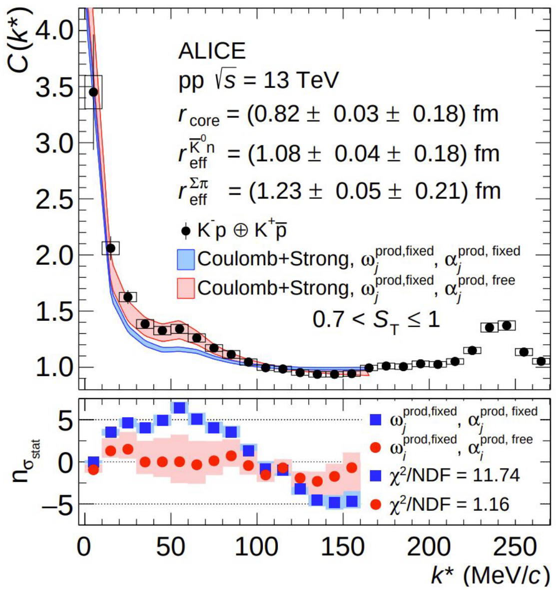
source $S(r)$: probability density of finding the two hadrons of the emitted pair at a given relative distance *r*

Kbar-N interaction

S = -1

K⁻p femtoscopy in pp collsions

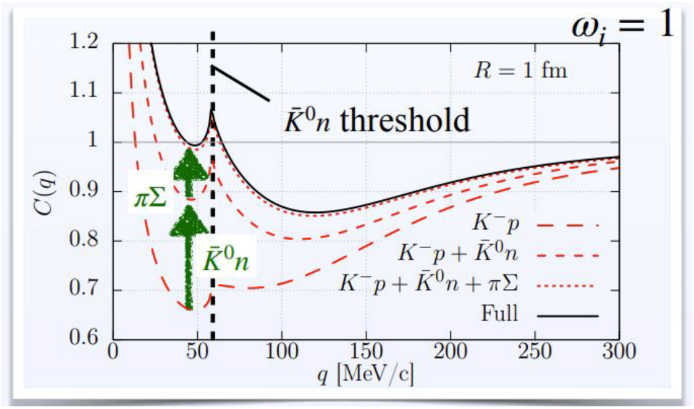
ALICE Coll. Phys. Rev. Lett. 124, 092301 (2020)
ALICE Coll. arXiv:2205.15176, EPJC in press (2022)



Strong interaction: Kyoto model
K. Miyahara, T. Hyodo, W. Weise, Phys. Rev. C98, 2, (2018) 025201

⇒ Provides a **quantitative test of coupled channels in the theory**
Effects of coupled channels enhanced by small source

Kamiya, Hyodo, Morita, Ohnishi, Weise, PRL 124 (2020) 13, 132501
⇒ Predicted to be negligible in larger sources



Otón Vázquez Doce, LNF -INFN
Present and future perspectives in Hadron Physics, Frascati, 2024

Talks at this conference….

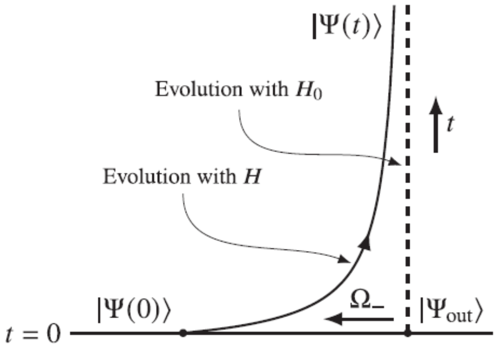
- *Femtoscopy probes of* $Z_c(3900)$ *and* $Z_{cs}(3985)$ *and the isovector* $X(3872)$ *partner from* $D^0D^{*-}, D^{*0}D^-, D^0D_s^{*-}$ *and ,* $D^{*0}D_s^-$ *correlations*, **Pan-Pan Shi**
- *The nature of* $\Sigma(1430)$ *from* $\Lambda_c^+ \to \Lambda\pi^+\pi^+\pi^-$ *reaction and correlation functions*, **Wei-Hong Liang**
- *Quarkonium–nucleon femtoscopy as a probe of nucleon gravitational form factors*, **Daisuke Fujii**
- *A unitary kaon-deuteron interaction for femtoscopy*, **Juan Torres-Rincon**
- *Femtoscopy in revealing the nature of exotic hadrons*, **Lisheng Geng**
- *Confusions in the strange baryons sector*, **Kanchan Pradeepkumar Khemchandani**
- *Constraining the low-energy* $S = -2$ *meson-baryon interaction with two-particle correlations*, **Albert Feijoo Aliau**
- *Femtoscopy as a tool to access the strong interaction among hadrons with ALICE*, **Oton Vazquez Doce**
- *Finite-Range Effects and Coulomb Interactions in Femtoscopic Correlation Functions*, **Pablo Encarnación**
- *Scattering and Femtoscopic Correlation Functions of the* $\Sigma_c\pi$ *Systems*, **Mikel F. Barbat**
- Coulomb interactions in hadronic femtoscopy: applications to the pp and $\alpha\alpha$ systems , **Miguel Albaladejo**

basic object: wave-function

Invariant mass distributions *versus* Koonin-Pratt CFs

$$\psi^*(\vec{r};\vec{k}) = e^{-i\vec{k}\cdot\vec{r}} + \int d^3\vec{p}\,\frac{e^{-i\vec{p}\cdot\vec{r}}}{E-\frac{\vec{p}^2}{2\mu_{ab}}+i\epsilon}\langle\vec{k}|\hat{T}^{\rm QM}(E+i\epsilon)|\vec{p}\rangle.$$

Albaladejo, Feijoo, Nieves, Oset, Vidaña, PRD 10 (2024) 114052



$$= e^{-i\vec{k}\cdot\vec{r}} + 4\pi\int dp\,p^2\frac{j_0(pr)}{E-\frac{p^2}{2\mu_{ab}}+i\epsilon}T^{\rm QM}(k\leftarrow p;E)$$

half off-shell T −matrix

Assume an S-wave interaction

$$|\psi(\vec{r};\vec{k})|^2 = 1 + 2{\rm Re}\left(e^{i\vec{k}\cdot\vec{r}}\int d^3\vec{p}\,\frac{j_0(pr)}{E-\frac{p^2}{2\mu_{ab}}+i\epsilon}T^{\rm QM}(k\leftarrow p;E)\right) + \left|\int d^3\vec{p}\,\frac{j_0(pr)}{E-\frac{p^2}{2\mu_{ab}}+i\epsilon}T^{\rm QM}(k\leftarrow p;E)\right|^2$$

spherically symmetric source

$$C(\vec{k}) = \int d^3\vec{r}\,S(\vec{r})\,|\psi(\vec{r};\vec{k})|^2 = 1 + 4\pi\int dr\,r^2 S(r)\left\{\left|j_0(kr) + \int d^3\vec{p}\,\frac{j_0(pr)}{E-\frac{p^2}{2\mu_{ab}}+i\epsilon}T^{\rm QM}(k\leftarrow p;E)\right|^2 - j_0^2(kr)\right\}$$

We follow a different approach and first perform the d^3r integration

$$S(r) = \frac{1}{(4\pi R^2)^{3/2}} \exp\left(-\frac{r^2}{4R^2}\right)$$

$$C(k) = 1 + 4\pi \int dr r^2 S(r) \left\{ \left| j_0(kr) + \int d^3\vec{p} \frac{j_0(pr)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} T^{\mathrm{QM}}(k \leftarrow p; E) \right|^2 - j_0^2(kr) \right\}$$

$$\boxed{\int d^3r \int d^3p \Rightarrow \int d^3p \int d^3r}$$

Gaussian source of radius R

$$\boxed{F_R(q,q') = \int d^3\vec{r}\, S(r) j_0(qr) j_0(q'r) = \frac{e^{-(q^2+q'^2)R^2} \sinh(2qq'R^2)}{2qq'R^2} \simeq 1 + O(q^2R^2, q'^2R^2)}$$

$$C(k) = 1 + 2\mathrm{Re}\left(\int d^3\vec{p} \frac{T^{\mathrm{QM}}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} F_R(k,p) \right) + \int d^3\vec{p}\, d^3\vec{p}\,' \frac{T^{\mathrm{QM}}(k \leftarrow p; E)[T^{\mathrm{QM}}(k \leftarrow p'; E)]^*}{\left(E - \frac{p^2}{2\mu_{ab}} + i\epsilon\right)\left(E - \frac{p'^2}{2\mu_{ab}} - i\epsilon\right)} F_R(p,p')$$

$$= C^{\mathrm{prod}}(k) + \delta C(k),$$

Albaladejo, Feijoo, Nieves, Oset, Vidaña, PRD 110 (2024) 114052

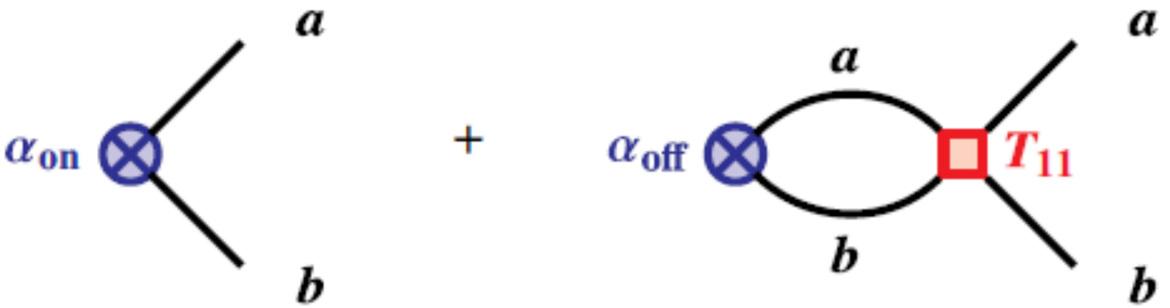
$$\boxed{C^{\mathrm{prod}}(k) = \left| 1 + \int d^3\vec{p} \frac{T^{\mathrm{QM}}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} \widetilde{F}_R(k,p) \right|^2,}$$

$$\tilde{F}_R(k,p) = \frac{F_R(k,p)}{F_R(k,k)}$$

$$\delta C(k) = 2\mathrm{Re}\left(\int d^3\vec{p} \frac{T^{\mathrm{QM}}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} \left[F_R(k,p) - \widetilde{F}_R(k,p)\right] \right)$$

$$+ \int d^3\vec{p}\, d^3\vec{p}\,' \frac{T^{\mathrm{QM}}(k \leftarrow p; E)[T^{\mathrm{QM}}(k \leftarrow p'; E)]^*}{\left(E - \frac{p^2}{2\mu_{ab}} + i\epsilon\right)\left(E - \frac{p'^2}{2\mu_{ab}} - i\epsilon\right)} \left[F_R(p,p') - \widetilde{F}_R(k,p)\widetilde{F}_R(k,p')\right]$$

Invariant mass distribution from a production experiment



half off-shell T −matrix

$$\frac{d\sigma}{dm_{ab}^2} \propto \left| 1 + \int d^3p \, \frac{T^{QM}(k \leftarrow p)}{E - \frac{\vec{p}^2}{2\mu_{ab}} + i\varepsilon} \, F(k,p) \right|^2$$

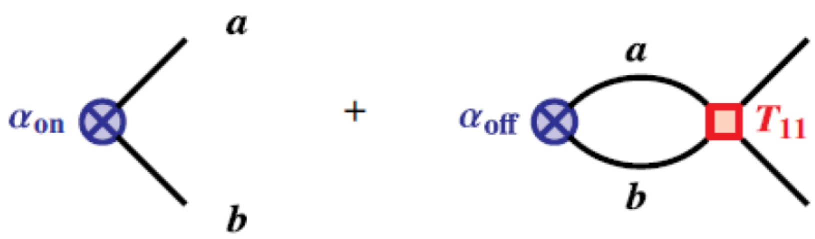
$$\frac{\alpha_{\text{on}}}{\alpha_{\text{off}}}$$

$k = \sqrt{2\mu_{ab}\, E}$ (on-shell momentum)

form-factor associated to the production of the virtual pair

$$C^{\mathrm{prod}}(k) = \left| 1 + \int d^3\vec{p}\, \frac{T^{\mathrm{QM}}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} \tilde{F}_R(k,p) \right|^2, \qquad \tilde{F}_R(k,p) = \frac{F_R(k,p)}{F_R(k,k)} = \frac{\int d^3r\, S(r) j_0(kr) j_0'(pr)}{\int d^3r\, S(r) j_0(kr) j_0'(kr)}$$

Invariant mass distribution from a production experiment



half off-shell T −matrix

$$\frac{d\sigma}{dm_{ab}^2} \propto \left| 1 + \int d^3p\, \frac{T^{QM}(k \leftarrow p)}{E - \frac{\vec{p}^2}{2\mu_{ab}} + i\varepsilon} F(k,p) \right|^2$$

$k = \sqrt{2\mu_{ab}\, E}$ (on-shell momentum)

form-factor associated to the production of the virtual pair

- $C(k) = C^{\mathrm{prod}}(k) + \delta C(k)$
- $\frac{d\sigma}{dm_{ab}^2} \propto C^{\mathrm{prod}}(k)$, with

  $F(k,p) = \frac{\alpha_{\mathrm{on}}}{\alpha_{\mathrm{off}}} = \tilde{F}_R(k,p)$
- $\tilde{F}_R(k,p)$: form-factor that accounts for the off-shell production of a and b in the first step, with a regulator determined by the size of the source R

Assuming that only the S-wave part of the wave function is modified by the hadronic interaction, $\boldsymbol{C^{\mathrm{prod}}(k)}$ can also be written as

$$C^{\mathrm{prod}}(k) = \left| \frac{1}{F_R(k,k)} \int d^3\vec{r}\, S(r)\, j_0(kr)\, \psi^*(\vec{r};\vec{k}\,) \right|^2 .$$

$$1 + \int d^3\vec{p}\, \frac{T^{\mathrm{QM}}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} \widetilde{F}_R(k,p) = \frac{1}{F_R(k,k)} \int d^3\vec{r}\, S(r) j_0(kr) \left[j_0(kr) + \int d^3\vec{p}\, j_0(pr) \frac{T^{\mathrm{QM}}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} \right]$$

$$= \frac{1}{F_R(k,k)} \int d^3\vec{r}\, S(r) j_0(kr) \left[e^{-i\vec{k}\cdot\vec{r}} + \int d^3\vec{p}\, e^{-i\vec{p}\cdot\vec{r}} \frac{T^{\mathrm{QM}}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} \right]$$

$$= \frac{1}{F_R(k,k)} \int d^3\vec{r}\, S(r)\, j_0(kr)\, \psi^*(\vec{r};\vec{k}\,)$$

Koonin–Pratt correlation functions and invariant-mass distributions are distinct observables; they coincide only in the zero-source-size limit.

