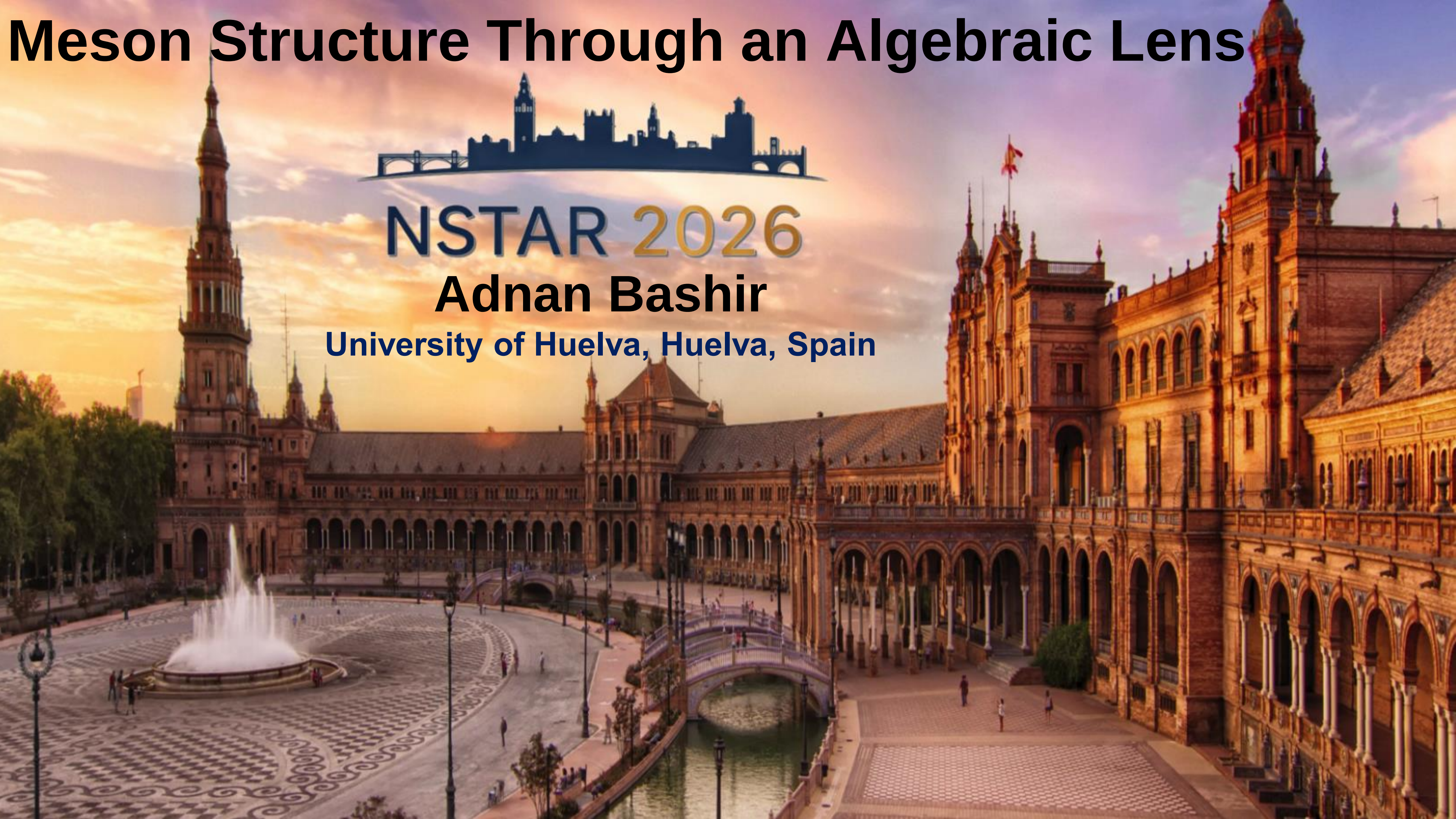
Meson Structure Through an Algebraic Lens

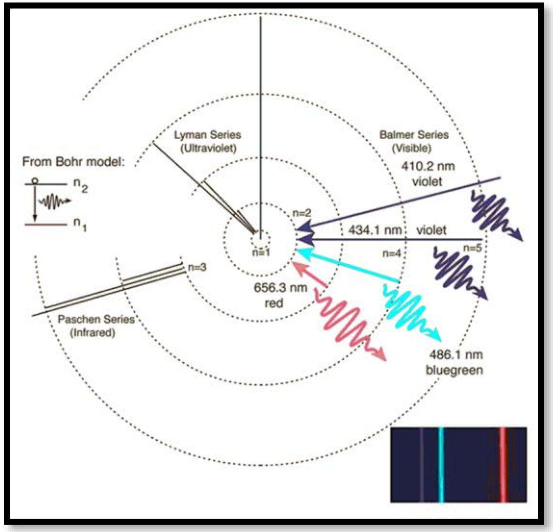


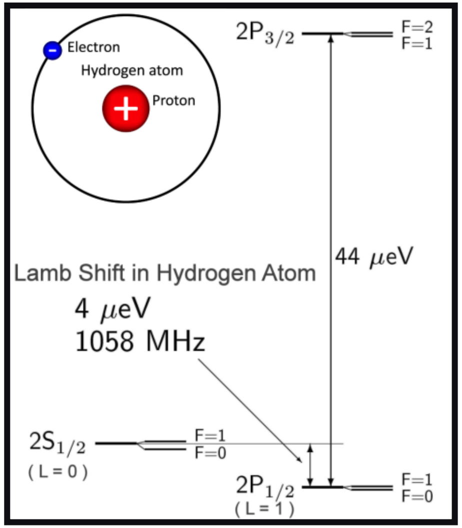
NSTAR 2026

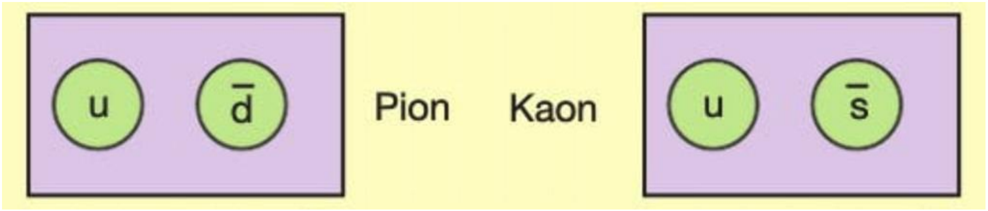
Adnan Bashir

University of Huelva, Huelva, Spain

Two particle bound states



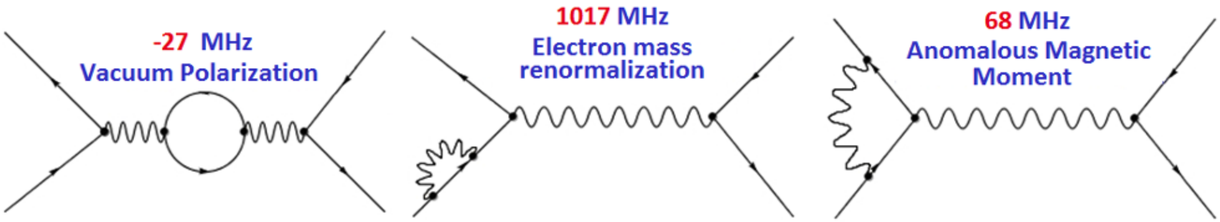




π and the **K** are the lightest two-body **bound states** in **QCD**. Unraveling their internal structure is a bigger challenge.

