

Preparations for Quantum Computing in Hadron Physics

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September 11th, 2026

Based on Int.J.Mod.Phys.A **41** (2026) 06, 2630003

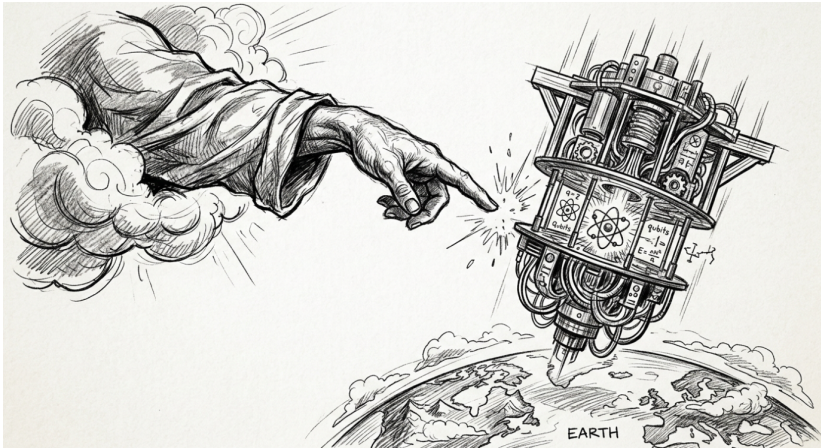


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Outline

- 1 Why quantum computing in hadron physics
- 2 Example problems (other authors)
- 3 Canonically quantized field theory
- 4 Fragmentation functions, and TMDs?
- 5 Particle Physics in Neutron Stars
- 6 Outlook

What would you do with a functional quantum computer?



Quantum Simulation Algorithm

- 1 Choose an encoding for the physical states (Here, particles, not fields)
- 2 Initiate the quantum computer memory to $|\psi(0)\rangle$
- 3 Decompose the unitary $U(t) = \exp(-itH/\hbar)$ into elementary gates and evolve $|\psi(t)\rangle = U(t)|\psi(0)\rangle$
- 4 Measure expectation values $\langle\psi(t)|\hat{O}|\psi(t)\rangle$

Minimization algorithm (more traditional many-body)

- 1 Formulate variational space $\{U(\theta_1, \theta_2, \dots, \theta_n)\psi_0\}$
- 2 Calculate $\langle H \rangle$ on quantum computer
- 3 Minimize over θ_i

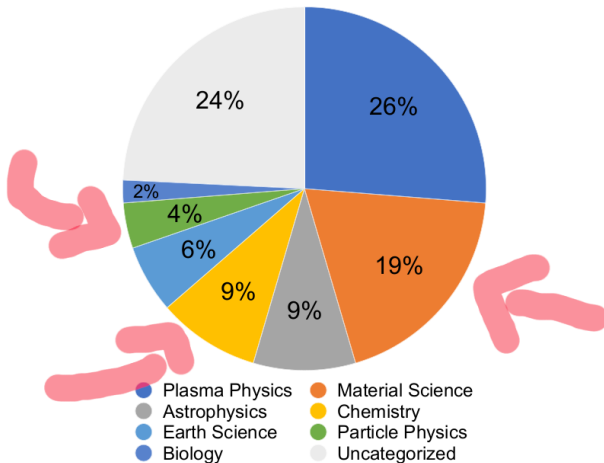
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1/3 of scientific computing might benefit (Los Alamos dixit)



A. Baertschi *et al.*, report LA-UR-26-24592 Los Alamos National Laboratory [\(Where is the nuclear slice?\)](#)

You could think of hadron spectroscopy...

Example: Triply heavy baryons
with QCD Cornell potential

$$V(r) = \frac{\alpha_s}{r} + \sigma r$$

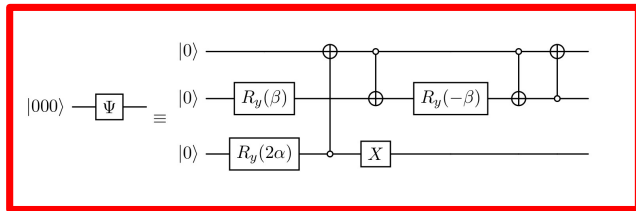
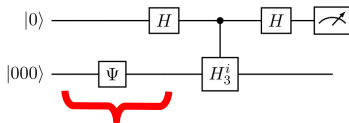


Eur.Phys.J.A 60 (2024) 216 with
Nicolás Martínez de Arenaza,

Baryon (composition)	$\Omega(bbb)$	$\Omega(bbc)$	$\Omega(bcc)$	$\Omega(ccc)$
This work	14270 ± 340	11210 ± 350	8100 ± 350	4940 ± 340
Variational pNRQCD	14700 ± 300	11400 ± 300	8150 ± 300	4900 ± 250
Coulomb variational	14370 ± 80	11190 ± 80	7980 ± 70	4760 ± 60
QCD sum rules	13280 ± 100	10460 ± 110	7443 ± 150	4670 ± 150
Quark counting	14760 ± 180	11480 ± 120	8200 ± 90	4925 ± 90
MIT bag model	14300	11200	8030	4790

With very few qubits (quantum computer = small diagonalizer)

State preparation & measurement of Hamiltonian

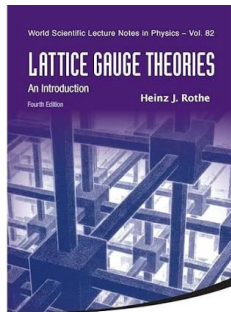


Simple calculations already working...

but Lattice can do it better (for now) at less cost! (forever?)

What do we need them for?

Wherever lattice can't go
perhaps quantum computers can



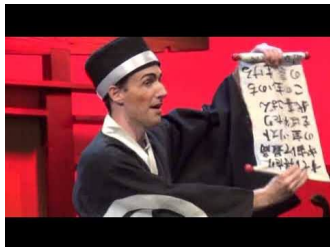
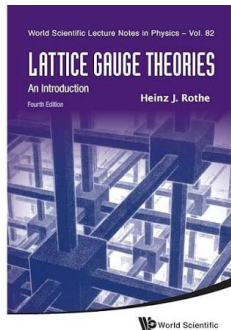
World Scientific

I've got a little list...

J. J. Galvez-Viruet, FJLE, Maria Gomez-Rocha Int.J.Mod.Phys.A 41 (2026) 06, 2630003

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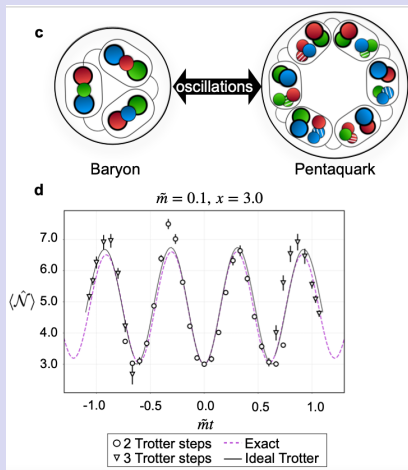


I've got a little list...