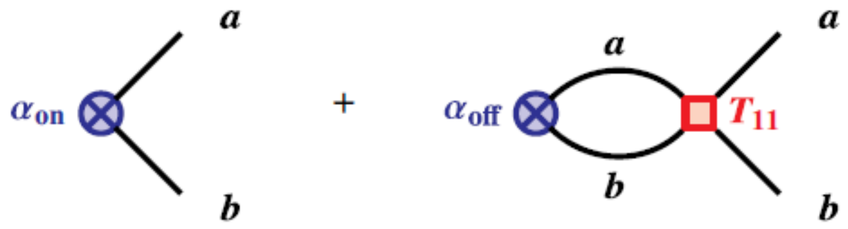
For <u>point-like sources</u> $\boldsymbol{\lim_{R\to 0} S(r) = \delta^3(\vec{r})}$

$$C(\vec{k}\,) = \int d^3\vec{r}\, S(\vec{r}\,)\, |\psi(\vec{r};\vec{k}\,)|^2$$

$$\boxed{\lim_{R\to 0} C(k) = |\psi(\vec{r}=\vec{0};\vec{k}\,)|^2 = \left| 1 + \int d^3\vec{p}\, \frac{T^{\mathrm{QM}}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} \right|^2 = \lim_{R\to 0} C^{\mathrm{prod}}(k)}$$

$$C(\vec{k}) = \int d^3\vec{r}\, S(\vec{r})\, |\psi(\vec{r};\vec{k})|^2 \qquad C^{\rm prod}(k) = \left| \frac{1}{F_R(k,k)} \int d^3\vec{r}\, S(r)\, j_0(kr)\, \psi^*(\vec{r};\vec{k}) \right|^2$$

$C^{\rm prod}(k)$: coherent sum of amplitudes. This is correct for exclusive processes where only the observed particles a and b are produced



This is obviously not necessarily the case in high-multiplicity event experiments. For inclusive reactions, for example initiated by pp collisions, the coherence is still preserved when the real and virtual production vertices, $\alpha_{\rm on}$ and $\alpha_{\rm off}$ stand for the $pp \to X + ab$ (real) and $pp \to X + ab$ (virtual) processes, respectively, with the rest of the particles (X) in the final state being the same for both reactions. In that case, the two Feynman amplitudes must be coherently added since both of them contribute to the same quantum amplitude of the reaction $pp \to X + ab$ (real).

In high-multiplicity events of pp, pA and AA collisions, where the hadron production yields are described by statistical models which lead to the extended sources $S(r)$, it is unlikely that the coherent sum of amplitudes, which is assumed in $C^{\rm prod}(k)$, is still appropriate.

The Koonin–Pratt scheme implies some kind of summation of probabilities, and not of amplitudes, since $C(k)$ is obtained from the spatial superposition between the extended source and the squared modulus of the wave-function pair $\left|\psi(\vec{r};\vec{k})\right|^2$

Limitations of the Koonin-Pratt formalism

Source function in pp collisions at the LHC

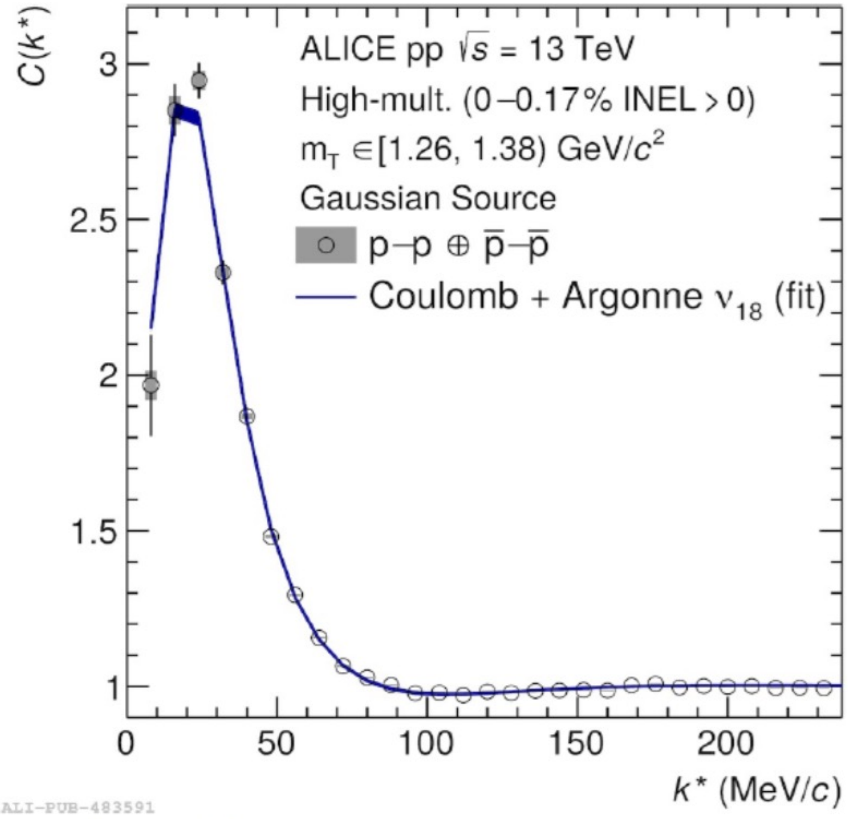
- Emitting source function anchored to p-p correlation function

$$\underbrace{C(k^*)}_{\text{measured}} = \int S(\vec{r}\,)\underbrace{\left|\psi(\vec{k}^*,\vec{r}\,)\right|^2}_{\text{known interaction}} d^3\vec{r}$$

- Gaussian parametrization

$$S(r) = \frac{1}{(4\pi r_{core}^2)^{3/2}} \exp\left(-\frac{r^2}{4r_{core}^2}\right) \times$$ Effect of short lived resonances (cτ ~ 1 fm)

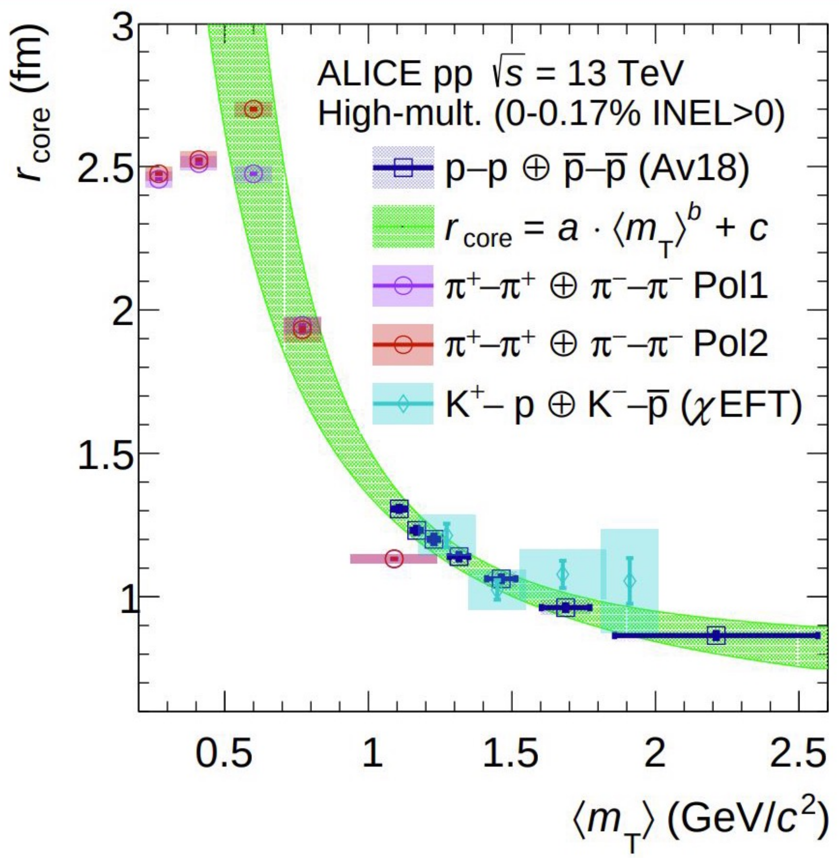
ALICE Coll., PLB, 811 (2020), 135849



ALICE Coll., PLB, 811 (2020)

Laura Fabbietti@HADRON25

A necessary first step: Setting the source



"Universal" dependence of the source size with pair transverse mass $<m_T>$

- related to hydrodinamics /collective phenomena ?

newst results:

ALICE Coll.,arXiv:2311.14527 [hep-ph] EPJC in press

Source size $<m_T>$ scaling in pp collisions confirmed as well with π-π and K-p pairs

$$m_T = \sqrt{k_T^2 + m^2} \qquad k_T = \frac{1}{2}|p_{T,1} + p_{T,2}|$$

Otón Vázquez Doce, LNF -INFN

Present and future perspectives in Hadron Physics, Frascati, 19 june 2024

Theoretical correlation function

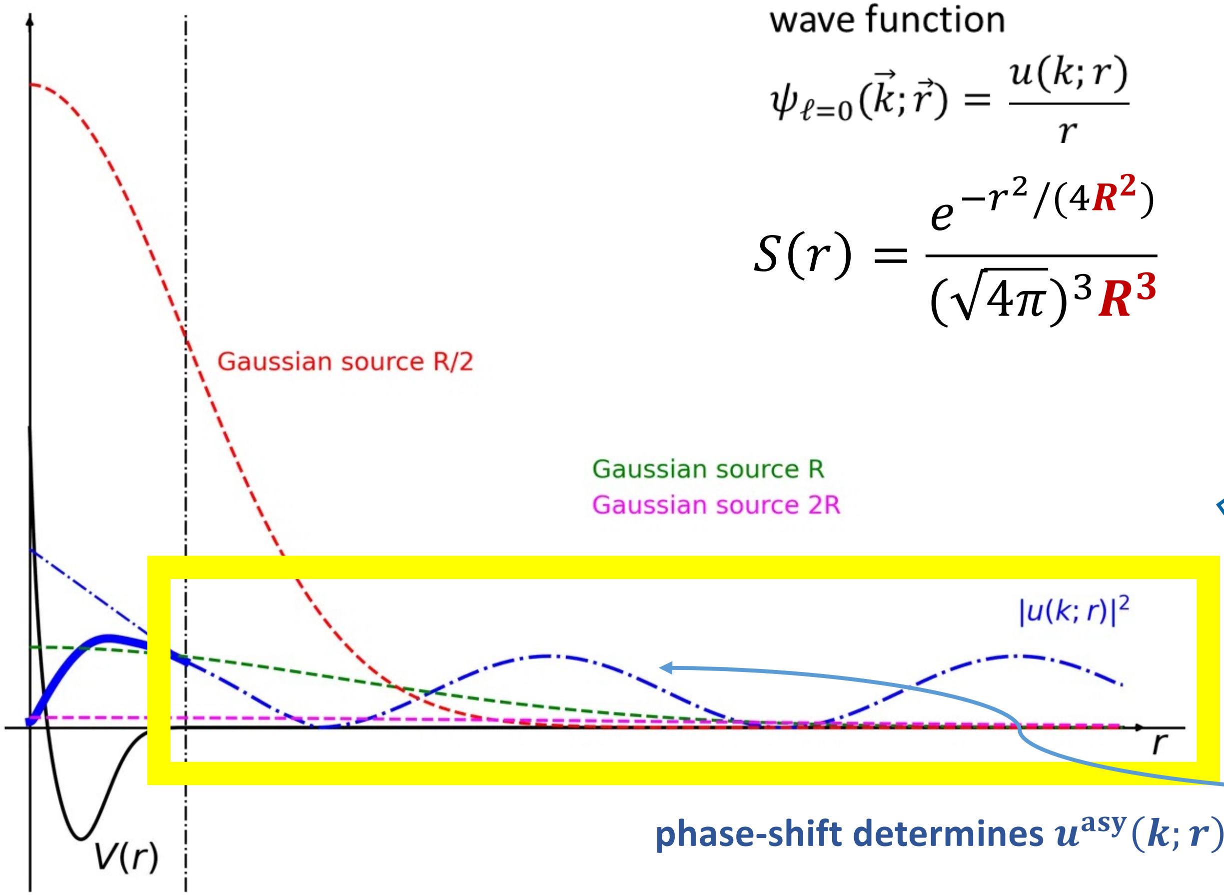
$$C(k^*) = \int S(r^*)\, \left|\Psi(k^*, \overrightarrow{r^*})\right|^2 d^3 r^*$$