If you don't understand the **pion**, you don't understand anything (in **QCD**).

Dyson: If you don't understand the **Hydrogen atom** you don't understand anything (in **QED**) .

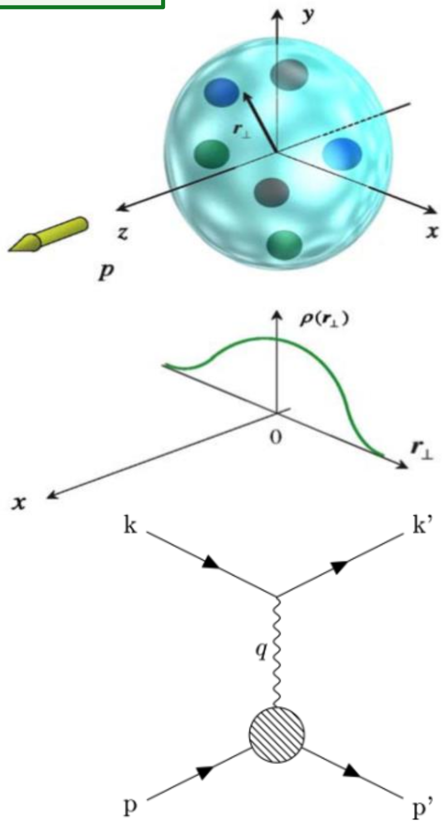


Renormalized QED

Pions are unique: Simultaneously, they are the lightest **bound states** of u and d quarks and also the Pseudo **Nambu-Goldstone-Salam-Weinberg bosons**, associated with **dynamical chiral symmetry breaking**.

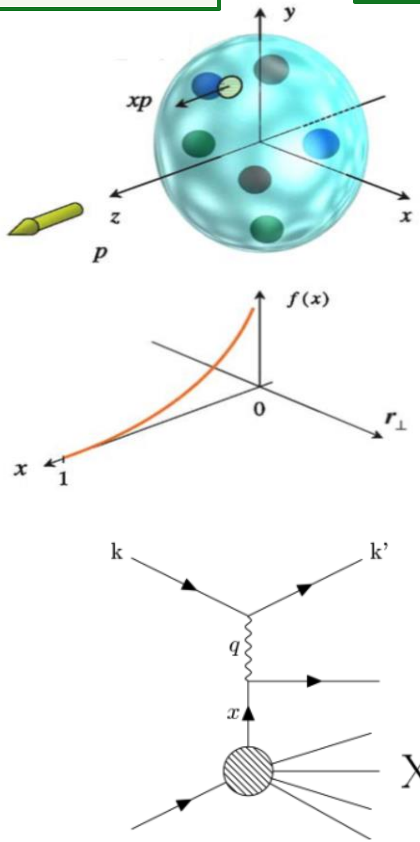
3D Structure of Mesons

Form Factors



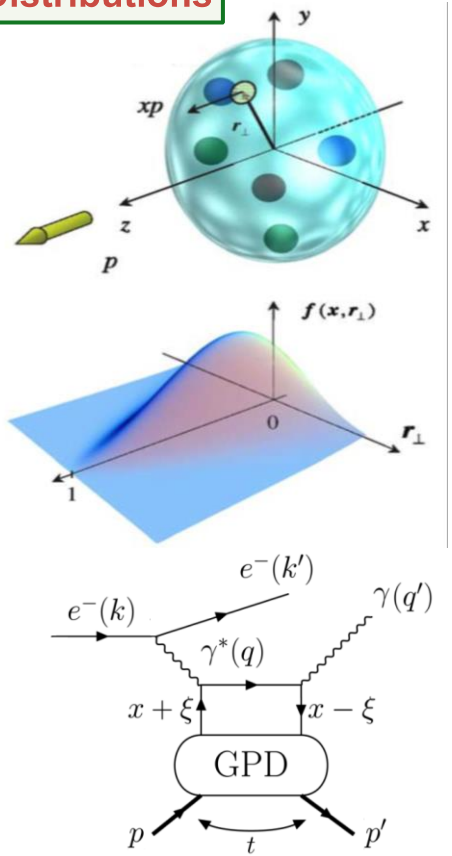
Hofstadter, et. al., Phys. Rev. 91, 422 (1953)
Hofstadter, Rev. Mod. Phys. 28, 214 (1956)

Parton Distributions



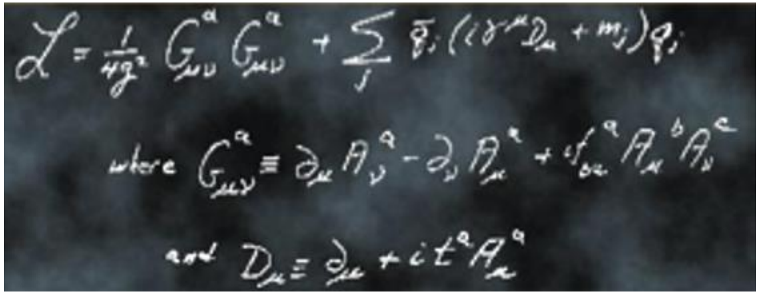
Feynman, Phys. Rev. Lett. 23, 1415 (1969)

Generalized Parton Distributions



Ji, Phys. Rev. D 55, 7114 (1997)
Radyushkin, Phys. Lett. B 380, 417 (1996)

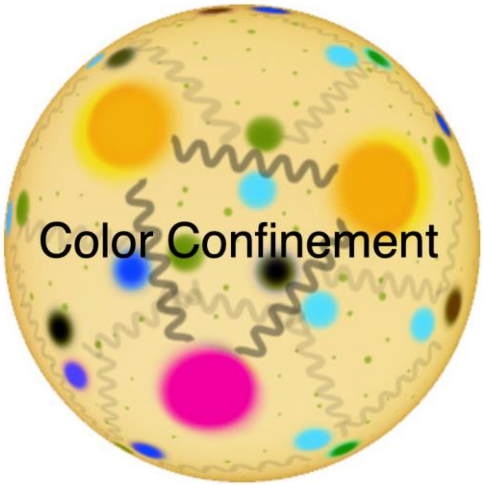
QCD: Emergent Phenomena



QCD is characterized by two **emergent** phenomena: **confinement** and **emergent hadron mass (EHM)**.

- Quarks and gluons do not reach detectors.
- Formation of color-singlet bound states: "**Hadrons**" mesons, baryons, tetraquarks, molecules

