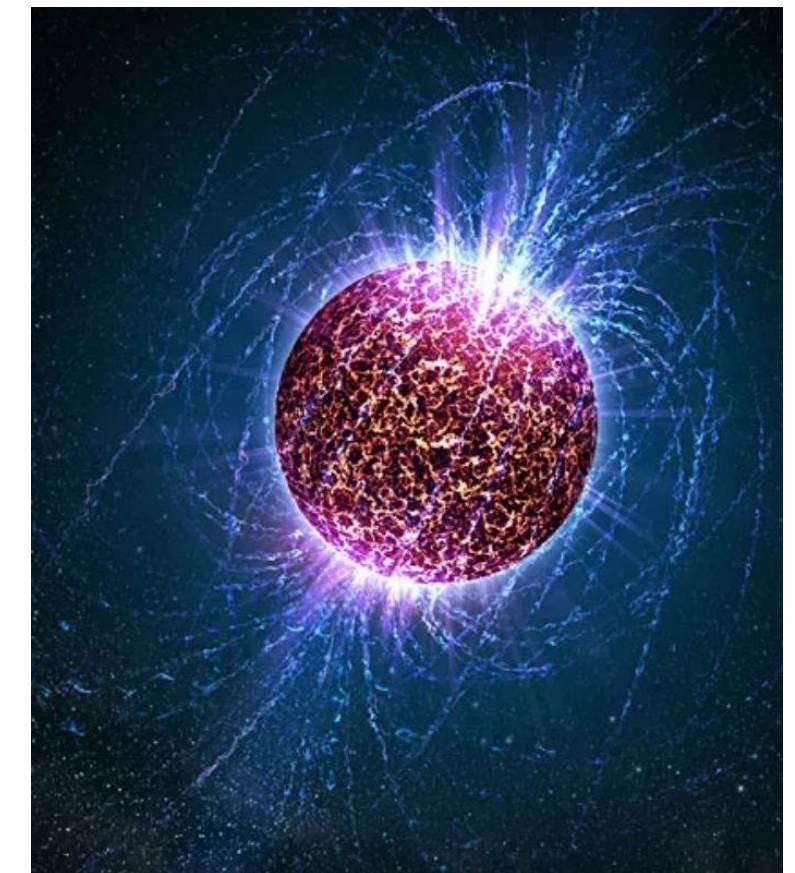
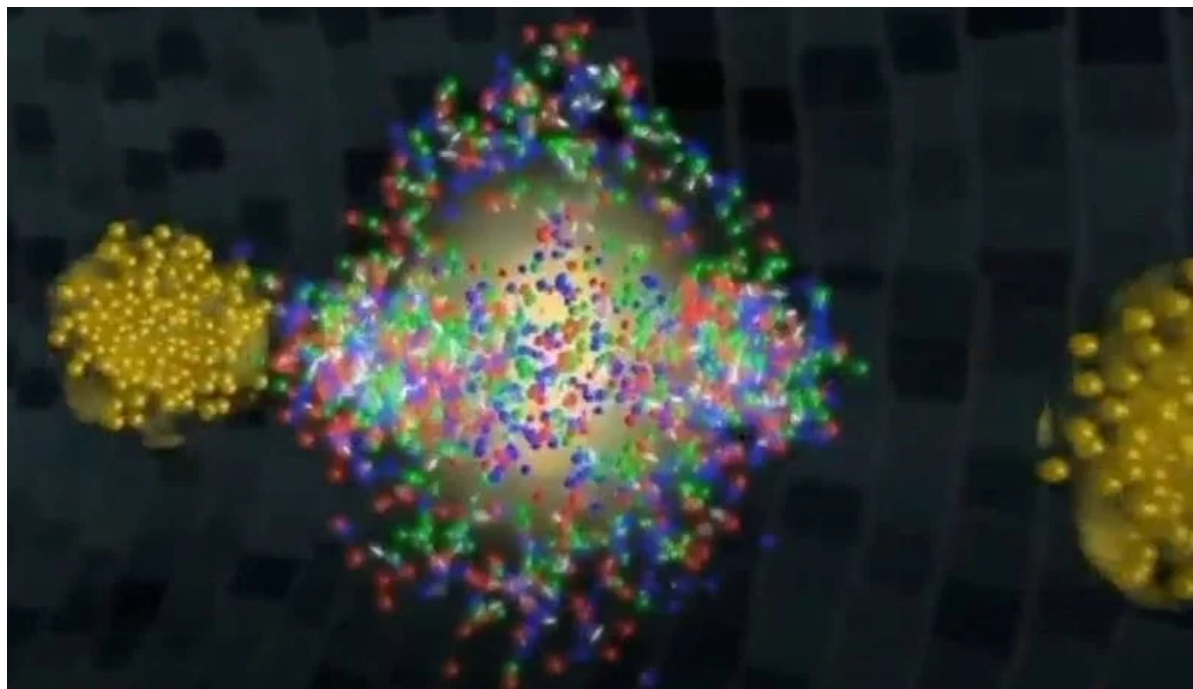




Criticality in Nature's Strongest Force

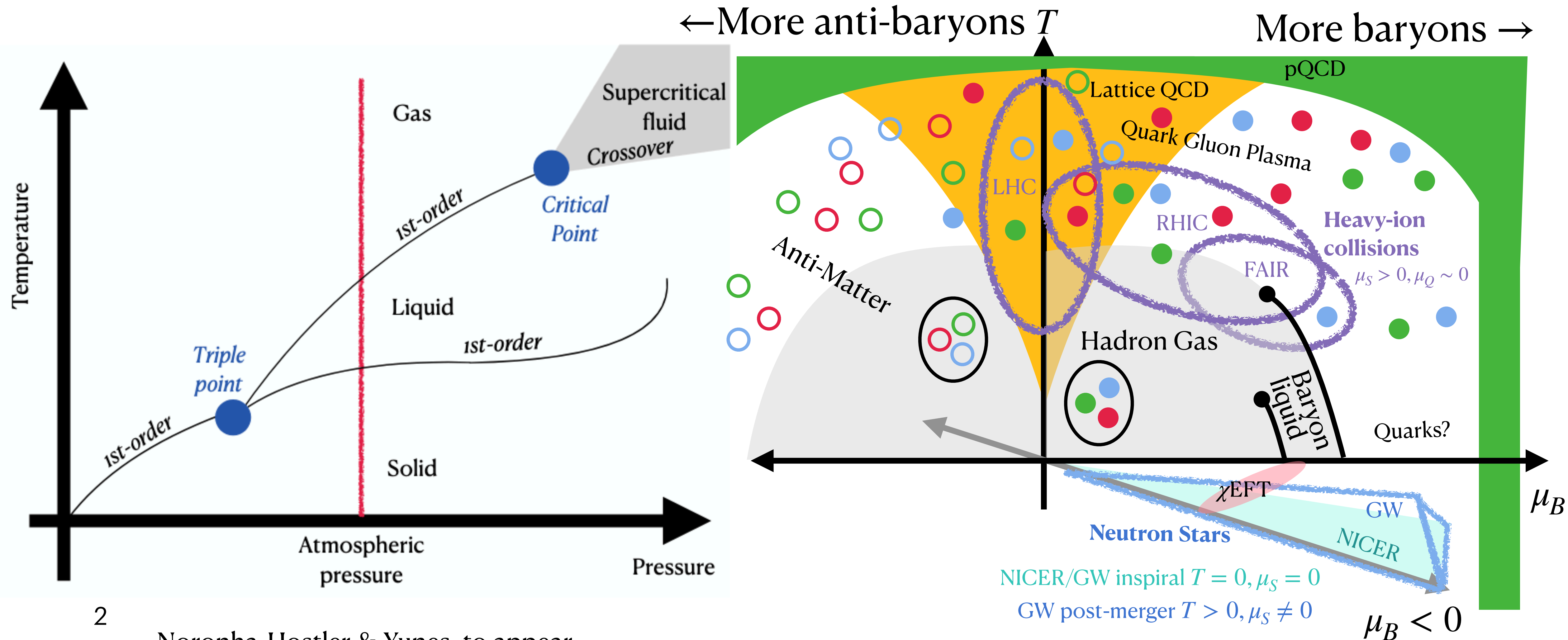
Jacquelyn Noronha-Hostler

**NSTAR 2026
Seville, Spain**



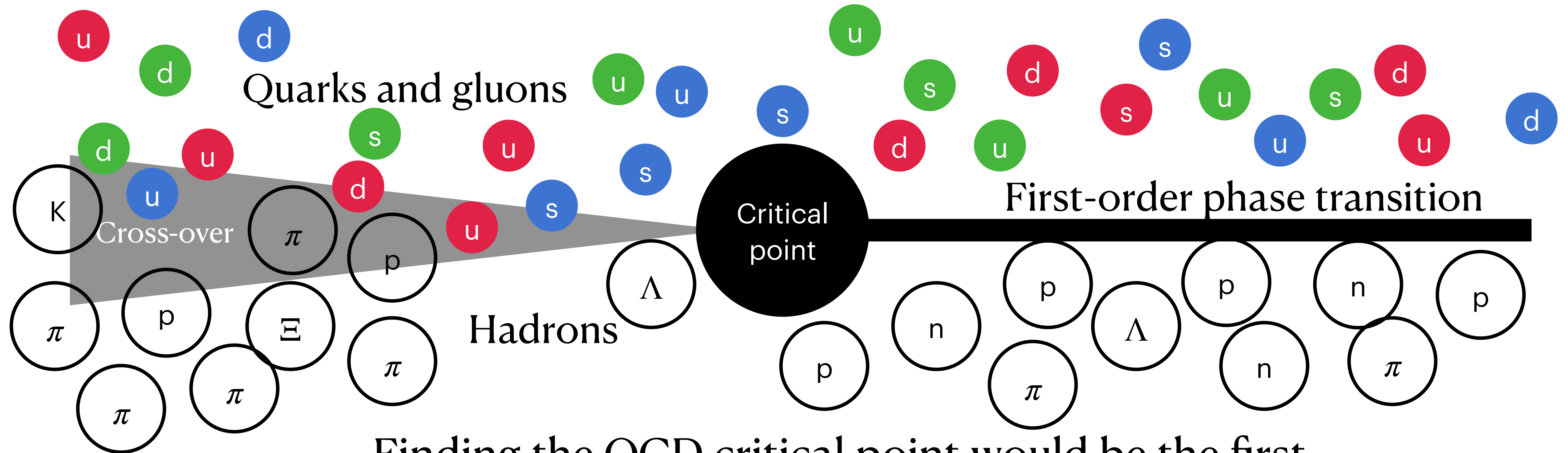
QCD phase diagram

Can we find a real phase transition into deconfinement?



Significance of the QCD critical point

Boundary between a first-order phase transition and cross-over



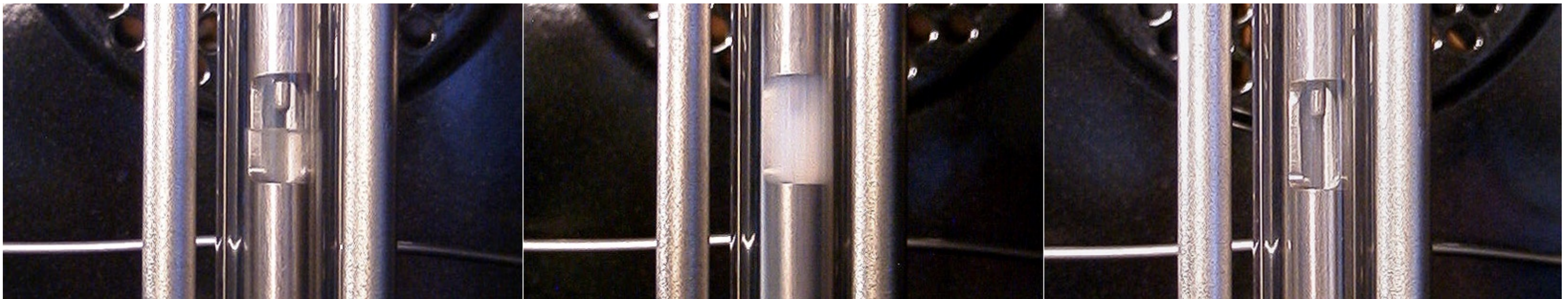
Finding the QCD critical point would be the first...

1. in a fundamental theory of nature,
2. in a truly ultra-relativistic many-body system,
3. accessed through relativistic hydrodynamic evolution



Critical opalescence

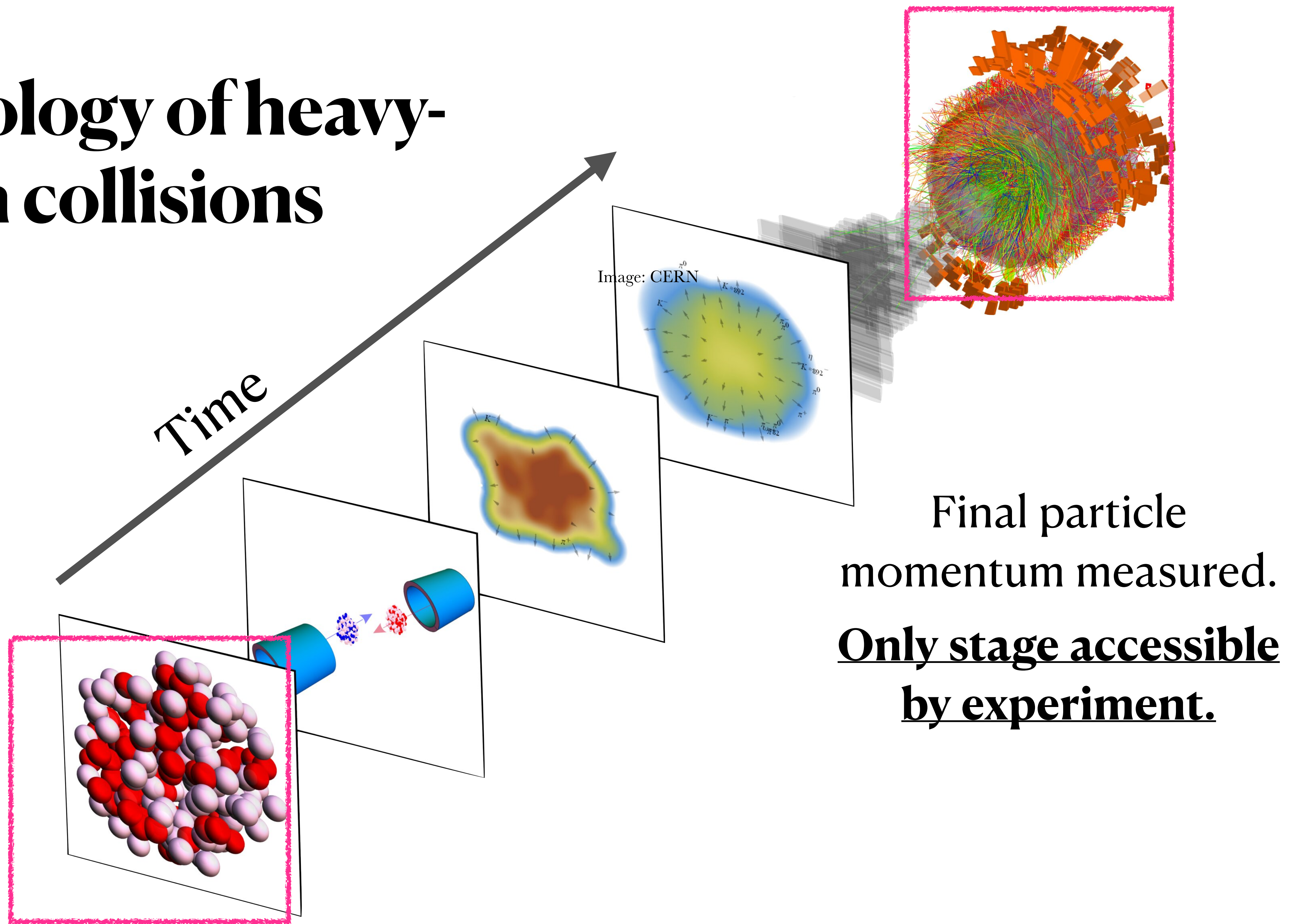
Could we have baryon opalescence?



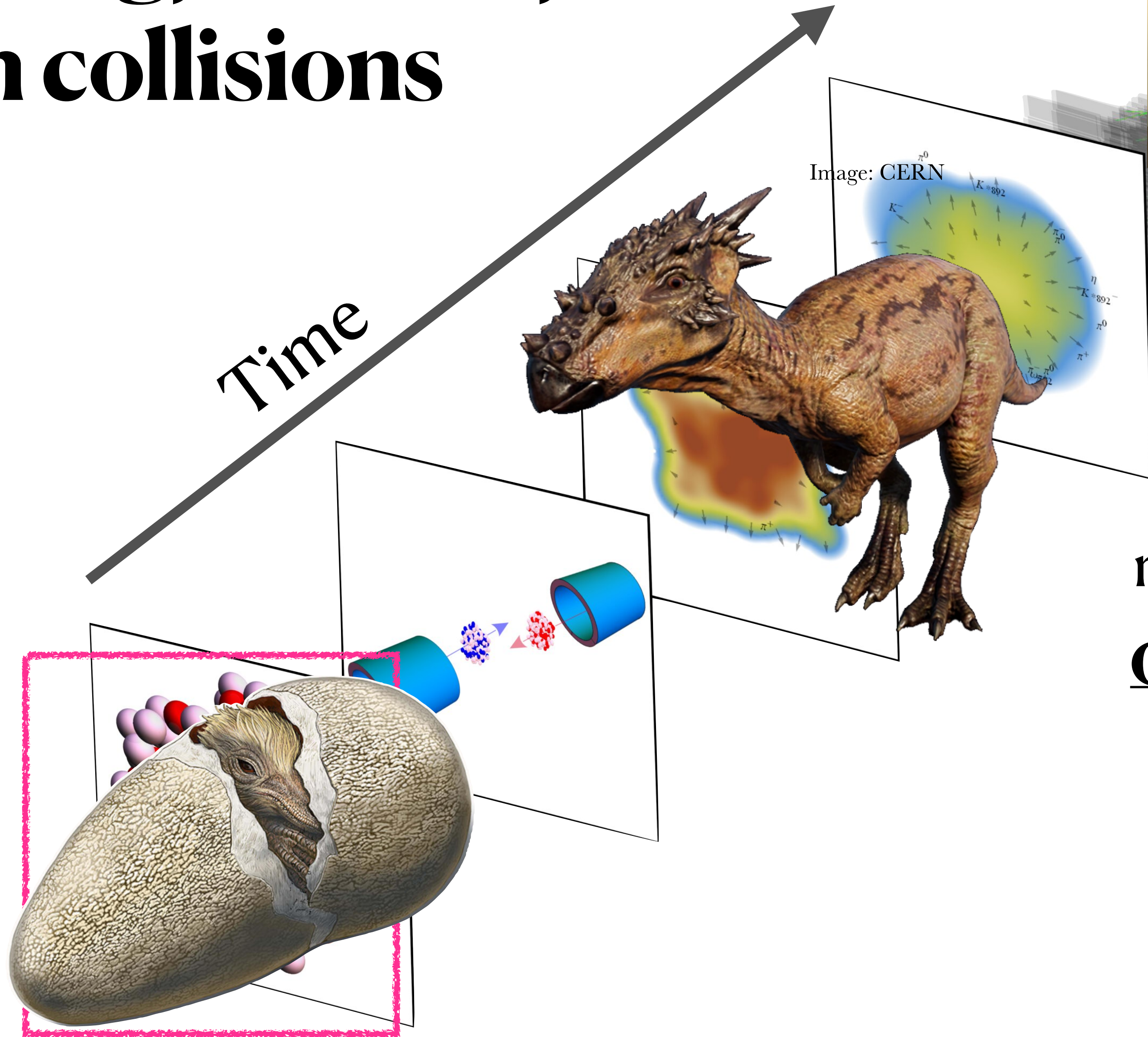
Due to large correlation length ξ , light gets trapped in the system at the critical point

How do we search for the QCD critical point?