S. E. Koonin, *Physics Letters B* **70** (1977) 43-47
S. Pratt, *Phys. Rev. C* **42** (1990) 2646-2652

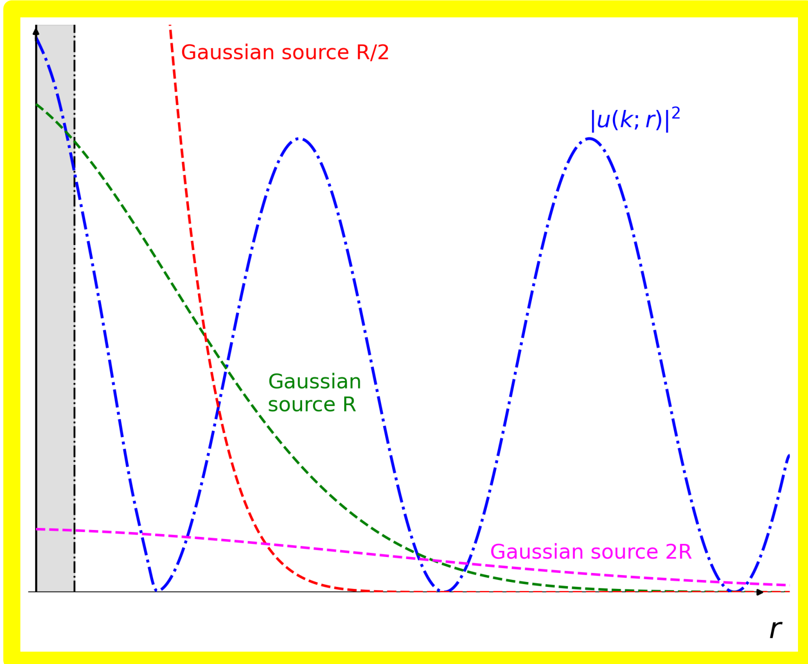
S-wave reduced radial wave function

$$\psi_{\ell=0}(\vec{k};\vec{r}) = \frac{u(k;r)}{r}$$

$$S(r) = \frac{e^{-r^2/(4\boldsymbol{R^2})}}{(\sqrt{4\pi})^3 \boldsymbol{R^3}}$$



phase-shift determines $u^{\text{asy}}(k;r)$



$$-u''(k;r) + 2\mu V(r) u(k;r) = k^2 u(k;r)$$

$$\lim_{r\to\infty} u(k;r) = u^{\text{asy}}(k;r) = f_0^{-1}(k)\frac{\sin(kr+\delta_0)}{k}$$

$$f_0^{-1}(k) = k\cot\delta_0 - ik$$

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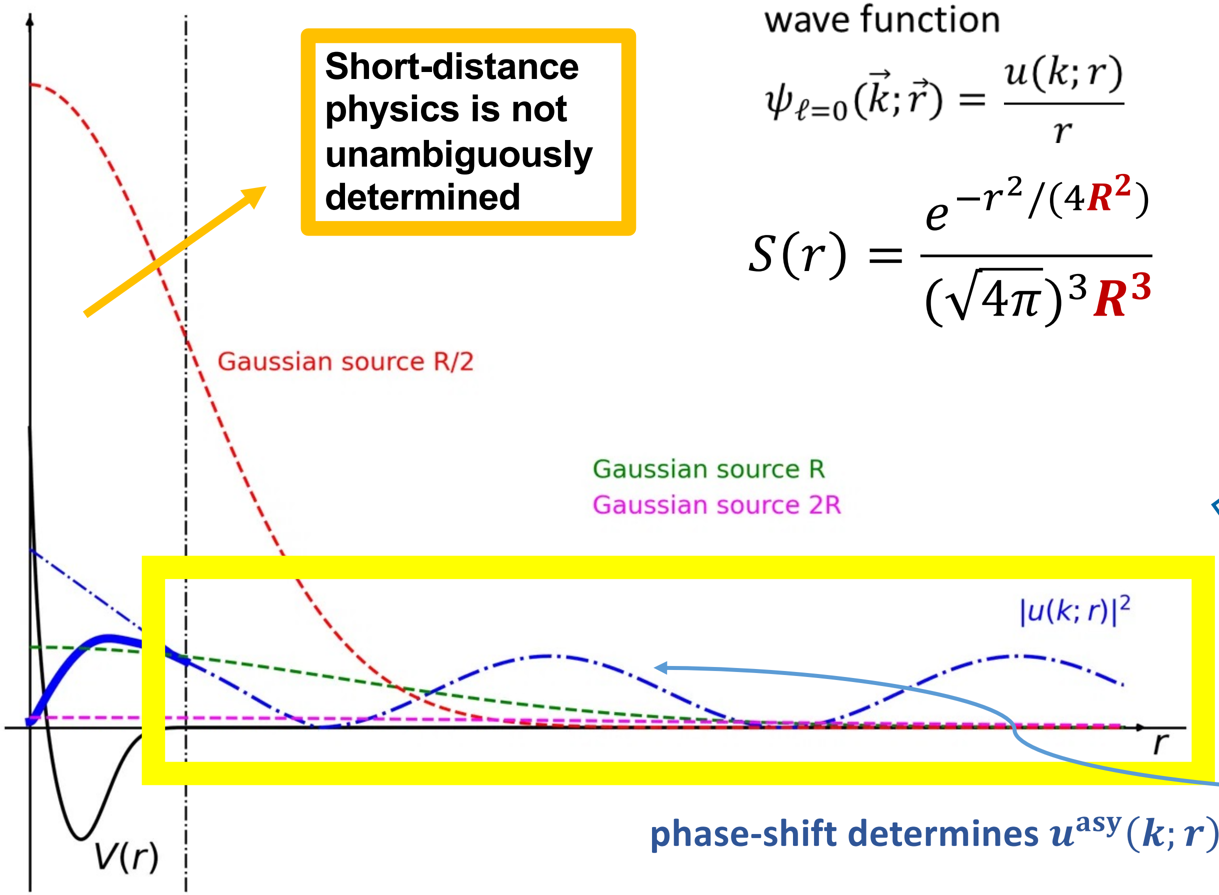
Short-distance physics is not unambiguously determined



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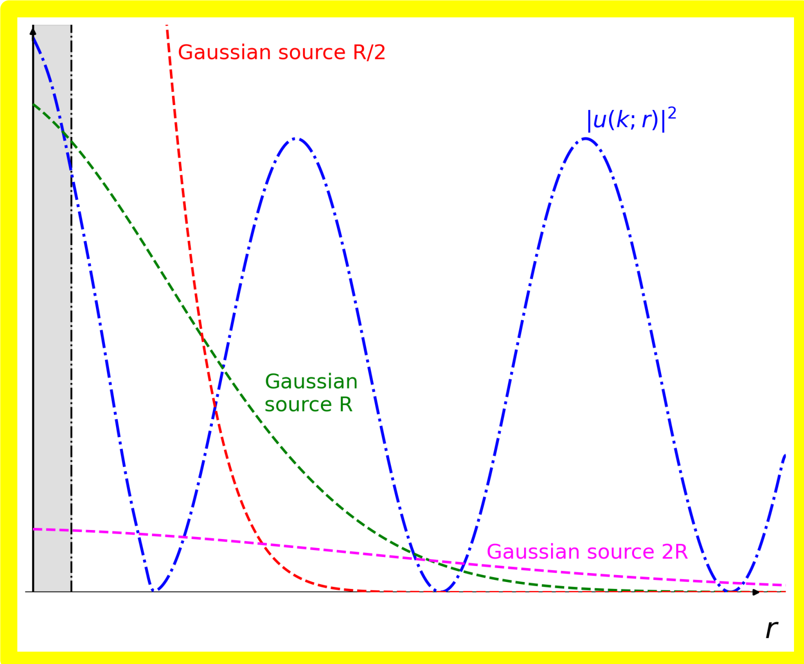

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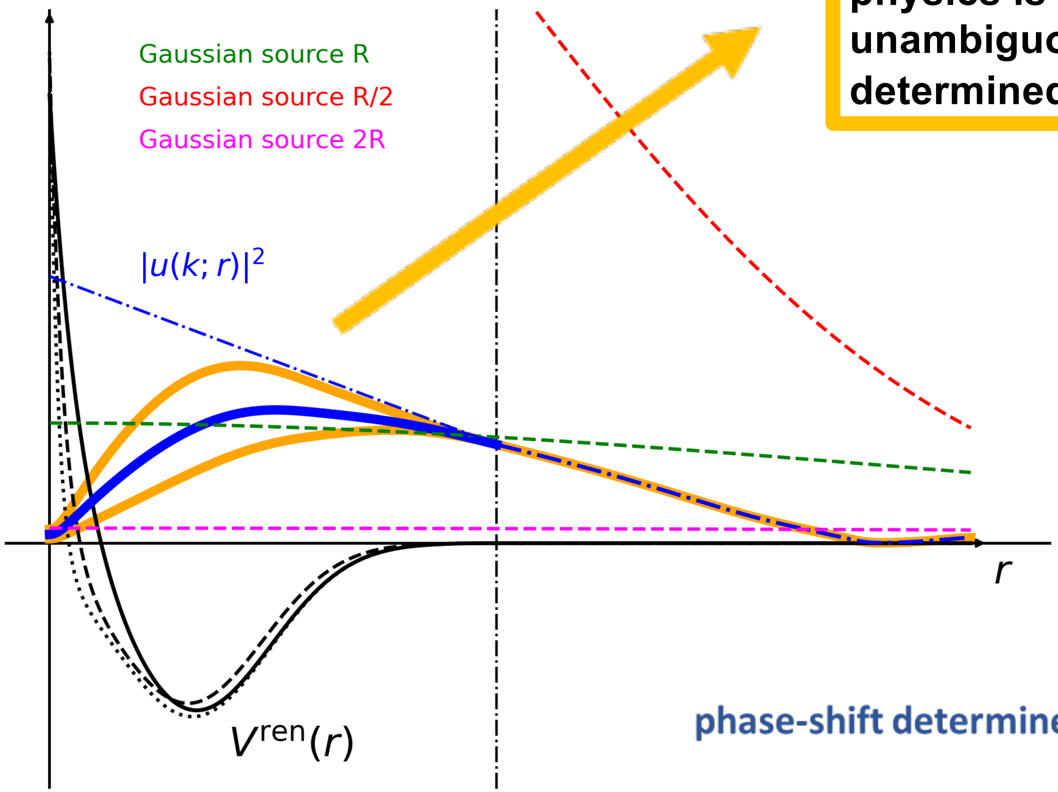

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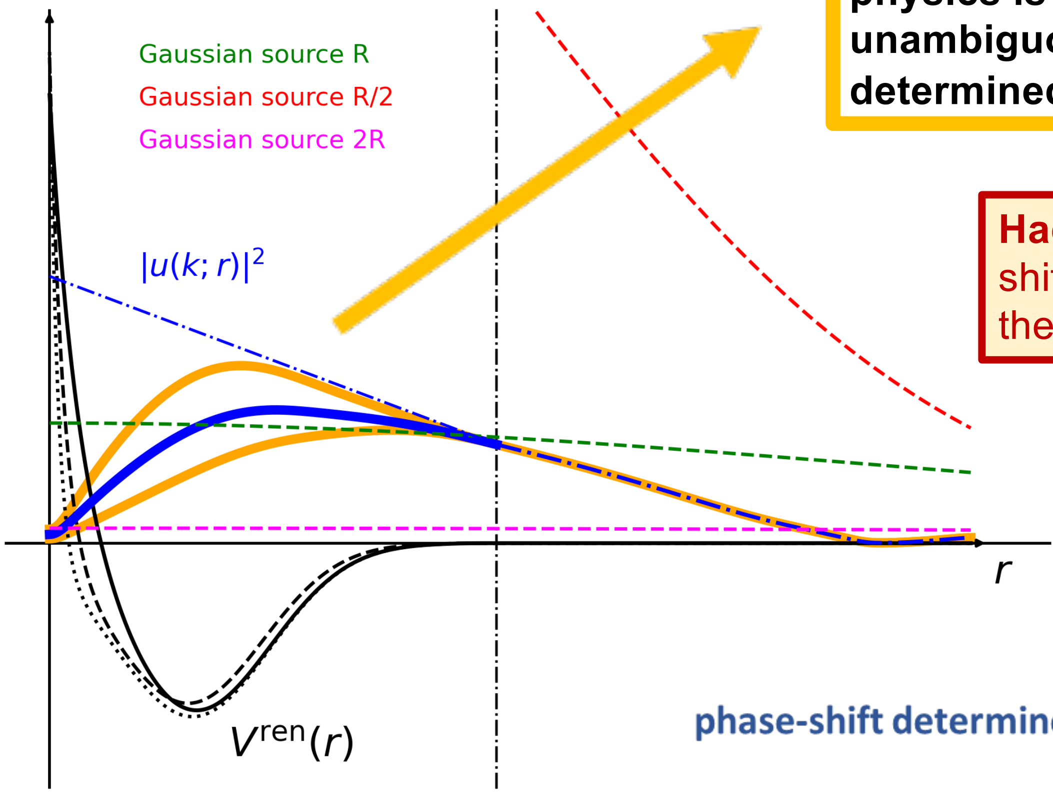

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Hadronic EFTs constrain only on-shell observables (phase shifts).The **off-shell behavior is scheme dependent**, as is the short-distance behavior of the wave function.

$$-u''(k;r) + 2\mu V(r)u(k;r) = k^2u(k;r)$$

$$\boxed{\lim_{r\to\infty} u(k;r) = u^{\text{asy}}(k;r) = f_0^{-1}(k)\frac{\sin(kr+\delta_0)}{k}}$$

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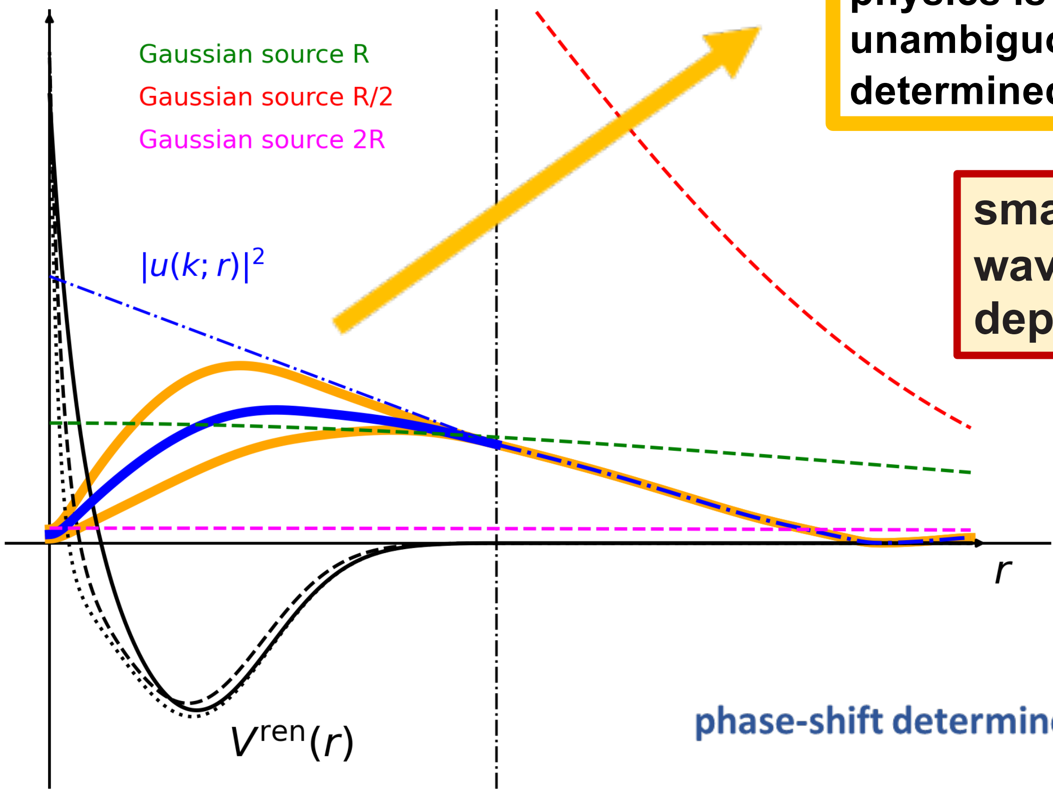