- Emergence of hadron masses **(EHM)** from **QCD dynamics**

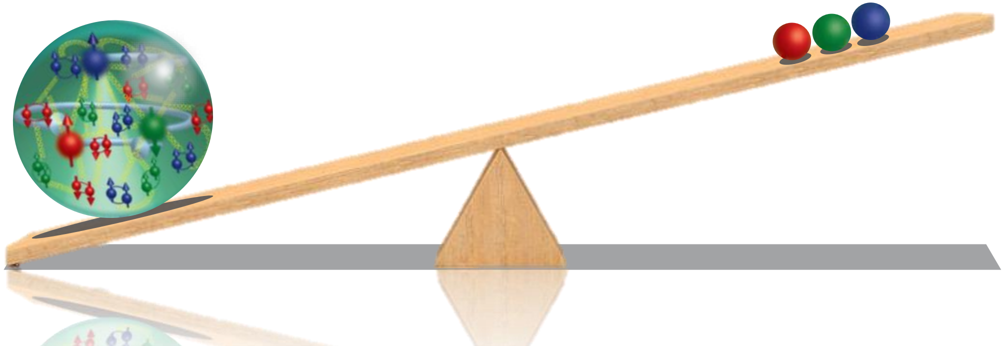


$\sim 2\%$ **of** M_p

$m_{u,d} \approx 5 - 10$ **MeV**

$\sim 98\%$ **of** M_p **strong QCD**

$M_{p,n} \approx 940$ **MeV**

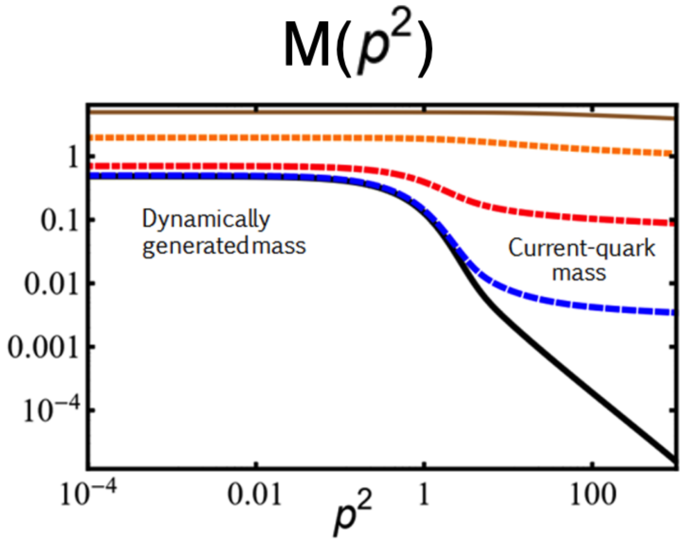


QCD: Emergent Phenomena

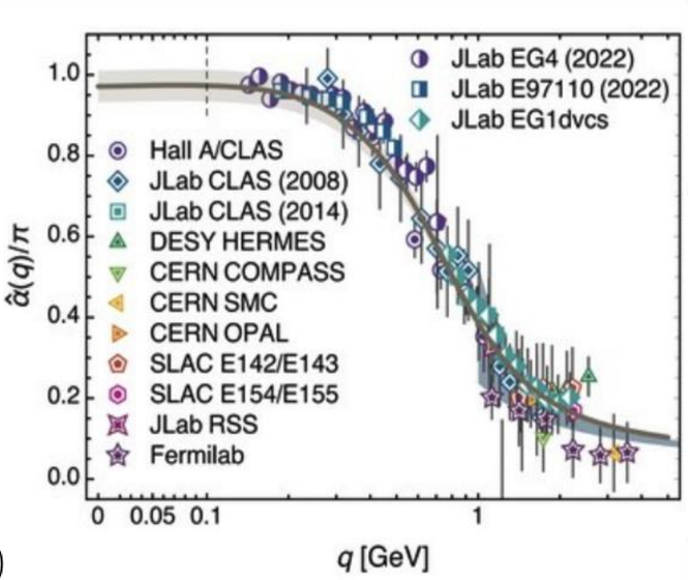
Origins of **confinement** and **dynamical mass generation** can perhaps be traced back to the Green functions of **quarks** and **gluons**.

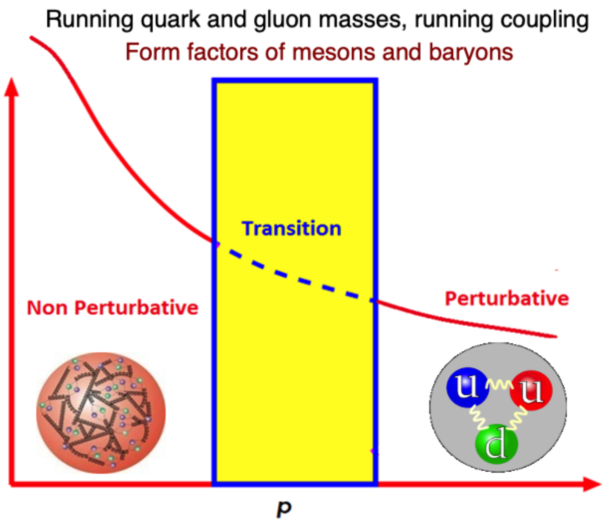
These emergent phenomena of **QCD**, non-existent in perturbation theory are naturally linked to the infrared enhancement of the **strong running coupling**.

The effects of of the pattern of **dynamical mass generation** are traceable in the **Q^2 evolution** of the π and **K form factors** explored and planned in the **JLab** and the **EIC**.



$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma \cdot p + \mathbf{M_f(p^2)})$$





Schwinger-Dyson Equations

Gauge Technique – Non Perturbative Solutios

- A. Salam, R. Delbourgo, Phys. Rev. 135 (1964) 6, B1398.

DCSB - Non-perturbative QED

- P.I. Fomin, V.A. Miransky, Phys. Lett. B64 (1976) 166.

DCSB – Non-abelian Gauge Theories

- V. Miransky, V. Gusynin, Y. Sitenko, Phys. Lett. B100 (1981) 157

DCSB – MT Model - Vector Mesons

- P. Maris, P. Tandy, Phys. Rev. C60 (1999)

PHYSICAL REVIEW VOLUME 135, NUMBER 6B 21 SEPTEMBER 1964

Renormalizable Electrodynamics of Scalar and Vector Mesons. II

ABDUS SALAM*
Imperial College, London, England

ROBERT DELBOURGO†
Imperial College, London, England

The "gauge technique" for solving field theories introduced in an earlier paper is applied to scalar and vector electrodynamics.

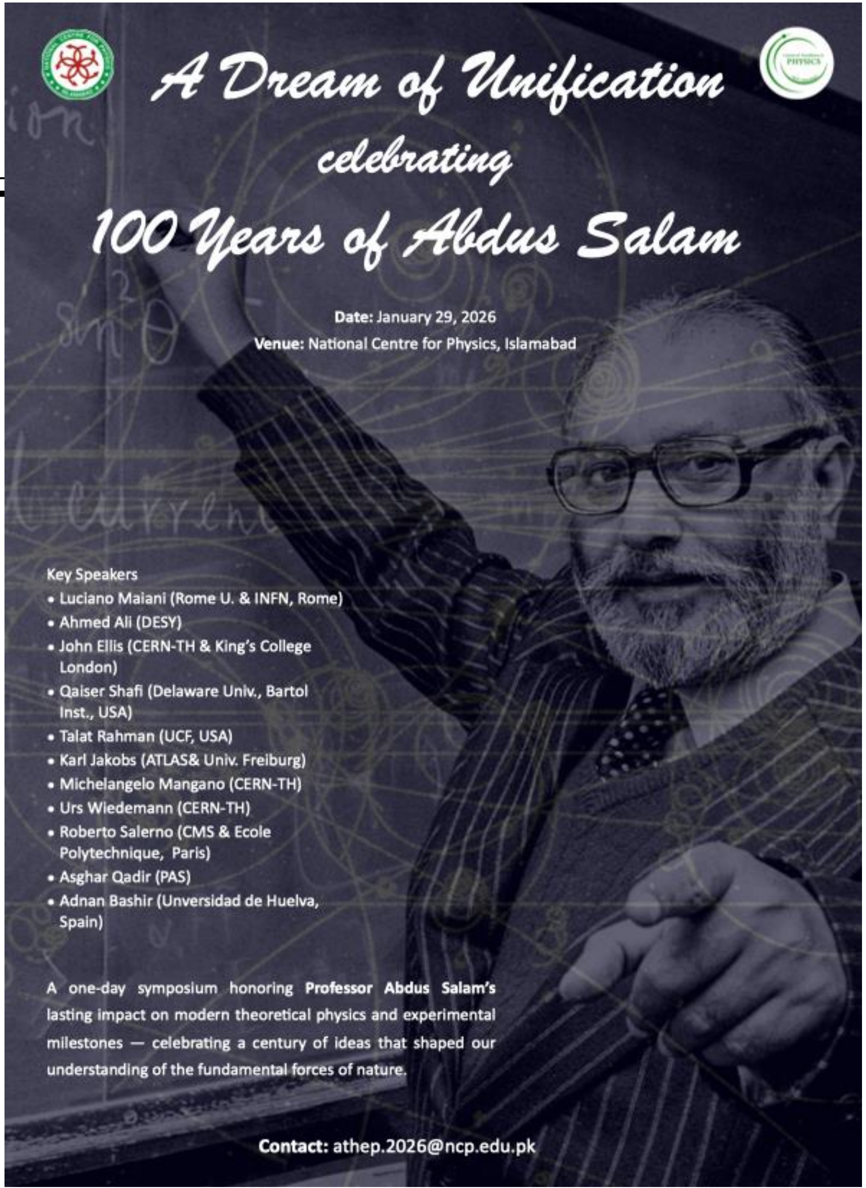
A. Dyson-Schwinger Set;

