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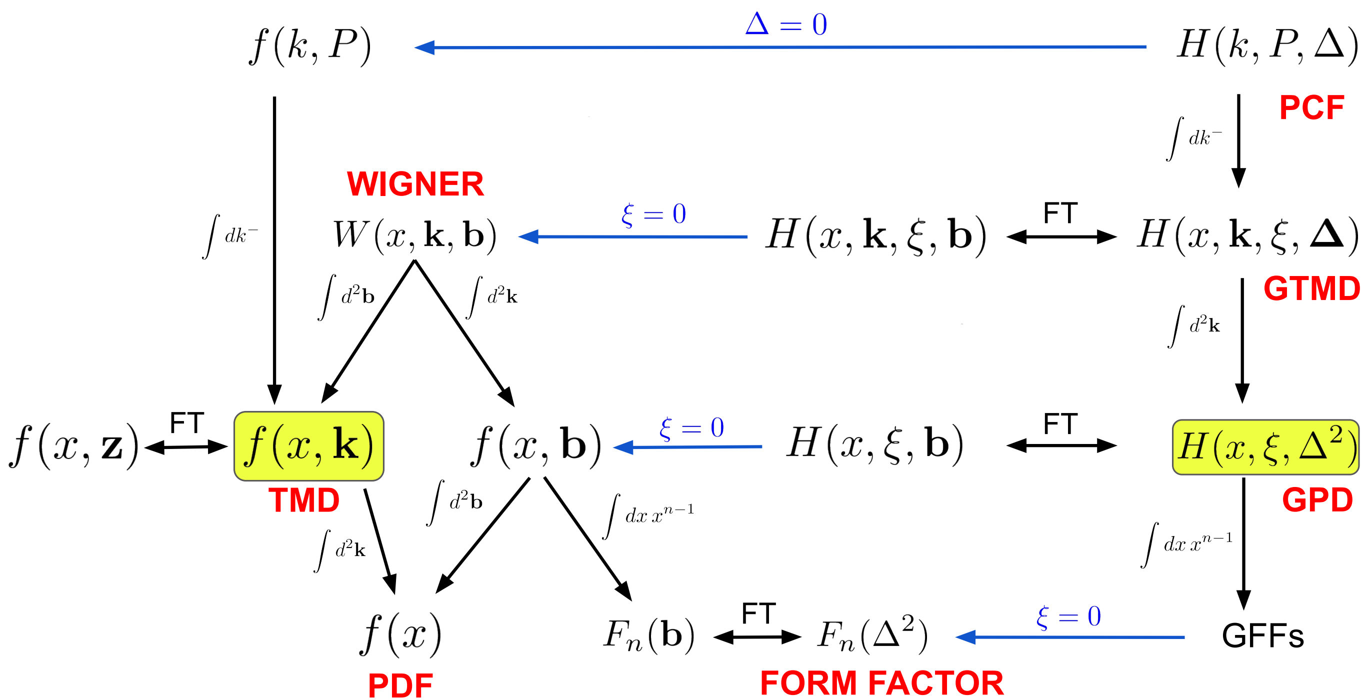
Tomography of hadrons in 3D

Outline

1. **Introduction: TMDs and GPDs**
2. **TMD formalism**
3. **Challenges and open questions**

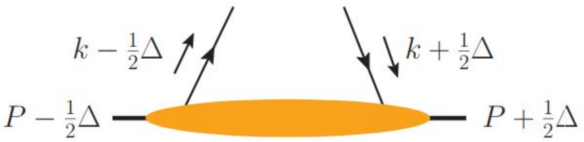
1. Introduction

The hadron structure landscape



See https://inspirehep.net/literature/3148063

TMDs and GPDs



$$x = k^+/P^+$$
$$\xi = -\Delta^+/2P^+$$

- **TMDs**: 3D picture in momentum space
- **GPDs**: picture in 1D momentum + 2D position space (**impact parameter distributions)**
- The transverse momentum **k** and transverse position **b** are **NOT related** to each other by a Fourier transform (**k** vs **z**)

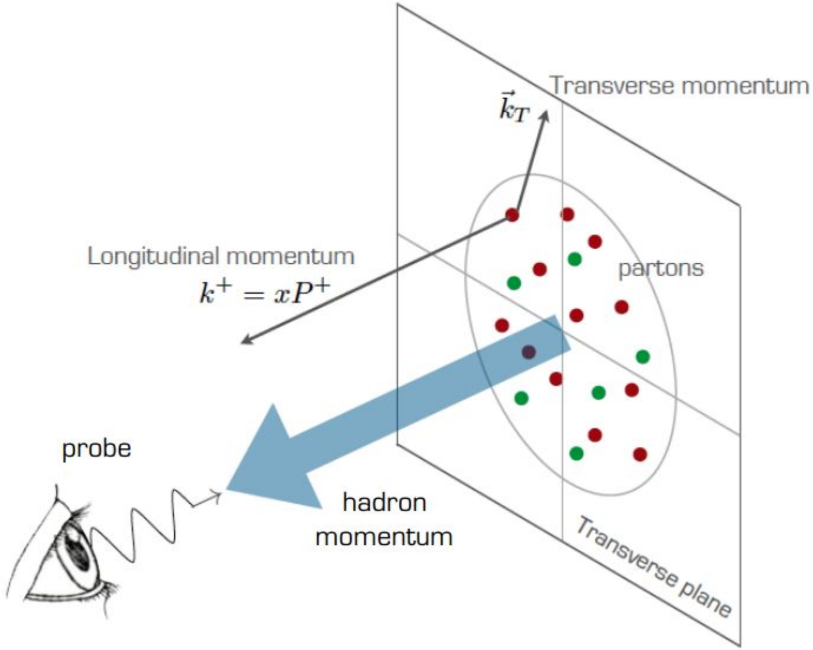
$$f(x, \mathbf{z}) \xleftrightarrow{\text{FT}} \boxed{f(x, \mathbf{k})} \qquad f(x, \mathbf{b}) \xleftarrow{\xi = 0} H(x, \xi, \mathbf{b}) \xleftrightarrow{\text{FT}} \boxed{H(x, \xi, \Delta^2)}$$

TMD PDF | **Impact parameter distribution** | **GPD**

For more details on GPDs, see C. Mezrag's talk and the associated contributions

Parton distribution functions

"Maps" of hadron structure in momentum space



$$f_1(x)$$

1D structure
in momentum space
("collinear")

$$f_1(x, k_T^2)$$

3D structure
in momentum space
("transverse momentum
dependent")

Credit picture: A. Bacchetta

Distributions with transverse hadron spin

$$f(x,\mathbf{b},\mathbf{s}) = H(x,\mathbf{b}^2) + \frac{\epsilon^{ij}\,\mathbf{b}_i\,\mathbf{s}_j}{M}\,\frac{\partial}{\partial\mathbf{b}^2}\,E(x,\mathbf{b}^2)$$

Unpolarized GPD (I.P.D.)