Short-distance physics is not unambiguously determined

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$$S(r)=\frac{e^{-r^2/(4\mathbf{R}^2)}}{(\sqrt{4\pi})^3\mathbf{R}^3}$$

small sources probe short distances, where the wave function carries renormalization-scheme dependence.

$$-u''(k;r) + 2\mu V(r)u(k;r) = k^2u(k;r)$$

$$\boxed{\lim_{r\to\infty} u(k;r) = u^{\mathbf{asy}}(k;r) = f_0^{-1}(k)\frac{\sin(kr+\delta_0)}{k}}$$

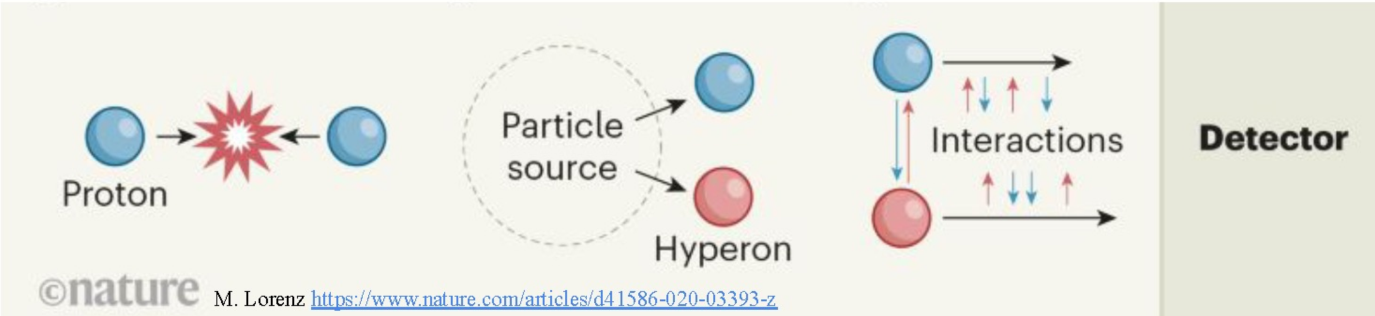
$$f_0^{-1}(k) = k\cot\delta_0 - ik$$

phase-shift determines $u^{\mathrm{asy}}(k;r)$

Femtoscopy method in nucleus-nucleus collision

Measurement of the <u>correlation function</u>
<u>of two particles emitted a nucleus-nucleus collision</u>

$$C(\overrightarrow{p_a},\overrightarrow{p_b}) = \frac{P(\overrightarrow{p_a},\overrightarrow{p_b})}{P(\overrightarrow{p_a})P(\overrightarrow{p_b})}$$



M. Lorenz https://www.nature.com/articles/d41586-020-03393-z

<u>"Non-traditional" femtoscopy</u>

- **Study the interaction** given a **known source**
- Applied to small collision systems, pp ⇒ Source size **~1fm**

L. Fabbietti, V. Mantovani, O. Vázquez Doce, Annu. Rev. Nucl. Part. Sci. (2021) 71:377

Otón Vázquez Doce, LNF -INFN
Present and future perspectives in Hadron Physics, Frascati, 2024

A ratio of two event yields, providing a **<u>robust observable</u> that contains highly valuable information on the underlying hadron–hadron interaction.**

The source must depend on the renormalization scheme (i.e. the off-shell prescription of the interaction) in such a way as to compensate the scheme dependence of the wave-function squared, ensuring a scheme-independent result and, hence, a physical observable.

$$C(\vec{k}) = \int d^3\vec{r} S(\vec{r}) |\psi(\vec{r};\vec{k})|^2$$

Hadronic EFTs constrain only on-shell observables (phase shifts).The **off-shell behavior is scheme dependent**, as is the short-distance behavior of the wave function.

(See also: E. Epelbaum, S. Heihoff, U.-G. Meißner, and A. Tscherwon, *Can the Strong Interactions between Hadrons Be Determined Using Femtoscopy?*, Phys. Rev. Lett. **136**, 212301 (2026), arXiv:2504.08631)

Small sources probe short distances, where the wave function is renormalization-scheme dependent. Therefore, the theoretical uncertainty of the KP calculation ***may become uncontrolled….***

Correlation function: ratio of two event yields, providing a **robust observable** that **contains highly valuable information on the underlying hadron–hadron interaction.**

$$C(\vec{k}) = \int d^3\vec{r} S(\vec{r}) |\psi(\vec{r};\vec{k})|^2$$

The source must depend on the renormalization scheme (i.e. the off-shell prescription of the interaction) in such a way as to compensate the scheme dependence of the wave-function squared, ensuring a scheme-independent result and, hence, a physical observable.

simultaneous fit of the ultraviolet cutoff and source radius (R. Molina and E. Oset, Phys. Rev. D **112**, 096006 (2025), arXiv:2506.03669 [hep-ph])

PHYSICAL REVIEW LETTERS **136,** 212301 (2026)

Can the Strong Interactions between Hadrons Be Determined Using Femtoscopy?

Evgeny Epelbaum,[1,*] Sven Heihoff,[1,†] Ulf-G. Meißner,[2,3,4,5,6,‡] and Alexander Tscherwon[1,§]

$$C(\mathbf{k}) = \int d\mathbf{r}\, S_{12}(\mathbf{r}) |\Psi(\mathbf{r},\mathbf{k})|^2,$$

$$C(\mathbf{k}) = \left\langle \Psi^{(+)}_{-\mathbf{k}} | \hat{S}_{12} | \Psi^{(+)}_{-\mathbf{k}} \right\rangle.$$

The Koonin-Pratt formula, Eq. (1), is the coordinate-space representation of Eq. (2), subject to the restriction that the source term is local, $\langle \mathbf{r}' | \hat{S}_{12} | \mathbf{r} \rangle = \delta(\mathbf{r}' - \mathbf{r}) S_{12}(\mathbf{r})$. Equation (2) makes it explicit that the correlation functions are invariant under a change of basis in the Hilbert space induced by unitary transformations (UTs) $\hat{U}$ as required for any observable quantity [22]:

$$C(\mathbf{k}) = \left(\left\langle \Psi^{(+)}_{-\mathbf{k}} \middle| \hat{U}^\dagger \right) \left(\hat{U} \hat{S}_{12} \hat{U}^\dagger \right) \left(\hat{U} \middle| \Psi^{(+)}_{-\mathbf{k}} \right\rangle \right)$$
$$\equiv \left\langle \Psi'^{(+)}_{-\mathbf{k}} \middle| \hat{S}'_{12} \middle| \Psi'^{(+)}_{-\mathbf{k}} \right\rangle.$$

See also the recent density-operator–measurement formulation of two-particle femtoscopic correlations, extending beyond the Koonin–Pratt formalism, by H.-N Liu, D.-L Ge, Z.-W- Liu, J.-X. Lu, and L.-S. Geng.

THEORETICAL OBJECTIVE

Develop an improved Koonin–Pratt framework to systematically and consistently extract hadron–hadron phase shifts as physical observables.

Correlation function: ratio of two event yields, providing a **robust observable that contains highly valuable information on the underlying hadron–hadron interaction.**

$$C(\vec{k}) = \int d^3\vec{r}S(\vec{r})|\psi(\vec{r};\vec{k})|^2$$

The source must depend on the renormalization scheme (i.e. the off-shell prescription of the interaction) in such a way as to compensate the scheme dependence of the wave-function squared, ensuring a scheme-independent result and, hence, a physical observable.

On-shell factorization scheme [R. Lednicky, V.L. Lyuboshits Yad. Fiz. 35, 1316 (1981)]

$$C(\vec{k}) = \int d^3\vec{r}\, S(\vec{r})\, |\psi(\vec{r};\vec{k})|^2$$

$$\psi^*(\vec{r};\vec{k}) = e^{-i\vec{k}\cdot\vec{r}} + \int d^3\vec{p}\,\frac{e^{-i\vec{p}\cdot\vec{r}}}{E-\frac{\vec{p}^2}{2\mu_{ab}}+i\epsilon}\langle\vec{k}|\hat{T}^{\mathrm{QM}}(E+i\epsilon)|\vec{p}\rangle.$$

$$= e^{-i\vec{k}\cdot\vec{r}} + 4\pi\int dp\,p^2\,\frac{j_0(pr)}{E-\frac{p^2}{2\mu_{ab}}+i\epsilon}\boxed{T^{\mathrm{QM}}(k\leftarrow p;E)}$$

half off-shell T −matrix

$\approx T^{QM}(k \leftarrow k)$ **on-shell T −matrix**

only S-wave interaction

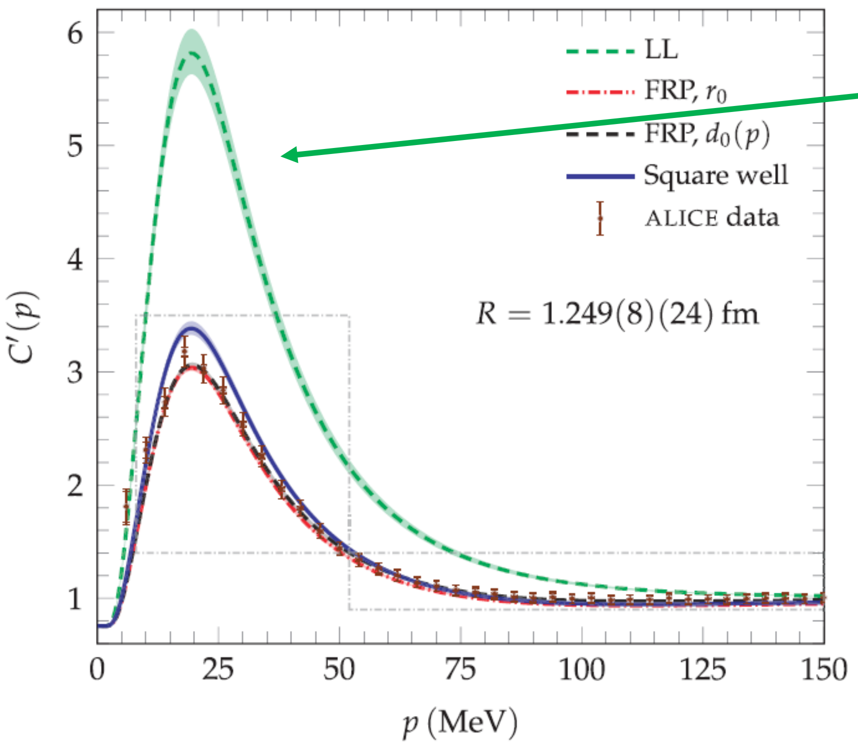
If the half off-shell $T^{QM}(k \leftarrow p)$ is **approximated by the on-shell amplitude** $T^{QM}(k \leftarrow k)$ the integrations can be done analytically….

asymptotic wave function

$$\psi^*_{LL}(\vec{r};\vec{k}) = e^{-i\vec{k}\cdot\vec{r}} + f_0(k)\frac{e^{ikr}}{r} \qquad f_0(k) = \frac{1}{k\cot\delta_0(k) - ik}$$

$x = 2kR$, $F_{1,2}$ analytical known functions

$$C_{LL}(k) = 1 + \frac{2\mathrm{Re}f_0(k)}{\sqrt{\pi}R}F_1(x) - \frac{\mathrm{Im}f_0(k)}{R}F_2(x) + \frac{|f_0(k)|^2}{2R^2} = 1 + \frac{2\mathrm{Re}f_0(k)}{\sqrt{\pi}R}F_1(x) + \frac{|f_0(k)|^2}{2R^2}e^{-x^2}$$



proton-proton CFs: deficiencies of on-shell **Lednicky & Lyuboshits model (green curve)** for $R \sim 1.25$ fm

ALICE data: PLB 805 (2020) 135419

$$-u''(k;r) + 2\mu V(r)u(k;r) = k^2 u(k;r)$$

$$\lim_{r\to\infty} u(k;r) = u^{\mathbf{asy}}(k;r) = f_0^{-1}(k)\frac{\sin(kr+\delta_0)}{k}$$

$$f_0^{-1}(k) = k\cot\delta_0 - ik$$

$$-\frac{d}{dk^2}[\mathrm{Re}(f_0^{-1}(k))] = -\frac{d}{dk^2}\left[-\frac{1}{a} + \frac{1}{2}r_0k^2 + vk^4 + \cdots\right] = \int_0^{+\infty} dr\,(u^2(k;r) - [u^{\mathbf{asy}}(k;r)]^2)$$

$$C(k) = C_{LL}(k) + \boxed{\delta C_{FR}(k)}$$

$$\propto \int_0^{+\infty} dr\, \boldsymbol{S(r)}\, (u^2(k;r) - [u^{\mathbf{asy}}(k;r)]^2)$$

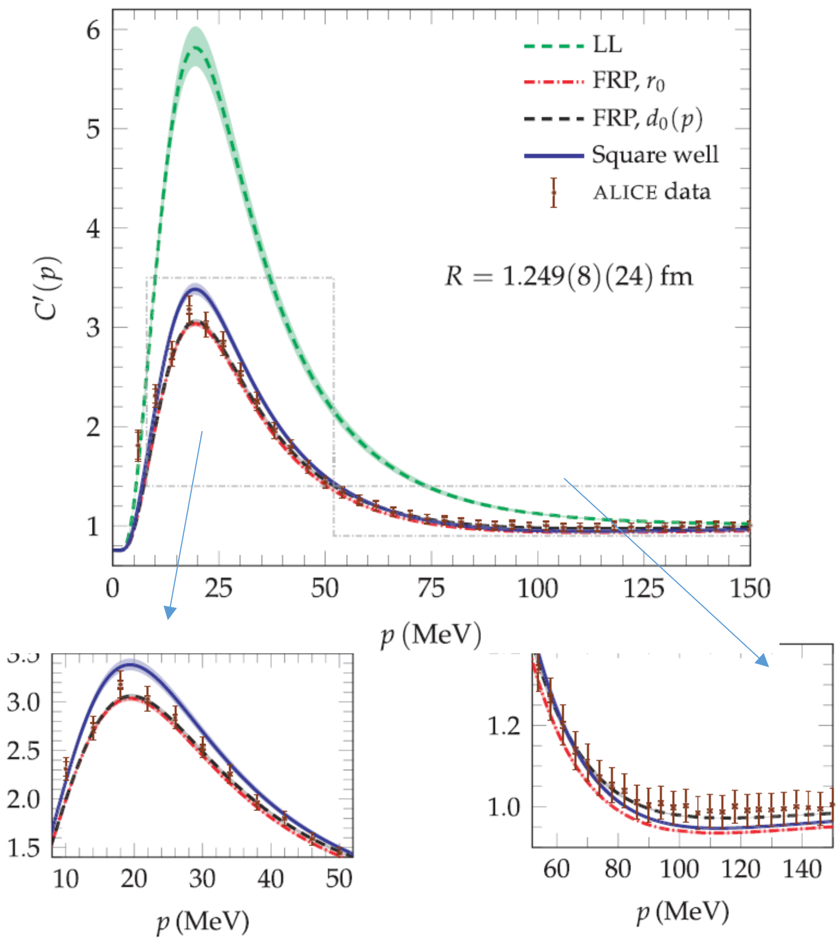
$$\approx -S(0) \times \frac{d}{dk^2}\left[\mathrm{Re}\left(f_0^{-1}(k)\right)\right]$$