For a typical 3-field (e.g., electron-photon) interaction the well-known Dyson equations

$$S^{-1}=Z_2S_0^{-1}+Z_1e^2\int \Gamma S\Gamma_0 D \qquad \text{(I.1)}$$

SDE: electron propagator

$$D^{-1}=Z_3D_0^{-1}+Z_1e^2\int \Gamma S\Gamma_0 S \qquad \text{(I.2)}$$



Towards Algebraic Models

Sophisticated Truncations

Numerically Demanding
QCD-akin Predictions
Talk: Angel Miramontes

QCD Green Functions
Mesons – Static, FFs, PDAs, PDFs
Precision Tests of the Standard Model

Schwinger-Dyson Equations

Most Observables
Mesons, Baryons, Excited States

Contact Interaction

Simple Implementation
Insightful – Mindful
Talks: Axel Ahiezer, Luis Albino

LFWFs, GPDs, TMDs,
PDFs, FFs

Algebraic Models

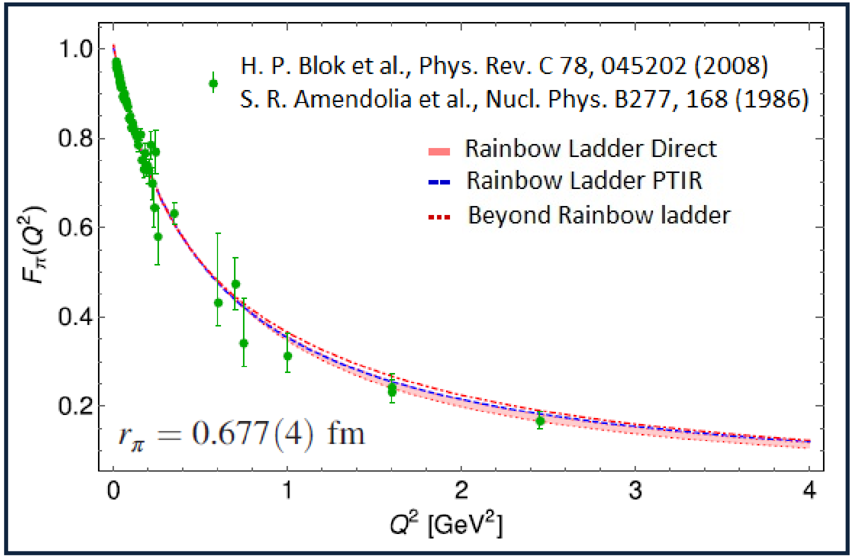
Moderate Simplicity
QCD-like Predictions
Luis Albino, Khépani Raya

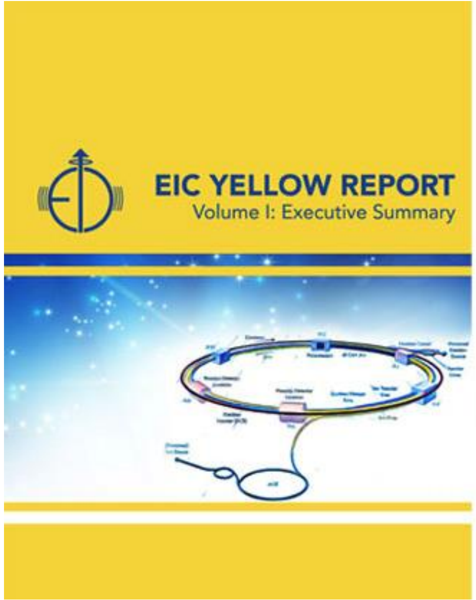
π Electromagnetic Form Factor

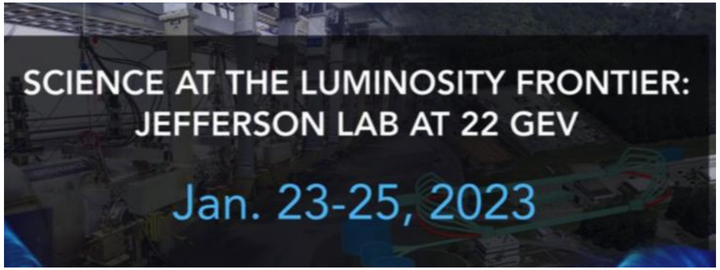
The **pion form factor** can be potentially be measured till **Q^2 ~ 6-8** in the **12 GeV** upgrade of the **Thomas Jefferson National Accelerator Facility**.

The **emergent pion mass** mechanism will be explored through the **pion form factor** measurements for **Q^2: 10-40 GeV^2** at the **Electron Ion Collider.**

A. Miramontes, AB, K. Raya, P. Roig,
Phys. Rev. D 105 (2022) 7, 074013







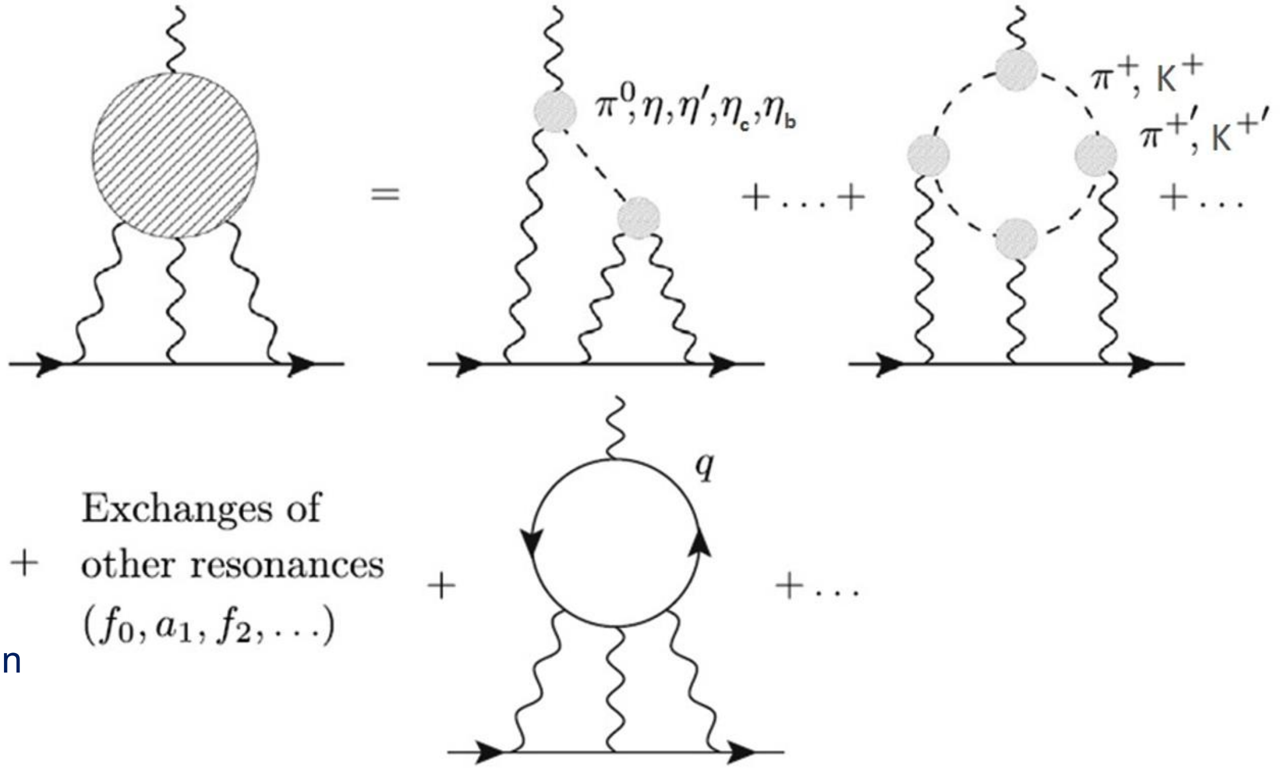
QCD - Hadronic Light by Light Scattering