Archeology of heavy-ion collisions



Archeology of heavy-ion collisions



Final particle
momentum measured.
**Only stage accessible
by experiment.**

Kinematics of heavy-ion collisions

Beam direction

Transverse direction

Lorentz
Contraction

$$\sqrt{s}_{LHC} = 5 \text{ TeV}$$

$$\frac{R_{beam}}{R_0} = \frac{2m_p}{\sqrt{s}} \sim 10^{-4}$$

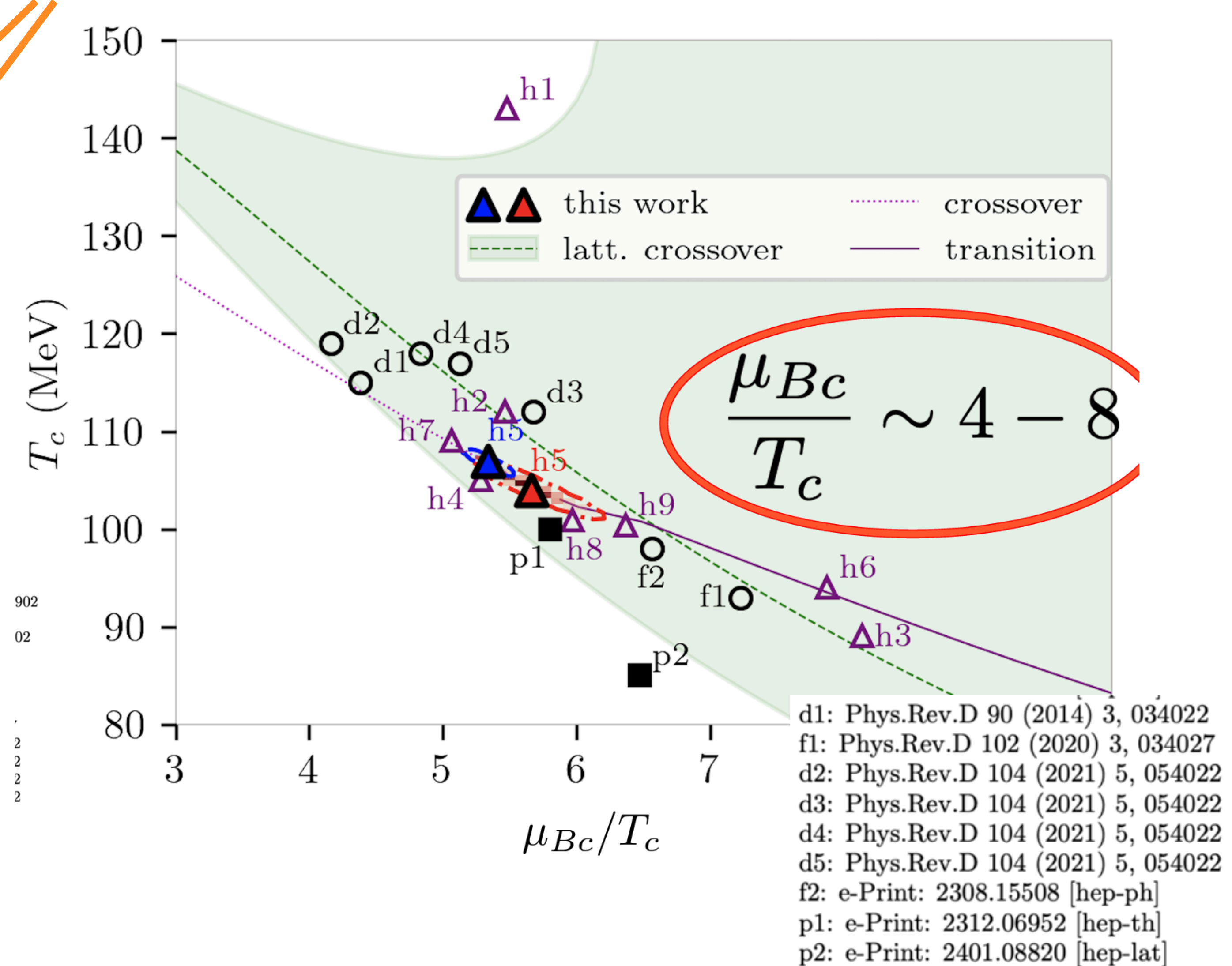
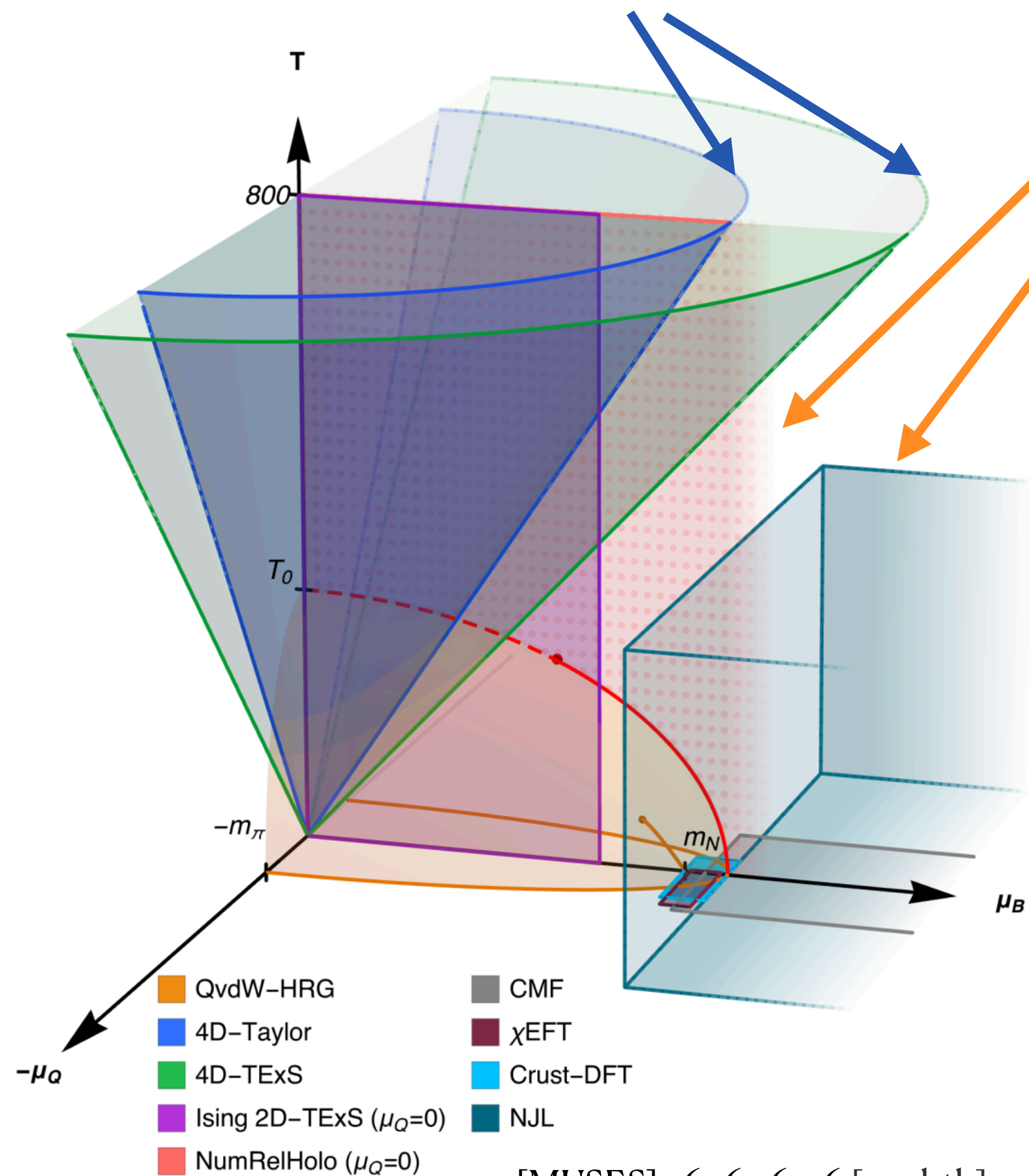
I Impact
Parameter

Quark Gluon
Plasma formed

$\mathcal{E}_n$
Eccentricity

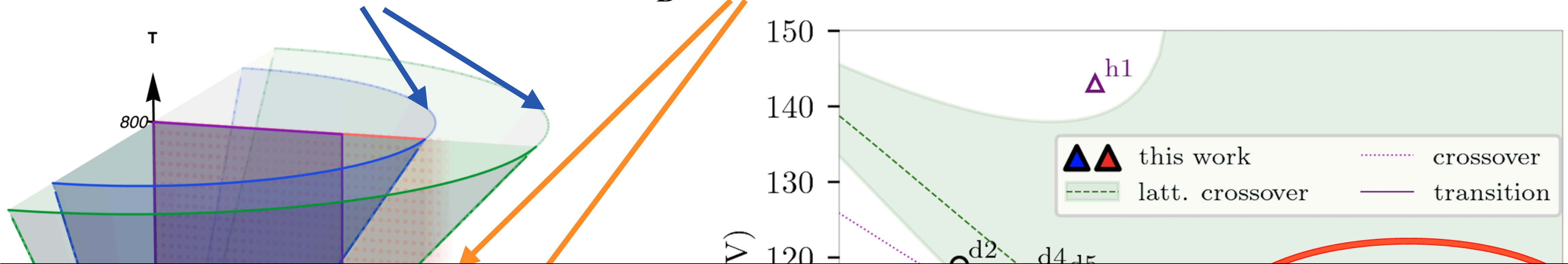
Challenge 1: Equation of state at finite μ_B

Expansions of lattice QCD at finite μ_B ; **effective models** and phenomenology

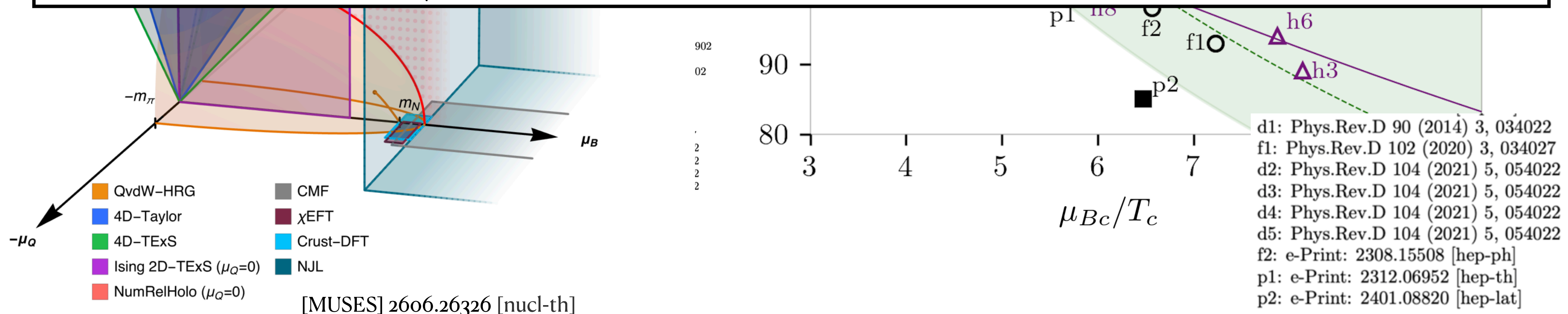


Challenge 1: Equation of state at finite μ_B

Expansions of lattice QCD at finite μ_B ; **effective models** and phenomenology



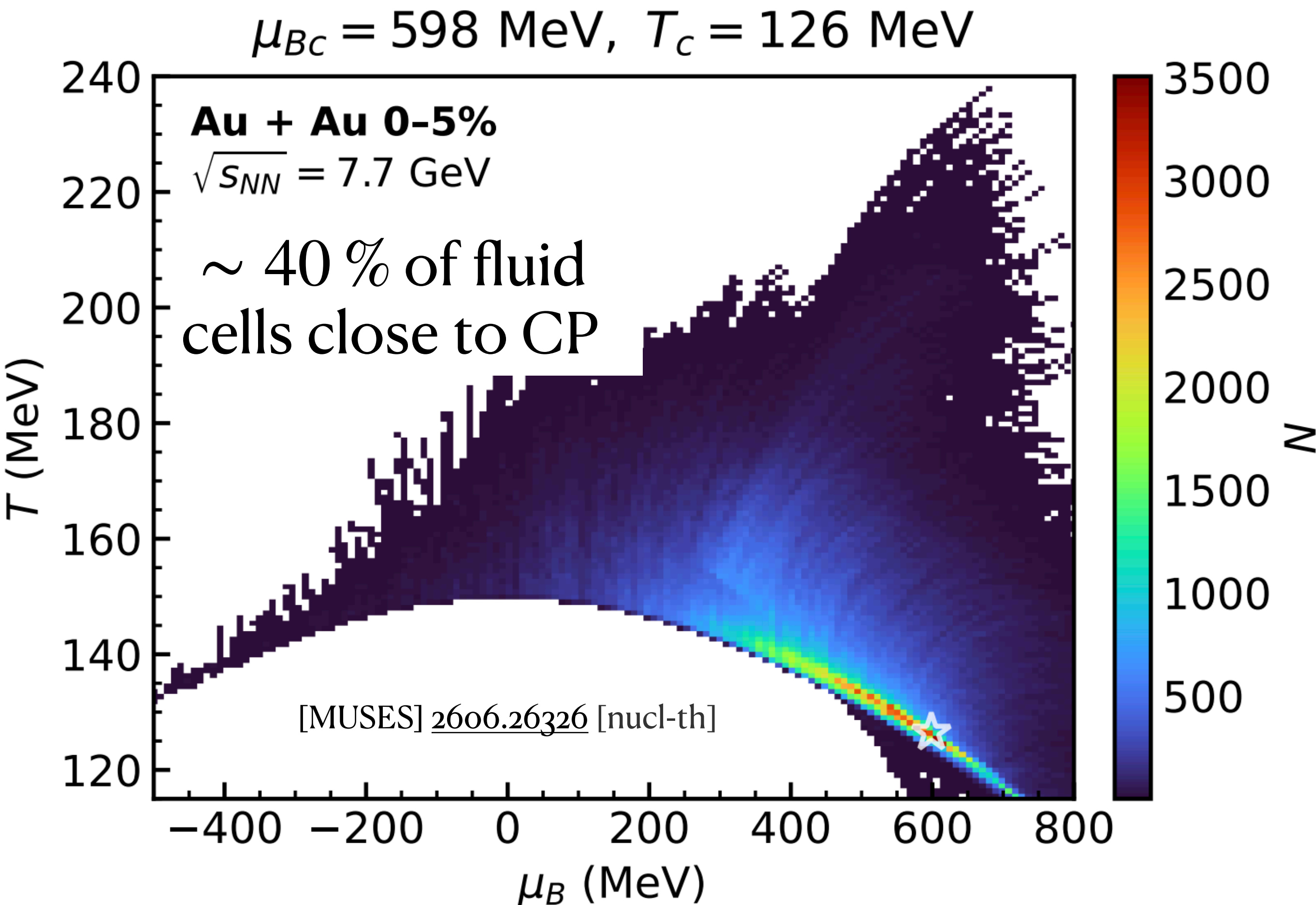
Models and phenomenological approaches hint at a critical point around $\mu_B^c \sim 600$ MeV, would appear around $\sqrt{s_{NN}} \sim 2 - 7.7$ GeV (fixed target)



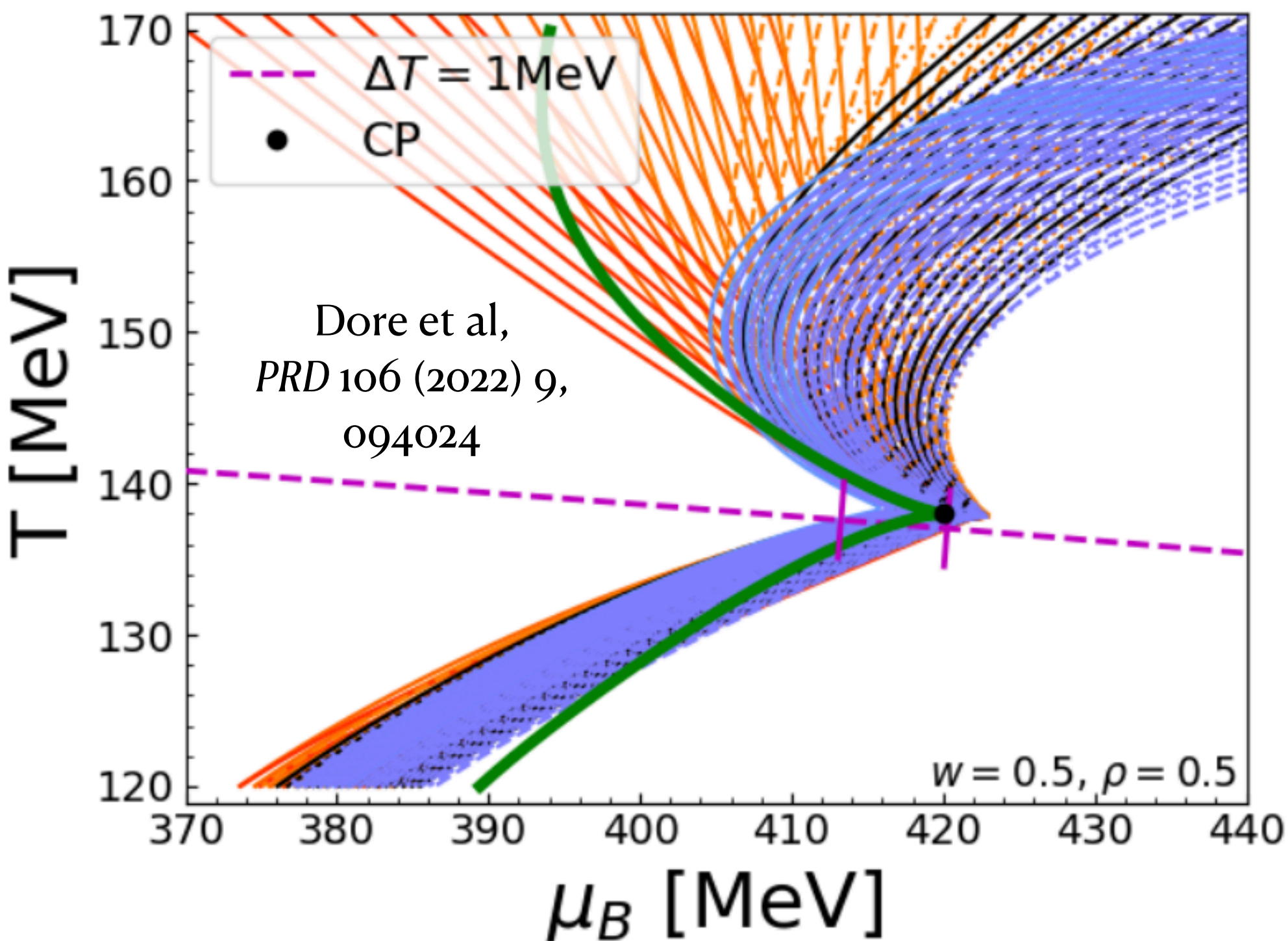
Challenge 2: heavy-ion collision are dynamical

Field in T, μ_B, μ_S, μ_Q

Amount of space-time spent close to the critical point



Some critical points exhibit
(dynamical) critical lensing: attracts
more fluid cells to CP