FINITE-RANGE CORRECTIONS

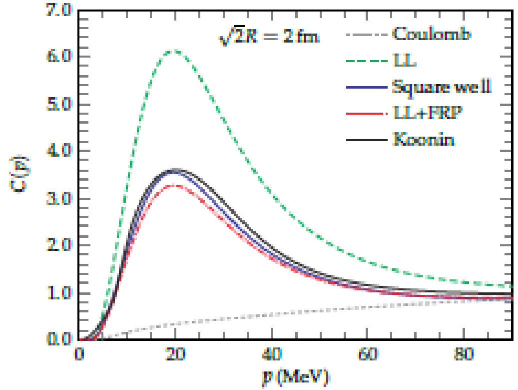
[R. Lednicky, V.L. Lyuboshits Yad. Fiz. 35, 1316 (1981)]

Lednicky & Lyuboshits

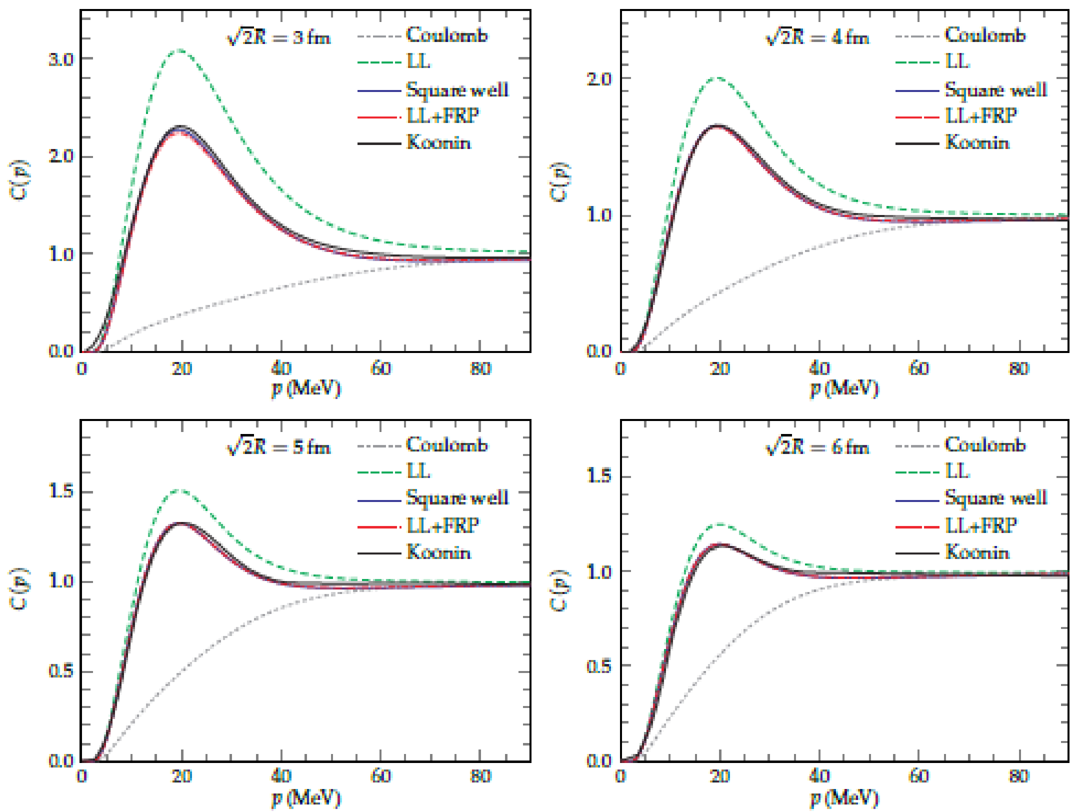
ALICE data: PLB 805 (2020)



Albaladejo, García-Lorenzo and Nieves, PTEP 2025 (2025) 093D02

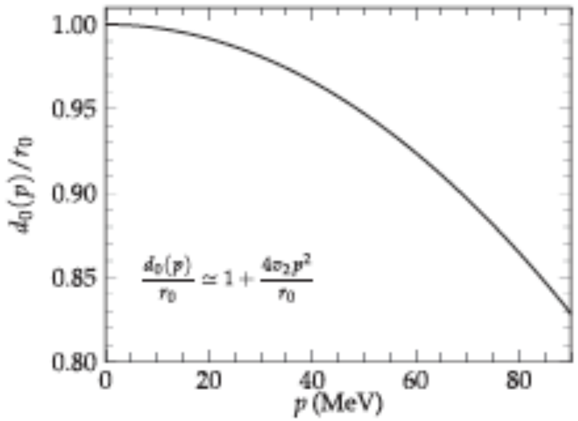


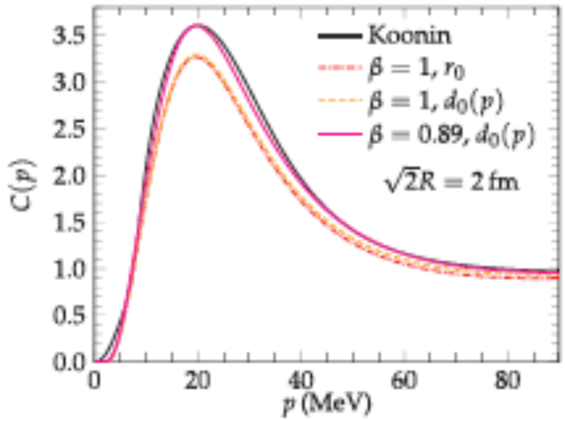
S. E. Koonin, PLB 70 (1977) 43 (NN Reid Soft Core potential)

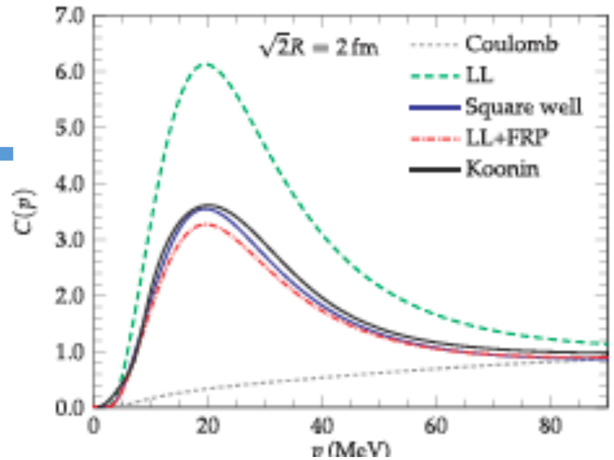


proton-proton CFs: deficiencies of on-shell **Lednicky & Lyuboshits model (green curves)** for small source radius!

$R \sim 0.7$ fm







S. E. Koonin, PLB 70 (1977) 43
(NN Reid Soft Core potential)

Phenomenological parameter

$$-\frac{d}{dk^2}\left[\mathrm{Re}\left(f_0^{-1}(k)\right)\right] = \int_0^{+\infty} dr\,\left(u^2(k;r) - [u^{\mathbf{asy}}(k;r)]^2\right)$$

$$C(k) = C_{LL}(k) + \boxed{\delta C_{FR}(k)}$$

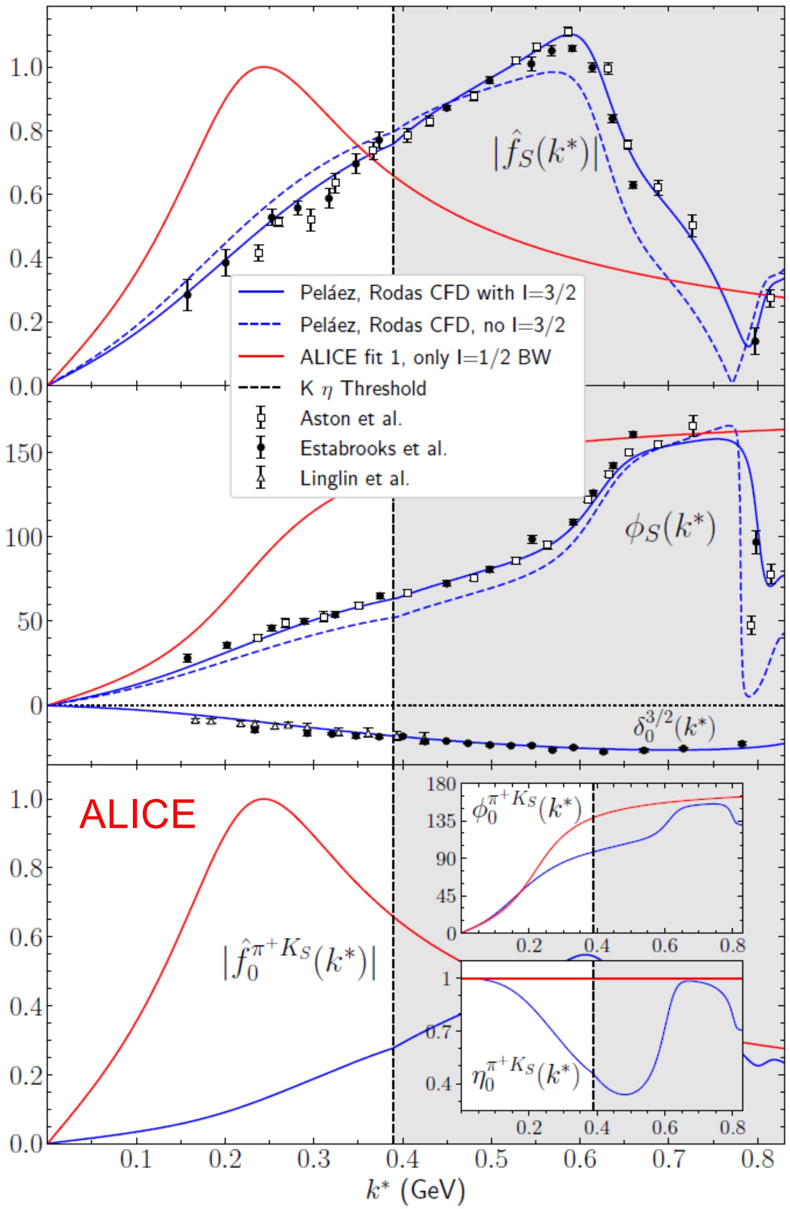
$$\delta C_{FR}(k) \to \int_0^{+\infty} dr\, \boldsymbol{S(r)}\left(u^2(k;r) - [u^{\mathbf{asy}}(k;r)]^2\right)$$

$$\approx -\boldsymbol{\beta} S(0) \times \frac{d}{dk^2}\left[\mathrm{Re}\left(f_0^{-1}(k)\right)\right]$$

FINITE-RANGE CORRECTIONS

[R. Lednicky, V.L. Lyuboshits Yad. Fiz. 35, 1316 (1981)]

Lednicky & Lyuboshits



Albaladejo, Canoa, Nieves, Peláez, Ruiz Arriola, Ruiz de Elvira, PLB 866 (2025) 139552

ALICE data $\pi^{\pm}K_S$ CFs [PLB 856 (2024) 138915]

$\kappa/K_0^*(700)$

Within **the on-shell scheme**:

i) improve on the ALICE πK BW interaction and use realistic $\pi^{\pm}K_S, \pi^{\pm}K_L, \pi^0K^{\pm}$ interactions obtained from a dispersive analysis of scattering data [J. R. Peláez and A. Rodas, Phys. Rept. **969** (2022) 1]

"QCD spontaneous chiral symmetry breaking threshold suppression"

ii) relativistic corrections

We get smaller-than-usual source radius which should raise concerns about its applicability

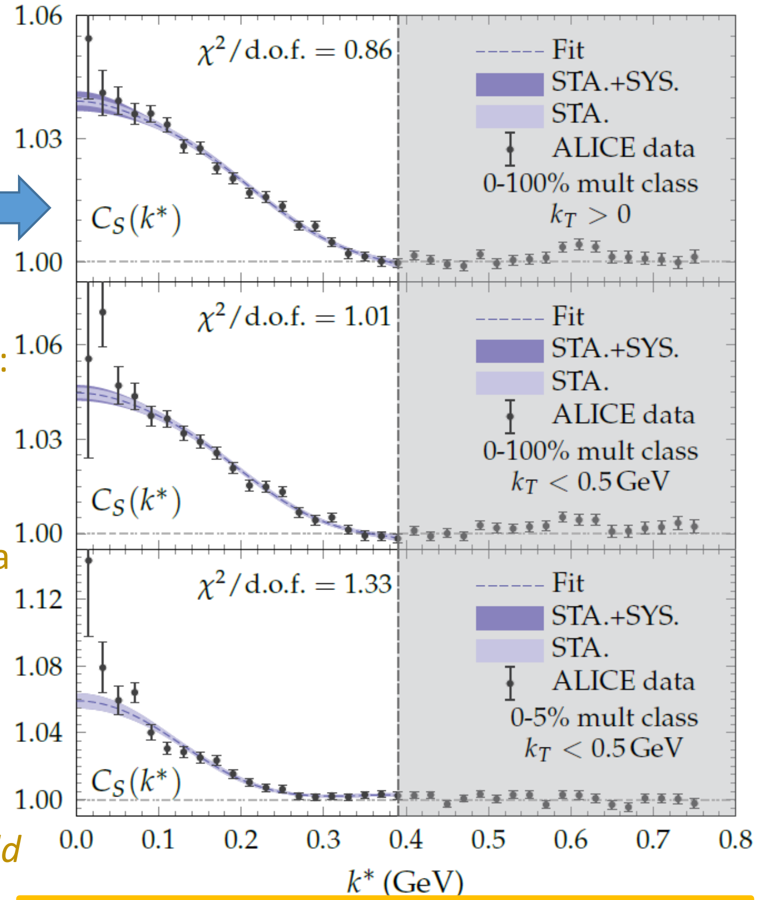
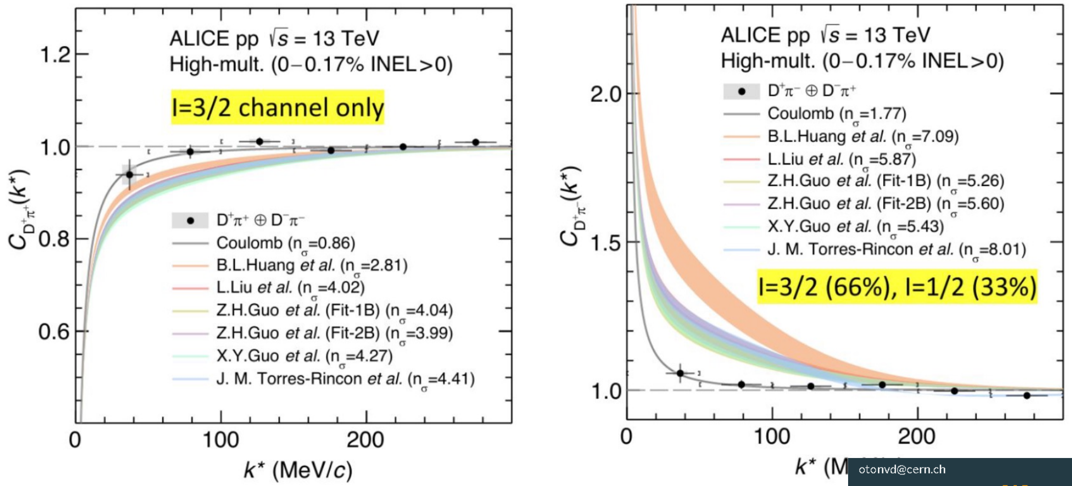


TABLE I. Parameters and χ^2/d.o.f. of the fits in Fig. 1.