Outline

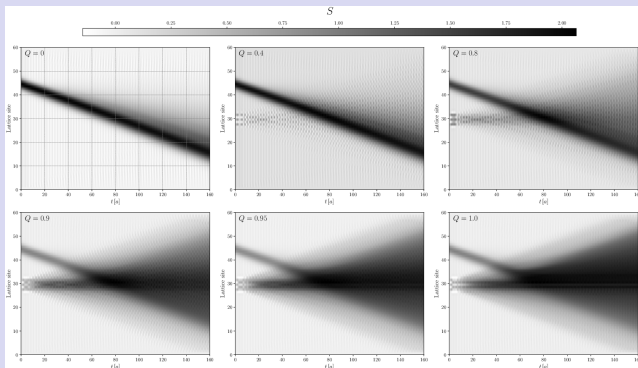
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Time evolution with H_{QCD}



Y. Y. Atas et al. Phys. Rev. Res. 5, 033184 (2023)

Jet energy loss in Schwinger model (1+1 QCD)



Shows S production for jet+medium, increasing coupling

J. Barata and E. Rico, arXiv:2502.17558 [hep-ph]

Also with the Schwinger model



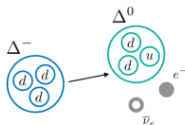
Marc Illa

beta decay

R. Farrell, I. Chernyshev, S. Powell, N. Zemlevskiy, M. S. 2209.1



$$H \approx H_{\text{QCD}} + H_{\text{lep}} + H_{\text{4-Fermi}}$$

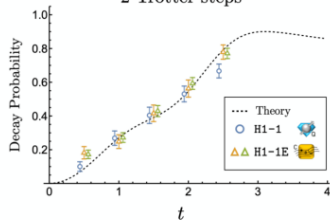


2 Trotter steps

$$H_{\text{4-Fermi}} \propto \frac{G}{\sqrt{2}} u^\dagger d e^\dagger \nu + \text{X} \cdot$$

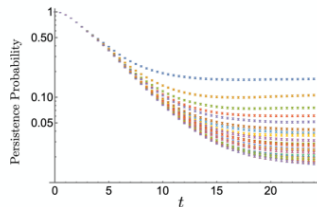
$\updownarrow$ "Valence" term

$$d \rightarrow u + e^- + \bar{\nu}$$

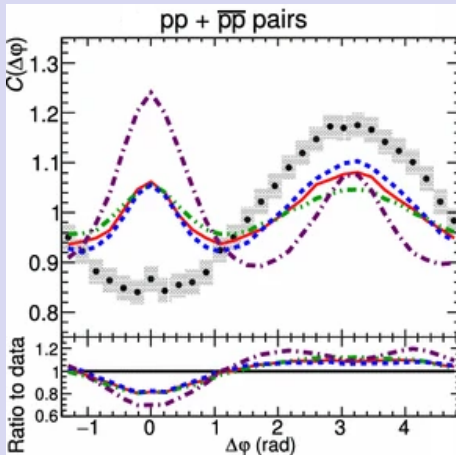


No exponential behavior

Not enough final states

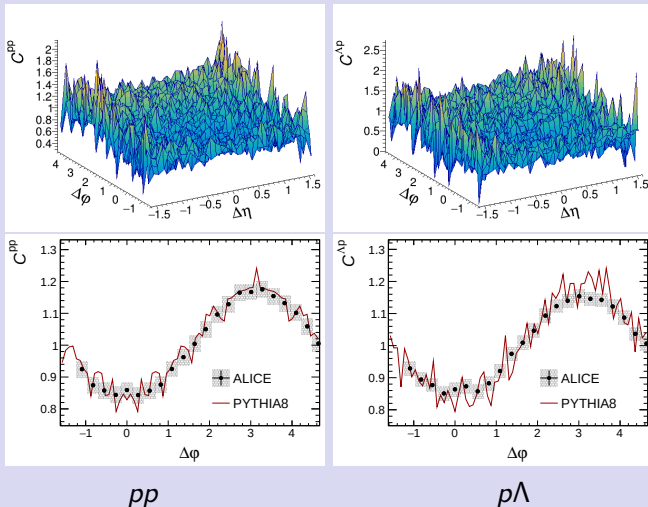


Monte Carlo generators use classical probability



Gross disagreement at $\varphi = 0$:
Pythia is missing baryon anticorrelations

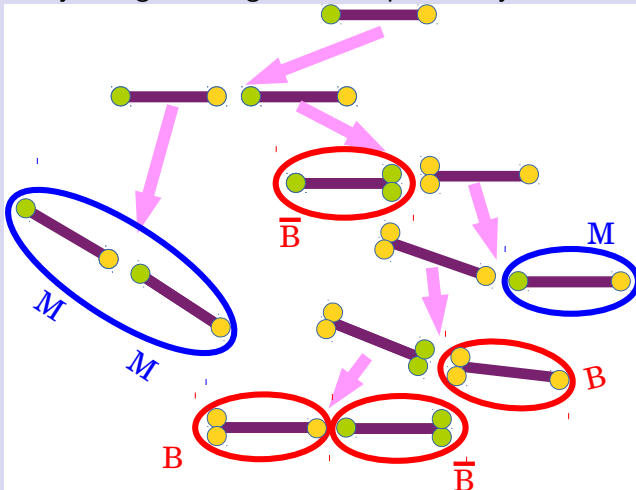
Ad hoc fix: One-baryon policy and All-baryon policy



Brings Pythia 8 in agreement with ALICE data

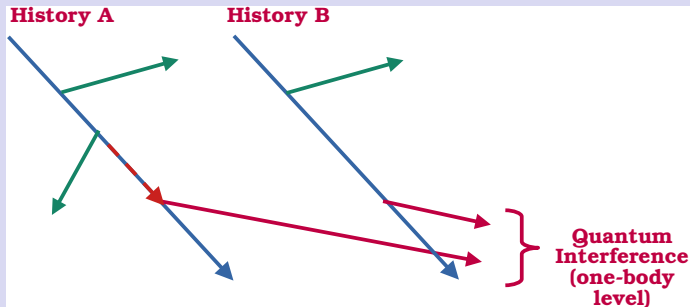
Rewrite parts of generators for a quantum chip?

Hadronization by string breaking: classical probability



- 1) Separate the baryons;
- 2) How about entanglement/Bell inequalities?

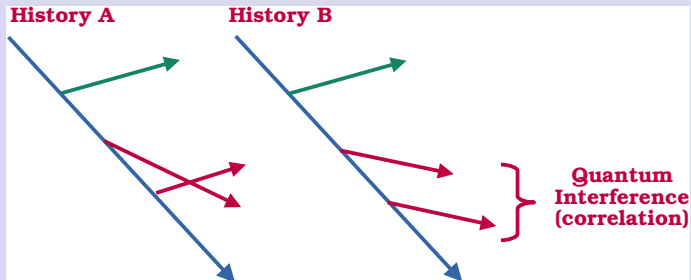
Interference of different histories



Incipient Quantum Computing efforts at one-body level

Masahito Yamazaki arxiv.org/pdf/2502.18059; B. Nachman *et al.* PRL 126, 062001 (2021)

Much to do...



- Two-body and higher correlations?
- Actually deploy QCD Hamiltonian?