Helicity-flip GPD (I.P.D.)

?

$$f^{[U]}(x,\mathbf{k},\mathbf{s}) = f_1^{[U]}(x,\mathbf{k}^2) - \frac{\epsilon^{ij}\,\mathbf{k}_i\,\mathbf{s}_j}{M}\,f_{1T}^{\perp[U]}(x,\mathbf{k}^2)$$

Unpolarized TMD PDF

Sivers TMD PDF

Process dependence!

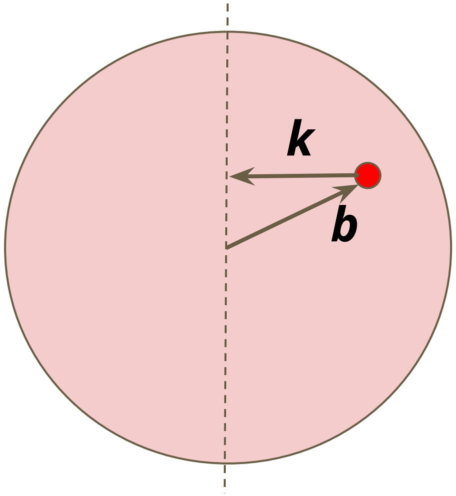
M. Diehl - https://inspirehep.net/literature/1408303

The lensing mechanism

$$f(x,\mathbf{b},\mathbf{s}) = H(x,\mathbf{b}^2) + \frac{\epsilon^{ij}\,\mathbf{b}_i\,\mathbf{s}_j}{M}\,\frac{\partial}{\partial\mathbf{b}^2}\,E(x,\mathbf{b}^2)$$

Symmetric distribution in transverse plane

Distorted distribution in transverse plane



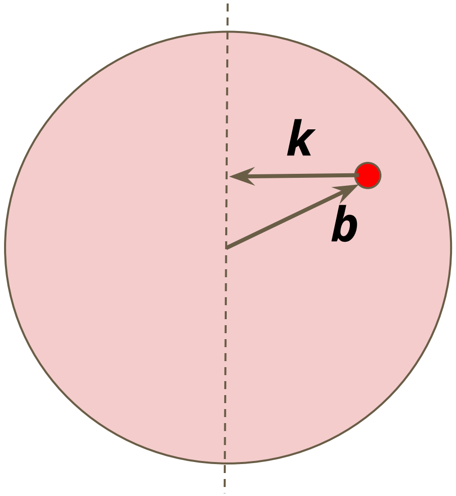
An attractive **color force** between the parton and the spectator turns the transverse position into a transverse momentum

M. Burkardt - https://inspirehep.net/literature/613471

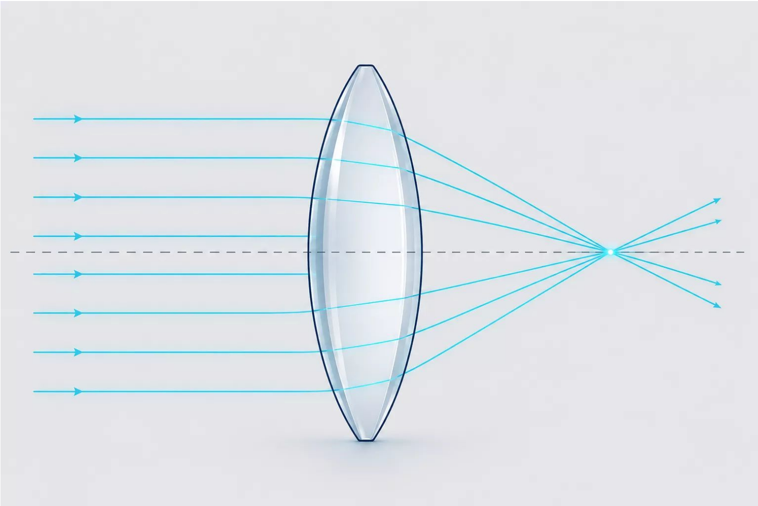
The lensing mechanism

$$f(x, \mathbf{b}, \mathbf{s}) = H(x, \mathbf{b}^2) + \frac{\epsilon^{ij}\, \mathbf{b}_i\, \mathbf{s}_j}{M} \frac{\partial}{\partial \mathbf{b}^2} E(x, \mathbf{b}^2)$$

Symmetric distribution
in transverse plane



Distorted distribution
in transverse plane



M. Burkardt - https://inspirehep.net/literature/613471

The lensing mechanism

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Symmetric distribution
in transverse plane

Distorted distribution
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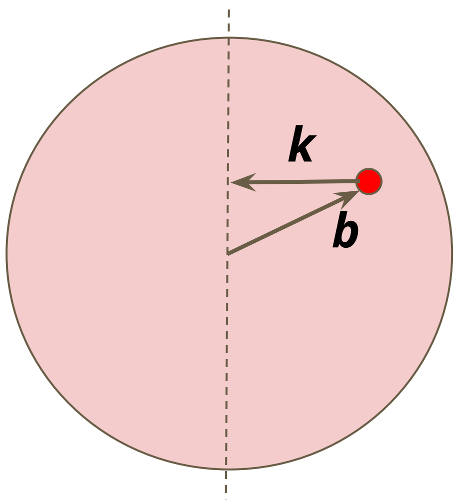


Lensing function $\mathbf{I}_\perp(\mathbf{b})$

$$\mathcal{P}(x, \mathbf{k}) = \int d^2\mathbf{b}\ q(x, \mathbf{b}) \boxed{\delta^{(2)}(\mathbf{k} - \mathbf{I}_\perp(\mathbf{b}))}$$

Lensing: a function of the **confining potential**

M. Burkardt - https://inspirehep.net/literature/613471

GPDs and angular momentum

Ji's sum rule

$$J^q(\mu) = \frac{1}{2}\int_{-1}^{1} dx\, x \left[H^q(x,\xi,\Delta^2;\mu) + E^q(x,\xi,\Delta^2;\mu)\right]_{\Delta^2=0}$$

Total angular momentum of (anti)quarks in proton polarized along the z direction

E flips the helicity of the target, while preserving the helicity of the quark: it is connected to **orbital angular momentum**

$$f_{1T}^{\perp(1)}(x) \sim \int d^2\mathbf{b}\; \mathbf{I}_\perp(x,\mathbf{b}_\perp)\, \frac{\partial}{\partial \mathbf{b}^2} E(x,\mathbf{b}^2)$$