	0-100% m. cl. $k_T > 0$	0-100% m. cl. $k_T < 0.5$ GeV	0-5% m. cl. $k_T < 0.5$ GeV
χ^2/d.o.f.	0.86	1.01	1.33
R (fm)	0.36(3)(3)	0.41(3)(3)	0.68(5)(3)
λ	0.19(2)(4)	0.29(4)(5)	0.80(14)(13)
$(N-1)\times10^2$	0.80(8)(7)	0.85(8)(6)	0.97(6)(4)

$D^{(*)}$–π femtoscopy



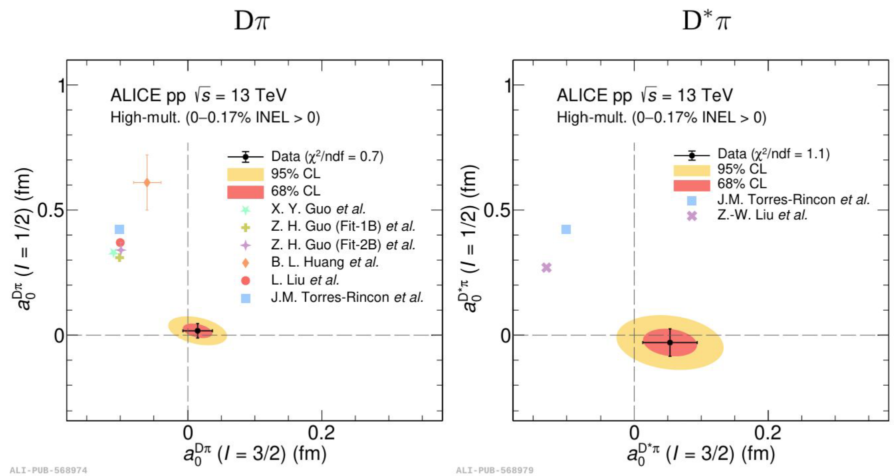
L. Liu et al, Phys. Rev. D87 (2013) 014508
X.-Y. Guo et al, Phys. Rev. D 98 (2018) 014510
Z.-H. Guo et al Eur. Phys. J. C 79 (2019) 13
B.-L. Huang et al, Phys. Rev. D 105 (2022) 036016
J. M. Torres-Rincon et al, Phys. Rev. D 108 (2023) 096008

Otón Vázquez Doce, LNF -INFN Present and future perspectives in Hadron Physics, Frascati, 2024

Femtoscopy results are in tension with robust, independent predictions from Unitarized ChPT, LQCD, and LHCb ($\bar{B} \longrightarrow D\pi\pi$) decay amplitudes!

$D^{(*)}$–π interaction

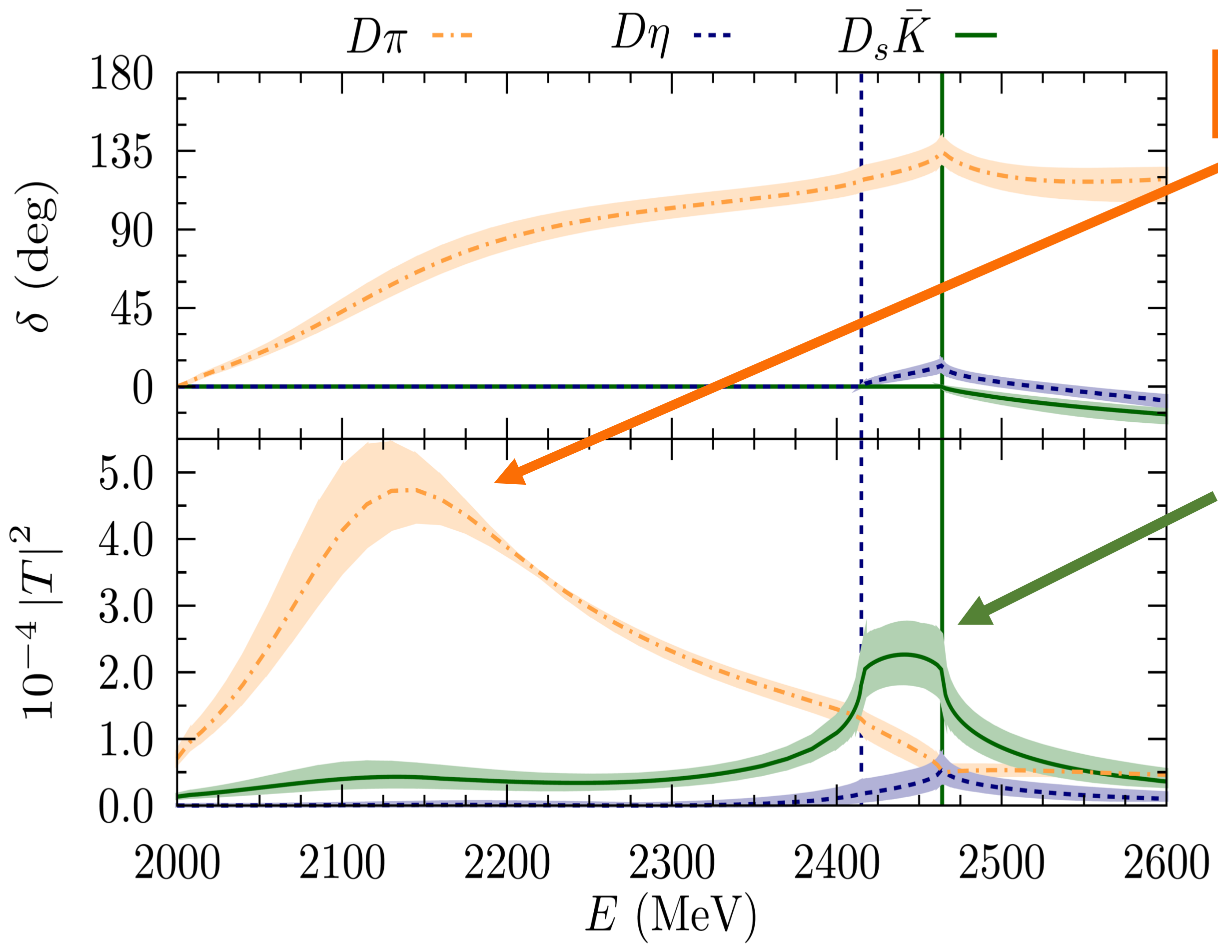
ALICE Coll. arXiv:2401.13541 [nucl-ex]



Small scattering lengths:
- compatible with Coulomb-only assumption
- theory overestimates interaction strength

Scattering lengths similar for Dπ and $D^*\pi$:
- heavy-quark spin symmetry

- DK data so far not precise enough



Lower pole, $\sqrt{s} = (2.1 \; - i\; 0.1)$ GeV

- $|T_{11}(s)|^2$ **peaks at** $\sqrt{s} \sim 2.1$ **GeV**
- $\delta_{11}(s) = \,^{\pi}/_{2}$ **at** $\sqrt{s} \sim 2.2$ **GeV**

Higher pole, $\sqrt{s} = (2.45 \; - i\; 0.13)$ GeV

- **small enhancement in** $|T_{11}(s)|$
- **clear peak in the** $D_s\bar{K}$ **amplitude.** **Narrow, non-conventional shape, stretched between thresholds cusps.**
- **Tests: FSI in** $B \longrightarrow D\phi\phi$ **decays**

Amplitudes

$$-i\; p_{ii}(s) T_{ii}(s)$$
$$= \; 4\pi\sqrt{s}\;\left(\eta_i(s) e^{2i\delta_{ii}(s)} - 1\right)$$

Two-pole structure of the $D_0^*(2400)$

Miguel Albaladejo[a,b], Pedro Fernandez-Soler[a], Feng-Kun Guo[c,d,*], Juan Nieves[a]

Phys. Lett. B767 (2017) 465

Conclusions

- **Koonin–Pratt vs. production observables:** Koonin–Pratt femtoscopic correlation functions and invariant-mass distributions from production experiments are **different observables**. They become equivalent only in the **zero-source-size limit**.

- **Short-distance sensitivity:** Small sources probe short distances, where the wave function is **renormalization-scheme dependent**. Consequently, the theoretical uncertainty of standard Koonin–Pratt calculations may become uncontrolled.

- **A consistent framework:**The source must consistently account for the renormalization scheme (i.e. the off-shell prescription of the interaction) so as to compensate the scheme dependence of the wave function and ensure a **scheme-independent, physical correlation function**.

- **Beyond Lednicky–Lyuboshits:** Off-shell and finite-range effects neglected in the on-shell Lednicky–Lyuboshits approximation can be **essential for realistic femtoscopic correlation functions**, particularly for small source radii.

- **Physics opportunity:** High-statistics femtoscopic data, such as those from ALICE for $\pi^{\pm}K_S$ and πD pairs, can provide valuable information on hadron–hadron interactions, especially where scattering data are scarce or unavailable. **A proper calibration requires testing the method in systems where phase shifts and inelasticities are known.**

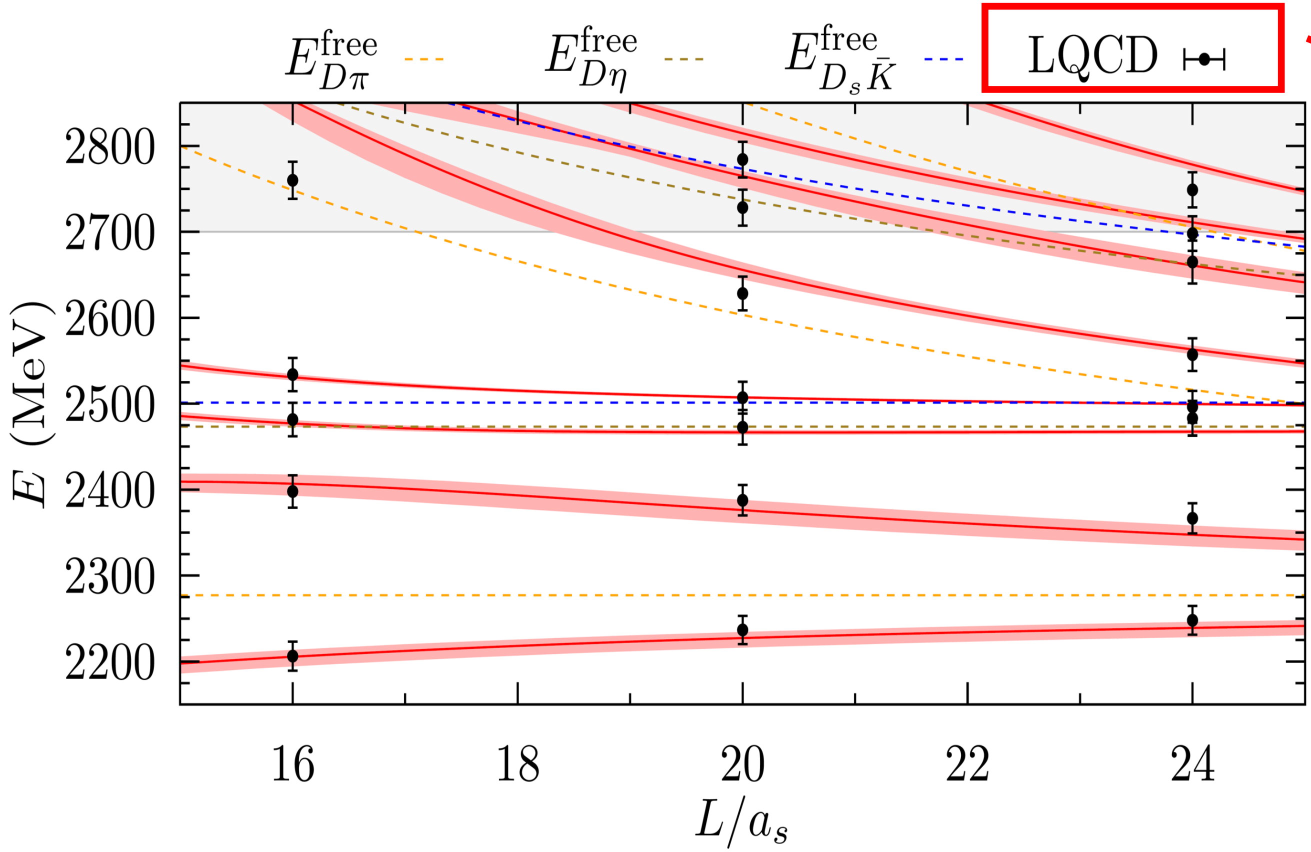
Theoretical objective: Develop an improved Koonin–Pratt framework to systematically and consistently extract hadron–hadron phase shifts as physical observables.