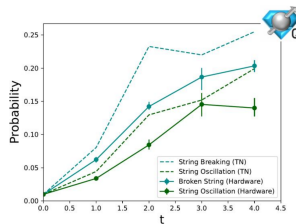
$SU(2)$ string-breaking simulation

$g = 1.1$: CLOSER TO THE CONTINUUM

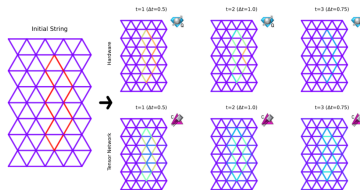
- Toward the continuum (small g), the electric vacuum is a poor approximation
- Variationally prepare the vacuum, minimizing $\langle \hat{H} \rangle$ (overlap ≈ 0.987 , 4×4):

$$|\psi(\theta)\rangle = \prod_p e^{-i\theta \hat{Y}_p} |0\rangle$$

- Adjoint string placed on this vacuum, then evolved
- Faster, breaking-dominated dynamics
- Electric energy drains into the magnetic term



$8 \times 8, g = 1.1$: breaking dominates (points: ibm_boston, dashed: MPS)



Link-energy evolution — hardware (top), MPS (bottom)

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Particle-register encoding at Complutense

Think Constructor & Destructor Methods of Object-Oriented Programming



Once you have the canonical operators...

Particle-register encoding at Complutense

Repeated application: entangled states with variable particle number



Particle into a “register” of several qubits



Particle into a “register” of several qubits

**Absence /
Presence**



Particle into a “register” of several qubits

**Absence /
Presence**

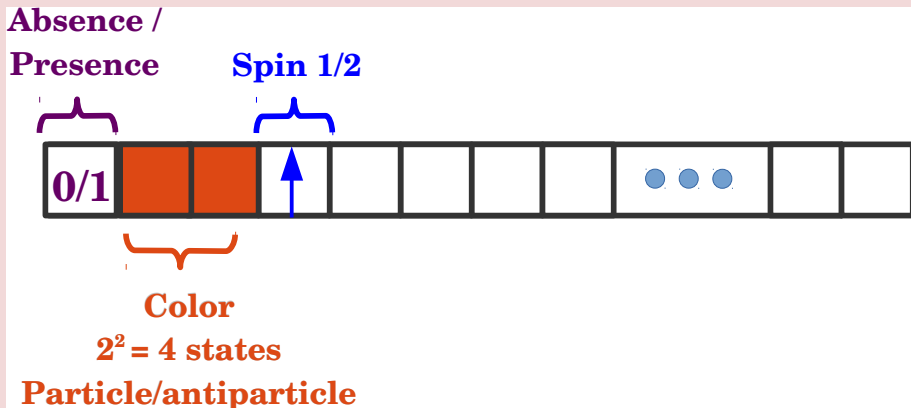


Color

$2^2 = 4$ states

Particle/antiparticle

Particle into a “register” of several qubits



Particle into a “register” of several qubits

**Absence /
Presence**

Spin 1/2



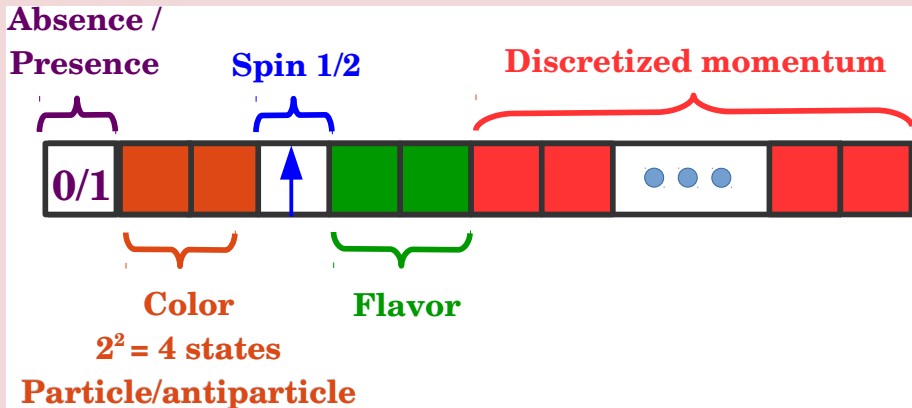
Color

2² = 4 states

Flavor

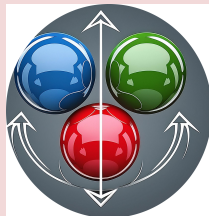
Particle/antiparticle

Momenta: N_p values $\rightarrow \log_2 N_p$ qubits



Particle into a “register” of several qubits

- 8^3 momentum grid $\implies$ 9 momentum qubits $\implies$ 15 qubits/quark
- IBM Eagle Chips with 127 qubits
- Already in business for quark model-size computations

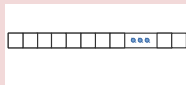


We would like $\left\{ b_{q_1}^{(n)}, b_{q_2}^{(n)\dagger} \right\} = \delta_{q_1, q_2}$

jth-Register implementation

$$b_{q,j}^{(n)\dagger} = \mathcal{A}_{j \leftarrow j-1} \cdot \mathbb{P}_0^{(n-j)} \otimes (\mathfrak{e}_{10} \otimes \mathfrak{s}_q^\dagger)_j \otimes \mathbb{P}_{j-1}^{(j-1)}$$

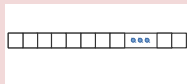
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Set / scrap operators

$$\mathfrak{S}_q^\dagger \quad / \quad \mathfrak{S}_q$$

set / scrap the quantum numbers

Apply them to

the first unoccupied / last occupied particle register

Control operators on the presence qubit

$$\mathfrak{C}_{00}|0\rangle = |0\rangle \text{ Empty} \rightarrow \text{Empty}$$

$$\mathfrak{C}_{11}|1\rangle = |1\rangle \text{ Full} \rightarrow \text{Full}$$

$$\mathfrak{C}_{10}|0\rangle = |1\rangle \text{ Empty} \rightarrow \text{Full}$$

$$\mathfrak{C}_{01}|1\rangle = |0\rangle \text{ Full} \rightarrow \text{Empty}$$

With usual computer
if ... then ...
else ... endif
switch ... case

Others zero

Wait... zero on a quantum computer?

Everything exponentiated later, $\mathbb{I}$ to the rescue!