Lensing of **E** GPD - **Sivers** TMD PDF → **OAM**

CAVEAT: this is a strictly **model-dependent** relation:
S. Rodini et al. - https://inspirehep.net/literature/1744103

2. TMD formalism

TMD and collinear PDFs for quarks in nucleon

quark pol. / nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

$$\Phi_{ij}(k,P) \;=\; \mathrm{F.T.}\,\langle P|\,\overline{\psi}_j(0)\,U\,\psi_i(\xi)\,|P\rangle$$

At leading twist: 8 TMD PDFs

(similar classification for gluons and for fragmentation functions)

- **Black**: time-reversal even AND collinear
- **Blue**: time-reversal even
- **Red**: time-reversal odd (*process dependence*)

The **symmetries of QCD** play a crucial role in this classification

Why should we study these distributions ?

f_1

- Test **factorization** and **universality**
- *Precise* knowledge: impact on **HEP**, e.g. **mW** determination

h_1

- Tensor charge of the nucleon: **CP** violation and access to **BSM** physics

$f_{1T}^{\perp}$, $h_1^{\perp}$

- **(TMD only)** Test the **symmetries** of QCD

---

e

- Quark-gluon correlations and quark contribution to **hadron mass**

E

- Quark-gluon correlations and **dynamical** generation of **quark mass**

TMD factorization

$$\boxed{q_T \ll Q}$$

$$pp \longrightarrow \gamma^{\cdot} \,/\, Z \longrightarrow l\,\bar{l} + X$$

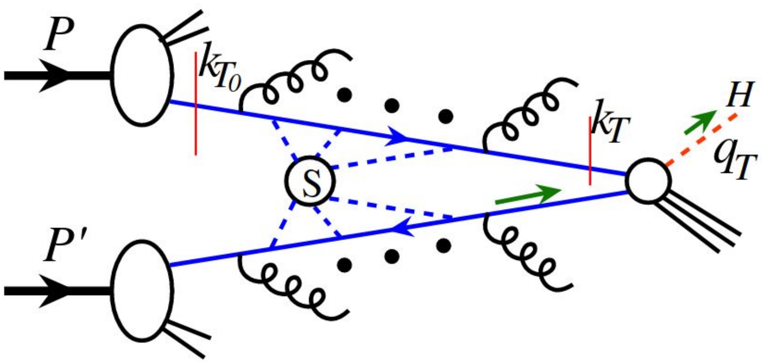
$$\frac{d\sigma}{dq_T} \sim \mathcal{H}\, f_1(x_a, k_{T\,a}, Q, Q^2)\, f_1(x_b, k_{T\,b}, Q, Q^2)\, \delta^{(2)}(q_T - k_{T\,a} - k_{T\,b}) \; + \mathcal{O}(q_T/Q) \, + \, \mathcal{O}(\Lambda/Q)$$

- TMDs & partonic cross section:
  same **IR poles** = same non-perturbative physics

- **observed transverse momentum qT** :
  transverse momenta of **quarks**

- quark transverse momentum :
  radiative (perturbative) and **intrinsic**
  (non-perturbative) components

- Renormalization = **evolution** equations tell us
  how to distinguish between the two



Quarks

Drell-Yan / Z / W production (hh)

Semi-Inclusive DIS (eh)

2h-inclusive
e+e- annihilation

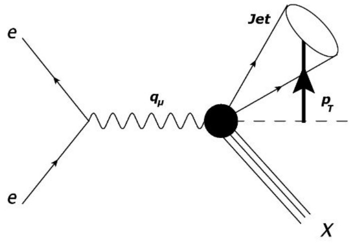
Gluons

Higgs production
in hadronic collisions

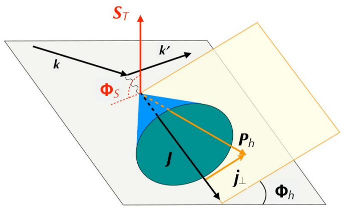
Quarkonium production (e.g. $\eta_{b,c}$,
double J/Psi in hadronic collisions)

TMD factorization

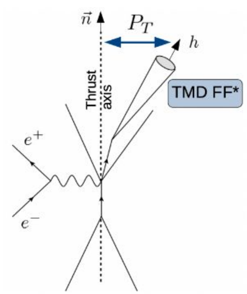
Jets:
(e.g. jet SIDIS,
di-jet SIDIS)



Hadron **"in jet"**:
(eh, hh, e+e-)



Y **"+ jet"**:
(e.g. Y = γ, h)



Quarks

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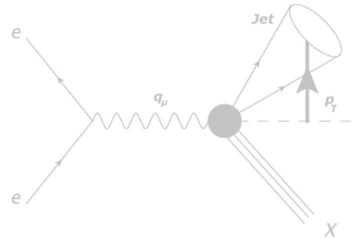
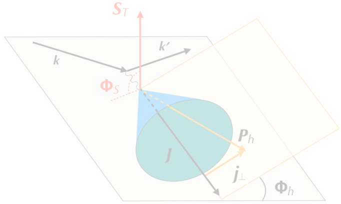
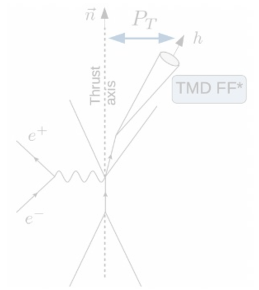


Global fits (unpolarized)

Hadron **"in jet"**:
(eh, hh, e+e-)



Y **"+ jet"**:
(e.g. Y = γ, h)



Not enough data (or not at all) for the other processes

Approximations

$$pp \longrightarrow \gamma^{\cdot} / Z \longrightarrow l\,\bar{l} + X$$

$$\frac{d\sigma}{dq_T} \sim \mathcal{H}\, f_1(x_a, k_{T\,a}, Q, Q^2)\, f_1(x_b, k_{T\,b}, Q, Q^2)\, \delta^{(2)}\big(q_T - k_{T\,a} - k_{T\,b}\big)$$

[TMD region, $q_T \ll Q$]

$$+\, \mathcal{O}(q_T/Q) + \mathcal{O}(\Lambda/Q)$$ [large q_T and low Q corrections]