"Contribution of neutral pseudoscalar mesons to a_μ^{HLbL} within a Schwinger-Dyson equations approach to QCD",
K. Raya, AB, P. Roig,
Phys. Rev. D101 (2020) 7, 074021.

"Pion and Kaon box contribution to a_μ^{HLbL}"
A. Miramontes, AB, K. Raya, P. Roig,
Phys. Rev. D105 (2022) 7, 074013.

"The anomalous magnetic moment of the muon in the Standard Model"
T. Aoyama et. al.,
Phys. Rept. 887 (2020) 1-166.

"The anomalous magnetic moment of the muon in the Standard Model: an update"
R. Aliberti et. al.,
Phys. Rept. 1143 (2025) 1-158.



From SDEs to the Precision Frontiers

Physics Reports

Volume 1143, 2 November 2025, Pages 1-158

Review article

The anomalous m… of the muon in th… an update

We dedi… …an.

…, A. Bashir [6,7], G. Benton [8,9], J. Bijnens [10], V. Biloshytskyi [1,2], … Bruno [14,15], E. Budassi [16,17], S. Burri [18], L. Cappiello [19], … M. Cè [14,15], V. Cirigliano [20,21], D.A. Clarke [22], G. Colangelo [18], L. …, M. Cottini [18]…A.S. Zhevlakov [28]

$$a_\mu^{\exp} - a_\mu^{\mathrm{SM}} = 38\,(63) \times 10^{-11}$$

which implies that there is no tension between the SM and experiment at the current level of precision.

Angel S. Miramontes, AB, Chrstian S. Fischer, Pablo Roig, e-Print: 2607.12918 [hep-ph]

The Algebraic Model (AM)

- It retains the **constant term** from original models, setting it to $\mathbf{M_q}$.

- There is a **term linear** in **w** with the coefficient $(\mathbf{M_h^2 - M_q^2})$. For same **flavored quarks**, it ceases to contribute by construction.

- There is a **quadratic term $\mathbf{w^2}$** with coefficient $\mathbf{m_M^2}$. The condition

$$|M_{\bar{h}} - M_q| \leq m_{\mathrm{M}} \leq M_{\bar{h}} + M_q$$

- It guarantees the **positivity** of

$$\Lambda^2(w)$$

The quark propagator:

$$S_{q(\bar{h})}(k) = [-i\gamma \cdot k + M_{q(\bar{h})}]\Delta(k^2, M^2_{q(\bar{h})})$$

$$\Delta(s,t) = (s+t)^{-1}$$

Bethe-Salpeter Amplitude:

$$n_{\mathrm{M}}\Gamma_{\mathrm{M}}(k,P) = i\gamma_5 \int_{-1}^{1} dw\, \rho_{\mathrm{M}}(w)[\hat{\Delta}(k_w^2, \Lambda_w^2)]^{\nu}$$

$$\hat{\Delta}(s,t) = t\Delta(s,t), \quad k_w = k + (w/2)P$$

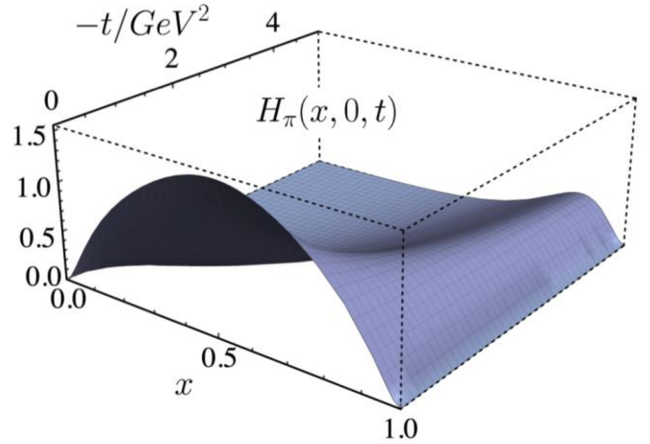
$M_{q(\bar{h})}$ is constituent quark mass for a given flavor

n_{M} is a normalization constant

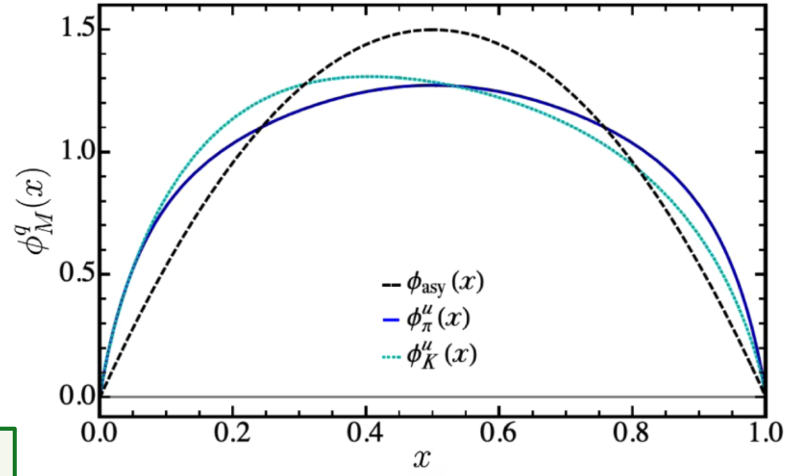
$\rho_{\mathrm{M}}(w)$ is a spectral density

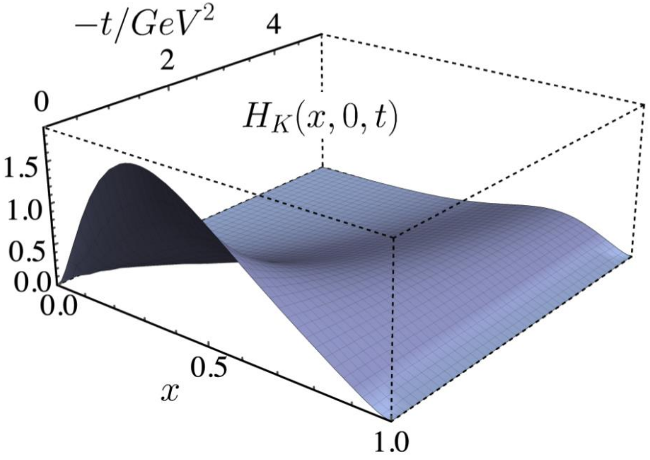
$$\Lambda^2(w) = M_q^2 - \frac{1}{4}(1-w^2)m_{\mathrm{M}}^2 + \frac{1}{2}(1-w)(M_{\bar{h}}^2 - M_q^2)$$

