

Gravitational form factors of pions, kaons and nucleons from dispersion relations

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Bing Wu, X.-K. Dong, M.-L. Du, FKG, B.-S. Zou, [Deciphering the mechanism of \$J/\psi\$ -nucleon scattering](#), Fund. Res. 5 (2025) 2531;

Xiong-Hui Cao, FKG, Q.-Z. Li, D.-L. Yao, [Precise determination of nucleon gravitational form factors](#), Nature Commun. 16 (2025) 6979;

Xiong-Hui Cao, FKG, Q.-Z. Li, D.-L. Yao, [Gravitational form factors of pions, kaons and nucleons from dispersion relations](#), Eur. Phys. J. ST 235 (2026) 2121.

Gravitational form factors

- Electro-magnetic form factors: matrix elements of em current R. Hofstadter, R. McAllister (1955); R. Hofstadter (1956)

□ Probes distributions of electric charge, magnetic moment densities, etc.

- Gravitational form factors: matrix elements of energy-momentum tensor

$$\left\langle N(p') \left| \hat{T}^{\mu\nu}(0) \right| N(p) \right\rangle = \frac{1}{4m_N} \bar{u}(p') \left[A(t) P^\mu P^\nu + J(t) \left(i P^{\{\mu} \sigma^{\nu\} \rho} \Delta_\rho \right) + D(t) (\Delta^\mu \Delta^\nu - t g^{\mu\nu}) \right] u(p)$$

□ Trace anomaly: origin of mass

□ Fundamental properties of the nucleon

➤ Energy/mass:

$$m_N = \int d^3r T^{00}(r), \quad A(0) = 1$$

➤ Spin:

$$J^i = \epsilon^{ijk} \int d^3r r^j T^{0k}(r), \quad J(0) = 1/2$$

➤ D-term:

$$D = -\frac{m_N}{2} \int d^3r \left(r^i r^j - \frac{1}{3} \delta^{ij} \right) T^{ij}(r), \quad D(0) : \text{“last global unknown property”}$$

Forces inside hadron, mechanical picture of quark confinement?

M. Polyakov, PLB 555 (2003) 57;
M. Polyakov, P. Schweitzer, IJMPA 33 (2018) 1830025;
X. Ji, G.A. Miller, C. Yang, Research 9 (2026) 1369

Remarks on J/ψ photoproduction and $J/\psi N$ scattering

- Near-threshold J/ψ photoprod. was proposed to probe nucleon GFFs. Main mechanism for this reaction?

□ Gluon exchange?

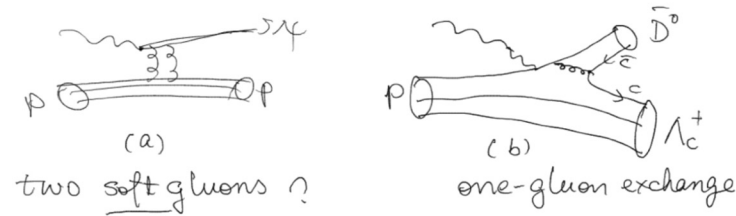
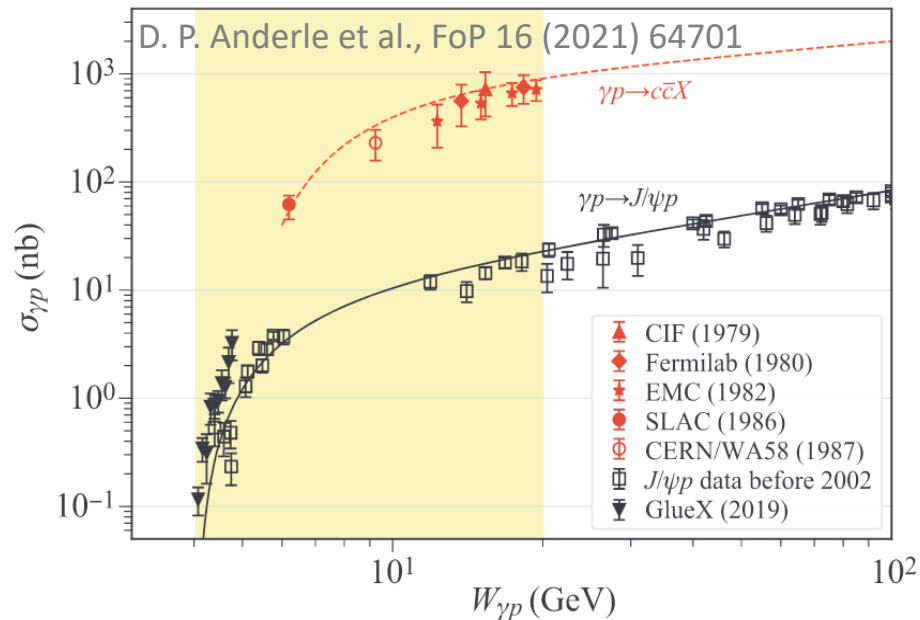
□ Coupled-channel mechanism?

➤ hinder extraction of gluonic matrix element in the regime

➤ if open-charm channels are produced with larger rates

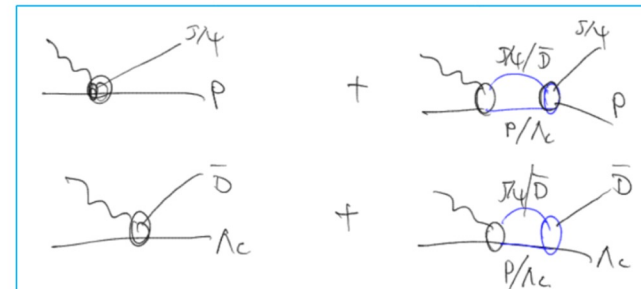
✧ Thresholds close to $J/\psi N$ threshold

✧ $\sigma_{c\bar{c}}$ inclusive $\gg \sigma_{J/\psi}$



Naïve estimate:

$$\sigma(\Lambda_c \bar{D}) \gg \sigma(J/\psi p)$$



For J/ψ near-threshold production, the $\Lambda_c \bar{D}$ channel is expected to be important!

Consequence: a cusp at the $\Lambda_c \bar{D}$ threshold

$$\begin{cases} \Lambda_c^+ + \bar{D}^- : 2286 + 1865 = 4151 \text{ MeV} \\ J/\psi + p : 3097 + 938 = 4035 \text{ MeV} \\ \Lambda_c^0 + B^+ : 5620 + 5280 = 10900 \text{ MeV} \\ \Upsilon + p : 9460 + 938 = 10398 \text{ MeV} \end{cases}$$

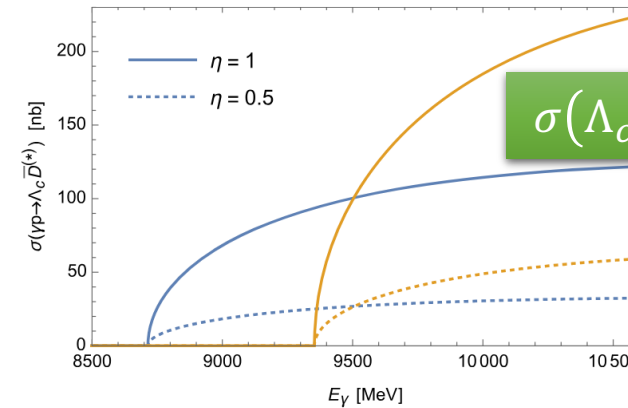
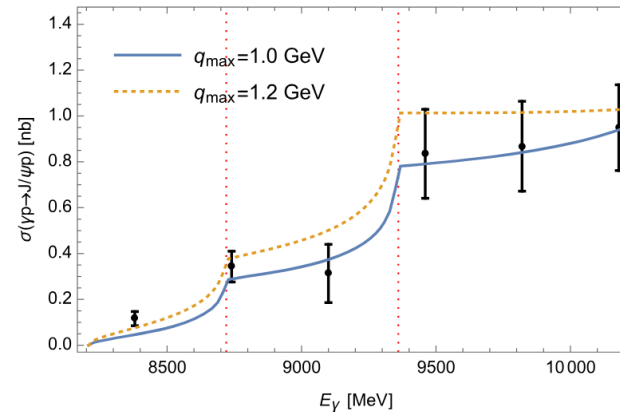
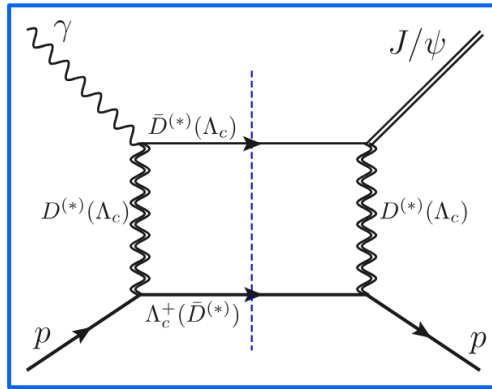
Remarks on J/ψ photoproduction and $J/\psi N$ scattering

- Coupled-channel mechanism [M.-L. Du et al., EPJC 80 \(2020\) 1053](#)

□ **Unitarity:** $J/\psi p \rightarrow J/\psi p$ enters coupled-channel scattering ($J/\psi p, \Lambda_c \bar{D}^{(*)}, \Sigma_c^{(*)} \bar{D}^{(*)}$) w/o VMD, but cannot be singled out

□ Simple model estimate: consider $\Lambda_c \bar{D}^{(*)}$ channels

➤ Charmed meson/baryon exchanges at 1-loop



Measure the open-charm production!

$$\sigma(\Lambda_c \bar{D}^{(*)}) \gg \sigma(J/\psi p)$$

➤ Threshold cusps (1-loop result is not robust)

✧ Needs to be analyzed with unitarity

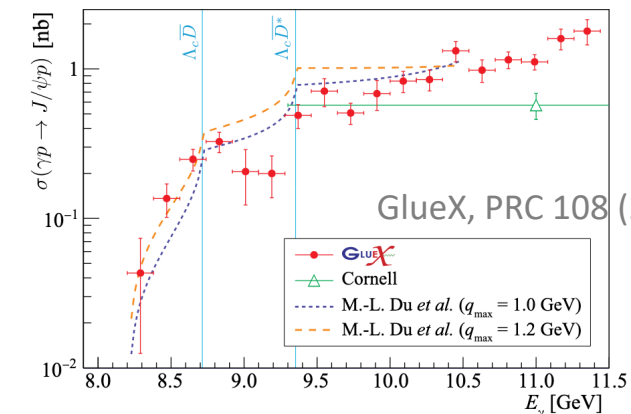
For such analyses, see, JPAC, PRD 108 (2023) 054018; X. Zhang, EPJC 85 (2025) 1120

✧ Nontrivial cusp only for attractive interaction

[X.-K. Dong, FKG, B.-S. Zou, PRL 126 \(2021\) 152001](#)