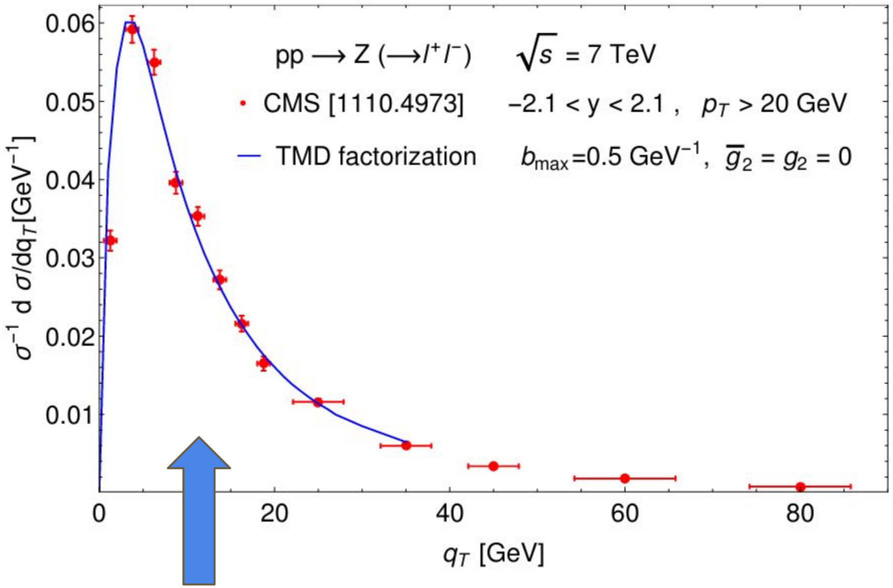
Large transverse momentum corrections: **"Matching" to collinear factorization at large qT**

Higher twist corrections: Multiparton correlations (?)

TMD region: low transverse momentum

$q_T \ll Q$

https://inspirehep.net/literature/1785810

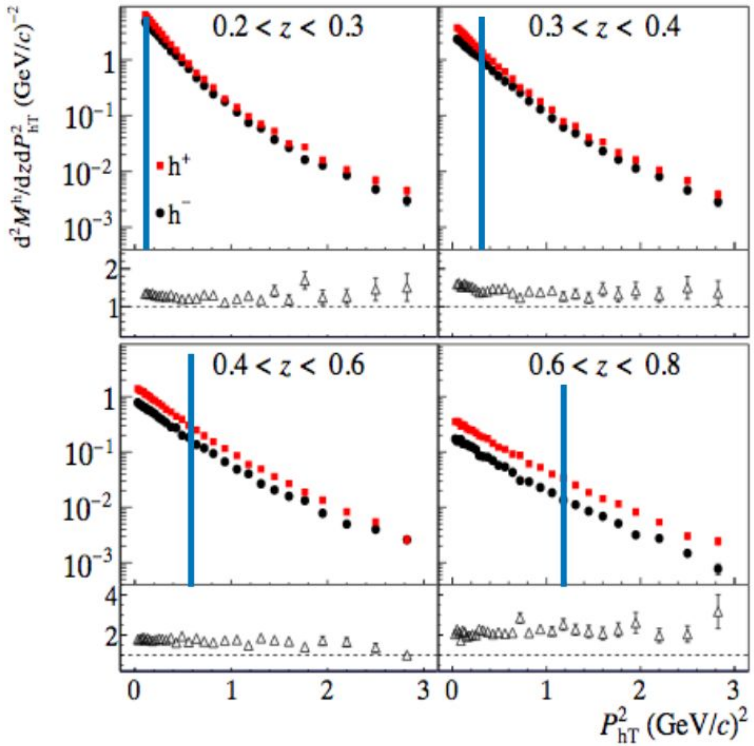


$$q_T \,/\, Q \; < \; 0.3$$

Hadronic collisions

TMD region: low transverse momentum

$\boxed{q_T \ll Q}$



COMPASS unpolarized SIDIS multiplicities - arxiv 1709.07374

SIDIS - TMD region

$$\boxed{P_{hT}^2/z^2 \ll Q^2}$$

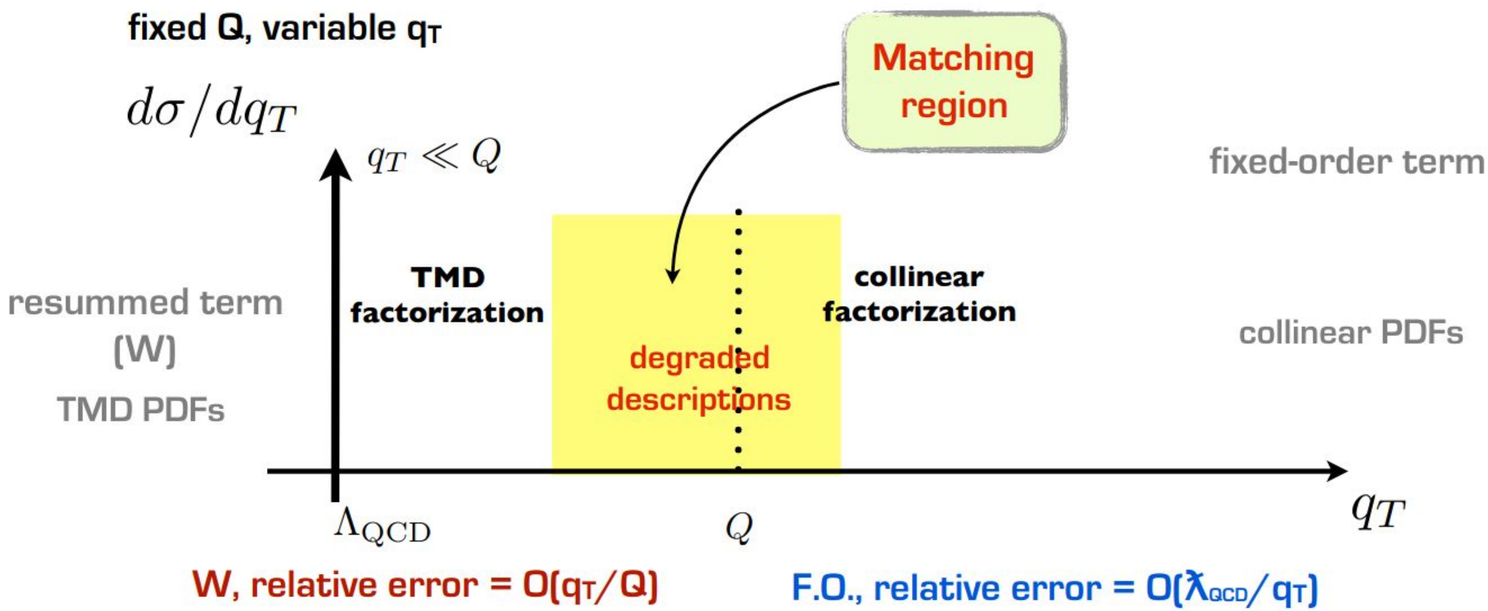
Let's highlight

$$P_{hT}^2/z^2 \sim 0.25\ Q^2$$

One of the bins with highest Q:

$$\langle Q^2 \rangle = 9.78\ \mathrm{GeV}^2$$
$$\langle x \rangle = 0.149$$

Matching TMD and collinear factorization



Evolution equations

$$F_a(x, b_T^2; \mu, \zeta) = \int d^2\boldsymbol{k}_T \, e^{i\boldsymbol{k}_T \cdot \boldsymbol{b}_T} F_a(x, k_T^2; \mu, \zeta)$$