From the GPDs to the PDFs



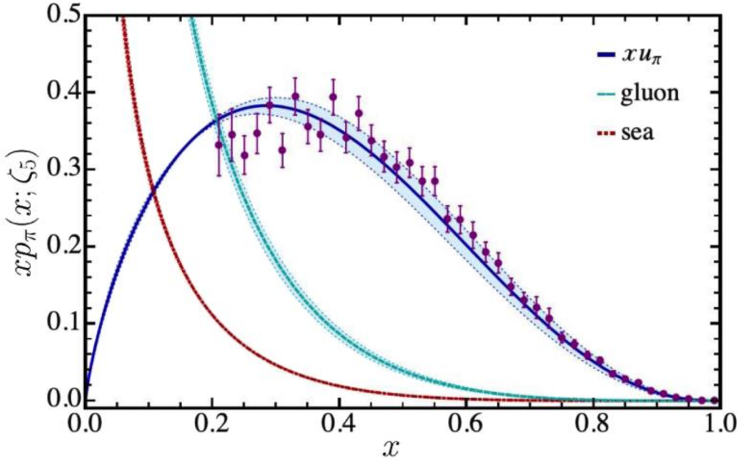
$$q_{\mathrm{M}}(x) \equiv H^q_{\mathrm{M}}(x,0,0)$$





DGLAP Evolution Equations

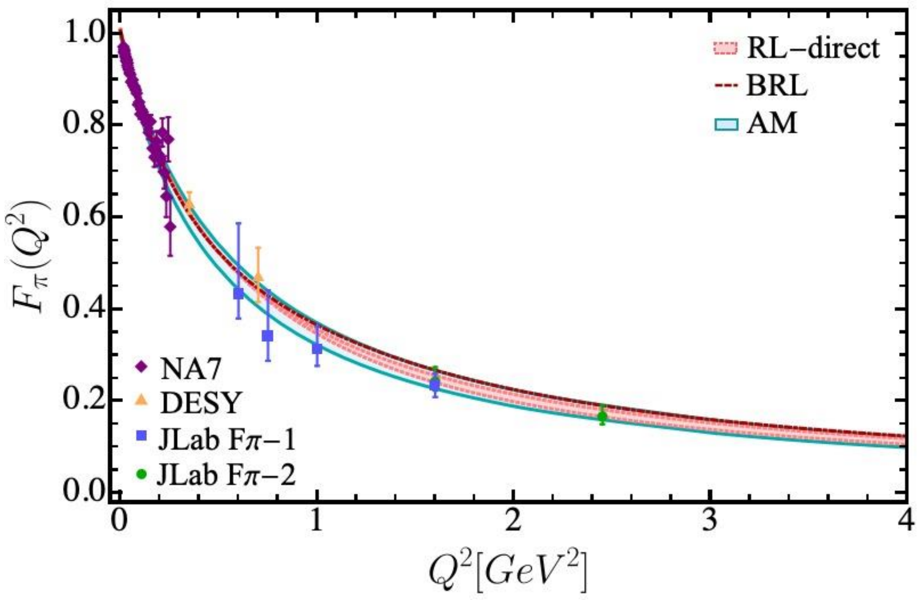
L. Albino, M. Higuera, K. Raya, AB
Phys. Rev. D 106 (2022) 3, 034003

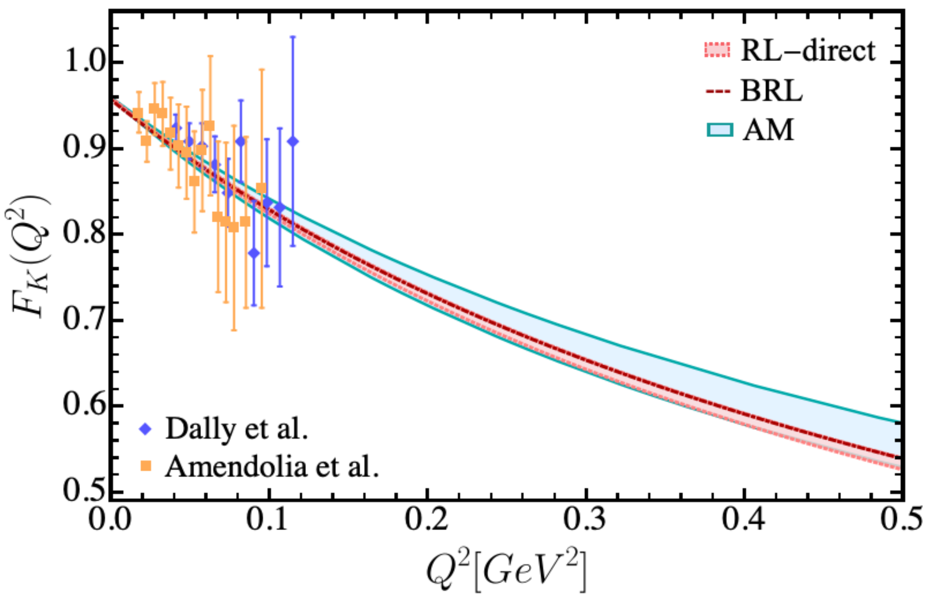


The Form Factors with Algebraic Models

The **electromagnetic form factors** using our **algebraic model** can be obtained either through the knowledge of the **GPDs** or the direct evaluation of the **triangle diagram**.

A. Miramontes, AB, K. Raya, P. Roig, Phys. Rev. D 105 (2022) 7, 074013
I.M. Higuera, R.J. Hernández, K.Raya, AB, Phys. Rev. D 110 (2024) 3, 034013