Backup

✓ Free energy levels: $E^{i}_{n,\text{free}}(L) = \omega_i\left(\frac{2\pi}{L}\vec{n}\right) + \omega'_i\left(\frac{2\pi}{L}\vec{n}\right)$

M. Albaladejo, P. Fernández-Soler, F.-K. Guo and JN, PLB767 (2017) 465

✓ Interacting energy levels $E_n(L)$ such that: $\boxed{\widetilde{T}^{-1}(E_n^2(L), L) = 0}$ **[poles of the $\widetilde{T}$ matrix]**



LQCD: G. Moir et al., JHEP 10 (2016) 011 (Hadron Spectrum Collaboration).
$D\pi, D\eta, D_s\overline{K}$ coupled-channels

M (MeV)	Latt.	Phys.
π	391	138
K	550	496
η	588	548
D	1886	1867
D_s	1952	1968

Chiral $D_{(s)}^{(*)}\phi$ molecular structure natural solution to three (experimental) puzzles:

✓ Why are the $M_{D_{s0}^*(2317)}$ & $M_{D_{s1}(2460)} \ll$ CQM $c\bar{s}$ 0^+ and 1^+ mass predictions

✓ Why $(M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)}) \sim (M_{D^*} - M_D)$ within 2 MeV (binding energies are independent of the heavy meson spin, as the leading spin symmetry breaking interaction is also of NLO in the chiral expansion).

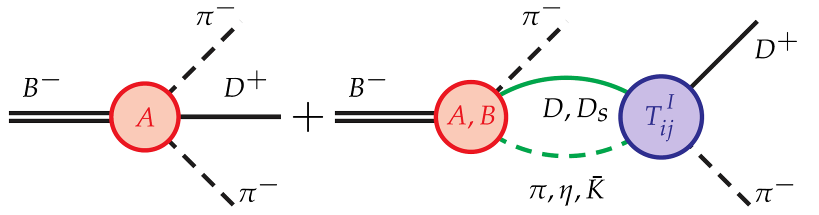
✓ Why are the $D_0^*(2300)$ $[0^+]$ and $D_1(2430)$ $[1^+]$ masses (higher members of the two-pole structure) almost equal to or even higher than their strange siblings despite of $\frac{m_s}{m_d} \sim 20$

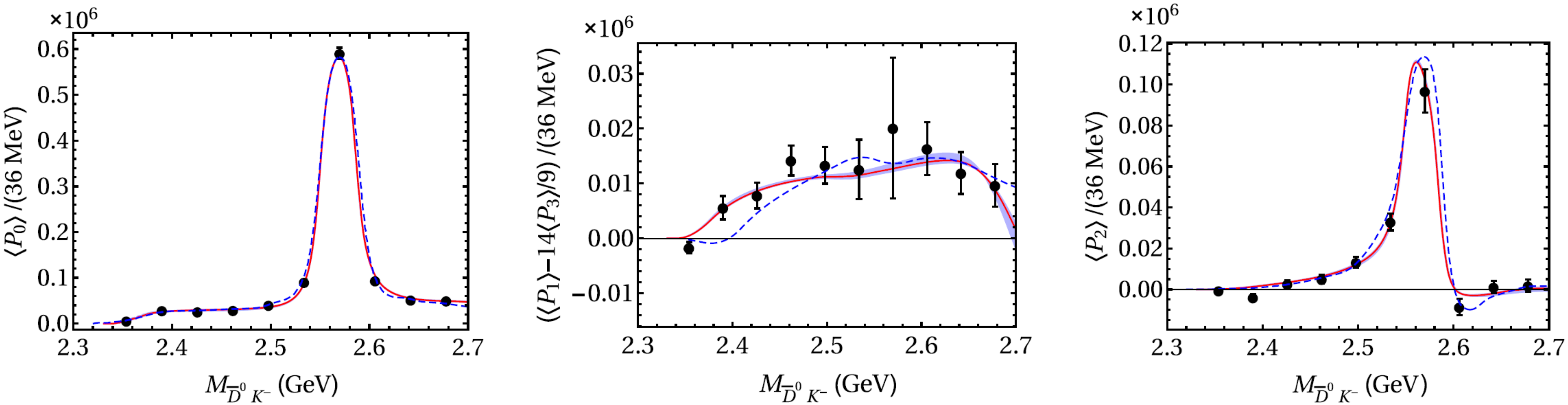
...confirmed by LHCb data [R. Aaij et al. PRD 94 (2016) 072001] for the $B^- \to D^+\pi^-\pi^-$ reaction

$(\pi^-\pi^-) \Rightarrow I = 2$

FSI between the two π^- is negligible

$B^- \to D^+\pi^-\pi^-$ provides access to the **$D\pi$ system and thus to the charm-nonstrange mesons**

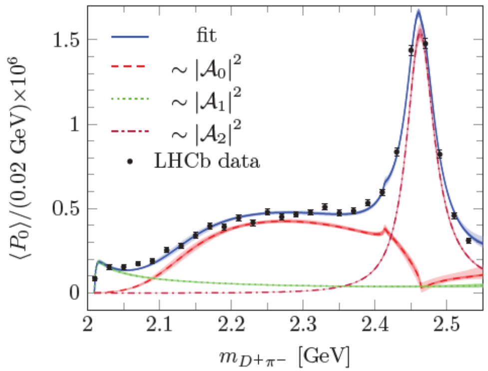


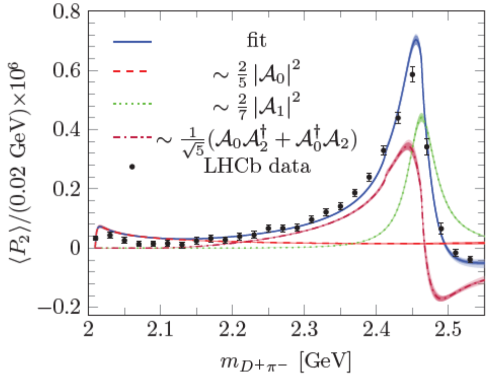


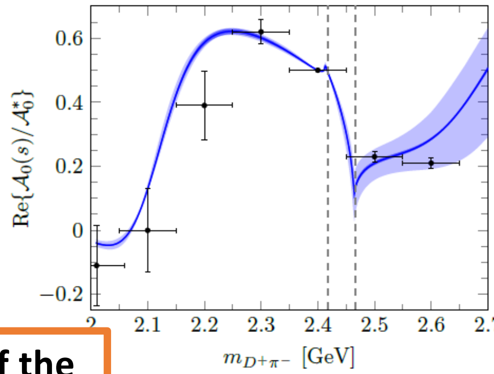
the LHCb data [R. Aaij et al. PRD 90 (2014) 072003] for the angular moments for $B_s^0 \longrightarrow \overline{D}^0 K^- \pi^+$ can be easily reproduced in the same framework with the untarized chiral $\overline{D}\,\overline{K}$ coupled-channels S-wave amplitude

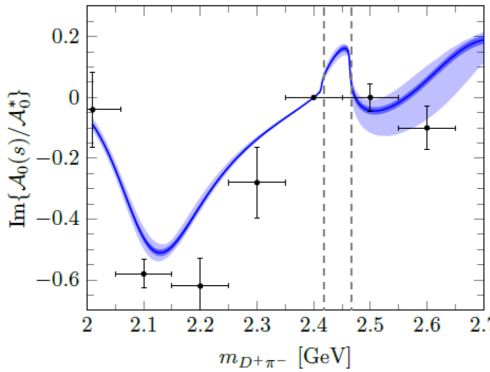
see also the works by M.-L Du, F.-K. Guo and U.-G. Meißner, PRD 99 (2019) 114002 (detailed combined chiral analysis of LHCb data for $B^- \longrightarrow D^+ K^- \pi^-, B_s^0 \longrightarrow \overline{D}^0 K^- \pi^+$, $B^0 \longrightarrow \overline{D}^0 \pi^- \pi^+$ and $B^0 \longrightarrow \overline{D}^0 K^+ \pi^-$) and M.-L. Du, F.-K. Guo, C. Hanhart , B. Kubis and U.-G. Meißner, PRL 126 (2021) 192001 (discussion of great deficiencies of BW parametrization to describe the LHCb data for $B^- \longrightarrow D^+ \pi^- \pi^-$)

$\mathcal{A}_0$ **(S-wave)**

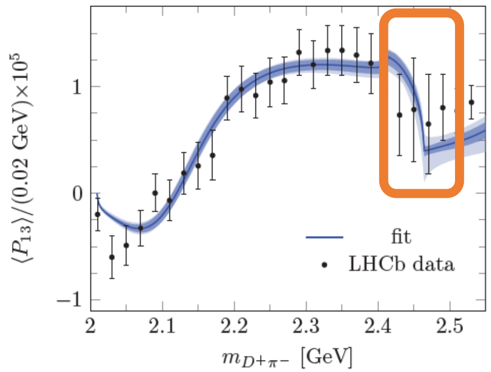




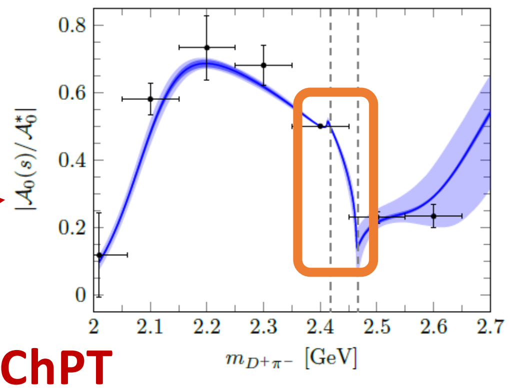




$< P_0 >$
$< P_2 >$
$< P_{13} >$



cusps: opening of the $D^0\eta$ and $D_s^+K^-$ thresholds enhanced by the higher $D_0^*(2300)$ pole



$$\langle P_0 \rangle \propto \boxed{|\mathcal{A}_0|^2} + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2,$$

$$\langle P_2 \rangle \propto \frac{2}{5}|\mathcal{A}_1|^2 + \frac{2}{7}|\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}}\boxed{|\mathcal{A}_0|}|\mathcal{A}_2|\cos(\Delta\delta_2),$$

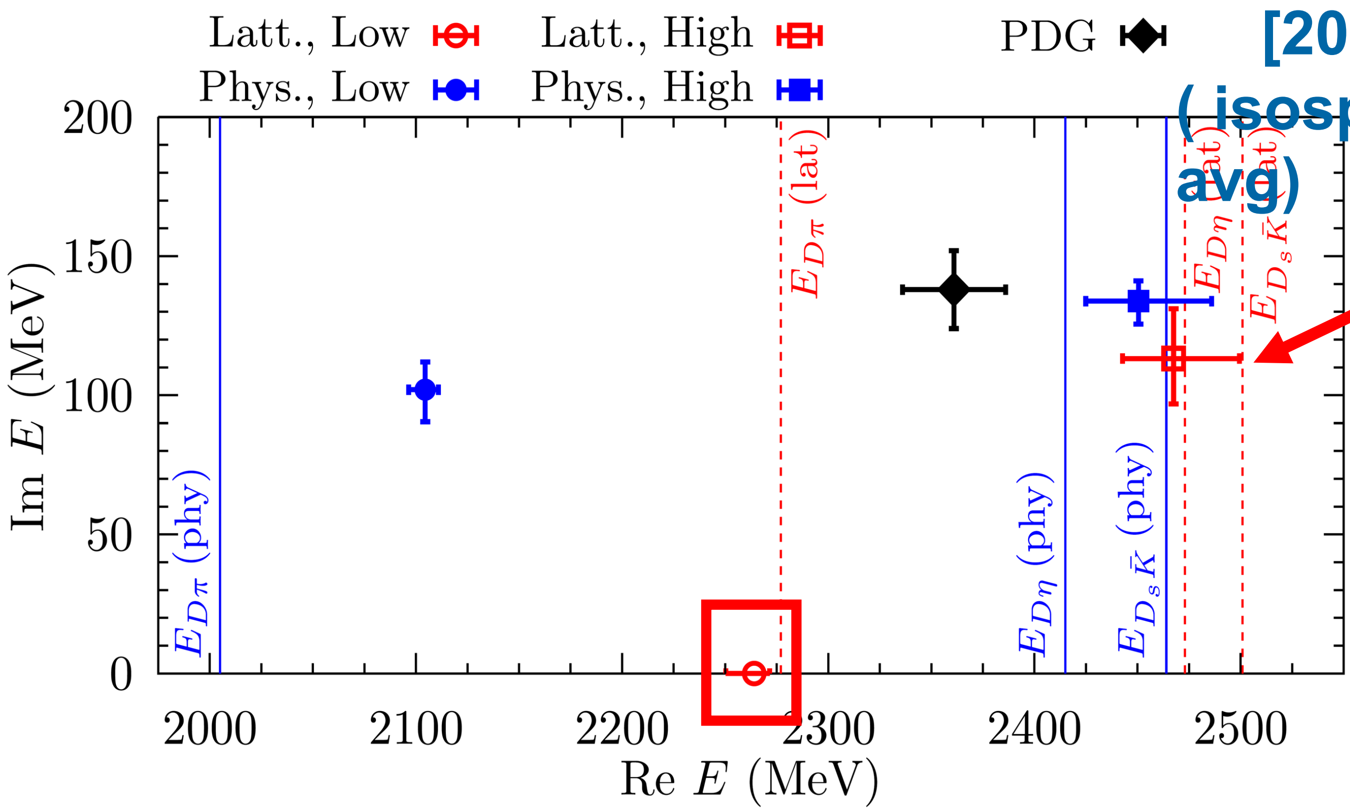
$$\langle P_{13} \rangle \equiv \langle P_1 \rangle - \frac{14}{9}\langle P_3 \rangle \propto \frac{2}{\sqrt{3}}\boxed{|\mathcal{A}_0|}|\mathcal{A}_1|\cos(\Delta\delta_1),$$

$\mathcal{A}_0$ **(S-wave) from UHMChPT**

M.-L.Du, M. Albaladejo, P. Fernández-Soler, F.-K. Guo, C. Hanhart, U.-G. Meißner, JN and D.-L. Yao, PRD 98 (2018) 094018

[2016]