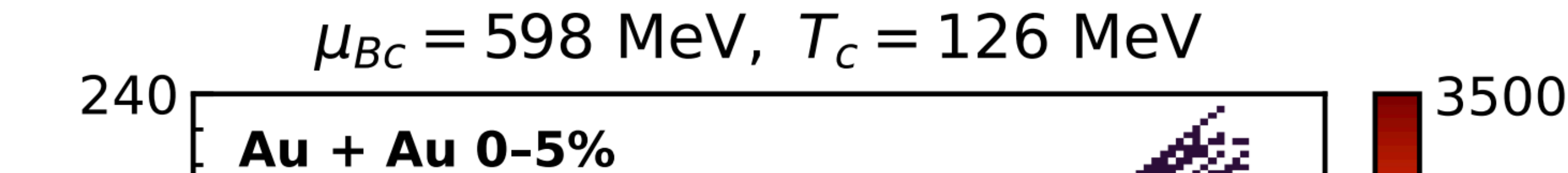


Challenge 2: heavy-ion collision are dynamical

Field in T, μ_B, μ_S, μ_Q

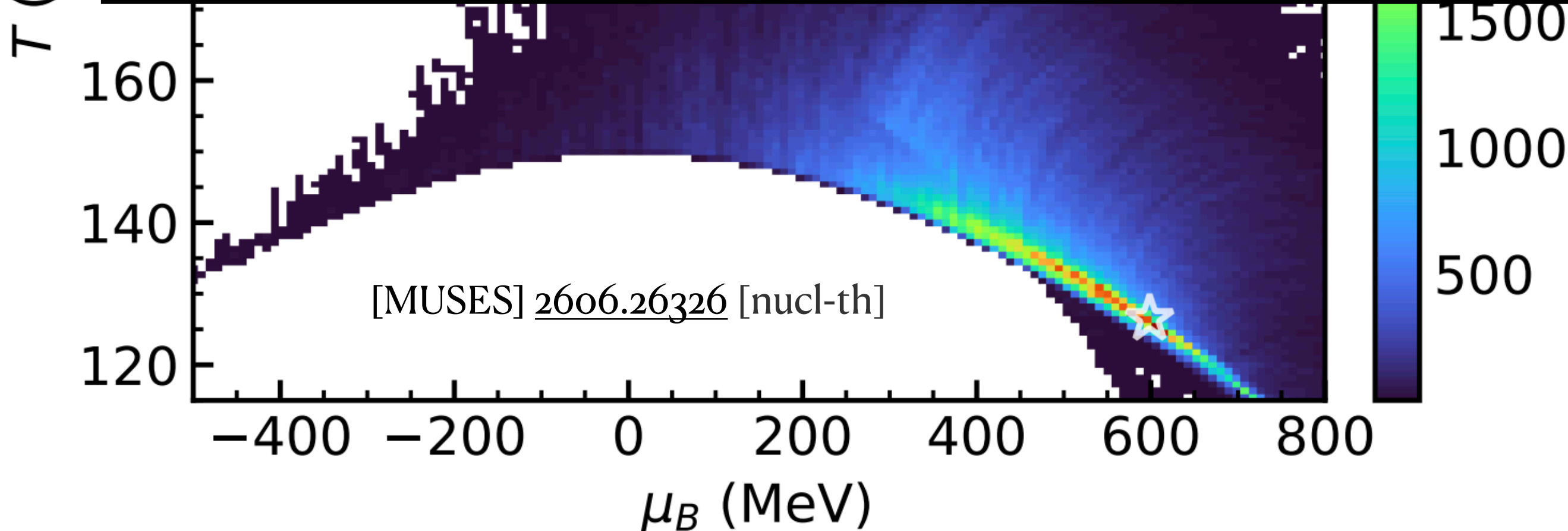
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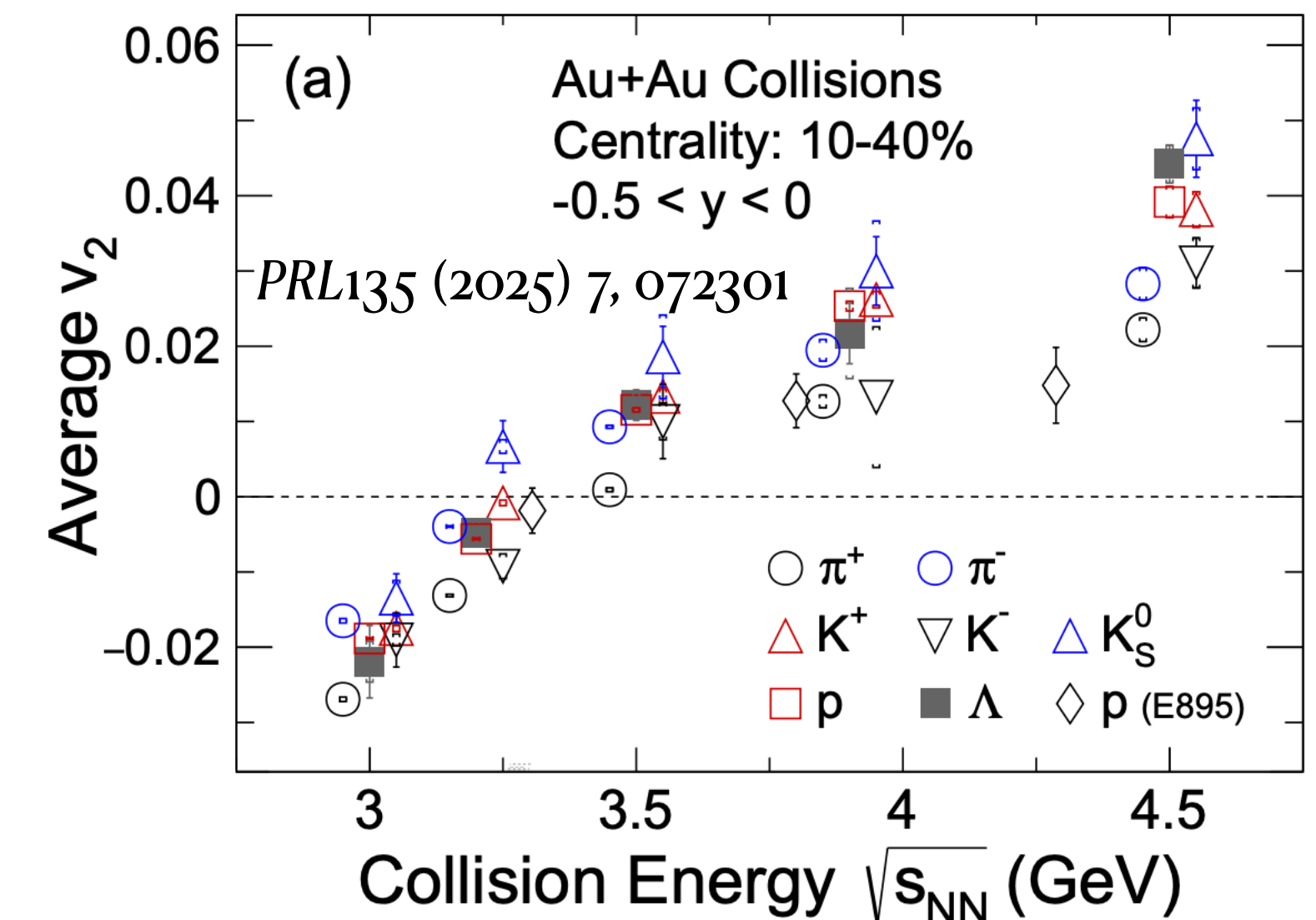
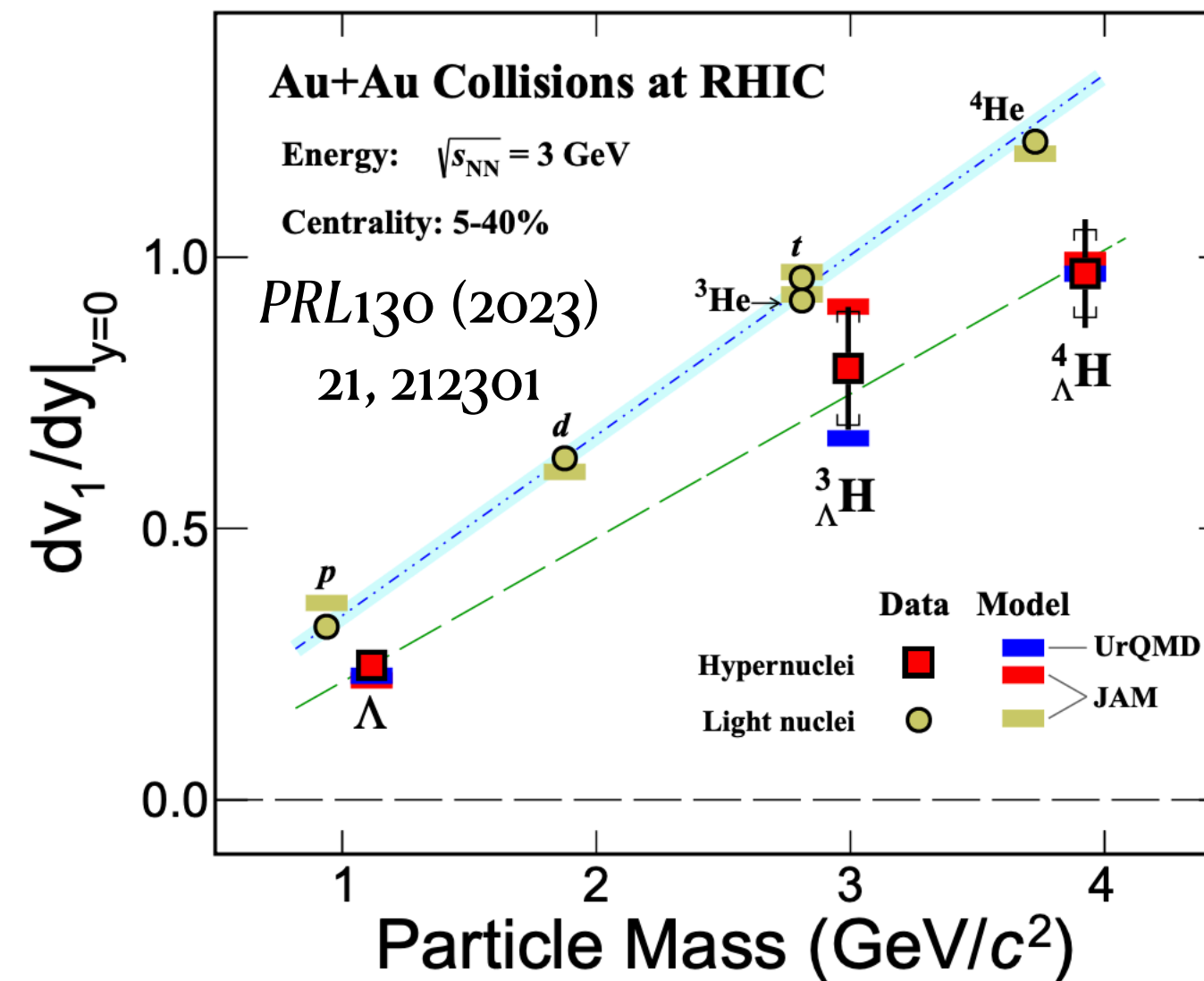
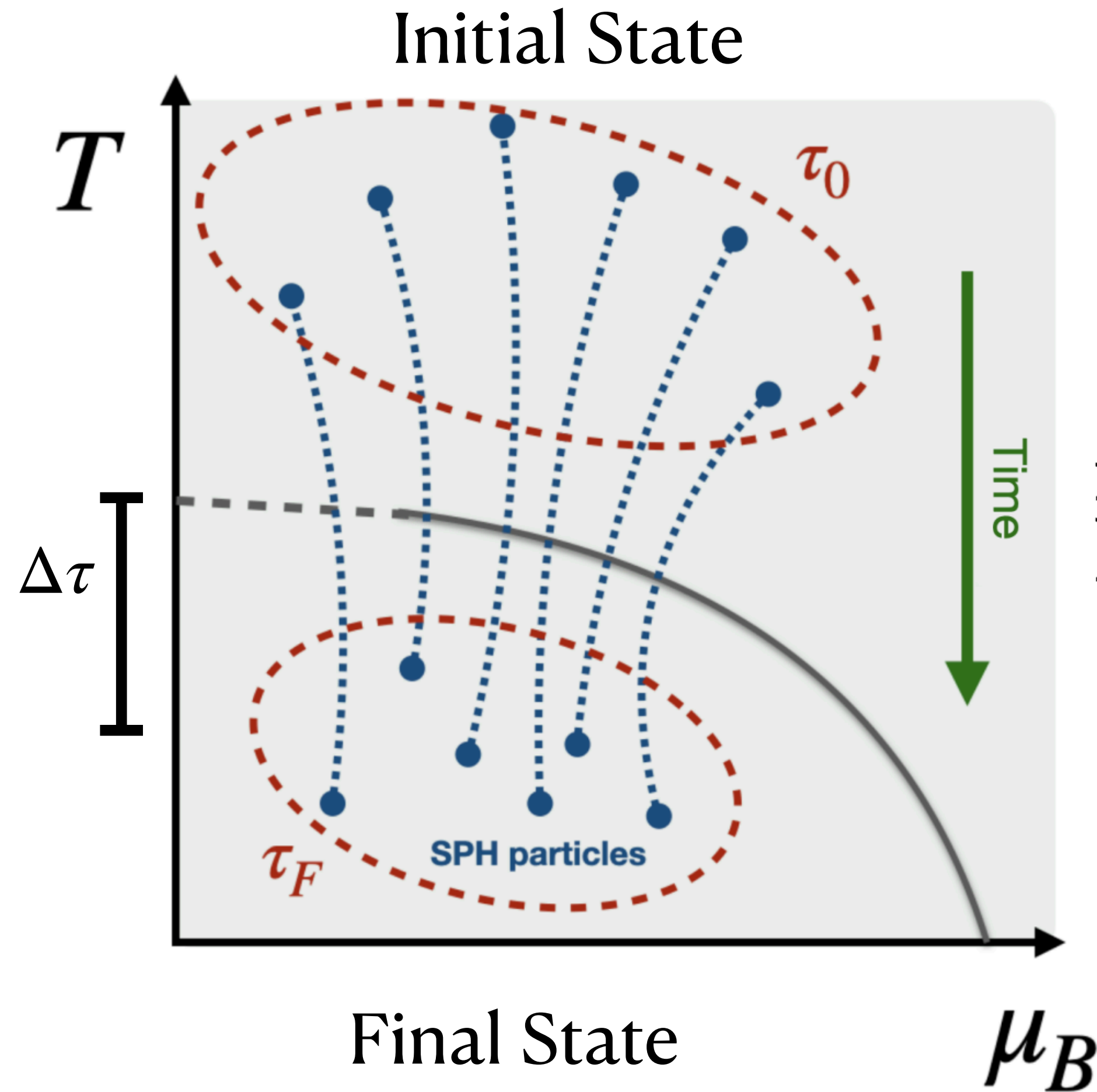
Require sophisticated dynamical models take into account far-from-equilibrium effects, critical scaling, and can make apples-to-apples comparisons to experiments.

See An et al, *Nucl.Phys.A* 1017 (2022) 122343



Challenge 3: experiments measure the final state

100s of observables possible to measure across $\sqrt{s_{NN}}$, each sensitive to different physics

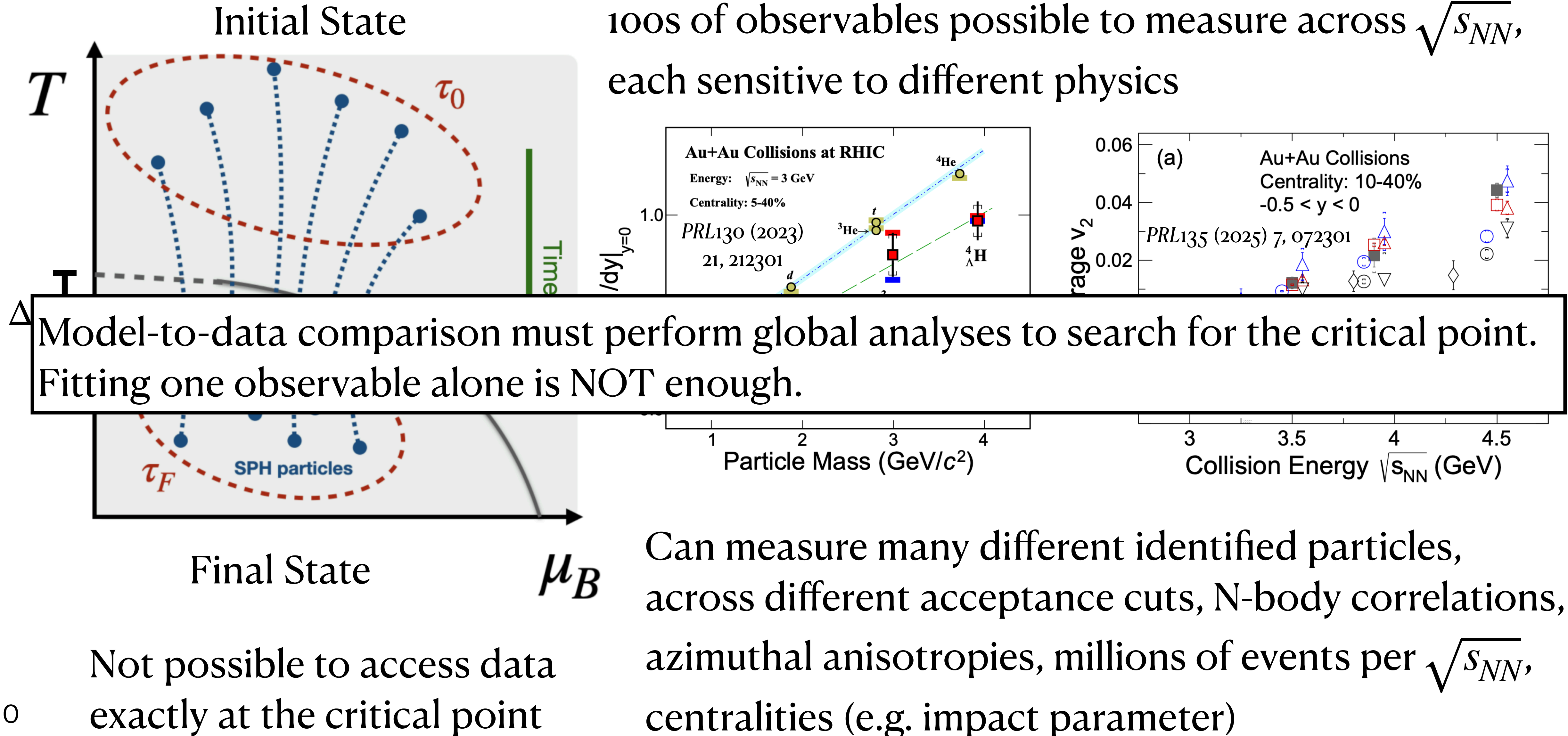


Can measure many different identified particles, across different acceptance cuts, N-body correlations, azimuthal anisotropies, millions of events per $\sqrt{s_{NN}}$, centralities (e.g. impact parameter)

Not possible to access data exactly at the critical point

Challenge 3: experiments measure the final state

100s of observables possible to measure across $\sqrt{s_{NN}}$, each sensitive to different physics

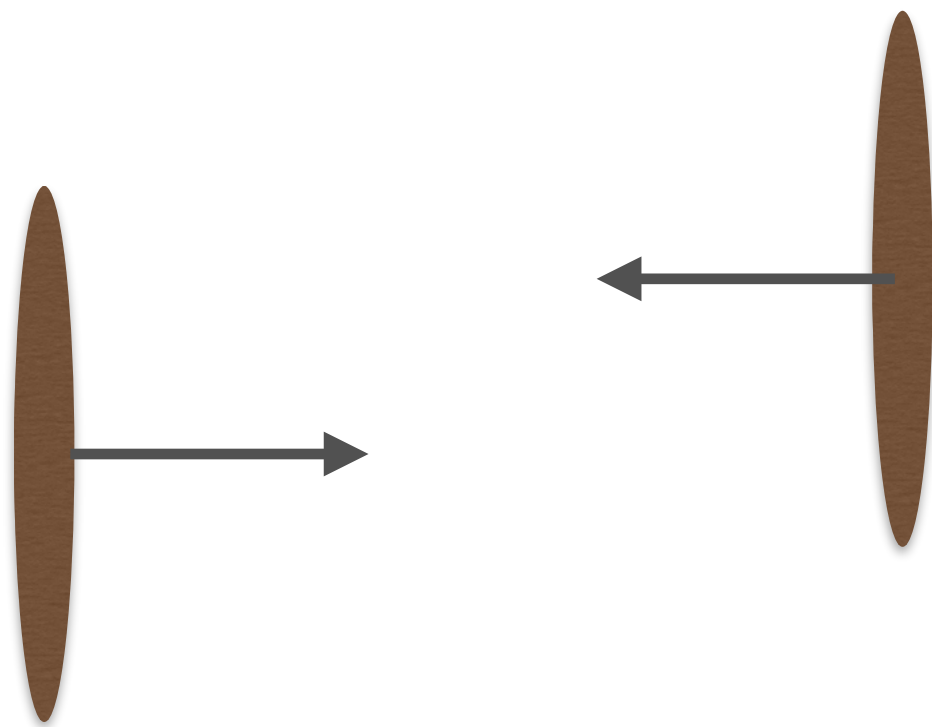


Experimental probes of the critical point

Doping the Quark gluon plasma with baryons

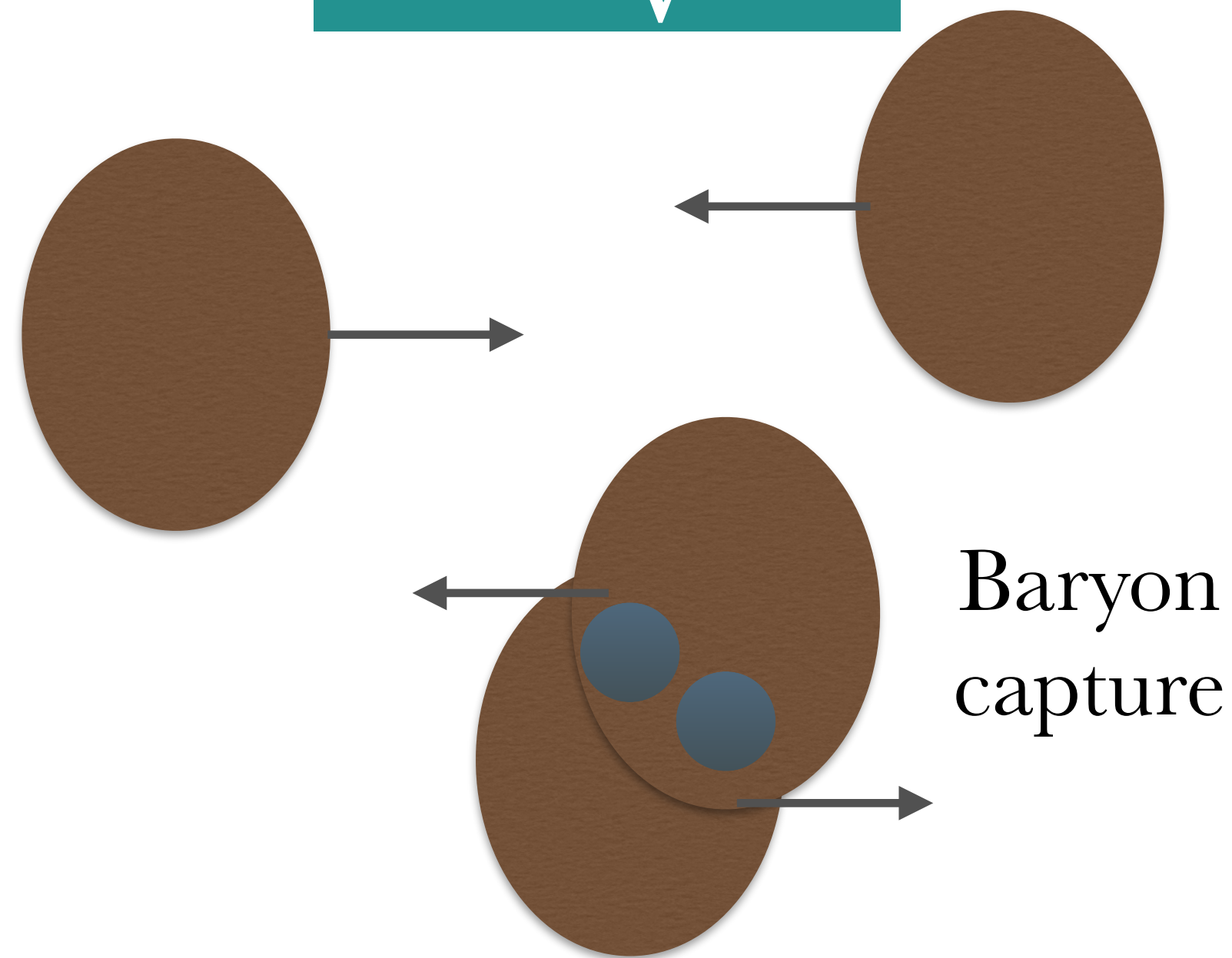
RHIC's Beam Energy Scan

Large $\sqrt{s_{NN}}$



- Lorentz contracted (2D)
- Nuclei pass through instantaneously
- Too quick to capture baryons

Small $\sqrt{s_{NN}}$

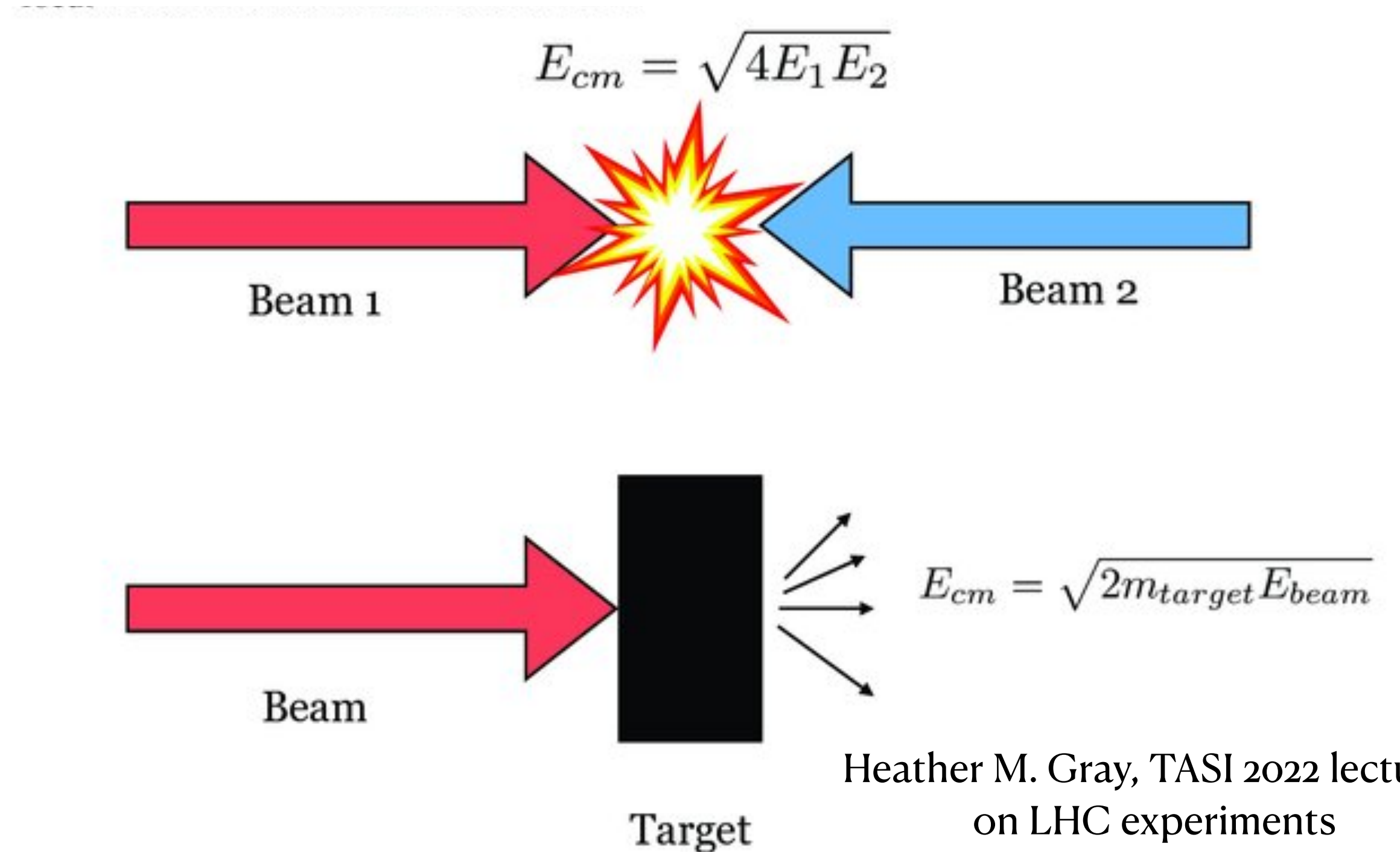
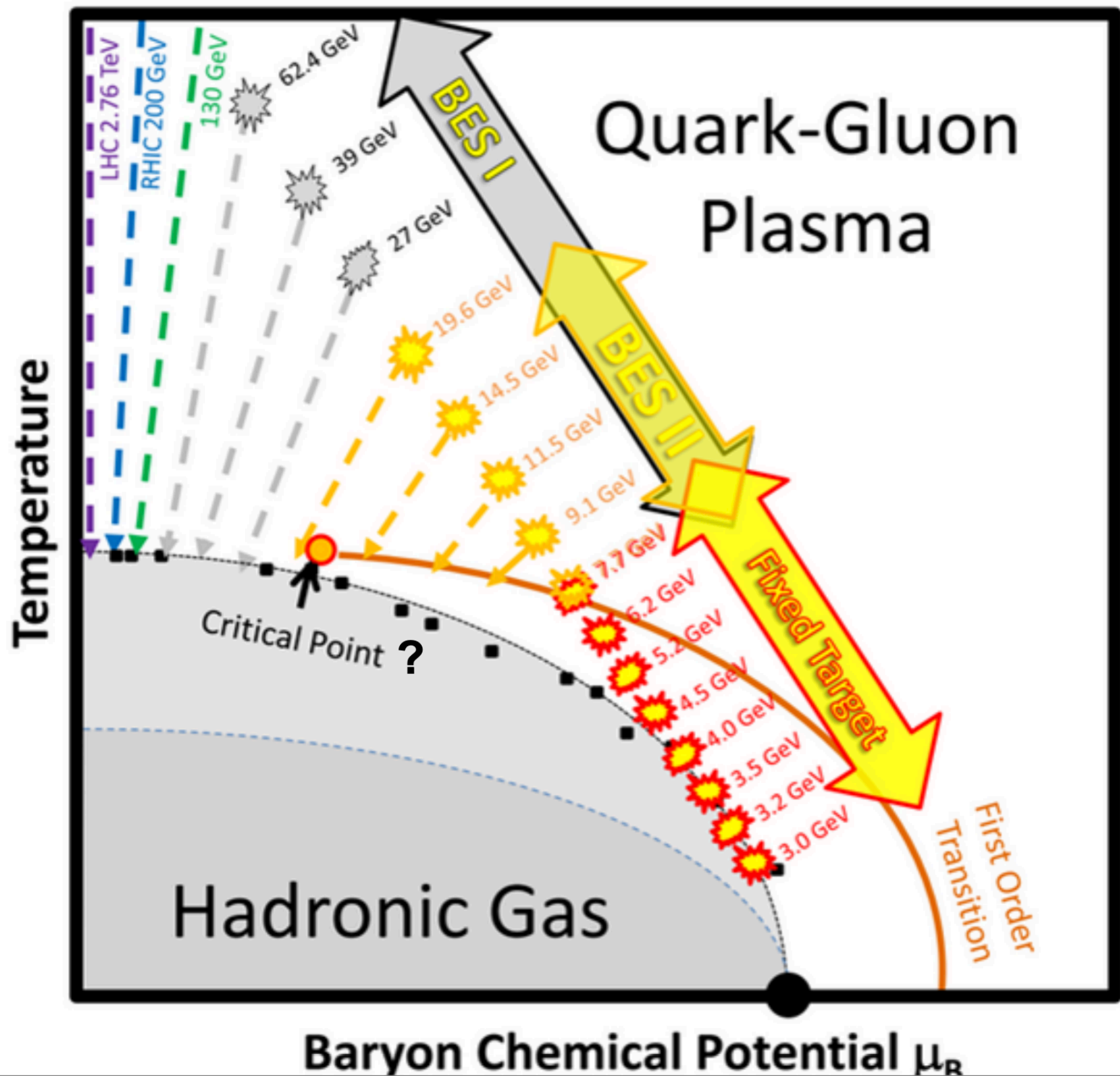