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Projector over j occupied registers

$\mathbb{P}_j^{(n)}$ constructible from those control operators

$$\mathbb{P}_j^{(n)} = \left[(\dots)_n \otimes \dots \otimes (\mathfrak{C}_{00} \otimes \mathfrak{i})_{j+1} \right] \otimes \left[(\mathfrak{C}_{11} \otimes \mathfrak{i})_j \otimes \dots \otimes (\dots)_1 \right] ,$$

(Anti)symmetrize only the last added particle



j th-Register implementation

$$b_{q,j}^{(n)\dagger} = \underbrace{\mathcal{A}_{j \leftarrow j-1}}_{\text{Step antisymmetrizer}} \cdot \mathbb{P}_0^{(n-j)} \otimes (\mathfrak{e}_{10} \otimes \mathfrak{s}_q^\dagger)_j \otimes \mathbb{P}_{j-1}^{(j-1)}$$



Step (anti)symmetrizer

Example: add 1 particle to a memory already containing 1 particle:

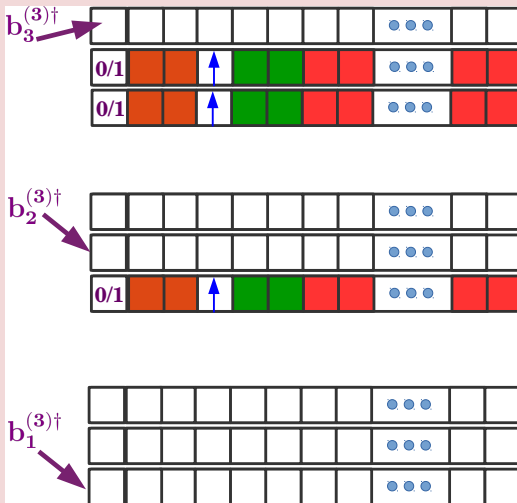
$$b_{q_1}^{(2)\dagger} = b_{q_1,1}^{(2)\dagger} + \underbrace{\frac{1}{\sqrt{2}} (\mathbb{I} \otimes \mathbb{I} - \mathcal{P}_{21})}_{\equiv \mathcal{A}_2} b_{q_1,2}^{(2)\dagger} .$$

More generally

$$\mathcal{A}_{n \leftarrow n-1} = \frac{1}{\sqrt{n}} (\mathbb{I}^{\otimes n} - \mathcal{P}_{n(n-1)} - \mathcal{P}_{n(n-2)} - \dots - \mathcal{P}_{n2} - \mathcal{P}_{n1})$$

All-register implementation

$$b_q^{(n)\dagger} = \sum_j b_{q,j}^{(n)\dagger}$$



Canonical commutation relations

We would like $\left\{ b_{q_1}^{(n)}, b_{q_2}^{(n)\dagger} \right\} = \delta_{q_1, q_2}$

Instead we get, with boundary condition $b^\dagger|n\rangle = 0$,

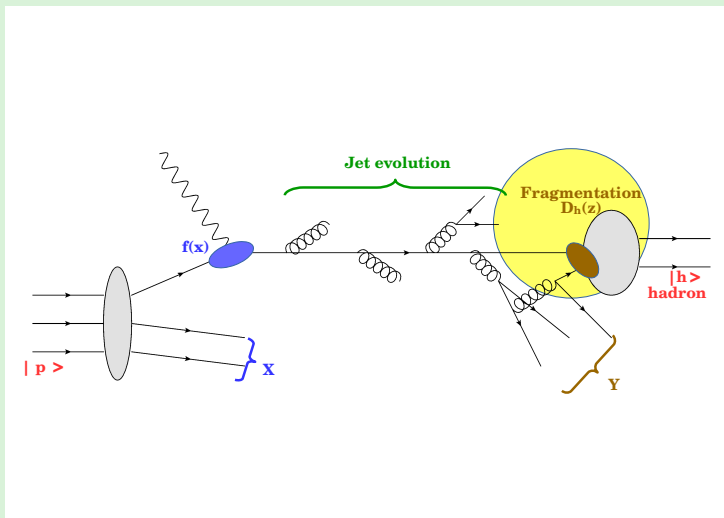
Boundary term @ full memory

$$\begin{aligned} \left\{ b_{q_1}^{(n)}, b_{q_2}^{(n)\dagger} \right\} &= \overbrace{\delta_{q_1, q_2} (\mathfrak{e}_{00} \otimes \mathbf{i})_n \otimes \mathbb{I}^{(n-1)}}^{\text{canonical up to } n-1 \text{ particles}} \\ &+ \underbrace{\mathcal{A}_{n \leftarrow n-1} \cdot \left(\mathfrak{e}_{11} \otimes \mathfrak{s}_{q_1}^\dagger \mathfrak{s}_{q_2} \right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{A}_{n \leftarrow n-1}}_{\text{finite, full memory with } n} \end{aligned}$$

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Fragmentation functions: from quarks to hadrons

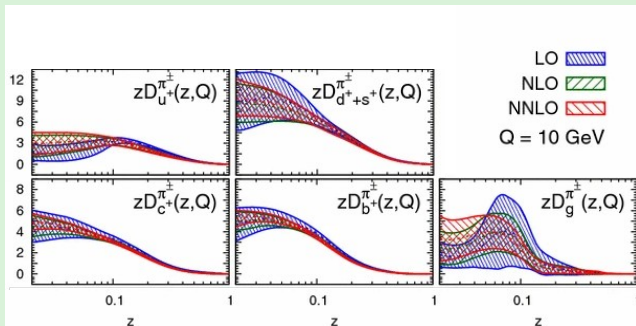


Fragmentation functions: evolution with light-front time

$$D_{(0) \ h/j}(z) = \frac{\text{Tr}}{4} \left(\gamma^- \times \frac{z}{z^{2\epsilon}} \sum_X \int \frac{dx^+}{2\pi} e^{i(zp^-)x^+} \right. \\ \left. \langle 0 | \psi_j^{(0)}(x) | h, X \rangle_{\text{out}} \langle h, X | \bar{\psi}_j^{(0)}(0) | 0 \rangle \right)$$

$x := (x^+, 0, 0_\perp)$ Not the simplest lattice computation

Fragmentation functions: normally extracted from data



NNPDF: EPJC 77 516 (2017)

Light-front QCD Hamiltonian, $A^+ = 0$ gauge

$$\begin{aligned}
 P^- = \int dx_+ d^2x_\perp \left\{ \frac{1}{2} \left(\bar{\tilde{\Psi}} \gamma^+ \frac{m^2 - \nabla_\perp^2}{i\partial^+} \tilde{\Psi} - \tilde{A}_r^\mu \nabla_\perp^2 \tilde{A}_{r,\mu} \right) \right. \\
 + g \tilde{J}_r^\mu \tilde{A}_{r,\mu} + \frac{g^2}{4} \tilde{B}_r^{\mu\nu} \tilde{B}_{r,\mu\nu} - \frac{g^2}{2} \tilde{J}_r^\mu \frac{1}{(\partial^+)^2} \tilde{J}_r^\mu \\
 \left. - \frac{g^2}{2} i \bar{\tilde{\Psi}} \gamma^\mu T_r \tilde{A}_{r,\mu} \frac{\gamma^+}{\partial^+} (\gamma^\nu T_s \tilde{A}_{s,\nu} \tilde{\Psi}) \right\} .
 \end{aligned}$$

Light-front QCD Hamiltonian

n	Sector	1 q $\bar{q}$	2 gg	3 q $\bar{q}$ g	4 q $\bar{q}$ q $\bar{q}$	5 gg g	6 q $\bar{q}$ gg	7 q $\bar{q}$ q $\bar{q}$ g	8 q $\bar{q}$ q $\bar{q}$ q $\bar{q}$	9 gg gg	10 q $\bar{q}$ gg g	11 q $\bar{q}$ q $\bar{q}$ gg	12 q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ g	13 q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ q $\bar{q}$
1	q $\bar{q}$					.		.	.	.	.	.	.	.
2	gg				.			.	.		.	.	.	.
3	q $\bar{q}$ g								.	.		.	.	.
4	q $\bar{q}$ q $\bar{q}$		.			.				.	.		.	.
5	gg g	.			.			.	.			.	.	.
6	q $\bar{q}$ gg								.				.	.
7	q $\bar{q}$ q $\bar{q}$ g	.	.			.				.				.
8	q $\bar{q}$ q $\bar{q}$ q $\bar{q}$	.	.	.		.	.			.	.			
9	gg gg	.		.	.			.	.			.	.	.
10	q $\bar{q}$ gg g	.	.		.				.				.	.
11	q $\bar{q}$ q $\bar{q}$ gg	.	.	.		.				.				.
12	q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ g	.	.	.	.	.				.	.			
13	q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ q $\bar{q}$	.	.	.	.	.	.			.	.	.		