VMD questionable for heavy quarkonium

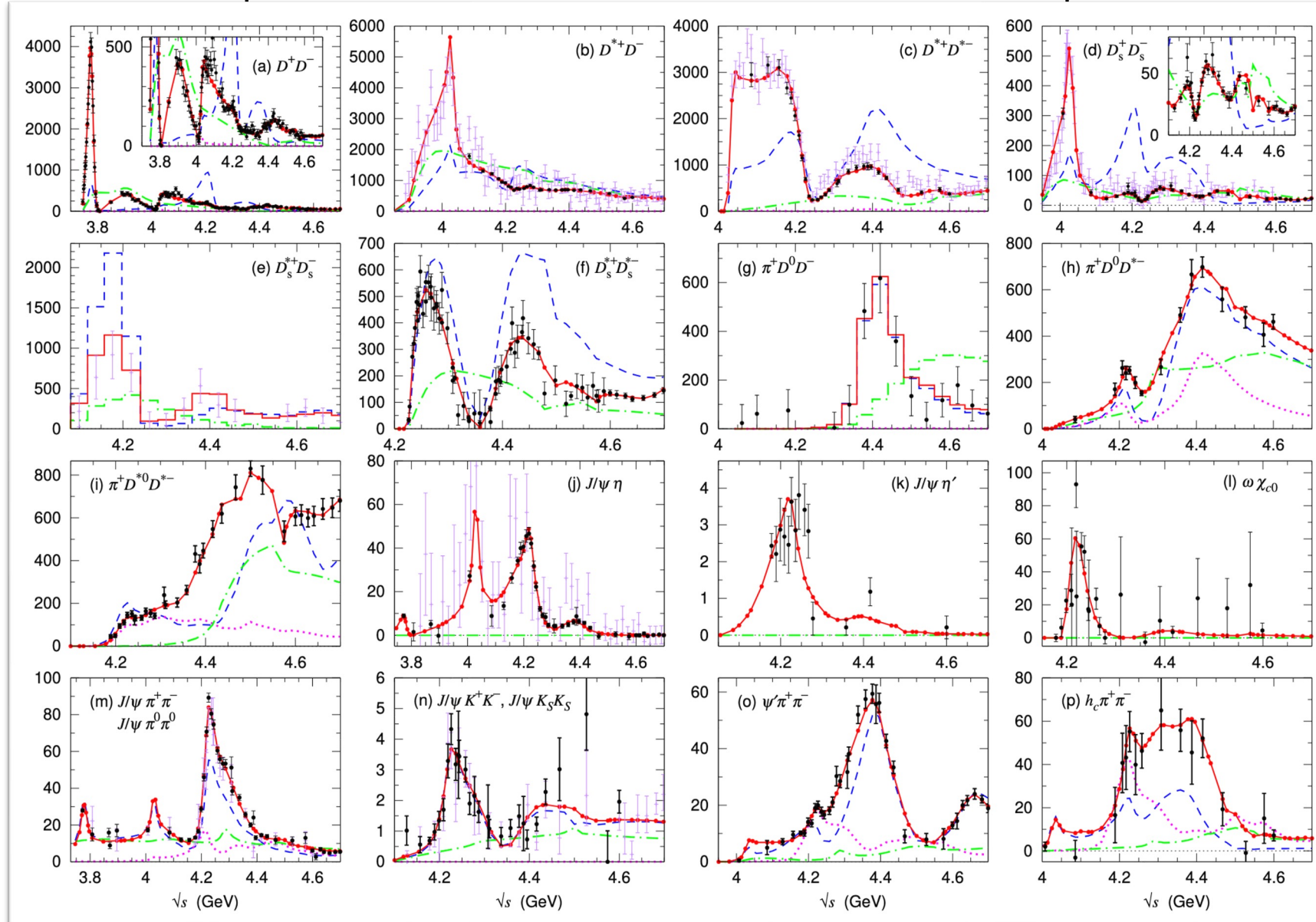
[Y.-Z. Xu et al., EPJC 81 \(2021\) 895; ...](#)



GlueX, PRC 108 (2023) 025201

Remarks on J/ψ photoproduction and $J/\psi N$ scattering

- Production rates of open-charm v.s. charmonium channels in electron-positron collisions (units: nb)



Plot taken from
S.X. Nakamura et al.,
PRD 112 (2025) 054027

Remarks on J/ψ photoproduction and $J/\psi N$ scattering

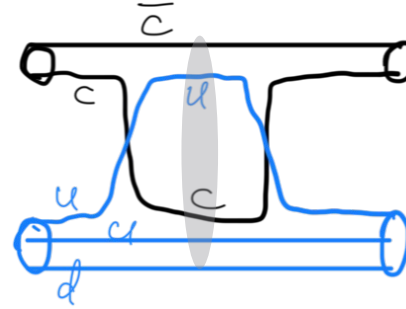
● $J/\psi N$ scattering length mechanisms

B. Wu et al., Fund. Res. 5 (2025) 2531

□ Open-charm coupled channels ($J/\psi N - \Lambda_c \bar{D}^{(*)} / \Sigma_c^{(*)} \bar{D}^{(*)} - J/\psi N$)

➤ Based on solution of coupled-channel LSE fitted to P_c data (direct $J/\psi N$ scattering neglected, only via coupled channels)

➤ Result: $\mathcal{O}(-(0.1 \dots 10) \times 10^{-3} \text{ fm})$

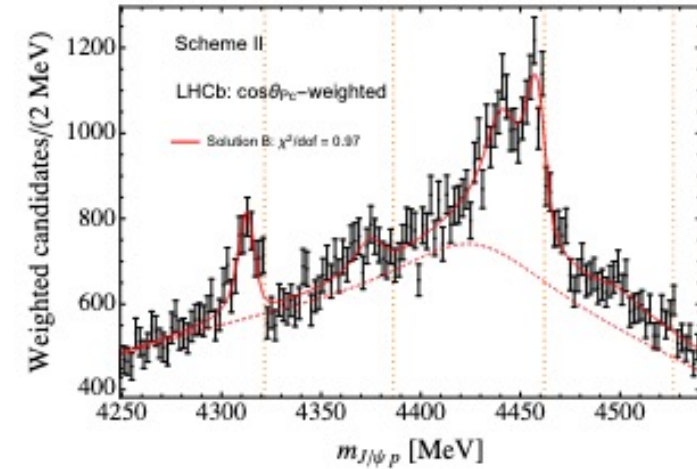
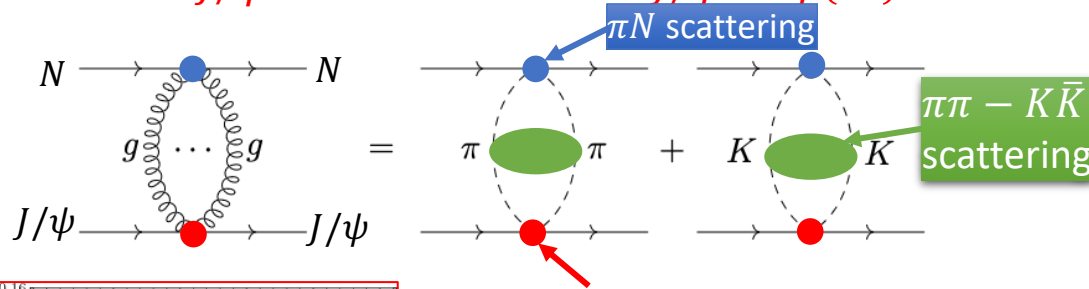


M.-L. Du et al., PRL 124 (2020) 072001; JHEP 08 (2021) 157

□ Soft-gluon exchange

➤ Based on dispersion relation

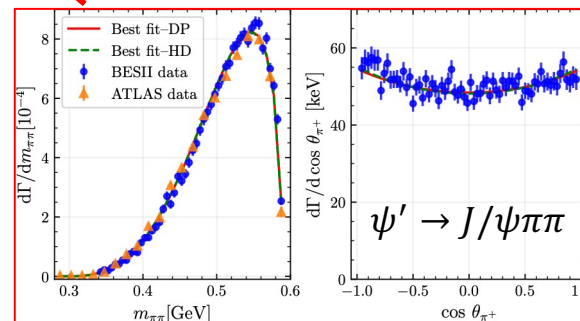
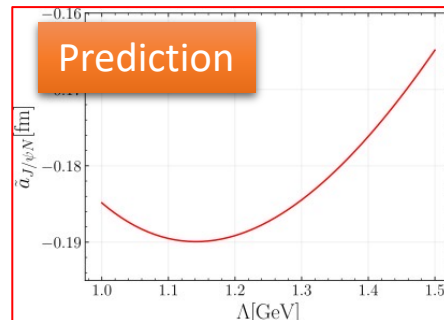
➤ Result: $a_{J/\psi N} \lesssim -0.16 \text{ fm}$, $a_{J/\psi N} a_{\psi(2S)N} \geq (-0.15 \text{ fm})^2$



➤ consistent with later lattice QCD result

Y. Lyu et al. [HALQCD], PLB 860 (2024) 139178

$$a_{J/\psi N (S=3/2)} = -0.30^{+0.02+0.00}_{-0.02-0.02} \text{ fm}$$



Low-energy $J/\psi N$ scattering dominated by gluonic exchange

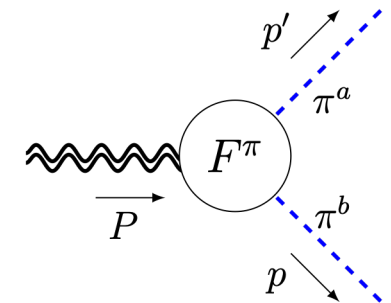
● Definitions

□ Gravitational form factors (GFFs) for spin-0 particles, e.g., for pion:

$$\langle \pi^a(p') | \hat{T}^{\mu\nu}(0) | \pi^b(p) \rangle = \frac{\delta^{ab}}{2} [A^\pi(t) P^\mu P^\nu + D^\pi(t) (\Delta^\mu \Delta^\nu - t g^{\mu\nu})]$$

$$P^\mu = p'^\mu + p^\mu, \Delta^\mu = p'^\mu - p^\mu \quad \downarrow \quad \text{Crossing, for constructing dispersion relations}$$

$$\langle \pi^a(p') \pi^b(p) | \hat{T}^{\mu\nu}(0) | 0 \rangle = \frac{\delta^{ab}}{2} [A^\pi(t) \Delta^\mu \Delta^\nu + D^\pi(t) (P^\mu P^\nu - t g^{\mu\nu})]$$

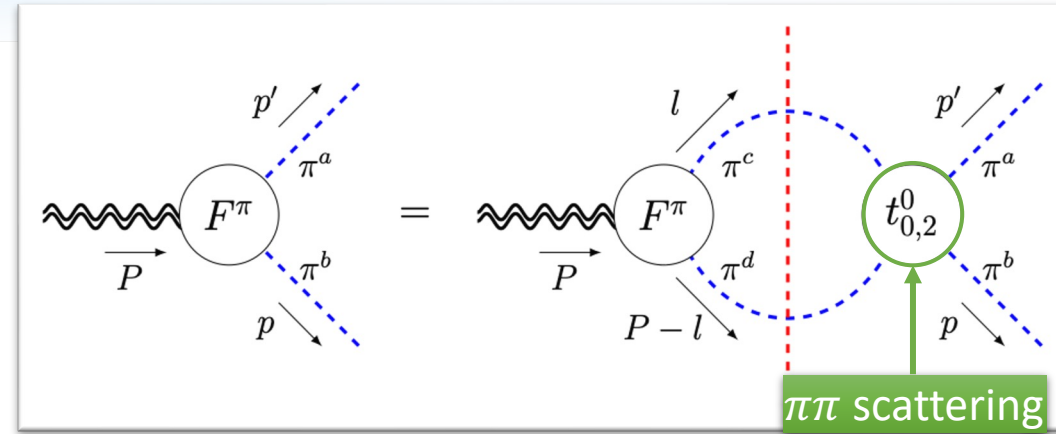


□ Nucleon GFFs

$$\langle N(p') \bar{N}(p) | \hat{T}^{\mu\nu}(0) | 0 \rangle = \frac{1}{4m_N} \bar{u}(p') \left[\hat{A}(t) \Delta^\mu \Delta^\nu + \hat{J}(t) \left(i \Delta^{\{\mu} \sigma^{\nu\} \rho} P_\rho \right) + \hat{D}(t) (P^\mu P^\nu - t g^{\mu\nu}) \right] u(p)$$