← Fourier transform of a TMD PDF (bT conjugated to kT)

$$\frac{d \ln F_a(x, b_T^2; \mu, \zeta)}{d \ln \zeta} = -D\left(b_T \mu, \alpha_s(\mu)\right) \,,$$

rapidity

$$\frac{d \ln F_a(x, b_T^2; \mu, \zeta)}{d \ln \mu} = \gamma_F\left(\alpha_s(\mu), \frac{\zeta}{\mu^2}\right) \,,$$

UV

$$\frac{d\, D\left(b_T \mu, \alpha_s(\mu)\right)}{d \ln \mu^2} = \frac{1}{2}\gamma_K\left(\alpha_s(\mu)\right) \,,$$

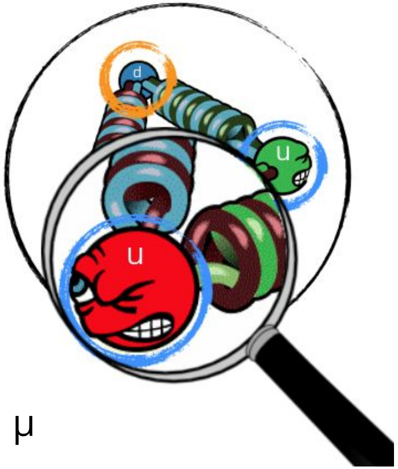
UV

$$\frac{d\, \gamma_F\left(\alpha_s(\mu), \frac{\zeta}{\mu^2}\right)}{d \ln \zeta} = -\gamma_K\left(\alpha_s(\mu)\right) \,.$$

rapidity

See J.C. Collins' book and many other references, e.g. https://inspirehep.net/literature/1393670

QCD evolution of a TMD PDF



$$F_a(x, b_T^2; \mu, \zeta) = F_a(x, b_T^2; \mu_0, \zeta_0)$$ → TMD distribution at initial scales

x $$\exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \boxed{\gamma_F\left(\alpha_s(\mu'), \frac{\zeta}{\mu'^2}\right)}\right]$$ → evolution in μ

Calculable in pQCD

x $$\left(\frac{\zeta}{\zeta_0}\right)^{-\boxed{D(b_T\mu_0, \alpha_s(\mu_0))}\boxed{+g_K(b_T;\lambda)}}$$ → evolution in ζ

Non-pert. corrections (large bT)

$$F_a(x, b_T^2; \mu_0, \zeta_0) = \sum_b \boxed{C_{a/b}(x, b_T^2, \mu_0, \zeta_0)} \otimes \underline{f_b(x, \mu_0)} \boxed{F_{NP}(b_T; \lambda)}$$

Prior knowledge assumed (?)

small bT (OPE) **large bT**

See J.C. Collins' book and many other references, e.g. https://inspirehep.net/literature/1393670

Non-perturbative TMD parts

$$F_a(x, b_T^2; \mu, \zeta) = F_a(x, b_T^2; \mu_0, \zeta_0)$$

$$\exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_F\left(\alpha_s(\mu'), \frac{\zeta}{\mu'^2}\right)\right]$$

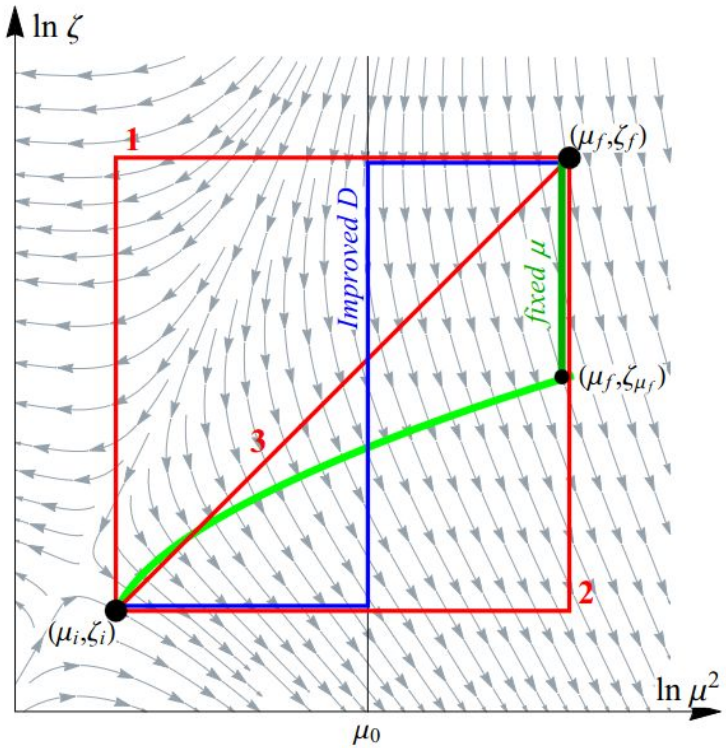
$$\left(\frac{\zeta}{\zeta_0}\right)^{-D(b_T \mu_0, \alpha_s(\mu_0)) \boxed{+ g_K(b_T; \lambda)}}$$

Non-pert. corrections (large bT)

$$F_a(x, b_T^2; \mu_0, \zeta_0) = \sum_b C_{a/b}(x, b_T^2, \mu_0, \zeta_0) \otimes f_b(x, \mu_0) \boxed{F_{NP}(b_T; \lambda)}$$

Intrinsic transverse momentum, potentially flavor dependent

A two-scale evolution



The combined evolution along the light green curve is absent.

Scimemi, Vladimirov - https://inspirehep.net/literature/1665218

Different frameworks, same observable

$$\left(\frac{d\sigma}{dq_T}\right)_{\mathrm{res.}} \begin{array}{l} \overset{q_T-\mathrm{res.}}{\underset{\mathrm{PB}}{\propto}} \quad e^{2S}\left[f_1 \otimes \mathcal{H} \otimes f_2\right] \\ \overset{\mathrm{TMD}}{\propto} \quad H \times F_1 \times F_2 \\ \overset{\mathrm{SCET}}{\propto} \quad H \times B_1 \times B_2 \times S \end{array} + \mathcal{O}\left[\left(\frac{q_T}{Q}\right)^m\right]$$

$$\mathcal{H} = HC_1C_2$$

$$F_i = e^S C_i \otimes f_i$$

$$F_i = \sqrt{S} \times B_i$$

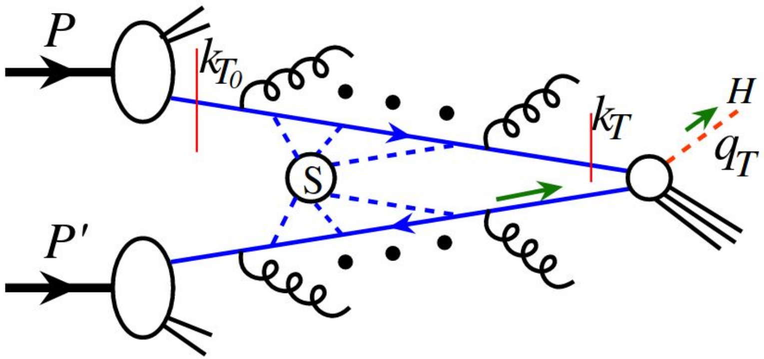
Dictionary to compare different factorization frameworks