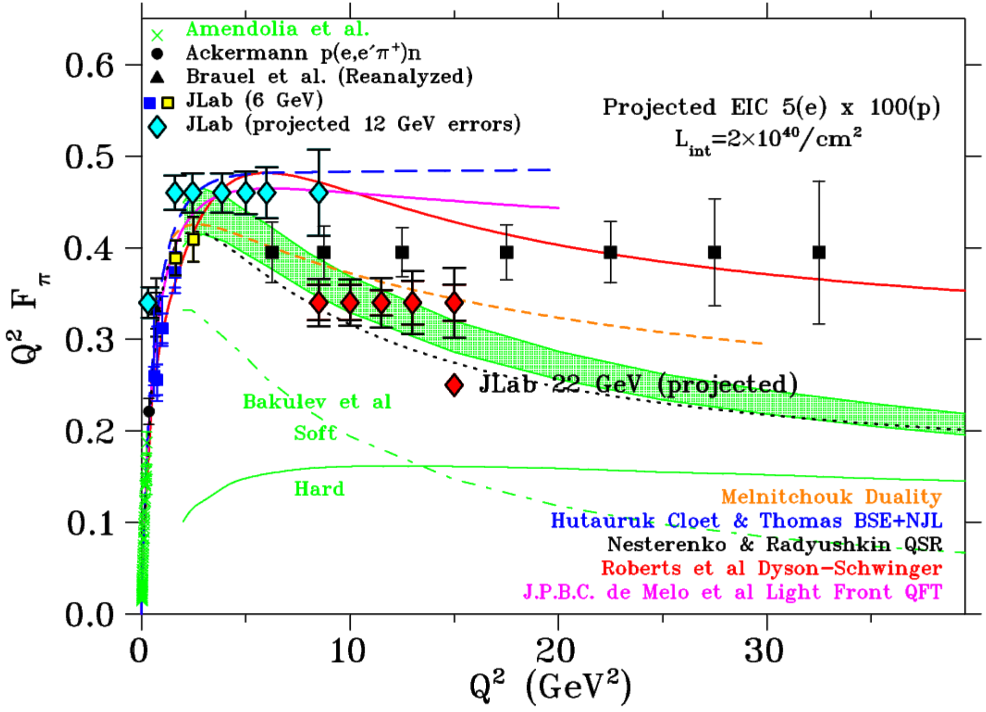


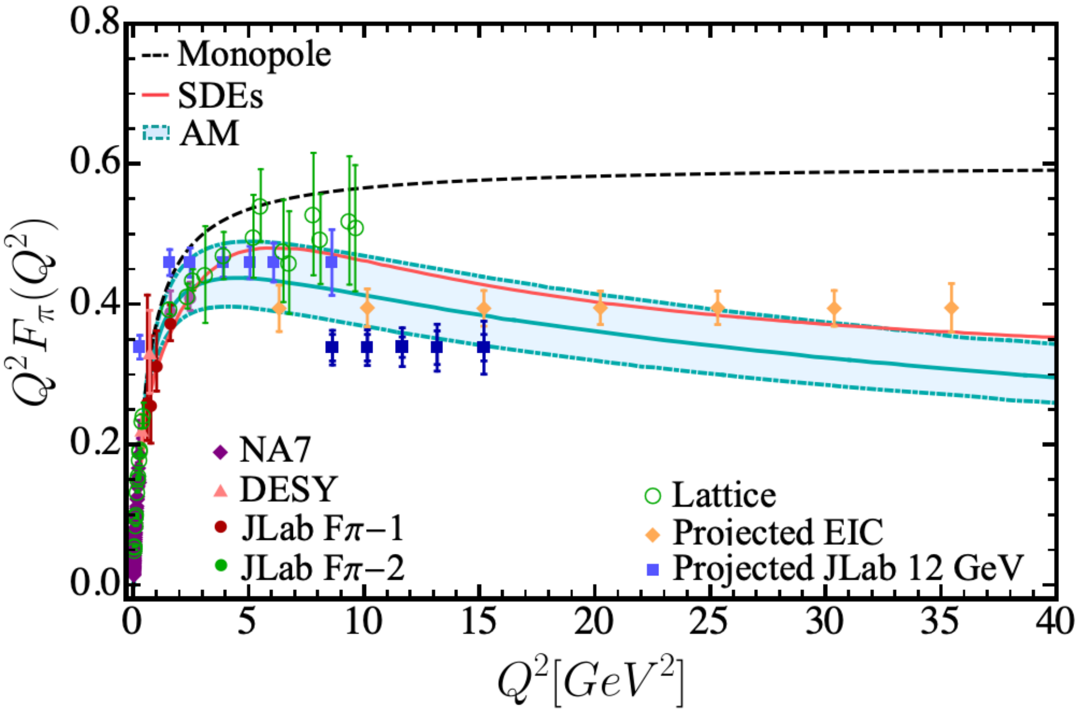
The Form Factors with Algebraic Models

We can extend the analysis of the **Algebraic Model** to larger **Q^2** range: **0-40 GeV^2.**

I.M. Higuera, R..J. Hernández, K.Raya, AB, Phys. Rev. D 110 (2024) 3, 034013

H-T Ding, X. Gao, A.D. Hanlon, S. Mukherjee, P. Petreczky, Q, Shi, et. al., Phys. Rev Lett. 133 (2024) 18, 181902