- 3D nuclei pass slowly
- Time to capture baryons

Sounds like a dynamics problem, but it changes the entire picture!

Beam Energy Scan I and II

Collider and Fixed Target



Heather M. Gray, TASI 2022 lectures
on LHC experiments

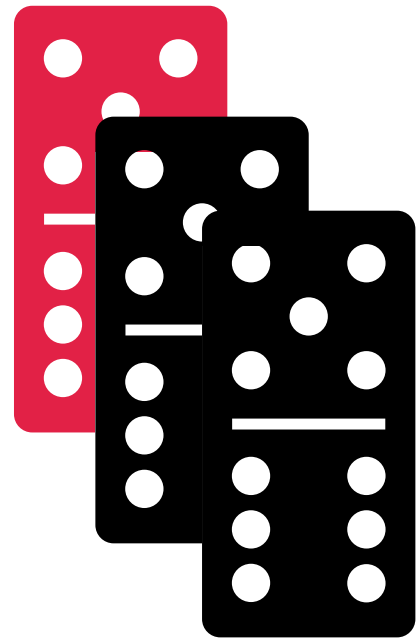
Lowering the beam energy $\sqrt{s_{NN}}$, gets at larger μ_B , but also lowers the temperature.

Lowest $\sqrt{s_{NN}}$ only possible with fixed target, changes the kinematics/acceptance

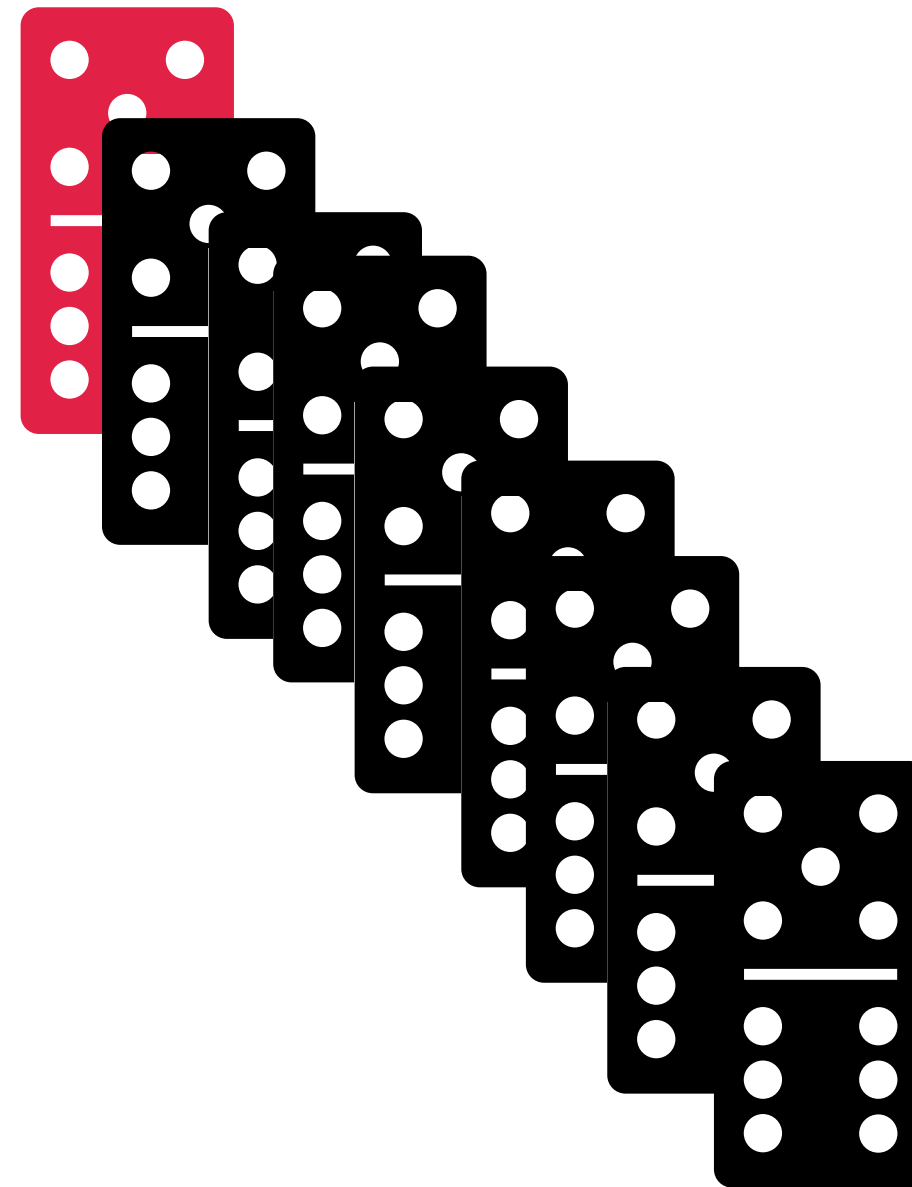
Signatures of the QCD critical point

At the critical point, the correlation length
diverges $\xi \rightarrow \infty$

Small ξ



Large ξ



Susceptibilities of the pressure scale with ξ

$$\chi_n \equiv \frac{\partial^n p}{(\partial \mu_B)^n}$$

Net-baryon number = first susceptibility

$$\chi_1 = n_B$$

Kurtosis $\kappa \propto \xi^7$ scales strongly with ξ at
the critical point, to cancel volume...

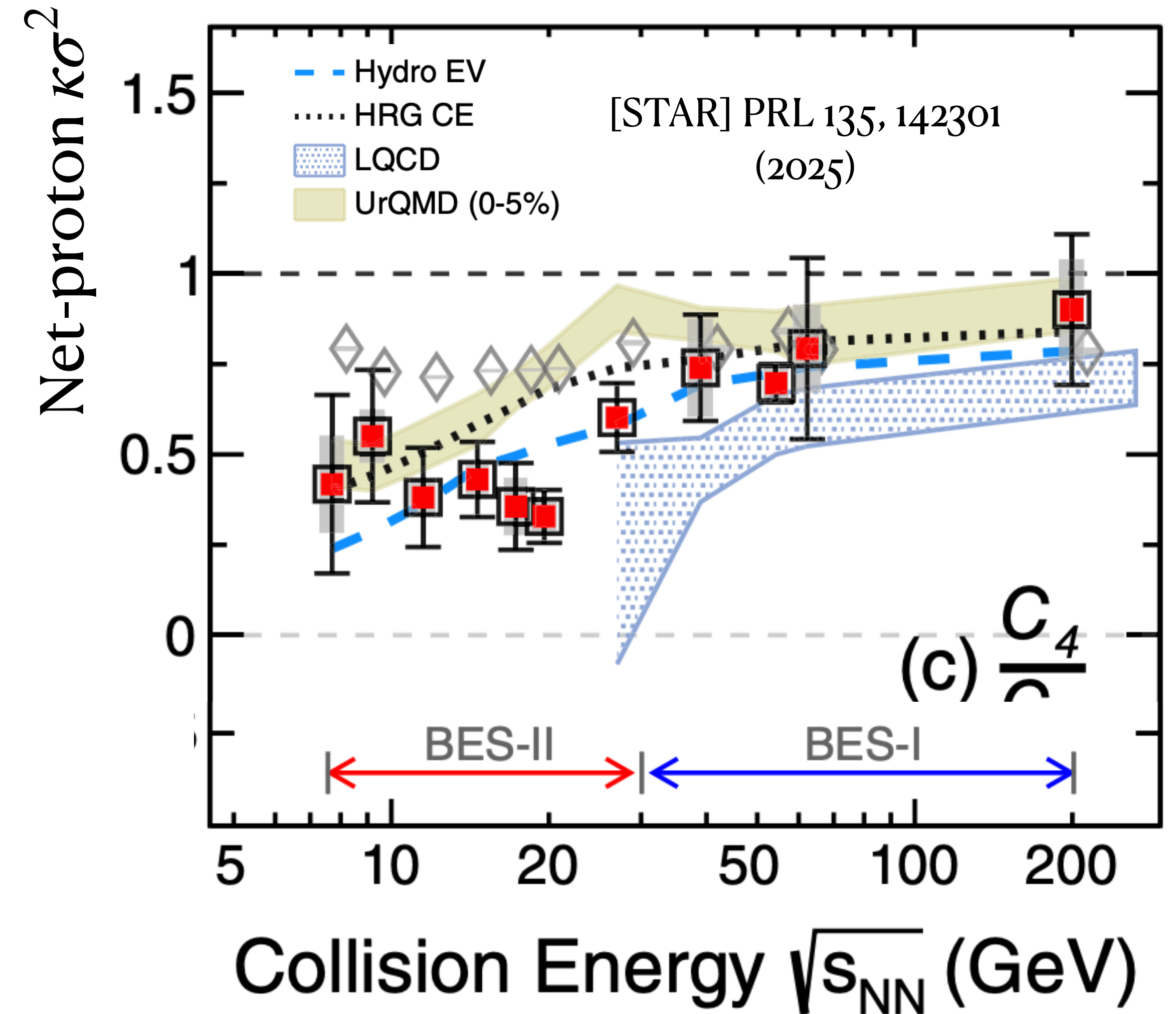
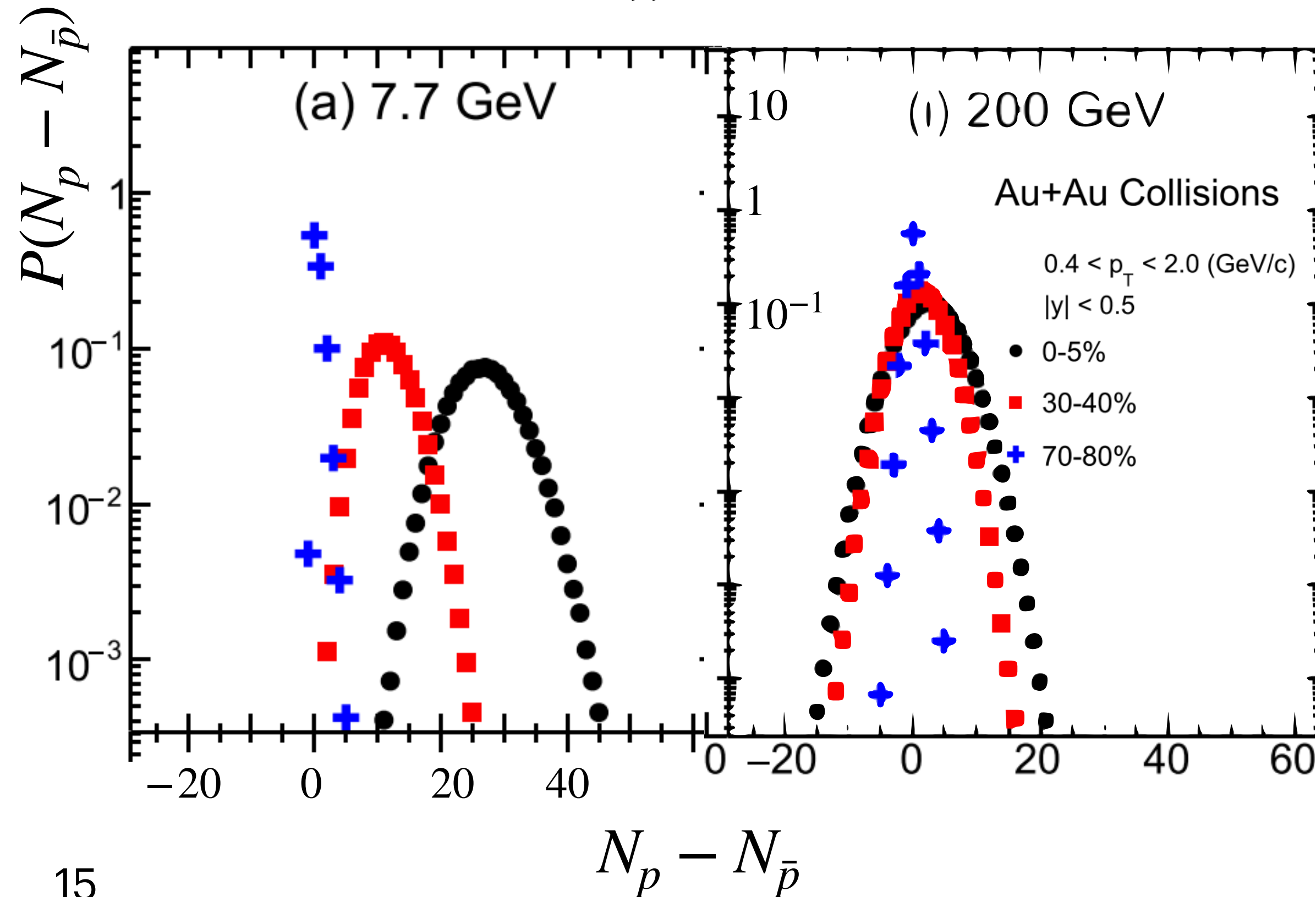
$$\kappa \sigma^2 = \frac{\chi_4}{\chi_2} \propto \xi^5$$

Leads to large fluctuations in the net-
baryon number!

Experimental measurements of the kurtosis

RHIC: measure net-proton distribution

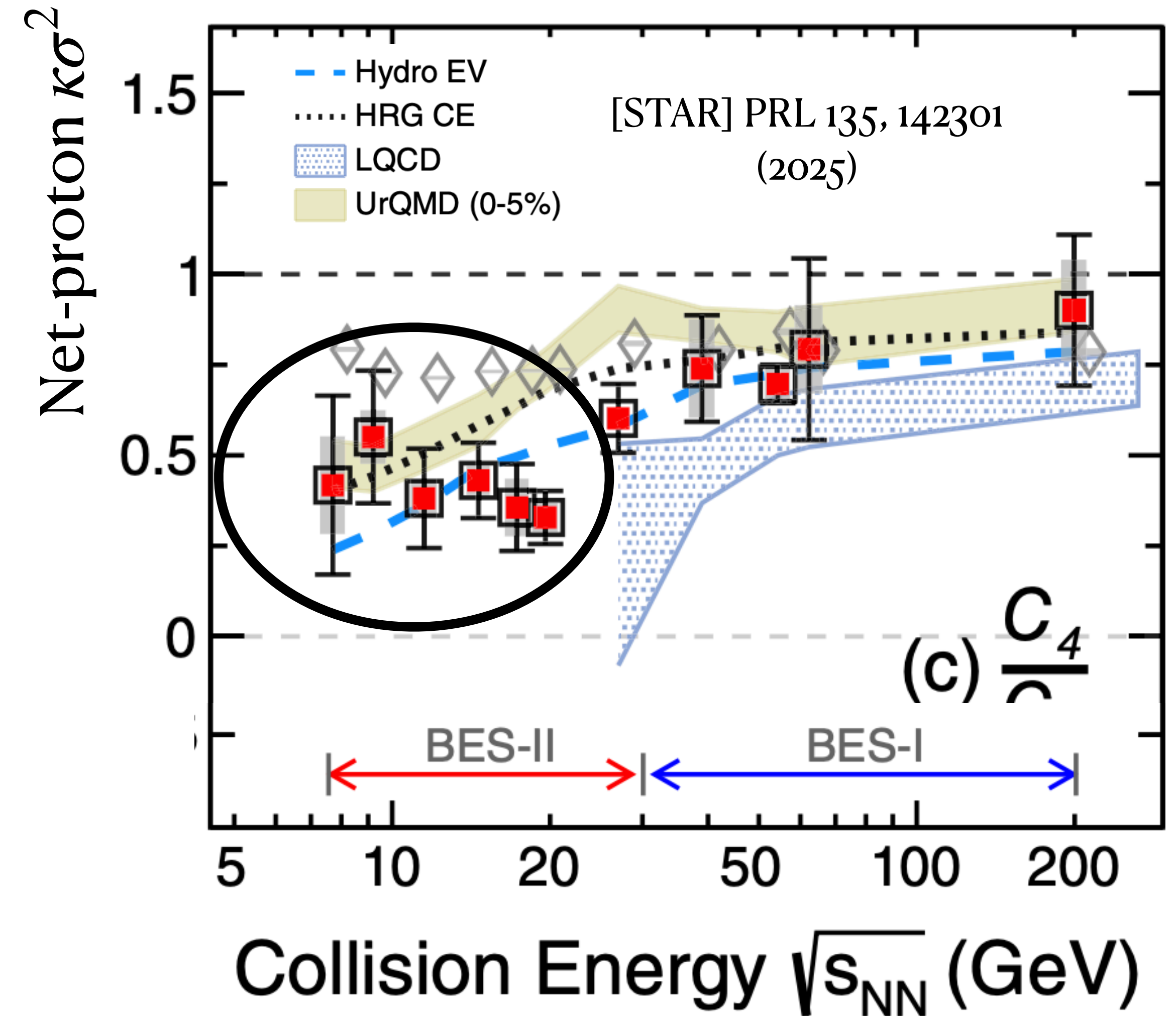
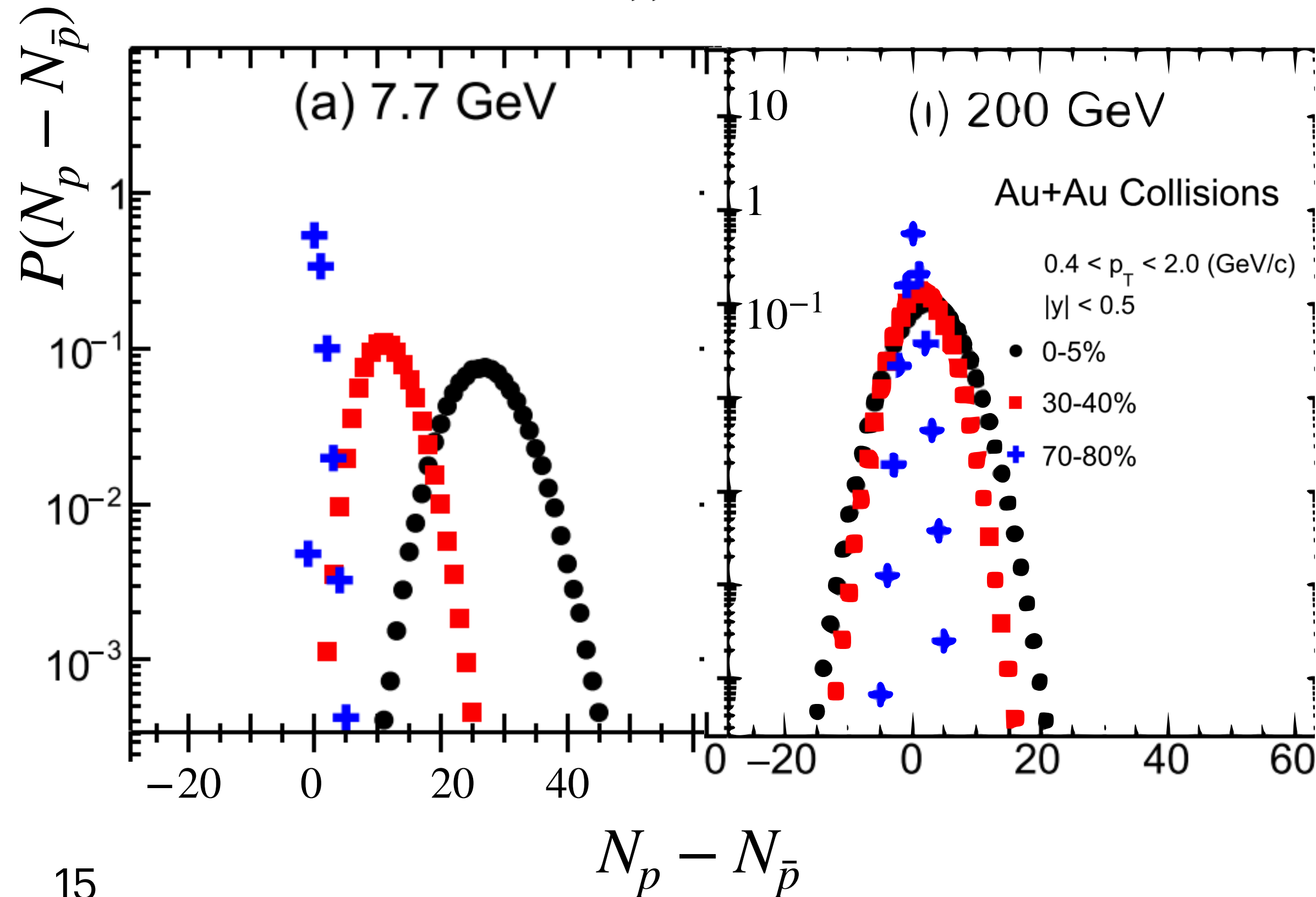
[STAR] Phys. Rev. C **111**, 029902