“equivalent” to the extent of describing TMD physics

See G. Bozzi’s talk at SarWors 2021 workshop - https://agenda.infn.it/event/27742/

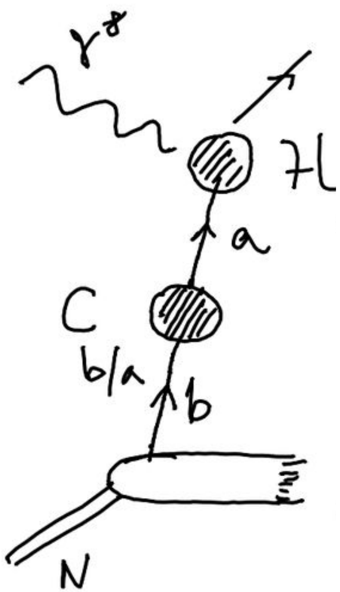
TMD evolution vs parton shower

- **Parton shower**: generates the radiation (**exclusively**) and then computes its recoil qT



- **TMD evolution**: sums (**inclusively**) over the radiation and predicts the recoil qT distribution.

$$F_a(x, b_T^2; \mu, \zeta)$$



3. Challenges

Challenges (TMD) as an outlook

- **Semi-inclusive DIS**: is the TMD factorization formalism under control?
  (see talks by M. Cerutti, S. Piloneta)

- **Parametrization bias**: are we using the best methodology to extract information from data?
  (see talk by M. Cerutti)

- **Factorization**: what can we say about its breaking? See also Gluon TMD-related observables
  (see talk by C. Flore)

- **Higher twists**: we are getting an improved picture (theory and pheno developments)
  (see talk by A. Vladimirov et al.)

- **Non-perturbative approaches**: what can we learn from LQCD and continuum methods?
  (see talk by M. Ding et al.)

- **Hadronization**: it deserves separate presentations
  (see talk by W. Brooks, F.G. Celiberto et al.)

- ...

Backup

TMD PDFs: traffic light notation

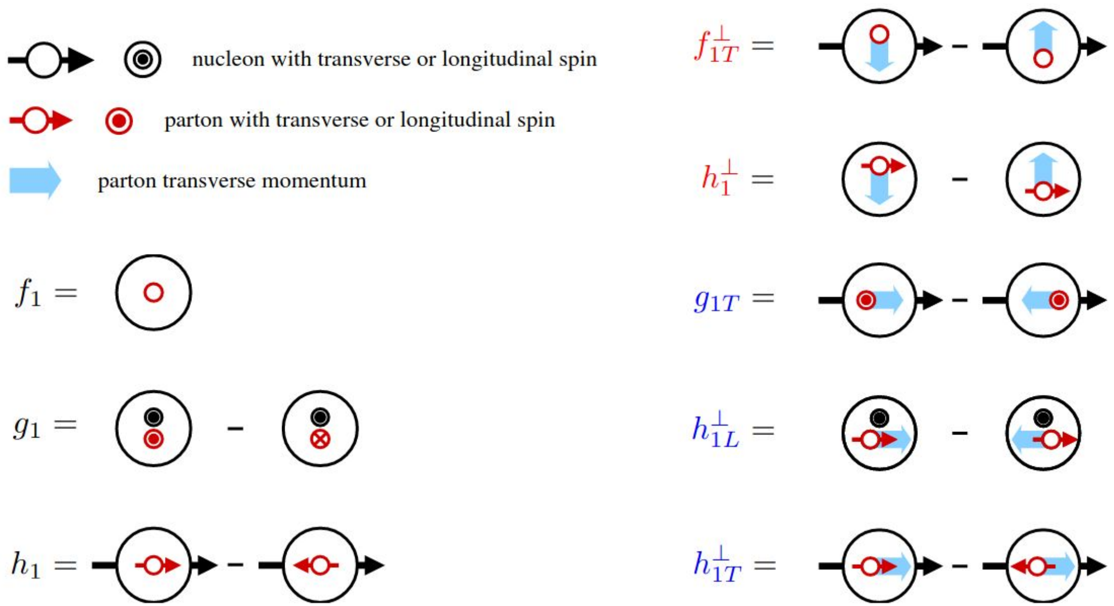
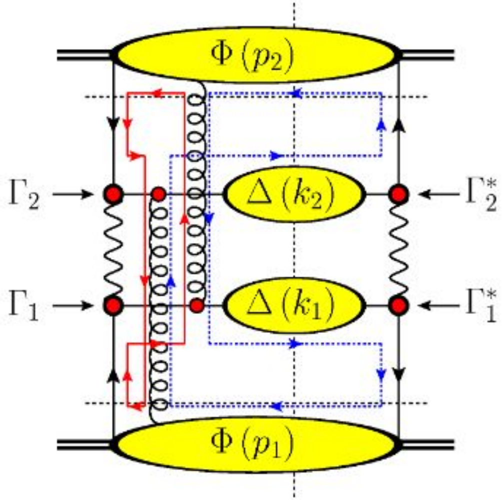


Figure 3.5: Probabilistic interpretation of twist-2 transverse-momentum-dependent distribution functions. To avoid ambiguities, it is necessary to indicate the directions of quark's transverse momemtum, target spin and quark spin, and specify that the proton is moving out of the page, or alternatively the photon is moving into the page.

See Bacchetta's "Trento" lecture notes

Non-TMD-factorizable processes



For **pp → h1 h2 X** TMD factorization is violated

See :

- Collins, Qiu (2007) : https://inspirehep.net/literature/750627
- Rogers, Mulders (2010) : https://inspirehep.net/literature/843028

- Buffing (2016) : https://inspirehep.net/literature/1391461 (see figure)

This **endangers** also other processes such as **pp → h X (and similar)**

Quantify factorization breaking effects ? See e.g. :

- Buffing, Kang, Lee, Liu (2018) : https://inspirehep.net/literature/1709823
- Aidala (2019) : https://inspirehep.net/literature/1772224
- LHCb collaboration (2021) : https://inspirehep.net/literature/1901628

Small ..?