Brodsky, Pauli & Pinsky, Phys. Rep. 301 (1998) 299

September 11th, 2026 Based on

Example extraction: $c \rightarrow J/\psi$ (known from NRQCD)

$$|J/\psi\rangle = \sum \delta_{c_q c_{\bar{q}}} \frac{\frac{1}{\sqrt{C}} x^{\beta/2} (z-x)^{\alpha/2} \vec{\sigma}_{ij}}{\sqrt{x(z-x)}} |x \ i \ c_q, (z-x) \ j \ c_{\bar{q}}\rangle ,$$

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Drastic simplifications from being able to simulate 30 qbits only:

- Very limited number of particles in the jet ($2 \times c, \bar{c}, g$)
- No renormalization group flow implemented
- Small k_+ grid, no zero modes ~~$k^+ = 0$~~
- Only one value of $k_\perp$

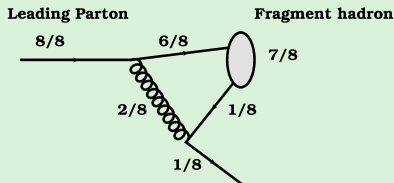
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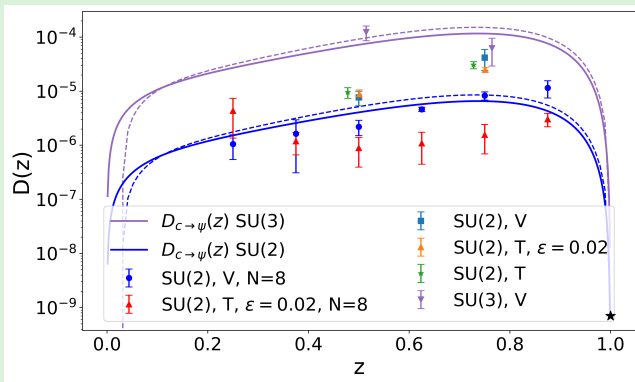
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So just a proof of concept, but give us 10^9 gates on 10^3 qubits and we talk again...



Example extraction: $c \rightarrow J/\psi$ as our twopence



J.J. Gálvez-Viruet et al. 2510.18869 [hep-ph] accepted at PRD

Thinking about: Transverse Momentum extensions

e.g. SIDIS

$$\frac{d\sigma}{dx dy dz d^2P_{hT}} = \frac{2\pi\alpha^2}{Q^2} (1 + (1-y)^2) \sum_q e_q^2 \times$$
$$\int d^2k_T d^2p_T \delta^{(2)}\left(k_T - p_T - \frac{P_{hT}}{z}\right) f_{q/H}(x, k_T) D_{h/q}(z, p_T)$$

$$f_{q/H}(x, k_T) = \int \frac{dz^- d^2z_T}{(2\pi)^3} e^{ixP^+z^- - ik_T \cdot z_T} \langle P | \bar{\psi}(0) \Gamma \mathcal{W}[0, z] \psi(z) | P \rangle \Big|_{\underbrace{z^+ = 0}}.$$

(Timelike separation unnatural in lattice QCD –Euclidean formulation!)

Outline

- 1 Why quantum computing in hadron physics
- 2 Example problems (other authors)
- 3 Canonically quantized field theory
- 4 Fragmentation functions, and TMDs?
- 5 Particle Physics in Neutron Stars**
- 6 Outlook

If your goal is to test gravity...

Einstein's equations in General Relativity

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Well tested in vacuo. And in matter? Neutron stars!

$$T_{\mu\nu} \rightarrow \text{diag}(\epsilon, P, P, P) + \underbrace{\tau_{ij}}_{\text{transport}} + \underbrace{\tau_{ij}^{NH}}_{\text{crystalline}} + \dots$$

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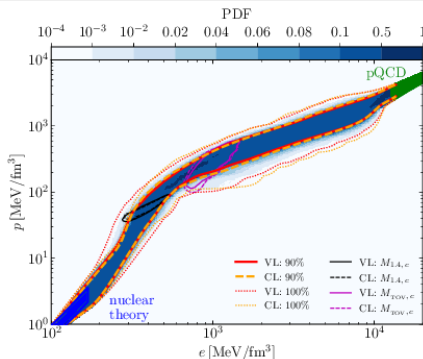
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$$T_{\mu\nu} \rightarrow \text{diag}(\epsilon, P, P, P) + \underbrace{\tau_{ij}}_{\text{transport}} + \underbrace{\tau_{ij}^{NH}}_{\text{crystalline}} + \dots$$

How about extracting the EoS from astrophysics?

Available data

- 1 Maximum neutron star mass
- 2 A few radius estimates
- 3 Upper bound on tidal deformability from aLIGO

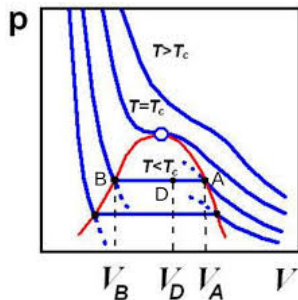


Tighter band

General Relativity assumed!

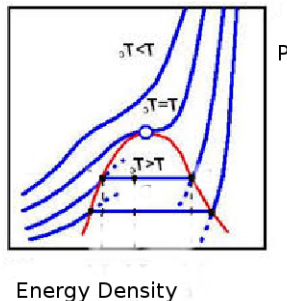
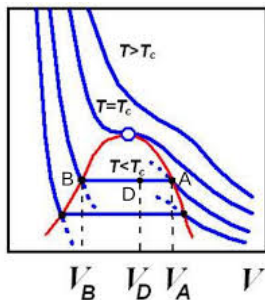
Inappropriate to test
Modified Gravity