Unitarity relation for the pion GFFs

● Discontinuity of A^π and D^π



$$\begin{aligned} \text{Disc} \left\langle \pi^a(p') \pi^b(p) \left| \hat{T}^{\mu\nu}(0) \right| 0 \right\rangle &= \frac{\delta^{ab}}{2} [\text{Disc } A^\pi(t) \Delta^\mu \Delta^\nu + \text{Disc } D^\pi(t) (P^\mu P^\nu - t g^{\mu\nu})] \\ &= 2i \frac{2p_\pi}{\sqrt{t}} \frac{\delta^{ab}}{2} \left[(A^\pi(t))^* \left(\frac{4}{3t} p_\pi^2 (t_0^0(t) - t_2^0(t)) (P^\mu P^\nu - t g^{\mu\nu}) + t_2^0(t) \Delta^\mu \Delta^\nu \right) + (D^\pi(t))^* t_0^0(t) (P^\mu P^\nu - t g^{\mu\nu}) \right] \end{aligned}$$

$$\text{Im } A^\pi(t) = \frac{2p_\pi}{\sqrt{t}} (t_2^0(t))^* A^\pi(t),$$

$$\text{Im } D^\pi(t) = \frac{2p_\pi}{\sqrt{t}} \left[\frac{4}{3} \frac{p_\pi^2}{t} (t_0^0(t) - t_2^0(t))^* A^\pi(t) + (t_0^0(t))^* D^\pi(t) \right] \Rightarrow \text{Im } \Theta^\pi(t) = \frac{2p_\pi}{\sqrt{t}} (t_0^0(t))^* \Theta^\pi(t)$$

● Decomposition into $J^{PC} = 0^{++}, 2^{++}$ matrix elements (conserved separately):

K. Raman (1971)

$$\left\langle \pi^a(p') \pi^b(p) \left| \hat{T}^{\mu\nu}(0) \right| 0 \right\rangle = \delta^{ab} \left\{ \frac{1}{3} \left(g^{\mu\nu} - \frac{P^\mu P^\nu}{P^2} \right) \Theta^\pi(t) + \left[\Delta^\mu \Delta^\nu + \frac{\Delta^2}{3t} (P^\mu P^\nu - t g^{\mu\nu}) \right] A^\pi(t) \right\}$$

$$\text{trace part: } \left\langle \pi^a(p') \pi^b(p) \left| \hat{T}_\mu^\mu(0) \right| 0 \right\rangle = \delta^{ab} \Theta^\pi(t), \quad \Theta^\pi(t) = -\frac{1}{2} (4p_\pi^2 A^\pi(t) + 3t D^\pi(t))$$

$\pi\pi$ - $K\bar{K}$ coupled channels

- $\pi\pi$ phase shifts known precisely from Roy(-like) equation analyses

Bern group; Madrid-Krakow group

- Generalization to **coupled channels**: isoscalar, scalar $\pi\pi$ - $K\bar{K}$; $f_0(500)$, $f_0(980)$ mesons

□ Unitarity relation for $\Theta^\pi(t) \Rightarrow$ matrix relation for coupled channels (both pion and kaon trace GFFs):

$$\text{Im } \Theta^\pi(t) = \frac{2p_\pi}{\sqrt{t}} (t_0^0(t))^* \Theta^\pi(t)$$

$$\text{Im } \Theta(t) = [\mathbf{T}_0^0(t)]^* \Sigma_0^0(t) \Theta(t), \quad \Theta(t) = \begin{pmatrix} \Theta^\pi(t) \\ \frac{2}{\sqrt{3}} \Theta^K(t) \end{pmatrix}$$

phase-space factor

$$\Sigma_0^0(t) \equiv \text{diag}(\sigma_\pi \theta(t - t_\pi), \sigma_K \theta(t - t_K))$$

with $\sigma_i(t) \equiv \sqrt{1 - 4m_i^2/t}$ ($i = \pi, K$)

$\pi\pi$ - $K\bar{K}$ T-matrix

$$\mathbf{T}_0^0(t) = \begin{pmatrix} \frac{\eta_0^0(t) e^{2i\delta_0^0(t)} - 1}{2i\sigma_\pi} & |g_0^0(t)| e^{i\Psi_0^0(t)} \\ |g_0^0(t)| e^{i\Psi_0^0(t)} & \frac{\eta_0^0(t) e^{2i(\Psi_0^0(t) - \delta_0^0(t))} - 1}{2i\sigma_K} \end{pmatrix}$$

$$\eta_0^0(t) = \sqrt{1 - 4\sigma_\pi \sigma_K |g_0^0(t)|^2 \theta(t - t_K)}$$

Muskhelishvili-Omnès representation

- Single-channel: Watson's theorem $\Rightarrow$ **phase of FF = scattering phase shift**

K.M. Watson (1952)

$$\text{disc } A^\pi(t) = 2iA^\pi(t)\theta(t - t_\pi) \sin \delta_2^0(t) e^{-i\delta_2^0(t)}$$

$$|t_2^0| e^{i\phi_2^0} = \frac{\eta_2^0 e^{2i\delta_2^0} - 1}{2i\sigma_\pi}$$

□ Omnès solution

$$A^\pi(t) = P_2^\pi(t) \Omega_2^0(t), \quad \Omega_2^0(t) \equiv \exp \left\{ \frac{t}{\pi} \int_{t_\pi}^{\infty} \frac{dt'}{t'} \frac{\phi_2^0(t')}{t' - t} \right\}$$

δ_2^0 replaced by the phase of $\pi\pi$ partial wave ϕ_2^0 to account for inelasticity

M. Hoferichter et al., Phys. Rept. 625 (2016) 1

- δ_2^0, η_2^0 up to $E_0 \simeq 2$ GeV from latest dispersive analysis

P. Bydžovský et al., PRD 94 (2016) 116013

- Beyond matching point E_0 , B. Moussallam, EPJC 14 (2000) 111

$$\delta_2^0(t) = \pi + (\delta_2^0(E_0^2) - \pi) \frac{2}{1 + (\sqrt{t}/E_0)^3}$$

$$\eta_2^0(t) = 1 + (\eta_2^0(E_0^2) - \pi) \frac{2}{1 + (\sqrt{t}/E_0)^3}$$

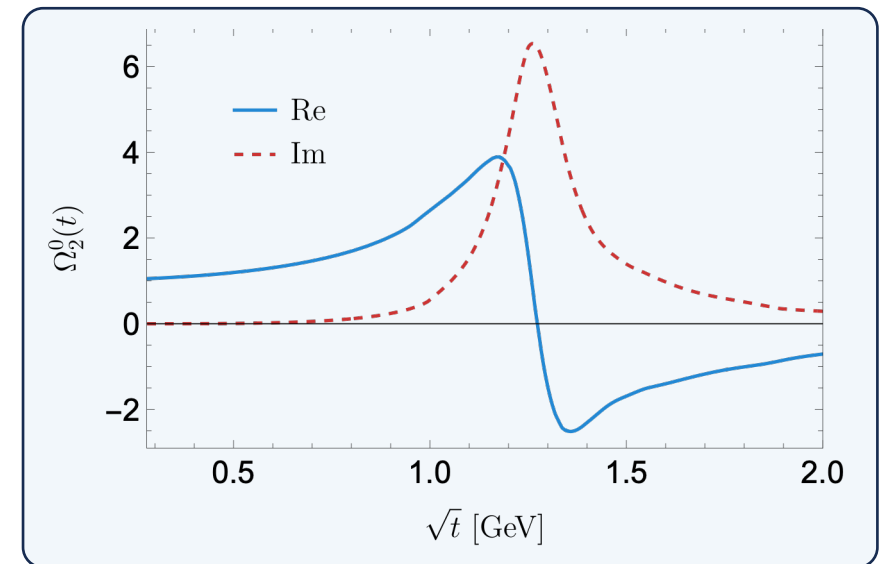
- Polynomial: $P_2^\pi(t) = 1 + \alpha t$ matched to NLO ChPT

$$A^{\pi,K}(t) = 1 - \frac{2L_{12}^r}{F_\pi^2} t \quad \text{J. Donoghue, H. Leutwyler, ZPC 52 (1991) 343}$$

tensor-meson dominance estimate: $L_{12}^r = -\frac{F_\pi^2}{2m_{f_2}^2}$

$$P_2^\pi(t) = 1 + \left(\frac{1}{m_{f_2}^2} - \dot{\Omega}_2^0(0) \right) t \simeq 1 - (0.01 \text{ GeV}^{-2}) t$$

Meson dominance picture for GFFs, W. Broniowski, E. Ruiz Arriola, PRD 112 (2025) 054028



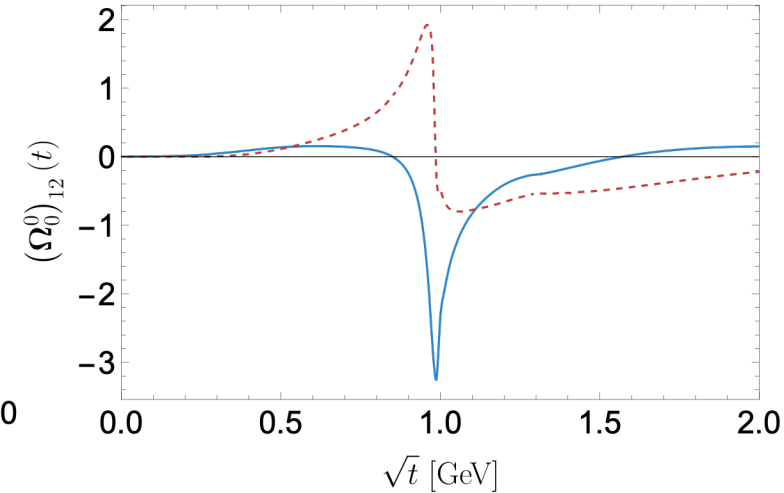
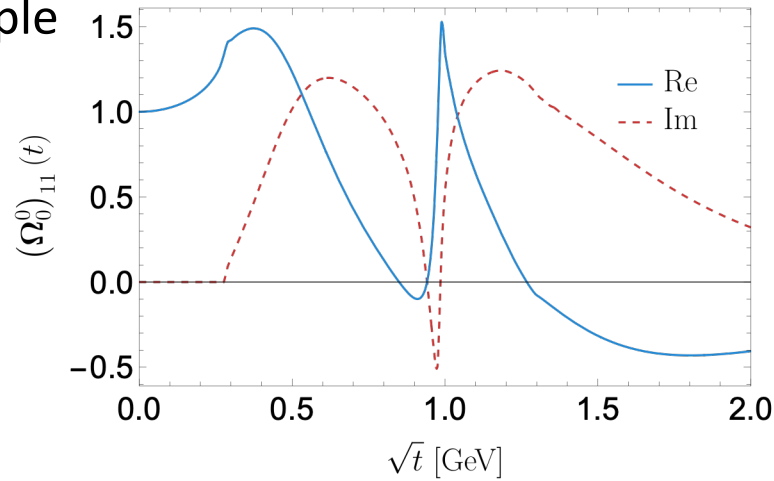
PDG average: $m_{f_2} = (1275.4 \pm 0.8) \text{ MeV}$ PDG2026