Experimental measurements of the kurtosis

RHIC: measure net-proton distribution

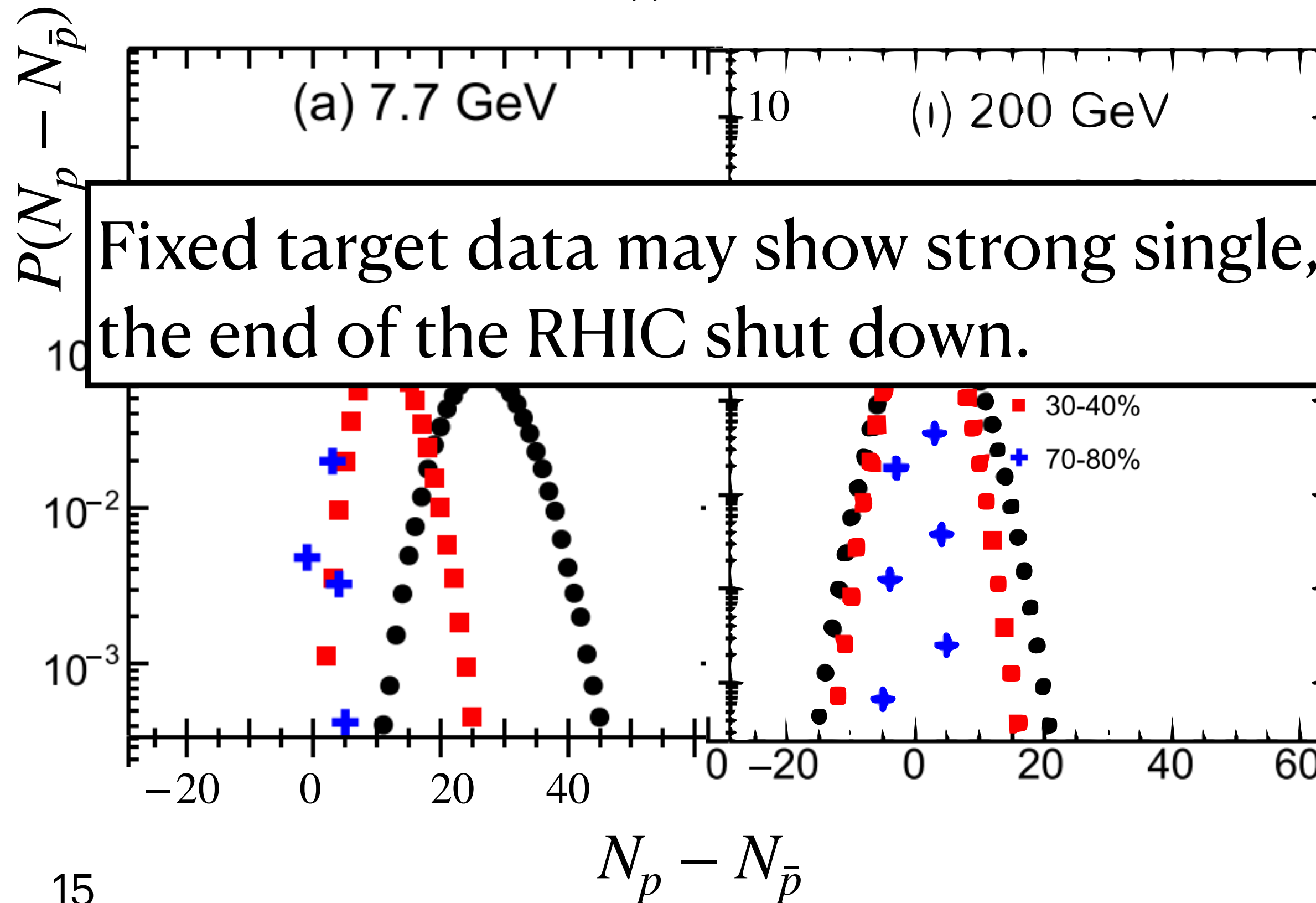
[STAR] Phys. Rev. C **111**, 029902



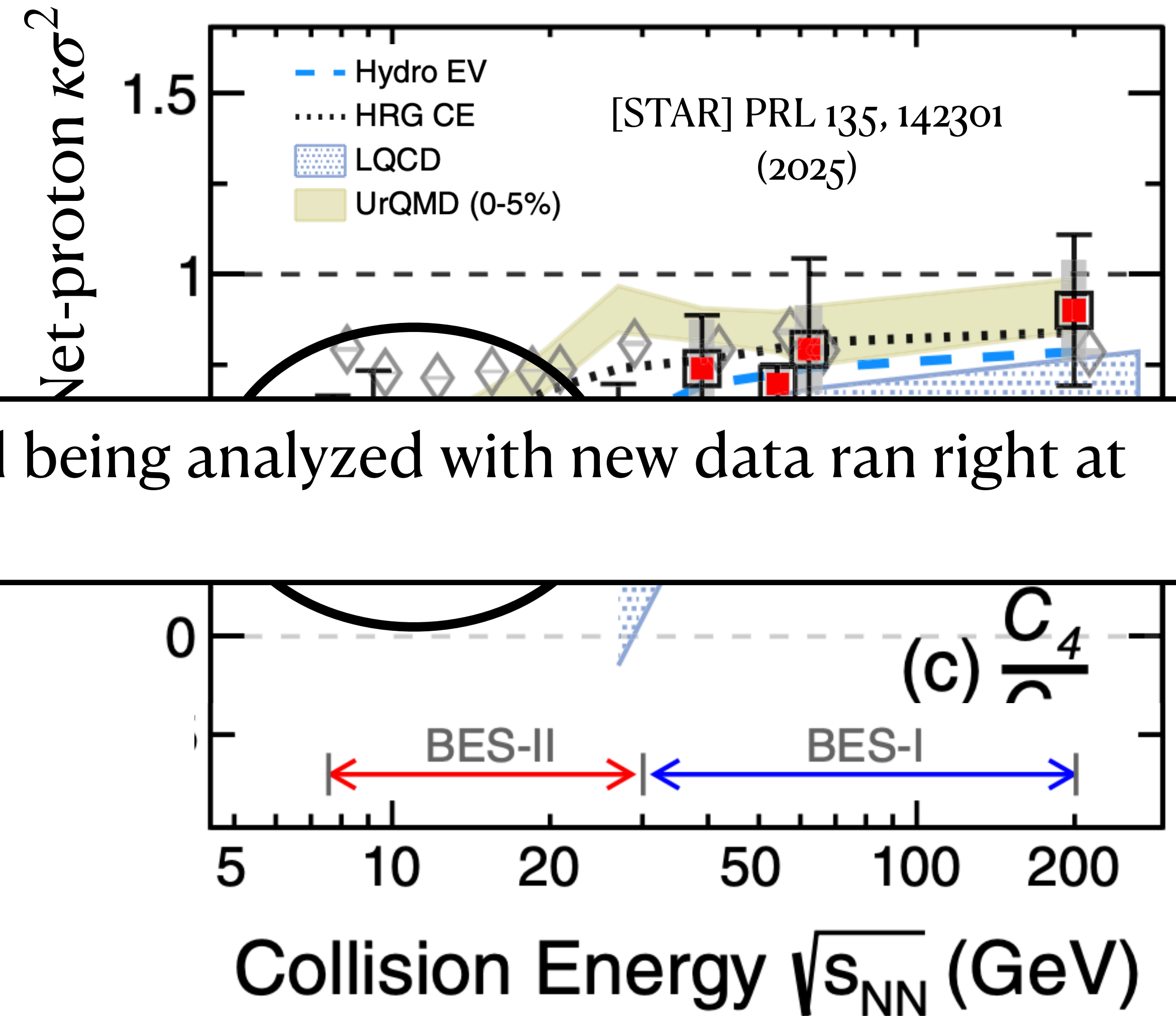
Experimental measurements of the kurtosis

RHIC: measure net-proton distribution

[STAR] Phys. Rev. C **111**, 029902



Fixed target data may show strong single, still being analyzed with new data ran right at the end of the RHIC shut down.

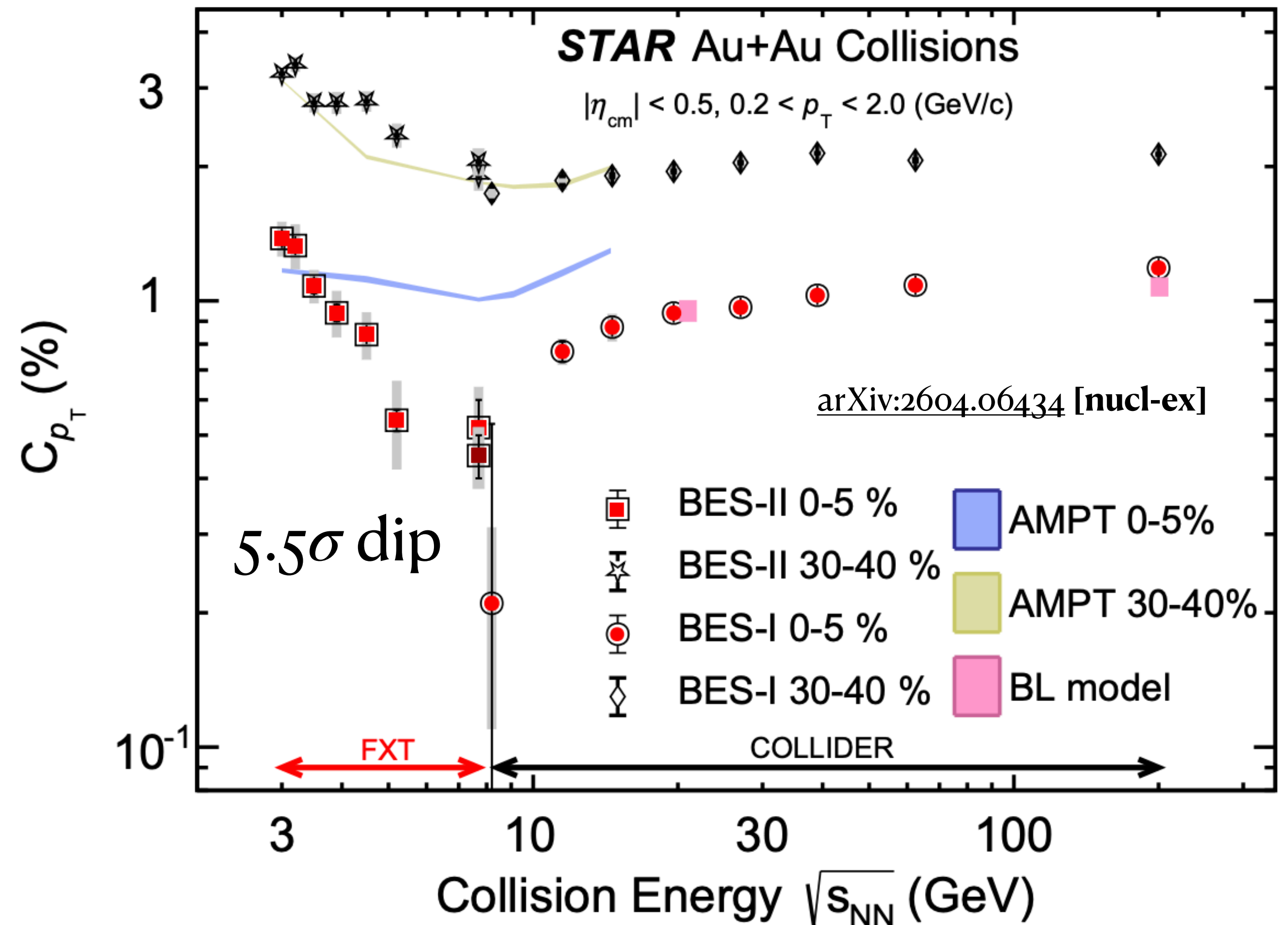


Fluctuations in the average momentum of protons

Possible hints in the fixed-target region?

Larger fluctuations in the the transverse momentum where we anticipate the critical point.

Could appear due to changes in the equation of state and/or critical scaling of transport coefficients

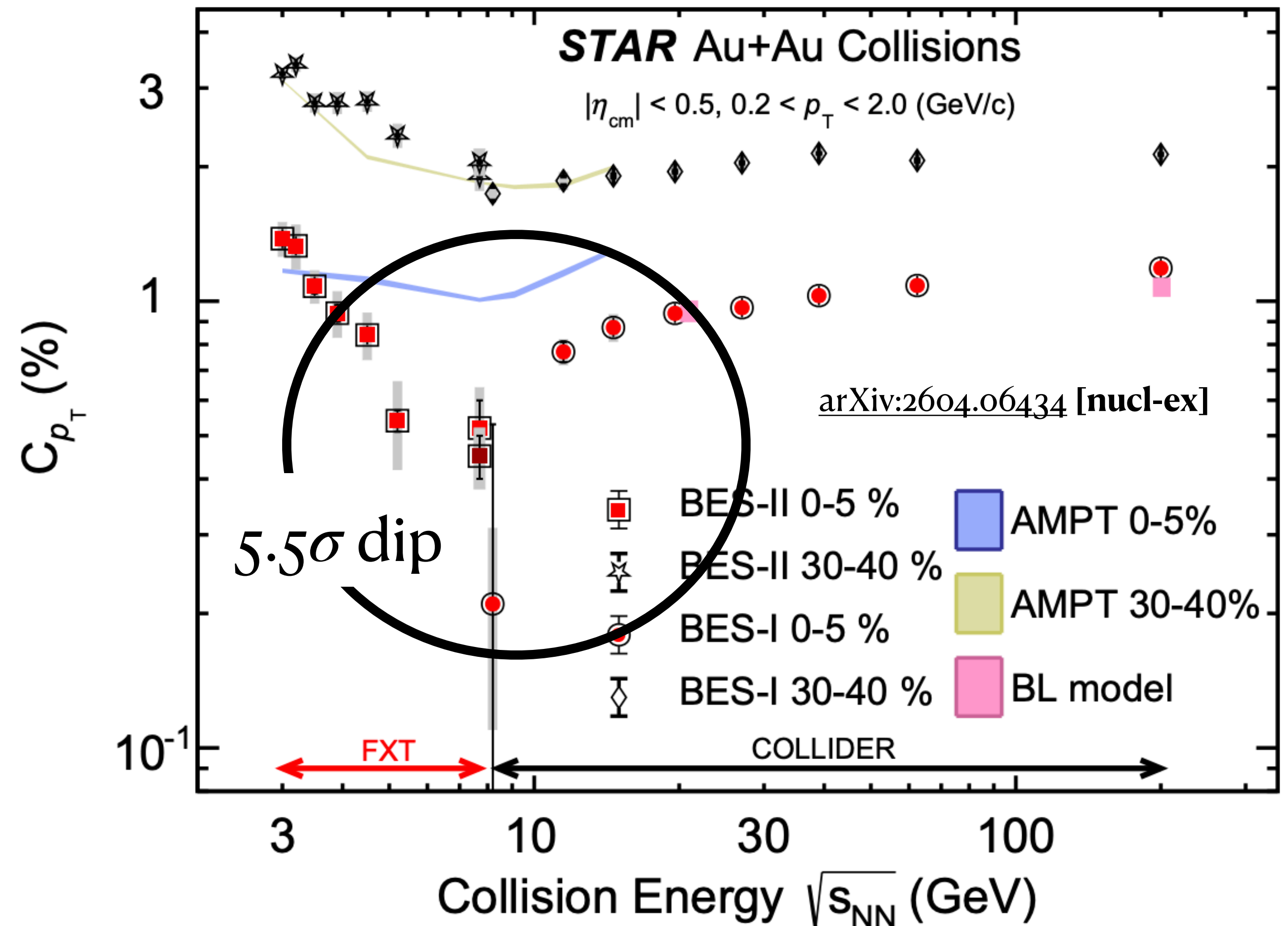


Fluctuations in the average momentum of protons

Possible hints in the fixed-target region?

Larger fluctuations in the the transverse momentum where we anticipate the critical point.

Could appear due to changes in the equation of state and/or critical scaling of transport coefficients



Decisive claims of claims need
theory input!

Are our theory comparisons under control?

No, absolutely not

Dynamical simulations are just now possible: moveable critical point in a equation of state, 3+1D relativistic viscous hydrodynamic code with 3 conserved charges and diffusion, analyses calculations

Open problem: criticality/phase transitions in relativistic viscous fluid dynamics

Next Steps: run a large-scale Bayesian analysis with dynamical simulations against the data

I don't have an answer today, but let me tell you about the state-of-the-art

Theory collaborations:

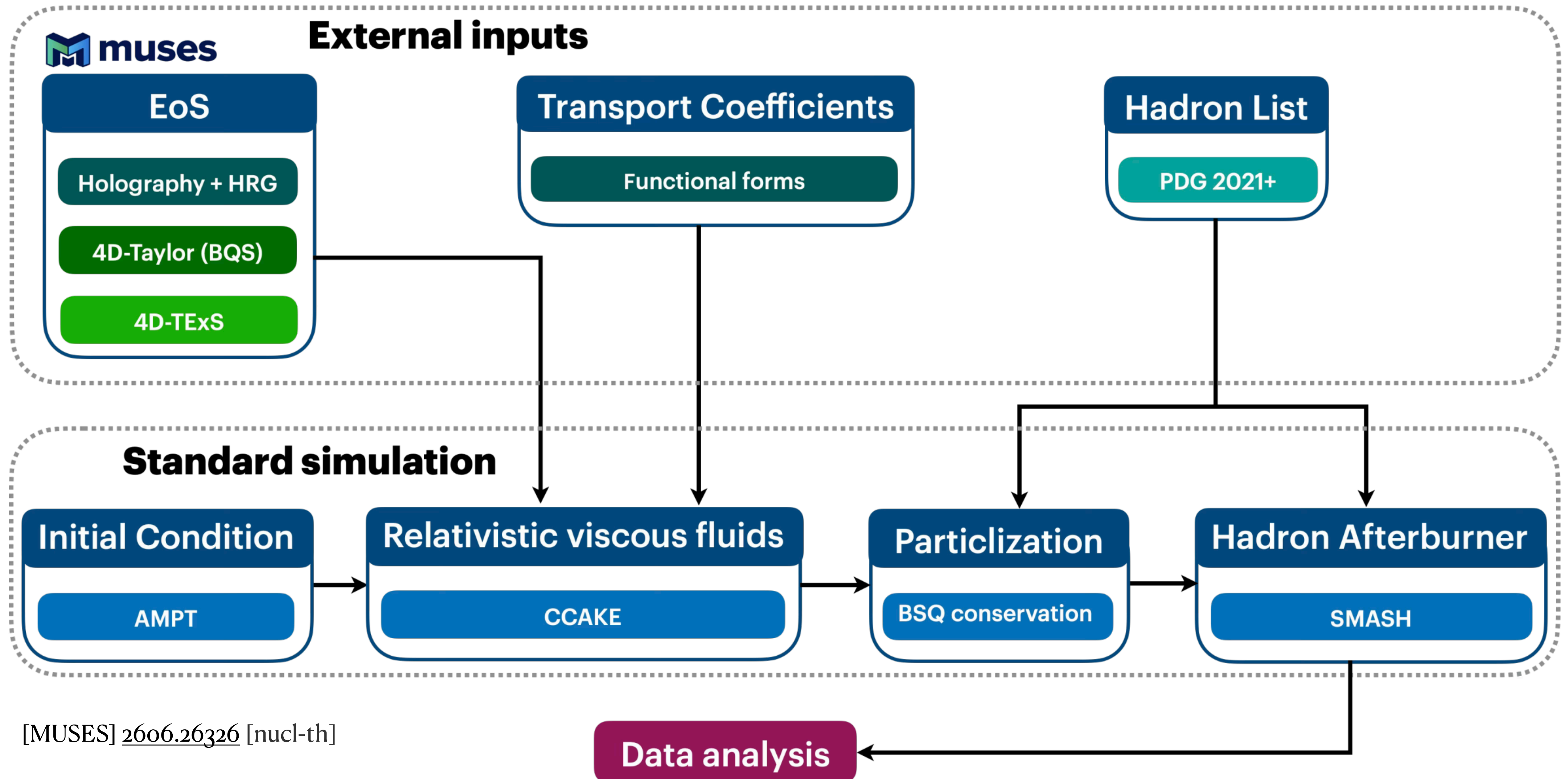
2016-2020



2021-???



Dynamical simulations



Relativistic viscous hydrodynamics

Largest contributor to simulations in terms of time and free parameters

Energy-Momentum Tensor

$$T^{\mu\nu} = T_0^{\mu\nu} + \Pi + \pi^{\mu\nu}$$

$$\text{Bulk Pressure } \Pi = \text{Tr} \left[T^{\mu\nu} - T_0^{\mu\nu} \right]$$

$$\text{Shear stress tensor } \pi^{\mu\nu} = T^{\mu\nu} - T_0^{\mu\nu} - \Pi$$

Equation
of State
(EOS)

Local rest
frame

$$T_0^{\mu\nu} = \begin{bmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{bmatrix}$$

In non-relativistic fluids, Navier-Stokes is used to describe viscous fluids. But Navier-Stokes is acausal and unstable in relativistic systems!

Hiscock & Lindblom *Phys.Rev.D* 31 (1985) 725-733; *Annals Phys.* 151 (1983) 466-496

Need a “relaxation time” to return to Navier-Stokes, e.g.

$$\tau_{\Pi} \dot{\Pi} + \Pi = -\zeta \theta + \dots$$

Israel & Stewart *Annals Phys.* 118 (1979) 341-372 + significant development from HICs!