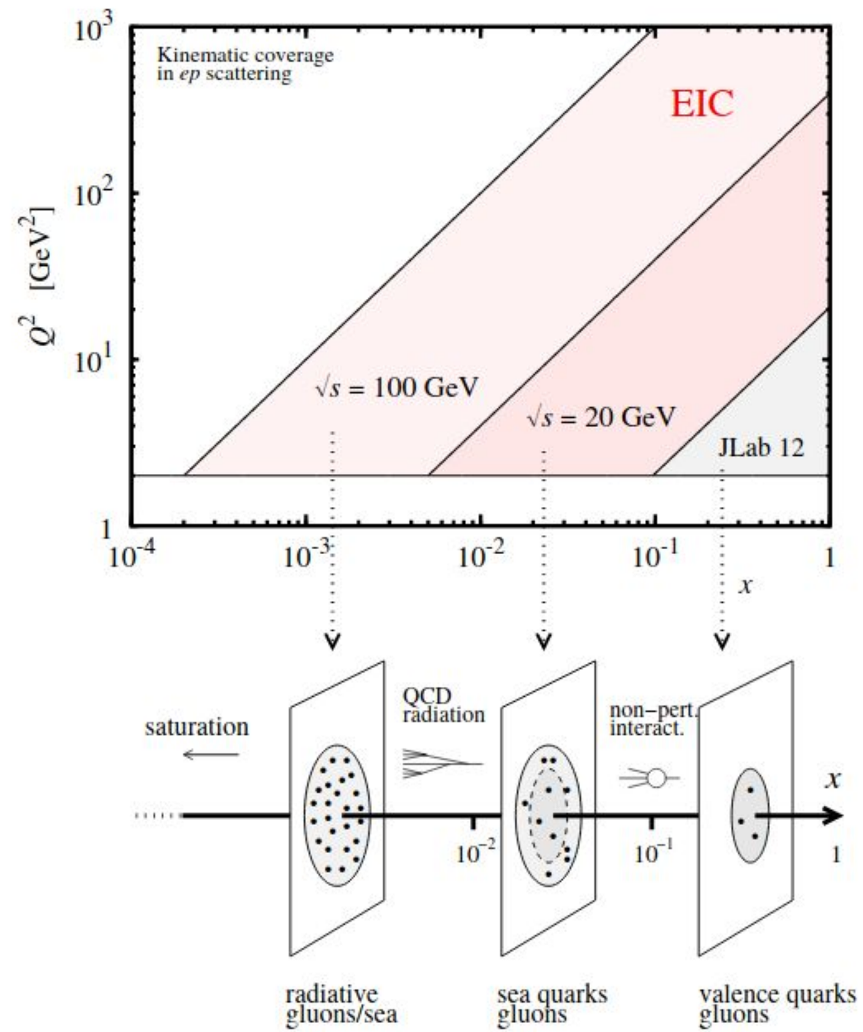
SIDIS coverage

Importance of complementary experiments

from JLab 12 GeV, Hermes, Compass
to the EIC

zooming into hadron structure

Credit picture: C. Weiss



Approximations

See https://inspirehep.net/literature/1732230

Semi-Inclusive Deep-Inelastic Scattering: Nachtmann variable(s)

$$x_{\mathrm{Bj}} = \frac{Q^2}{2P\cdot q}\,, \qquad x_{\mathrm{N}} = -\frac{q^+}{P^+} = \frac{2x_{\mathrm{Bj}}}{1+\sqrt{1+\boxed{\frac{4x_{\mathrm{Bj}}^2 M^2}{Q^2}}}}\,,$$

Nachtmann and Bjorken-x are
the same up to **target mass corrections**

$$\mathcal{O}(q_T^2/Q^2)$$

And there are also
qT-dependent corrections!

$$z_{\mathrm{N}} = \frac{P_{\mathrm{B}}^-}{q^-}$$

$$z_{\mathrm{N}} = \frac{Q^4 x_{\mathrm{N}} z_{\mathrm{h}}\left(1 \pm \sqrt{1-\frac{4M^2M_{\mathrm{B}}^2 x_{\mathrm{Bj}}^2 (Q^4+\boxed{x_{\mathrm{N}}^2M^2\mathbf{q}_{\mathrm{T}}^2)}}{Q^8 z_{\mathrm{h}}^2}}\right)}{2x_{\mathrm{Bj}}(Q^4+x_{\mathrm{N}}^2M^2\mathbf{q}_{\mathrm{T}}^2)} \;\text{Fixed}\overset{\mathrm{x_N,z_h,q_T}}{=}\; z_{\mathrm{h}}\left(1+O\left(\frac{m^4}{Q^4}\right)\right)$$

Approximations

See https://inspirehep.net/literature/1732230

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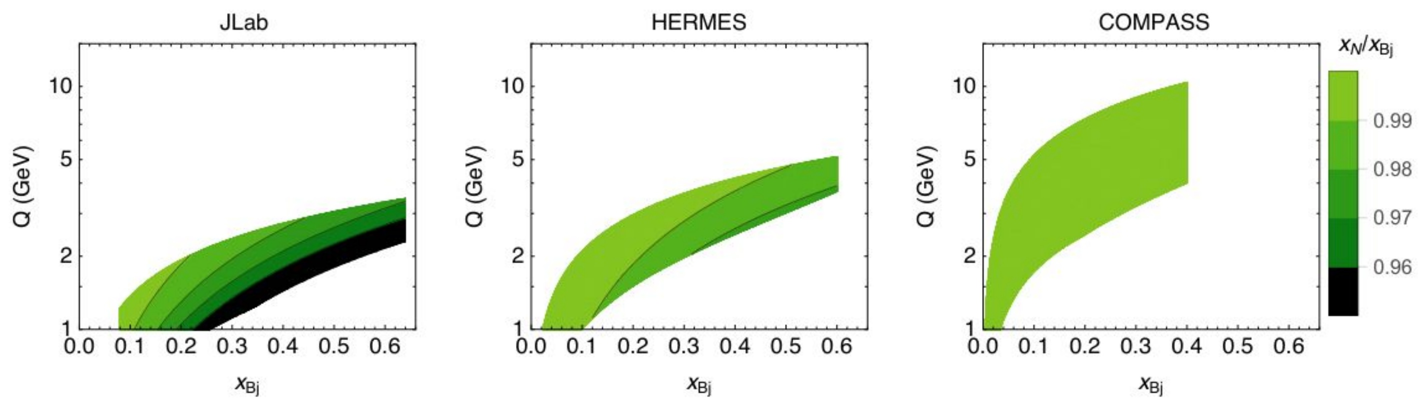
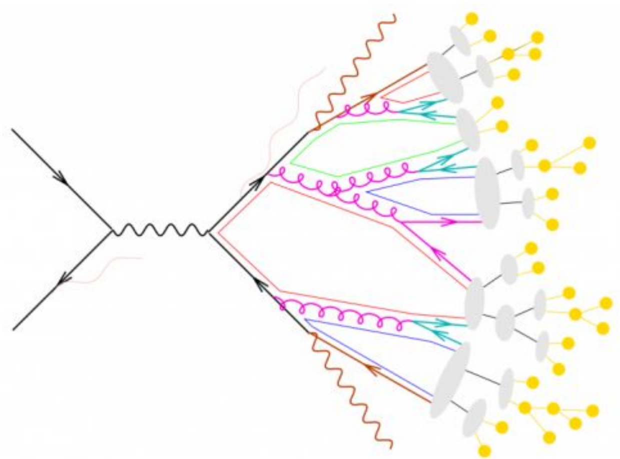