We take $m_{f_2} = (1275 \pm 20) \text{ MeV}$ for a conservative error estimate

Muskhelishvili-Omnès representation

- Coupled-channel: solution known as the Muskhelishvili-Omnès (MO) representation
- ▣ The above can be generalized to $\pi\pi-K\bar{K}$ coupled channels (matching point: ~ 1.3 GeV)
- ▣ Take **isoscalar scalar** $\pi\pi-K\bar{K}$ as example

$$\Omega_0^0(t) = \frac{1}{\pi} \int_{t_\pi}^{\infty} \frac{dt'}{t' - t} [\mathbf{T}_0^0(t)]^* \Sigma_0^0(t) \Omega_0^0(t')$$



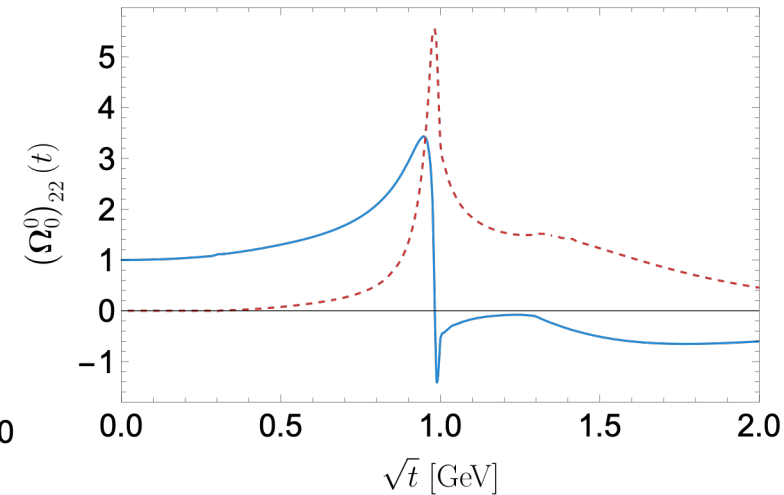
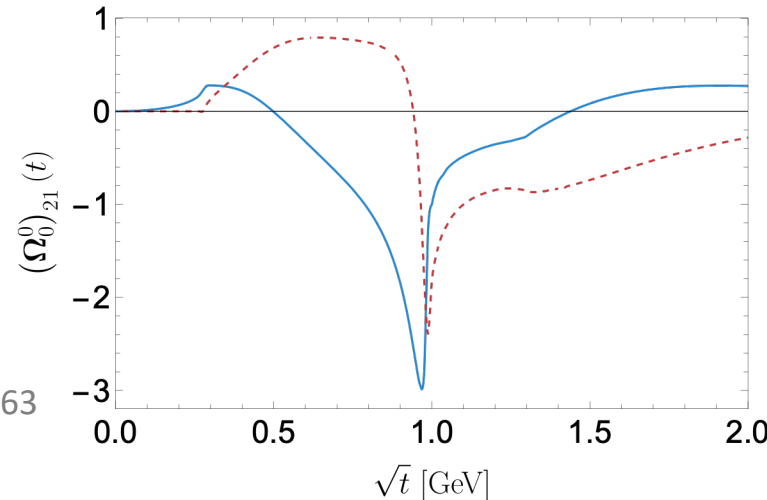
$\pi\pi$ phase shifts: Roy equation

I. Caprini et al. (2012);

$\pi\pi \rightarrow K\bar{K}$: Roy-Steiner equation

P. Büttiker et al. (2004);

M. Hoferichter et al., JHEP 06 (2012) 063



Muskhelishvili-Omnès representation

- Pion and kaon trace GFFs:

$$[\Theta(t)]^T = [P_0(t)]^T \Omega_0^0(t), \quad P_0(t) = \begin{pmatrix} 2m_\pi^2 + \beta_\pi t \\ \frac{2}{\sqrt{3}} (2m_K^2 + \beta_K t) \end{pmatrix}$$

$$\beta_\pi = \dot{\Theta}^\pi(0) - 2m_\pi^2 \left(\dot{\Omega}_0^0 \right)_{11}(0) - \frac{4m_K^2}{\sqrt{3}} \left(\dot{\Omega}_0^0 \right)_{12}(0),$$

$$\beta_K = \dot{\Theta}^K(0) - \sqrt{3}m_\pi^2 \left(\dot{\Omega}_0^0 \right)_{21}(0) - 2m_K^2 \left(\dot{\Omega}_0^0 \right)_{22}(0)$$

- Matching to NLO ChPT

J. Donoghue, H. Leutwyler, ZPC 52 (1991) 343

$$\dot{\Theta}^\pi(0) = 1 - 4L_{12}^r \frac{m_\pi^2}{F_\pi^2} - 24(L_{11}^r - L_{13}^r) \frac{m_\pi^2}{F_\pi^2} - \frac{3}{2} \frac{m_\pi^2}{F_\pi^2} I_\pi + \frac{m_\pi^2}{2F_\pi^2} I_\eta = 0.98(2),$$

$$\dot{\Theta}^K(0) = 1 - 4L_{12}^r \frac{m_K^2}{F_\pi^2} - 24(L_{11}^r - L_{13}^r) \frac{m_K^2}{F_\pi^2} - \frac{m_K^2}{F_\pi^2} I_\eta = 0.94(14)$$

Chiral logs: $I_i = \frac{1}{48\pi^2} \left(\ln \left(\frac{\mu^2}{m_i^2} \right) - 1 \right)$

- Similar coupled-channel analysis for D-wave $\pi\pi-K\bar{K} \Rightarrow$ coupled-channel results for A^π, A^K and D^π, D^K

Earlier work on dispersive studies of pion GFFs:

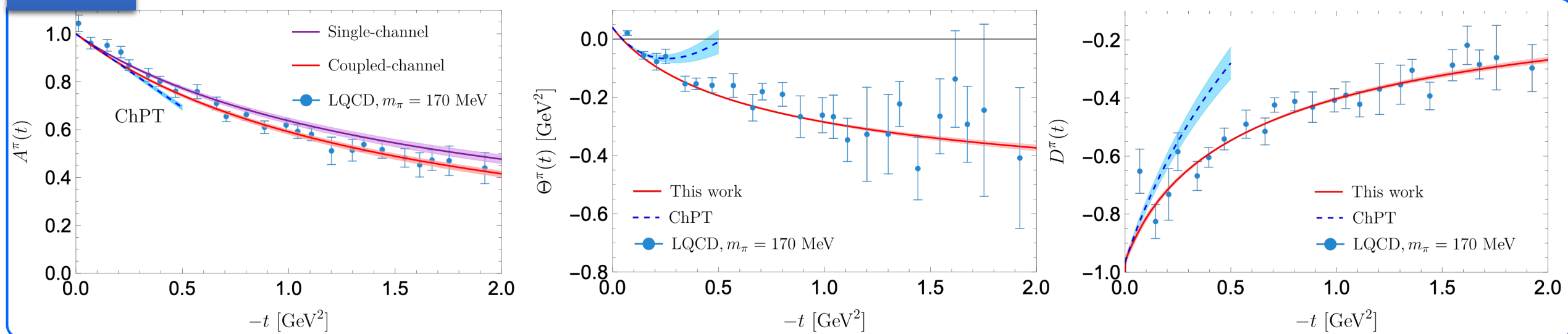
J.F. Donoghue, J. Gasser, H. Leutwyler, NPB 343 (1990) 341; W. Broniowski, E. Ruiz Arriola, PLB 859 (2024) 139138