Transport coefficients in the Quark Gluon Plasma

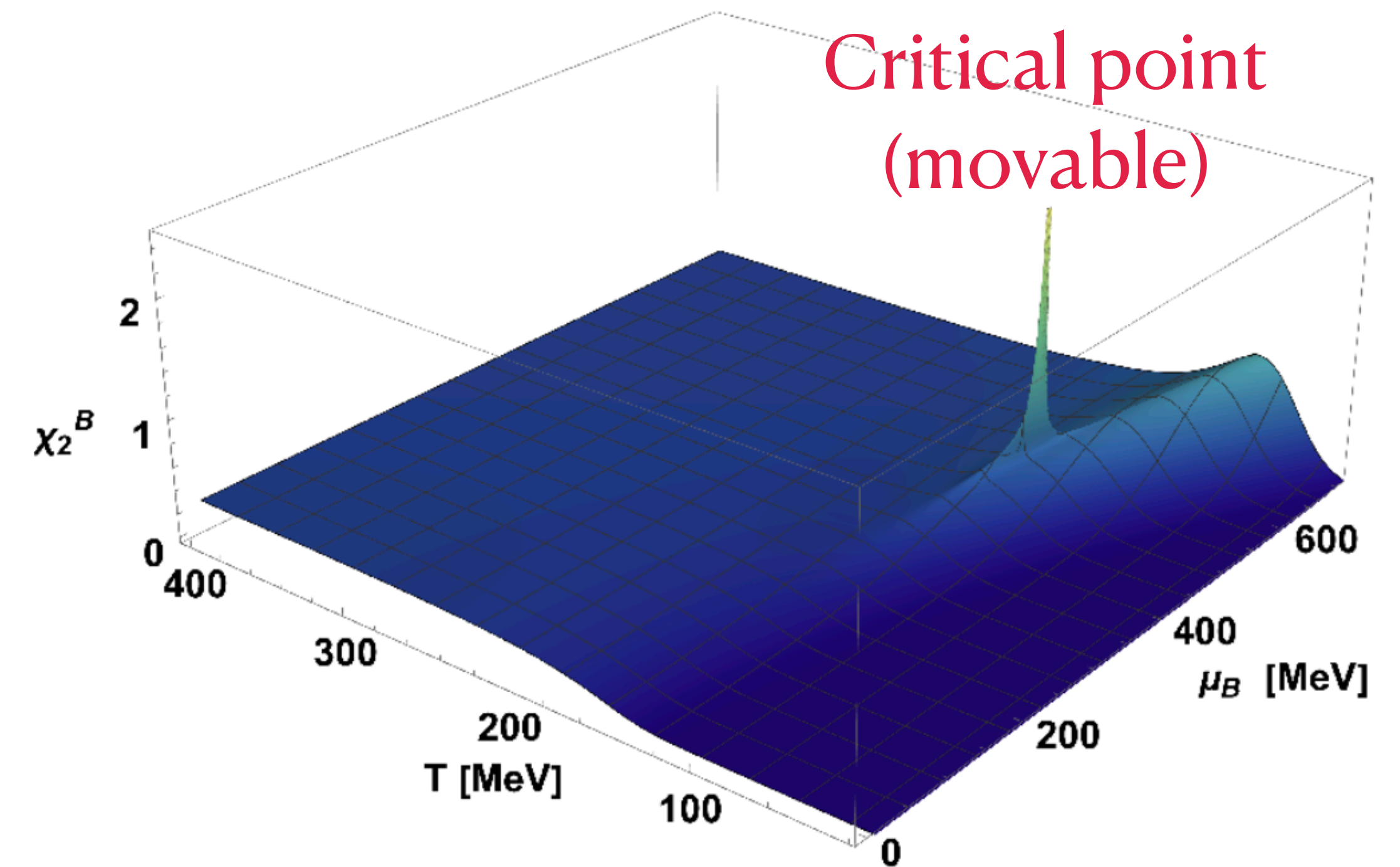
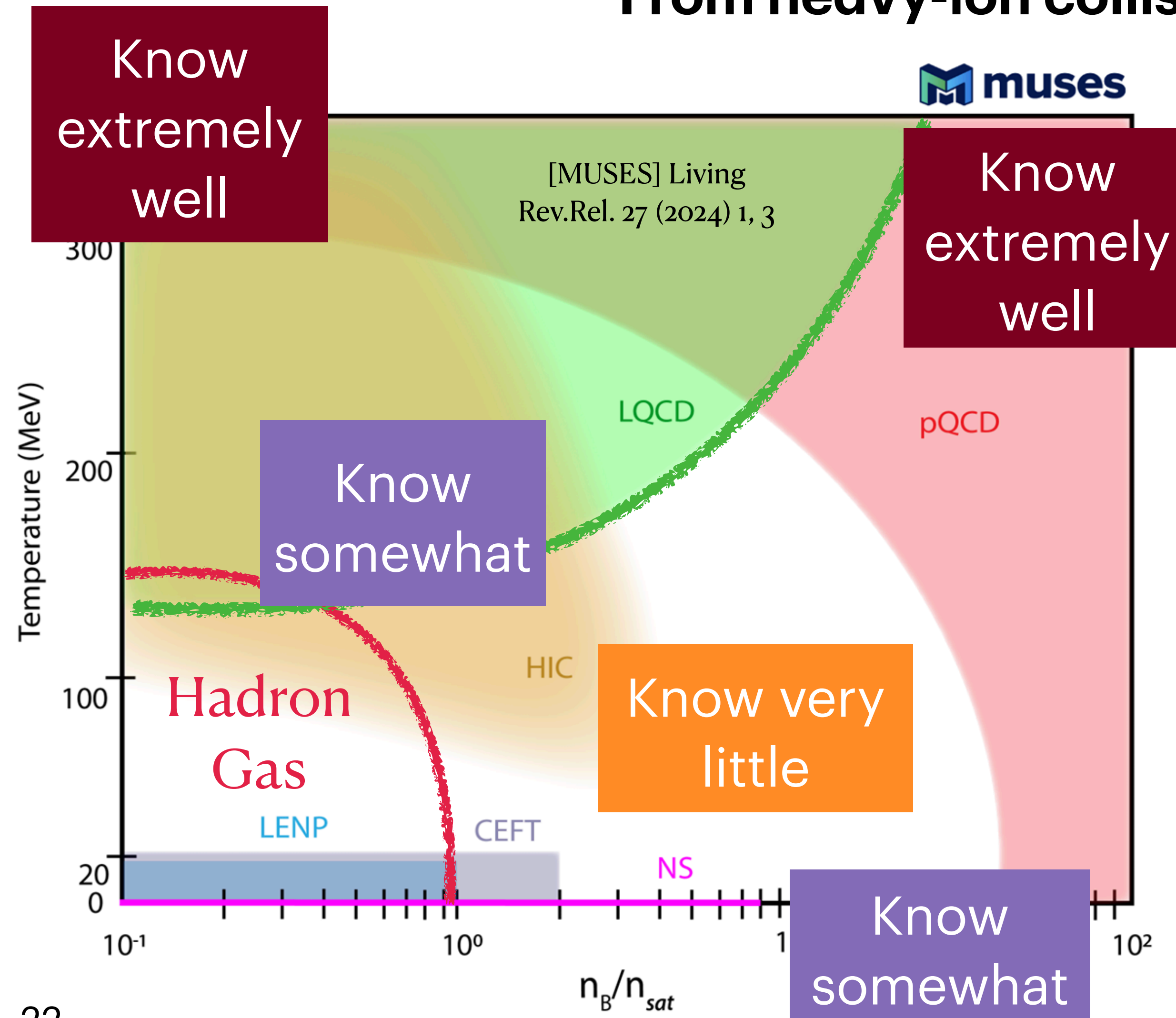
Bulk	Shear	Diffusion
$\Pi \sim -\zeta \Theta$	$\pi^{\mu\nu} \sim 2\eta\sigma^{\mu\nu}$	$q_{\perp}^{\mu} \sim \kappa \nabla_{\perp}^{\mu} (\mu/T)$
 Θ <i>expansion</i>	 $\sigma_{\mu\nu}$ <i>shear</i>	QCD conserved charges (B,S,Q)  <i>Diffusion</i>

Credit: Rezzolla & Zanotti Relativistic Hydrodynamics

21 **Viscosity** - resistance to deformation or “thickness” of liquid

muses: open-source equations of state

From heavy-ion collisions to neutron stars

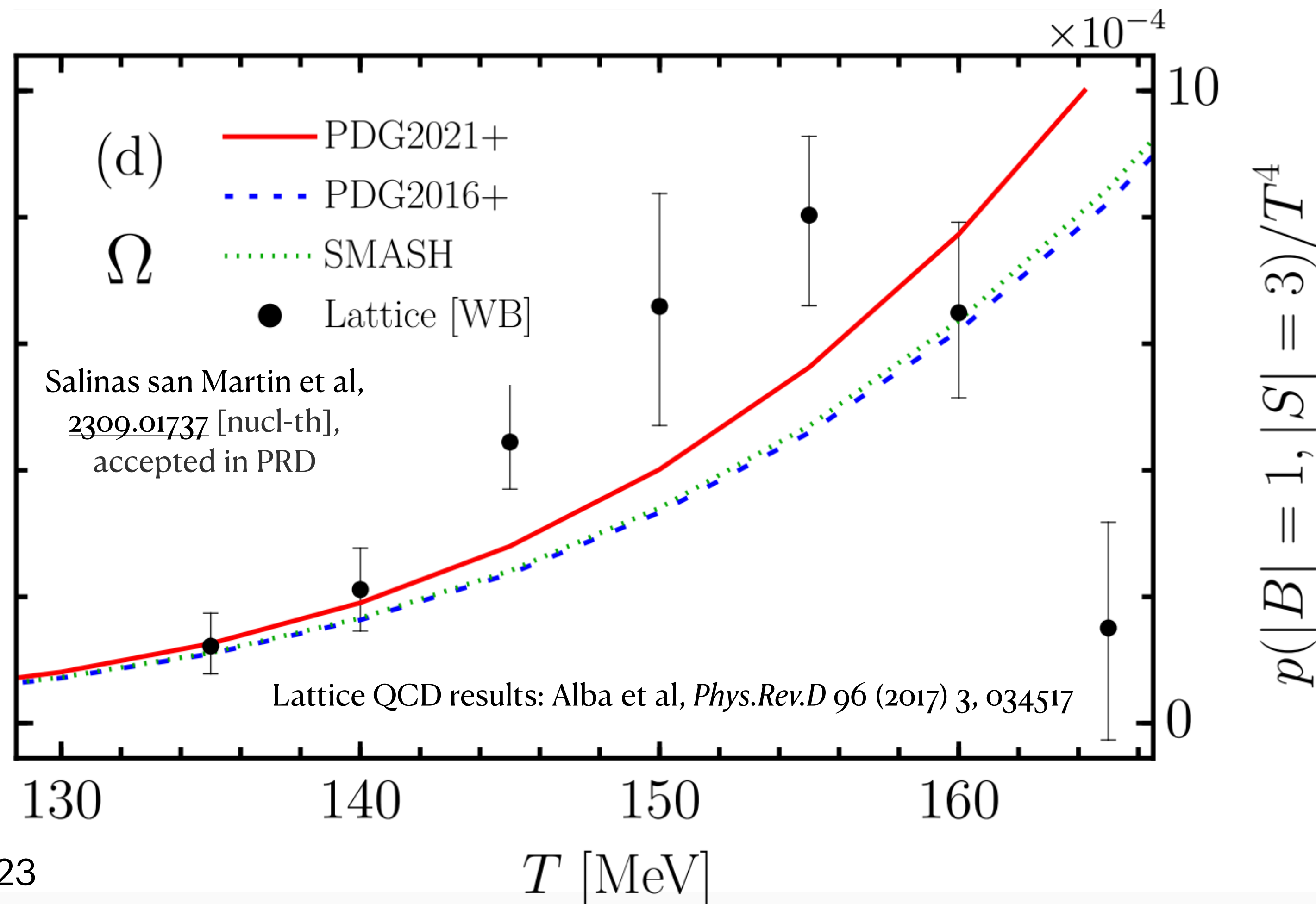


No first-principle critical point calculations exist, but models predict its location around $\sqrt{s_{NN}} \sim 2 - 4 \text{ GeV}$

Hadronic afterburners - need input!

Influence of hadron spectroscopy

Partial pressure from lattice need more strange baryons



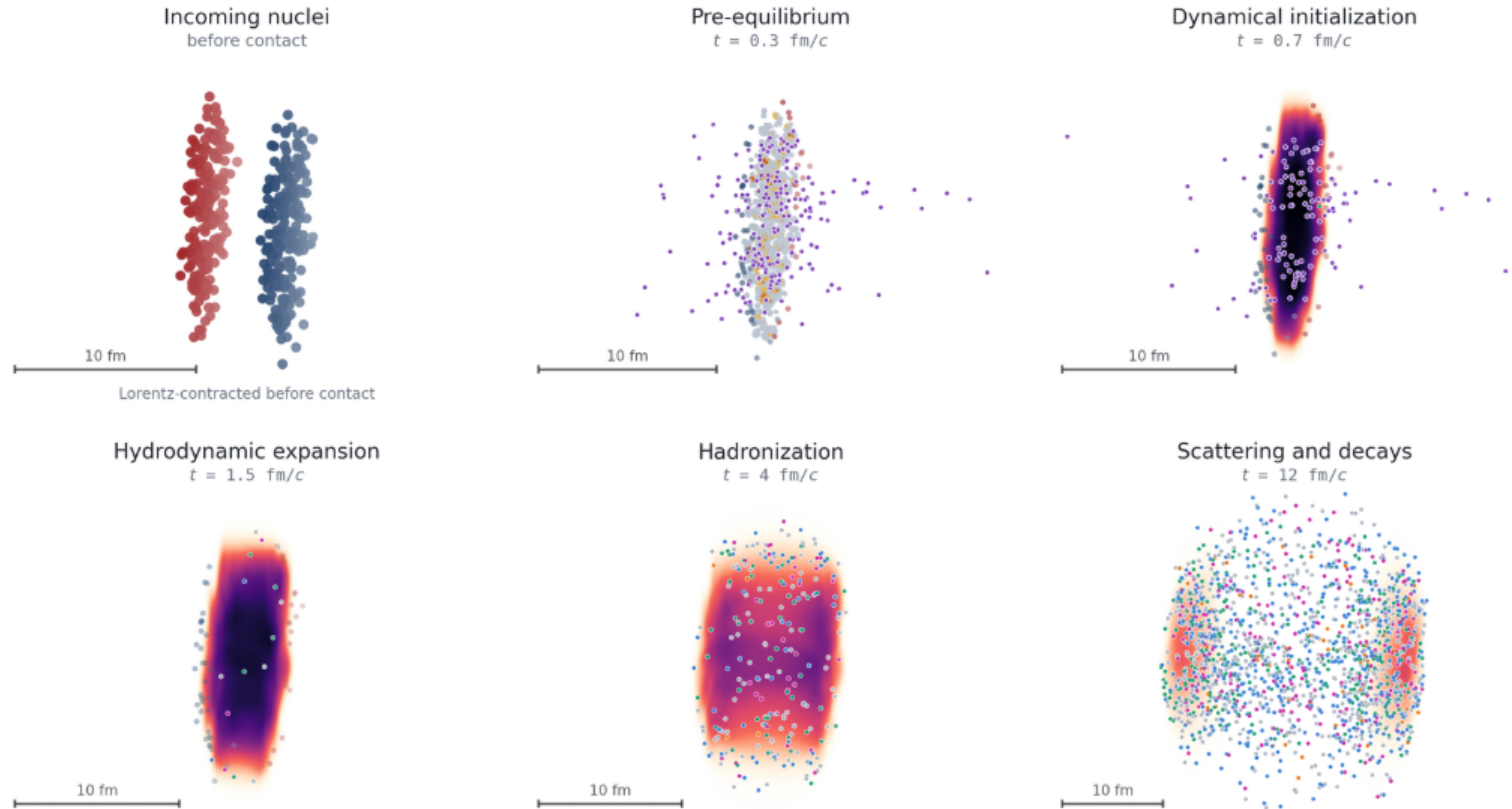
S-matrix approach *Nucl.Phys.A* 1010 (2021) 122176

- Missing resonances and/or incorrect branching ratios can influence: yields, spectra, average transverse momentum etc
- Require formats used in heavy-ion collision simulation codes
- Hadronic phase more important at low $\sqrt{s_{NN}}$, longer lifetime
- Insights into K^+/π^+ horn?

<https://arxiv.org/pdf/2608.04758>

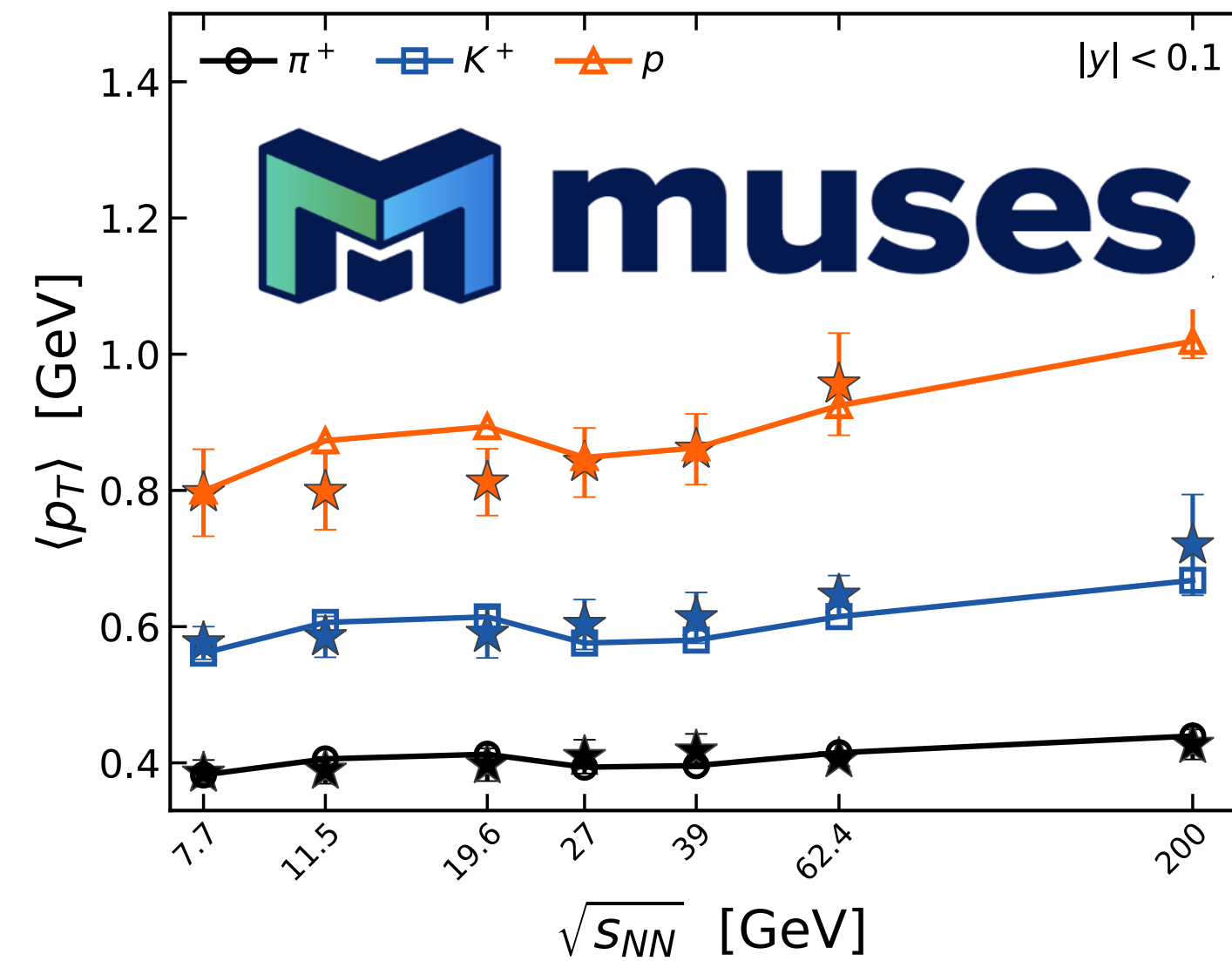
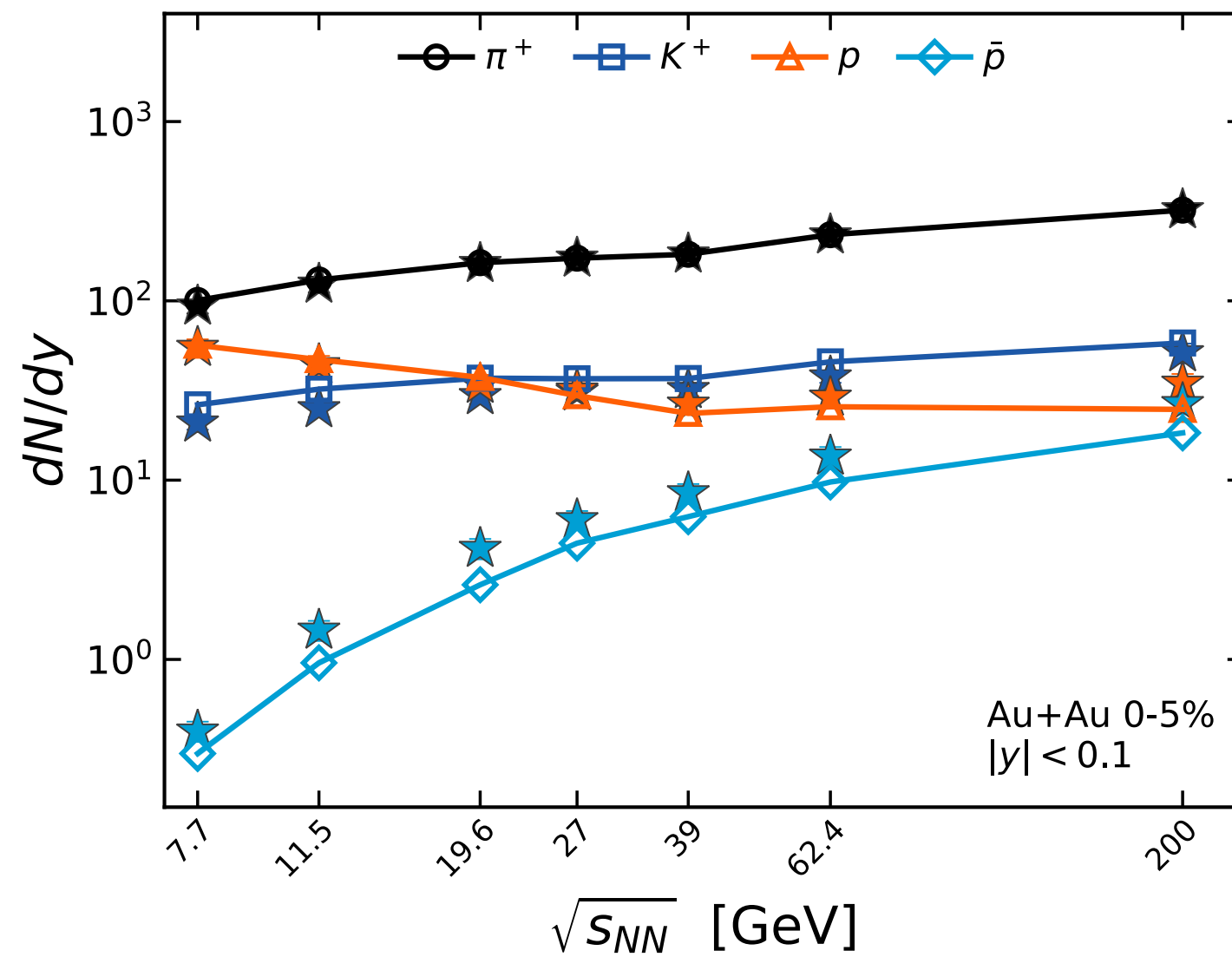
Simulations of heavy-ion collisions in practice