Equation of state of neutron-star matter

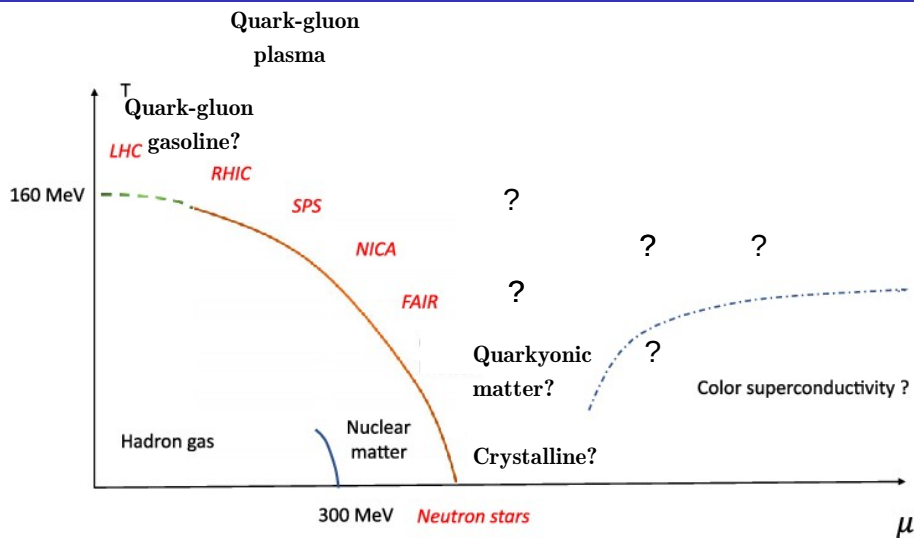


Equation of state of neutron-star matter

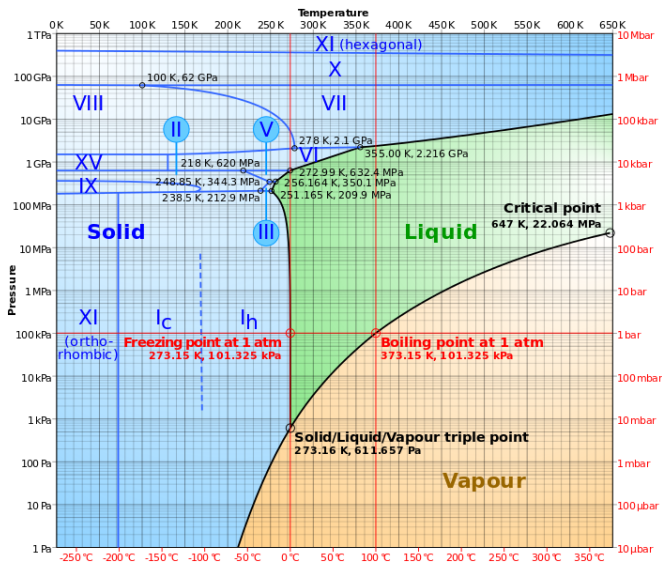
p



Microscopic physics: QCD phase diagram



Why would it be simpler than water...



From minimum of $\langle H \rangle$ to EoS

- Lattice gauge theory stumbles:
finite chemical potential μ (sign problem)
- Put the QCD EoS on a Quantum Computer

$$p(T \rightarrow 0, \mu, \varepsilon) \sim \varepsilon \left[\frac{\partial}{\partial \varepsilon} \left(\langle \hat{H} \rangle - \mu \langle \hat{N} \rangle \right)_{min} \right]_{T \rightarrow 0, \mu}$$

No difficulty in introducing a chemical potential

With usual number operator

$$\hat{N}_Q = \int \frac{d^3 \vec{k}}{(2\pi)^{3/2}} \sum_{s,c,f} \left(b_{\vec{k},s,c,f}^\dagger b_{\vec{k},s,c,f} - d_{\vec{k},s,c,f}^\dagger d_{\vec{k},s,c,f} \right) .$$

From minimum of $\langle H \rangle$ to EoS

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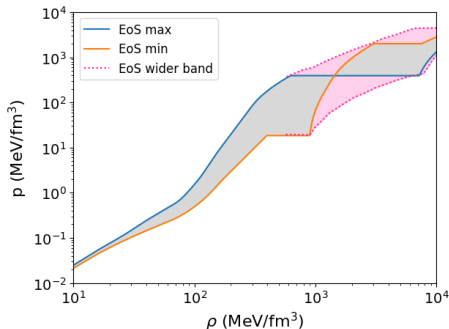
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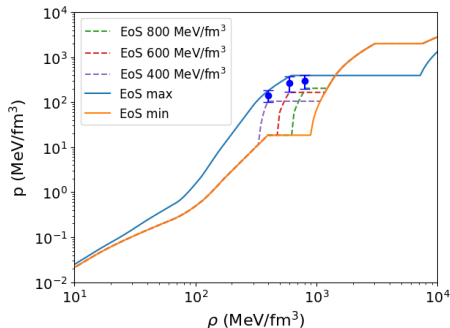
Equation of state of neutron-star matter



- Low density: Chiral PT
 - High density: pQCD
 - Flat edges: stability ($\dot{P} \geq 0$)
 - Steep edges: causality ($\dot{P} \leq 1$)
- +Integral constraints in the $n(\mu)$ plane from Komoltsev & Kurkela

E. Lope-Oter and A. Wojnar, JCAP 02 (2024) 017

Equation of state of neutron-star matter



Can we put a point from nonperturbative QCD in this graph?

Have the ingredients to code a many-body theory...

Gauge fixing

- Neutron star: privileged frame $\implies$ Weyl gauge
 $A_0 = 0$
- No 0-eq. of motion $\implies$ need to impose Gauss's law
 $\mathcal{G}|\psi\rangle = 0 \quad \forall \psi \text{ physical}$

In terms of canonical operators,

$$\begin{aligned} \mathcal{G}^a(x) = & - \int [k] E_k \sum_i k^i a_{kia}^\dagger e^{-ikx} - g f^{abc} \int [k, p] E_k \sum_i [(i a_{kib}^\dagger a_{pic}^\dagger e^{-i(k+p)x} + (i a_{kib}^\dagger a_{pic} e^{-i(k-p)x}) \\ & + g \int [k, p] \sum_{\lambda c f \lambda' c' f'} T_{cc'}^a [(u_{\lambda f}(k)^\dagger v_{\lambda' f'}(p) e^{-i(k+p)x} b_{k\lambda c f}^\dagger d_{p\lambda' c' f'}^\dagger) \\ & + u_{\lambda f}^\dagger(k) u_{\lambda' f'}(p) e^{-i(k-p)x} b_{k\lambda c f}^\dagger b_{p\lambda' c' f'} - v_{\lambda f}^\dagger(k) v_{\lambda' f'}(p) e^{-i(p-k)x} d_{p\lambda' c' f'}^\dagger d_{k\lambda c f}] + h.c. \end{aligned}$$

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We actually proceed by minimization

$$\delta \left(\langle H - \mu N - \frac{\text{penalty}}{\Lambda^2} \mathcal{G}^a \mathcal{G}^a \rangle_\psi \right) \rightarrow 0$$

Quark part

$$\begin{aligned} \hat{H}_{quark} = & \int d^3 \vec{k} \, k^0 \sum_{s,c,f} \left(b_{\vec{k},s,f,c}^\dagger b_{\vec{k},s,f,c} + d_{\vec{k},s,f,c}^\dagger d_{\vec{k},s,f,c} \right) + \\ & + g \int d^3 \vec{x} \int d\vec{k} \, d\vec{p} \, d\vec{q} \, (T^a)_{cc'} \sum_{s,s'} \sum_{\sigma,c,f} \left\{ a_{\vec{k},\sigma,a}^\dagger e^{i(\vec{k}+\vec{p}+\vec{q}) \cdot \vec{x}} \left(\bar{u}_{\vec{q},s,c} \gamma^\sigma u_{-\vec{p},s',c'} b_{\vec{q},s,c,f}^\dagger b_{-\vec{p},s',c',f} + \right. \right. \\ & + \bar{u}_{\vec{q},s,c} \gamma^\sigma v_{\vec{p},s',c'} b_{\vec{q},s,c,f}^\dagger d_{\vec{p},s',c',f}^\dagger + \bar{v}_{-\vec{q},s,c} \gamma^\sigma v_{-\vec{p},s',c'} d_{-\vec{q},s,c,f}^\dagger b_{-\vec{p},s',c',f}^\dagger + \\ & \left. \left. \bar{v}_{-\vec{q},s,c} \gamma^\sigma v_{\vec{p},s',c'} d_{\vec{p},s',c',f}^\dagger d_{-\vec{q},s,c,f} \right) + h.c. \right\} + \text{constant} . \end{aligned}$$