Pion and kaon GFFs

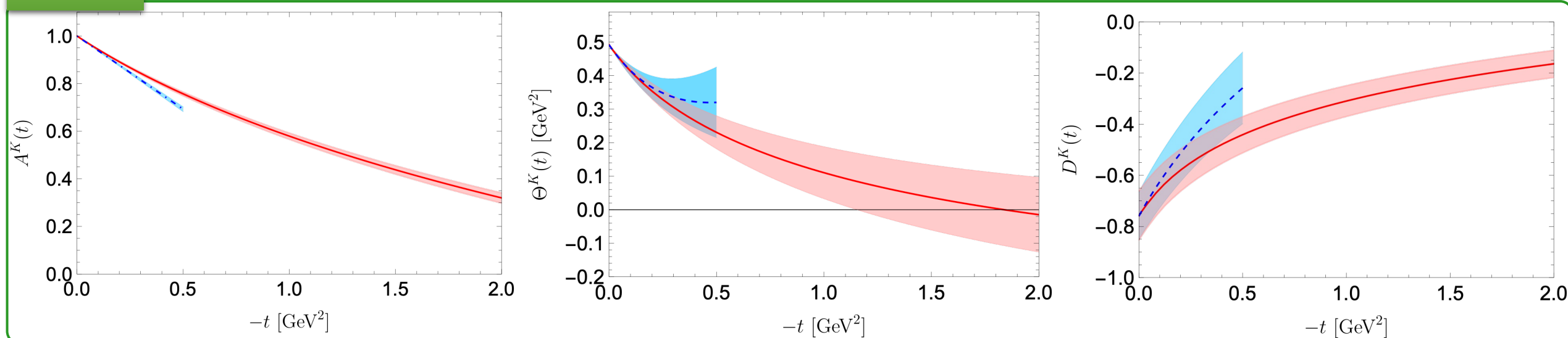
Prediction, NOT fit

LQCD ($m_\pi = 170$ MeV): D.C. Hackett et al., PRL 132 (2024) 251904

Pion



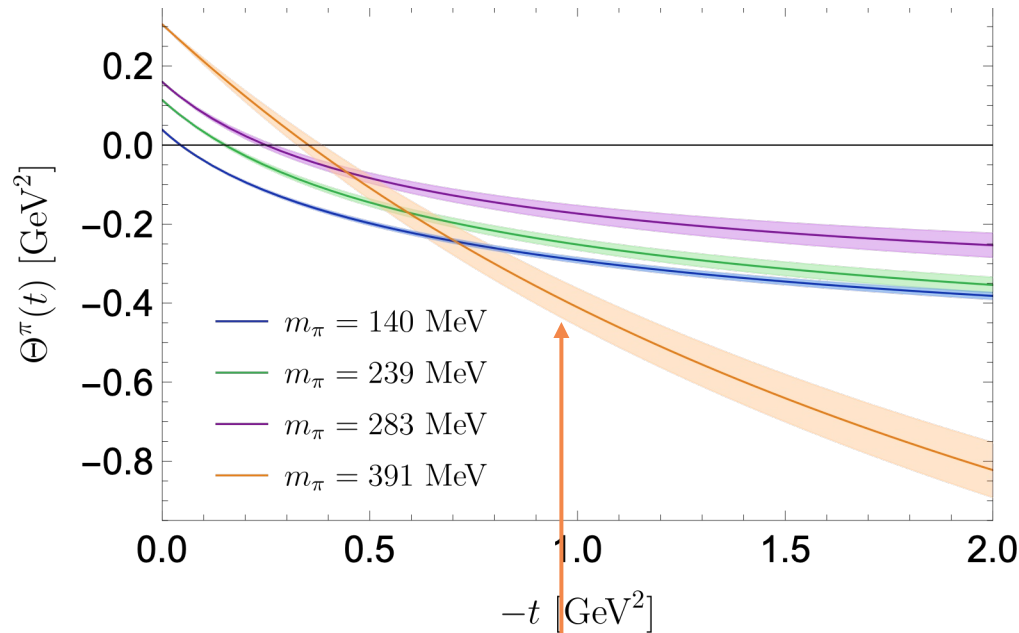
Kaon



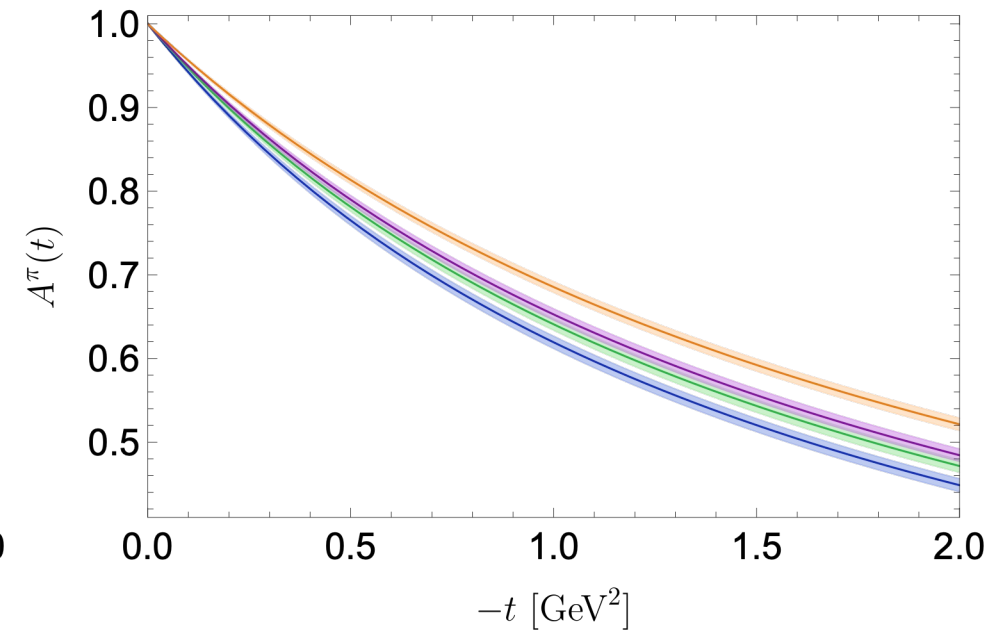
Pion GFFs: pion mass dependence

- Using the $\pi\pi$ scattering phase shifts at unphysical pion masses (239 MeV, 283 MeV, 391 MeV) obtained from Roy equation analyses

X.-H. Cao et al., PRD 108 (2023) 034009; A. Rodas et al., PRD 109 (2024) 034513



Fast change before $m_\pi = 391$ MeV:
 ➤ σ meson becomes a $\pi\pi$ bound state from lattice HadSpec, PRL 118 (2017) 022002



- Singularity in quark-mass dependence at such critical point:

FKG, C. Hanhart, F. Llanes-Estrada, U.-G. Meißner, PLB 703 (2011) 510;
 FKG, U.-G. Meißner, PRL 109 (2012) 062001

Unitarity relation for nucleon GFFs

● Partial-wave amplitudes for $\pi\pi \rightarrow N\bar{N}$

W. Frazer, J. Fulco (1960); G. Höhler (1983)

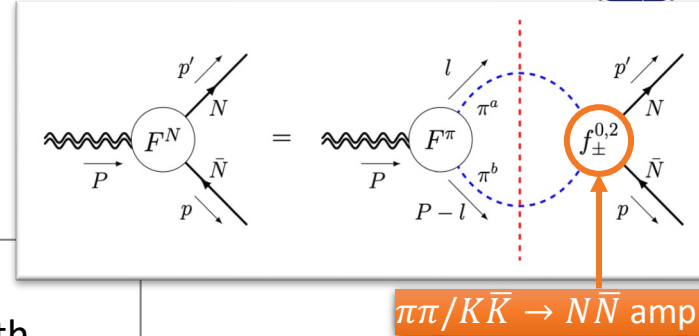
$$A^I(t, s) = -\frac{8\pi}{p_N^2} \sum_{J=0}^{\infty} \left(J + \frac{1}{2}\right) (p_\pi p_N)^J \left\{ P_J(\cos \theta) f_+^J(t) - \frac{m_N \cos \theta}{\sqrt{J(J+1)}} P_J'(\cos \theta) f_-^J(t) \right\},$$

$$B^I(t, s) = 8\pi \sum_J \frac{J + \frac{1}{2}}{\sqrt{J(J+1)}} (p_\pi p_N)^{J-1} P_J'(\cos \theta) f_-^J(t)$$

$I = +/ -$ for even/odd J ;

$f_\pm^J$: $\pi\pi \rightarrow N\bar{N}$ partial-wave amp. with

$+/-$ for parallel/antiparallel $N\bar{N}$ helicities



● Discontinuity of the nucleon GFFs

$$\text{Im } A^s(t) = \frac{3p_\pi^5}{\sqrt{6t}} \left[f_-^2(t) + \sqrt{\frac{3}{2}} \frac{m_N}{p_N^2} \left(m_N \sqrt{\frac{2}{3}} f_-^2(t) - f_+^2(t) \right) \right]^* A^\pi(t)$$

$$\text{Im } J^s(t) = \frac{3p_\pi^5}{2\sqrt{6t}} (f_-^2(t))^* A^\pi(t),$$

$$\text{Im } D^s(t) = -\frac{3m_N p_\pi}{2p_N^2 \sqrt{t}} \left[\frac{4p_\pi^2}{3t} \left((f_+^0(t))^* - (p_\pi p_N)^2 (f_+^2(t))^* \right) A^\pi(t) + (f_+^0(t))^* D^\pi(t) \right]$$

$$\text{Im } \Theta^s(t) = -\frac{3p_\pi}{4p_N^2 \sqrt{t}} (f_+^0(t))^* \Theta^\pi(t)$$

● Decomposition into $J^{PC} = 0^{++}, 2^{++}$ matrix elements

$$\langle N(p') \bar{N}(p) | \hat{T}^{\mu\nu}(0) | 0 \rangle = \bar{u}(p') (T_S^{\mu\nu} + T_T^{\mu\nu}) v(p)$$

$$T_S^{\mu\nu} = \frac{1}{3} \left(g^{\mu\nu} - \frac{P^\mu P^\nu}{P^2} \right) \Theta^s(t),$$

$$\Theta^s(t) = \frac{1}{4m_N} [-4p_N^2 A^s(t) + 2t J^s(t) - 3t D^s(t)]$$

$$T_T^{\mu\nu} = \frac{1}{4m_N} \left[\Delta^\mu \Delta^\nu + \frac{\Delta^2}{3t} (P^\mu P^\nu - t g^{\mu\nu}) \right] A^s(t) + \left[i \Delta^{\{\mu} \sigma^{\nu\} \rho} P_\rho + \frac{2i \sigma^{\rho\kappa} \Delta_\rho P_\kappa}{3t} (P^\mu P^\nu - t g^{\mu\nu}) \right] J^s(t)$$

$$\text{Im } \Theta^s(t) = -\frac{3}{4p_N^2 \sqrt{t}} \left[p_\pi (f_+^0(t))^* \Theta^\pi(t) \theta(t - t_\pi) + \frac{4}{3} p_K (h_+^0(t))^* \Theta^K(t) \theta(t - t_K) \right]$$

$K\bar{K} \rightarrow N\bar{N}$ amplitude

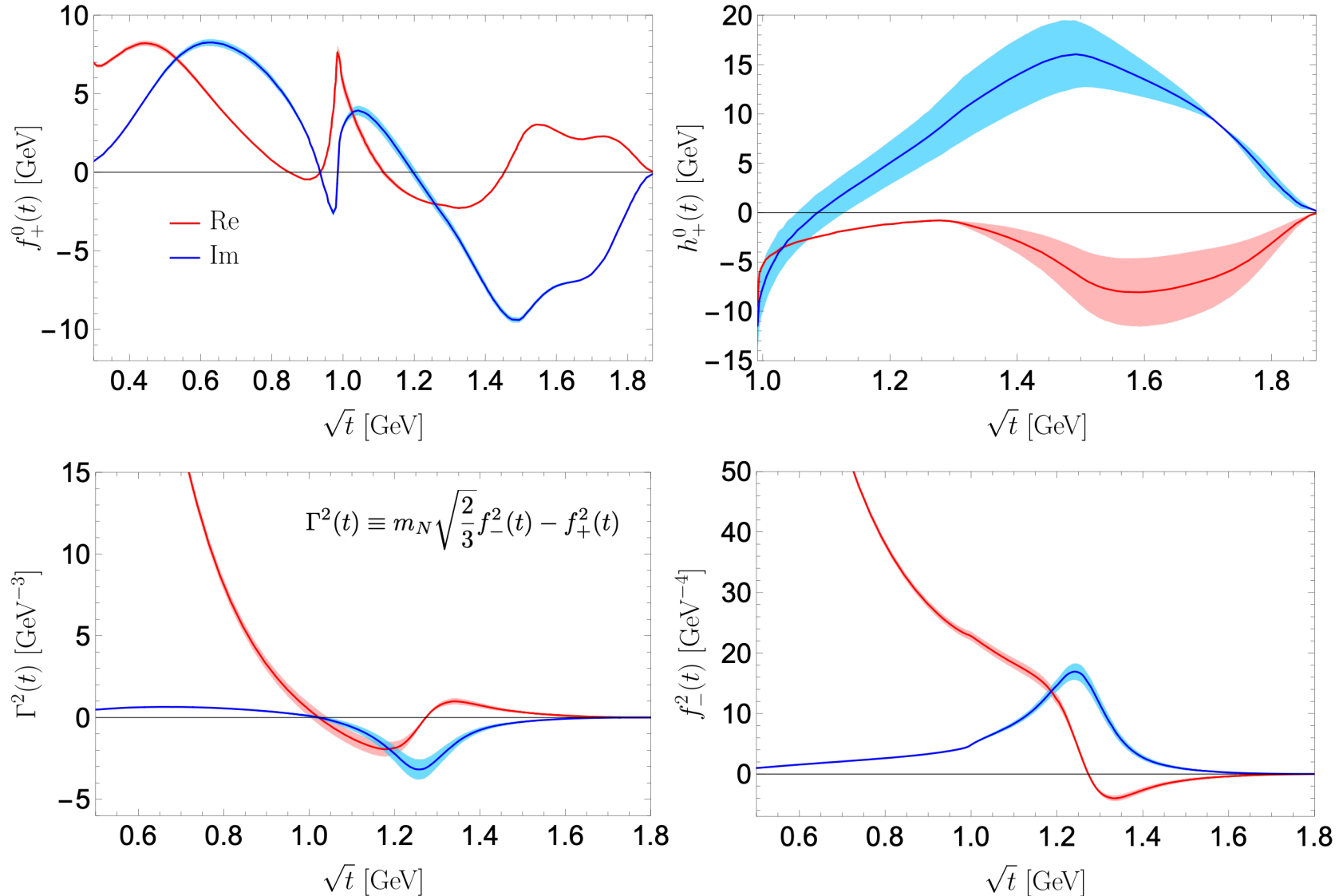
coupled-channel

$\pi\pi/K\bar{K} \rightarrow N\bar{N}$ S-wave amplitudes

G.E. Hite, F. Steiner (1973)

- Inputs: $\pi\pi/K\bar{K} \rightarrow N\bar{N}$ S-wave amplitudes $f_{\pm}^{0,2}, h_{+}^0$ from Roy-Steiner equation analyses

M. Hoferichter et al., Phys. Rept. 625 (2016) 1; PLB 853 (2024) 138698; X.-H. Cao et al., JHEP 12 (2022) 073



Nucleon GFFs

G.E. Hite, F. Steiner (1973)