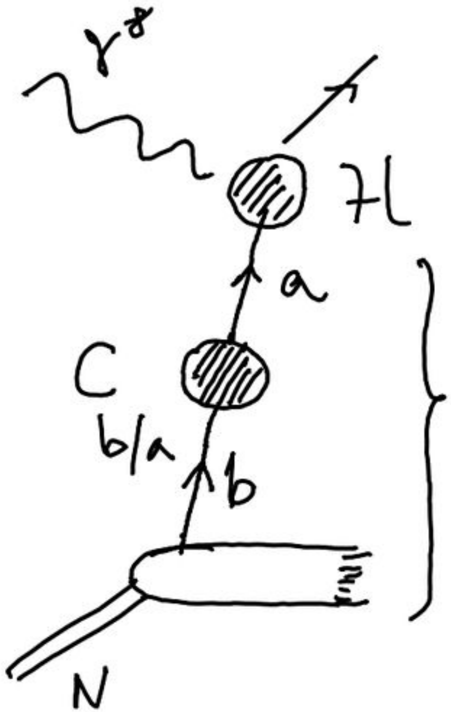
Hadronization and fragmentation functions (FFs)

"Maps" of hadron formation in momentum space



$D_1^h(z)$	single-hadron collinear FF
$D_1^h(z, P_T^2)$	single-hadron TMD FF
$D_1^{h_1\,h_2}(z, \zeta)$	di-hadron FF
$J(s)$	inclusive jet FF
$\mathcal{G}^h(s, z)$	in-jet FF

Intrinsic transverse momentum



Provides fixed-order (perturbative) transverse-momentum structure at the matching scale (**DGLAP** splitting and **logs** of bT)

$$f_1(x, b_T)^{a/N} = \sum_b \mathcal{C}_{b/a} \otimes f_1(x)^{b/N} + \mathcal{O}(b_T \Lambda)^{a/N}$$

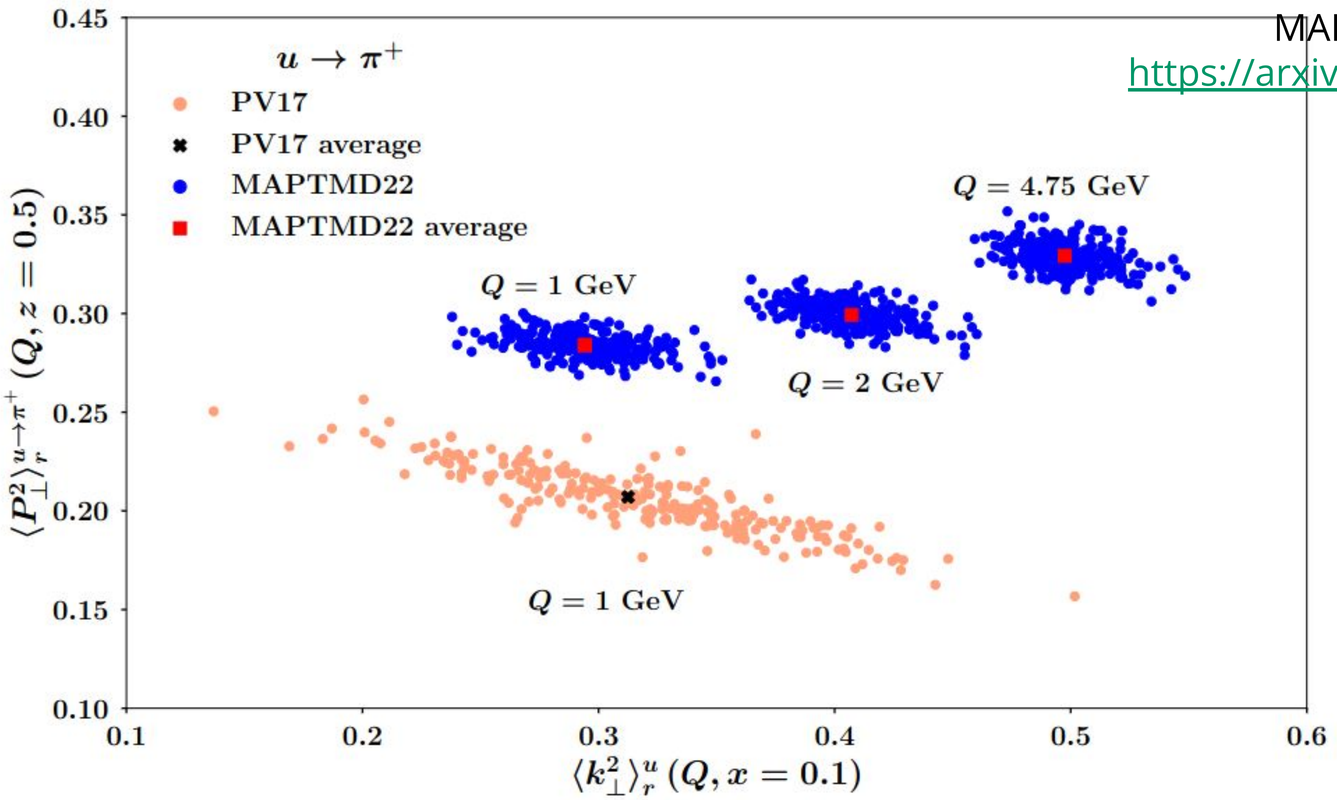
$\boxed{F_{NP}(b_T; \lambda)}$

For quark flavor *a*, not *b*

Intrinsic transverse momentum

Kinematic dependence of average intrinsic transverse momenta

MAP coll. 2022 - https://arxiv.org/pdf/2206.07598



Intrinsic transverse momentum

Kinematic dependence of average intrinsic transverse momenta

MAP coll. 2024 - https://arxiv.org/pdf/2405.13833