AMPT+CCAKE+SMASH includes MUSES Holography+HRG equation of state



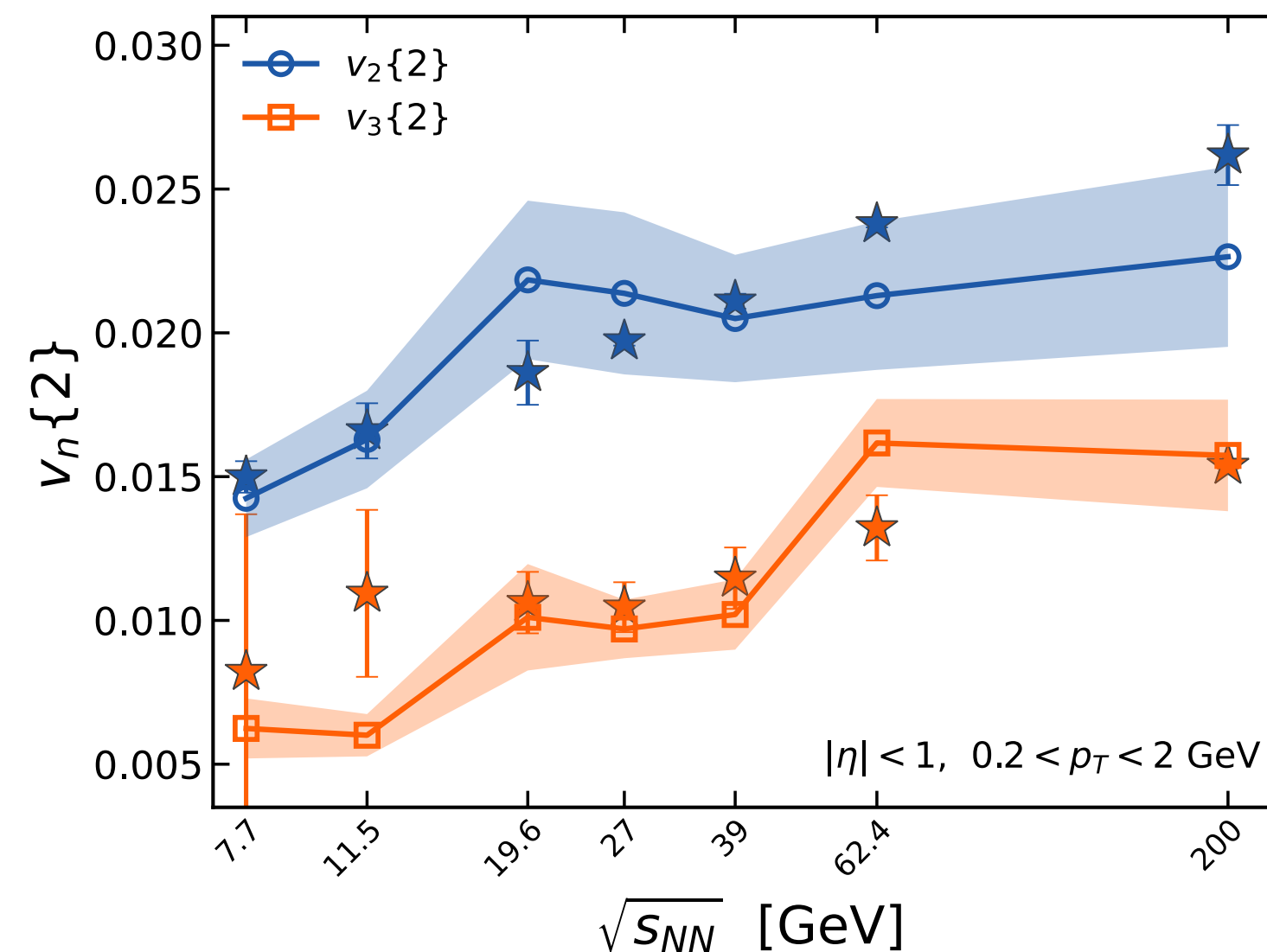
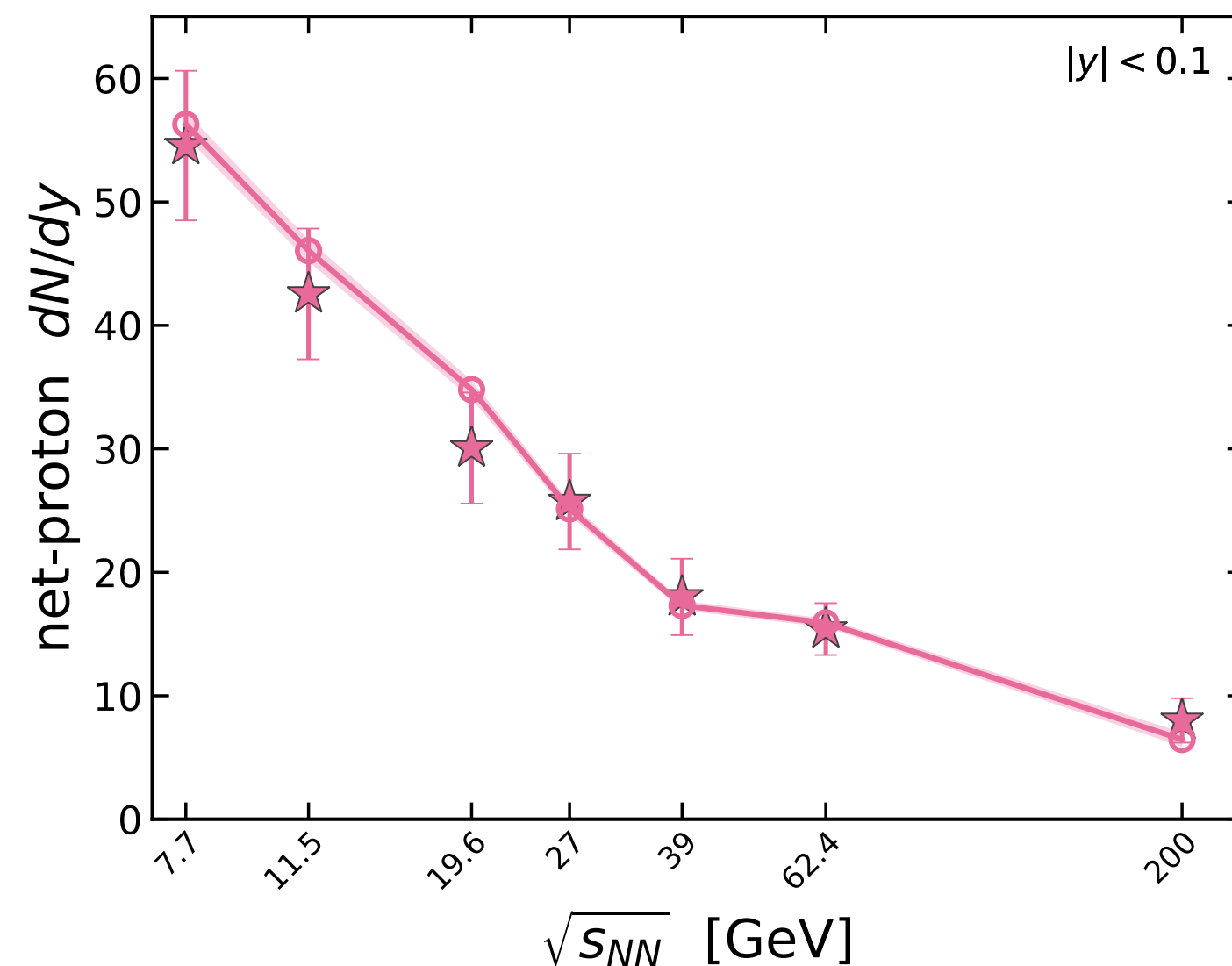
Simulations that probe the critical point

★ STAR ⊠ NuclearConfectionery



Simulations have a number of free parameters, must tune to the data before searches

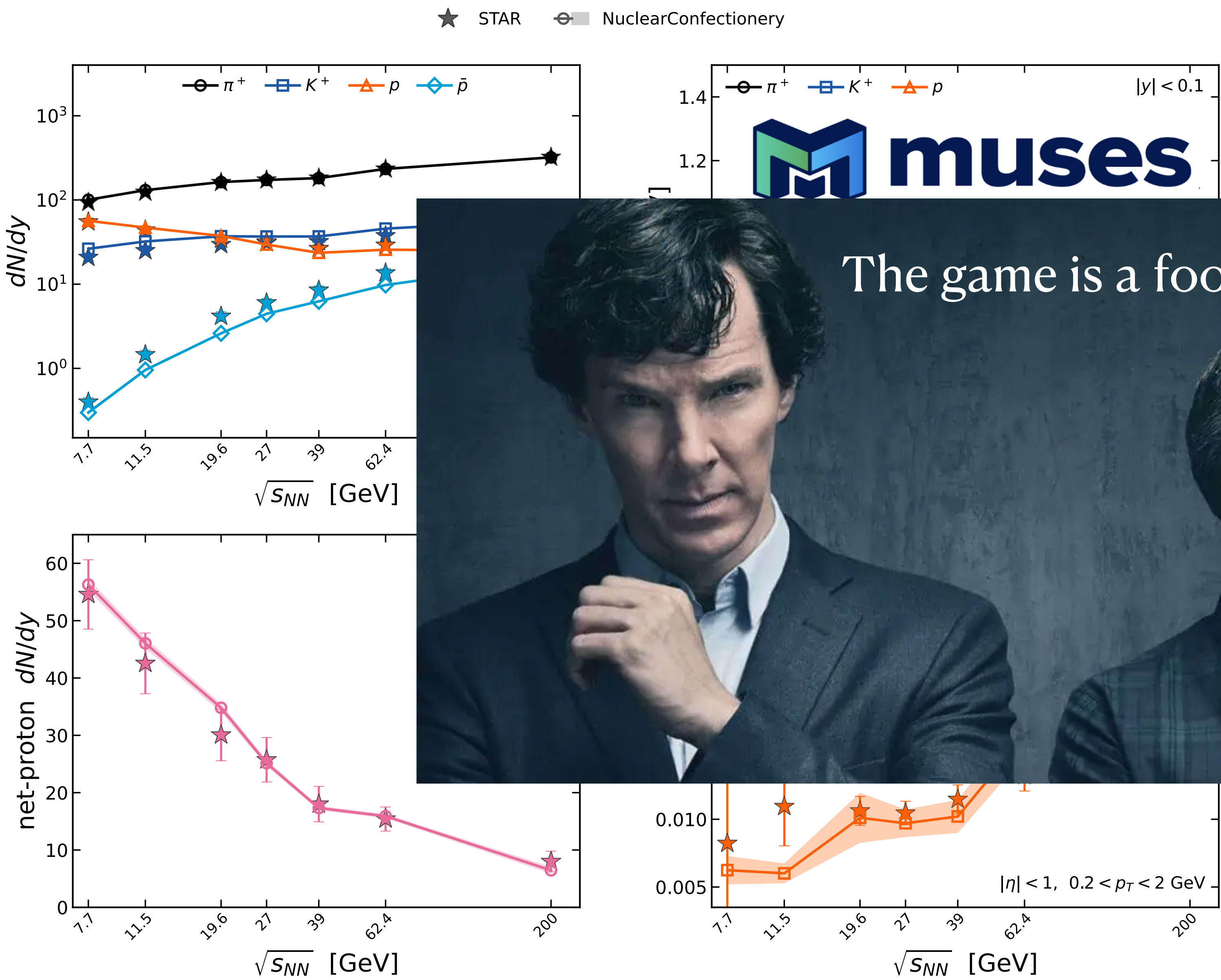
Large scale searches will require emulators and a Bayesian analysis



Caveat: equation of state is still missing strangeness and electric charge

[MUSES] to appear soon, AMPT+CCAKE+SMASH with Holography+HRG EOS

Simulations that probe the critical point



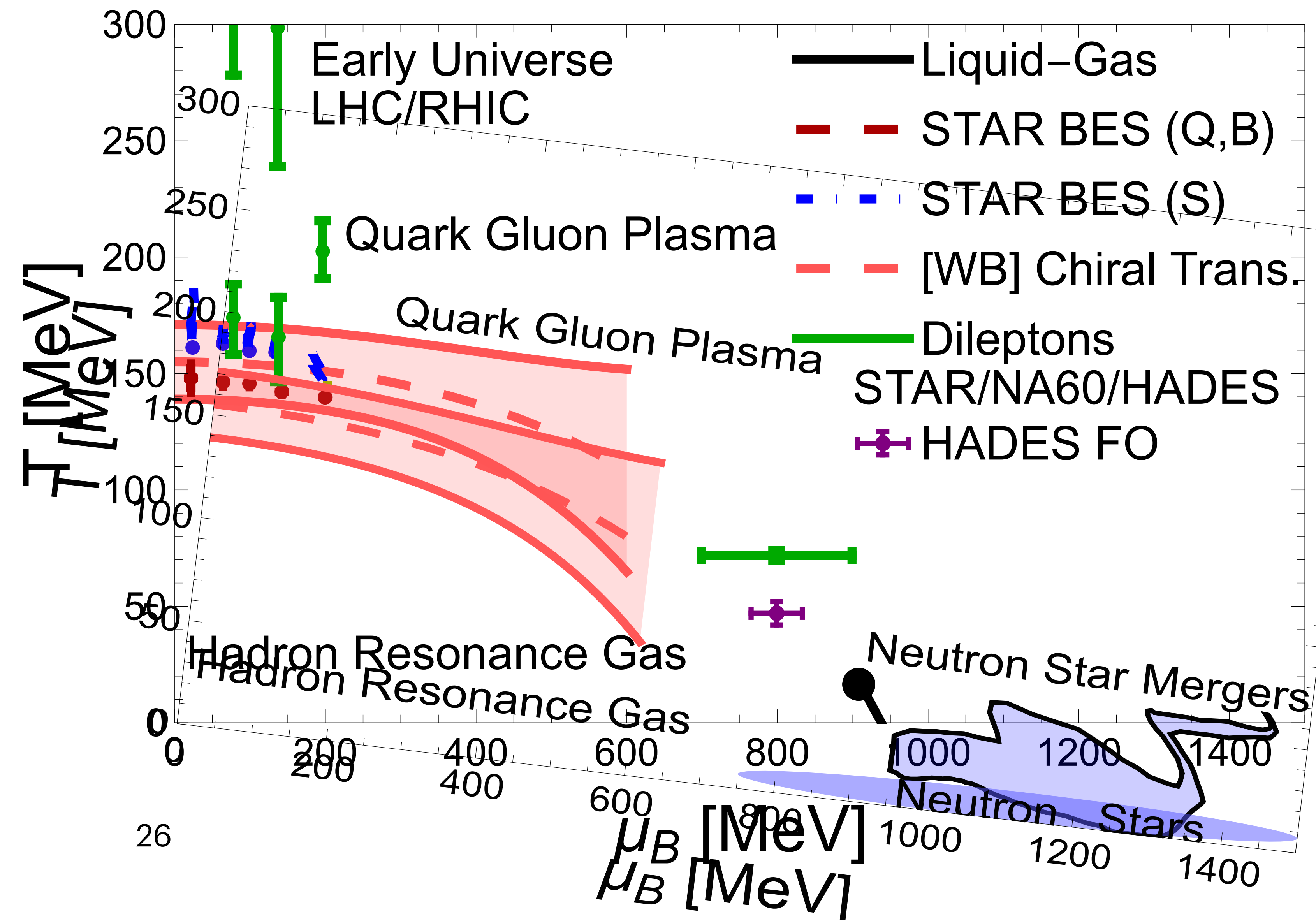
Simulations have a number of free parameters, must tune to the data before searches

The searches will require a Bayesian analysis

Equation of state is still unknown, strangeness and electric charge

[MUSES] to appear soon, AMPT+CCAKE+SMASH with Holography+HRG EOS

Consequences of criticality for neutron stars



Heavy-ion collisions

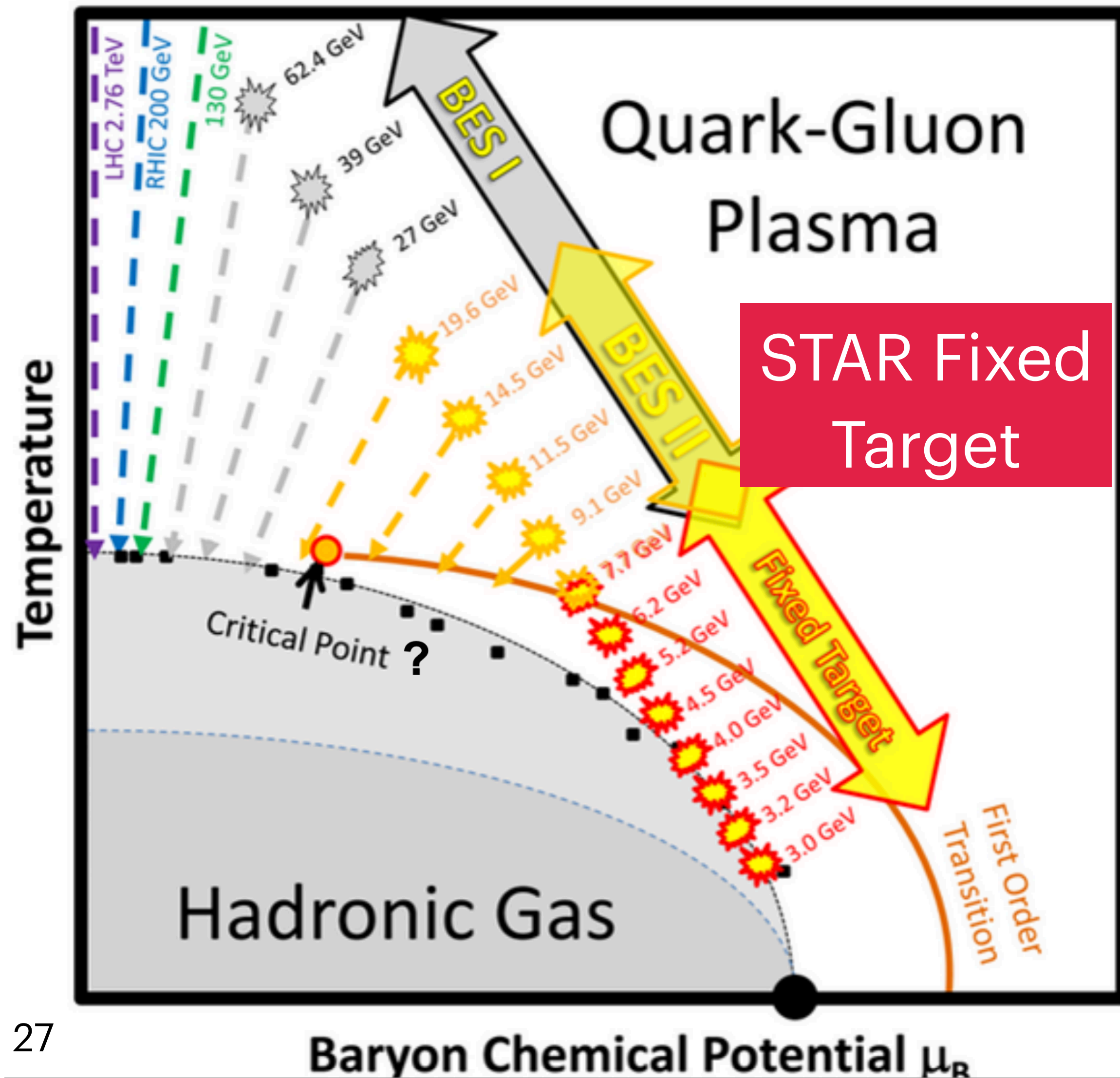
$$\frac{n_Q}{n_B} \sim [0.4, 0.5]$$

Neutron Stars

$$\frac{n_Q}{n_B} \sim [0.01, 0.2]$$

Caution: if/where a phase transition appears can change with n_Q/n_B

Answer soon: more data + theory are coming!



Ongoing discussions on a potential fixed-target experiment at the Electron-Ion Collider

Joint analyses with neutron stars may provide new insights!