Gluon part

$$\begin{aligned}
 H_{gluon} = & \int d^3\vec{k} \sum_{\sigma,\rho,a} \left\{ \left(k^0 \delta^{\sigma\rho} - \frac{1}{2} \frac{k^\sigma k^\rho}{k^0} \right) a_{\vec{k},\sigma,a}^\dagger a_{\vec{k},\rho,a} - \frac{1}{4} \frac{k^\sigma k^\rho}{k^0} \left(a_{\vec{k},\sigma,a}^\dagger a_{-\vec{k},\rho,a}^\dagger + h.c. \right) \right\} + \\
 & + igf^{abc} \int d^3\vec{x} \int d\vec{k} d\vec{q} d\vec{p} \sum_{\sigma,\rho} k^\sigma \left\{ e^{-i(\vec{k}+\vec{q}+\vec{p})\cdot\vec{x}} \left(a_{\vec{k},\sigma,c}^\dagger a_{\vec{q},\rho,a}^\dagger a_{\vec{p},\rho,b}^\dagger + a_{\vec{q},\rho,a}^\dagger a_{\vec{p},\rho,b}^\dagger a_{-\vec{k},\sigma,c}^\dagger \right. \right. \\
 & \left. \left. - a_{\vec{k},\sigma,c}^\dagger a_{\vec{q},\rho,a}^\dagger a_{-\vec{p},\rho,b}^\dagger - a_{\vec{k},\sigma,c}^\dagger a_{\vec{p},\rho,b}^\dagger a_{-\vec{q},\rho,a}^\dagger \right) + h.c. \right\} + \\
 & + \frac{1}{4} g^2 f^{abc} f^{aef} \int d^3\vec{x} \int d\vec{k} d\vec{q} d\vec{p} d\vec{r} \sum_{\sigma,\rho} \left\{ e^{-i(\vec{k}+\vec{q}+\vec{p}+\vec{r})\cdot\vec{x}} \left(a_{\vec{k},\sigma,b}^\dagger a_{\vec{q},\rho,e}^\dagger a_{\vec{p},\rho,c}^\dagger a_{\vec{r},\rho,f}^\dagger \right. \right. \\
 & - a_{\vec{k},\sigma,b}^\dagger a_{\vec{q},\rho,e}^\dagger a_{\vec{r},\rho,f}^\dagger a_{-\vec{p},\rho,c}^\dagger - a_{\vec{q},\rho,e}^\dagger a_{\vec{q},\rho,c}^\dagger a_{\vec{r},\rho,f}^\dagger a_{-\vec{k},\sigma,b}^\dagger - a_{\vec{k},\sigma,b}^\dagger a_{\vec{p},\rho,c}^\dagger a_{\vec{r},\rho,f}^\dagger a_{-\vec{q},\rho,e}^\dagger \\
 & + a_{\vec{k},\sigma,b}^\dagger a_{\vec{p},\rho,c}^\dagger a_{\vec{q},\rho,e}^\dagger a_{-\vec{r},\rho,f}^\dagger + a_{\vec{k},\sigma,b}^\dagger a_{\vec{q},\rho,e}^\dagger a_{-\vec{p},\rho,c}^\dagger a_{-\vec{r},\rho,f}^\dagger + a_{\vec{q},\rho,e}^\dagger a_{\vec{p},\rho,c}^\dagger a_{-\vec{k},\sigma,b}^\dagger a_{-\vec{r},\rho,f}^\dagger \\
 & \left. \left. + a_{\vec{q},\rho,e}^\dagger a_{\vec{r},\rho,f}^\dagger a_{-\vec{k},\sigma,b}^\dagger a_{-\vec{p},\rho,c}^\dagger \right) + h.c. \right\} + \text{constant} \quad ,
 \end{aligned}$$

Busy students...



Need higher-level objects, gate-level coding is for superduper nerds.

Busy students...

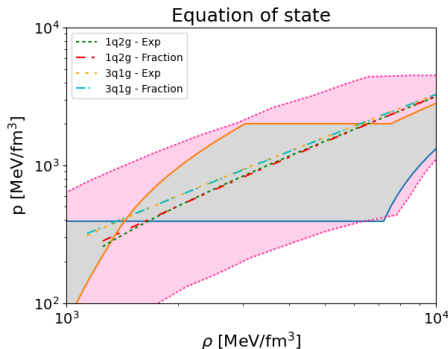


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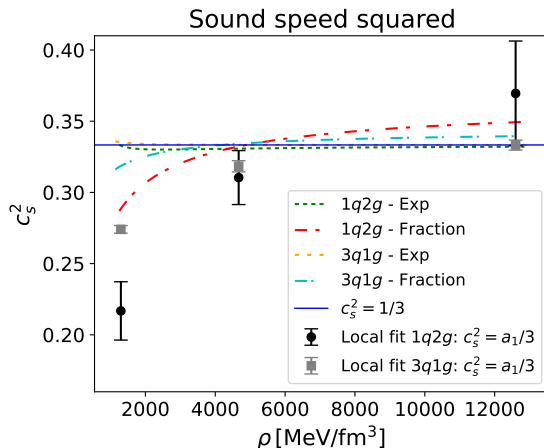
Example: Few-body variational wf + chemical potential

$$|\Psi_1(\theta)\rangle = \cos(\theta_1)|\Psi^{3q}\rangle + \sin(\theta_1) e^{i\theta_2} |\Psi^{3q1g}\rangle$$

Subtract $H \rightarrow H' = H - H_0 - (h_0 + h_2 g^2)\Lambda$
and match to pQCD at $\mu = 2.6$ GeV



Numbers immaterial for now: just demonstrate that this might well work!



Numbers immaterial for now: just demonstrate that this might well work!