- Inputs: $\pi\pi/K\bar{K} \rightarrow N\bar{N}$ S-wave amplitudes $f_{\pm}^{0,2}, h_{+}^0$ from Roy-Steiner equation analyses

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- Dispersive relations for the nucleon GFFs

$$(A, J, \Theta)(t) = \frac{1}{\pi} \int_{4m_{\pi}^2}^{\infty} dt' \frac{\text{Im}(A, J, \Theta)(t')}{t' - t}$$

- Normalization $\Rightarrow$ sum rules

$$\frac{1}{\pi} \int_{4m_{\pi}^2}^{\infty} dt' \frac{\text{Im}(A, J, \Theta)(t')}{t'} = \left(1, \frac{1}{2}, m_N\right)$$

- Introduce S-wave (0^{++}) and D-wave (2^{++}) poles to the spectral functions: $\pi c_{S,D} m_{S,D}^2 \delta(t - m_{S,D}^2)$ to satisfy the sum rules

➤ 0^{++} : $m_S \in (1.5, 1.8)$ GeV to cover $f_0(1500)$ and $f_0(1710)$

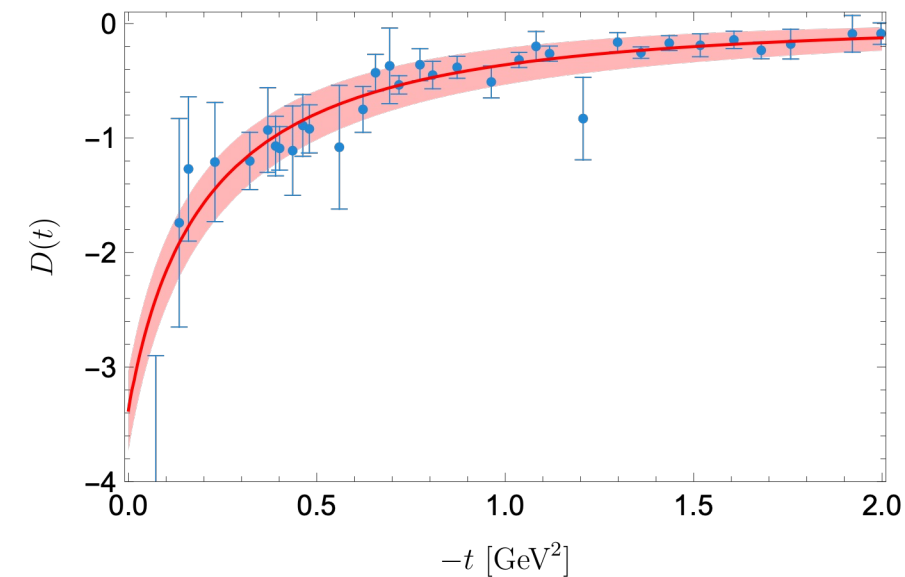
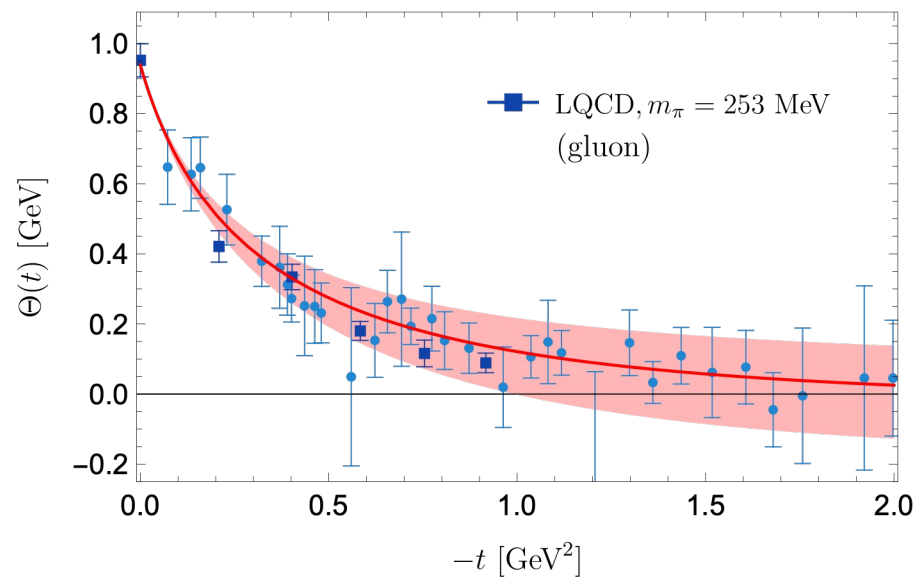
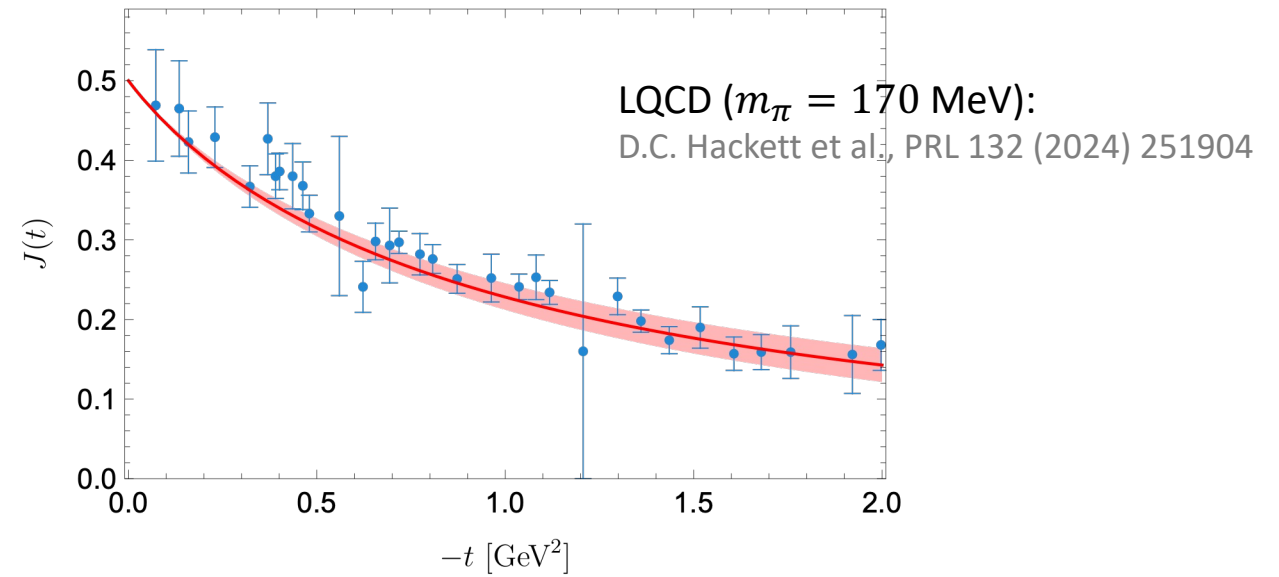
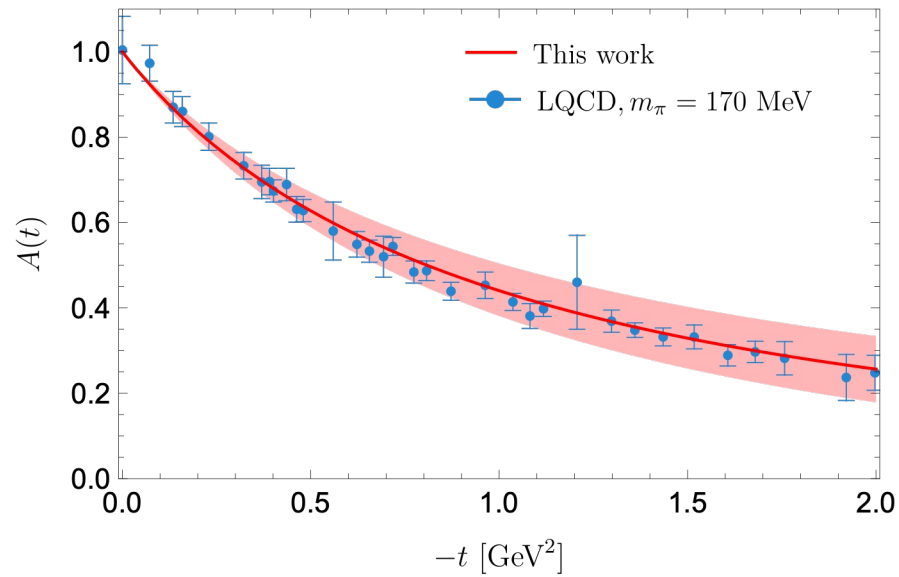
➤ 2^{++} : $m_D \in (1.5, 2.2)$ GeV to cover $f_2(1565), f_2(1950)$ and $f_2(2010)$

error estimate

M.A. Belushkin et al., PRC 75 (2007) 035202; M. Hoferichter et al., EPJA 52 (2016) 331;
J.M. Alarcón, C. Weiss, PLB 784 (2018) 373

Nucleon GFFs: results

Prediction, NOT fit



LQCD ($m_\pi = 253$ MeV), gluon part only:

B. Wang et al. [χ QCD], PRD 109 (2024) 094504

Nucleon GFFs: results

● D-term: $D \equiv D(0)$

● Various radii in the Breit frame

□ From the trace FF (scalar radius):

$$\langle r_{\Theta}^2 \rangle = \frac{6\dot{\Theta}(0)}{m_N} = 6\dot{A}(0) - \frac{9D}{2m_N^2}$$

□ Radius of the energy density (mass radius):

$$\langle r_{\text{Mass}}^2 \rangle = 6\dot{A}(0) - \frac{3D}{2m_N^2}$$

Quantity	Result	Error budget
D-term	$-3.38^{+0.34}_{-0.35}$	$+(0.18)_{\text{ChPT}}(0.12)_{\text{pwa}}(0.26)_{\text{eff}}$
		$-(0.16)_{\text{ChPT}}(0.12)_{\text{pwa}}(0.29)_{\text{eff}}$
$\sqrt{\langle r_{\Theta}^2 \rangle}$ [fm]	$0.97^{+0.03}_{-0.03}$	$+(0.01)_{\text{ChPT}}(0.01)_{\text{pwa}}(0.03)_{\text{eff}}$
		$-(0.02)_{\text{ChPT}}(0.01)_{\text{pwa}}(0.02)_{\text{eff}}$
$\sqrt{\langle r_{\text{Mass}}^2 \rangle}$ [fm]	$0.70^{+0.03}_{-0.04}$	$+(0.02)_{\text{ChPT}}(0.01)_{\text{pwa}}(0.02)_{\text{eff}}$
		$-(0.02)_{\text{ChPT}}(0.01)_{\text{pwa}}(0.03)_{\text{eff}}$

Definition of various radii:

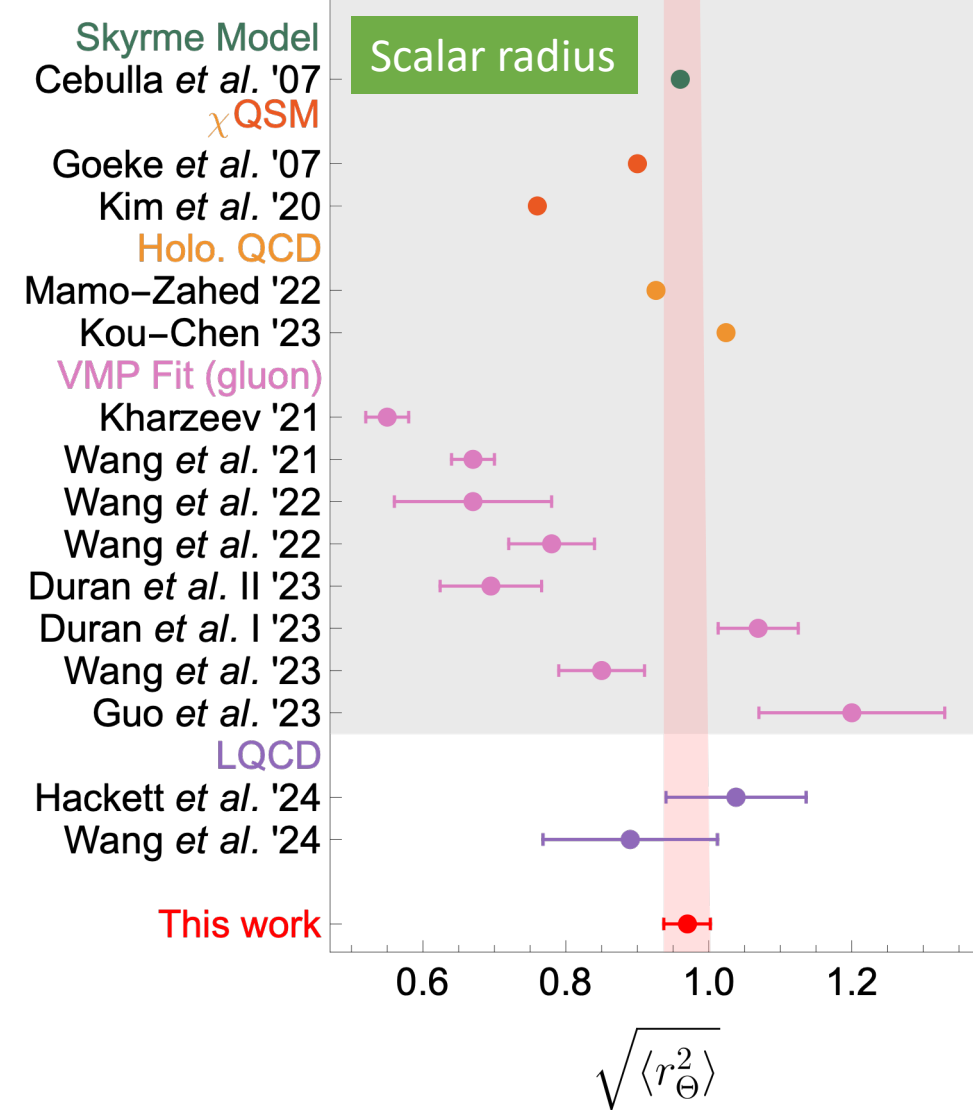
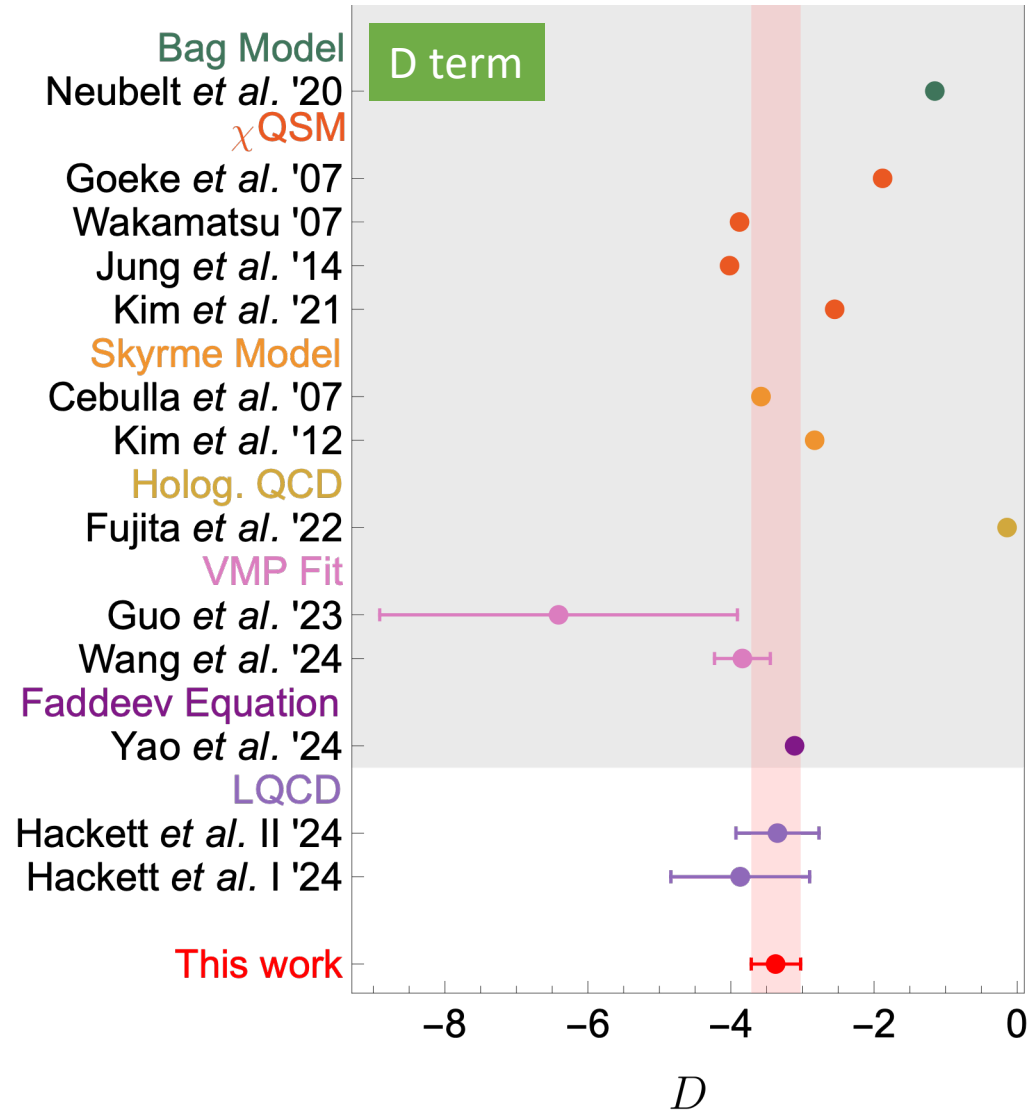
M. Polyakov, PLB 555 (2003) 57; M. Polyakov, P. Schweitzer, JIMPA 33 (2018) 183005; C. Lorcé et al., EPJC 79 (2019) 89

M. Polyakov, PLB 555 (2003) 57; C. Lorcé et al., PLB 776 (2018) 38

- ChPT: NLO ChPT inputs
- pwa: $\pi\pi/K\bar{K} \rightarrow N\bar{N}$
- eff: effective poles m_S, m_D

Nucleon GFFs: results

● Comparison with other results



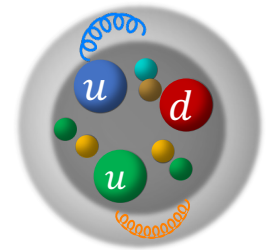
Summary and outlook

- The pion, kaon and nucleon GFFs are precisely determined using dispersive method with inputs:
 - ▣ Coupled-channel $\pi\pi\text{-}K\bar{K}$, $\pi\pi/K\bar{K} \rightarrow N\bar{N}$ amplitudes
 - ▣ Low energy: NLO ChPT with LECs estimated using resonance saturation, improvable with lattice calculations
 - ▣ High energy: highly excited meson resonances

$$\bullet D^N = -3.38^{+0.34}_{-0.35}, \quad \sqrt{\langle r_\Theta^2 \rangle} = 0.97^{+0.03}_{-0.03} \text{ fm} > \sqrt{\langle r_{E,p}^2 \rangle} \simeq 0.84 \text{ fm} > \sqrt{\langle r_{\text{Mass}}^2 \rangle} = 0.70^{+0.03}_{-0.04} \text{ fm}$$