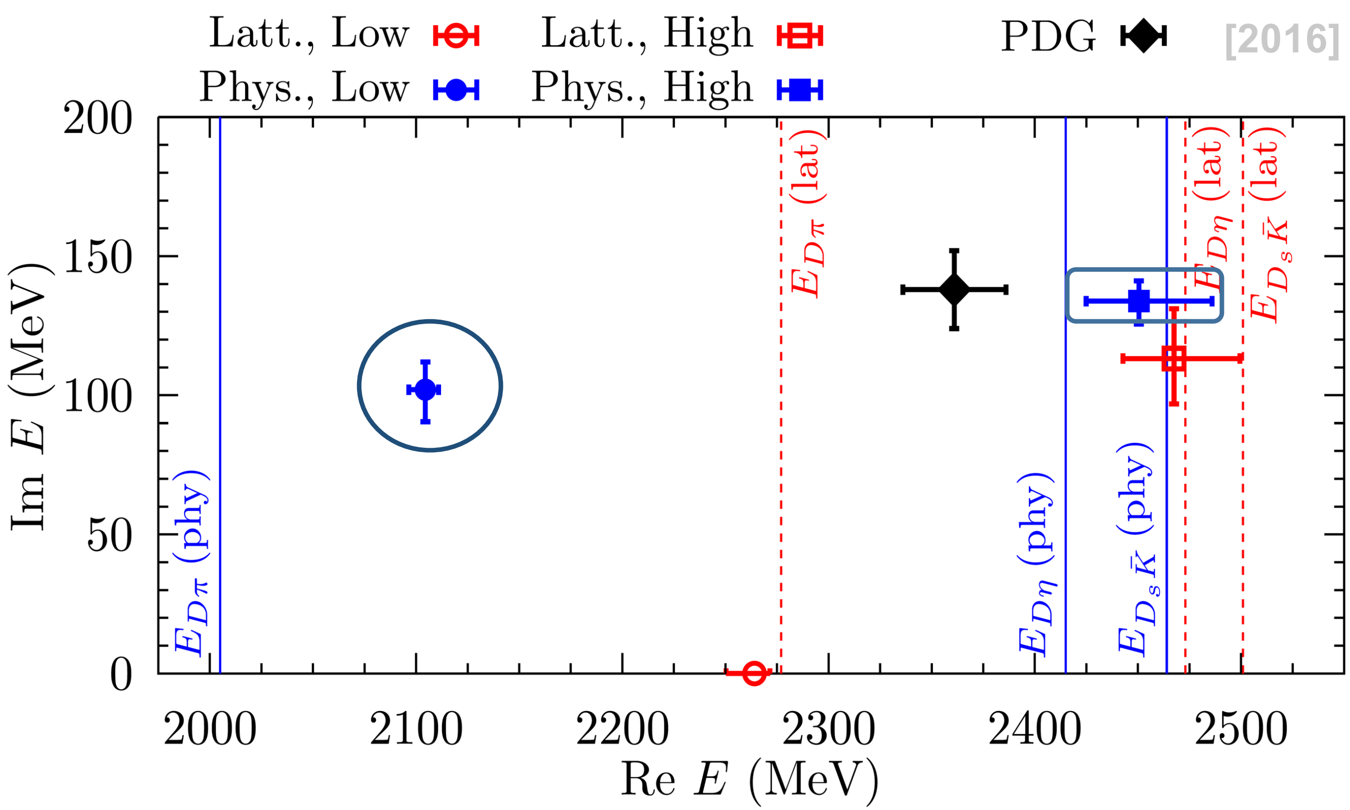
(isospin avg)



✓ **For lattice masses, we find a bound state (000) and a resonance (110)**

M. Albaladejo, P. Fernández-Soler, F.-K. Guo and JN, PLB767 (2017) 465

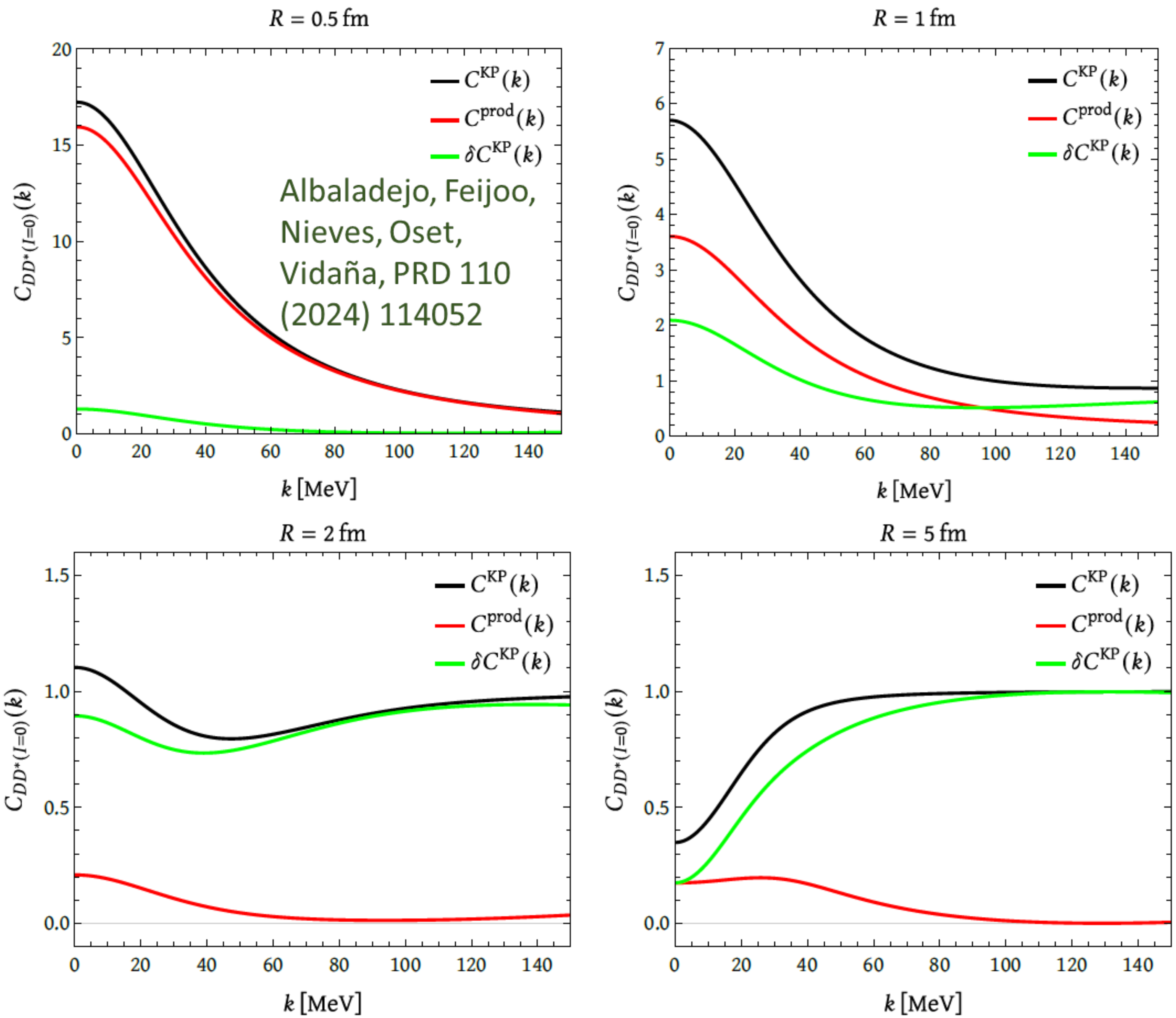
Masses	M (MeV)	$\Gamma/2$ (MeV)	RS	$\lvert g_{D\pi}\rvert$	$\lvert g_{D\eta}\rvert$	$\lvert g_{D_s\bar{K}}\rvert$
lattice	2264^{+8}_{-14}	0	(000)	$7.7^{+1.2}_{-1.1}$	$0.3^{+0.5}_{-0.3}$	$4.2^{+1.1}_{-1.0}$
	2468^{+32}_{-25}	113^{+18}_{-16}	(110)	$5.2^{+0.6}_{-0.4}$	$6.7^{+0.6}_{-0.4}$	$13.2^{+0.6}_{-0.5}$
physical	2105^{+6}_{-8}	102^{+10}_{-12}	(100)	$9.4^{+0.2}_{-0.2}$	$1.8^{+0.7}_{-0.7}$	$4.4^{+0.5}_{-0.5}$
	2451^{+36}_{-26}	134^{+7}_{-8}	(110)	$5.0^{+0.7}_{-0.4}$	$6.3^{+0.8}_{-0.5}$	$12.8^{+0.8}_{-0.6}$



The $D_0^*(2300)$ structure is actually produced by two different states (poles), together with complicated interferences with thresholds. This two-pole structure was previously reported, and receives now a robust support

Masses	M (MeV)	$\Gamma/2$ (MeV)	RS	$\lvert g_{D\pi}\rvert$	$\lvert g_{D\eta}\rvert$	$\lvert g_{D_s\bar{K}}\rvert$
lattice	2264^{+8}_{-14}	0	(000)	$7.7^{+1.2}_{-1.1}$	$0.3^{+0.5}_{-0.3}$	$4.2^{+1.1}_{-1.0}$
	2468^{+32}_{-25}	113^{+18}_{-16}	(110)	$5.2^{+0.6}_{-0.4}$	$6.7^{+0.6}_{-0.4}$	$13.2^{+0.6}_{-0.5}$
physical	2105^{+6}_{-8}	102^{+10}_{-12}	(100)	$9.4^{+0.2}_{-0.2}$	$1.8^{+0.7}_{-0.7}$	$4.4^{+0.5}_{-0.5}$
	2451^{+36}_{-26}	134^{+7}_{-8}	(110)	$5.0^{+0.7}_{-0.4}$	$6.3^{+0.8}_{-0.5}$	$12.8^{+0.8}_{-0.6}$

Kolomeitsev, & Lutz, PLB 582 (2004) 39
Guo *et al.*, PLB 641 (2006) 278; Guo *et al.*, EPJA 40 (2009) 171



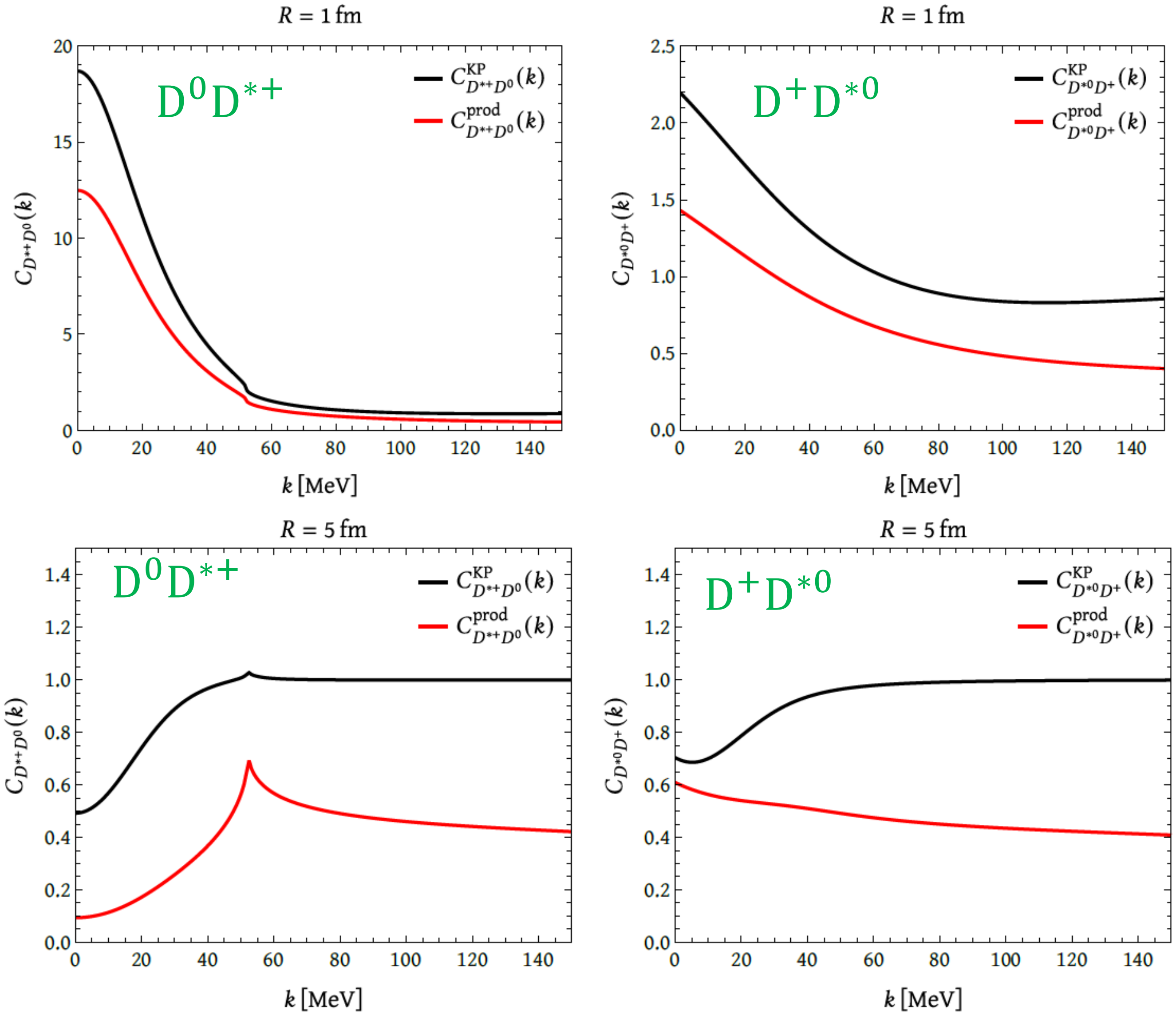
$$C(k) = \boldsymbol{C^{prod}(k)} + \delta C(k)$$

- isoscalar DD^* contact interaction regularized with hard-cutoff of around 400 MeV, which gives rise to a $T_{cc}(3875)^+$ bound by 860 keV

- The differences arise mostly from the T–matrix quadratic terms and increase with the source size

DD^* SINGLE channel

Albaladejo, Feijoo, Nieves, Oset, Vidaña, PLB 846 (2023) 138201



$C(k) = C^{\mathbf{prod}}(\boldsymbol{k}) + \delta C(k)$

- D^0D^{*+} and D^+D^{*0} correlation functions of interest to unravel the dynamics of the exotic $T_{cc}(3875)^+$.

Interaction inspired from the realistic model of A. Feijoo, W. H. Liang and E. Oset, Phys. Rev. D104, 114015 (2021). It gives rise to a $T_{cc}(3875)^+$ bound by 354 keV

$D^0D^{*+} - D^+D^{*0}$ COUPLED channels