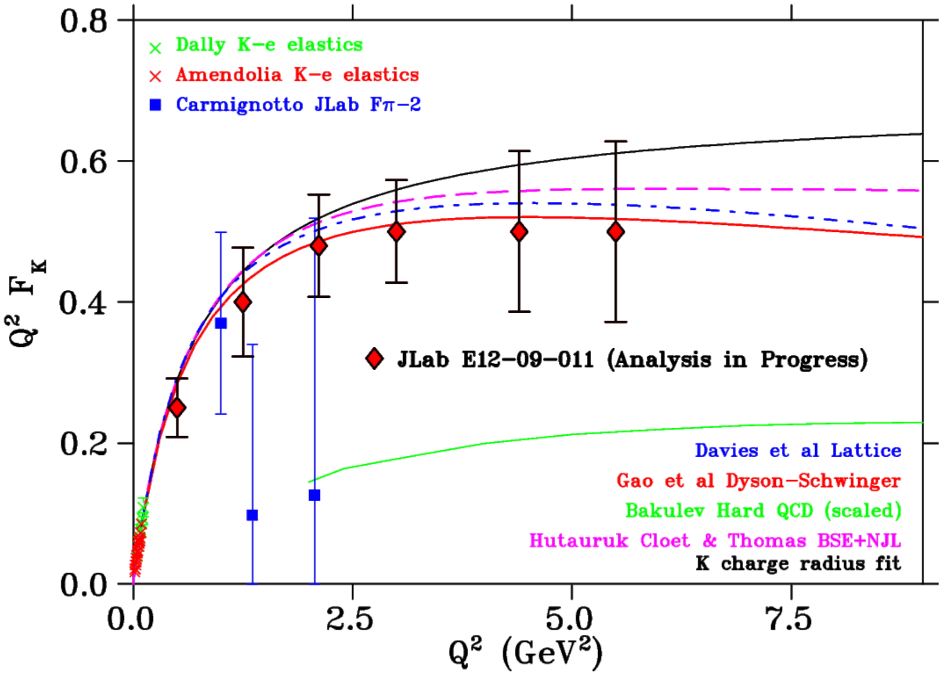


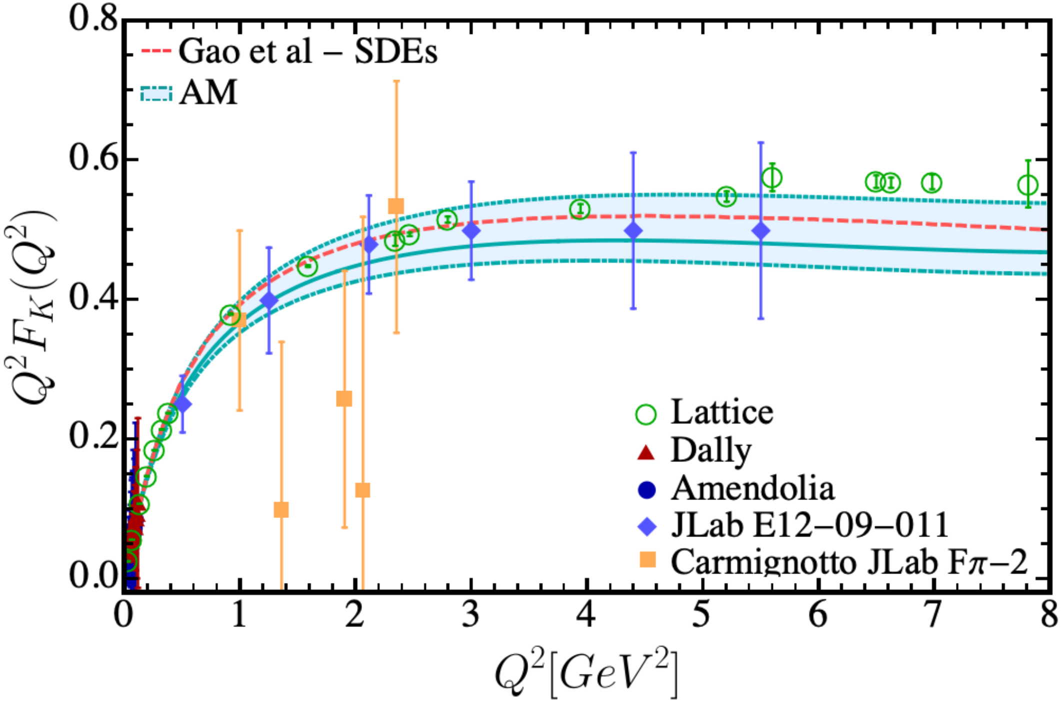
The Form Factors with Algebraic Models

There is an analysis underway of the **kaon electromagnetic form factor** till **5.5 GeV²** of the data obtained in **JLab E12-09-011** experiment.

I.M. Higuera, R..J. Hernández, K.Raya, AB, Phys. Rev. D 110 (2024) 3, 034013

H-T Ding, X. Gao, A.D. Hanlon, S. Mukherjee, P. Petreczky, Q, Shi, et. al., Phys. Rev Lett. 133 (2024) 18, 181902





Algebraic Models – Further Applications

- **Electromagnetic** and two-photon **transition form factors** of the **pseudoscalar mesons**: An **algebraic model** computation

  I.M. Higuera-Angulo, R.J. Hernández-Pinto, K. Raya, AB,
  Phys. Rev. D 110 (2024) 3, 034013 • e-Print: 2407.06461 [hep-ph]

- **Algebraic model** to study the internal structure of **pseudoscalar mesons** with **heavy-light** quark content

  B. Almeida-Zamora, J.J. Cobos-Martínez, AB, K. Raya, J. Rodríguez-Quintero, J. Segovia,
  Phys. Rev. D 109 (2024) 1, 014016 • e-Print: 2309.17282 [hep-ph]

- Light-front wave functions of **vector mesons** in an **algebraic model**

  B. Almeida-Zamora, J.J. Cobos-Martínez, AB, K. Raya, J. Rodríguez-Quintero, J. Segovia,
  Phys. Rev. D 107 (2023) 7, 074037 • e-Print: 2303.09581 [hep-ph]

- Unified description of **pseudoscalar meson structure** from light to **heavy quarks**

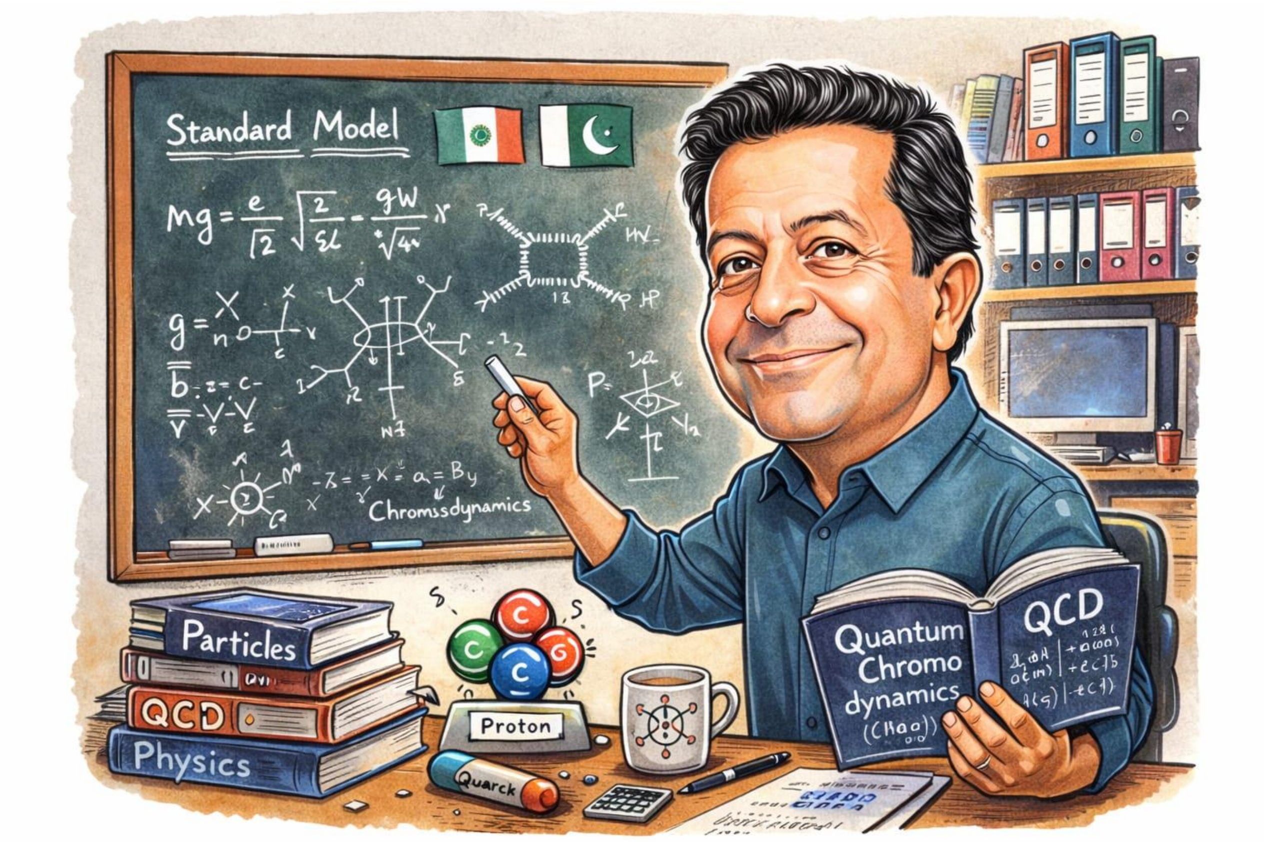
  Bilgai Almeida-Zamora, Luis Albino, AB, Jesús Javier Cobos-Martínez, Jorge Segovia,
  Symmetry 18 (2026) 6, 1017 • e-Print: 2602.10775 [hep-ph]

- **Light-Front wave functions** of Open-Flavor **vector mesons** in an **algebraic model**

  B. Almeida-Zamora, Luis Albino, et. al. in progress

Summary and Outlook

- We have shown how **algebraic models** inspired by **Schwinger–Dyson** and **Bethe–Salpeter equations** provide a simple yet powerful framework for describing **meson structure**, from **LFWFs** and **GPDs** to **PDFs** and **electromagnetic form factors**.
- The same interaction that produces the **pion $q\bar{q}$** channel generates strong **scalar 0+ diquark** correlations.
- Correspondingly, the **vector-meson 1^-** channel is related through the **Bethe–Salpeter kernel** to the **axial-vector 1^+ diquark** channel.
- These diquark correlations are the essential building blocks of the **quark–diquark Faddeev** description of the **nucleon** and its **excitations**.
- Therefore, if the algebraic framework can be extended to realistic scalar and axial-vector **diquark amplitudes** and their **internal structure**, the machinery we have developed for mesons—**LFWFs, form factors, PDFs** and **GPDs** — provides a natural route toward **N→N* transition form factors**. [David Rodríguez]



Thank you for your attention