May be regarded as a
confinement radius

proton charge radius



Bag radius in MIT bag model X. Ji, Front. Phys. (Beijing) 16 (2021) 64601

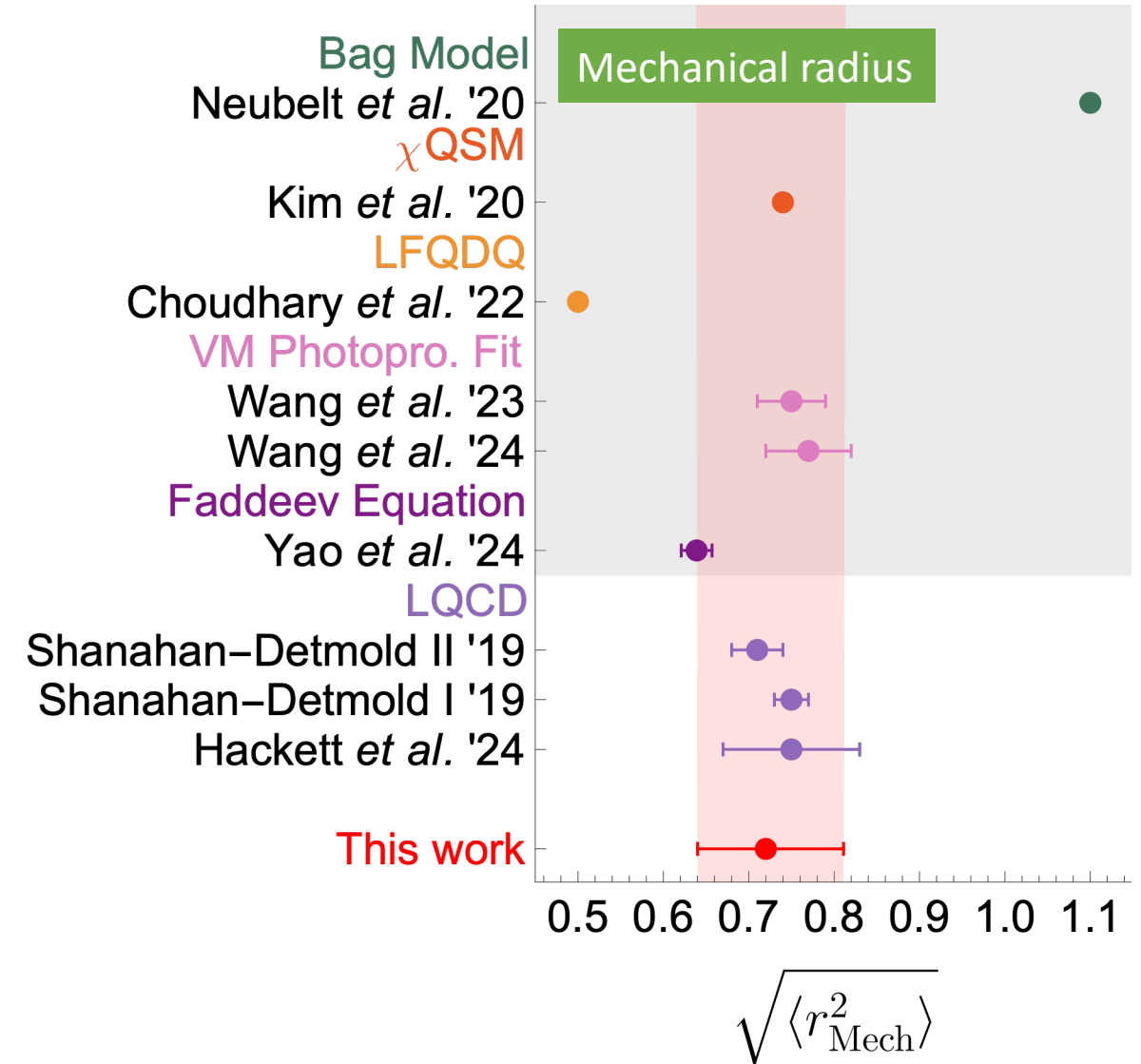
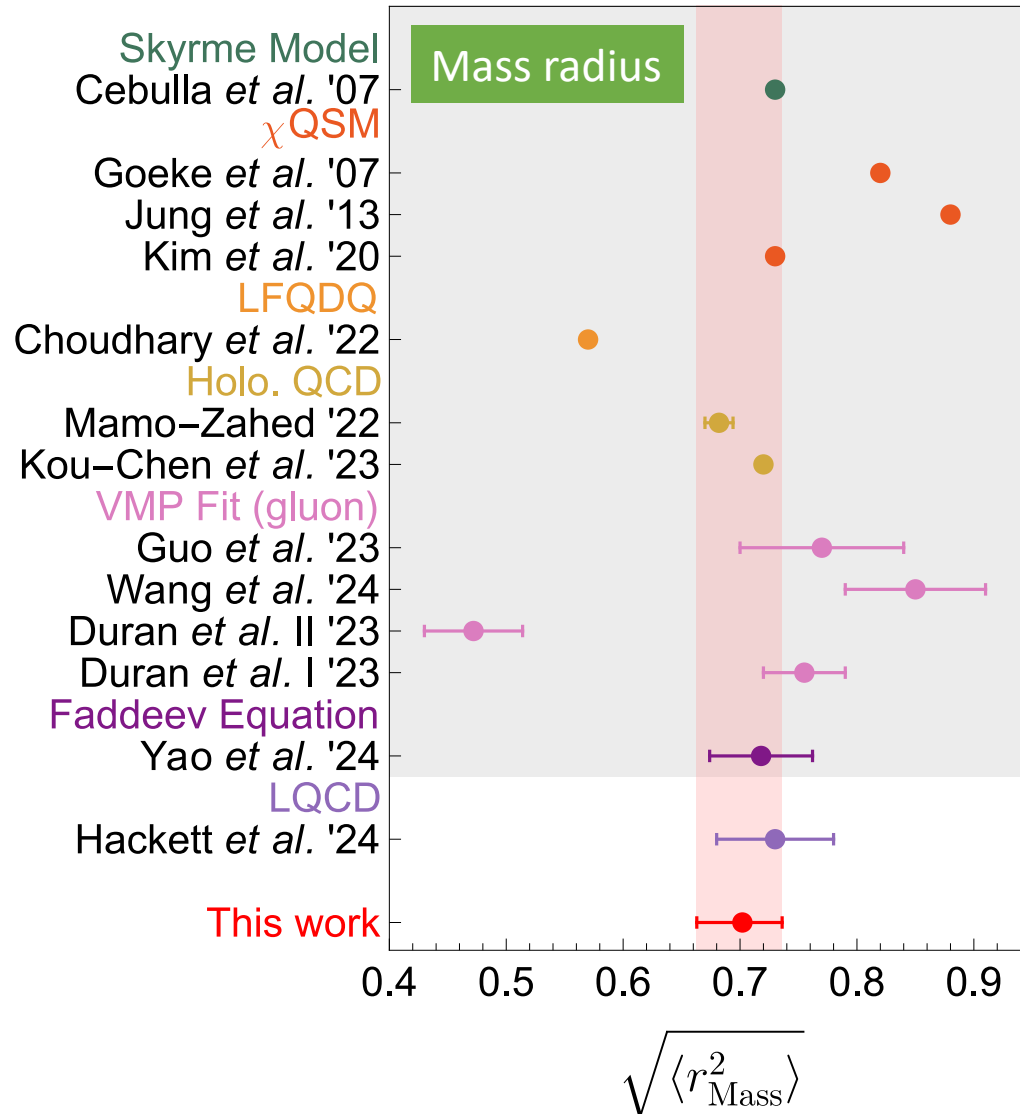
- Foundation for further analyses, interpretations, etc.
- More results to come. Outlook:
 - ▣ Pion mass dependence
 - ▣ Extension to hyperons; ...

Thank you for your attention!

Nucleon GFFs: results

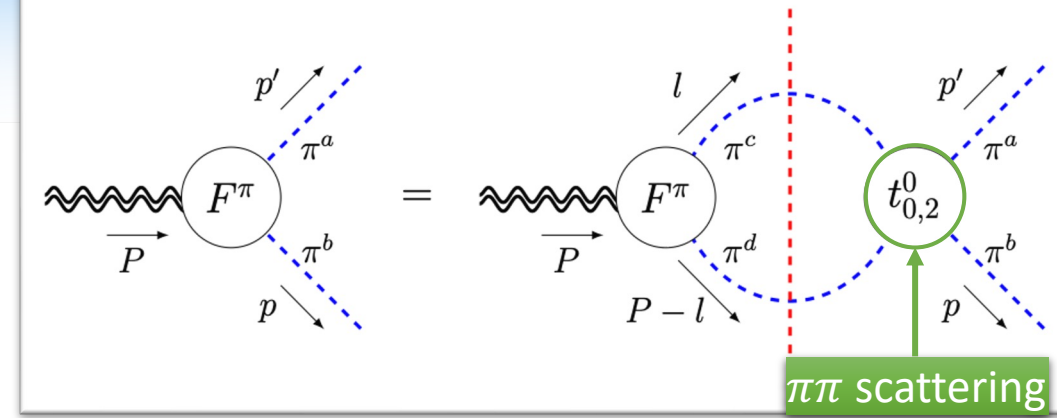
● Comparison with other results

$$\langle r_{\text{Mech}}^2 \rangle = \frac{6D}{\int_{-\infty}^0 dt D(t)}$$



Unitarity relation for the pion GFFs

- Unitarity $\Rightarrow$ discontinuity (imaginary part) of the pion GFFs



$$\begin{aligned}
 & \text{Disc} \left\langle \pi^a(p') \pi^b(p) \left| \hat{T}^{\mu\nu}(0) \right| 0 \right\rangle \\
 &= \frac{\delta^{ab}}{2} [\text{Disc } A^\pi(t) \Delta^\mu \Delta^\nu + \text{Disc } D^\pi(t) (P^\mu P^\nu - t g^{\mu\nu})] \\
 &= \frac{1}{2} \frac{i}{(4\pi)^2} \frac{p_\pi}{\sqrt{t}} \int d\Omega_l \left\langle \pi^a(p') \pi^b(p) \left| \pi^c(l) \pi^d(P-l) \right\rangle \left\langle \pi^c(l) \pi^d(P-l) \left| \hat{T}^{\mu\nu}(0) \right| 0 \right\rangle^* \right. \\
 &= \frac{1}{2} \frac{i}{(4\pi)^2} \frac{p_\pi}{\sqrt{t}} \int d\Omega_l \frac{\delta^{ab}}{2} [3A(t, s, u) + A(s, t, u) + A(u, s, t)] \\
 &\quad \times [(A^\pi(t))^* (2l - P)^\mu (2l - P)^\nu + (D^\pi(t))^* (P^\mu P^\nu - t g^{\mu\nu})] \\
 &= \frac{1}{2} \frac{i}{(4\pi)^2} \frac{p_\pi}{\sqrt{t}} \int d\Omega_l \frac{\delta^{ab}}{2} A^{I=0}(t, s, u) [(A^\pi(t))^* \boxed{(2l - P)^\mu (2l - P)^\nu} + (D^\pi(t))^* \boxed{(P^\mu P^\nu - t g^{\mu\nu})}] \\
 &\hspace{15em} \text{S-, D-waves} \hspace{15em} \text{S-wave}
 \end{aligned}$$

Mandelstam variables: $t = P^2, s = (p' - l)^2, u = (p - l)^2$; only two independent, $A^{I=0}(t, s) \equiv A^{I=0}(t, s, u)$

Lorentz structures: $\int d\Omega_l A^{I=0}(t, s) (2l - P)^\mu (2l - P)^\nu = A_1 \Delta^\mu \Delta^\nu + A_2 (P^\mu P^\nu - t g^{\mu\nu})$

Contract with $\Delta_\mu \Delta_\nu$ and $g_{\mu\nu} \Rightarrow A_1, A_2 \quad A^I(t, s) = 32\pi \sum_J (2J+1) P_J(\cos \theta) t_J^I(t)$

Unitarity relation for nucleon GFFs

● **Discontinuity:** $\text{Disc} \langle N(p') \bar{N}(p) | \hat{T}^{\mu\nu}(0) | 0 \rangle$

$$\propto \sum_n \langle N(p') \bar{N}(p) | n \rangle \langle n | \hat{T}^{\mu\nu}(0) | 0 \rangle^* \delta^4(p + p' - p_n)$$

● In the region $t \in (4M_\pi^2, 16M_\pi^2)$, only $\pi\pi$ intermediate state

$$\begin{aligned} & \text{Disc} \langle N(p') \bar{N}(p) | \hat{T}^{\mu\nu}(0) | 0 \rangle \\ &= \frac{1}{4m_N} \bar{u}(p') \left[\text{Disc} \hat{A}(t) \Delta^\mu \Delta^\nu + \text{Disc} \hat{J}(t) \left(i \Delta^{\{\mu} \sigma^{\nu\} \rho} P_\rho \right) + \text{Disc} \hat{D}(t) (P^\mu P^\nu - t g^{\mu\nu}) \right] v(p) \\ &= \frac{1}{2} \frac{i}{(4\pi)^2} \frac{p_\pi}{\sqrt{t}} \int d\Omega_l \langle N(p') \bar{N}(p) | \pi^a(l) \pi^b(P-l) \rangle \langle \pi^a(l) \pi^b(P-l) | \hat{T}^{\mu\nu}(0) | 0 \rangle^* \\ &= \frac{1}{2} \frac{i}{(4\pi)^2} \frac{p_\pi}{\sqrt{t}} \int d\Omega_l \bar{u}(p') \left[\underbrace{\delta^{ab} \mathbf{1} \left(A^+ + \frac{(\not{P} - 2\not{l})}{2} B^+ \right)}_{\text{Isospin-even}} + i \epsilon_{bac} \tau^c \underbrace{\left(A^- + \frac{(\not{P} - 2\not{l})}{2} B^- \right)}_{\text{Isospin-odd}} \right] v(p) \\ & \quad \times \frac{\delta^{ab}}{2} \left[(A^\pi(t))^* (2l - P)^\mu (2l - P)^\nu + (D^\pi(t))^* (P^\mu P^\nu - t g^{\mu\nu}) \right] \\ &= \frac{1}{2} \frac{i}{(4\pi)^2} \frac{p_\pi}{\sqrt{t}} \int d\Omega_l \bar{u}(p') \frac{3}{2} \left(A^+ + \frac{(\not{P} - 2\not{l})}{2} B^+ \right) v(p) \left[(A^\pi(t))^* (2l - P)^\mu (2l - P)^\nu + (D^\pi(t))^* (P^\mu P^\nu - t g^{\mu\nu}) \right] \end{aligned}$$

$A^\pm, B^\pm$: Lorentz invariant πN scattering amplitudes G. Höhler (1983)

$\pi N, \pi\pi/K\bar{K} \rightarrow N\bar{N}$ scattering