Conclusions and Outlook

- Beam Energy Scan II data may be showing hints of a critical point in the fixed target regime, future FAIR data may place a decisive role here!
- Search is underway for the QCD critical point. Between upcoming data and theoretical improvements, we will likely know the answer in the years to come
 - **Theory development desperately needed to interpret results**
- HADES, NA61, FAIR, NICA can play a crucial role in understanding the (potential) critical point and first-order line
- Dense QCD matter: combined analyses with heavy-ion collisions, neutron stars, and gravitational waves.
 - **Care needed in understanding data from multiple sources, interdisciplinary collaborations key**

Legacy 1: Description of relativistic viscous fluids

Different methods to derive the equations of motion

Israel-Stewart

Annals Phys. 118 (1979) 341-372

Enforce 2nd law of thermo

$$\tau_\pi \pi^{\dot{\mu}\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \frac{\tau_\pi \dot{\beta}_\pi}{2\beta_\pi} \pi^{\mu\nu}$$

Israel-Stewart only

DNMR

Denicol, et al, *Phys.Rev.D* 85 (2012) 114047

Expansion in applicability of hydro

$$\tau_\pi \pi^{\dot{\mu}\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \frac{\tau_\pi}{2} \pi^{\mu\nu} \theta + \frac{\eta \delta_{\Pi\pi}}{2} \Pi \sigma^{\mu\nu}$$

Couples shear-bulk viscosity

BDNK

Bemfica et al, *Phys.Rev.D* 98 (2018) 10, 104064

1st-order, generalized frame

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}$$

Picks up acceleration terms

ADNH=IS+DNMR

Almaalol et al, *Phys.Rev.D* 111 (2025) 1, 014020

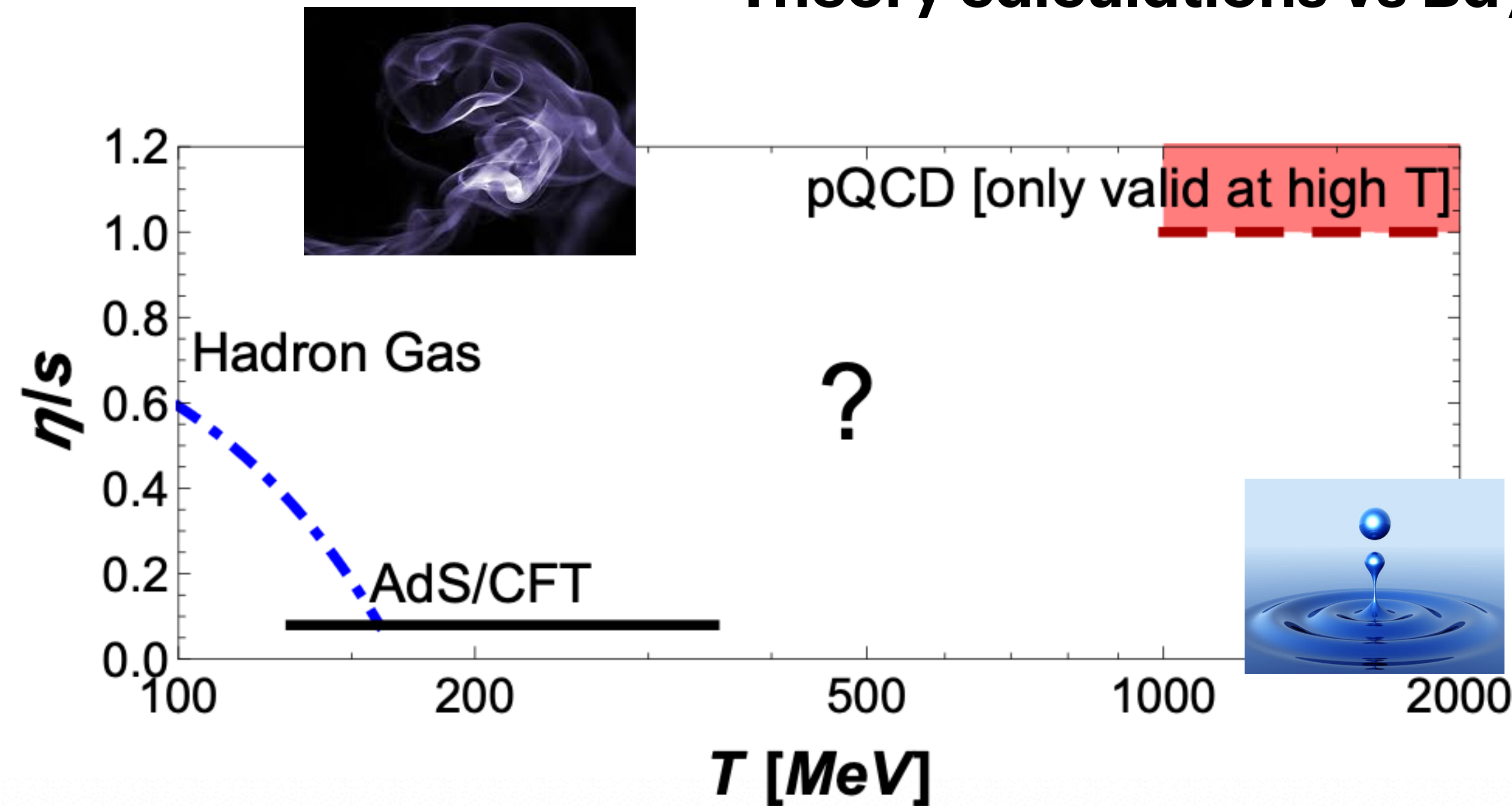
Recovers most DNMR terms from 2nd law method

$$\tau_\pi \pi^{\dot{\mu}\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \frac{\tau_\pi \dot{\beta}_\pi}{2\beta_\pi} \pi^{\mu\nu} + \frac{\tau_\pi}{2} \pi^{\mu\nu} \theta + \frac{\eta \delta_{\Pi\pi}}{2} \Pi \sigma^{\mu\nu}$$

Larger body of work, studying attractors (see original Heller Spalinski *PRL* 115 (2015) 7, 072501), applicability in different systems, finite densities, magnetic fields, coupled to general relativity etc

Shear viscosity over entropy density

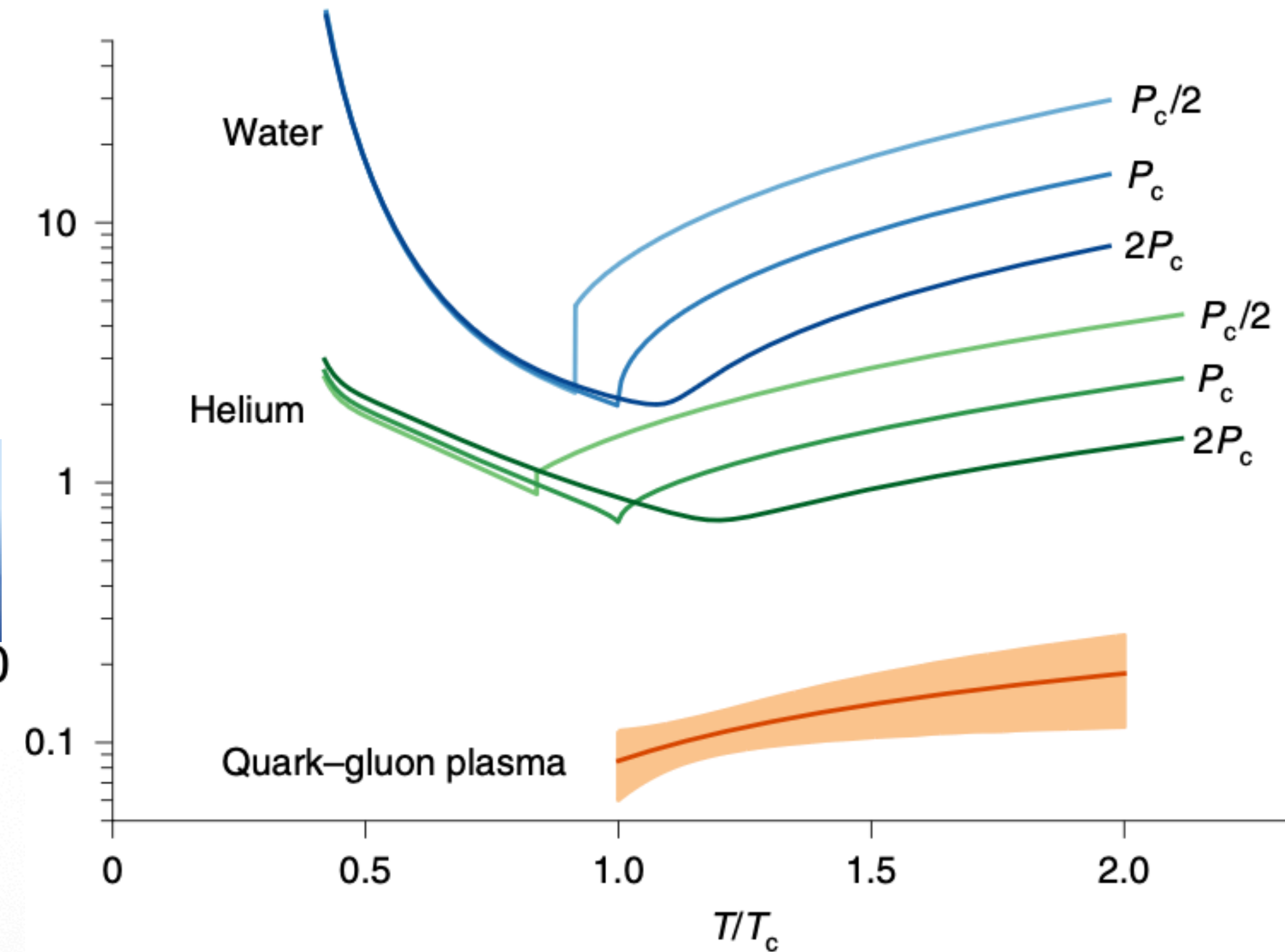
Theory calculations vs Bayesian posteriors



Hadron Gas: JNH et al, PRL103(2009)172302; PRC86(2012)024913

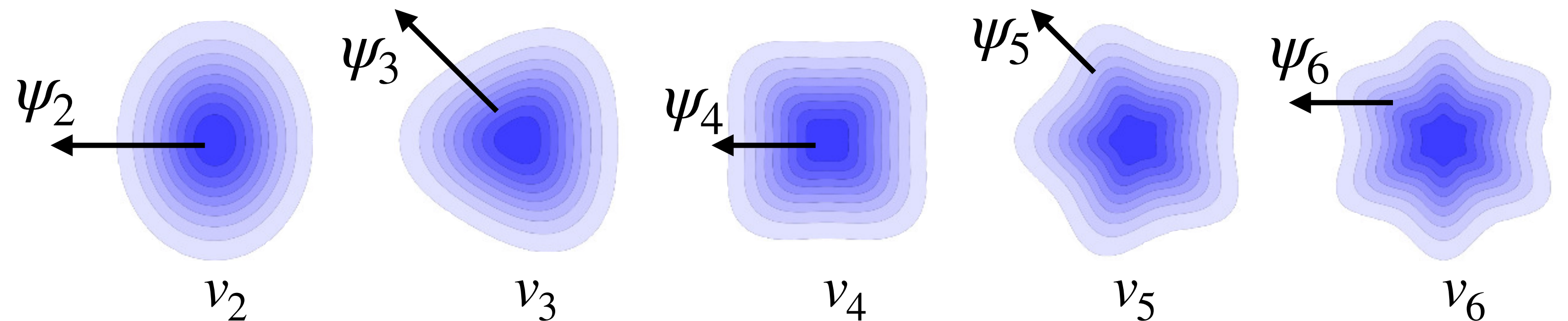
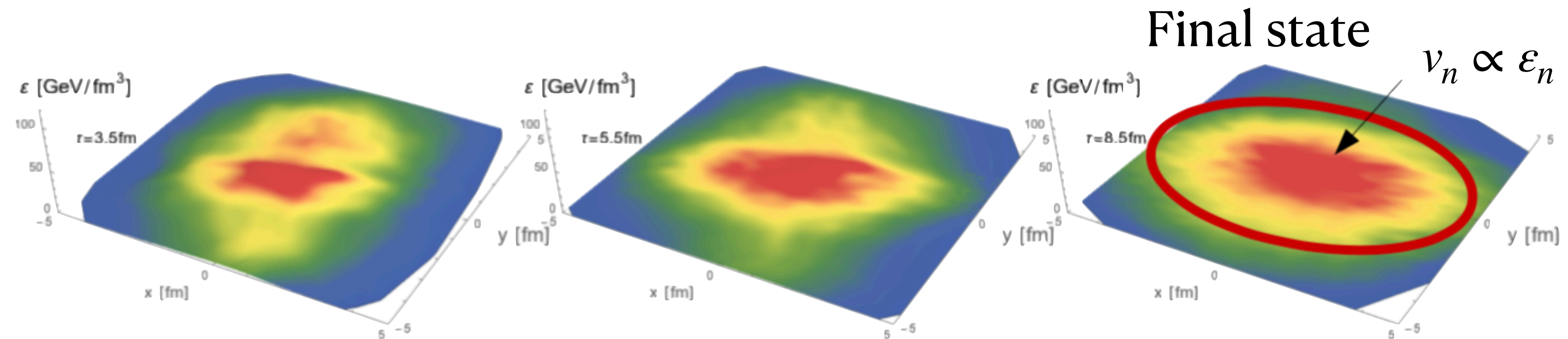
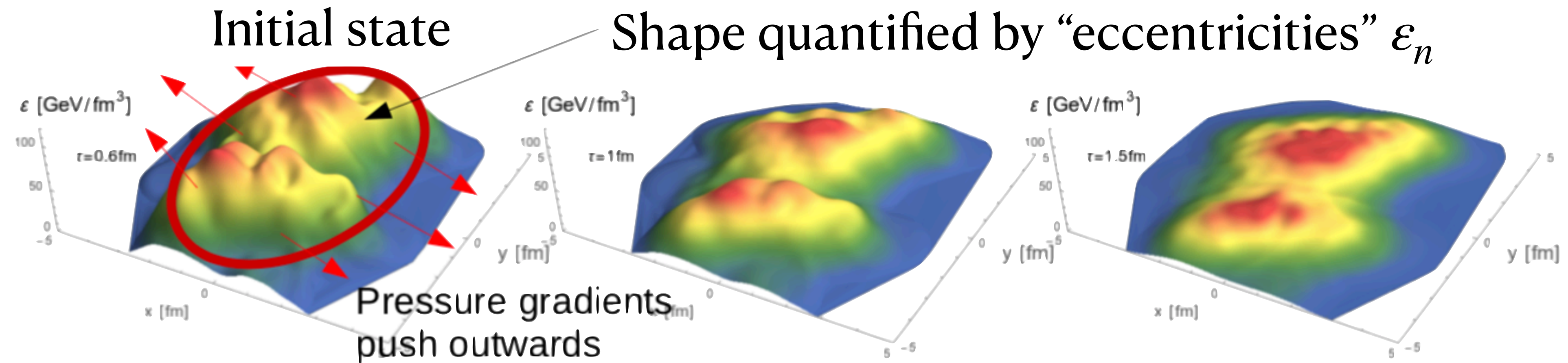
AdS/CFT: Kovtun, Son, Starinets PRL94(2005)111601

pQCD: Arnold, Moore, Yaffe JHEP 0011(2000)001 ; JHEP0305(2003)051



Bernhard et al, *Nature Phys.* 15 (2019) 11, 1113-1117

Flowing like a “good fluid”

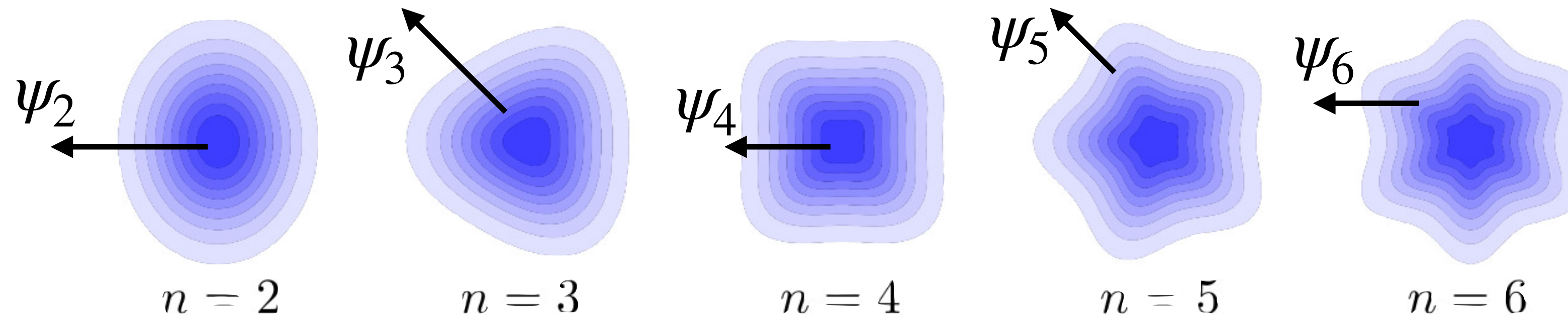


Fourier Decomposition:

Fourier Decomposition of final particle number

Collective flow

The distribution of particles can be written as a Fourier series



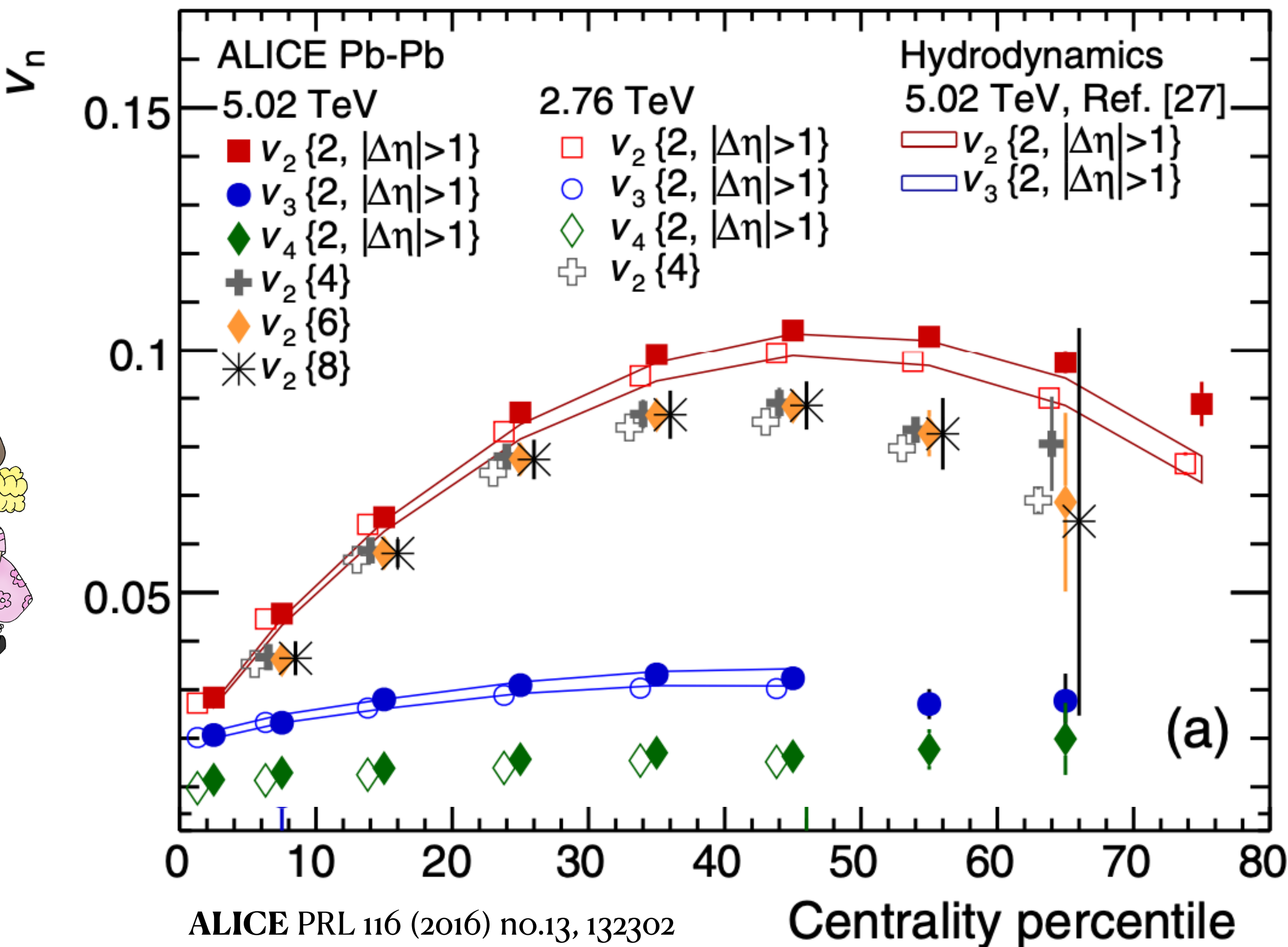
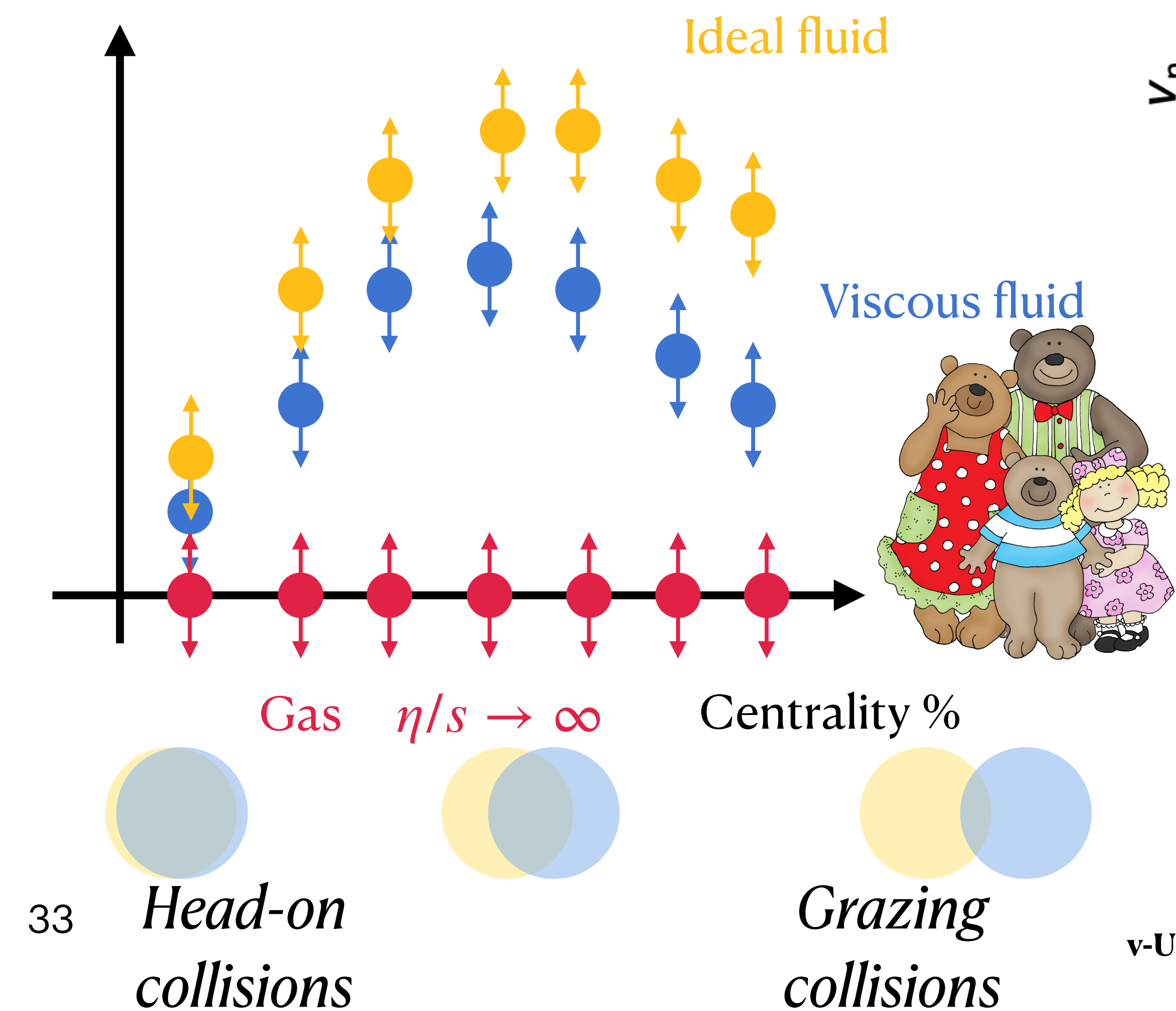
Collective flow: Flow harmonics, $v_n\{m\}$, are calculated by correlating $m = [2,4,6,8,\dots]$ particles $\rightarrow$ collective behavior

NOTE: V_n , $\mathcal{E}_n$ are vectors, v_n , ε_n are magnitudes, and Ψ_n , Φ_n are angles

Constraining shear viscosity with collective flow

Models constrained by RHIC, precisely
predicted LHC

Elliptical flow v_2



ALICE PRL 116 (2016) no.13, 132302
v-USPhydro (CCAKE) predictions: PRC93 (2016) no.3, 034912
EKRT predictions: PRC 93, 014912 (2016)