2608.06515, submitted to Phys. Rev. C

Analyzing results phase

- Quantum advantage? Only a quantum computer can do it!
- No quantum computer can run this yet (Noise!)
- On classical simulators thereof, $\#$ qubits ≤ 30
- Simulate 3 – 4 particles with few momenta each

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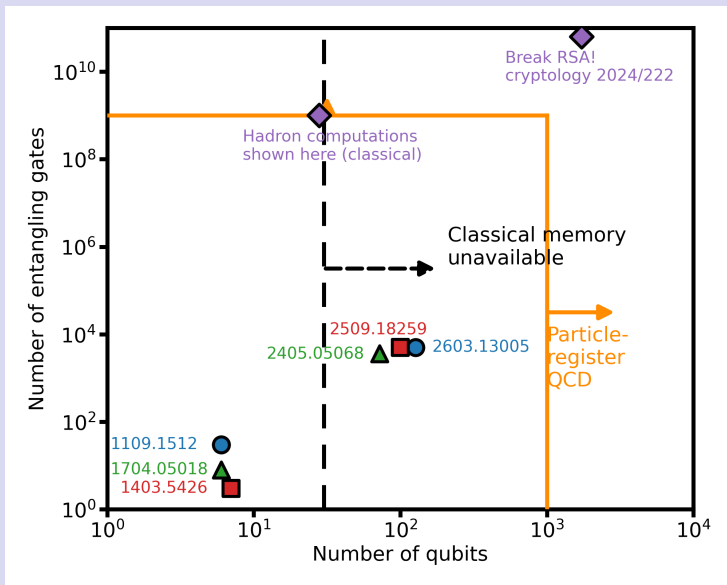
Gate costs: polynomial in n particles and N_p momenta

Operator	Costs	
	CNOT & single-qubit	Order oracle
$\mathcal{U}_{11}$	$\mathcal{O}(nN_p)$	None
$\mathcal{U}_{22}$	$\mathcal{O}(n^2 N_p^3 \log_2^2 N_p)$	None
$\mathcal{U}_{10}$	$\mathcal{O}(N_p \log_2^2 N_p)$	$\mathcal{O}(n^3 N_p)$
$\mathcal{U}_{21}$	$\mathcal{O}(n_b n_f^2 N_p^2 \log_2^2 N_p)$	$\mathcal{O}(n_b^3 N_p)$
$\mathcal{U}'_{21}$	$\mathcal{O}(n_b n_f N_p^3 \log_2 N_p)$	$\mathcal{O}(n_b n_f^3 N_p^2)$

Conclusions

- Niche problems in hadron physics resisting lattice:
Quantum Compute them!
- Number of qubits adequate; error rate and decoherence time, not yet
- @ UCM: investigating hadron structure and quark/hadron matter
- Canonical gluon quantization (not lattice link) route

So how far along are quantum computers?



Acknowledgments

Funded by research grant PID2022-137003NB-I00 from spanish MCIN/AEI/10.13039/501100011033/ and EU FEDER



Test Calculation $\langle \phi | \mathcal{G} q_{11}^1 | \phi \rangle$, a piece of the Gauss law

mode = (k_z, λ, c, f)	classical simulator	'ibm_sherbrooke' (quantum)
$(-2, -1/2, 2, 1)$	0.0263	-0.00400
$(-2, -1/2, 2, 2)$	0.000786	-0.00408
$(-1, -1/2, 2, 1)$	0.00174	-0.0000480
$(-1, -1/2, 2, 2)$	-0.000466	-0.00221
$(1, -1/2, 2, 1)$	-0.000635	0.00214
$(1, -1/2, 2, 2)$	0.000456	-0.00808
$(2, -1/2, 2, 1)$	0.000465	0.00621
$(2, -1/2, 2, 2)$	-0.00000575	-0.00931

ibm_sherbrooke probably calculated noise

Depth needed for this one element: 5k-11k layers

Trustworthy depth @ IBM, 2024-25: 28 2-qubit layers

Comparison of UCM's particle-register encoding to Jordan-Wigner

Encode a momentum set $\{p_1, \dots, p_{N_p}\}$:

Jordan-Wigner: Each qubit a momentum. Up to N_p particles

$$\overbrace{|\downarrow\downarrow\downarrow \dots \downarrow\rangle}^{N_p} = |\Omega\rangle \quad |\uparrow\downarrow\uparrow \dots \downarrow\rangle = |p_1 p_3\rangle \quad |\uparrow\uparrow\uparrow \dots \uparrow\rangle = |p_1 \dots p_{N_p}\rangle$$

requires $\mathcal{O}(N_p)$ qubits

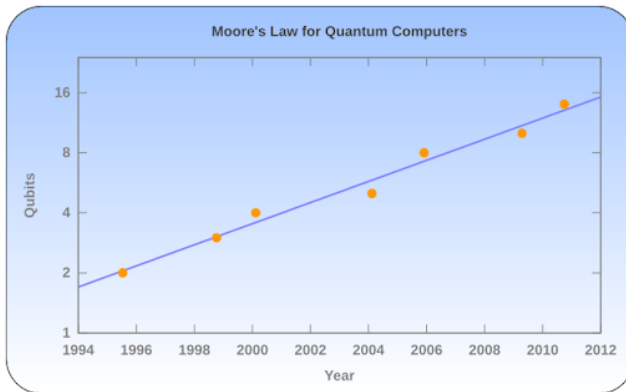
Register-Particle: Each binary number a momentum. One register per particle

$$|\downarrow\rangle \overbrace{|\downarrow\downarrow \dots \downarrow\rangle}^{\log_2 N_p} = |\Omega\rangle \quad |\uparrow\rangle |\uparrow\downarrow\uparrow \dots \downarrow\rangle = |p_3\rangle \quad |\uparrow\rangle |\uparrow\uparrow\uparrow \dots \uparrow\rangle = |p_{N_p}\rangle$$

requires $\mathcal{O}(n \log_2 N_p)$ qubits if n particles

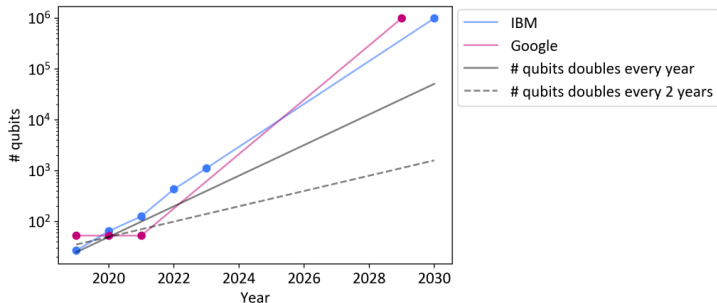
If n small $\Rightarrow$ **RP** encoding preferable

Moore's (Rose's) law working for a long time



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Moore's (Rose's) law and more



<https://www.quantum.amsterdam/>

And let's not forget

- $2^{\text{\#bits}}$ = size of a representable **number**
- $2^{\text{\#qubits}}$ = size of basis for a **linear space**

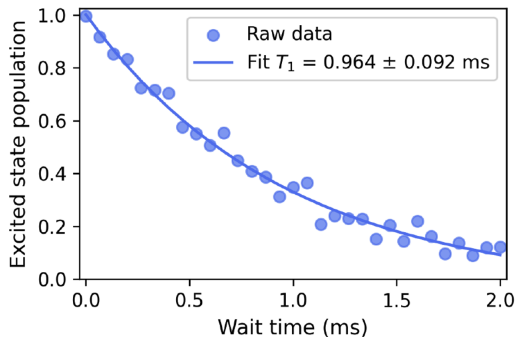
We will need circuits many thousands of layers deep...

- T. Rudolph on photon devices: 10^{-5} done, 10^{-8} achievable, APL Photonics 2, 030901 (2017)
- Quantinuum (trapped ions) claims error rate 10^{-5} - 10^{-10} by 2029.
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Decoherence time



- Decoherence time hitting the millisecond
- 10^3 gates in that time

Milestone by IQM company